Quark saturation and quarkyonic matter in a quark-meson coupling (QMC) model

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OMEG
Origin of Matter and Evolution of Galaxies
우주 물질 중점연구소

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— Introduction —

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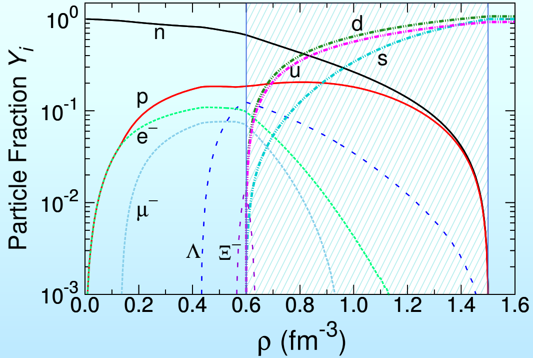
- Quark saturation mechanism and quarkyonic (quarks+baryons) matter
- Quark-hadron duality

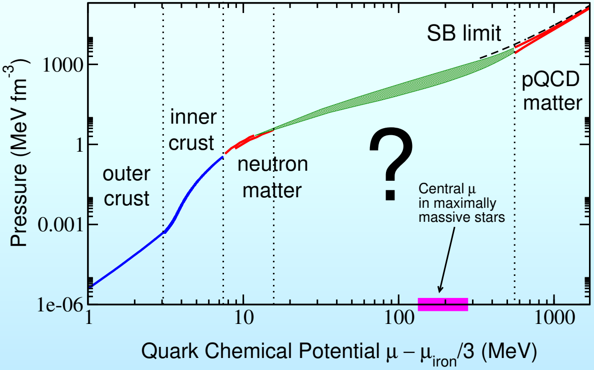
Hadrons and quarks in dense matter

Connection between nuclear physics and pQCD

Combining the EoSs for hadron and quark phases
under the first-phase transition, crossover etc.



A. Kurkela *et al.*, Astrophys.J. 789 (2014) 127.



SB (Stefan-Boltzmann) limit: an ideal gas of quarks and gluons
$\mu_B \sim 2.6$ GeV, $n_B = 40 n_0$ in the perturbative QCD region

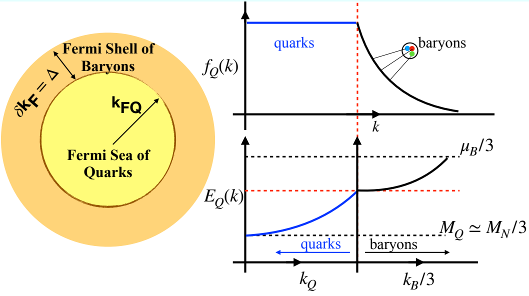
Quark saturation and quarkyonic matter

Hard and soft realizations of deconfinement from nuclear to quark matter

Momentum shell of baryons

L. McLerran and S. Reddy, Phys. Rev. Lett. 122 (2019) 122701.

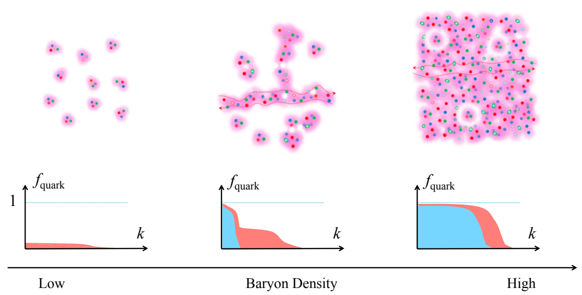
$$n_B = \frac{2}{3\pi^2}[k_{\rm FB}^3 - (k_{\rm FB}-\Delta)^3 + k_{\rm FQ}^3], \quad k_{\rm FQ} = \frac{(k_{\rm FB}-\Delta)}{N_c}\Theta(k_{\rm FB}-\Delta),$$



Soft deconfinement

Quantum percolation of quark wave functions in a chiral soliton model

K. Fukushima *et al.*, Phys. Rev. D. 102 (2022) 096017.



Quarkyonic matter based on quark-hadron duality

L. McLerran and S. Reddy, Phys. Rev. Lett. 122 (2019) 122701, T. Kojo, Phys. Rev. D. 104 (2021) 074005.

The ideal dual quarkyonic (IdylliQ) model:

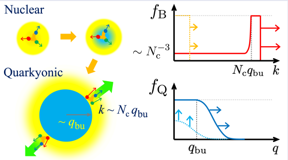
Y. Fujimoto, T. Kojo, and L. D. McLerran, Phys. Rev. Lett. 132 (2024) 112701.

- The nucleon is composed of quarks whose single quark momentum distribution is supposed to be a Yukawa-type function:

$$\varphi(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda},$$

with the typical momentum scale $\Lambda \sim \Lambda_{\mathrm{QCD}}$.

- This choice makes it possible to solve the IdylliQ model exactly.



Our motivation:

- We investigate a dual description of quarks and baryons in dense QCD matter, focusing on the quarkyonic regime with $\boxed{\text{a relativistic confinement potential}}$.
- We incorporate realistic nucleon-nucleon interactions using the quark-meson coupling (QMC) model.

$\rho_{\mathrm{sat}} > \rho_0$ or $\rho_{\mathrm{sat}} < \rho_0$ (ρ_0: nuclear saturation density)

— Theoretical framework —

- Relativistic gaussian quarkyonic (GQ) model
  quarkyonic regime
- Quarkyonic quark-meson coupling (QQMC) model
  quarkyonic regime + N-N interactions

Relativistic quark model of nucleon
with harmonic oscillator potential

K. Saito, T. Miyatsu, and M.-K. Cheoun, Phys. Rev. D 110 (2024) 113001.

- The quark wavefunction satisfies the Dirac equation:

$$\left[i\gamma_\mu\partial^\mu - m - U(r)\right]\psi_Q^\alpha(\boldsymbol{r},t) = 0,$$

with the scalar-vector harmonic oscillator potential, $U(r) = \frac{c}{2}(1+\gamma_0)\,r^2$ $(c>0)$, and the constituent quark mass, m.

- The lowest-state ($\alpha = 0$) solution is given by

$$\psi_Q(\boldsymbol{r}) = \frac{N}{\sqrt{4\pi a}}\begin{pmatrix} 1 \\ i\boldsymbol{\sigma}\cdot\hat{r}\,\frac{1}{\lambda a}\left(\frac{r}{a}\right)\end{pmatrix}\exp\left(-r^2/2a^2\right)\chi,$$

$$\text{with} \qquad N^2 = \frac{8}{\sqrt{\pi}a}\frac{\lambda^2 a^2}{2\lambda^2 a^2+3}, \quad \lambda = \epsilon + m, \quad a^2 = \frac{1}{\sqrt{c\lambda}} = \frac{3}{\epsilon^2 - m^2},$$

where a is the length scale and χ is the quark spinor.

- The single-particle quark energy, ϵ, is determined by $\sqrt{\epsilon+m}\,(\epsilon - m) = 3\sqrt{c}$.
- In the limit $\lambda \to \infty$, this wave function turns out to be the nonrelativistic (NR) one.
- We assume that the strength parameter, "c", respects flavor symmetry. $\Rightarrow$ The Hilbert space is common for all quarks.

Nucleon mass

K. Saito, T. Miyatsu, and M.-K. Cheoun, Phys. Rev. D 110 (2024) 113001.

- The zeroth-order energy of the low-lying nucleon, E_N^0, is then simply given by the sum of the quark energies, $E_N^0 = 3\epsilon$.
- The spin correlations, E_N^{spin}, due to the quark-gluon and quark-pion interactions.
  Here we do not calculate the spin correlation explicitly, but we assume that it can be treated as a constant parameter which is fixed so as to reproduce the nucleon mass.
- The center-of-mass correction to the spurious motion: $E_N^{c.m.} = \dfrac{77\epsilon + 31m}{3a^2(3\epsilon + m)^2}$.

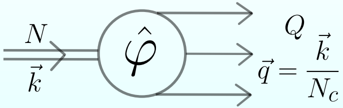
$$\boxed{M_N = E_N^0 + E_N^{\mathrm{spin}} - E_N^{c.m.}}$$

r_p (fm)	a (fm)	c (fm^{-3})	ϵ (MeV)	E_N^0 (MeV)	E_N^{spin} (MeV)	$E_N^{c.m.}$ (MeV)
0.75	0.719	0.856	562.1	1686.2	−412.7	344.6
0.80	0.769	0.675	536.3	1608.8	−365.0	304.8
0.85	0.819	0.539	514.0	1542.0	−324.2	278.8

We fix the root-mean-square radius of proton including the c.m. correction with $m = 300$ MeV.

Quark-baryon duality

Introducing the mapping operators



- $|N\rangle \in \mathbb{N}$ vector space (nucleon set in momentum space)
- $|Q\rangle \in \mathbb{Q}$ vector space (quark set in momentum space)

✓ $|k\rangle$: basis of nucleon momentum, $|q\rangle$: basis of quark momentum

Closures: $\int_k |k\rangle\langle k| = \int_k \frac{d\boldsymbol{k}}{(2\pi)^3} |k\rangle\langle k| = 1$ (unit matrix), $\int_q |q\rangle\langle q| = \int_q \frac{d\boldsymbol{q}}{(2\pi)^3} |q\rangle\langle q| = 1$.

✓ Mapping operators: a mapping matrix $\hat{\varphi}$ and its inverse mapping matrix $\hat{\varphi}^{-1}$

$$|Q\rangle = \hat{\varphi}\,|N\rangle\,, \qquad |N\rangle = \hat{\varphi}^{-1}\,|Q\rangle\,,$$

where $|Q\rangle = \hat{\varphi}\,|N\rangle = \hat{\varphi}\hat{\varphi}^{-1}\,|Q\rangle = |Q\rangle \Longrightarrow \hat{\varphi}\hat{\varphi}^{-1} = \hat{\varphi}^{-1}\hat{\varphi} = 1.$

$$\mathbb{N} \xrightarrow{\hat{\varphi}} \mathbb{Q}: \qquad \langle q|\hat{\varphi}|k\rangle = \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_c}\right) = (2\pi)^3 \left(\frac{2\bar{\lambda}^2}{2\bar{\lambda}^2 + 3}\right) \frac{a^3}{\pi^{3/2}} \left(1 + \frac{(\bar{\boldsymbol{q}} - \bar{\boldsymbol{k}})^2}{\bar{\lambda}^2}\right) \exp(\bar{\boldsymbol{q}} - \bar{\boldsymbol{k}})^2,$$

with the dimensionless variables, $\bar{\lambda} = \lambda a$, $\bar{\boldsymbol{q}} = \boldsymbol{q}a$, and $\bar{\boldsymbol{k}} = \boldsymbol{k}\frac{a}{N_c}$. Nucleon momentum, k, is divided by N_c (the numbers of quarks, high momentum limit), namely $\boldsymbol{q} = \boldsymbol{k}/N_c$ ($N_c = 3$).

Gaussian quarkyonic (GQ) model

Quark-baryon duality via the simple sum rule

NR: T. Kojo, Phys. Rev. D. 104 (2021) 074005.

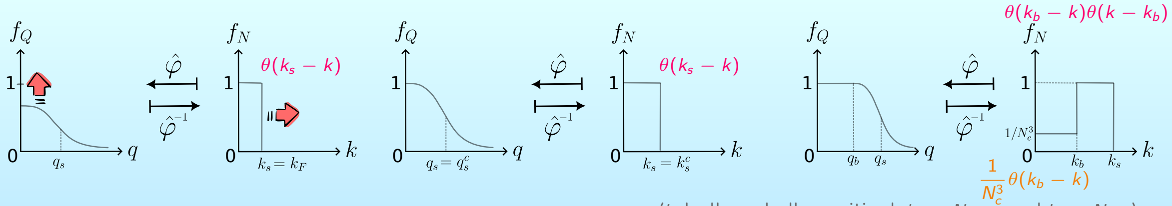
- Using the gaussian quark wavefunction, the single momentum distribution in a single nucleon is given by

$$\varphi(\boldsymbol{q}) = \pi^{3/2} a^3 \left(\frac{16\lambda^2 a^2}{2\lambda^2 a^2 + 3} \right) \left(1 + \frac{q^2}{\lambda^2} \right) \exp\left(-a^2 q^2\right), \qquad \int_q \varphi(\boldsymbol{q}) := \int \frac{d^3 \boldsymbol{q}}{(2\pi^3)} \varphi(\boldsymbol{q}) = 1.$$

pre-saturation

quark saturation

post-saturation



(b: bulk, s: shell, c: critical, $k_b = N_c q_b$, and $k_s = N_c q_s$)

$$f_Q(q) = \int_k \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_c}\right) f_N(k) \quad \text{and} \quad \rho_N = \langle \psi_N^\dagger \psi_N \rangle = \int_k f_N(k).$$

Quark distribution below quark saturation density

In symmetric nuclear matter

In the confinement dominate region ($\rho_N \le \rho_N^c$),

$$\begin{cases} \rho_N = \frac{2}{3\pi^2} k_s^3 = \frac{2}{3\pi^2} k_F^3 \\ \rho_N^c = \frac{2}{3\pi^2} (k_s^c)^3 \quad \text{(at quark saturation density)} \end{cases}, \quad k_s \le k_s^c, \quad f_Q(q) \le 1, \quad f_Q(q=0) \le 1.$$

$$f_Q(q) = \int_{k'} \langle q|\hat{\varphi}|k'\rangle f_N(k') = \int \frac{d\boldsymbol{k}'}{(2\pi)^3} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}'}{N_c}\right) \theta(k_s - k') =: \mathcal{I}_0^{k_s}(q)$$

$$= \frac{N_c^3}{\sqrt{\pi}} \Big[\mathrm{Er}(\bar{q}) + \mathrm{Er}(-\bar{q}) - \mathrm{Er}(\bar{k}_s + \bar{q}) - \mathrm{Er}(\bar{k}_s - \bar{q}) + \frac{1}{2\bar{q}} \left\{ \exp[-(\bar{k}_s + \bar{q})^2] - \exp[-(\bar{k}_s - \bar{q})^2] \right\}$$

$$+ \frac{2\bar{k}_s^2 + 1}{2\bar{\lambda}^2 + 3} \frac{1}{2\bar{q}} \left\{ \exp[-(\bar{k}_s + \bar{q})^2] - \exp[-(\bar{k}_s - \bar{q})^2] \right\} + \frac{\bar{k}_s}{2\bar{\lambda}^2 + 3} \left\{ \exp[-(\bar{k}_s + \bar{q})^2] + \exp[-(\bar{k}_s - \bar{q})^2] \right\} \Big],$$

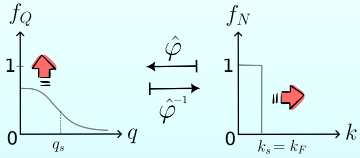
where the error function is defined as $\mathrm{Er}(x) = \int_x^\infty \exp(-t^2)\, dt$. $\Rightarrow$ Numerical calculations!!!

$$f_Q(q=0) = \mathcal{I}_0^{k_s}(0) = \frac{N_c^3}{\sqrt{\pi}} \left[\sqrt{\pi} - 2\mathrm{Er}(\bar{k}_s) - 2\bar{k}_s \exp(-\bar{k}_s^2) - \frac{4\bar{k}_s^3}{2\bar{\lambda}^2 + 3} \exp(-\bar{k}_s^2) \right] = 1 \implies \bar{k}_s = \bar{k}_s^c.$$

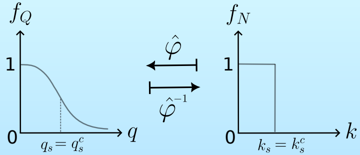
Quark saturation

at the critical density, ρ_N^c

Below the quark saturation density

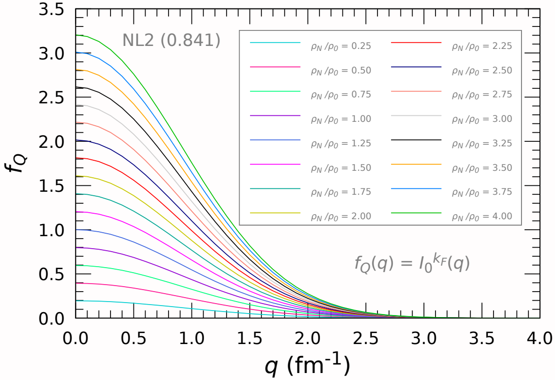


At the quark saturation density



$m_q = 300$ MeV, $\langle r^2 \rangle_p = 0.841$ fm, $M_N = 939$ MeV

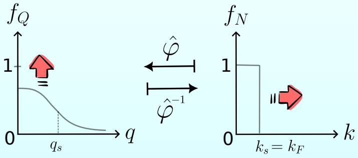
$f_Q(q) \lesssim 1 \Rightarrow$ quarkyonic picture



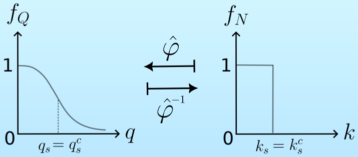
Quark saturation

at the critical density, ρ_N^c

Below the quark saturation density



At the quark saturation density



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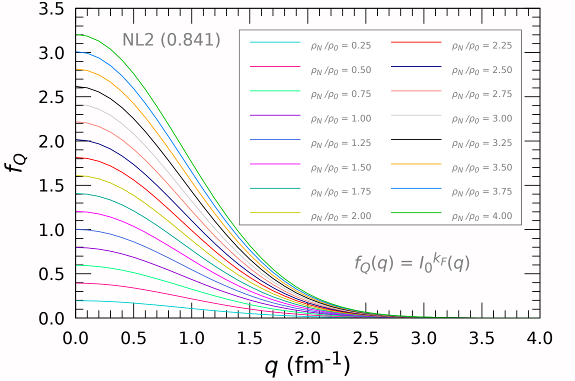


Quark distribution above the quark saturation

Connecting the two segments: a complete dual description

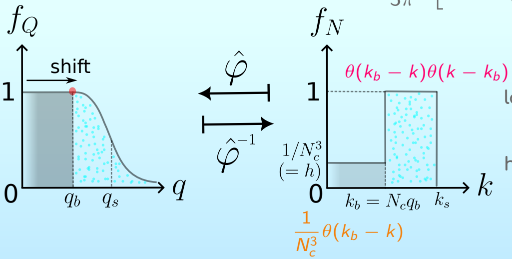
Y. Fujimoto *et al.*, Phys. Rev. Lett. 132 (2024) 112701.

- At the quark saturation density, $f_Q(q=0, k_s=k_F)=1 \Rightarrow \rho_{\rm sat} = \frac{2}{\pi^2}k_F^3$.
- Above $\rho_N^{\rm sat}$, we assume the following nucleon momentum-space distributions for symmetric nuclear matter:

$$f_N(k) = \frac{1}{N_c^3}\theta(k_b-k) + \theta(k_b-k)\theta(k-k_b), \quad \rho_N = \frac{2}{3\pi^2}\left[k_s^{*3} - \left(1-\frac{1}{N_c^3}\right)k_b^{*3}\right].$$

Quark-level Pauli blocking
$\Rightarrow$ neighboring nucleons begin to overlap at the quark level.

quarkyonic matter



low-momentum sector: bulk
softly deconfined/delocalized quark modes
"soft deconfinement"

high-momentum sector: shell
quarks are still confined in baryonic modes.

crossover regime

Continuity condition: $f_Q(0,k_s) - \left(1-\frac{1}{N_c^3}\right)f_Q(0,k_b) = 1$ $\Rightarrow$ determine k_b and k_s at a given ρ_N

Quark-meson coupling (QMC) model

with a gaussian quark wavefunction

K. Saito, K. Tsushima, and A. Thomas, Prog. Part. Nucl. Phys. 58 (2007) 1.

- The Dirac equation in the quark-meson coupling model:

$$\left[i\gamma_\mu\partial^\mu - (m - V_s) - \gamma_0 V_0 - \frac{c}{2}(1+\gamma_0)\, r^2\right]\psi(\boldsymbol{r}, t) = 0,$$

where $V_s = g^q_\sigma \bar{\sigma}$ and $V_0 = g^q_\omega \bar{\omega}$ with the mean-field values of σ and ω mesons, $\bar{\sigma}$ and $\bar{\omega}$.

- The effective quark mass and the quark single-particle energy are respectively given by $m^* = m - V_s$ and $\epsilon^* = \epsilon - V_0$.

- The total energy per nucleon in symmetric nuclear matter (SNM) is then obtained by

$$\varepsilon_{tot}/\rho_N = \frac{2}{\pi^2\rho_N}\int dk\; k^2 f_N(k)\sqrt{M_N^{*2}(\bar{\sigma}) + k^2} + g_\omega\bar{\omega} + \frac{1}{2\rho_N}\left(m_\sigma^2\bar{\sigma}^2 - m_\omega^2\bar{\omega}^2\right) + \frac{1}{2\rho_N} g_2\bar{\sigma}^3.$$

with $M_N^* = E_N^{0*} + E_N^{\mathrm{spin}} - E_N^{c.m.*}$ $(E_N^{0*} = 3\epsilon^*)$.

- The mean fields are self-consistently calculated through

$$\left(m_\sigma^2 + g_2\bar{\sigma}\right)\bar{\sigma} = g_\sigma G_\sigma(\bar{\sigma})\rho_N^s, \qquad m_\omega^2\bar{\omega} = g_\omega\rho_N$$

with the scalar and number density, ρ_N^s and ρ_N, and the quark scalar polarizability, $G_\sigma(\bar{\sigma})$.

Quarkyonic quark-meson coupling (QQMC) model

K. Saito, T. Miyatsu, and M.-K. Cheoun, arXiv:2512.04505.

- Above $\rho_{\rm sat}$, we incorporate the quarkyonic mechanism into the QMC model.
- The matter effect is taken into account through m^* and ϵ^*.
- The boundary condition is also modified by using the effective shell and bulk momenta,

$$f_Q(0, k_s^*) - \left(1 - \frac{1}{N_c^3}\right) f_Q(0, k_b^*) = 1.$$

- The ε_{tot}, ρ_N^s, ρ_N are also modified:

$$\varepsilon_{tot}/\rho_N = \frac{2}{\pi^2 \rho_N}\left[\int_0^{k_s^*} - \left(1 - \frac{1}{N_c^3}\right)\int_0^{k_b^*}\right] dk\, k^2 \sqrt{M_N^{*2}(\bar{\sigma}) + k^2} + \frac{1}{2\rho_N}\left(m_\sigma^2 \bar{\sigma}^2 + m_\omega^2 \bar{\omega}^2\right) + \frac{1}{2\rho_N} g_2 \bar{\sigma}^3,$$

$$\rho_N^s = \frac{2}{\pi^2}\left[\int_0^{k_s^*} - \left(1 - \frac{1}{N_c^3}\right)\int_0^{k_b^*}\right] dk\, k^2 \frac{M_N^*(\bar{\sigma})}{\sqrt{M_N^{*2}(\bar{\sigma}) + k^2}}, \quad \rho_N = \frac{2}{3\pi^2}\left[k_s^{*3} - \left(1 - \frac{1}{N_c^3}\right) k_b^{*3}\right].$$

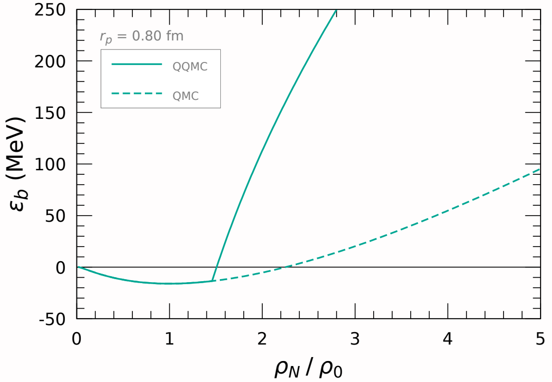
momentum shell and bulk effects

— Numerical results —

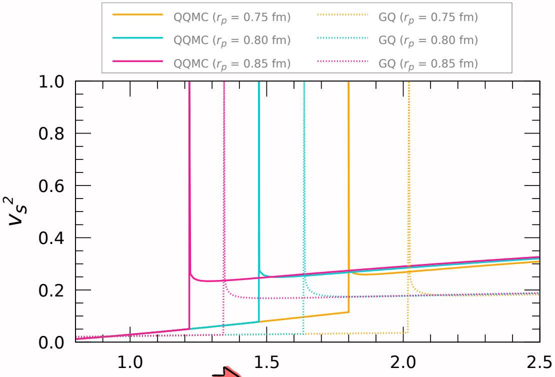
Quarkyonic matter properties

Binding energy and sound velocity in the QQMC model

Binding energy per nucleon, $\varepsilon_b = \varepsilon_{\rm tot}/\rho_N - M_N$



Sound velocity, $v_s^2 = \dfrac{\rho_N}{\mu_N}\dfrac{\mathrm{d}\mu_N}{\mathrm{d}\rho_N}$



relativistic effect $\Rightarrow$ $\boxed{\rho_{\rm sat} > \rho_0}$

✓ Quarkyonic matter leads to singular behavior in thermodynamic quantities around the quark saturation density.

Smearing the sharp Fermi surface in f_Q

to remove the divergence at ρ_N^{sat}

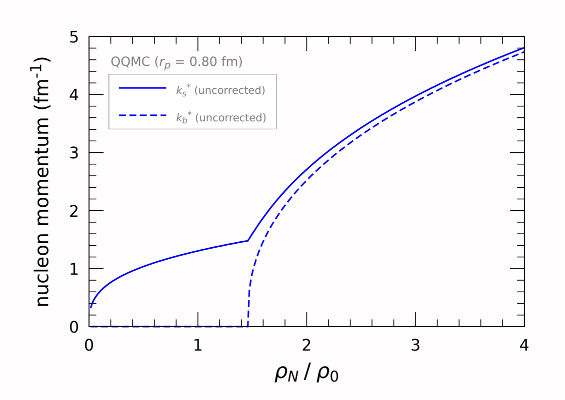
- The divergence appears in v_s^2 at ρ_N^{sat}.
- The gap around ρ_N^{sat} also occurs in μ_N and pressure (P).
- A minimal correction to this problem is to smear the sharp Fermi surface, $\theta(k_b^* - k)$, in f_Q.
- We introduce an infrared regulator to the boundary condition:

$$f_Q(0, k_s^*) - \left(1 - \frac{1}{N_c^3}\right) \alpha(k_b^*) f_Q(0, k_b^*) = 1,$$

with

$$\alpha(k_b^*) = 1 + \left(\frac{w}{k_b^*}\right)^{\nu} \to \infty(1) \text{ as } k_b^* \to 0(\infty).$$

where w is a width parameter and ν : is a positive, real number.



Smearing the sharp Fermi surface in f_Q

to remove the divergence at $\rho_N^{\rm sat}$

- The divergence appears in v_s^2 at $\rho_N^{\rm sat}$.
- The gap around $\rho_N^{\rm sat}$ also occurs in μ_N and pressure (P).
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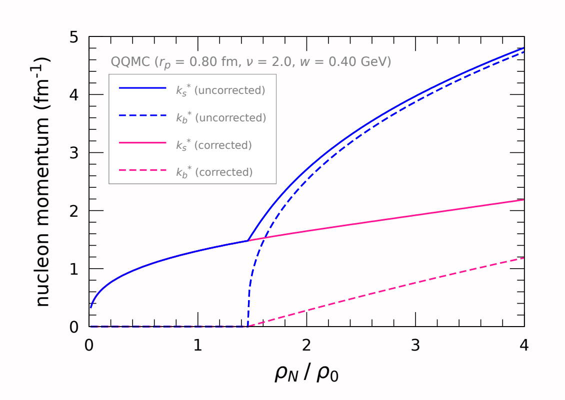
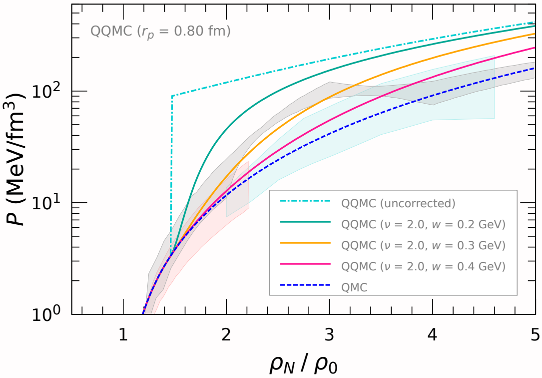


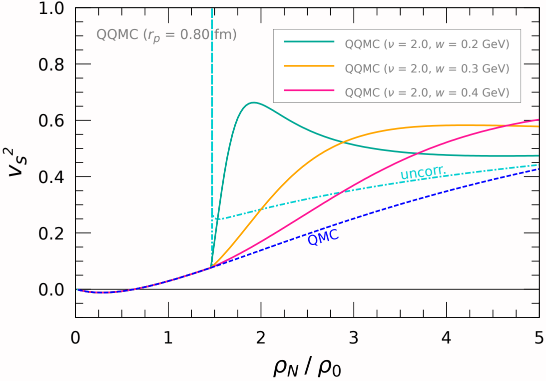
Pressure and sound velocity

in the quarkyonic quark-meson coupling (QQMC) model

Pressure, $P = \mu_N \rho_N - \epsilon_{tot}$



Sound velocity, v_s^2



✓ The shaded areas represent the constraints from analyses of heavy-ion collision (HIC) data:

skyblue: P. Danielewicz, et al., Science 298 (2002) 1592, pink: C. Fuchs, Prog. Part. Nucl. Phys. 56 (2006) 1, gray: D. Oliinychenko, et al., Phys. Rev. C 108 (2023) 034908.

— Summary —

Summary

Quarkyonic matter and quark saturation density: $\boxed{\rho_{\mathrm{sat}} > \rho_0}$

- We have constructed a novel, practical nuclear model based on the quark degrees of freedom, namely **the quarkyonic quark-meson coupling (QQMC) model**.
- Not only **the variation of nucleon structure** but also **the effect of Pauli blocking at the quark level** are taken into account.
- A **relativistic**, **gaussian** **quark wavefunction** is adopted to describe the structure of nucleon.

Open problems and future work:

- **The boundary condition** (the IdylliQ model) and a general rule for constructing **the quark momentum distribution, $f_Q(q)$, in the post-saturation region**
- The singular behavior of physical quantities around ρ_{sat} (**regulator**); basically, such singular behavior should instead be resolved by the fundamental dynamics, i.e. QCD.
- **Isospin-asymmetric nuclear matter** and neutron-star equation of state with hyperons

Thank You for Your Attention.

Summary

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- We have constructed a novel, practical nuclear model based on the quark degrees of freedom, namely **the quarkyonic quark-meson coupling (QQMC) model**.
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Open problems and future work:

- **The boundary condition** (the IdylliQ model) and a general rule for constructing **the quark momentum distribution, $f_Q(q)$, in the post-saturation region**
- The singular behavior of physical quantities around $\rho_{\rm sat}$ (**regulator**); basically, such singular behavior should instead be resolved by the fundamental dynamics, i.e. QCD.
- **Isospin-asymmetric nuclear matter** and neutron-star equation of state with hyperons

Thank You for Your Attention.