Carroll geometry and field theories

ÖPG Mitgliederversammlung

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Graz, Austria

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General motivations

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- Address conceptual issues of quantum gravity

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- Is holography realized in nature?

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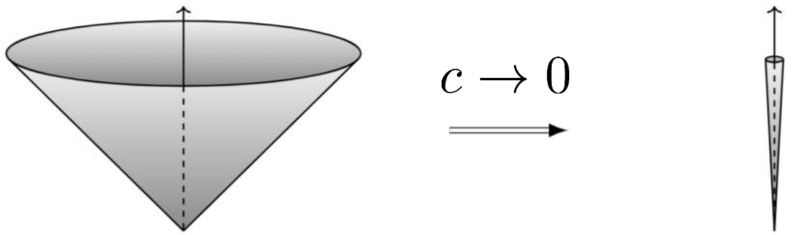
- Address conceptual issues of quantum gravity
- Is holography realized in nature?
- Does it generalize beyond AdS/CFT?

$\Rightarrow$ flat space holography?

Carroll group

Limit of the Poincaré group



[Lévy-Leblond ’65, Gupta ‘66]

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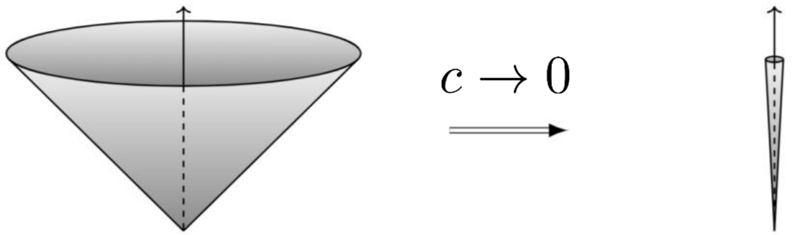

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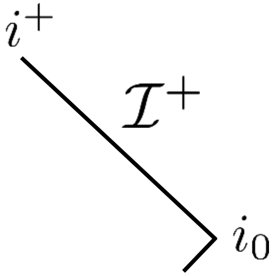
Flat space holography

Asymptotic symmetries

$$\mathrm{BMS}_{D+1} \cong \mathrm{CCarr}_D$$



are global symmetries of dual theory [Duval et al ‘14]

any null hypersurface carries Carroll structure

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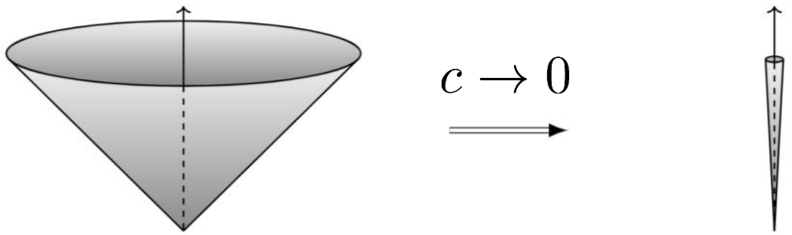

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Make Carroll symmetries local

- Electric [Henneaux ’79]
- Magnetic [Bergshoeff et al ‘17]

$\Rightarrow$ massive solutions: Carroll black holes [FE, Grumiller, Hartong, Pérez, Prohazka, Troncoso ‘23]

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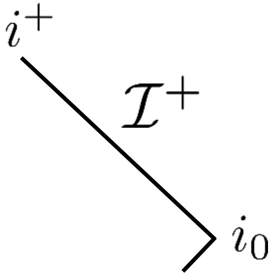

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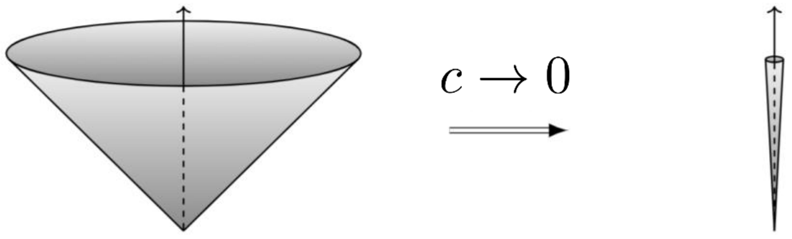

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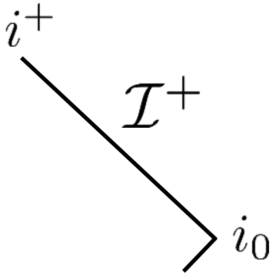

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Carroll hydrodynamics

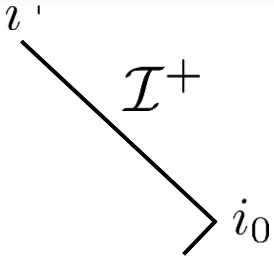
- Bjorken flow [Bagchi et al ‘23]
- BH membrane paradigm [Donnay et al ‘19]
- Cosmology [de Boer et al ‘22]

Carroll Workshop

A workshop dedicated to Carrollian Physics will take place in-person in Vienna from Tuesday 15/02/22 to Tuesday 22/02/22 at the Technische Universität Wien (TU Wien). This workshop will be the first part of a series of workshops on Carrollian physics, the second edition will take place at Université de Mons (UMONS) in September 2022.

Lately, Carrollian physics has been playing a key role in several research directions, including fluid/gravity and holographic correspondences, asymptotically flat spacetimes at null infinity and the related BMS symmetries, cosmology and geometries of null hypersurfaces such as black hole horizons.

The main purpose of the CARROLL WORKSHOP is to gather people who are currently participating to this research endeavor. Anyone engaged in the fascinating Carrollian research effort is most welcome to join us to take this opportunity to share innovative ideas and perhaps create new collaborative motions.



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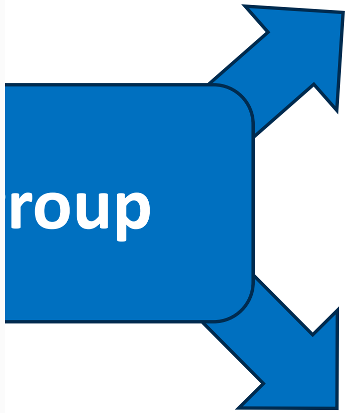

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Carrollian Physics and Geometry

Workshop

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Carrollian Physics and Geometry

Workshop

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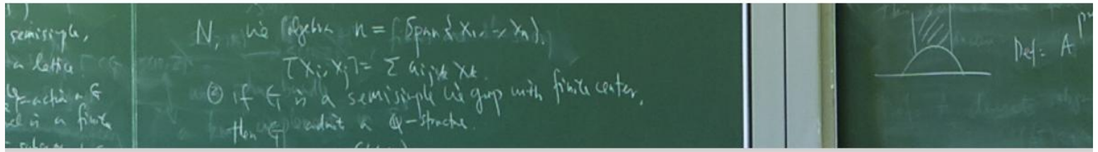

Carrollian Physics and Holography

Description | Schedule | Associated Events | Participants | Recordings

Carrollian physics and holography

Carrollian physics appeared for the first time out of a mathematical curiosity in a work of Lévy-Leblond: what if, instead of taking the usual non-relativistic limit of the Poincaré group where the speed of light c is taken to ∞, one considers the opposite limit, namely $c \to 0$? While the first limit gives rise to the familiar Galilean group, the second leads to the so-called Carroll group. In a Carrollian limit ($c \to 0$), the lightcones collapse along the time axis, which means that particles can no longer move and two spacetime events are causally disconnected. This absence of causality, as well as arbitrarinesses of time intervals, reminded Lévy-Leblond of Lewis Carroll's book *Through the looking-glass*, which is the origin for the name of this ultralocal limit of the Poincaré group.

While such spacetimes seem a priori to be too strange to be of any physical relevance, there has been recently a surge of interest in Carrollian physics. It is quite a remarkable fact that, indeed, Carrollian physics naturally appear in the context of the geometry of null surfaces and the associated Bondi-Metzner-Sachs symmetries, but also black hole horizons, inflationary cosmology, fluid/gravity and holographic correspondences, as well as limits of Einstein gravity.

The ESI programme is expected to foster further progress on the following developments:

- Carrollian fluids
- Flat space holography
- Carrollian gravity theories
- Lower-dimensional models
- Fracton physics

Outline

- Carroll geometry
- Carroll field theories
- Towards holography

[J. Tenniel 1865]

Carroll geometry

Carroll geometry

- Take $c \to 0$ limit of a Lorentzian metric

$$\begin{aligned} \mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 \quad &\to \quad \mathrm{d}s^2 = \mathrm{d}\vec{x}^2 = h_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\ g^{\mu\nu} \quad &\to \quad -v^\mu v^\nu = -\delta^\mu_t \delta^\nu_t \end{aligned}$$

Carroll metric data

$$\boxed{h_{\mu\nu} \; , \; v^\mu}$$

$$h_{\mu\nu} v^\nu = 0$$

Carroll geometry

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$$ds^2 = -c^2 dt^2 + d\vec{x}^2 \quad \to \quad ds^2 = d\vec{x}^2 = h_{\mu\nu} dx^\mu dx^\nu$$
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- On the level of isometries: $\mathfrak{poin}_{d+1} \to \mathfrak{carr}_{d+1}$ (contraction)

unchanged: $H = \partial_t \qquad P = \partial_i \qquad J_{ij} = x_i \partial_j - x_j \partial_i$

changed: $B_i = c^2 t \partial_i - x_i \partial_t \quad \to \quad B_i = -x_i \partial_t$

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- Brackets involving B_i

$$[B_i, H] = 0 \qquad [B_i, B_j] = 0 \qquad [B_i, P_j] = \delta_{ij}H \qquad [B_k, J_{ij}] = 2\delta_{k[i}B_{j]}$$

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- In the following: Formulate Carroll spacetimes as Cartan geometries

$\Rightarrow$ Practical for modelling geometry

Carroll geometry

- Geometry described by Cartan connection A with curvature $F = \mathrm{d}A + \frac{1}{2}[A \wedge A]$ $\quad (a, b = 1, \dots, d)$

$$A = \boxed{\tau H + e^a P_a} + \boxed{\omega^a B_a + \tfrac{1}{2}\omega^{ab} J_{ab}} \qquad F(A) = TH + T^a P_a + \Omega^a B_a + \tfrac{1}{2}\Omega^{ab} J_{ab}$$

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may be partly chosen as dependent by imposing vanishing torsion $(T, T^a) = 0$

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$$\omega^{ab} = \omega^{ab}[\tau, e^c]$$

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$$\boxed{C^{\mu\nu} = e^\mu_a e^\nu_b C^{ab}}$$

$$C^{[\mu\nu]} = 0 = C^{\mu\nu}\tau_\nu$$

Carroll field theories

- Description by fields A_μ inherits a redundancy: Local Carroll boosts λ^a + rotations $\lambda^a{}_b$

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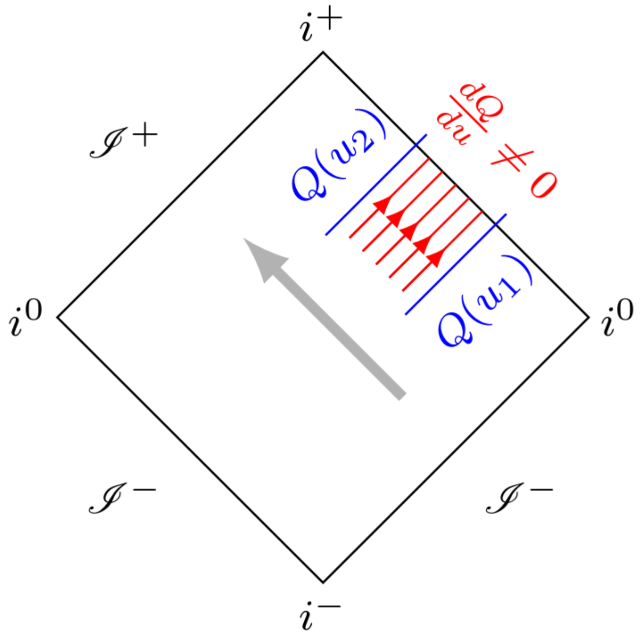
- An interacting model: Mutualistic scalar fields (“Carroll Swiftons”) $\delta_\lambda h^{\mu\nu} = -2v^{(\mu}\lambda^{\nu)}$

$$S[\phi, \chi] = \int \mathrm{vol}\, \left((v^\mu \partial_\mu \phi)^2 + (v^\mu \partial_\mu \chi)^2 + g B_\mu B^\mu\right) \qquad B_\mu = 2 v^\alpha \partial_{[\alpha}\phi \partial_{\mu]}\chi$$

[FE, D. Grumiller, M. Henneaux, P. Salgado-Rebolledo ‘24]

Towards holography

- Asymptotically flat spacetimes $\Rightarrow$ open systems (e.g. radiation at $\mathscr{I}$)



Towards holography

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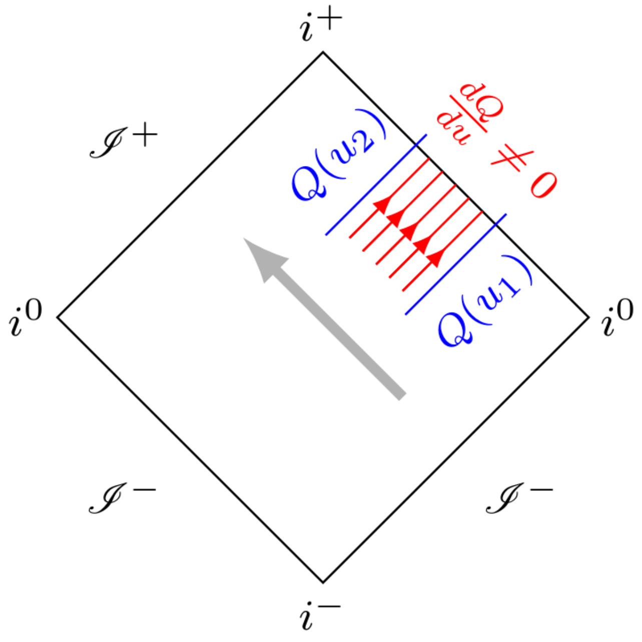
- Goal: Description of dual field theory living at $\mathscr{I}$

$$\delta W[\tau, h, C] = \int_{\mathscr{I}} \mathrm{d}^3x \left(\langle T^{\mu} \rangle \delta\tau_{\mu} + \langle T^{\mu\nu} \rangle \delta h_{\mu\nu} + \langle \Psi^{\mu\nu} \rangle \delta C_{\mu\nu} \right)$$



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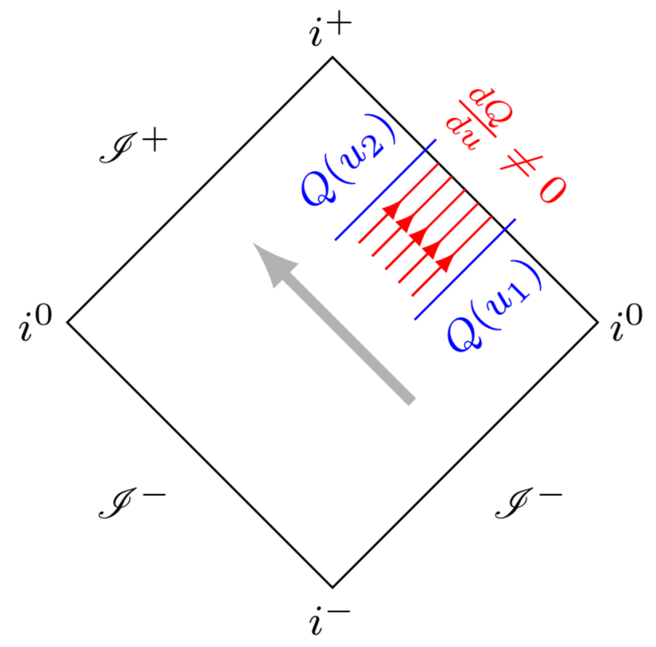

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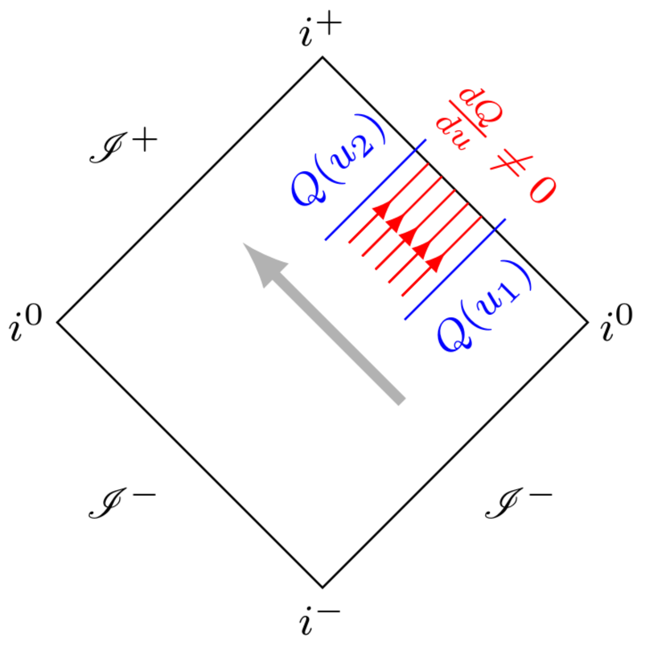
- Impose boundary covariance, Weyl invariance and local Carroll boost invariance of $W[\tau, h, C]$
- Implies a current (non-)conservation equation for $j^\mu = (T^\mu \tau_\nu + T^{\mu\sigma} h_{\sigma\nu} + \Psi^{\mu\sigma} C_{\sigma\nu})\xi^\nu - \Psi^{\mu\nu}\lambda_\nu$

$$\boxed{\partial_\mu \langle j^\mu \rangle = \langle T^\mu \rangle \delta_{\xi,\lambda,\rho}\tau_\mu + \langle T^{\mu\nu} \rangle \delta_{\xi,\lambda,\rho} h_{\mu\nu} + \langle \Psi^{\mu\nu} \rangle \delta_{\xi,\lambda,\rho} C_{\mu\nu}}$$

Towards holography

coordinates (u, x^i)



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- Fix $\tau = \mathrm{d}u, \quad h_{\mu\nu} = h_{ij}(x^k)\delta^i_\mu \delta^j_\nu$ but leave $C_{\mu\nu}$ unconstrained $\quad 2D_{(i}Y_{j)} = h_{ij} D_k Y^k$

$$\Rightarrow \quad \xi = \Big(f(x^k) + \tfrac{u}{2} D_i Y^i \Big)\partial_u + Y^i(x^k)\partial_i, \qquad \rho = \rho(f, Y), \qquad \lambda_\mu = \lambda_\mu(f, Y)$$

supertranslations — superrotations

$\Rightarrow$ realizes BMS algebra

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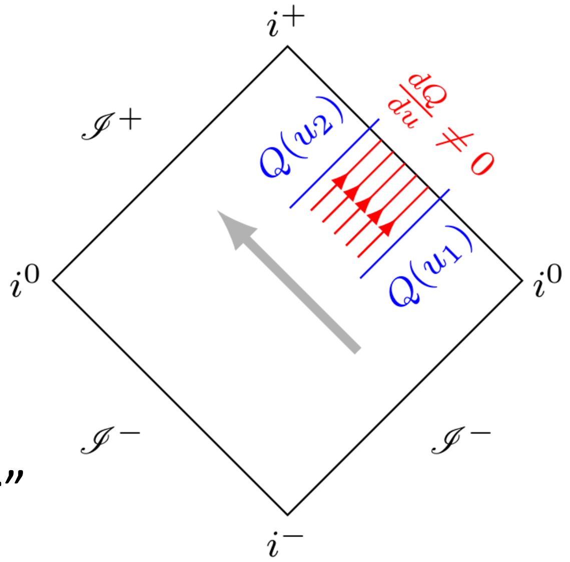

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"Bondi shear"

Towards holography

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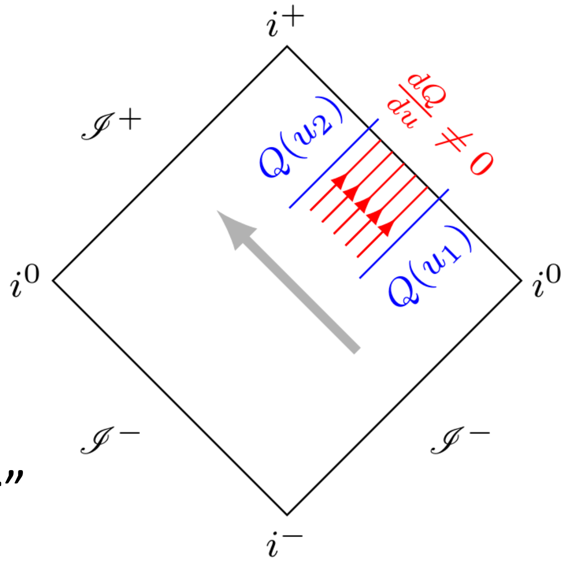

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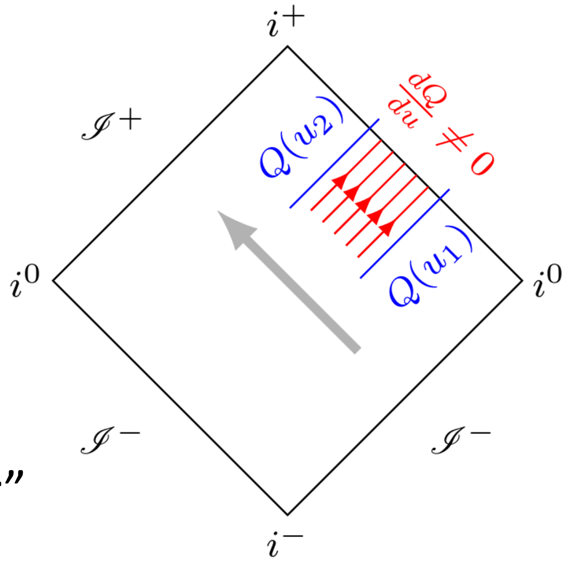

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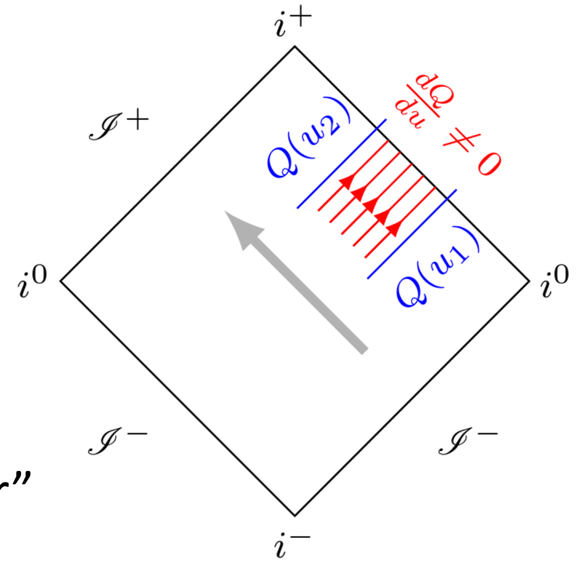

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- May be integrated to charges $Q_{(f,Y)} = \int_{S^2} d^2 x \, j^u$ to give representation of BMS

Boundary Ward identities $\leftrightarrow$ Einstein equations in the bulk

Universal brackets [FE, L. Montecchio, S. Prohazka, J. Salzer WIP] $\mathbf{T}^{\mu}{}_{\nu} = T^{\mu}\tau_{\nu} + T^{\mu\sigma}h_{\sigma\nu}$

- From the conservation identities we may determine equal time brackets by varying again, e.g.

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$$i[\mathbf{T}^0{}_0(x), \mathbf{T}^0{}_0(y)] = \tau^0{}_0{}^0{}_0(x,y) \qquad (1)$$

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- Universal structure + theory dependent contact terms $\tau^\mu{}_\nu{}^\sigma{}_\rho(x,y)$ [Schwinger ‘59, Dirac ’62]

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  coulombic data in $T^{\mu}, \quad T^{\mu\nu} \quad \leftrightarrow \quad m, \quad N_i$

  radiative data in $\Psi^{\mu\nu} \quad \leftrightarrow \quad N^{ij}$
- Possible to extract universal structure in $[T, T]$ and $[T, \Psi]$ brackets dictated by symmetry

  $\Rightarrow$ Carroll analogue of what Schwinger called

  ``the most fundamental equation of relativistic quantum field theory'' [Schwinger '63]

Thank you!

Carroll swiftons [FE, D. Grumiller, M. Henneaux, P. Salgado-Rebolledo '24]

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- Energy is still bounded from below: with $\phi_A = (\phi,\chi)$, $\quad \pi^A = \frac{\partial\mathcal{L}}{\partial\dot{\phi}_A}$

$\Rightarrow$ No instabilities

$$\mathcal{H} = \tfrac{1}{2}H_{AB}(\partial\phi)\pi^A\pi^B \qquad \det(H_{AB})>0,\ \text{Tr}(H_{AB})>0 \quad \text{if} \quad g > -\tfrac{1}{(\partial\phi)^2+(\partial\chi)^2}$$

Carroll 2d dilaton gravity

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$$\mathcal{M} \xrightarrow{X^I(x^\mu)} \mathcal{P}$$

- Write as Poisson-Sigma model with potential $\mathcal{V}(X, X_\mathrm{H})$

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Another example: $\mathcal{V}_{\mathrm{CJT}} = X$

“Carroll-Jackiw-Teitelboim”

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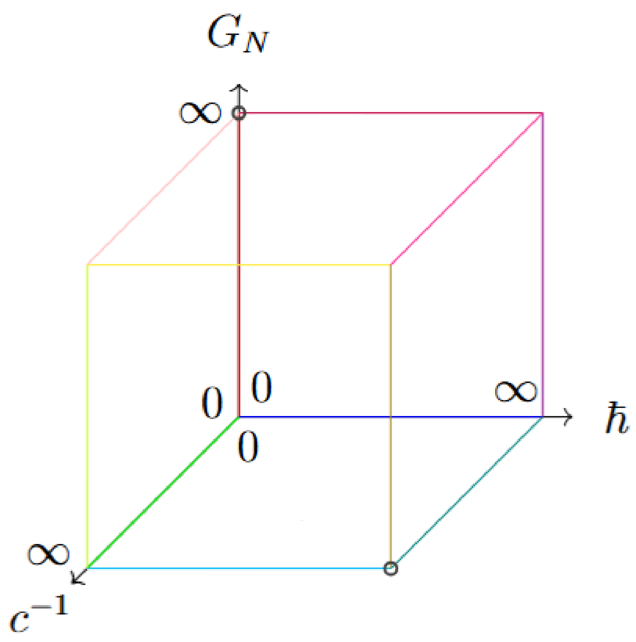
➢ Bronstein cube: Visualizing various limits of quantum gravity



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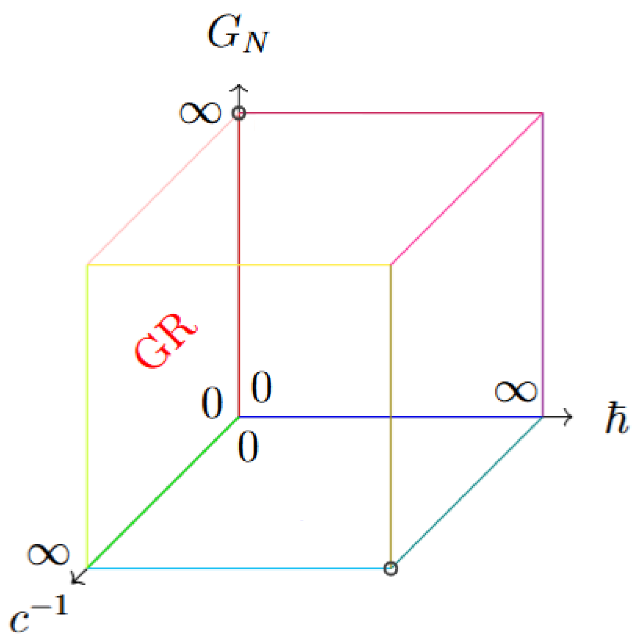

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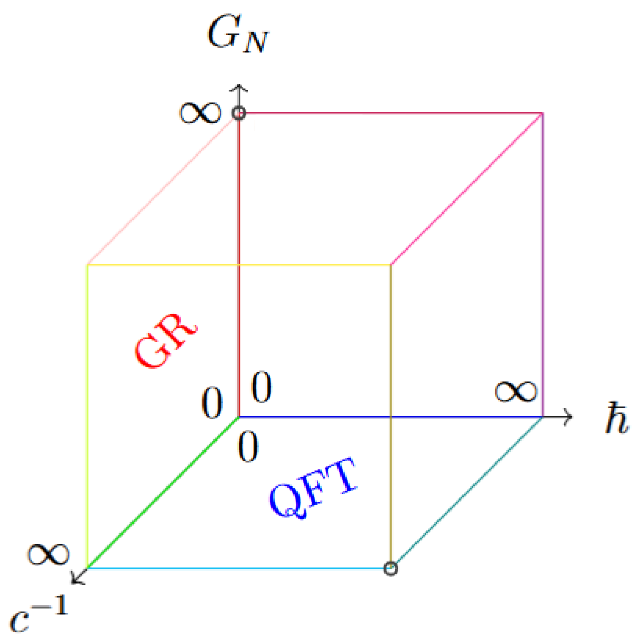

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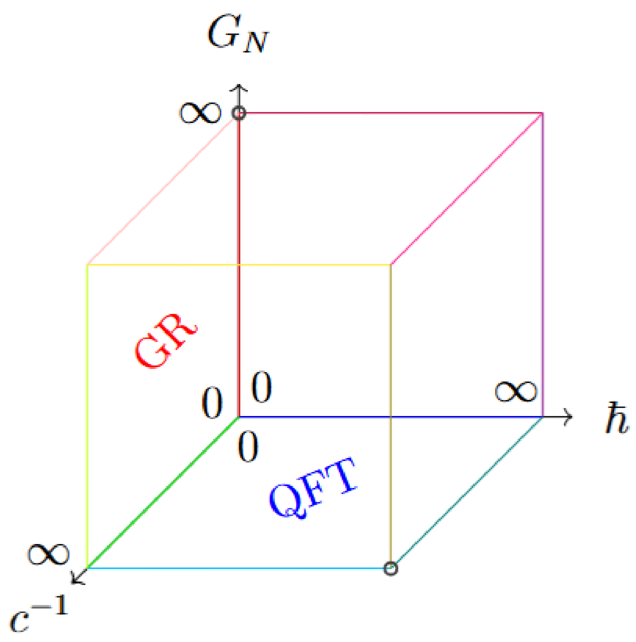

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$$T = \frac{\hbar c^5}{8\pi G_N E} \qquad \mathrm{r}_S = \frac{2G_N E}{c^4} \qquad E = mc^2$$

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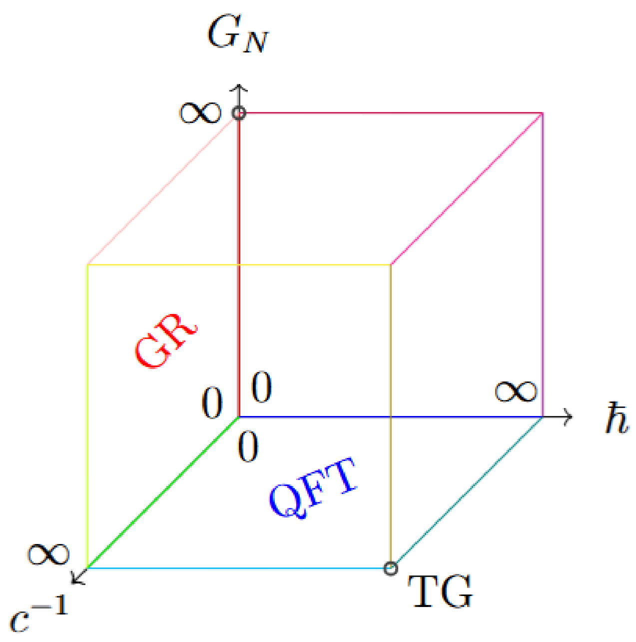

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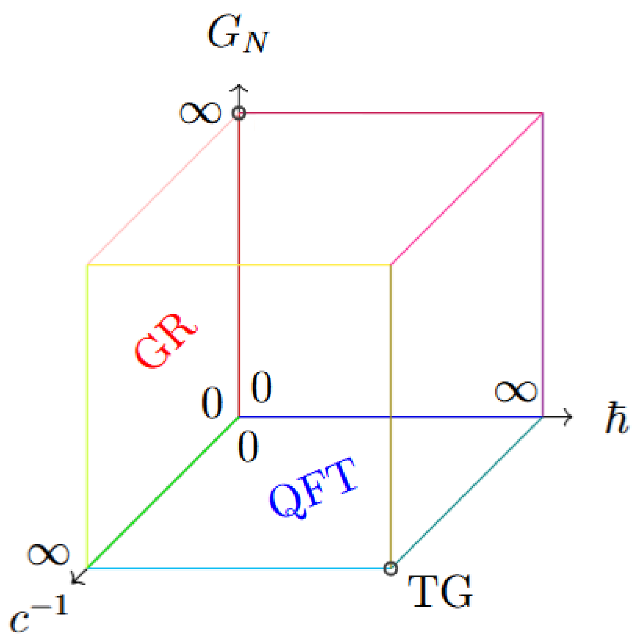

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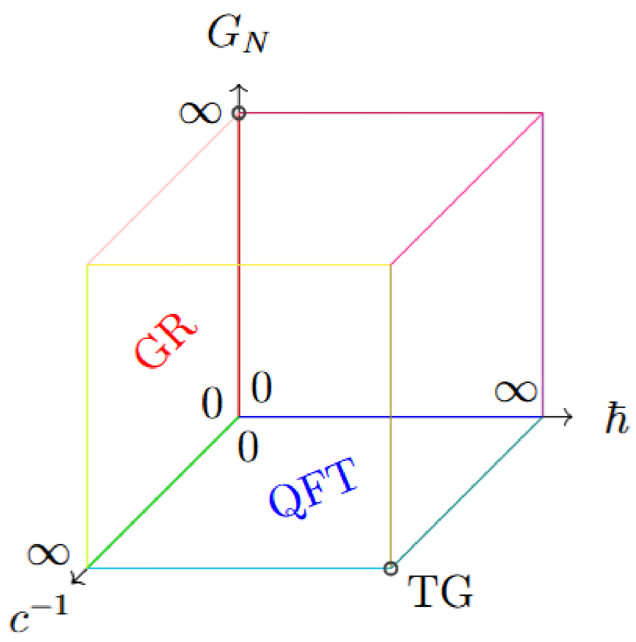

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$\longrightarrow$ magnetic Carroll gravity is saddle-point approximation of tantum gravity for $\kappa \to 0$

Carroll black holes [FE, J. Hartong, D. Grumiller, A. Pérez, S. Prohazka, R. Troncoso ‘23]

- Solution space of Carroll-Jackiw-Teitelboim model:

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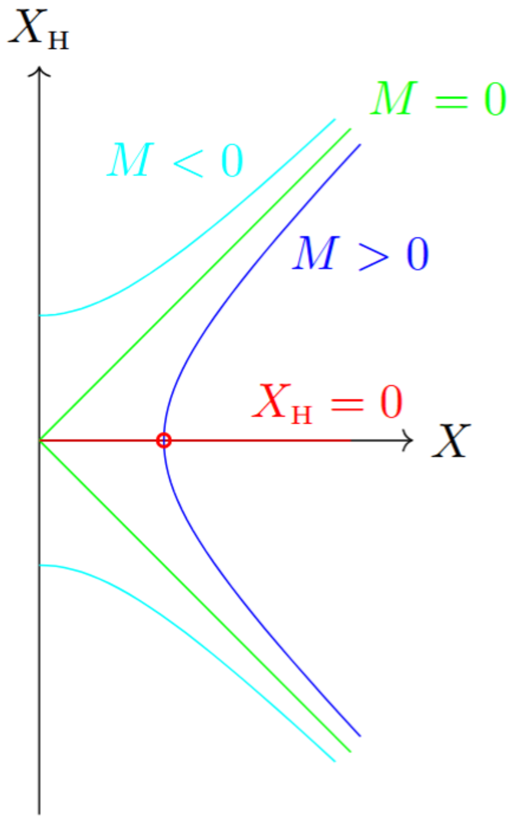
Target space of CJT



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- $M > 0$ sector admits **fixed-points** under Carroll boosts $X_{\mathrm{H}} = 0$

$$\boxed{\text{Carroll extremal surface (CES)}} \qquad (i)$$

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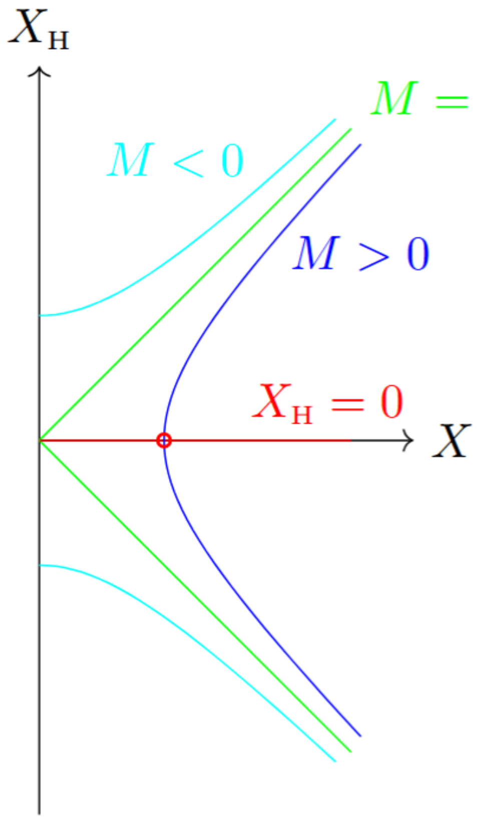

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$$\mathrm{d}s^2 = \mathrm{d}r^2 \qquad X_H = \tfrac{1}{2}e^r - Me^{-r}$$

$$\tau = \left(\tfrac{1}{2}e^r - Me^{-r}\right)\mathrm{d}t \qquad X = \tfrac{1}{2}e^r + Me^{-r}$$

- $M > 0$ sector admits **fixed-points** under Carroll boosts $X_{\mathrm{H}} = 0$

Carroll extremal surface (CES) (i)

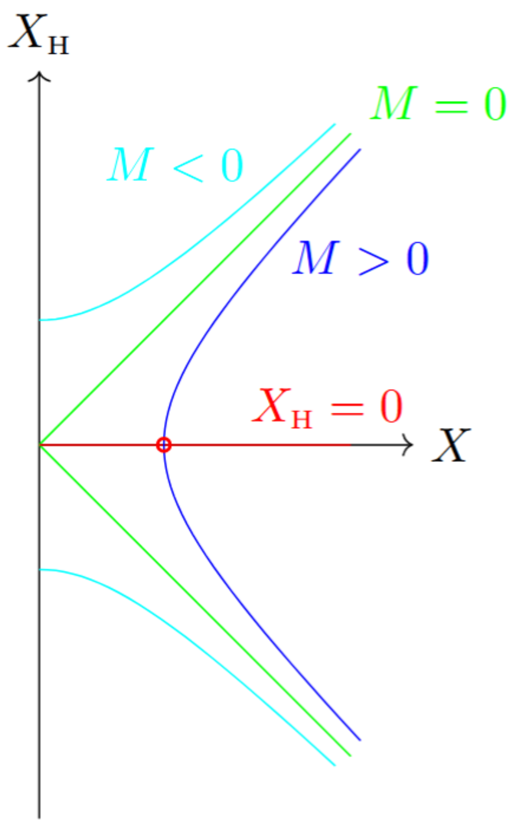
- May associate **thermal properties** to these sol's

$E \sim M$ $\swarrow$ boundary charge Q_{∂_t}

$S(M)$ $\downarrow$ charge at CES

$T(M)$ $\searrow$ no conical defects

Target space of CJT



Carroll black holes [FE, J. Hartong, D. Grumiller, A. Pérez, S. Prohazka, R. Troncoso '23]

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- $M > 0$ sector admits **fixed-points** under Carroll boosts $X_{\mathrm{H}} = 0$

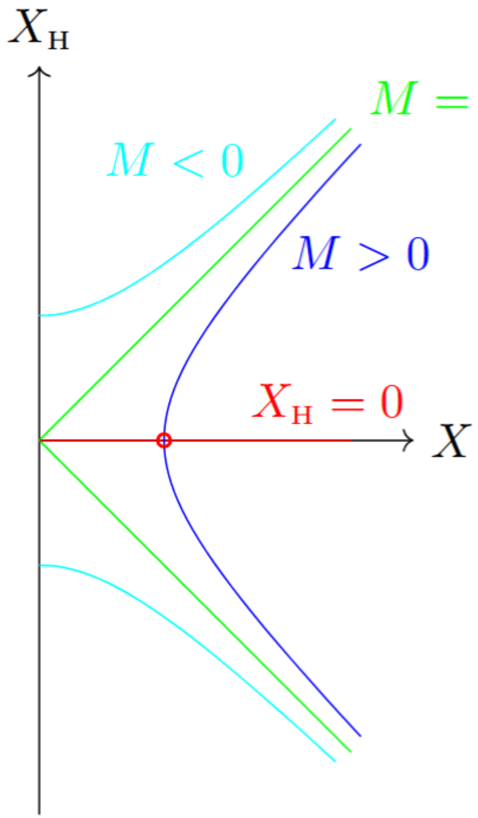
  Carroll extremal surface (CES) $\quad (i)$

- They satisfy a **first law**

  $$\boxed{\delta E = T\delta S} \qquad (ii)$$

$(i) + (ii)$ define a **Carroll black hole**

Target space of CJT



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Magnetic Carroll gravity

- Hamiltonian action:

$$S[\pi^{ij}, g_{ij}, N, N^i] = \int \mathrm{d}t\mathrm{d}^d x \left(\pi^{ij}\dot{g}_{ij} - N\mathcal{H} - N^i\mathcal{H}_i\right)$$

- First class constraints:

$$\mathcal{H} = -\frac{\sqrt{g}}{16\pi G_M}\left(R^{(d)}[g] - 2\Lambda\right) \qquad \mathcal{H}_i = -2\nabla_j \pi^j{}_i$$

$$\{\mathcal{H}(x), \mathcal{H}(y)\} = 0$$

$$\{\mathcal{H}(x), \mathcal{H}_i(y)\} = \mathcal{H}(y)\partial_i \delta^d(x-y)$$

$$\{\mathcal{H}_i(x), \mathcal{H}_j(y)\} = (\mathcal{H}_i(y)\partial_j + \mathcal{H}_j(x)\partial_i)\delta^d(x-y)$$

Units

- Case of Carroll-Schwarzschild:

$$[k_B] = 1$$

$$[\kappa] = J \cdot L$$

$$[G_M] = J^{-1} \cdot L$$

$$[\mathrm{r}_S] = L$$

$$[S_{rel}] = \left[\frac{Ac^3}{4\hbar G_N}\right] = \left[\frac{A}{4\kappa G_M}\right] = 1$$

$$[T] = \left[\frac{\hbar c}{4\pi \mathrm{r}_S}\right] = \left[\frac{\kappa}{4\pi \mathrm{r}_S}\right] = J$$

$$[E] = \left[\frac{c^4 \mathrm{r}_S}{2G_N}\right] = \left[\frac{\mathrm{r}_S}{2G_M}\right] = J$$