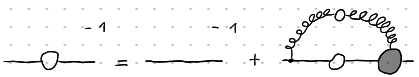
The analytic properties of the Quark propagator for timelike momenta



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Institute of Physics, University of Graz



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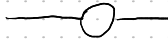
- Quark Propagator as Basic Building Block
- Computing the Propagator
- Results
- Outlook

Quark Propagator as Basic Building Block



free propagator:

- spherical wave, starting at x
- measured at y
- used to build more complex processes

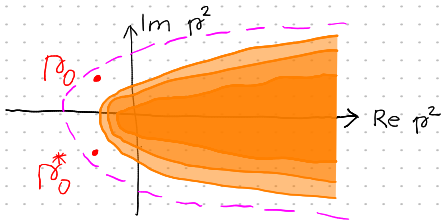


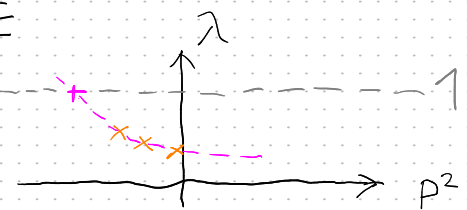
dressed propagator

- includes all possible interactions occuring between x and y
- effect of interactions absorbed in the dressing functions

Quark Propagator as Basic Building Block

- full propagator usually computed and used in momentum space
- use case: input for BSE





Computing the quark propagator

Quark propagator: $S(p) = -i\not{p}\,\sigma_V(p^2) + \mathbb{1}\,\sigma_S(p^2)$

tensor structures — dressing functions

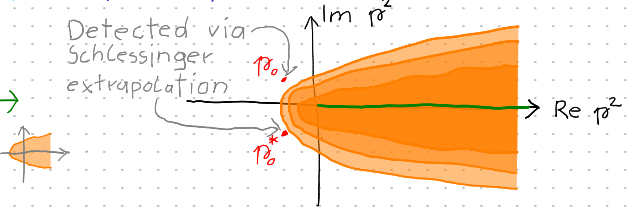
Compute $S(p)$ via Quark DSE

$S^{-1}(p) = S_0^{-1}(p) + \Sigma(p)$

dressed quark — free quark — self energy

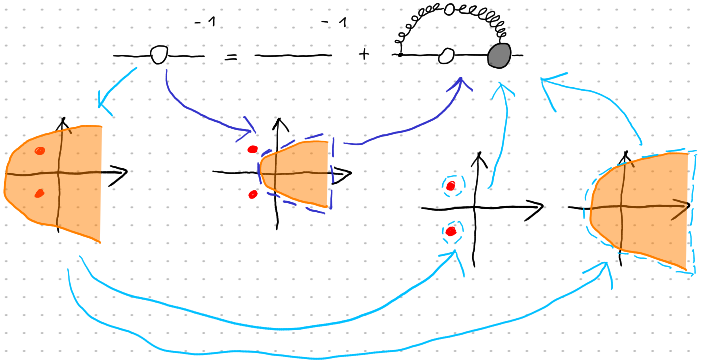
Solution domain:

- positive real axis →
- complex parabolas



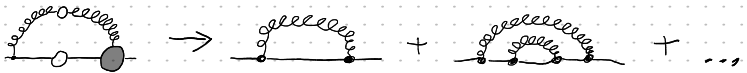
Computing the quark propagator

Sketch of iteration cycle



Computing the quark propagator

- rainbow ladder truncation



- Maris-Tandy interaction (k Gluon momentum)

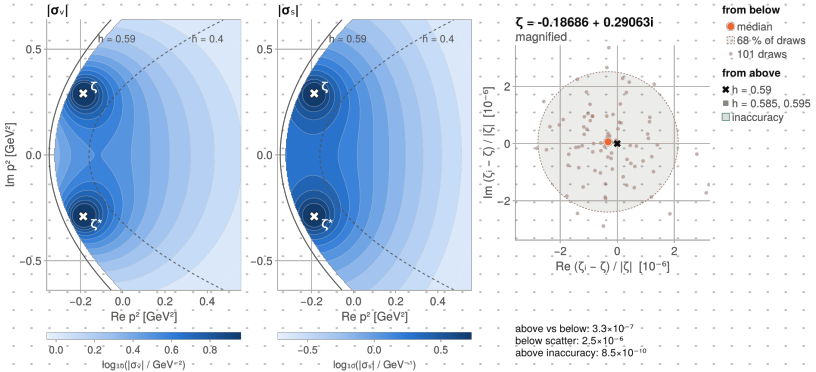
$$\alpha(k^2) = \text{IR-Term} + \text{UV-Term}$$

$$\text{IR-Term} \sim x^2 \exp(-\eta x^2) \quad , \quad x^2 = \frac{k^2}{\Lambda^2}$$

$$\text{UV-Term} \sim 1 / \ln[\dots]$$

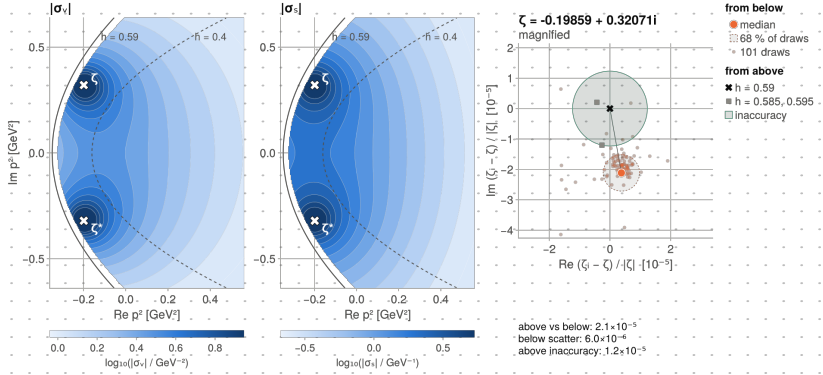
Results

σ_v and σ_s in the complex p^2 plane — infrared term only; parabolas Im $\sqrt{p^2}$ = h [GeV]



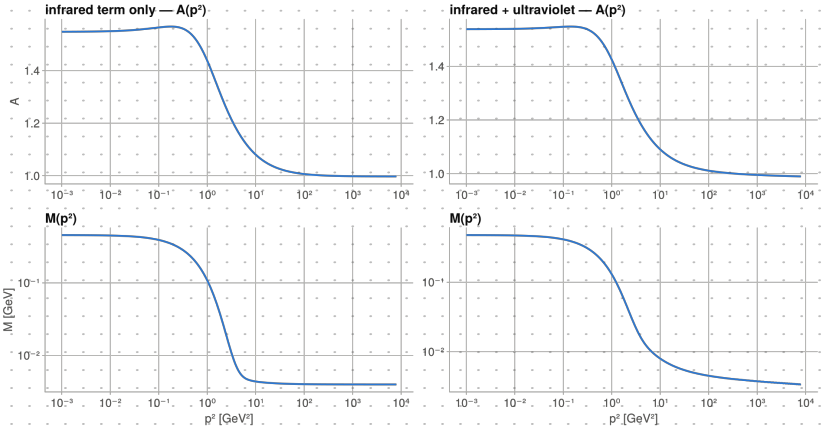
Results

σᵥ and σₛ in the complex p² plane — infrared + ultraviolet; parabolas Im √p² = h [GeV]



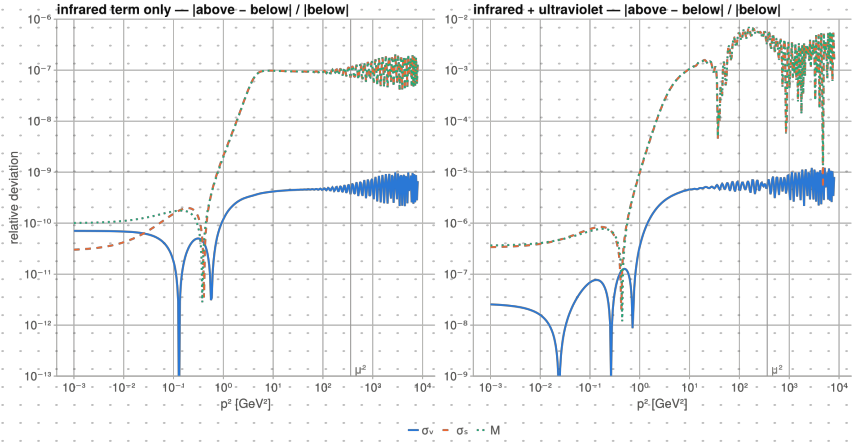
Results

Quark dressing functions above the poles, on the real axis



Results

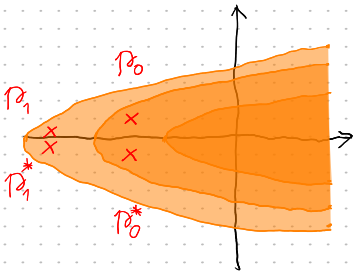
Above the poles against below the poles: relative deviation



Summary & Outlook

Solution procedure

1. solve Quark DSE on  ✓
2. find nearest poles ✓
3. extend  beyond poles ✓



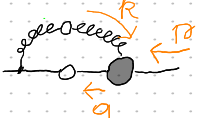
Change model input

- 
- do poles change?
  - 3. is nice, ~~but optional~~

Rigidity of solution domain

Solution domain emerges from

- momentum conservation

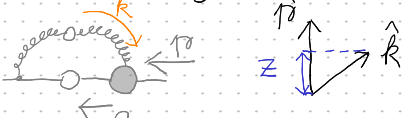


$q = p + k$: Vector form

$q^2 = p^2 + k^2 + 2\underbrace{\sqrt{p^2}\sqrt{k^2}}_{p \cdot k} z$

: Invariant form

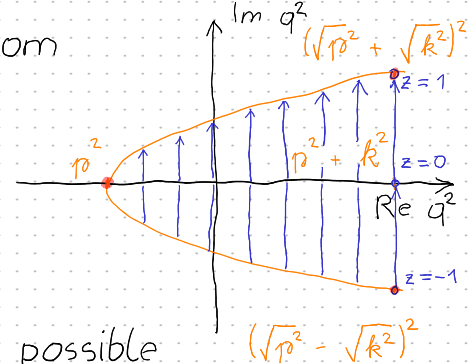
- loop integration



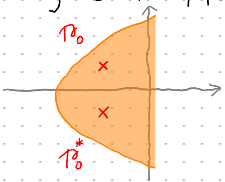
$$\int d^4k \propto \int_0^\infty dk^2 k^2 \int_{-1}^{1} dz \sqrt{1-z^2}$$

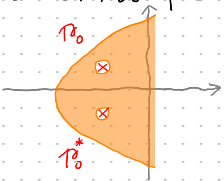
Rigidity of solution domain

Solution domain emerges from
- momentum conservation
- loop integration



Rigidity of solution domain
- pole evasion challenging
- „cutting out" tiny holes still possible

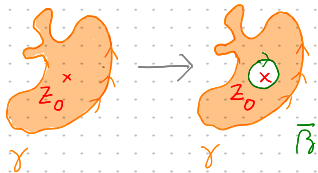




Modification of solution domain

„cutting out" tiny holes

$$\oint_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta \to \oint_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta - \oint_\beta \frac{f(\zeta)}{\zeta - z}\,d\zeta$$



$$= \oint_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta - 2\pi i \operatorname*{Res}_{\zeta = z_0} \frac{f(\zeta)}{\zeta - z}$$

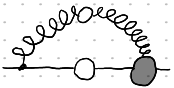
$$= \oint_\gamma \frac{f(\zeta)}{\zeta - z}\,d\zeta - 2\pi i \frac{\operatorname*{Res}_{\zeta = z_0} f(\zeta)}{z_0 - z} + \left(\begin{array}{l}\text{other terms if}\\ f \text{ has higher poles}\\ \text{at } z_0\end{array}\right)$$

Recap: From poles to cuts

„cutting out" tiny holes from  at pole position

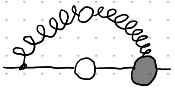
$\Longleftrightarrow$ adding pole terms to the Cauchy formula

$$\sigma_{v,s}(p^2) = \frac{1}{2\pi i} \oint \frac{\sigma_{v,s}(s)}{s - p^2}\, ds + \frac{r_{v,s}}{p^2 - p_0^2} + \frac{r_{v,s}^*}{p^2 - (p_0^*)^2}$$

$\Longrightarrow$ poles enter , which enters 

$\Longrightarrow$ { momentum conservation  } strike again
{ loop integration  }

From poles to cuts

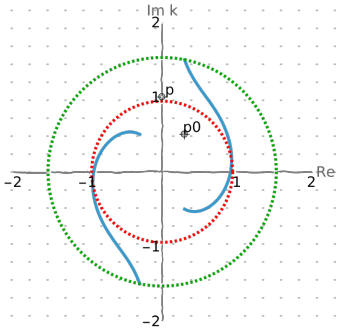
Pole $\frac{r_{v,s}}{p^2 - p_0^2}$ appears in  via

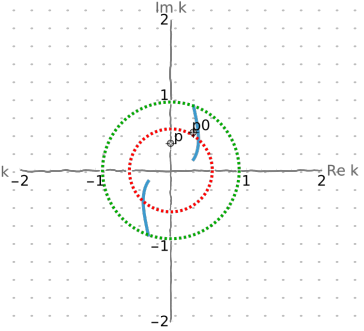
$$\int_0^\infty dk^2 \int_{-1}^{1} dz\,(\ldots)\,\frac{r_{v,s}}{p^2 + k^2 + 2pkz - p_0^2}$$

momentum conservation

loop integration

Visualize cut in k-plane $(k \equiv \sqrt{k^2})$





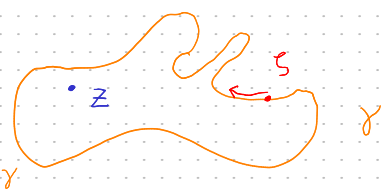
Cauchy formula

Solution domain

- positive real axis →
  - no interpolation necessary
- complex parabolas 
  - compute propagator on boundary
  - interpolate via Cauchy formula



$$f(z) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(\zeta)}{\zeta - z} \, d\zeta$$



Contour deformations

Cut parametrized by

- pole position p_0
- external momentum p

p_0 separates p-plane in 3 different regions for contour deformation

(I) no deformation necessary
(II) deformation towards $-p$
(III) deformation towards p

