What is Negative Coupling Field Theory?

Paul Romatschke
Institut für Theoretische Physik, TU Wien

Field Theory

$$S = \int dx \left[\frac{1}{2} \partial_\mu \phi \partial_\mu \phi + V(\phi) \right]$$

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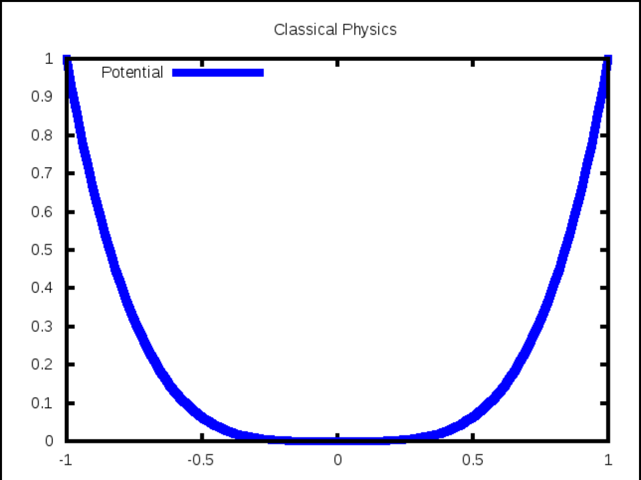
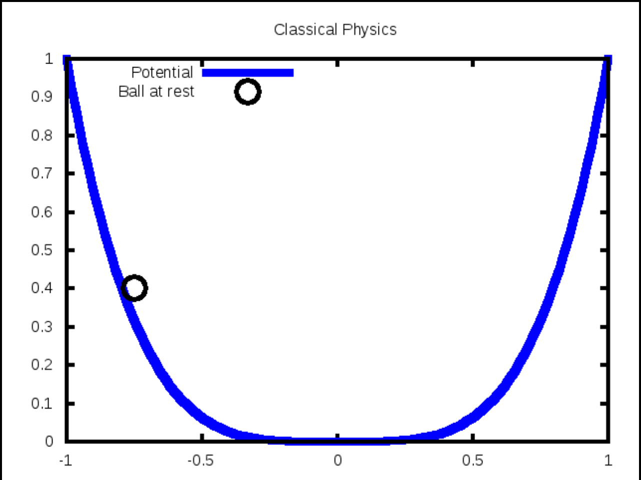
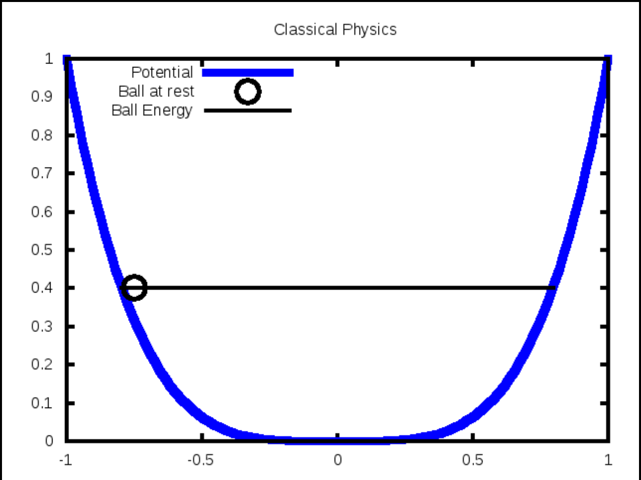
Textbook case: potential real and bounded

$$V(\phi) = m^2 \phi^2 + \lambda \phi^4, \quad \lambda > 0$$



Classical Physics
Potential
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.5
0
0.5
1


Classical Physics
Potential
Ball at rest
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.5
0
0.5
1


Classical Physics
Potential
Ball at rest
Ball Energy
1
0.9
0.8
0.7
0.6
0.5
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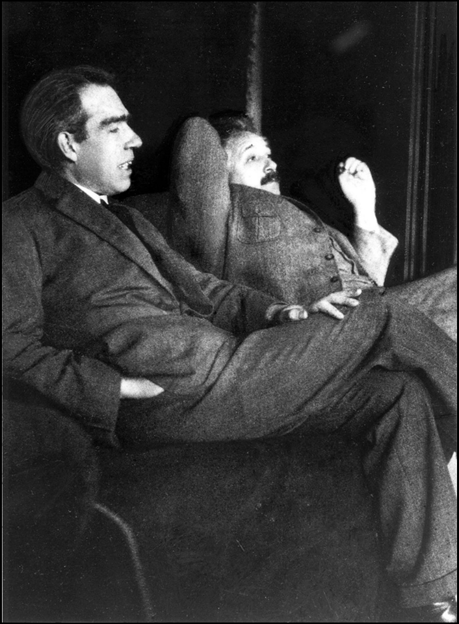
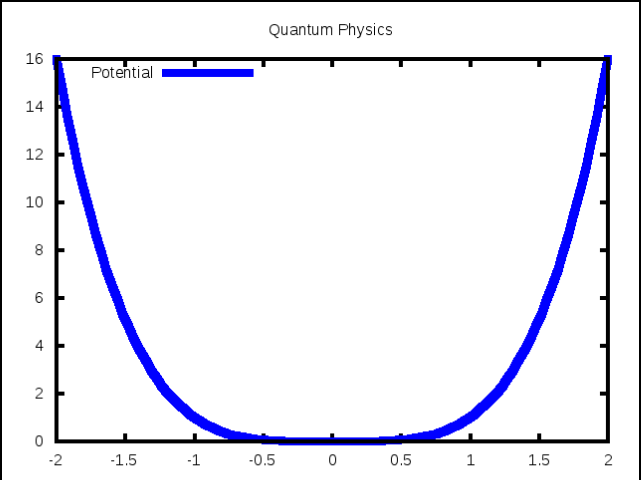
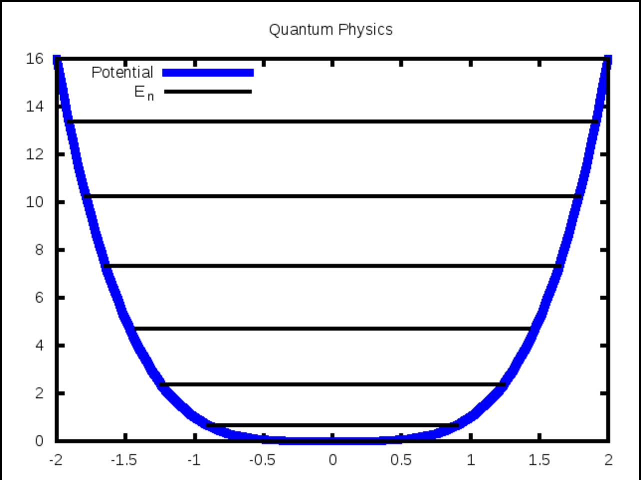
0.5
1



Classical is so 19th century...what about Quantum Mechanics????


Quantum Physics
Potential
16
14
12
10
8
6
4
2
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Quantum Physics
Potential
E_n
16
14
12
10
8
6
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Negative Coupling Field Theory

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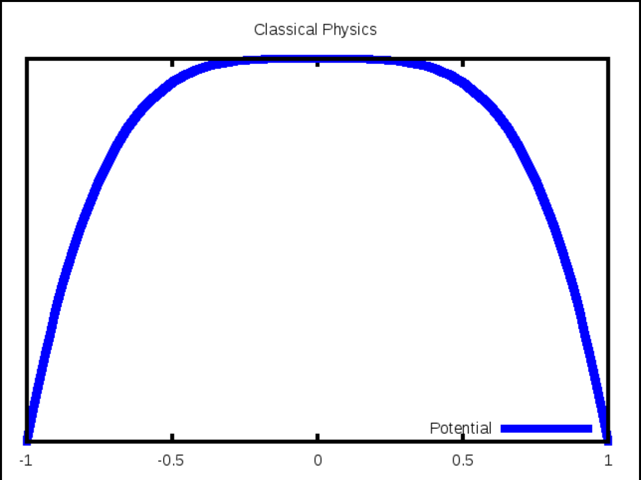
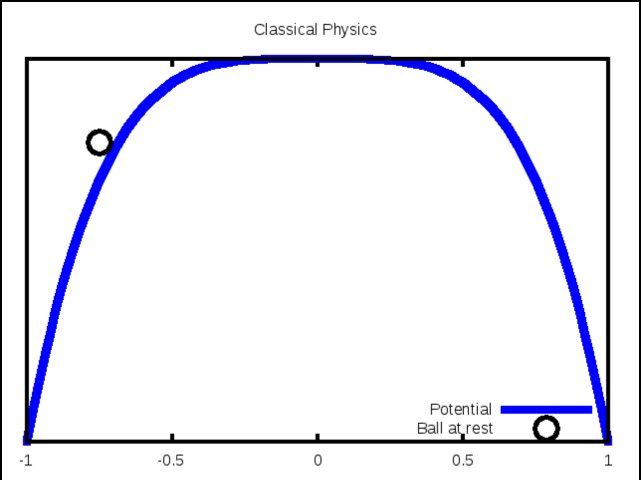
Negative Coupling

$$V(\phi) = m^2 \phi^2 - g\phi^4, \quad g > 0$$


Classical Physics
Potential
-1
-0.5
0
0.5
1



Classical Physics
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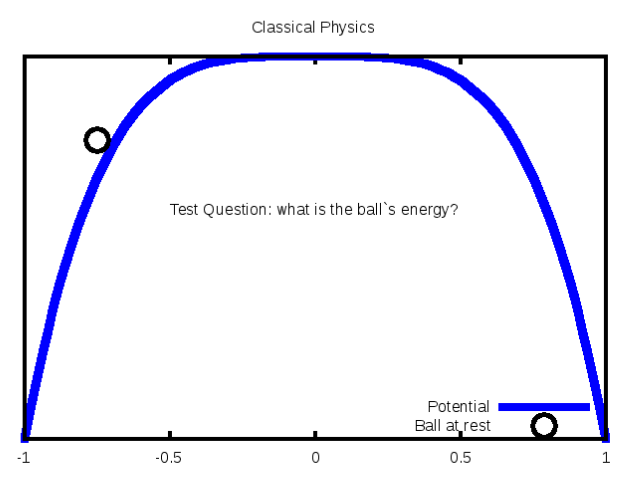
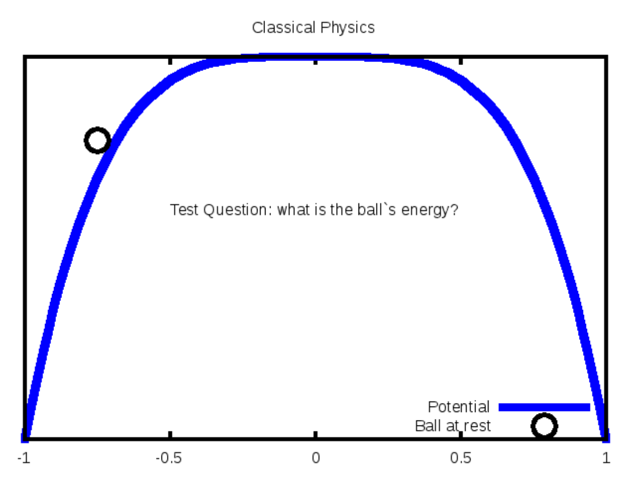
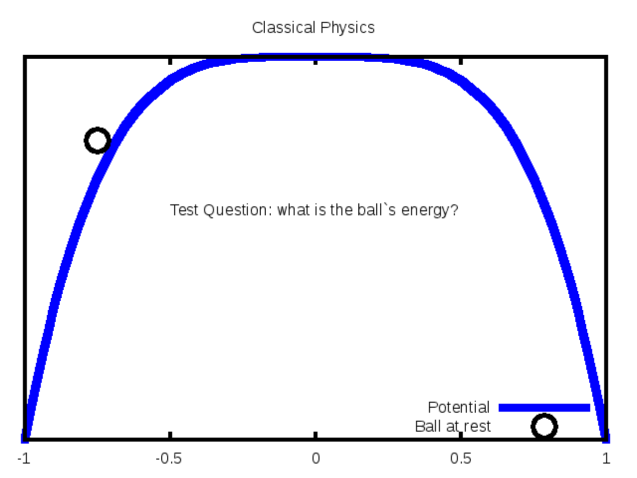
Ball at rest
-1
-0.5
0
0.5
1



Classical Physics
Test Question: what is the ball`s energy?
Potential
Ball at rest
-1
-0.5
0
0.5
1



In classical physics: $E_{\mathrm{ball}} = \infty$



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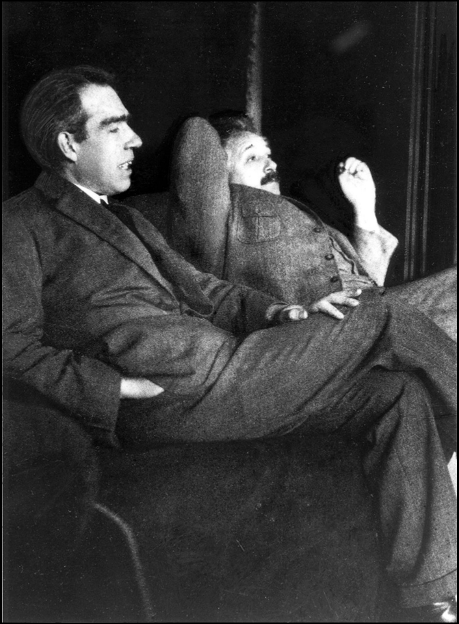
'Endless Hill' potentials make no sense in classical physics



Classical is so 19th century...what about Quantum Mechanics????

Consider

$$V(\phi) = -g\phi^4$$

in Quantum Mechanics

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$$V(\phi) = -g\phi^4$$

in Quantum Mechanics
(without any classical prejudice)

PHYSICAL REVIEW D **73,** 085002 (2006)

Equivalent Hermitian Hamiltonian for the non-Hermitian $-x^4$ potential

H. F. Jones
Physics Department, Imperial College, London SW7, United Kingdom

J. Mateo
Departamento de Física Teórica, Facultad de Ciencias, E-47011, Valladolid, Spain
(Received 2 February 2006; published 5 April 2006)

The potential $V(x) = -x^4$, which is unbounded below on the real line, can give rise to a well-posed bound state problem when x is taken on a contour in the lower-half complex plane. It is then PT-symmetric rather than Hermitian. Nonetheless it has been shown numerically to have a real spectrum, and a proof of reality, involving the correspondence between ordinary differential equations and integrable systems, was subsequently constructed for the general class of potentials $-(ix)^N$. For such Hamiltonians the natural PT metric is not positive definite, but a dynamically-defined positive-definite metric can be defined, depending on an operator Q. Further, with the help of this operator an equivalent Hermitian

$V(x) = -gx^4$ isospectral to $V(x) = 4gx^4 - \sqrt{2g}x$

Known since 1997: "$\mathcal{PT}$-symmetric Quantum Mechanics"

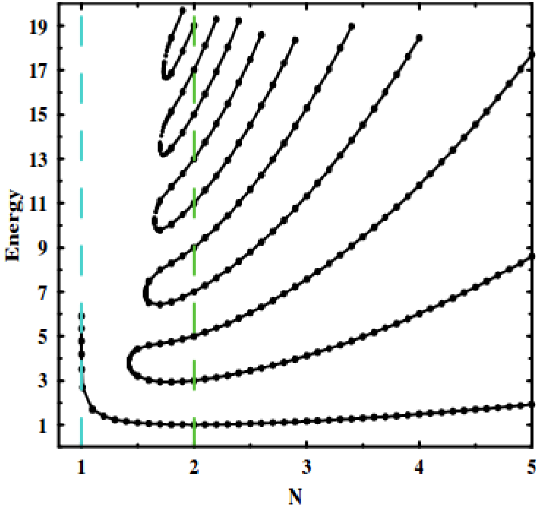

FIG. 1. Energy levels of the Hamiltonian $H = p^2 - (ix)^N$ as a function of the parameter N. There are three regions:

[Bender & Böttcher, 1997]

What does $\mathcal{PT}$ symmetry stand for?

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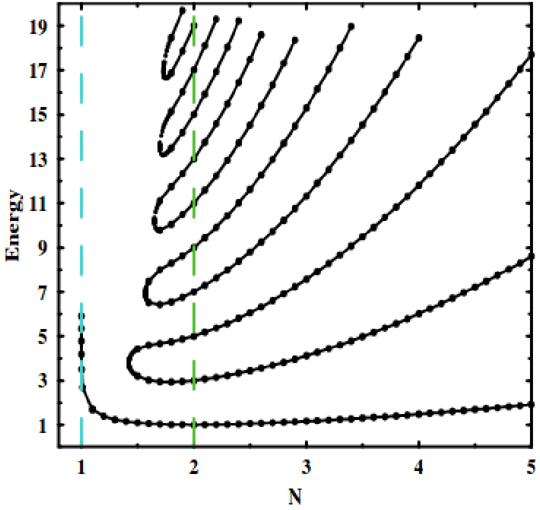
***P**utin **T**rump*

Discovered by accident



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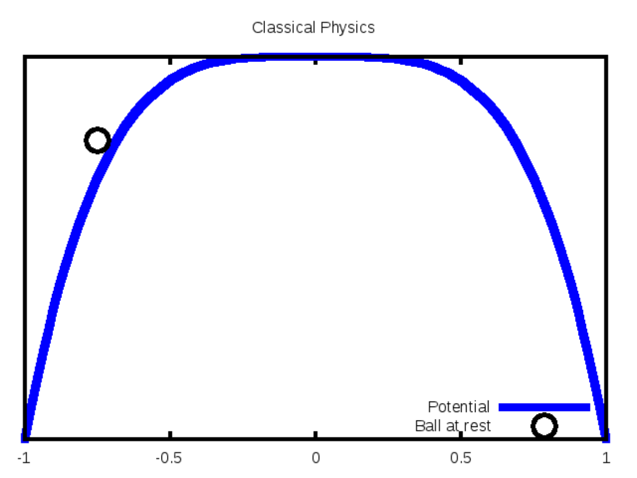
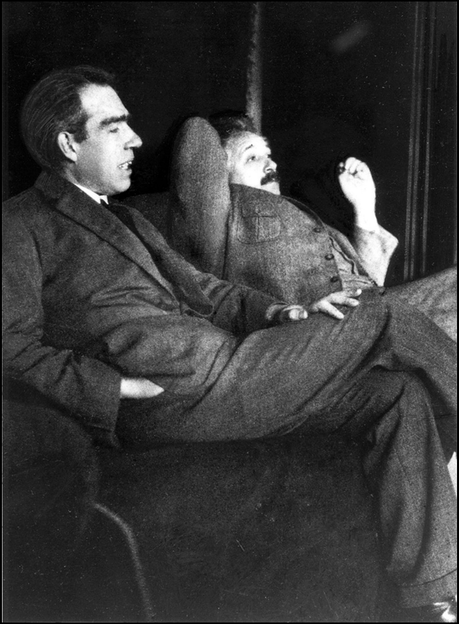
[Bender & Böttcher, 1997]

No stable *classical* ground state!





Purely Quantum Phenomenon!

Can be tested in AMO experiments, e.g. at TU Wien: Schmiedmayer group

Quantum Field Theory for Higgs Field

$$Z = \int \mathcal{D}\phi e^{-S_E}\,, \quad S_E = \int_x \left[\frac{1}{2}\partial_\mu\phi \cdot \partial_\mu\phi - \frac{1}{2}m^2\phi^2 + \lambda\phi^4\right]$$

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"Normal" solution approach:

- small perturbations around classical theory ($\lambda \ll 1$)
- Systematically improvable, phenomenologically successful
- Requires classical ground state to be stable

Investigating the near-criticality of the Higgs boson

Dario Buttazzo[a,b], Giuseppe Degrassi[c], Pier Paolo Giardino[a,d],
Gian F. Giudice[a], Filippo Sala[b,e], Alberto Salvio[b,f],
Alessandro Strumia[d]

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unexpected feature of the SM that opens new avenues for theoretical speculation. The intriguing result is that, assuming the validity of the SM up to very high energy scales, the measured value of M_h is near-critical, in the sense that it places the EW vacuum right at the border between absolute stability and metastability. Because of the present experimental uncertainties on the SM parameters (mostly the top quark mass), we cannot conclusively establish the fate of the EW vacuum, although metastability is now preferred at 99.3% CL.

Classical ground state is **not** stable

Is the Higgs Field unstable? | Vacuum Decay

Negative Coupling To the Rescue!

Negative Coupling To the Rescue!

No time for details, but Dec 2024 colloquium recording available:

Link to Recording

... and now some thoughts about HEP theory in Europe...

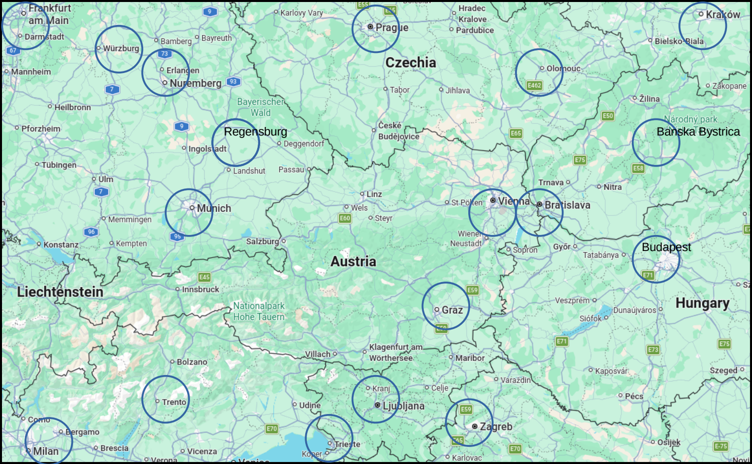
HEP Theory in Austria

universität wien

TU WIEN TECHNISCHE UNIVERSITÄT WIEN

MBI Marietta Blau Institute for Particle Physics

UNI GRAZ


Frankfurt am Main
Darmstadt
Mannheim
Würzburg
Bamberg
Bayreuth
Erlangen
Nuremberg
Karlovy Vary
Prague
Hradec Kralove
Pardubice
Czechia
Tabor
Jihlava
Olomouc
Kraków
Bielsko-Biala
Zakopane
Žilina
Heilbronn
Pforzheim
Tübingen
Ulm
Bayerischer Wald
Regensburg
Deggendorf
Ingolstadt
Landshut
Passau
České Budějovice
Banska Bystrica
Národný park
Trnava
Nitra
Linz
Wels
Steyr
St Pölten
Vienna
Bratislava
Munich
Memmingen
Kempten
Konstanz
Salzburg
Wiener Neustadt
Sopron
Győr
Tatabánya
Budapest
Austria
Liechtenstein
Innsbruck
Nationalpark Hohe Tauern
Graz
Veszprém
Dunaújváros
Siófok
Hungary
Villach
Klagenfurt am Wörthersee
Maribor
Bolzano
Kaposvár
Szeged
Varaždin
Celje
Kranj
Pécs
Trento
Udine
Ljubljana
Zagreb
Como
Bergamo
Milan
Brescia
Vicenza
Verona
Trieste
Koper
Osijek
Karlovac

Central European Supercluster for High Energy Theory
(CENTAURUS)

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- Virtual collaboration of HEP theory groups

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- Identify common goals/strategy
- Offer common teaching for special topics
- Mentioned as “very good idea” by Restricted European Committee for Future Accelerators (RECFA Austria visit, Sep 2026)

Want to know more? Contact: centaurus@quantengesellschaft.at

Summary and Conclusions

- Quantum mechanics needs less symmetry than often assumed

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- Same Standard Model theory allows for new physical solutions
- Particularly important for cosmology/Higgs
- Policy level: push to set up European network of HEP theory departments

Thank You

Path Integral Perspective – Contour Deformations

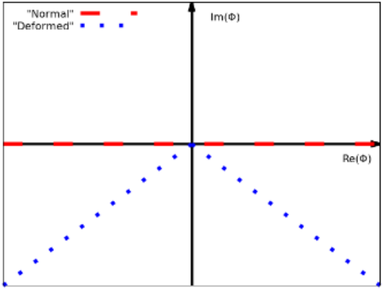

Figure 1: Contour deformation for integrations in the complex domain of $\phi(x)$. "Normal" integration for $\phi(x)$ is for along the real line, but the "Deformed" integration contour is along two segments at an angle to the real line.

Negative Coupling ϕ^4 on the Lattice

Paul Romatschke*

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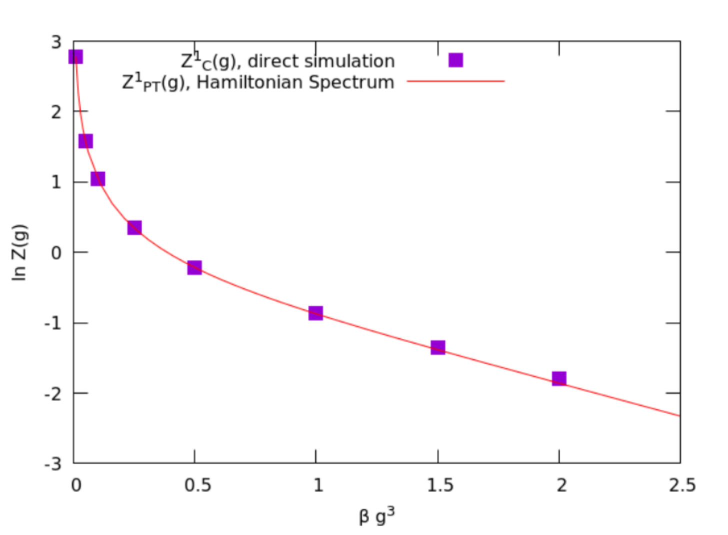
Center for Theory of Quantum Matter, University of Colorado, Boulder, Colorado 80309, USA

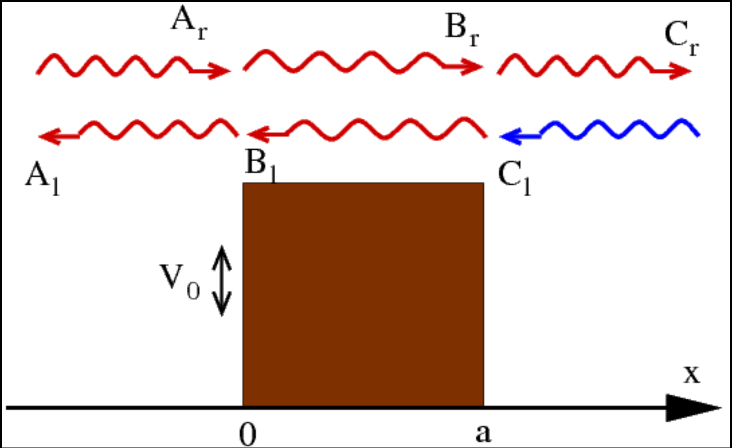
E-mail: paul.romatschke@colorado.edu

Triviality of ϕ^4 theory in four dimensions can be avoided if the bare coupling constant is negative in the UV. Theories with negative coupling can be put on the lattice if the integration domain for $\phi(x)$ is contour-deformed from the real to the complex domain. In 0+1d (quantum mechanics), one can recover results from $\mathcal{PT}$-symmetric quantum mechanics in this way. In this work, I report on an attempt to put negative coupling ϕ^4 theory in 4 dimensions on the lattice.

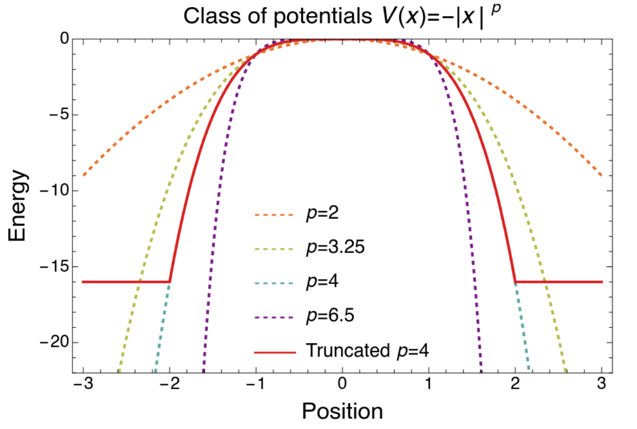
Negative Coupling Quantum Mechanics



Physics Perspective: Scattering

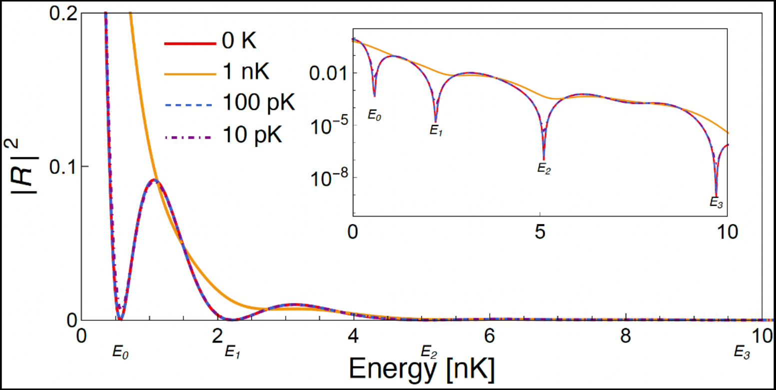


Special Potential Barriers: "Upside-Down" Potentials



[Micheline B. Soley, C. M. Bender, A. D. Stone, PRL (2023)]

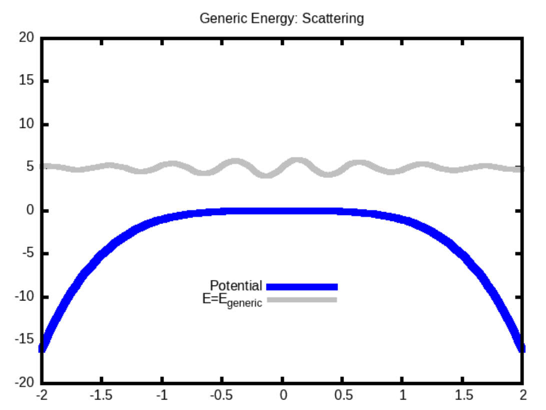
Theory prediction: reflectionless scattering for some energies!



Scattering on quasi 1d truncated $V(x) = -x^4$

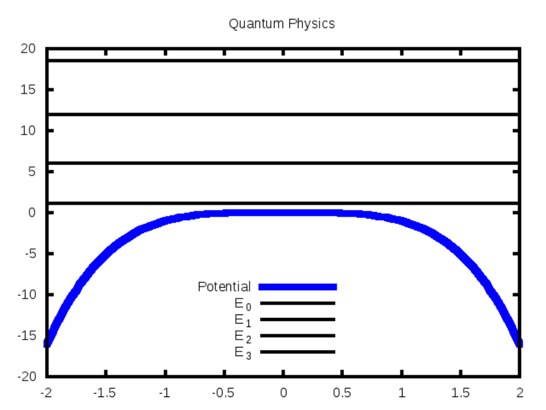
[Micheline B. Soley, C. M. Bender, A. D. Stone, PRL (2023)]

What does reflectionless scattering mean?



- Sending in wavepacket $\psi(x)$ with generic $E > 0$ will scatter

What does reflectionless scattering mean?



- For *special* values $E = E_n$, no scattering at all!

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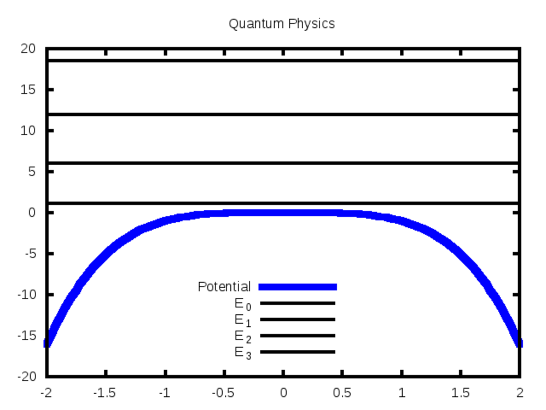

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For $E = E_n > 0$ $\psi_n(x)$ is exact solution of $\mathcal{H}\psi = E\psi$