Origin of Mass and Matter in the Universe

APCTP International Workshop

July 21 - July 24, 2026

In-medium mass shifts and nucleus bond states of $B_c^{(*)}$, $B_s^{(*)}$ and $D_s^{(*)}$ mesons ($B_c^{\pm}$-^{12}C, -^{208}Pb)

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Guilherme N. Zeminiani, Samuel L. P. G. Beres

- G. N. Zeminiani, S. L. P. G. Beres and K. Tsushima, Phys. Rev. D 110, 094045 (2024); GZ, JJCM, KT, Phys. Rev. C 111, 055202 (2025) [B_c-A]
GNZ and KT, PoS HADRON2025, 119 (2026) [arXiv:2508.18261 [nucl-th]]
- S. L. P. G. Beres, [arXiv:2407.03377 [hep-ph]] (Master Thesis)
- K. T, PTEP 2022 043D02 (effective hadron mass parametrization)
- K. Saito, K. T, and A. W. Thomas, Prog. Part. Nucl. Phys. 58, 1 (2007) (QMC)

1. Motivation, Focus
2. Introduction: QMC Model
3. Effective Hadron Masses and Potentials (Parametrization)
4. Two-flavored heavy meson self-energies, mass shift
5. Summary and comparison of mass shifts
6. Focus: $B_c^{\pm}$-nucleus bound states
7. Summary of $B_c^{\pm}$-^{12}C bound states and prospect

Motivation, Focus

- In-medium mass shifts of two non-light quark flavor mesons (B_c, B_c^*, B_s, B_s^*, D_s and D_s^*) ! m_b, m_c
$\leftrightarrow$ Higgs mechanism, not directly with chiral symmetry

- Effects of chiral symm. (partial) restoration via self-energy loops ?! on two-non-light quark flavor mesons (B_c, B_c^*, B_s, B_s^*, D_s and D_s^*)

- $m_{B_c^*} \simeq$6339.0 (Expt. ATLAS: arXiv.2605.16228 (2026))

- Focus! $B_c^{\pm}$-nucleus bound states

- Propaganda!
Density dependent parametrization: (for easy usage)
Baryon and Meson $m^*_{B,M}$ and vector $V^{q,B,M}_{\omega,\rho}$ potentials

Quark-meson coupling (QMC) model

[ρ-field is suppressed (symmetric nuclear matter $\rightarrow$).]

- The relativistically moving confined **light quarks (*u*, *d*)** in the nucleon bag interact self-consistently with the **scalar-isoscalar σ**, **vector-isoscalar ω** and **vector-isovector ρ** mean fields generated by the light quarks in the other nucleons and itself.

The QMC model P. Guichon, PLB 200, 235 (1988)

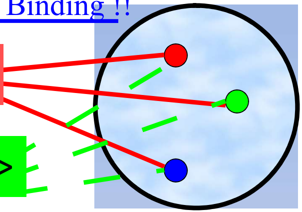
(For a review, PPNP 58, 1 (2007))

Light (u,d) quarks interact **self-consistently** with mean **σ** and **ω** fields

Nuclear Binding !!

< σ >

< ω >



$m^*_q = m_q - g_\sigma^q \sigma = m_q - V_\sigma^q$

↓ nonlinear in **σ**

$M^*_N \approx M_N - g_\sigma^N \sigma + \underline{(d/2)\,(g_\sigma^N \sigma)^2}$

$M^*_N = M_N - V_\sigma^N$

$[\, i\gamma\cdot\partial - (m_q - V_\sigma^q) - \gamma_0 V_\omega^q \,]\; q = 0$

1. Start

$[i\gamma\cdot\partial - M^*_N - \gamma_0 V_\omega^N]\; N = 0$

$V_\omega^N = 3V_\omega^q$

Self-consistent !

(Applied quark model !)

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Mesons in nuclear medium in **QMC**

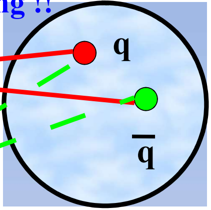
(For a review, PPNP 58, 1 (2007))

Light (u,d) quarks interact self-consistently with mean **σ** and **ω** fields

Nuclear Binding !!

< σ >

< ω >



$m^*_q = m_q - g^q_\sigma \sigma = m_q - V^q_\sigma$

↓ **nonlinear** in **σ**

$M^*_M \approx M_M - g^M_\sigma \sigma + (d^M/2)\ (g^M_\sigma \sigma)^2$

$$[\ i\gamma\cdot\partial - (m_q - V^q_\sigma) - \gamma_0 V^q_\omega\ ]\ q = 0$$

σ,ω fields: **no couplings** with **s,c,b** quarks!!

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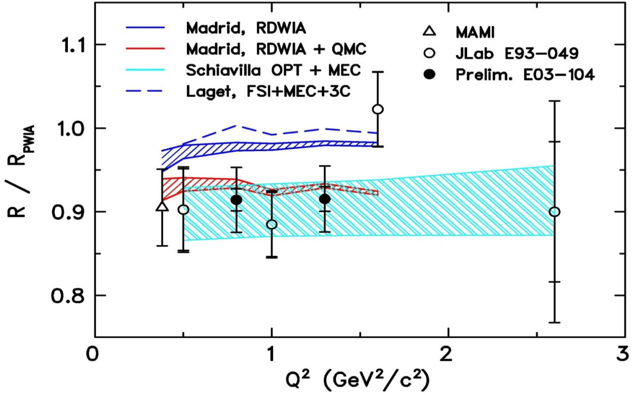
•**Experimentally extracted η'-^{12}C potential of $\simeq$ -61 MeV [arXiv:2509.07824v1 [nucl-ex]] $\simeq$ QMC prediction**

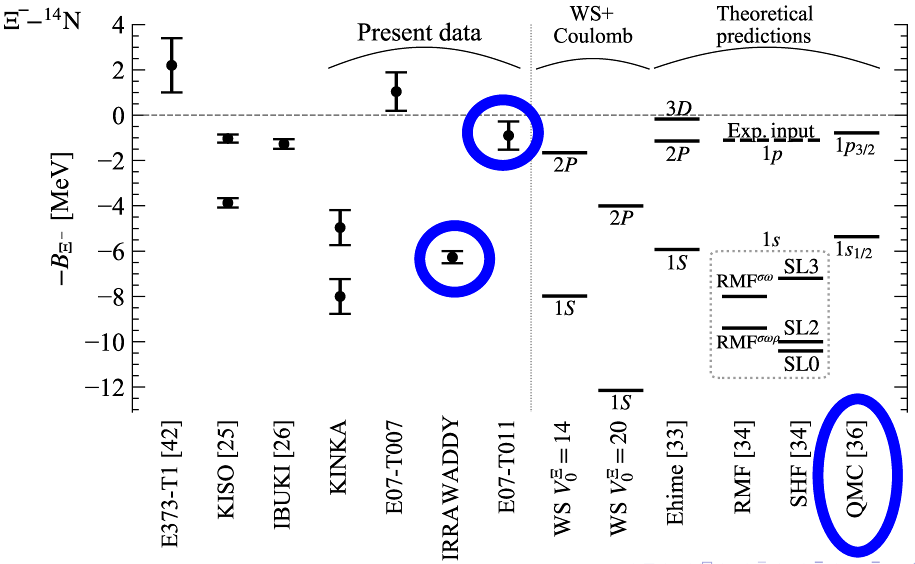
$R=(p'_x/p'_z)=(G_E^p/G_M^p)$: $^4He/^1H$

S. Malace, M. Paolone and S. Strauch, arXiv:0807.2251 [nucl-ex]

S. Strauch *et al., Phys. Rev. Lett.* ***91, 052301 (2003)***



Ξ^--^{14}N energy levels (Prog. Theor. Exp. Phys. 2021, 073D02)



QMC model 1: Hadron level

$$\mathcal{L} = \bar{\psi}[i\gamma\cdot\partial - m_N^*(\sigma) - g_\omega\omega^\mu\gamma_\mu]\psi + \mathcal{L}_{\rm meson},$$

$$m_N^*(\sigma) \equiv m_N - g_\sigma\underline{(\sigma)}\sigma \simeq m_N - g_\sigma\underline{[1-(a_N/2)(g_\sigma\sigma)]}\sigma$$

$$g_\sigma \equiv g_\sigma(\sigma=0) \qquad \text{(can well be parametrized)}$$

$$\mathcal{L}_{\rm meson} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2 - \frac{1}{2}\partial_\mu\omega_\nu(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu \quad \text{(no higher order meson self-interactions)}$$

$$\rho_B = \frac{4}{(2\pi)^3}\int d^3k\,\theta(k_F - |\vec{k}|) = \frac{2k_F^3}{3\pi^2},$$

$$\rho_s = \frac{4}{(2\pi)^3}\int d^3k\,\theta(k_F - |\vec{k}|)\frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma)+\vec{k}^2}},$$

QMC model 2: Quark level

$x = (t, \vec{r})$ ($|\vec{r}| \leq$ bag radius), $V^q_\sigma = g^q_\sigma \sigma$

$$\left[i\gamma \cdot \partial_x - (m_q - V^q_\sigma) \mp \gamma^0 \left(V^q_\omega + \frac{1}{2} V^q_\rho \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} = 0$$

$$\left[i\gamma \cdot \partial_x - (m_q - V^q_\sigma) \mp \gamma^0 \left(V^q_\omega - \frac{1}{2} V^q_\rho \right) \right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} = 0$$

$$[i\gamma \cdot \partial_x - m_Q] \psi_Q(x) \ (\text{or } \psi_{\overline{Q}}(x)) = 0$$

$$m^*_h = \sum_{j=q,\bar{q},Q\overline{Q}} \frac{n_j \Omega^*_j - z_h}{R^*_h} + \frac{4}{3}\pi R^{*3}_h B, \quad \left. \frac{dm^*_h}{dR_h} \right|_{R_h = R^*_h} = 0$$

$$j_0(x_{q,Q}) = \beta^*_{q,Q}\, j_1(x_{q,Q}), \ \beta^*_q = \sqrt{(\Omega^*_q - R^*_h m^*_q)/(\Omega^*_q + R^*_h m^*_q)}$$

$$\Omega^*_q = \Omega^*_{\bar{q}} = [x_q^2 + (R^*_h m^*_q)^2]^{1/2}, \text{with } m^*_q = m_q - g^q_\sigma \sigma$$

$$\Omega^*_Q = \Omega^*_{\overline{Q}} = [x_Q^2 + (R^*_h m_Q)^2]^{1/2} \quad (Q = s, c, b)$$

QMC model 3: From quarks

$$\omega = \frac{g_\omega \rho_B}{m_\omega^2},$$

$$\sigma = \frac{g_\sigma}{m_\sigma^2} C_N(\sigma) \frac{4}{(2\pi)^3} \int d^3k\, \theta(k_F - |\vec{k}|) \frac{m_N^*(\sigma)}{\sqrt{m_N^{*2}(\sigma) + \vec{k}^2}}$$

$$= \frac{g_\sigma}{m_\sigma^2} C_N(\sigma) \rho_s \quad (g_\sigma \equiv g_\sigma(\sigma = 0)),$$

$$C_N(\sigma) = \frac{-1}{g_\sigma(\sigma = 0)} \left[\frac{dm_N^*(\sigma)}{d\sigma} \right], \leftrightarrow [j_0(x_q) = \beta_q^*(x_q, \sigma) j_1(x_q)]$$

$$E^{\mathrm{tot}}/A - m_N = \frac{4}{(2\pi)^3 \rho_B} \int d^3k\, \theta(k_F - |\vec{k}|) \sqrt{m_N^{*2}(\sigma) + \vec{k}^2}$$

$$+ \frac{m_\sigma^2 \sigma^2}{2\rho_B} + \frac{g_\omega^2 \rho_B}{2 m_\omega^2} - m_N.$$

QMC model 4: Couplings etc.

m_q(MeV)	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	m_N^*	K	Z_N	$B^{1/4}$(MeV)
5	5.39	5.30	754.6	279.3	3.295	170
220	6.40	7.57	698.6	320.9	4.327	148

$$\frac{d\mathrm{m}_\mathrm{N}^*(\sigma)}{d\sigma} = -3g_\sigma^q \int_{bag} d^3r\, \overline{\psi}_q(\vec{r})\psi_q(\vec{r}) \quad \text{(the lowest bag w.f.)}$$

$$\equiv -\underline{3g_\sigma^q S_N(\sigma)} = -\frac{\partial}{\partial\sigma}\left[g_\sigma(\sigma)\sigma\right],$$

$$C_N(\sigma) = \frac{-1}{g_\sigma(\sigma=0)}\left[\frac{d\mathrm{m}_\mathrm{N}^*(\sigma)}{d\sigma}\right],$$

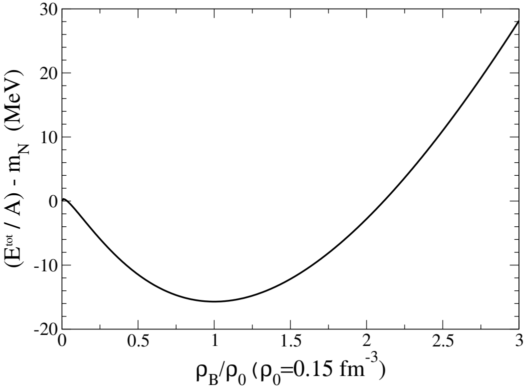
$$g_\sigma \equiv g_\sigma^N \equiv 3g_\sigma^q S_N(\sigma=0).$$

Parameters

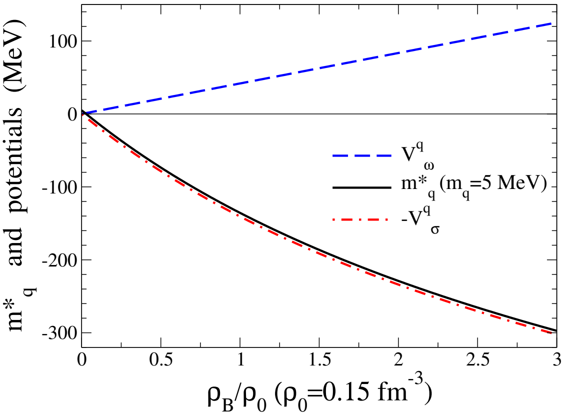
Table: Current quark mass values (inputs), quark-meson coupling constants and the bag constant B_p, obtained with the inputs: free nucleon bag radius $R_N = 0.8$ fm, empirical values $E^{\rm tot}/A - m_N = -15.7$ MeV ($m_N = 939$ MeV) at the saturation density $\rho_0 = 0.15$ fm^{-3}, and the symmetry energy, 35 MeV.

$m_{u,d}$	5 MeV	g_σ^q	5.69
m_s	250 MeV	g_ω^q	2.72
m_c	1270 MeV	g_ρ^q	9.33
m_b	4200 MeV	$B_p^{1/4}$	170 MeV

Results: Quark Meson Coupling Model (Standard)



- **Symmetric Nuclear Matter - Binding Energy per Nucleon**
- $m_q = 5$ **MeV**, $K = 279.3$ **MeV**



- **Effective mass of constituent quarks: $m_q = 5$ MeV**
- **All the light-quarks in any hadrons feel the same potentials !!**

QMC: In-medium hadron masses (remind again!)

$x = (t, \vec{r})$ ($|\vec{r}| \leq$ bag radius), $V^q_\sigma = g^q_\sigma \sigma$

$$\left[i\gamma \cdot \partial_x - (m_q - V^q_\sigma) \mp \gamma^0 \left(V^q_\omega + \frac{1}{2} V^q_\rho \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} = 0$$

$$\left[i\gamma \cdot \partial_x - (m_q - V^q_\sigma) \mp \gamma^0 \left(V^q_\omega - \frac{1}{2} V^q_\rho \right) \right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} = 0$$

$$\left[i\gamma \cdot \partial_x - m_Q \right] \psi_Q(x) \ (\text{or } \psi_{\overline{Q}}(x)) = 0$$

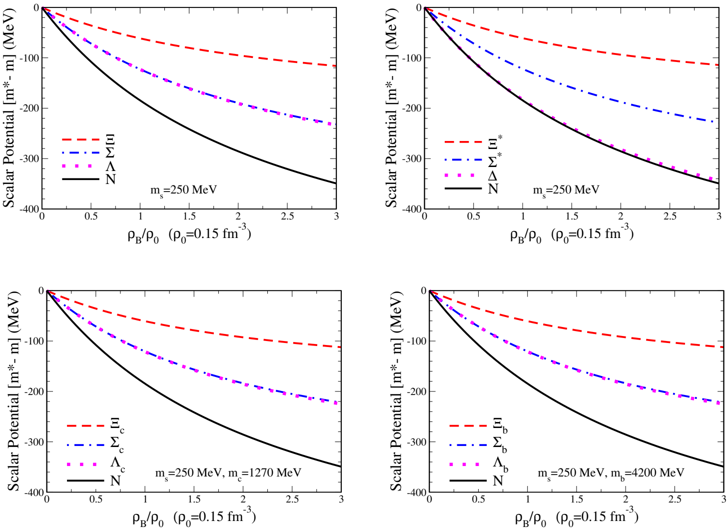
$$m^*_h = \sum_{j=q,\bar{q},Q\overline{Q}} \frac{n_j \Omega^*_j - z_h}{R^*_h} + \frac{4}{3}\pi \mathrm{R}^{*3}_\mathrm{h} B, \quad \left. \frac{d\mathrm{m}^*_\mathrm{h}}{d\mathrm{R}_\mathrm{h}} \right|_{R_h = R^*_h} = 0$$

$$j_0(x_{q,Q}) = \beta^*_{q,Q}\, j_1(x_{q,Q}), \ \ \beta^*_q = \sqrt{(\Omega^*_q - R^*_h m^*_q)/(\Omega^*_q + R^*_h m^*_q)}$$

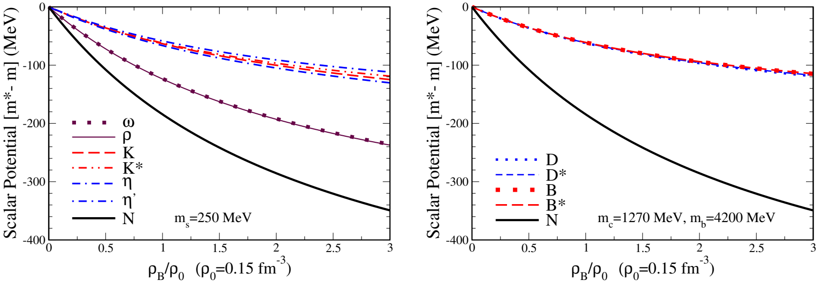
$$\Omega^*_q = \Omega^*_{\bar{q}} = [x^2_q + (R^*_h m^*_q)^2]^{1/2}, \text{with } m^*_q = m_q - g^q_\sigma \sigma$$

$$\Omega^*_Q = \Omega^*_{\overline{Q}} = [x^2_Q + (R^*_h m_Q)^2]^{1/2} \quad (Q = s, c, b)$$

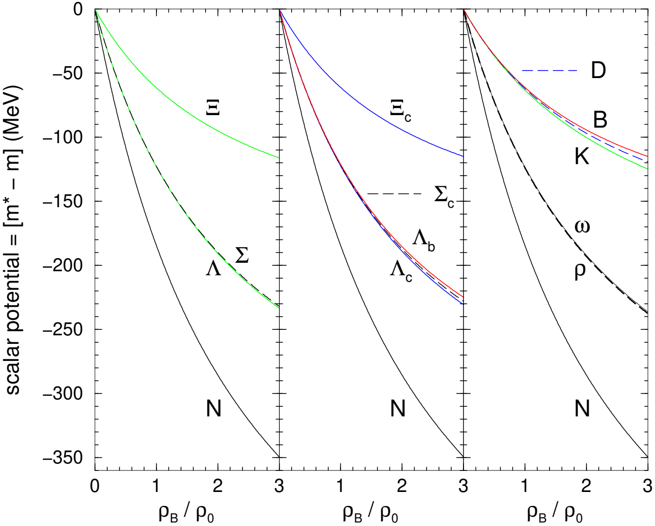
Baryon (with light quark) Scalar potential $[m_B^* - m_B]$



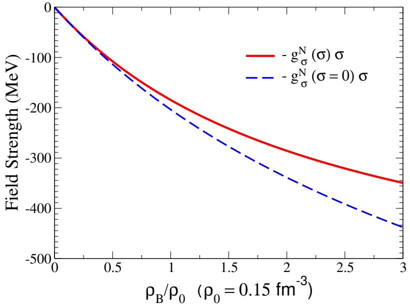
Meson (with light quark) Scalar potential $[m^*_M - m_M]$

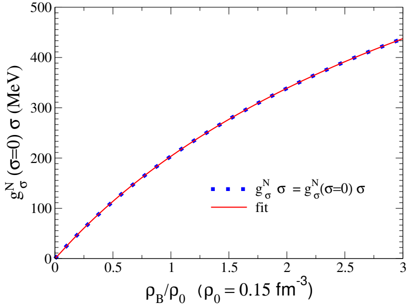


Scalar potential (with light quark): $m^*_{B,M} - m_{B,M}$



Scalar field σ and fit





$$x = (\rho_B/\rho_0), \quad \rho_0 = 0.15\mathrm{fm}^{-3}$$

$$(g_\sigma^N\sigma)(x) = 1.60828 - 23.9107\sqrt{x} + 350.631\,x - 144.309\,x\sqrt{x}$$
$$+19.4750\,x^2 \;\; (x > 0, [x > 0.001])$$
$$(g_\sigma^N\sigma)(x) = 0 \;\; (x = 0)$$

$M^*_{B,M}$: Parametrization

$$
\begin{aligned}
m_B^* &\simeq m_B - \frac{n_q}{3} g_\sigma^N \left[1 - \frac{a_B}{2} (g_\sigma^N \sigma) \right] \sigma, \\
&= m_B - \frac{n_q}{3} \left[(g_\sigma^N \sigma) - \frac{a_B}{2} (g_\sigma^N \sigma)^2 \right], \\
&\quad (B = N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Lambda_c, \Sigma_c, \Xi_c, \Lambda_b, \Sigma_b, \Xi_b).
\end{aligned}
$$

$$
\begin{aligned}
m_M^* &\simeq m_M - \frac{n_q^M}{3} g_\sigma^N \left[1 - \frac{a_M}{2} (g_\sigma^N \sigma) \right] \sigma, \\
&= m_M - \frac{n_q^M}{3} \left[(g_\sigma^N \sigma) - \frac{a_M}{2} (g_\sigma^N \sigma)^2 \right],
\end{aligned}
$$

$$
(M = \omega, \rho, K, K^*, \eta, \eta', D, D^*, B, B^*, \ \text{with } n_q^M \to 1 \text{ for } \eta \text{ and } \eta').
$$

Vector Potentials: Parametrization

$x = \rho_B/\rho_0$ ($\rho_0 = 0.15$ fm^{-3})

$$
\begin{aligned}
V^B_\omega(x) &= b_B\, x, \\
V^q_\omega(x) &= 41.77\, x \\
V^h_\omega(x) &= V^h_\omega = (n_q - n_{\bar{q}})\, V^q_\omega = (n_q - n_{\bar{q}}) \times 41.77\, x \\
V^K_\omega(x) &\simeq 1.96 \times 41.77\, x
\end{aligned}
$$

(baryon octet $\rightarrow$ Pauli potentials)

$y \equiv \rho_3/\rho_0 = (\rho_p - \rho_n)/\rho_0$, isospin-third component of h, I^h_3,

$$I^h_3 V^h_\rho(y) = I^h_3 \times 84.61\, y$$

Parameters

Table: **Effective mass slope parameter $a_{B,M}$, and b_B.**
($[m^*_\Sigma - m_\Sigma] + V^\Sigma_v \simeq +30$ MeV at ρ_0.)

a_B	$\times 10^{-4}$ MeV^{-1}	a_B	$\times 10^{-4}$ MeV^{-1}	a_B	$\times 10^{-4}$ MeV^{-1}	a_B	$\times 10^{-4}$ MeV^{-1}
a_N	9.15	a_Δ	10.08	—	—	—	—
a_Λ	9.35	—	—	a_{Λ_c}	9.90	a_{Λ_b}	10.78
a_Σ	9.59	a_{Σ^*}	10.15	a_{Σ_c}	10.34	a_{Σ_b}	11.22
a_Ξ	9.52	a_{Ξ^*}	10.15	a_{Ξ_c}	9.99	a_{Ξ_b}	10.83
b_B	MeV	b_B	MeV	b_B	MeV	b_B	MeV
b_N	125.30	b_Δ	125.30	—	—	—	—
b_Λ	92.57	—	—	b_{Λ_c}	83.54	b_{Λ_b}	83.54
b_Σ	100.12	b_{Σ^*}	83.54	b_{Σ_c}	83.54	b_{Σ_b}	83.54
$\tilde{b}_\Sigma$	152.42						
b_Ξ	46.29	b_{Ξ^*}	41.77	b_{Ξ_c}	41.77	b_{Ξ_b}	41.77
a_M	$\times 10^{-4}$ MeV^{-1}	a_M	$\times 10^{-4}$ MeV^{-1}	a_M	$\times 10^{-4}$ MeV^{-1}	a_M	$\times 10^{-4}$ MeV^{-1}
a_ω	8.73	a_K	6.66	a_D	8.61	a_B	9.92
a_ρ	8.70	a_{K^*}	8.60	a_{D^*}	9.09	a_{B^*}	10.04
—	—	$a_\eta\,(n_q^\eta \to 1)$	7.03	—	—	—	—
—	—	$a_{\eta'}\,(n_q^{\eta'} \to 1)$	8.81	—	—	—	—

Two-nonlight flavor meson self-energies:

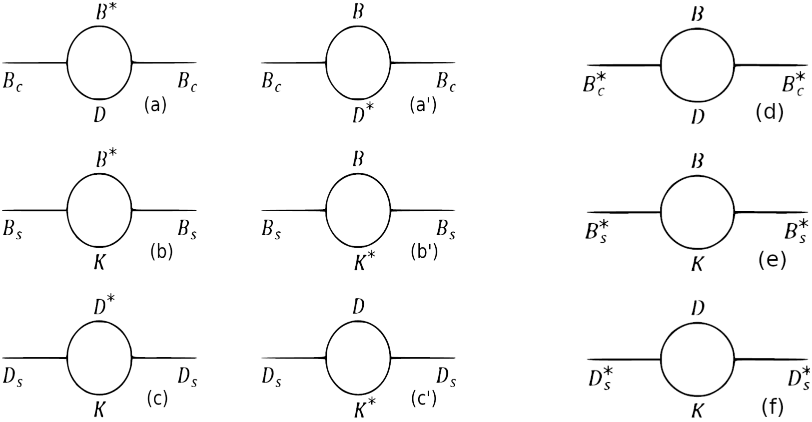
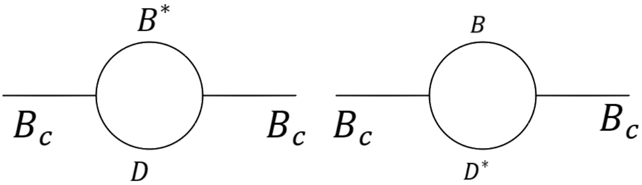


Figure: **Self-energies of pseudoscalar mesons B_c, B_s and D_s, and vector mesons B_c^*, B_s^* and D_s^* (from top to bottom) in the lowest order.**

B_c mass shift in nuclear medium

- $\Delta_{m_h} = m_h^* - m_h, \qquad h = B_c(B_s, D_s)$
- $m_h^2 = \left(m_h^0\right)^2 + \Sigma_h(k^2 = m_h^2)$



- Only lowest order (”minimal” contribution)
- **SU(5) effective Lagrangian densities** used to calculate the self-energy effect of the mesons

B, B^*, D, D^*, K, K^* effective masses

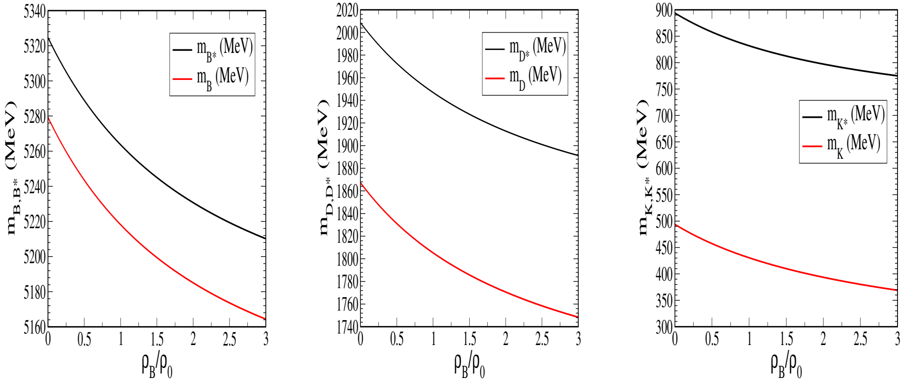


Figure: B and B^* (left), D and D^* (middle) and K and K^* (right) meson effective Lorentz-scalar masses in symmetric nuclear matter versus baryon density.

Effective Lagrangian

$$\mathcal{L}_0 = Tr\left(\partial_\mu P^\dagger \partial^\mu P\right) - \frac{1}{2} Tr\left(F^\dagger_{\mu\nu} F^{\mu\nu}\right)$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\partial_\mu P \rightarrow \partial_\mu P - \tfrac{ig}{2}[V_\mu, \mathrm{P}],\ F_{\mu\nu} \rightarrow \partial_\mu V_\nu - \partial_\nu V_\mu - \tfrac{ig}{2}[V_\mu,\ V_\nu]$$

$$\begin{aligned} \mathcal{L} \ &= \mathcal{L}_0 + igTr(\partial^\mu P[P,\ V_\mu]) - \tfrac{g^2}{4} Tr\left([\mathrm{P},\ \mathrm{V}_\mu]^2\right) \\ &+ igTr(\partial^\mu V^\nu [V_\mu,\ V_\nu]) + \tfrac{g^2}{8} Tr\left([V_\mu,\ V_\nu]^2\right) \end{aligned}$$

Z. w. Lin and C. M. Ko, Phys. Lett. B 503, 104 (2001)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & \pi^+ & K^+ & \overline{D}^0 & B^+ \\ \pi^- & P_2 & K^0 & D^- & B^0 \\ K^- & \overline{K}^0 & P_3 & D_s^- & B_s^0 \\ D^0 & D^+ & D_s^+ & P_4 & B_c^+ \\ B^- & \overline{B}^0 & \overline{B}_s^0 & B_c^- & P_5 \end{pmatrix}$$

$$P_1 = \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_2 = \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_3 = \frac{-2\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_4 = \frac{-3\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_5 = \frac{-2\eta_b}{\sqrt{5}}$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} V_1 & \rho^+ & K^{*+} & \overline{D}^{*0} & B^{*+} \\ \rho^- & V_2 & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \overline{K}^{*0} & V_3 & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & V_4 & B_c^{*+} \\ B^{*-} & \overline{B}^{*0} & \overline{B}_s^{*0} & B_c^{*-} & V_5 \end{pmatrix}$$

$$\begin{aligned} V_1 &= \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_2 &= \frac{-\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_3 &= \frac{-2\omega}{\sqrt{6}} + \frac{J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_4 &= \frac{-3J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}} \\ V_5 &= \frac{-2\Upsilon}{\sqrt{5}} \end{aligned}$$

Effective lagrangian (example): B_c, B_c^*

$$
\begin{aligned}
\mathcal{L}_{B_cB^*D} &= ig_{B_cB^*D}[(\partial_\mu B_c^-)D - B_c^-(\partial_\mu D)]B^{*\mu} + h.c. \\
\mathcal{L}_{B_cBD^*} &= ig_{B_cBD^*}[(\partial_\mu B_c^+)\overline{B} - B_c^+(\partial_\mu \overline{B})]\overline{D^*}^\mu + h.c. \\
\mathcal{L}_{B_c^*BD} &= -ig_{B_c^*BD}B_c^{*+\mu}[\overline{B}(\partial_\mu \overline{D}) - (\partial_\mu \overline{B})\overline{D}] + h.c.
\end{aligned}
$$

$$
B = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix}, \quad \overline{B} = \begin{pmatrix} B^- & \overline{B^0} \end{pmatrix}, \quad B^* = \begin{pmatrix} B^{*+} \\ B^{*0} \end{pmatrix}, \quad \overline{B^*} = \begin{pmatrix} B^{*-} & \overline{B^{*0}} \end{pmatrix}.
$$

$$
\overline{D} = \begin{pmatrix} \overline{D}^0 \\ D^- \end{pmatrix}, \quad D = \begin{pmatrix} D^0 & D^+ \end{pmatrix}, \quad \overline{D}^* = \begin{pmatrix} \overline{D}^{*0} \\ D^{*-} \end{pmatrix}, \quad D^* = \begin{pmatrix} D^{*0} & D^{*+} \end{pmatrix}.
$$

$$
g_{\Upsilon BB} = \frac{5g}{4\sqrt{10}} \approx 13.2
$$

$$
g_{B_cB^*D} = g_{B_cBD^*} = g_{B_c^*BD} = \frac{g}{2\sqrt{2}} \approx 11.9
$$

B_c self-energy (example): $B^* D$ loop (only case)

$$V_{B_c} = \Delta m_{B_c} \equiv m^*_{B_c} - m_{B_c} \quad (m^*_{B_c} < m_{B_c}, \text{ Lorentz scalar}),$$

$$m^2_{B_c} = \left(m^0_{B_c}\right)^2 - |\Sigma_{B_c}(k^2 = m^2_{B_c})|,$$

$$m^{*2}_{B_c} = \left(m^0_{B_c}\right)^2 - |\Sigma^*_{B_c}(k^2 = m^{*2}_{B_c})|,$$

$$\Sigma^{B^* D}_{B_c}(m^*_{B_c}) = \frac{-4g^2_{B_c B^* D}}{\pi^2} \int d|\mathbf{k}||\mathbf{k}|^2 I_{B^* D}(|\mathbf{k}|) F_{B_c B^* D}(\mathbf{k}^2),$$

$$I^{B^* D}_{B_c}(|\mathbf{k}|) = \left.\frac{m^{*2}_{B_c}\left(-1 + k_0^2/m^{*2}_{B^*}\right)}{(k_0 - \omega^*_{B^*})(k_0 - m^*_{B_c} + \omega^*_D)(k_0 - m^*_{B_c} - \omega^*_D)}\right|_{k_0 = -\omega^*_{B^*}}$$

$$+ \left.\frac{m^{*2}_{B_c}\left(-1 + k_0^2/m^{*2}_{B^*}\right)}{(k_0 + \omega^*_{B^*})(k_0 - \omega^*_{B^*})(k_0 - m^*_{B_c} - \omega^*_D)}\right|_{k_0 = m^*_{B_c} - \omega^*_D}$$

Total potential $V_s^{B_c}$ $(B^*D + BD^*)$ loops

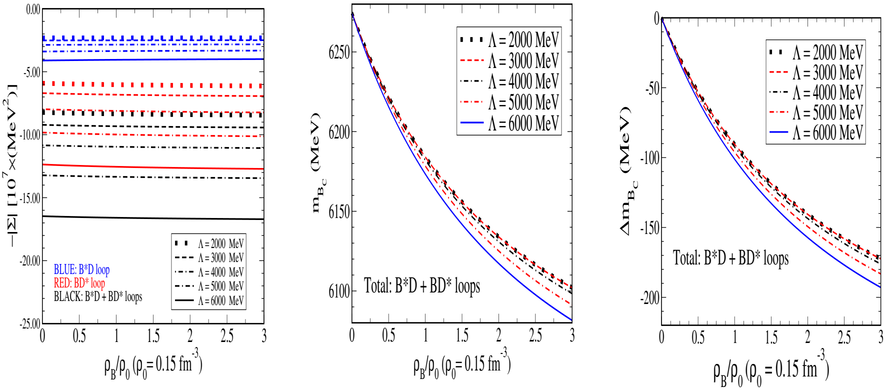


Figure: $|\Sigma_{B_c}|$ (left), $m^*_{B_c}$ (middle) and Δm_{B_c} (right)

Total potential $V_s^{B_c^*}$ (BD) loop

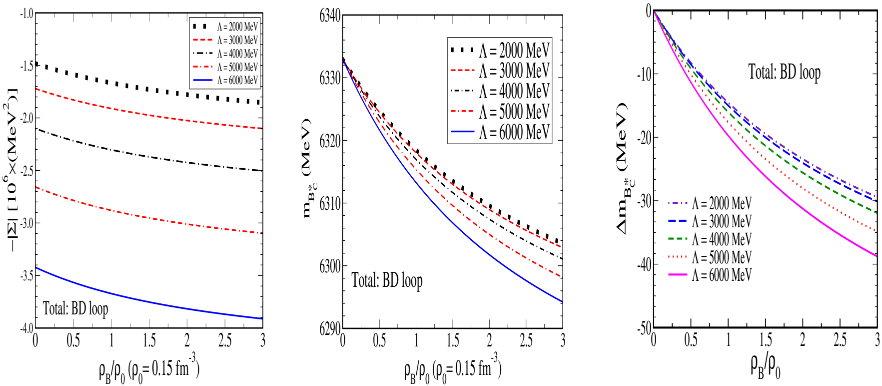


Figure: $|\Sigma_{B_c^*}|$ (left), $m^*_{B_c^*}$ (middle) and $\Delta m_{B_c^*}$ (right)

Total potentials: $\Delta m_{\eta_b}, \Delta m_{B_c}, \Delta m_{\eta_c}$

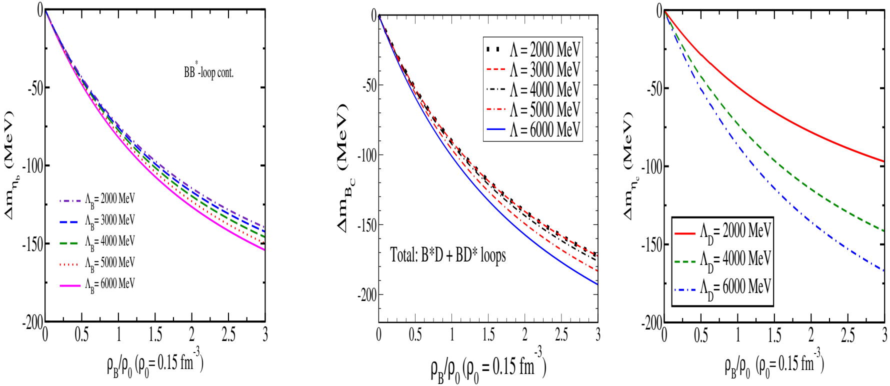


Figure: Δm_{η_b}(BB*) (left), Δm_{B_c}(B*D + BD*) (middle) and Δm_{η_c}(DD*) (right)

Total potentials: $\Delta m_{\Upsilon}, \Delta m_{B_c^*}, \Delta m_{J/\Psi}$

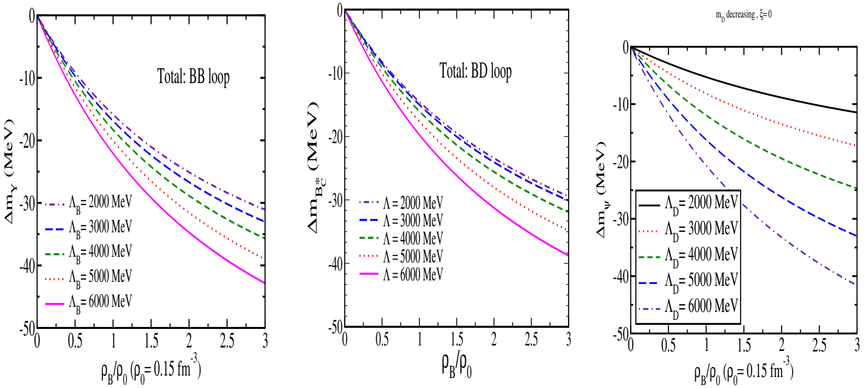


Figure: Δm_{Υ}(BB) (left), $\Delta m_{B_c^*}$(BD) (middle) and $\Delta m_{J/\psi}$(DD) (right)

Summary of mass shifts for the two-flavored mesons

• $|\Delta m_{B_c}| < |\Delta m_{B_s^0}| > |\Delta m_{D_s}|$
• $|\Delta m_{\eta_b}| < |\Delta m_{B_c}| > |\Delta m_{\eta_c}|$

• $|\Delta m_{B_c^*}| \simeq |\Delta m_{B_s^*}| < |\Delta m_{D_s^*}|$
• $|\Delta m_{\Upsilon}| \simeq |\Delta m_{B_c^*}| \simeq |\Delta m_{J/\psi}|$

• $B_c^{\pm}$-nucleus bond states (^{12}C done, and more! $\rightarrow$ next)

Other studies (include KT below):
• A. W. Thomas, G. Krein, J. J. Cobos-Multinez: J/ψ- and η_c-nucleus bound states, Prog. Part. Nucle. Phys. 100, 161 (2018)
• G. Zeminiani, J. J. Cobos-Multinez: Υ- and η_b-nucleus bound states, Phys. Rev. C 105, 025204 (2022)
• J. J. Cobos-Multinez: η- and η'-nucleus bound states with widths, Phys. Rev. C 109, 025202 (2024)
• $B_c^{\pm}$-^{12}C bound states! Phys. Rev. C 111, 055202 (2025)

$B_c^{\pm}$-nucleus bound states

$B_c^{\pm}$-^{12}C bound states, Phys. Rev. C 111, 055202 (2025)

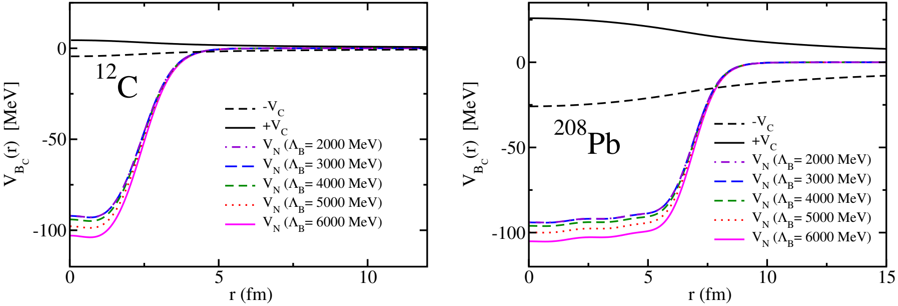
Solve the K.G. equation in momentum space for $\boxed{B_c^{\pm}}$

$$\left[\vec{P}^2 + \left(m + V_N^S\right)^2\right]\psi = \underline{\left[\varepsilon - (\mathrm{V_N^V} = 0 + \mathrm{V_C})\right]^2}\psi$$

Bound-state energies $E_{n\ell}$ for each energy level

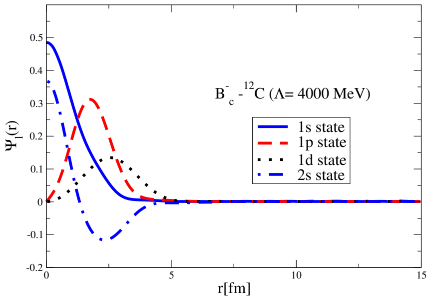
$$\boxed{E_{n\ell} = \varepsilon_{n\ell} - m}$$

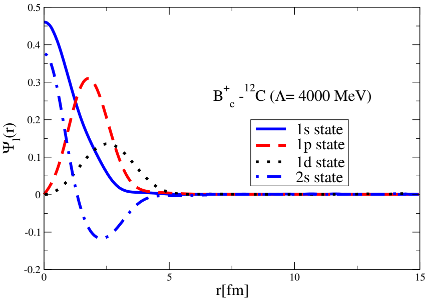
B_c-^{12}C and B_c-^{208}Pb potentials (wo Coulomb pots.)



- **Poetntial depths are similar !**

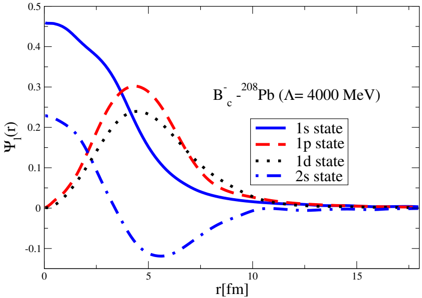
$B_c^{\pm}$-^{12}C wave functions including the Coulomb pot.

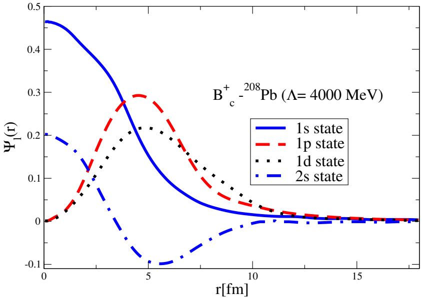




- **Vertical scale difference !**

$B_c^{\pm}$-^{208}Pb wave functions including the Coulomb pot.





B_c-^{12}C bound state energies w/wo Coulomb pot.

		Bound state energies (MeV)		
		B_c-^{12}C (Strong only)		
	$n\ell$	$\Lambda_B = 2000$ MeV	$\Lambda_B = 4000$ MeV	$\Lambda_B = 6000$ MeV
$^{12}_{B_c}$C	1s	-74.55	$\boxed{-76.06}$	-82.43
	1p	-51.53	-52.88	-58.64
		B_c^--^{12}C		
	$n\ell$	$\Lambda_B = 2000$ MeV	$\Lambda_B = 4000$ MeV	$\Lambda_B = 6000$ MeV
$^{12}_{B_c^-}$C	1s	-79.12	-80.63	-87.03
	1p	-56.15	-57.53	-63.38
		B_c^--^{12}C ($2EV_c \to 0$)		
	$n\ell$	$\Lambda_B = 2000$ MeV	$\Lambda_B = 4000$ MeV	$\Lambda_B = 6000$ MeV
$^{12}_{B_c^-}$C	1s	-75.06	$\boxed{-76.58}$	-82.98
	1p	-52.63	-54.01	-59.85
		B_c^+-^{12}C		
	$n\ell$	$\Lambda_B = 2000$ MeV	$\Lambda_B = 4000$ MeV	$\Lambda_B = 6000$ MeV
$^{12}_{B_c^+}$C	1s	-71.01	-72.53	-78.94
	1p	-49.11	-50.49	-56.32

Summary of $B_c^{\pm}$-^{12}C bound states and prospect

•Importance of the Coulomb-strong intermittence! for the $B_c^{\pm}$-^{12}C bound states (explicitly for the first time by the full KG equation)

Future plans:

•Complete for other nuclei, $B_c^{\pm}$-nucleus bound states

•Complete the study of two-flavored-heavy-heavy meson-nucleus bound state studies (B_c^*, $B_s^{(*)}$ and $D_s^{(*)}$ mesons)

•Complete for other nuclei, two-flavored-heavy-light meson-nucleus bound state studies $B^{(*)}$, $D^{(*)}$, and $K^{(*)}$ mesons)

•Studies of the mesons with non-zero momenta cases (vector mesons !)

Support from:

- **APCTP** (Korea)
- **OMEG Institute, Soongsil University** (Seoul, Korea)
- **Pukyong National University** (Pusan, Korea)
- **COREnet, RCNP, Osaka University** (Japan)
- **INCT-FNA** (Brazil)
- **CNPq** (Brazil)

Thank You Very Much !!!

Bound quark Dirac spinor (1s$_{1/2}$)

Quark Dirac spinor in a **bound hadron:**

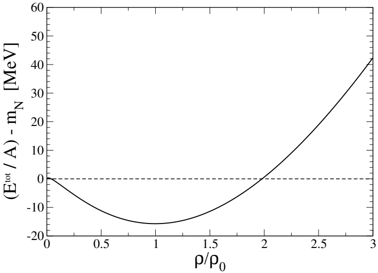
$$q_{1s}(r) = \begin{pmatrix} U(r) \\ i\sigma \cdot \hat{r}\, L(r) \end{pmatrix} \chi$$

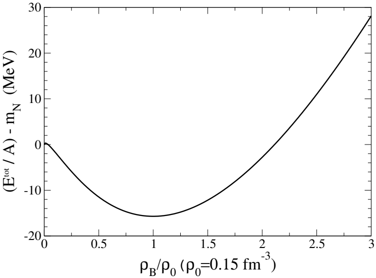
Lower component is **enhanced** !

$\Rightarrow$ $g_A^* < g_A$: $\sim |U|^{**}2 - (1/3)\ |L|^{**}2$,

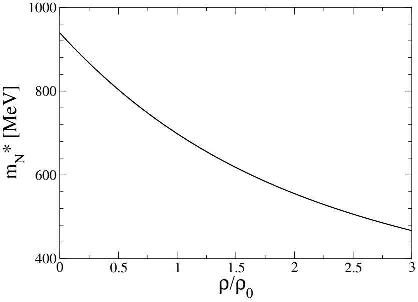
$\Rightarrow$ **Decrease** of scalar density $\Rightarrow$

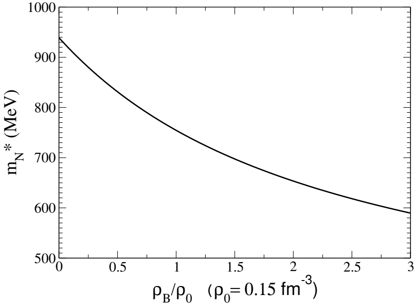
Comparison of Energy/nucleon





- Symmetric Nuclear Matter - Binding Energy per Nucleon (scale !!)
- LF pion model (left): $m_q = 220$ MeV, $K = 320.9$ MeV
- Standard QMC (right): $m_q = 5$ MeV, $K = 279.3$, MeV

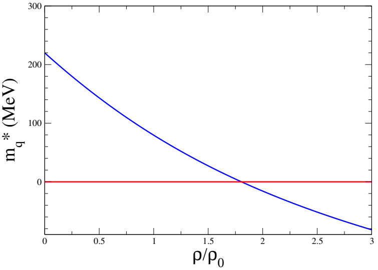


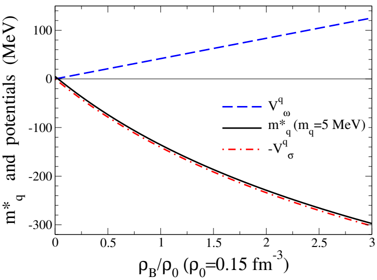


Nucleon effective mass

- **LF pion model** (left: $m_q = 220 MeV$)
- **Standard QMC** (right: $m_q = 5 MeV$)

LF pion model and Standard QMC: m_q^* (potentials)





- Effective mass of constituent quarks, up and down
- LF pion model: $m_q = 220$ MeV (left)
- Standard QMC $m_q = 5$ MeV (right)