Higher-spin baryons with functional methods

Luca Oberguggenberger[†]

Gernot Eichmann[†], Christian S. Fischer[*]

[†]Institut für Physik, Universität Graz
[*]Institut für Theoretische Physik, Justus-Liebig-Universität Giessen

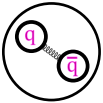
23.09.2026

Introduction

QCD: Quarks and gluons

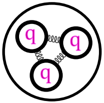
→ flavors: u,d,s,c,t,b and intrinsic spin s = 1/2

mesons



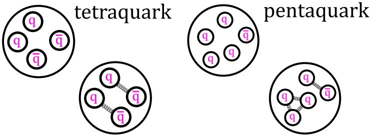
$S = 0,1$

baryons



$S = 1/2,3/2$

exotics



...

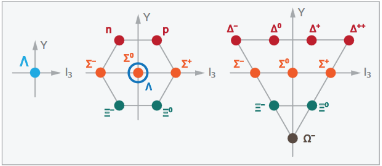
Gluons keep hadrons together!

Introduction

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{7}{2}^-$	$\frac{9}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$	$\frac{11}{2}^-$	$\frac{13}{2}^+$	$\frac{13}{2}^-$	$\frac{15}{2}^+$
N	N(1535)	N(1720)	N(1520)	N(1680)	N(1675)	N(1990)	N(2190)	N(2220)	N(2250)		N(2600)	N(2700)		
N(1440)	N(1650)	N'(1720)	N(1700)	N(1860)	N(2060)									
N(1710)	N(1895)	N(1900)	N(1875)	N(2000)	N(2570)									
N(1880)		N(2040)	N(2120)											
N(2100)														
N(2300)														
Δ(1750)	Δ(1620)	Δ(1232)	Δ(1700)	Δ(1905)	Δ(1930)	Δ(1950)	Δ(2200)	Δ(2300)	Δ(2400)	Δ(2420)			Δ(2750)	Δ(2950)
Δ(1910)	Δ(1900)	Δ(1600)	Δ(1940)	Δ(2000)	Δ(2350)	Δ(2390)								
	Δ(2150)	Δ(1920)												
Λ(1116)	Λ(1380)	Λ(1890)	Λ(1520)	Λ(1820)	Λ(1830)	Λ(2085)	Λ(2100)	Λ(2350)						
Λ(1600)	Λ(1405)	Λ(2070)	Λ(1690)	Λ(2110)	Λ(2080)									
Λ(1710)	Λ(1670)		Λ(2050)											
Λ(1810)	Λ(1800)		Λ(2325)											
	Λ(2000)													
Σ(1189)	Σ(1620)	Σ(1385)	Σ(1580)	Σ(1915)	Σ(1775)	Σ(2030)	Σ(2100)							
Σ(1660)	Σ(1750)	Σ(1780)	Σ(1670)	Σ(2070)										
Σ(1880)	Σ(1900)	Σ(1940)	Σ(1910)											
	Σ(2110)	Σ(2080)	Σ(2010)											
		Σ(2230)												
Ξ(1315)		Ξ(1530)	Ξ(1820)											
		Ω(1672)												

light baryons

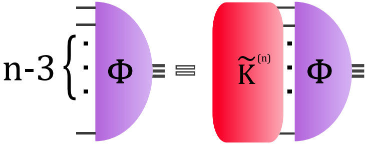
*S. Navas, et al., Review of particle physics, 2024,



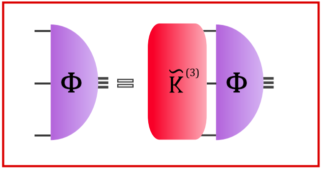
- Most of the mass of light baryons comes from quark dynamics.
- Total angular momentum J = L + S.
- Flavor states have roughly the same masses.

Introduction

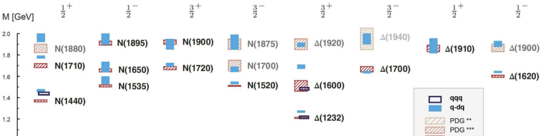
– How to calculate baryons



Bethe-Salpeter equation.
QCD equivalent of Schrödinger eq.

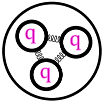


Baryons from three-body equation

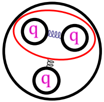


*Eichmann, Hadron Physics with Functional methods, 2025

qqq numerically intensive ⟶ q-dq two-body equation



⟶



Quark-diquark model

- Leading three-body contributions negligible – only two-body interaction kernel remains:

$$\Phi = \tilde{K}^{(3)}\Phi \to \Phi = \sum_{i=1}^{3} \tilde{K}_i^{(2)}\Phi$$

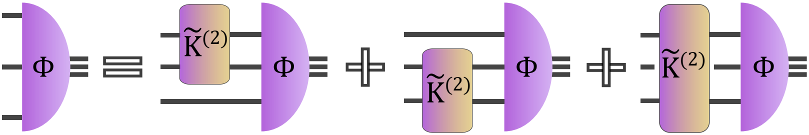


Figure: Faddeev approximation of BSE.

- Three-body equation $\to$ diquarks "freeze" 1 DoF $\leftrightarrow$ (pseudo-/scalar and axial-/vector) diquark BSE with amplitude Γ^a, propagator D^{ab}.

Quark-diquark Bethe-Salpeter equation

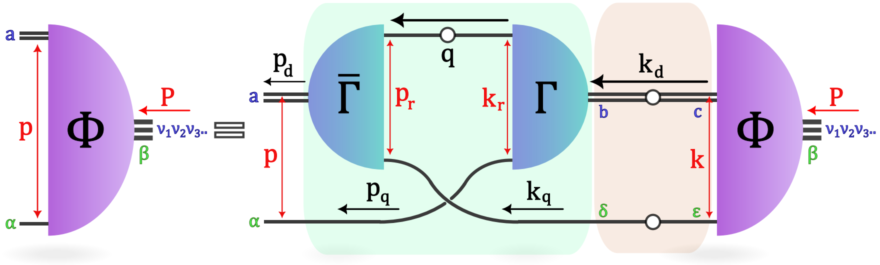


Figure: Quark-Diquark BSE.

- Solve eigenvalue equation for $\lambda(P^2 = -M^2) = 1$:

$$\Phi^{a;\nu_1\nu_2..}(p,P) = \lambda(P)\;[K\,G](P)\Phi^{a;\nu_1\nu_2..}(k,P)$$

Rarita-Schwinger – Structure

- Higher spins fields from the Rarita-Schwinger formalism.
- Example: $\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)=$
$\left[\left(1,\frac{1}{2}\right)\oplus\left(\frac{1}{2},1\right)\right]\oplus\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]\overset{\mathrm{SO(3)}}{\rightarrow}\frac{3}{2}+\frac{1}{2}+\frac{1}{2}$
- Remove redundant components by imposing some constraints:

$$(i\gamma\cdot\partial-m)\Psi^{\mu_1..\mu_n}(x)=0 \tag{1}$$

$$\gamma^{\mu}\Psi^{\mu..\mu_n}=0 \tag{2}$$

$$\partial^{\mu}\Psi^{\mu..\mu_n}=0 \tag{3}$$

$$\Psi^{\mu..\mu_n},\ \mu..\mu_n \text{ symmetric} \tag{4}$$

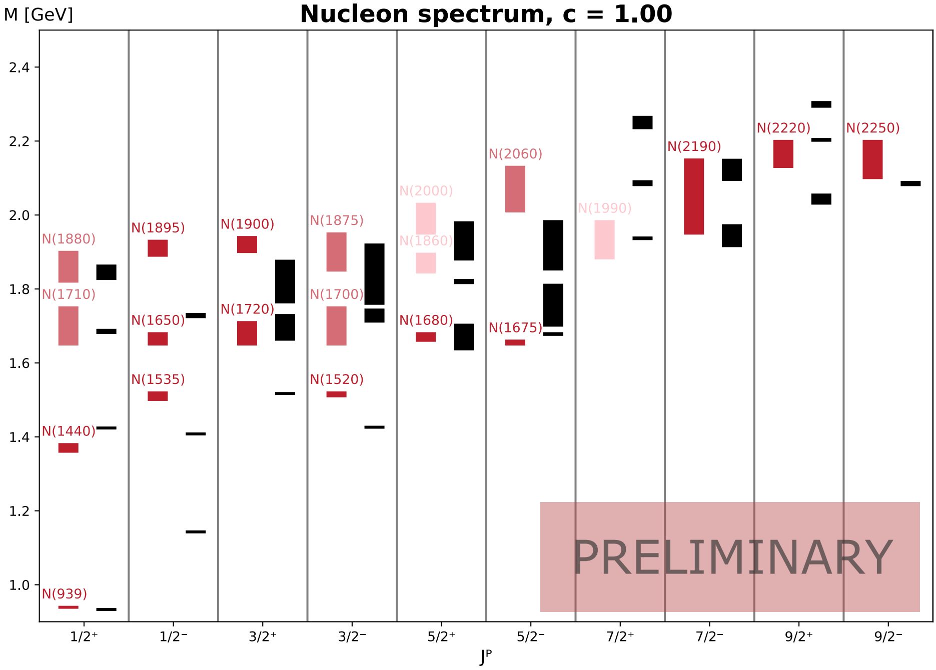
$$\Psi^{\mu..\mu_n},\ \mu..\mu_n \text{ traceless} \tag{5}$$

Results

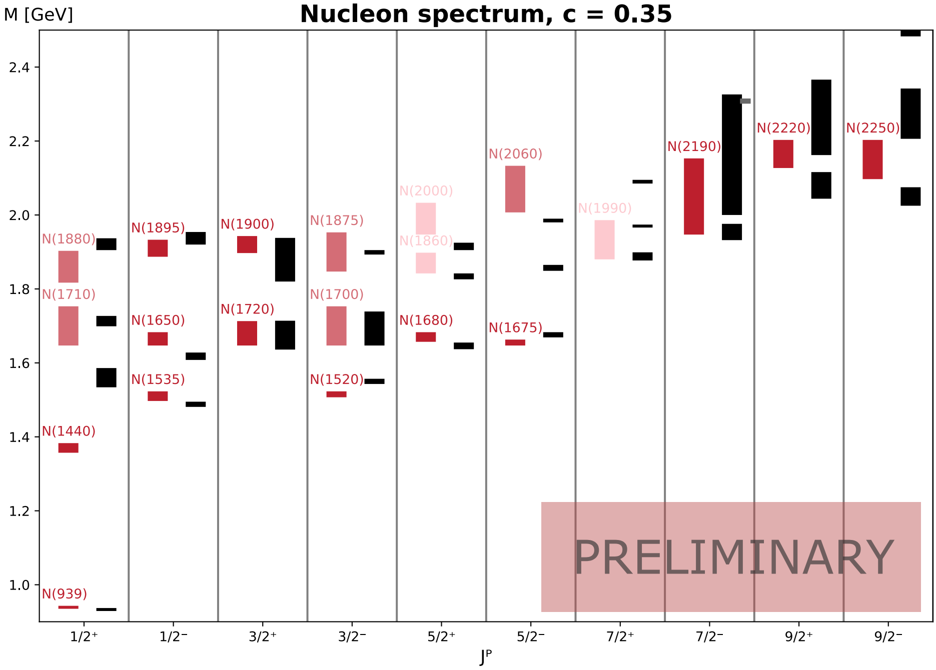
- Poles in the propagators limit the integration domain. We extrapolate beyond the threshold.
- Extrapolation of eigenvalue curves with linear, quadratic and cubic fits in least squares or Schlessinger-point-method [1] with random sampling [2].
- The choice of extrapolation method is given by the standard deviation and curvature. If the choice is ambiguous, the average is used.
- Factor c in front of pseudo-scalar and vector kernel reproduces beyond-rainbow-ladder effects.

[1] L. Schlessinger. *Physical Review*, 1968

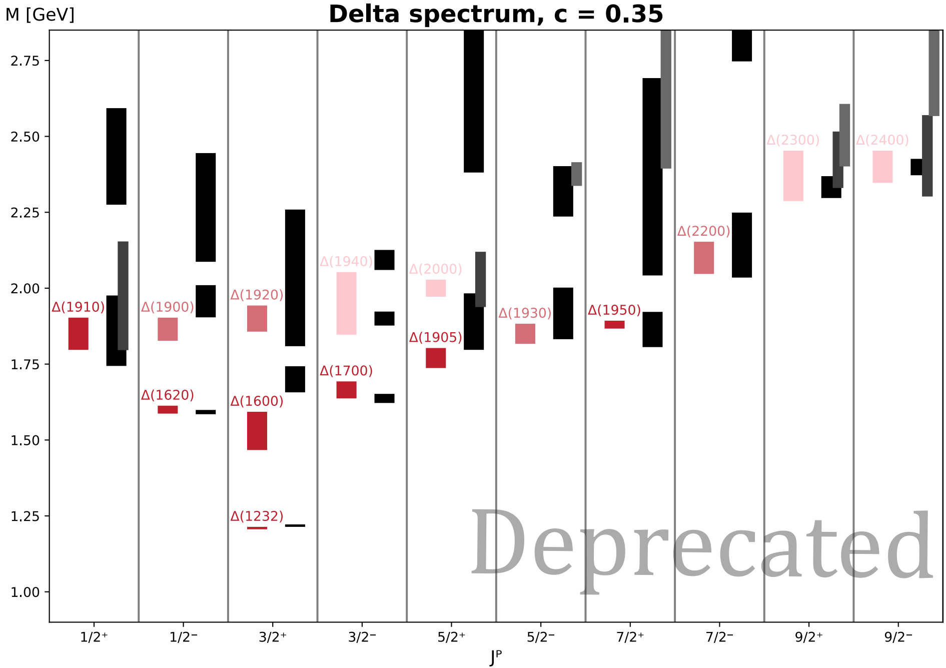
[2] S. Hagel et al. *The European Physical Journal A*, 2026



Luca Oberguggenberger



Luca Oberguggenberger



Delta spectrum, c = 0.35
M [GeV]
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
Δ(1910)
Δ(1900)
Δ(1620)
Δ(1920)
Δ(1600)
Δ(1232)
Δ(1940)
Δ(1700)
Δ(2000)
Δ(1905)
Δ(1930)
Δ(1950)
Δ(2200)
Δ(2300)
Δ(2400)
1/2⁺
1/2⁻
3/2⁺
3/2⁻
5/2⁺
5/2⁻
7/2⁺
7/2⁻
9/2⁺
9/2⁻
J^P
Deprecated

Summary and outlook

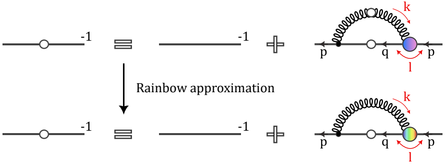
- Improve our diquark ingredients.
- Higher-spin baryons in three-quark picture, beyond rainbow-ladder.
- Expand our calculations to the heavy baryon sector.
- Implement contour deformations to avoid extrapolations.

Bonus Slides

1. Quark Propagator
2. Color-flavor matrices
3. Tensor parity
4. Spin projection
5. Rarita-Schwinger
6. Angular momentum operator
7. More

Dyson-Schwinger equations

DSEs encode the dynamical behavior of quarks and gluons. Rainbow-Ladder is a simplification with the effective coupling $\alpha(k^2)$:



$$S(p)^{-1} = S_0^{-1}(p) + \frac{16\pi}{3} Z_2^2 \int_q \frac{\alpha(k^2)}{k^2} T_k^{\mu\nu} \gamma^\mu S(q) \gamma^\nu$$

$$g^2 Z_\Gamma D^{\mu\nu}(k) \to 4\pi Z_2^2 \alpha(k) \frac{T^{\mu\nu}}{k^2},$$
$$\Gamma^\mu \to i\gamma^\mu.$$

$$\alpha(k) = \pi\eta^7 \frac{k^2}{\Lambda^2} e^{-\eta^2 x} + \text{UV}$$

$\alpha(k)$ from ansatz by Maris & Tandy[a]. Parameters set by Pion mass and decay constant, from its BSE.

[a] P. Maris and P. C. Tandy.

Physical Review, 1999

Tensor decomposition

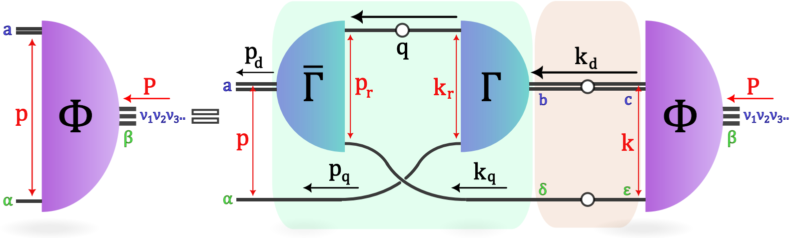


Figure: Quark-Diquark BSE.

- Expand BSA in tensor basis.
- $\Phi^{a;\mu_1..\mu_n} = \Phi_{\text{color}} \otimes \Phi_{\text{flavor}} \otimes \sum_k f_k(p^2, p \cdot P, ..)\{\sigma_k^{a;\mu_1..\mu_n}(p,P)\}.$
- $\sigma_i^{a,\mu_1..\mu_n}(p,P) = \underbrace{(\gamma^5)}_{\text{parity}} \underbrace{\tau_i^{a,\nu_1..\nu_n}(p,P)}_{\text{dynamic}} \underbrace{\Lambda^+(P)}_{\text{energy}} \underbrace{\mathcal{P}^{\nu_1..\nu_n;\mu_1..\mu_n}(P)}_{\text{spin}}.$

Tensor decomposition – Momenta

- $$\sigma_i^{a,\mu_1..\mu_n}(p,P) = \underbrace{(\gamma^5)}_{\text{parity}} \boxed{\underbrace{\tau_i^{a,\nu_1..\nu_n}(p,P)}_{\text{dynamic}}} \underbrace{\Lambda^+(P)}_{\text{energy}} \underbrace{\mathcal{P}^{\nu_1..\nu_n;\mu_1..\mu_n}(P)}_{\text{spin}}.$$

- Search orthonormal basis!

- Construct general basis from Dirac tensors $\{\mathbb{1}, \not r, \not P, \not p \not P\}$ with set of $\{\gamma_T^\mu, r^\mu, d^\mu\}$ for every index.

- Momentum products $r^\mu r^\nu..r^\omega \to \ell^{\mu\nu..\omega}$ are replaced by eigenstates of the orbital angular momentum operator L^2.

- Operators constrain basis to at most 10 elements for any spin.

Tensor decomposition – Parity

- The parity transformation is $\Lambda^{\mu\nu} = \mathrm{diag}(-1,-1,-1,1)$.
- Contracting the Bethe-Salpeter amplitude $\Phi^{a;\mu..\nu}$ with the Rarita-Schwinger spinor $u^{\mu..\nu}$ gives the baryon spinor $\Phi_{\alpha\beta\gamma}$.

$$u^{\mu_1..\mu_n}(p,s) = \sum_{M,s'} \left\langle n, M; \frac{1}{2}, s' \middle| n, \frac{1}{2}, n+\frac{1}{2}, s \right\rangle \epsilon_M^{\mu_1..\mu_n}(p) u(p,s').$$

- Parity behavior of $u^{\mu_1..\mu_n}$ translates to parity behavior of the baryon spinor and gives conditions for the basis,

$$\gamma^4 \mathbf{\Phi}^{0;\rho_1..\rho_n}(\Lambda p, \Lambda P)\Lambda^{\rho_1\nu_1}..\Lambda^{\rho_n\nu_n}\gamma^4 \stackrel{!}{=} (-1)^n \; \mathbf{\Phi}^{0;\nu_1..\nu_n}(p,P),$$

$$\gamma^4 \Lambda^{\mu\delta}\mathbf{\Phi}^{\delta;\rho_1..\rho_n}(\Lambda p, \Lambda P)\Lambda^{\rho_1\nu_1}..\Lambda^{\rho_n\nu_n}\gamma^4 \stackrel{!}{=} (-1)^{n+1} \; \mathbf{\Phi}^{\mu;\nu_1..\nu_n}(p,P).$$

- Append γ^5 matrices to have scalar and axialvector components transform the same.

Tensor decomposition – Angular momentum

- Defining $L^{\mu_1..\mu_\ell}(\ell) = \ell^{\mu_1..\mu_\ell}\ \mathbb{1}$ and $\not{L}^{\mu_1..\mu_{\ell-1}}(\ell) = \gamma^{\mu_\ell}\ell^{\mu_1..\mu_\ell}$;

orthogonal spin $n+\frac{1}{2}$ scalar and and axial basis:

$$\tau_1^{0;\nu..\nu_n} = b_1 L^{\nu_1..\nu_n}(n)$$
$$\tau_2^{0;\nu..\nu_n} = b_2 \not{L}^{\nu_1..\nu_n}(n+1)$$

$$\begin{aligned}
\tau_3^{\mu;\nu_1..\nu_n} &= b_3 \delta^{\mu\nu_1} L^{\nu_2..\nu_n}(n-1)\\
\tau_4^{\mu;\nu_1..\nu_n} &= b_4 \left[\delta^{\mu\nu_1}\not{L}^{\nu_2..\nu_n}(n) - \frac{2}{3}\gamma_T^{\mu} L^{\nu_1..\nu_n}(n)\right]\\
\tau_5^{\mu;\nu_1..\nu_n} &= b_5 d^{\mu} L^{\nu_1..\nu_n}(n)\\
\tau_6^{\mu;\nu_1..\nu_n} &= b_6 d^{\mu} L^{\nu_1..\nu_n}(n+1)\\
\tau_7^{\mu;\nu_1..\nu_n} &= b_7 \gamma_T^{\mu} L^{\nu_1..\nu_n}(n)\\
\tau_8^{\mu;\nu_1..\nu_n} &= b_8 \gamma_T^{\mu} L^{\nu_1..\nu_n}(n+1)\\
\tau_9^{\mu;\nu_1..\nu_n} &= b_9 \left[L^{\mu\nu_1..\nu_n}(n+1) - \frac{1}{3}\gamma_T^{\mu}\not{L}^{\nu_1..\nu_n}(n+1)\right]\\
\tau_{10}^{\mu;\nu_1..\nu_n} &= b_{10}\not{L}^{\mu\nu_1..\nu_n}(n+2).
\end{aligned}$$

Color and flavor

- $\Phi^{a;\mu_1..\mu_n} = \boxed{\Phi_{\text{color}} \otimes \Phi_{\text{flavor}}} \otimes \sum_k f_k(p^2, p{\cdot}P, ..)\{\sigma_k^{a;\mu_1..\mu_n}(p, P)\}$
- Include scalar 0^+, axialvector 1^+, pseudoscalar 0^-, and vector 1^- diquarks..
- With isospin symmetry $u = d$, extract color-flavor factors from su$(3)_c$ and su$(2)_f$. Matrix components correspond to diquarks in the kernel. Example with just sc. and av.:

$$[\Phi_{\text{color}} \otimes \Phi_{\text{flavor}}] \leftrightarrow \begin{pmatrix} SS & SA \\ AS & AA \end{pmatrix}$$

$$N : \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}, \qquad \Delta : \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}.$$

Spin projection operators

- Label relativistic states by eigenvalues of Poincaré invariants $P^\mu P^\mu$ and $\mathcal{W} = W^\mu W^\mu$.
- General projection operator[3]

$$\mathcal{P}(j) = \prod_{j' \neq j} \frac{\mathcal{W} + j'(j'+1)p^2}{-j(j+1)p^2 + j'(j'+1)p^2}$$

- $\mathcal{W}$ for each irreducible sector $\rightarrow$ hard to generalize.
- Highest spin only contained once within RS representation.

[3] A. Aurilia and H. Umezawa. *Physical Review*, 1969

Spin projection operators – Rarita-Schwinger

- $\Rightarrow$ Ansatz with $\delta^{\mu\nu}, \gamma^\mu, p^\mu$ from constraints[4].

Using $T_p^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{p^2} p^\mu p^\nu$ and $a_l^{(n)} = -\frac{1}{2l} a_{l-1}^{(n)} \frac{(n-2l+2)(n-2l+1)}{2n+1-2l}$, $\quad a_0^{(n)} = 1$, for even $j = n$,

$$\boxed{\begin{aligned}\mathcal{P}^{\mu_1 \dots \mu_n}_{\nu_1 \dots \nu_n}(j,p) = \left(\frac{1}{n!}\right)^2 \sum_{\tilde{P}(\mu),\tilde{P}(\nu)} \Bigg[& a_0^{(n)} \prod_{i=1}^{n} T_p^{\mu_i \nu_i} + a_1^{(n)} T_p^{\mu_1 \mu_2} T_p^{\nu_1 \nu_2} \prod_{i=3}^{n} T_p^{\mu_i \nu_i} + \dots \\ & + a_{\frac{n}{2}}^{(n)} T_p^{\mu_1 \mu_2} T_p^{\nu_1 \nu_2} \times \dots \times T_p^{\mu_{n-1} \mu_n} T_p^{\nu_{n-1} \nu_n} \Bigg],\end{aligned}}$$

for odd integer $j = n$

$$\boxed{\begin{aligned}\mathcal{P}^{\mu_1 \dots \mu_n}_{\nu_1 \dots \nu_n}(j,p) = \left(\frac{1}{n!}\right)^2 \sum_{\tilde{P}(\mu),\tilde{P}(\nu)} \Bigg[& a_0^{(n)} \prod_{i=1}^{n} T_p^{\mu_i \nu_i} + a_1^{(n)} T_p^{\mu_1 \mu_2} T_p^{\nu_1 \nu_2} \prod_{i=3}^{n} T_p^{\mu_i \nu_i} + \dots \\ & + a_{\frac{n-1}{2}}^{(n)} T_p^{\mu_1 \mu_2} T_p^{\nu_1 \nu_2} \times \dots \times T_p^{\mu_{n-2} \mu_{n-1}} T_p^{\nu_{n-2} \nu_{n-1}} T_p^{\mu_n \nu_n} \Bigg],\end{aligned}}$$

and, for half-integer spin $j = n - \frac{1}{2}$,

$$\boxed{\mathcal{P}^{\mu_1 \dots \mu_{n-1}}_{\nu_1 \dots \nu_{n-1}}(j,p) = \frac{n}{2n+1} \gamma^\mu \gamma^\nu \mathcal{P}^{\mu \mu_1 \dots \mu_{n-1}}_{\nu \nu_1 \dots \nu_{n-1}}\left(j + \frac{1}{2}, p\right).}$$

[4] C. Fronsdal. On the theory of higher spin fields. *Il Nuovo Cimento*, 1958

Rarita-Schwinger – structure

- Four vector $V^\mu \in \left(\frac{1}{2}, \frac{1}{2}\right)$.
- Dirac Bispinor $\xi_\delta \in \left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$.
- Construct general tensor
  $\Psi_\delta^{\mu_1..\mu_n} \in \left(\frac{1}{2}, \frac{1}{2}\right) \otimes .. \otimes \left(\frac{1}{2}, \frac{1}{2}\right) \otimes \left(\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)\right)$.
- $d(V^\mu) = 4$, $d(\psi_\delta) = 4 \Rightarrow 4^{(n+1)}$ components. Multiple spin sectors from decomposition into SO(3).
- Irreducible $\left(n + \frac{1}{2}, 0\right) \oplus \left(0, n + \frac{1}{2}\right)$ representation of spin $n + \frac{1}{2}$ with $d = 4n + 4$.

Angular momentum eigenstates

- $L^2 = 2r^\alpha \frac{\partial}{\partial r^\alpha} + (r^\alpha r^\beta - T_P^{\alpha\beta}) \frac{\partial}{\partial r^\alpha} \frac{\partial}{\partial r^\beta}$ with eigenstates

$$
\begin{aligned}
r^{\alpha_1} \times \ldots \times r^{\alpha_\ell} + a_{(1,\ell)} \quad & \big(T^{\alpha_1\alpha_2} r^{\alpha_3} \times \ldots \times r^{\alpha_\ell} + T^{\alpha_1\alpha_3} r^{\alpha_2} r^{\alpha_4} \times .. \\
& + \ldots + T^{\alpha_{\ell-1}\alpha_\ell} r^{\alpha_1} \times \ldots \times r^{\alpha_{\ell-2}} \big) \\
+ a_{(2,\ell)} \quad & \big(T^{\alpha_1\alpha_2} T^{\alpha_3\alpha_4} r^{\alpha_5} \times \ldots \times r^{\alpha_\ell} + \ldots \big) \\
+ \ldots &
\end{aligned}
$$

- The normalized coefficients are

$$a_{(k,\ell)} = -\frac{2a_{(k-1,\ell)}}{2k(1-2k+2\ell)}, \quad 1 \leq k \leq \text{floor}\left(\frac{\ell}{2}\right); \quad a_{(l,0)} = 1$$

On-shell decaying particles?

- Our "resonances" are bound states in that they have no hadronic decay widths.

- But we can actually still calculate decay widths from the residues of the transition form factors (for example from $N \Rightarrow \Delta$). Feeding these back into the BSE shifts the poles into the complex plane, however, the real parts of the masses remain basically unchanged[5].

- Beyond rainbow-ladder effects (meson-baryon dynamics) should, in principle, also shift our poles into the complex plane and should have an effect on our masses[6]. Their exact impact remains to be analysed.

---

[5] V. Mader et al. *Physical Review D*, 2011

[6] G. Eichmann et al. *Physical Review D*, 2016