Infrared Quark–Gluon Vertex and Pion Structure in Minkowski Space

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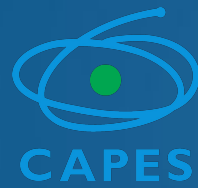
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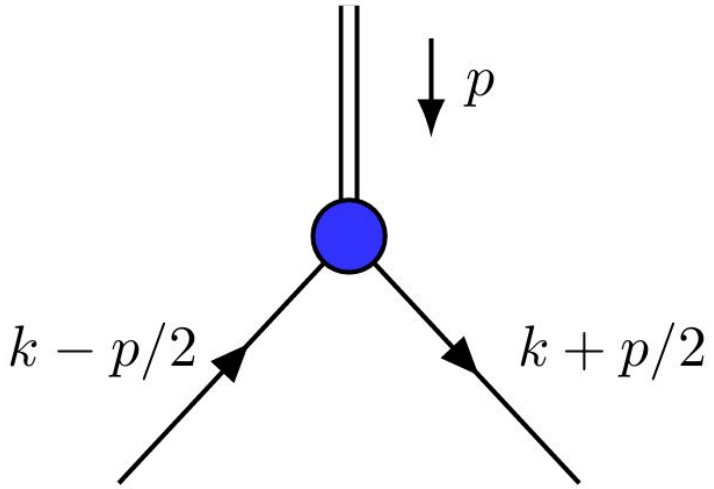


Pion as a Bound State

- The pion is the lightest hadron and is recognized as the Goldstone boson associated with spontaneous chiral symmetry breaking (SCSB)

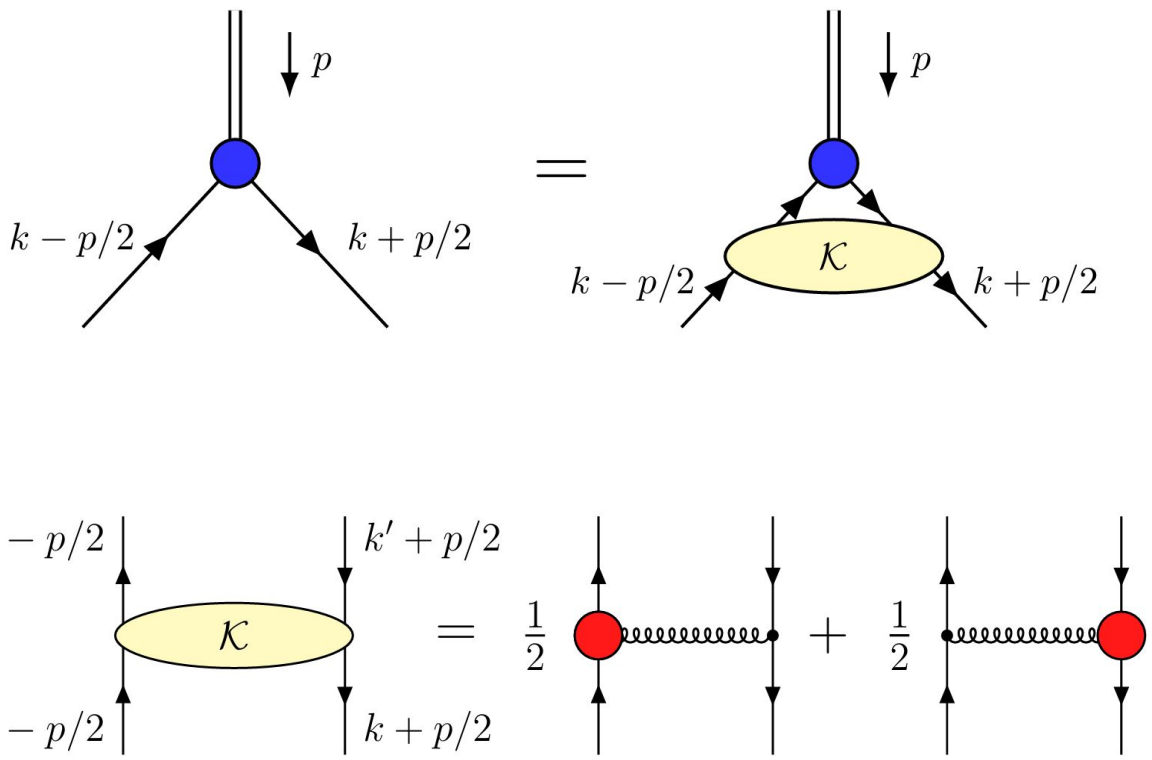
- Fundamentally a quark–antiquark state ($J^P = 0^-$) in a color singlet.

$$\Psi_\pi(k,p) = S(k+p/2)\Gamma_\pi(k,p)S(k-p/2) \quad (2)$$



$$\Psi_\pi(k,p) = \int d^4x\, e^{ik\cdot x} \left\langle 0 \left| T\left\{ q\left(\frac{x}{2}\right) \bar{q}\left(-\frac{x}{2}\right)\right\}\right| \pi(p)\right\rangle \quad (1)$$

Pion Bethe-Salpeter Equation (BSE)



Pion Vertex BSE:

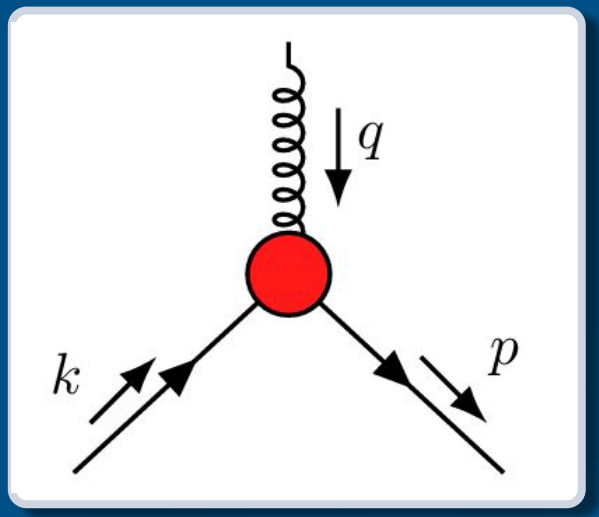
$$\Gamma_\pi(k,p) = \frac{-ig}{2}\int \frac{d^4k'}{(2\pi)^4} D^{\mu\nu}(k'-k)$$

$$\times\left[\Gamma_\mu\left(k+\frac{p}{2},k-k'\right) S\left(k'+\frac{p}{2}\right)\Gamma_\pi(k',p)\, S\left(k'-\frac{p}{2}\right)\gamma_\nu\right.$$

$$\left.+\gamma_\mu\, S\left(k'+\frac{p}{2}\right)\Gamma_\pi(k',p)\, S\left(k'-\frac{p}{2}\right)\Gamma_\nu\left(k'-\frac{p}{2},k'-k\right)\right] \quad (3)$$

Dressed Quark-Gluon Vertex

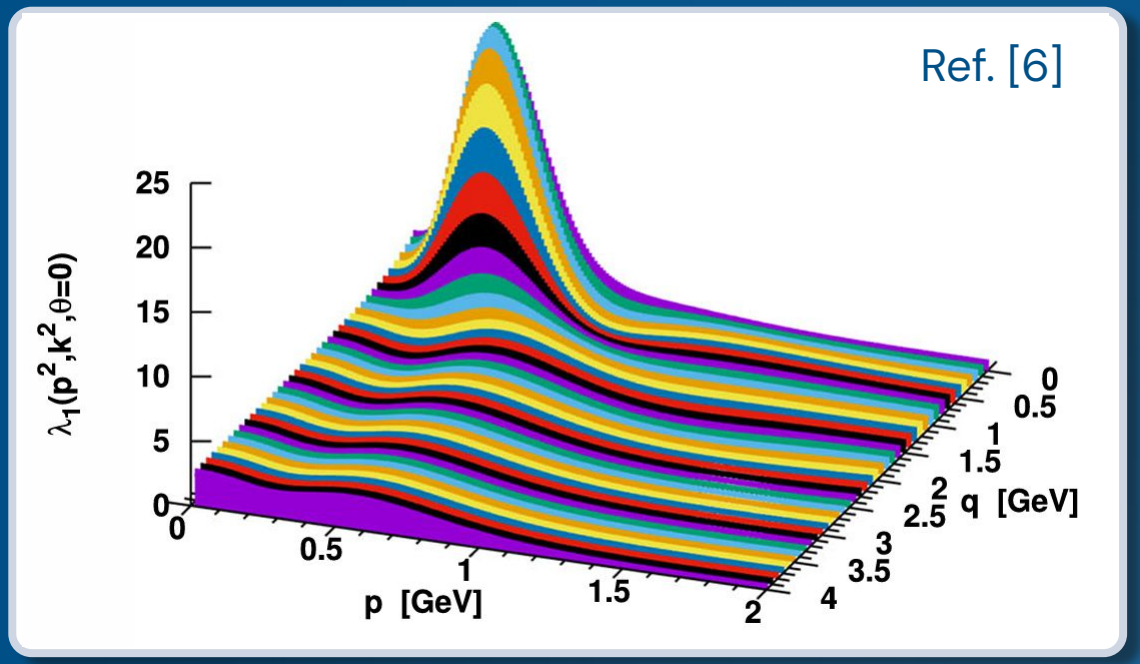
- An analytical ansatz inspired by the combined analysis of the Schwinger–Dyson equations (SDE) and Lattice-QCD (LQCD) [6].
- Modeling the first component of the Ball-Chiu longitudinal form factor [4].



(4) $$\Gamma_\mu(p,k,q) = \Gamma_\mu^{(L)}(p,k,q) + \Gamma_\mu^{(T)}(p,k,q)$$

(5) $$q^\mu \Gamma_\mu^{(T)}(p,k,q) = 0$$

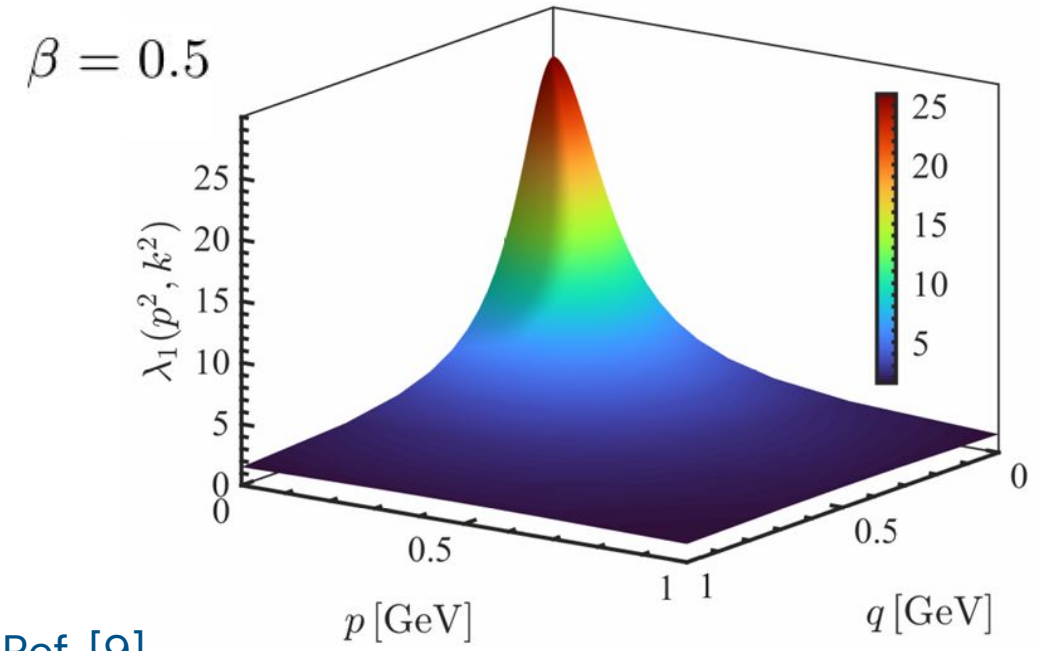
(6) $$\Gamma_\mu^{(L)}(p,k,q) = -\imath g \Big(\lambda_1 \gamma_\mu - \lambda_2 (\not{p} + \not{k})(p+k)_\mu - \lambda_3 (p+k)_\mu + \lambda_4 \imath \sigma_{\mu\nu} (p+k)^\nu \Big)$$



Ansatz for the quark-gluon Vertex

$$\lambda_1(p^2, k^2) = \frac{\beta\ \Lambda}{(p^2 + k^2 + q^2 + \varrho^2)} + 1 \qquad (7)$$

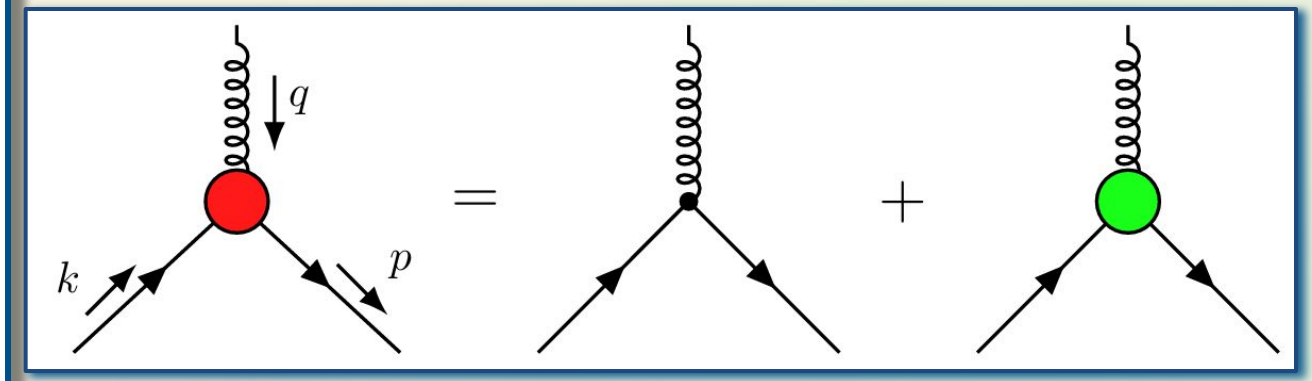
where $\varrho = 0.2\,\mathrm{GeV}$, $\Lambda = 2.0\,\mathrm{GeV}^2$ and the dimensionless parameter $\beta \in [0,\ 1]$ controls the IR enhancement.



Ref. [9]

Minkowski Space:

$$\Gamma_\mu(p, q) = \left(\frac{-\beta\ \Lambda}{p^2 + k^2 + q^2 - \varrho^2 + \imath\varepsilon} + 1\right)(-\imath g\gamma_\mu) \qquad (8)$$



Dressed Quark Propagator

- General form of the dressed quark propagator [3]:

$$S(p) = \imath\, Z(p^2)\, \frac{\not{p} + \mathcal{M}(p^2)}{p^2 - \mathcal{M}^2(p^2) + \imath\varepsilon} \tag{9}$$

where $Z(p^2)$ is the wavefunction renormalization and $\mathcal{M}(p^2)$ the dressed quark-mass.

- Mass function fitted to Landau gauge LQCD results [1]:

$$\mathcal{M}(p^2) = m_0 - \frac{m^3}{p^2 - \lambda^2 + \imath\varepsilon} \tag{10}$$

with $Z(p^2) = 1$ and parameters from Ref. [1] are given by $m_0 = 0.008$ GeV, $m = 0.648$ GeV and $\lambda = 0.900$ GeV.

Dressed Quark Propagator

$$S(p) = \imath\, Z(p^2)\, \frac{\not{p} + \mathcal{M}(p^2)}{p^2 - \mathcal{M}^2(p^2) + \imath\varepsilon}$$

$$\mathcal{M}(p^2) = m_0 - \frac{m^3}{p^2 - \lambda^2 + \imath\varepsilon}$$

- Vector and Scalar components:

$$S(p) \equiv \imath\Big(\not{p} S_v(p^2) + S_s(p^2)\Big) = \imath\Big(\not{p}\mathcal{A}(p^2) - \mathcal{B}(p^2)\Big)^{-1}$$

where $\mathcal{A}(p^2) = 1/Z(p^2)$, $\mathcal{B}(p^2) = \mathcal{M}(p^2)/Z(p^2)$ **(11)**

- Decomposing in partial fractions:

$$S_v(p^2) = \sum_{j=1}^{3} \frac{A_j}{p^2 - m_j^2 + \imath\varepsilon}, \quad S_s(p^2) = \sum_{j=1}^{3} \frac{B_j}{p^2 - m_j^2 + \imath\varepsilon} \qquad \textbf{(12)}$$

$$A_j = \frac{(m_j^2 - \lambda^2)^2}{(m_j^2 - m_k^2)(m_j^2 - m_l^2)}, B_j = m_0 A_j - \frac{m^3(m_j^2 - \lambda^2)}{(m_j^2 - m_k^2)(m_j^2 - m_l^2)}$$

with $j \neq k \neq l$ and $m_j(m_j^2 - \lambda^2) = \pm[m_0(m_j^2 - \lambda^2) - m^3]$

TABLE 1: Poles and Residues for the fit

j	m_j[GeV]	A_j	B_j[GeV]
1	0.469	3.7784	1.7742
2	0.573	-2.8863	-1.6546
3	1.035	0.1079	-0.1115

Gluon Propagator

- Gluon propagator in arbitrary covariant gauge implemented with a massive vector boson model:

$$D^{\mu\nu}(q) = \frac{-\imath}{q^2 - \mu^2 + \imath\varepsilon}\left(g^{\mu\nu} - \frac{(1-\zeta)q^\mu q^\nu}{q^2 - \zeta\mu^2 + \imath\varepsilon}\right) \quad (13)$$

where μ is the gluon mass and the parameter ζ labels the gauge.

$\zeta = 1$ Feynman gauge

$\zeta = 0$ Landau gauge

Integral Representation

- Decomposition of the vertex function [7]:

$$\begin{aligned}\Gamma_\pi(k,p) = \gamma^5 \Big[& \imath\, E_\pi(k,p) + \not{p}\, F_\pi(k,p) \\ & + k^\mu p_\mu \not{k}\, G_\pi(k,p) + \sigma_{\mu\nu} k^\mu p^\nu\, H_\pi(k,p) \Big] \\ \gamma^5 \equiv \imath\gamma^0\gamma^1\gamma^2\gamma^3 \text{ and } \sigma^{\mu\nu} & \equiv \tfrac{\imath}{2}[\gamma^\mu, \gamma^\nu]\end{aligned} \tag{14}$$

- A model that includes only the dominant component with a integral representation:

$$E_\pi(k,p) = \imath \int_{-1}^{1} dz \int_{-\infty}^{+\infty} d\gamma\, \frac{\rho(\gamma,z)}{k^2 + z(k\cdot p) - \gamma + \imath\,\varepsilon} \tag{15}$$

Nakanishi Form

- After introducing the integral representation to the pion vertex function, the BSE becomes:

$$\int_{-1}^{1} dz \int_{-\infty}^{+\infty} d\gamma \, \frac{\rho(\gamma, z)}{k^2 + z(k \cdot p) - \gamma + \imath\varepsilon} = \int_{-1}^{1} dz' \int_{-\infty}^{+\infty} d\gamma' \rho(\gamma', z') \mathcal{K}(k, p; \gamma', z') \tag{16}$$

$$\mathcal{K}(k, p; \gamma', z') = \sum_n \mathcal{I}_n(k, p; \gamma', z') \tag{17}$$

$$\mathcal{I}_n(k, p; \gamma', z') = \mathcal{D}_{r_n} \left[\frac{\omega_n}{\left(\Delta_n(k, p\,; \underline{x}, \gamma', z') + \imath\varepsilon\right)^{t_n}} \right], \; r_n, t_n \in \mathbb{N} \tag{18}$$

$$\Delta_n(k, p\,; \underline{x}, \gamma', z') \equiv a_n(\underline{x}) k^2 + b_n(\underline{x}, z')(k \cdot p) + c_n(\underline{x}, z') p^2 + d_n(\underline{x}, \gamma') \tag{19}$$

$$\mathcal{D}_n[f(\underline{x})] \equiv (n-1)! \left\{ \prod_j^n \int_0^1 dx_j \right\} \delta\left(1 - \sum_j^n x_j\right) f(\underline{x}) \tag{20}$$

Nakanishi Form

- Constructing the Nakanishi Integral Representation (NIR) [5]:

$$\mathcal{I}_n(k,p;\gamma',z') = \mathcal{N}_{t_n}\left[g_n(\gamma,z;\gamma',z')\right] \tag{21}$$

where the functional is given by:

$$\mathcal{N}_t[g(\gamma,z)] \equiv \int_{-1}^{1} dz \int_{-\infty}^{+\infty} d\gamma \, \frac{g(\gamma,z)}{\left(k^2 + z(k\cdot p) - \gamma + \imath\varepsilon\right)^t} \tag{22}$$

$$\mathcal{N}_{t+1}\left[g(\gamma,z)\right] = \frac{(-1)^t}{t!}\mathcal{N}_1\left[\frac{\partial^t g(\gamma,z)}{\partial\gamma^t}\right] \tag{23}$$

Nakanishi Form

- The new integral equation:

$$\rho(\gamma, z) = g^2 \int_{-1}^{1} dz' \int_{-\infty}^{+\infty} d\gamma'\, G(\gamma, z; \gamma', z')\rho(\gamma', z') \tag{24}$$

where the kernel is given by:

$$G(\gamma, z; \gamma', z') = \sum_n \frac{(-1)^{t_n - 1}}{(t_n - 1)!} \left(\frac{\partial^{t_n - 1} g_n(\gamma, z; \gamma', z')}{\partial \gamma^{t_n - 1}} \right) \tag{25}$$

- Chiral Limit ($p^2 = 0$)

(Eq. 24) ⟶

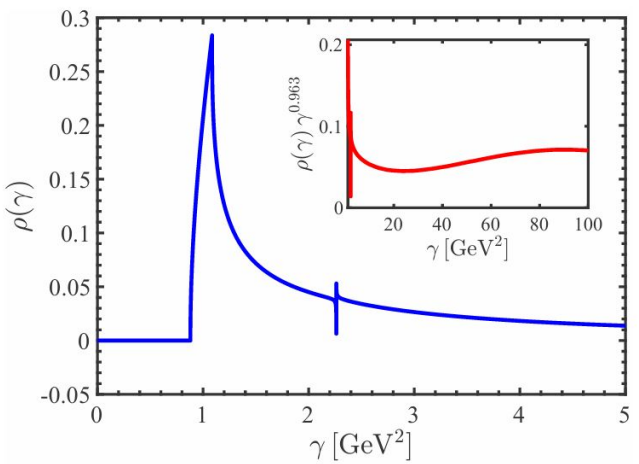
$$\rho(\gamma, z) \to \rho(\gamma)\delta(z)$$

$$\rho(\gamma) = g^2 \int_{0}^{+\infty} d\gamma'\, G(\gamma, \gamma')\rho(\gamma') \tag{26}$$

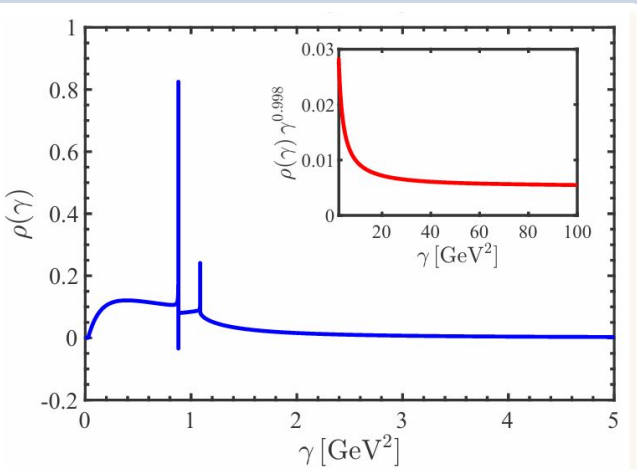
Spectral function and coupling

$$\rho(\gamma) = g^2 \int_0^{+\infty} d\gamma' \, G(\gamma, \gamma') \rho(\gamma')$$

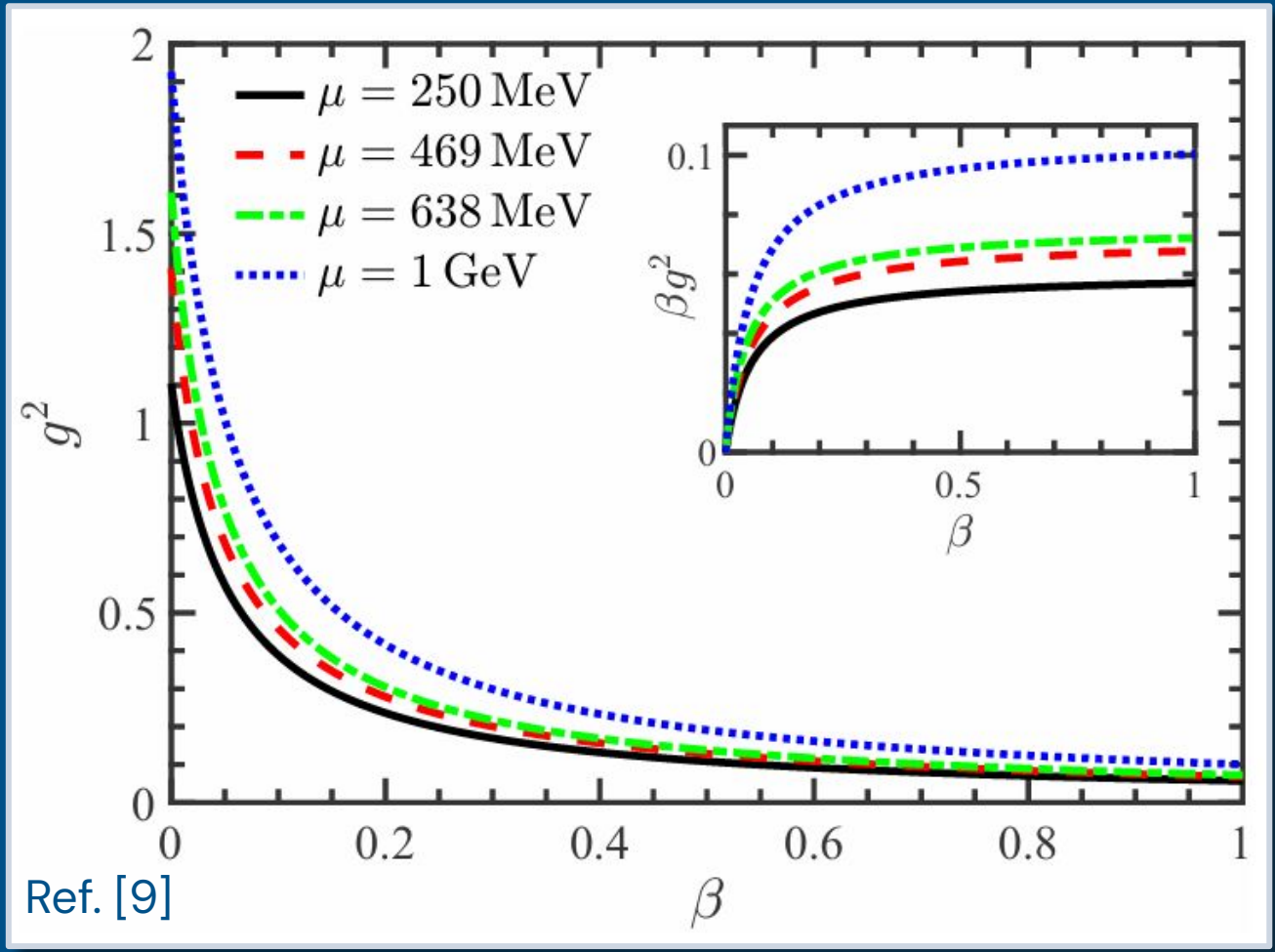
$$\rho_i = g^2 G_{ij} \rho_j = g^2 \left(L_{ij} + \beta M_{ij} \right) \rho_j$$



μ = 469 MeV
β = 0



μ = 469 MeV
β = 1



Ref. [9]

BSA Normalisation

- Pion decay constant directly from the BSA [8]:

$$\imath\, p^\mu f_\pi = N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma^\mu \gamma^5 \Psi(k,p)] \quad \textbf{(27)}$$

Nc = 3 is the number of colors

- Axial Ward–identity in the chiral limit [7]:

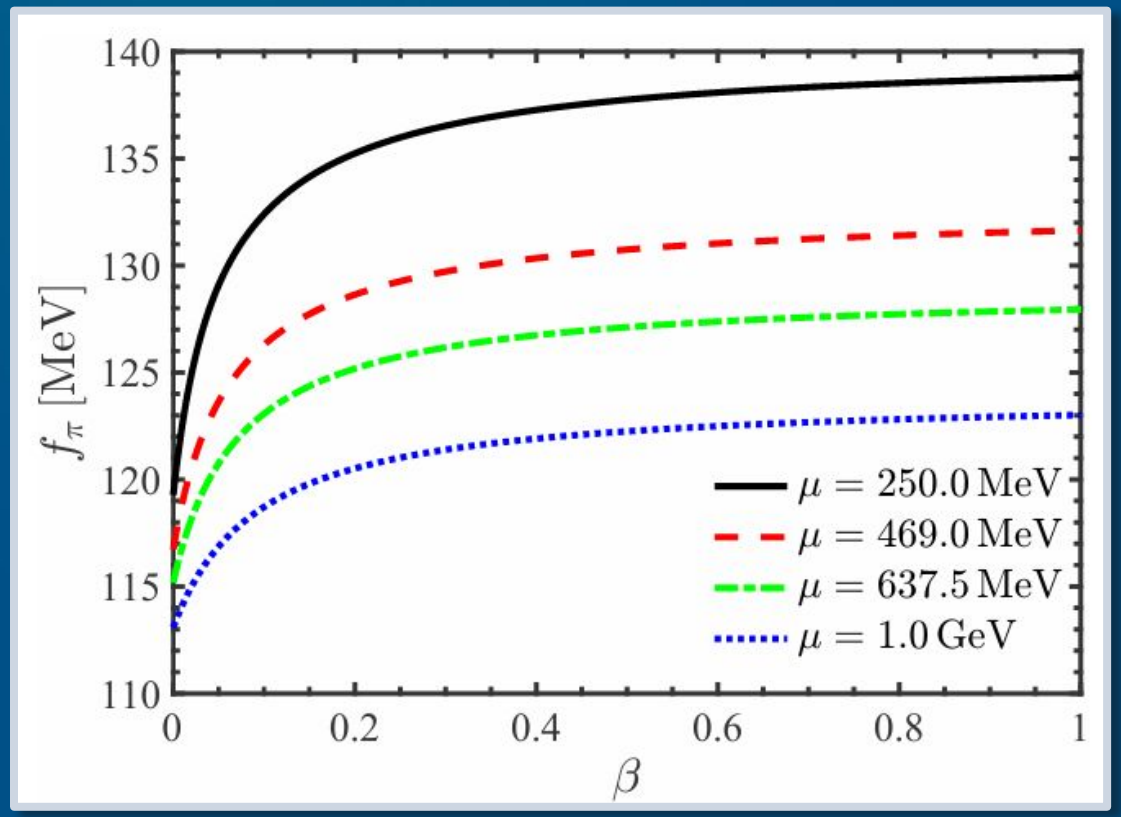
$$f_\pi\, E_\pi(k^2) = \imath \sqrt{2}\, \mathcal{B}(k^2) \quad \textbf{(28)} \quad \Rightarrow \quad f_\pi\, E_\pi(-\lambda^2) = \imath \sqrt{2}\, \mathcal{B}_{\mathrm{Latt}}(-\lambda^2)$$

λ = 0.9 GeV, as the typical scale of the model set by the pole of the quark running mass

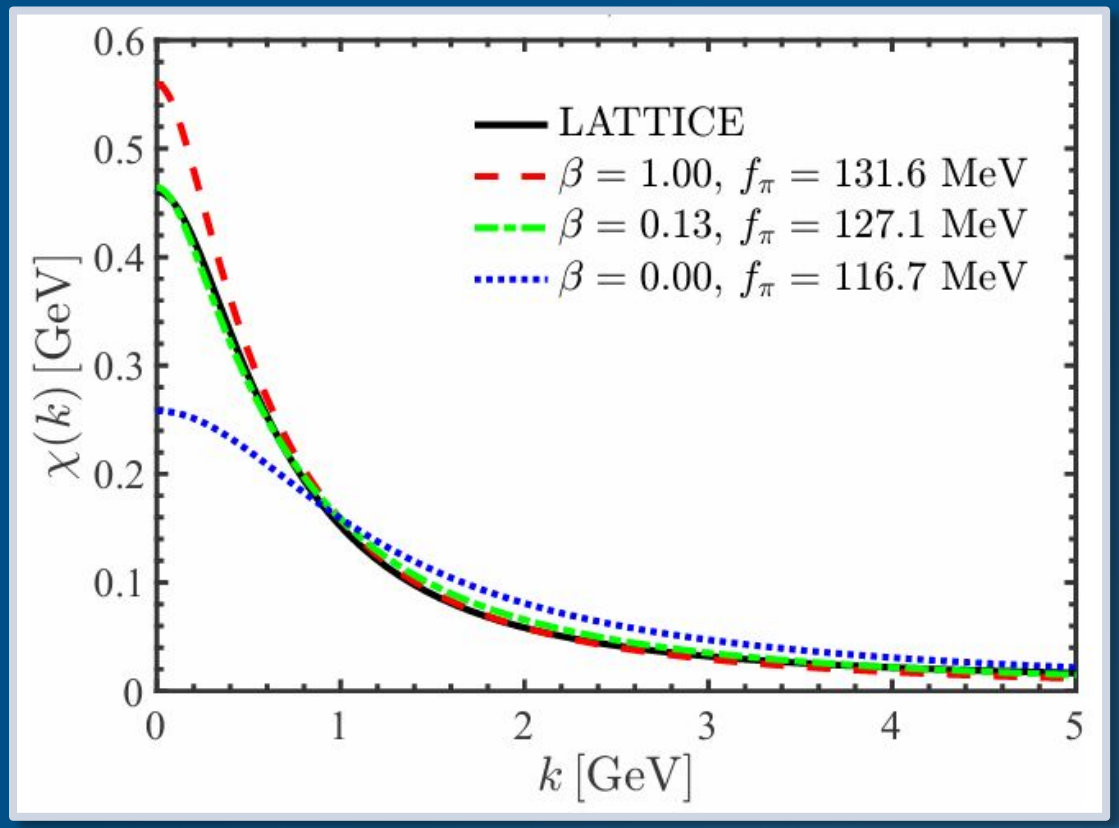
Pion Decay Constant

$$f_\pi \, E_\pi(k^2) = \imath \sqrt{2} \, \mathcal{B}(k^2)$$

Ref. [9]



$$\chi(k) \equiv -\frac{\imath}{\sqrt{2}} f_\pi E_\pi(k^2) \qquad (29)$$



$f_{\pi^+}|V_{ud}| = (127.13 \pm 0.01 \pm 0.13)$ MeV [PDG 2026]

$f_\pi \approx 130$ MeV

Ref. [9]

Light-Front (LF)

Conventions:

$$k^+ = k^0 + k^3,\ k^- = k^0 - k^3,\ \vec{k}_\perp = (k^1, k^2)$$

- Projecting the BSA onto the light-front [2]:

Spin-0 component:

$$\psi_{\uparrow\downarrow}(k^+, \vec{k}_\perp) = -\frac{1}{4}\int \frac{dk^-}{2\pi}\mathrm{Tr}\left[\gamma^+\gamma^5\Psi(k,p)\right] \quad \textbf{(30)}$$

Spin-1 component:

$$-\imath k^i \psi_{\uparrow\uparrow}(k^+, \vec{k}_\perp) = \frac{\sqrt{\gamma}}{4}\int \frac{dk^-}{2\pi}\mathrm{Tr}\left[\sigma^{+i}\gamma^5\Psi(k,p)\right] \quad \textbf{(31)}$$

Light-Front Wavefunction (LFWF)

new variables:

$$\xi \equiv \frac{k^+}{p^+} + \frac{1}{2} = \frac{1-z}{2}$$

$$\gamma \equiv |\vec{k}_\perp|^2$$

Spin-0:

$$\psi_{\uparrow\downarrow}(\xi,\gamma) = \imath \sum_{j,k} \int_0^{+\infty} d\gamma' \left[\frac{\xi\left(\xi A_j B_k + (1-\xi) A_k B_j\right) \rho(\gamma') \theta(\xi) \theta(\frac{1}{2}-\xi)}{\left(\gamma + (1-\xi) m_j^2 + \xi m_k^2\right)\left(\frac{1}{2}\gamma + (\frac{1}{2}-\xi) m_j^2 + \xi\gamma'\right)} + (\xi \leftrightarrow 1-\xi) \right]$$

(32)

Spin-1:

$$\psi_{\uparrow\uparrow}(\xi,\gamma) = -\imath\sqrt{\gamma} \sum_{j,k} \int_0^{+\infty} d\gamma' \left[\frac{\xi A_j A_k \rho(\gamma') \theta(\xi) \theta(\frac{1}{2}-\xi)}{\left(\gamma + (1-\xi) m_j^2 + \xi m_k^2\right)\left(\frac{1}{2}\gamma + (\frac{1}{2}-\xi) m_j^2 + \xi\gamma'\right)} + (\xi \leftrightarrow 1-\xi) \right]$$

(33)

Transverse-momentum Dependent Distributions (TMDs)

- Decomposition in terms of the unpolarized twist−2 TMDs that have the meaning of probabilities densities for each component:

$$\mathcal{P}_{\rm val}(\xi,\gamma) = \mathcal{P}_{\uparrow\downarrow}(\xi,\gamma) + \mathcal{P}_{\uparrow\uparrow}(\xi,\gamma) \quad (34)$$

$$\mathcal{P}_{\uparrow\downarrow(\uparrow\uparrow)}(\xi,\gamma) = \frac{N_c}{8\pi^2}\left|\psi_{\uparrow\downarrow(\uparrow\uparrow)}(\xi,\gamma)\right|^2 \quad (35)$$

- Valence probabilities are given by:

$$P_{\rm val} = \int_0^1 d\xi \int_0^\infty d\gamma\, \mathcal{P}_{\rm val}(\xi,\gamma) \quad (36)$$

$$P_{{\rm val}\uparrow\downarrow(\uparrow\uparrow)} = \int_0^1 d\xi \int_0^\infty d\gamma\, \mathcal{P}_{\uparrow\downarrow(\uparrow\uparrow)}(\xi,\gamma) \quad (37)$$

Valence Probabilities

$$P_{\rm val} = \int_0^1 d\xi \int_0^\infty d\gamma\, \mathcal{P}_{\rm val}(\xi,\gamma)$$

$$P_{{\rm val}\uparrow\downarrow(\uparrow\uparrow)} = \int_0^1 d\xi \int_0^\infty d\gamma\, \mathcal{P}_{\uparrow\downarrow(\uparrow\uparrow)}(\xi,\gamma)$$

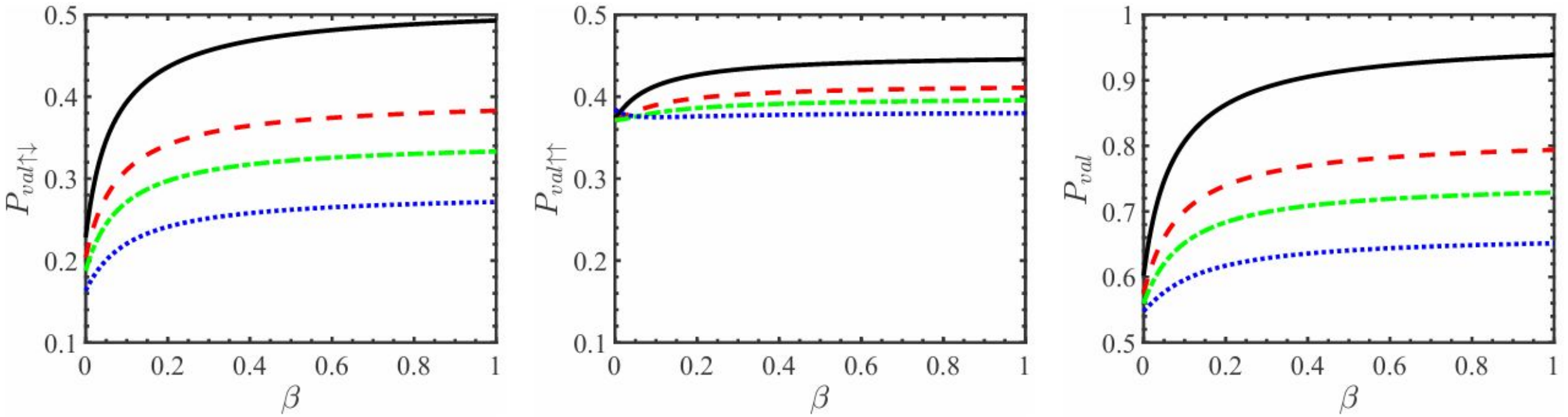


FIG. Valence probabilities as a function of the parameter β for different values of the gluon mass μ. The left, middle and right panels display the anti-aligned, aligned and total valence probabilities. The curves correspond to $\mu = 250\,\text{MeV}$ (black solid), $\mu = 469\,\text{MeV}$ (red dashed), $\mu = 637.5\,\text{MeV}$ (green dashed-point) and $\mu = 1\,\text{GeV}$ (blue dotted) lines, respectively.

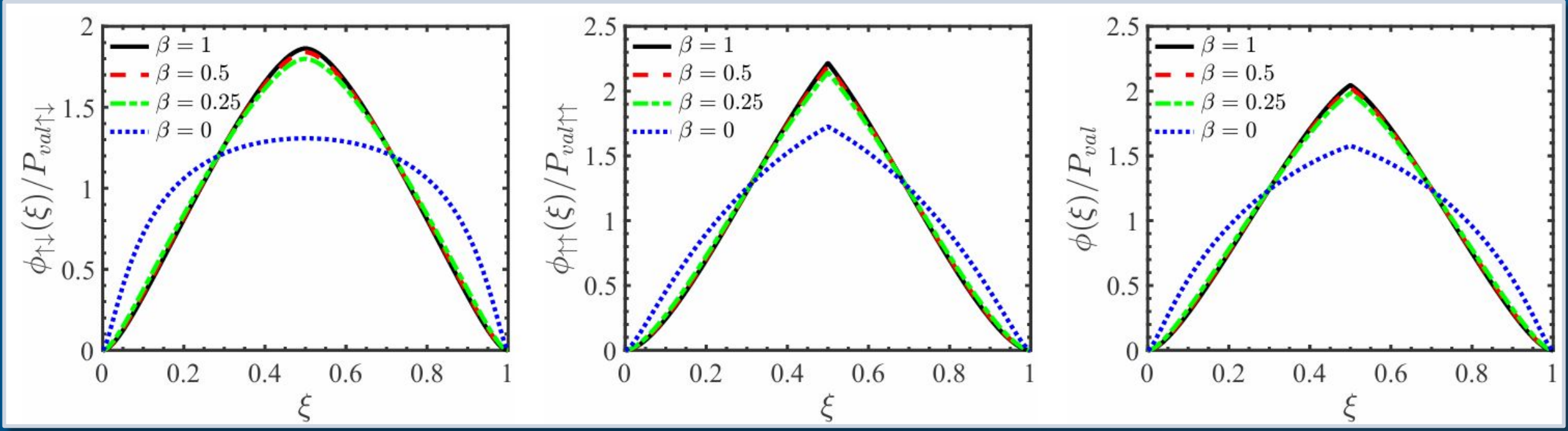
Ref. [9]

Longitudinal Distributions

$$\phi_{\uparrow\downarrow(\uparrow\uparrow)}(\xi) = \int_0^\infty d\gamma\, \mathcal{P}_{\uparrow\downarrow(\uparrow\uparrow)}(\xi,\gamma) \quad (38)$$

$$\phi(\xi) = \phi_{\uparrow\downarrow}(\xi) + \phi_{\uparrow\uparrow}(\xi) \quad (39)$$

μ = 469 MeV



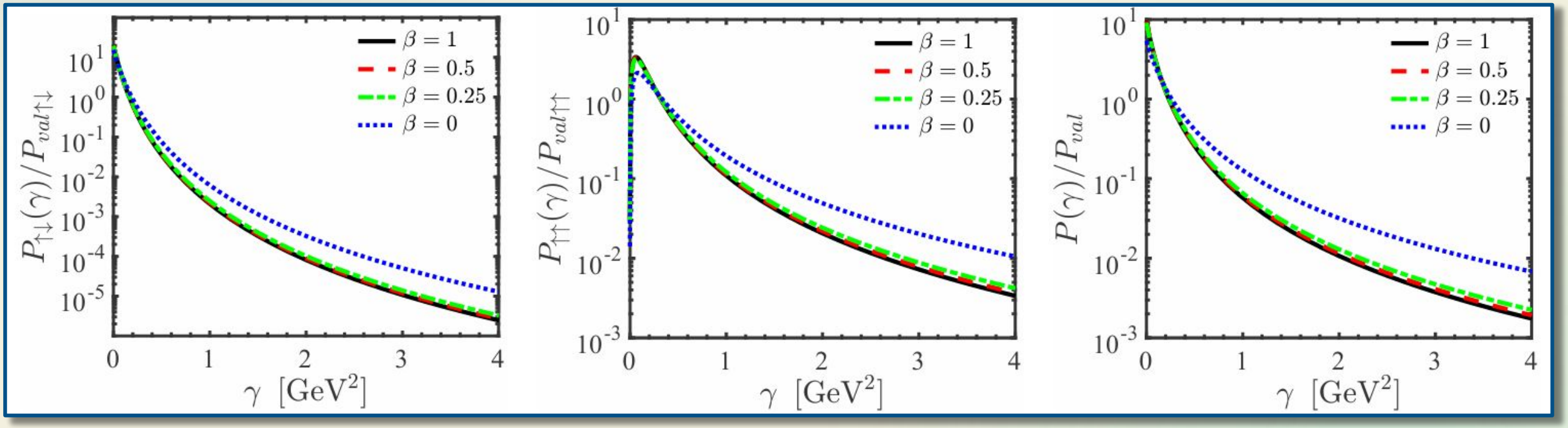
Ref. [9]

Transverse Distributions

$$P(\gamma) = P_{\uparrow\downarrow}(\gamma) + P_{\uparrow\uparrow}(\gamma) \quad (40)$$

$$P_{\uparrow\downarrow(\uparrow\uparrow)}(\gamma) = \int_0^1 d\xi\, \mathcal{P}_{\uparrow\downarrow(\uparrow\uparrow)}(\xi, \gamma) \quad (41)$$

μ = 469 MeV



Ref. [9]

Transverse-momentum Dependent Distributions (TMDs)

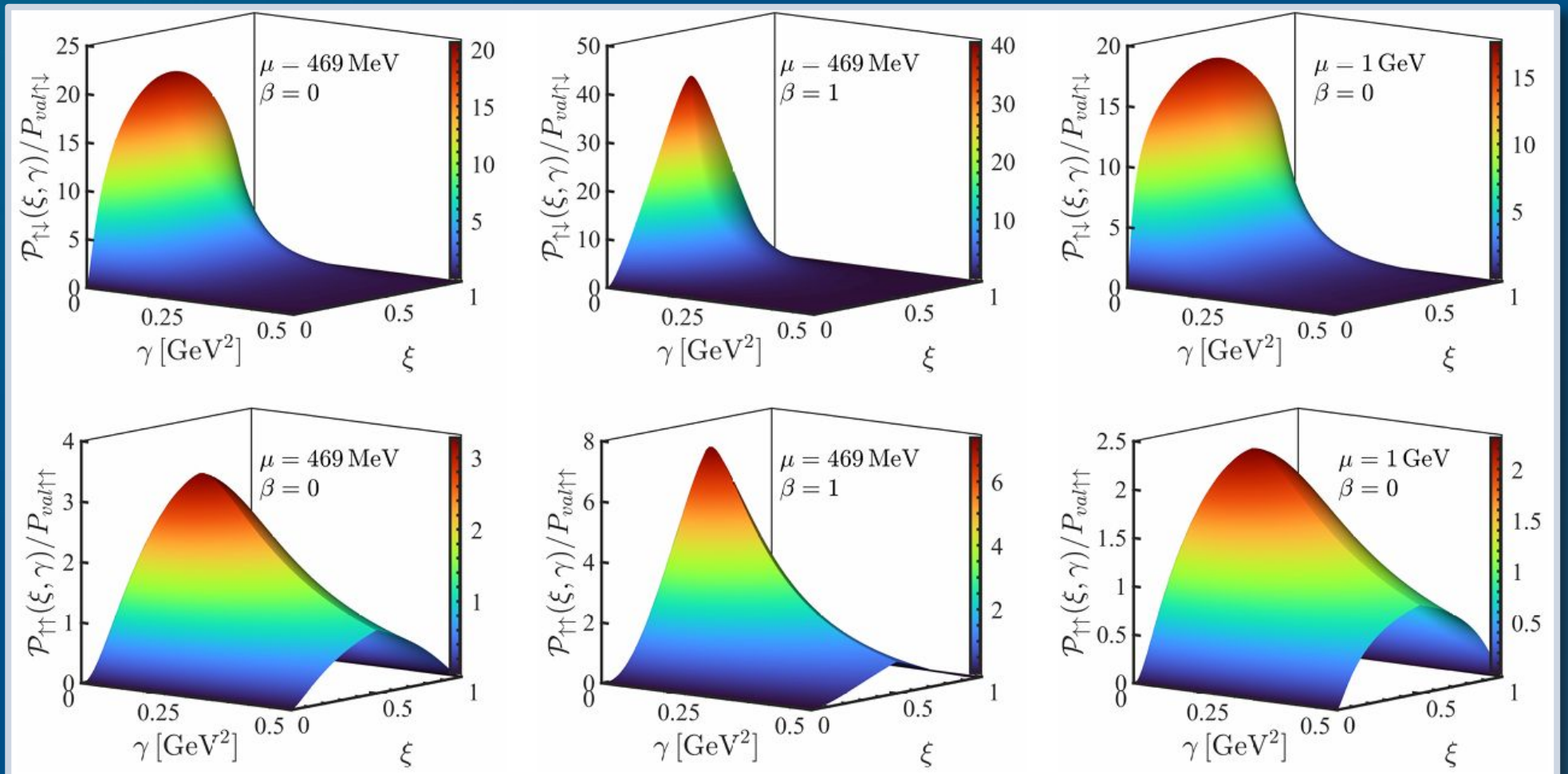


FIG. Normalized valence TMDs for the anti-aligned spin component (top) and the aligned spin one (bottom). The results shown in the left, middle and right panels are computed for $\mu = 469$ MeV and $\beta = 0$, $\mu = 469$ MeV and $\beta = 1$, $\mu = 1$ GeV and $\beta = 0$, respectively.

Ref. [9]

Summary

- A simple model for the quark–gluon vertex was proposed;
- This new IR structure of the quark–gluon interaction affects the valence spin–dependent partonic structure of the pion;
- As the IR interaction becomes stronger:
  - spin–0 component of the valence LFWF is enhanced;
  - spin–1 contribution is progressively depleted;
- Leaves clear signatures in both pion PDFs and TMDs at the pion scale, while the model provides a decay constant close to experimental data.

References

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[9] V. Raimundo, M. Pedroso, T. Frederico, and W. de Paula, Infrared QCD Dynamics and the Structure of the Pion (to be published)

Thank You

Extra: UV Limit

- Asymptotic form of the spectral function

Consistent with:
O. Oliveira, T. Frederico, and W. de Paula, On the conformal limit of a QED-inspired model, Eur. Phys. J. C 84, 851 (2024)

$$\rho(\gamma) = g^2 \int_0^{+\infty} d\gamma' \, G(\gamma, \gamma') \rho(\gamma') \quad \xrightarrow{\gamma \to \infty} \quad \rho(\gamma) \sim \gamma^{\eta} \tag{42}$$

$$\eta = -\frac{1 \pm \sqrt{1 - g^2(3+\zeta)/(4\pi^2)}}{2} \tag{43}$$

- Using the scaling of the spectral function on the valence wavefunctions (Eq. 32 and Eq. 33).

$$\psi_{\uparrow\downarrow}(\xi, \gamma)\Big|_{\gamma \to \infty} \sim \frac{1}{\gamma^{1-\eta}} \tag{44}$$

$$\psi_{\uparrow\uparrow}(\xi, \gamma)\Big|_{\gamma \to \infty} \sim \frac{1}{\gamma^{\frac{1}{2}-\eta}} \tag{45}$$

$$\psi_{\uparrow\downarrow}(\xi, \gamma)\Big|_{\xi \to 0_+} \sim \xi^{-\eta} \tag{46}$$

$$\psi_{\uparrow\uparrow}(\xi, \gamma)\Big|_{\xi \to 0_+} \sim \xi^{-\eta} \tag{47}$$

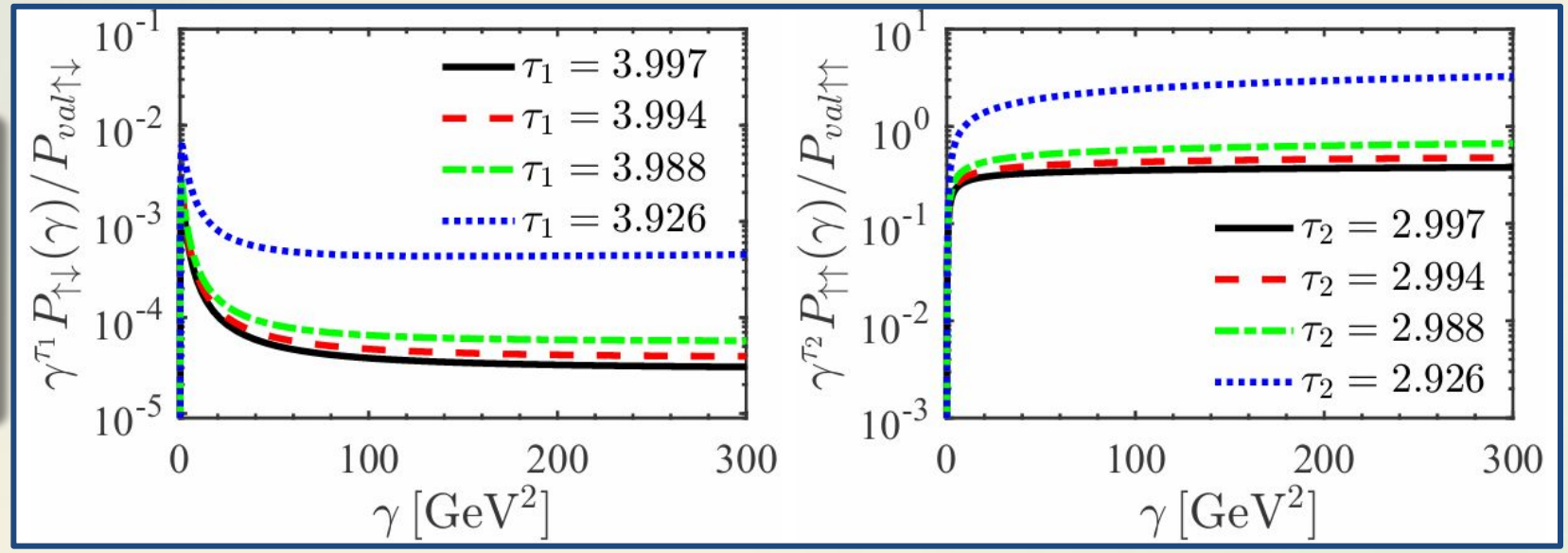
Endpoint Behavior

- From the wavefunctions one gets the endpoint behavior of the correspondent distribution:

$$P_{\uparrow\downarrow}(\gamma) \sim \gamma^{-\tau_1}, \quad \tau_1 \equiv 2 - 2\eta$$

$$P_{\uparrow\uparrow}(\gamma) \sim \gamma^{-\tau_2}, \quad \tau_2 \equiv 1 - 2\eta$$

(48)



$$\phi_{\uparrow\downarrow(\uparrow\uparrow)}(\xi) \sim \xi^{-2\eta}$$

(49)

