The FMS mechanism as a tool for semiclassical calculations in quantum gauge theories

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Austria

UNI GRAZ

NAWI Graz
Natural Sciences

FWF Österreichischer Wissenschaftsfonds

Setup

Consider a gauge theory with gauge fields *A*, matter fields ω, and gauge-invariant measure

$$Z = \int_{\Omega} DA\, D\omega\, e^{iS[A,\phi]}$$

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[Reviews: 1712.04721, 2305.01960]

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Integration over full gauge orbits!

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Integration over full gauge orbits!

The gauge symmetry cannot be broken spontaneously (Elitzur's theorem) – all gauge-dependent correlation functions vanish without gauge-fixing

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But the action could be extremalized by non-vanishing field configurations for some (renormalized) parameters

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But the action could be extremalized by non-vanishing field configurations for some (renormalized) parameters

These minima are invariant under gauge transformations

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Brout-Englert-Higgs (BEH) effect:
The configurations dominating the path integral allow for a **common gauge** in which **fluctuations** around a minimizing solutions **are small.**

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The configurations dominating the path integral allow for a **common gauge** in which **fluctuations** around a minimizing solutions **are small.**
Dynamical question, dependent on parameters!
Still: Gauge symmetry is unbroken, and asymptotic states need to be gauge-invariant

The split

[Reviews: 1712.04721, 2305.01960]

$$h_i^a = \eta_a^i + \nu_a^i$$

The split

[Reviews: 1712.04721, 2305.01960]

Unbroken
global symmetries

Full field

$$h_i^a = \eta_a^i + v_a^i$$

Gauge index

The split

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Unbroken
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Full field

$$h^{a}_{i} = \eta^{i}_{a} + v^{i}_{a}$$

Gauge index

Vacuum expectation value

The split

[Reviews: 1712.04721, 2305.01960]

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Gauge index

Vacuum expectation value:
- Extremalizes action
- Satisfies gauge condition
- May be space-time dependent
- Acts as projector

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Fluctuation field
By construction: Small amplitude

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Insert this into gauge-invariant correlation functions:
Fröhlich-Morchio-Strocchi mechanism (FMS)

[Fröhlich et al.'80,'81
Dudal et al.'20, Maas et al.'20]

Off-shell correlation functions

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1) Formulate gauge-invariant operator

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Local, composite,
invariant operator

Any other field(s)

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$$\langle (h\psi)^+(x)(h\psi)(y)\rangle = v^2\langle \psi^+(x)\psi(y)\rangle$$
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Manifestly gauge-invariant

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Only sum gauge-invariant,
but not individual terms

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Contains projection

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Perturbatively calculable

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Bound state mass

Matter mass

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Trivial two-particle state

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Standard perturbation theory – up to projection

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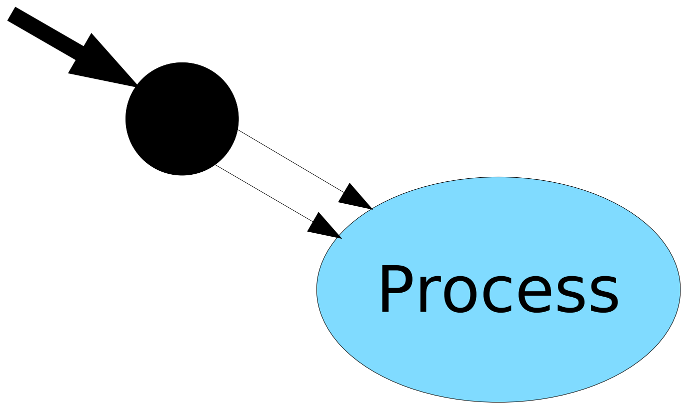
Asymptotic states

[Maas et al.'17
Maas & Reiner '22
Maas, Plätzer et al.'24, unpublished]

Incoming (asymptotic) particle
Gauge-invariant LSZ: Bound state



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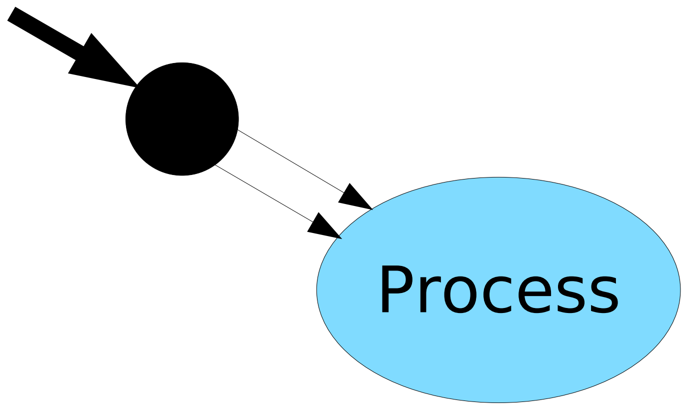

$$\langle (h\,\psi)(p)\ldots\rangle$$

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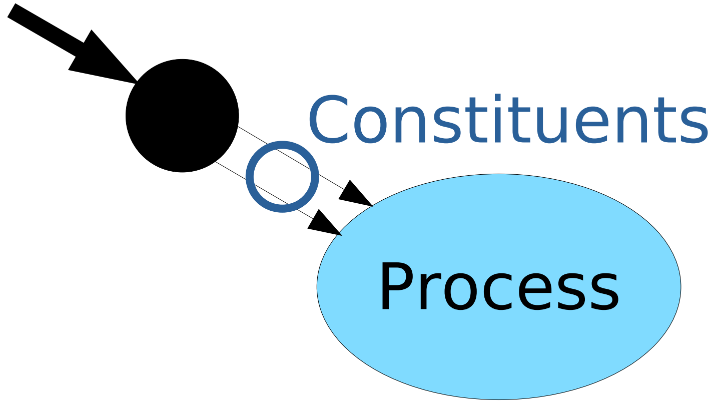

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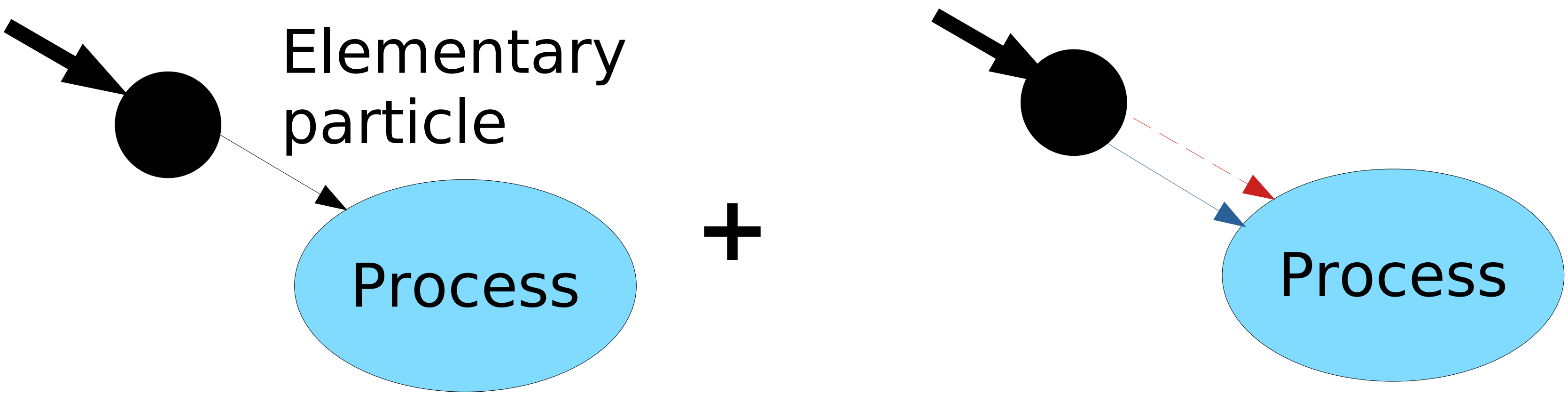

$$v\langle \psi(p)\ldots\rangle + \langle (\eta\psi)(p)\rangle$$

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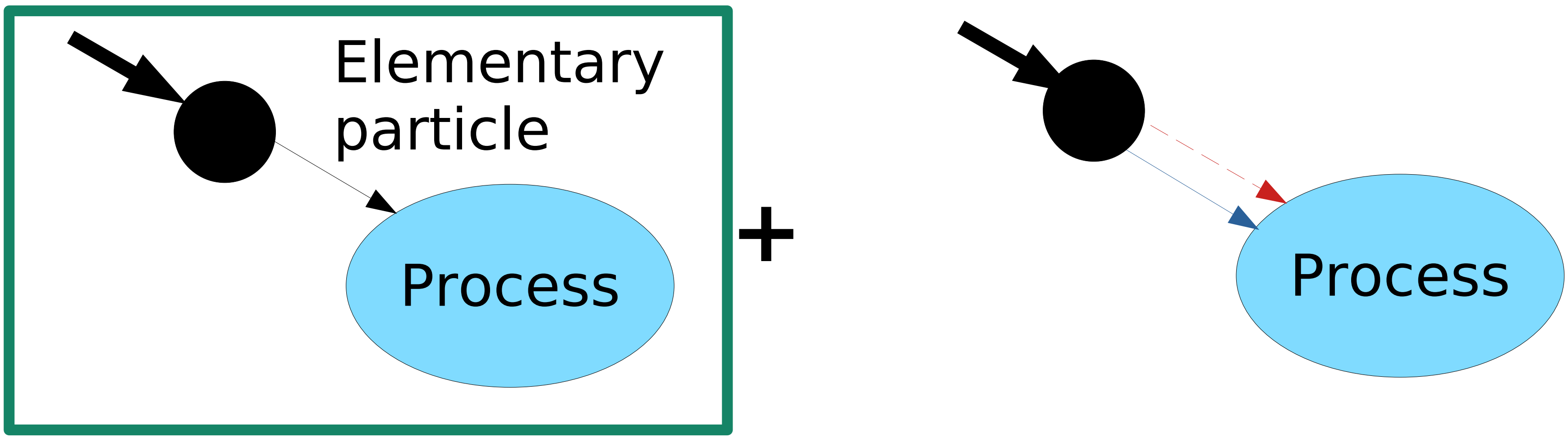

Standard perturbation theory

$$\boxed{v\langle \psi(p)\ldots\rangle} + \langle(\eta\psi)(p)\rangle$$

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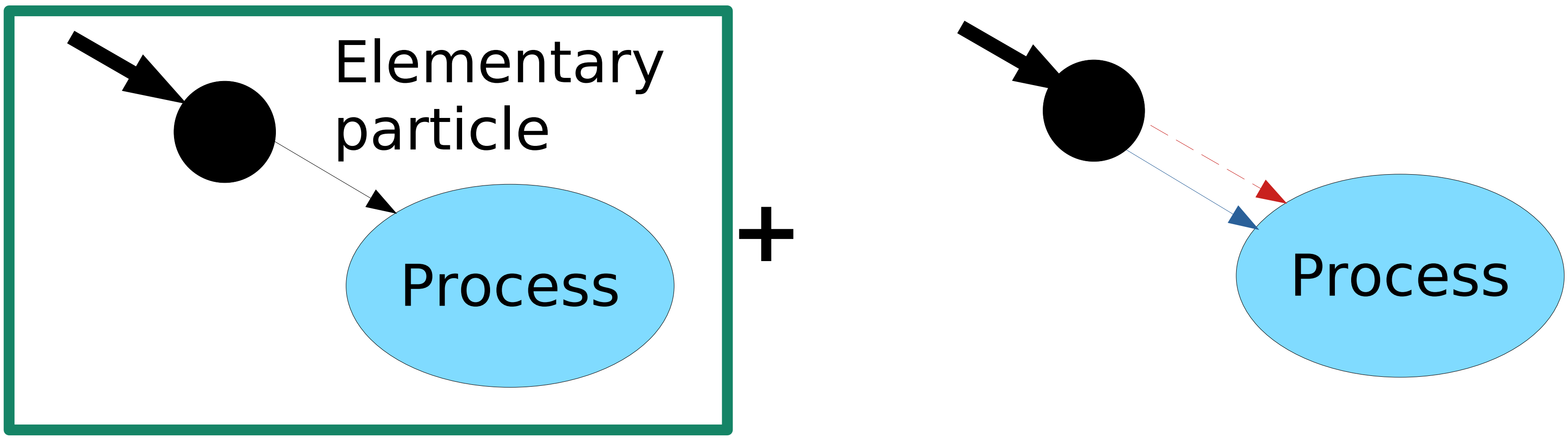

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$$\boxed{v\langle\psi(p)\ldots\rangle}+\langle(\eta\psi)(p)\rangle$$

Note: If the projector is not trivial, this term can already be different from standard perturbation theory!

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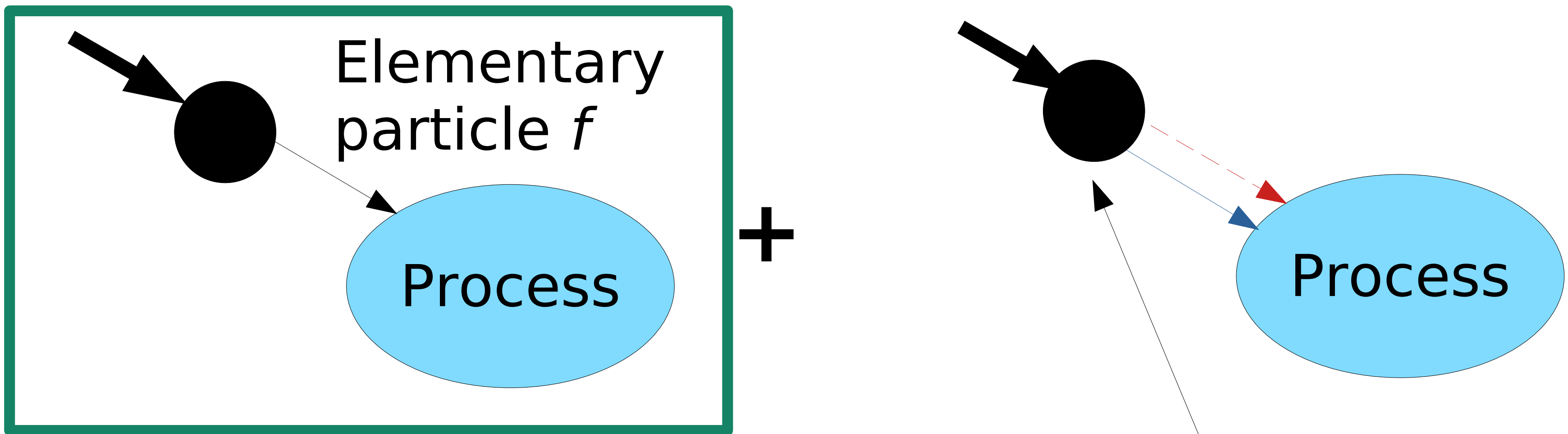

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$$\boxed{v\langle\psi(p)\ldots\rangle}+\boxed{\langle(\eta\psi)(p)\rangle}$$

Can this contribute
after LSZ truncation?

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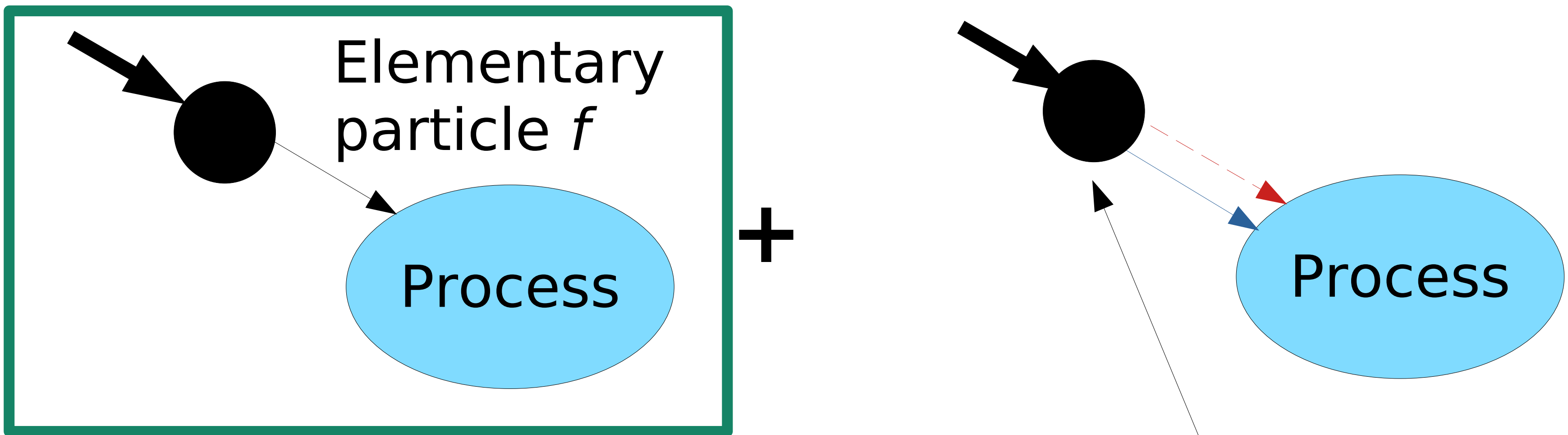

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Can this contribute
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Yes!

Bubble sum

[Maas et al.’17
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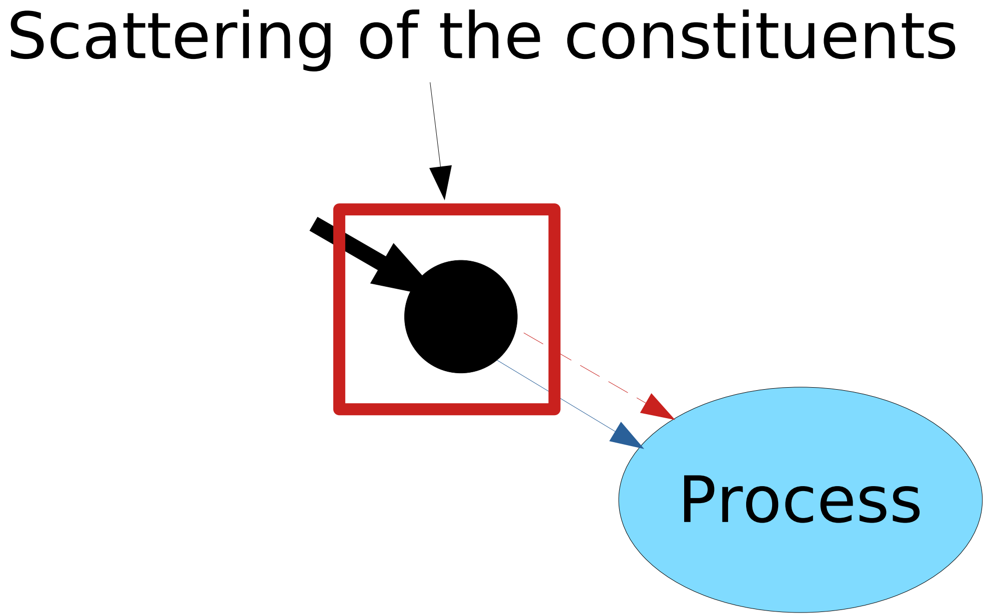

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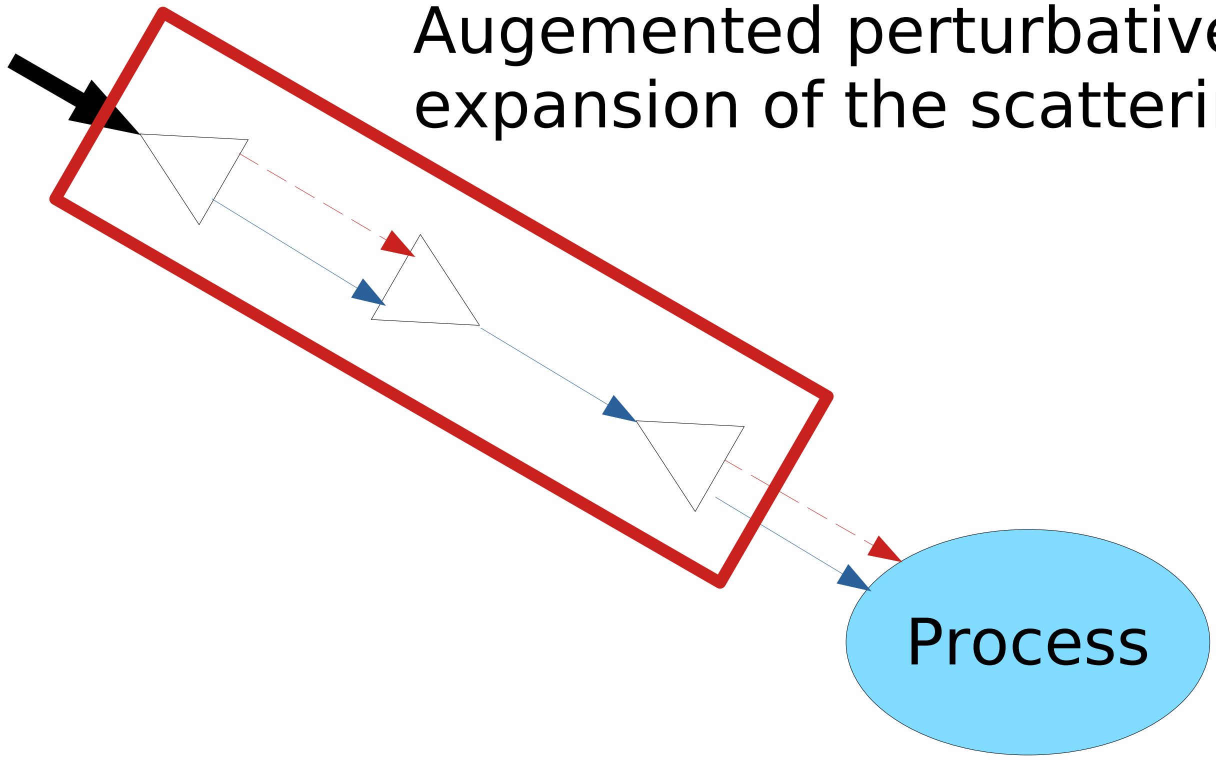
Augemented perturbative
expansion of the scattering kernel



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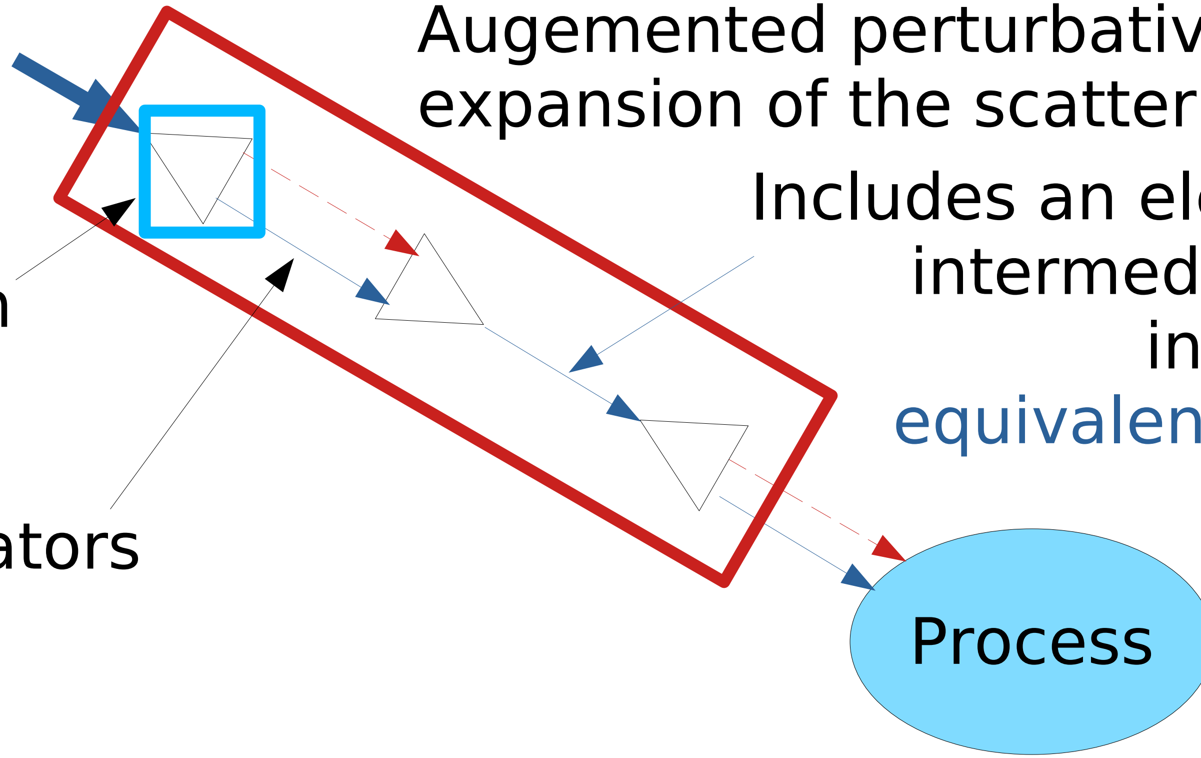
Augemented perturbative
expansion of the scattering kernel

Includes an elementary
intermediate state
in the FMS-
equivalent channel

Vertex function



Propagators

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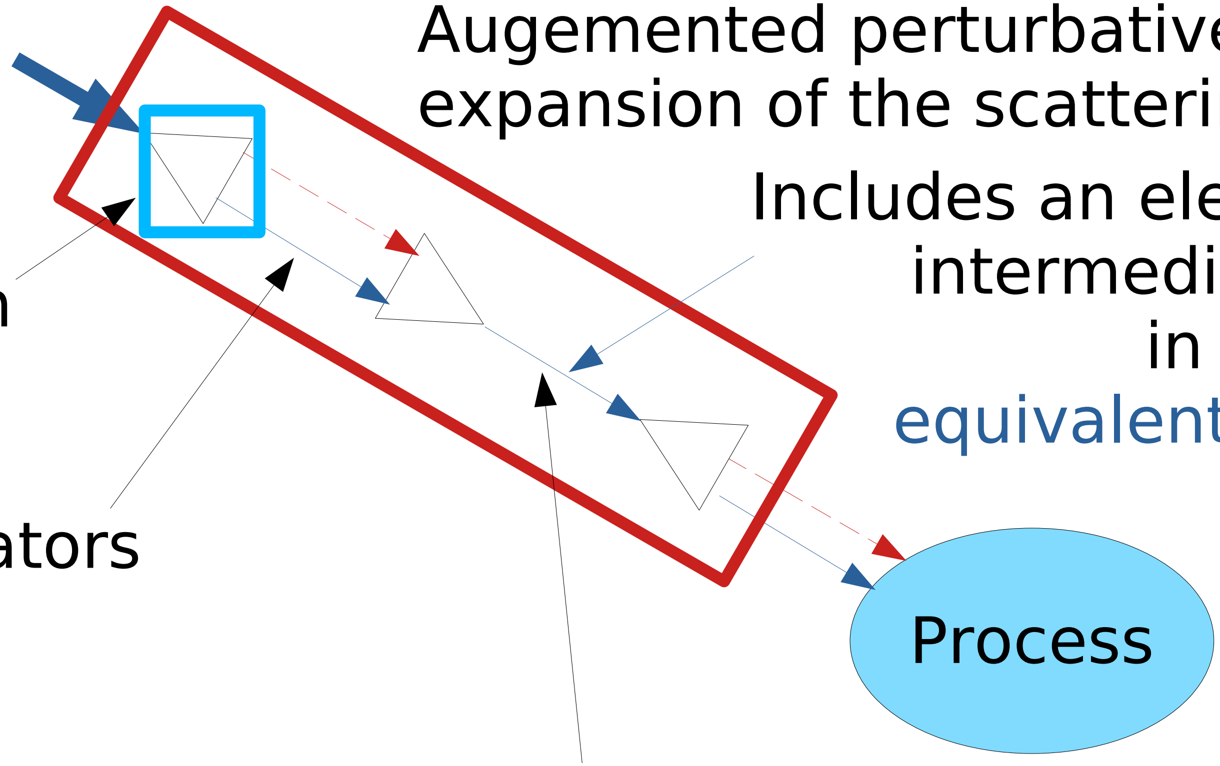

Propagators

This intermediate state
carries the necessary pole
to let this expression survive the
LSZ truncation

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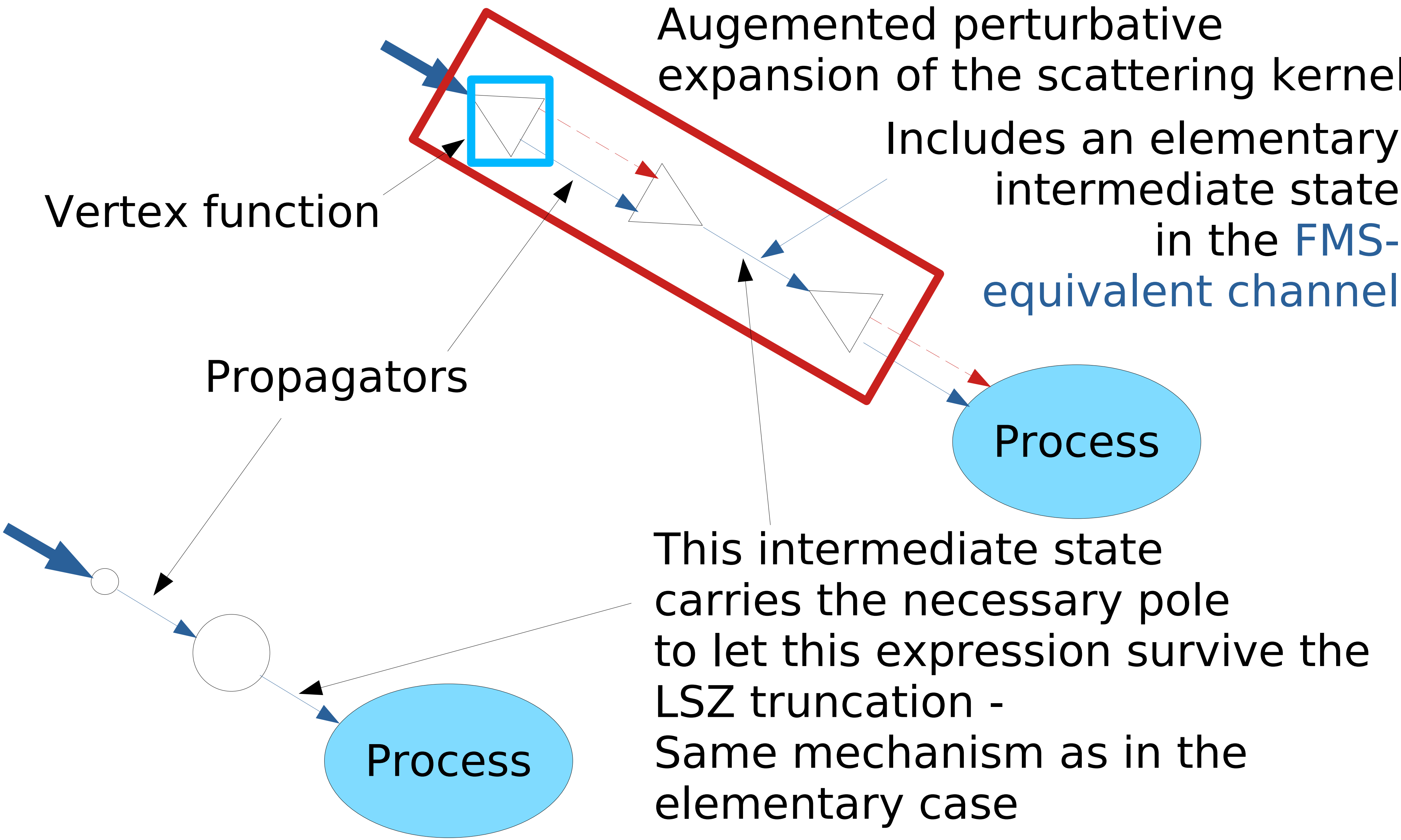

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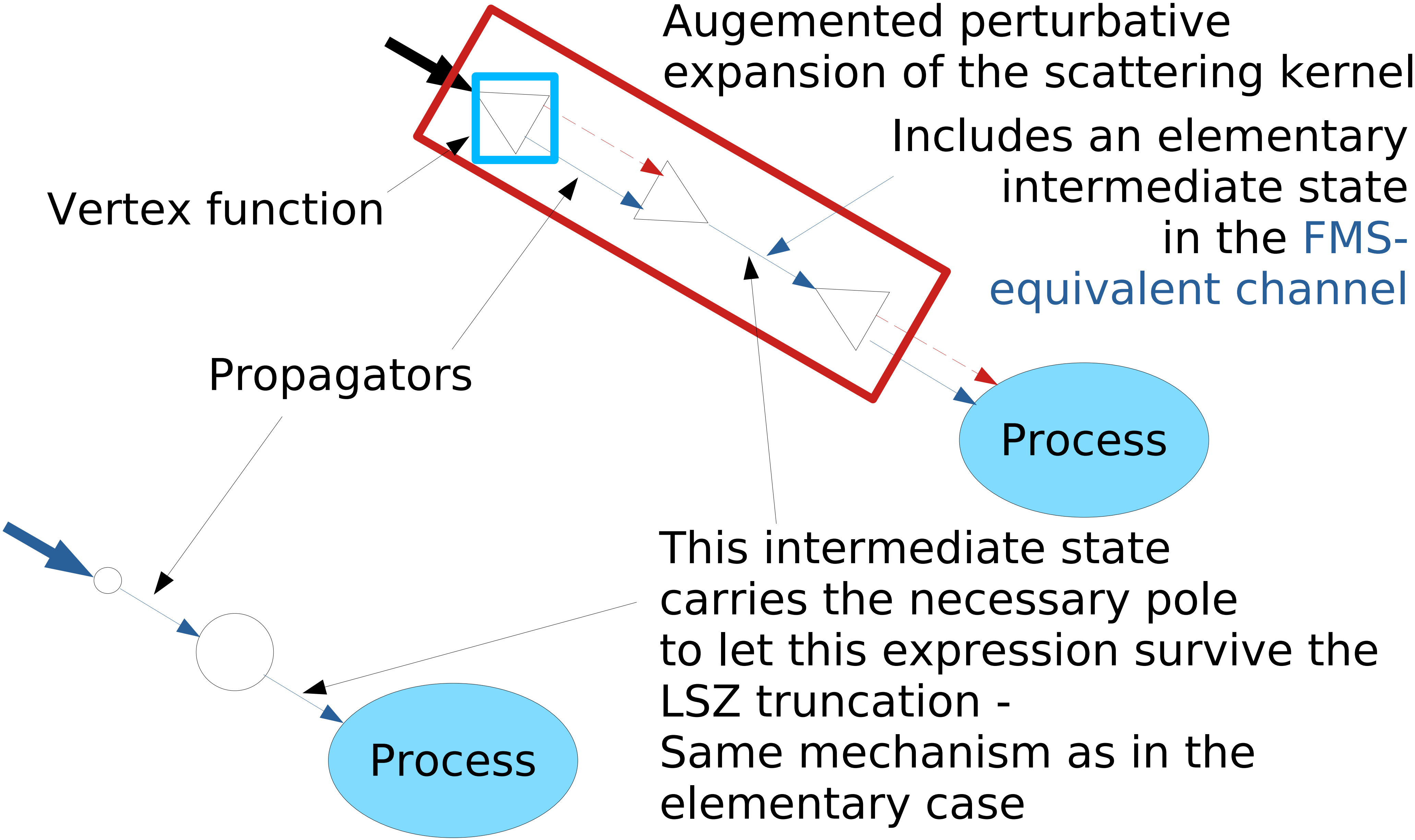

Can be perturbatively resummed!

Asymptotic states

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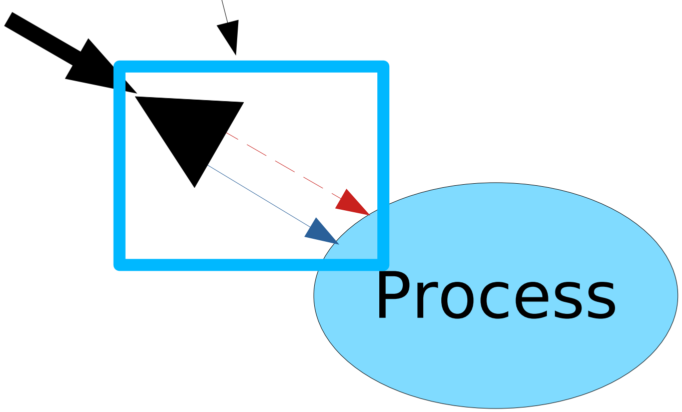
Becomes the perturbatively calculable,
Process-independent Bethe-Salpeter amplitude



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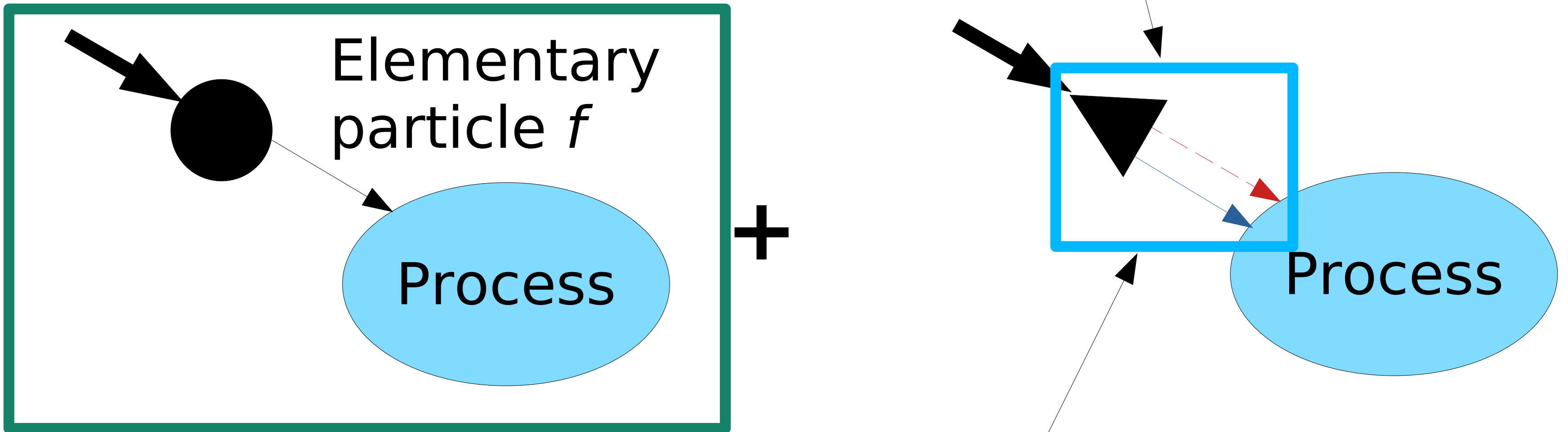

Standard perturbation theory

$$\boxed{v\langle\psi(p)\ldots\rangle}+\int\boxed{dq\,\Gamma(P,q)\,D_\psi(p-q)\,D_\eta(q)}\langle\eta(q)\,\psi(P-q)\ldots\rangle$$

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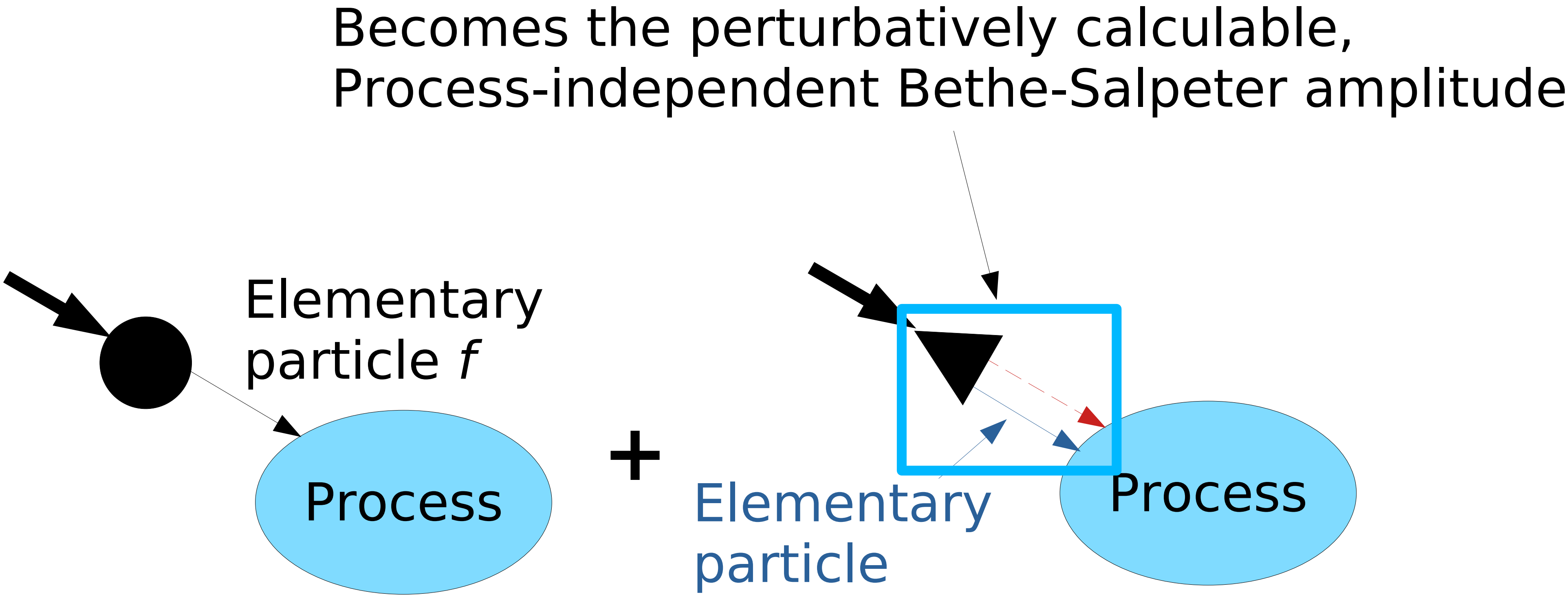

$$v\langle \psi(p)\ldots\rangle + \int dq\, \Gamma(P,q)\, D_\psi(p-q)\, D_\eta(q)\, \langle \eta(q)\, \psi(P-q)\ldots\rangle$$

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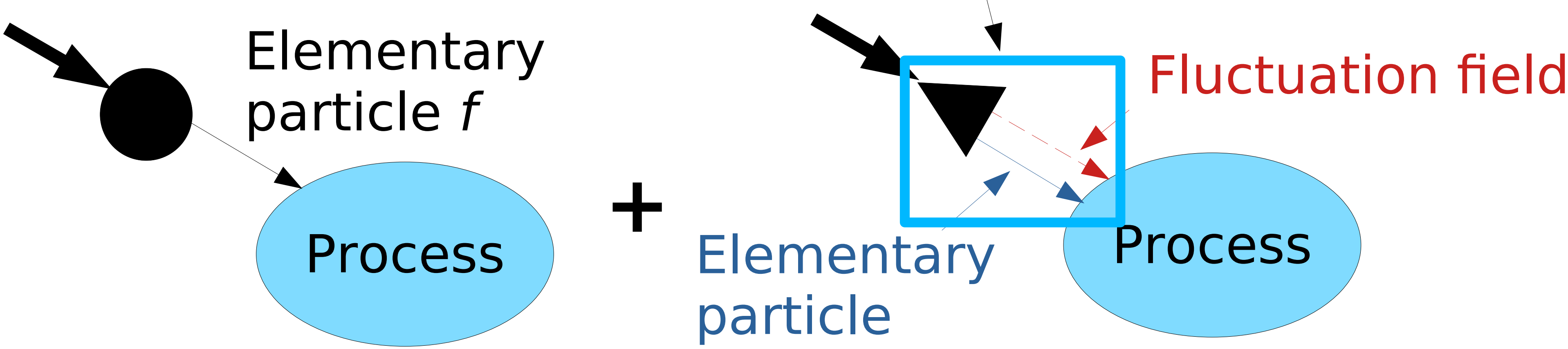

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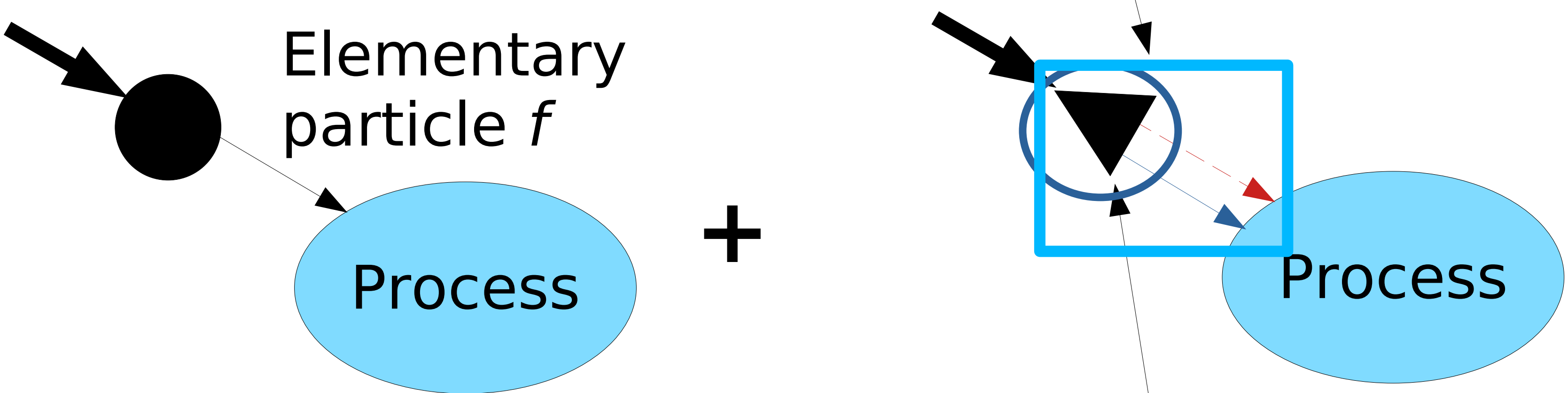

3-point vertex

$$v\langle\psi(p)\ldots\rangle+\int dq\,\Gamma(P,q)D_{\psi}(p-q)D_{\eta}(q)\langle\eta(q)\psi(P-q)\ldots\rangle$$

Applications

[Reviews: 1712.04721, 2305.01960]

- Theories with trivial projector: SM, 2HDM, MSSM
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- Theories with non-trivial projector: Almost all GUTs
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Maas'19,Maas et. al.'25]

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- Theories with non-trivial projector: Almost all GUTs
  - Deviations at tree-level
- Theories with non-trivial condensate: Quantum gravity
  - Yet unclear
- Substantial lattice support for many theories

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Maas'19,Maas et. al.'25]

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- Substantial lattice support for many theories
- Experiment: Elitzur's theorem and FMS yield correctly that for the SM perturbation theory should be good

Summary

[Reviews: 1712.04721, 2305.01960]

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  - Together: Augmented perturbation theory

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- Promises to become a relevant approach to quantum gravity