

Quantum Simulation of Generalized Parton Distributions: A Qudit-Based Approach

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in collaboration with

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Introduction

Nuclear Structure as QCD laboratory

- Nuclear structure measurements allow us to test fundamental QCD predictions at the highest precision
- Tremendous effort in the past to extract Parton Distribution Functions (PDFs)
- Current state-of-the-art are off-forward Generalized Parton Distributions (GPDs) and Light-Cone Wave Functions (LCWFs)
 - Spin & mass decomposition, spin-orbit correlations, higher Fock components,...

Current limitations and new avenues

- Computation of required correlators using Euclidean lattice QCD (sign problem, analytic continuation,...)
- Quantum hardware enables simulation and measurement using the Hamiltonian formulation of lattice QCD

QCD₂ on a $d = 8$ Color Site

Staggered fermions [Kogut and Susskind, 1975]
with $c \in \{r, g, b\}$ give (cf. Schwinger model)

$$H_{\text{QCD}_2} = H_{\text{hop}} + H_m + H_C$$

$$H_{\text{hop}} = \frac{1}{2a} \sum_{n,c} (\psi_{n,c}^\dagger \psi_{n+1,c} + \text{h.c.}), \quad H_m = m \sum_{n,c} (-1)^i \psi_{n,c}^\dagger \psi_{n,c},$$

$$H_C = \frac{g^2 a}{2} \sum_n \left(\sum_{n' \leq n} Q_{n'}^A \right)^2, \quad Q_n^A = \psi_n^\dagger T^A \psi_n.$$

- Gauss's law eliminates the non-dynamical gauge field.
- The resulting long-range color Coulomb interaction confines quarks.

Local Hilbert space

- Three occupation numbers $n_r, n_g, n_b \in \{0, 1\}$ give $\dim \mathcal{H}_{\text{site}} = 2^3 = 8$.
- Jordan-Wigner map translates encoding to quantum hardware

Baryon sector

- The color-singlet and $N_B = 1$ sectors can be enforced exactly by using quantum-number-conserving tensor networks.
- A similar framework is possible on quantum hardware.

Tensor-Network Calculation

Rest baryon

DMRG minimizes $\langle \Psi | \mathbb{H} | \Psi \rangle$ in the exact baryon sector [White, 1992].

Boosted baryon

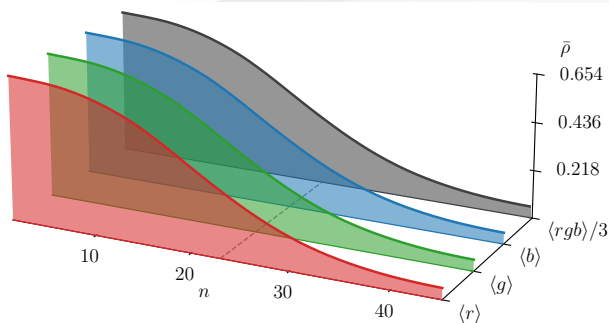
TDVP applies $e^{-i\chi\mathbb{K}}$ on the baryon state [Haegeman et al., 2016].

Alternatively:

DMRG of boosted Hamiltonian

Independent checks

- State quality is tested against the relativistic dispersion relation.
- Sum rules test the observables.



Color density profile of the ground state baryon

Large Momentum Effective Theory

Large Momentum Effective Theory (LaMET) uses an equal-time spatial bilocal correlator in boosted baryon states [Ji, 2013, Ji et al., 2021]

$$H_{\Gamma}(x, \xi) = \int \frac{dz}{2\pi} e^{ixzP} \langle B(\chi_2) | \bar{\psi}(-\frac{z}{2}) \Gamma W \psi(\frac{z}{2}) | B(\chi_1) \rangle,$$

where $\Gamma \in \{\gamma^0, \gamma^1 = \gamma^z\}$ (LC GPD uses γ^+).

The rest baryon is boosted with

$$|B(\chi)\rangle = e^{-i\chi\mathbb{K}} |B(0)\rangle, \quad P = M \sinh \chi.$$

- The in- and out-state rapidities fix ξ .
- In $1+1$ dimensions, ξ and t are kinematically linked.

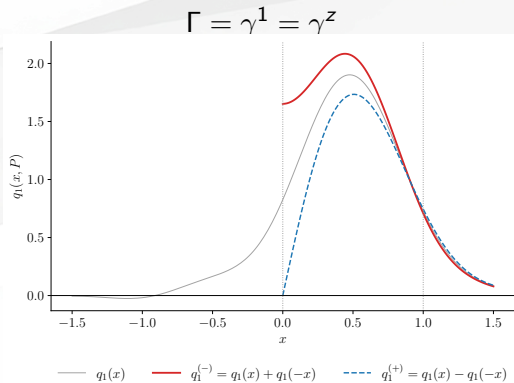
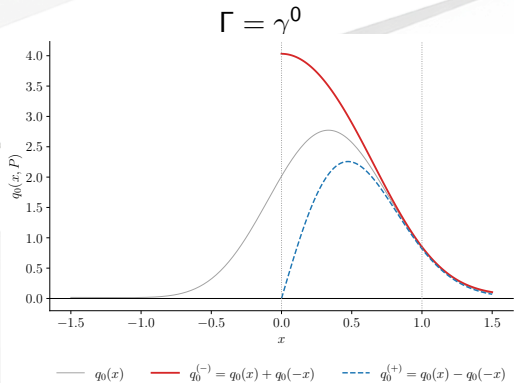
LaMET limit

The finite-momentum quasi-distribution matches the LC distribution as $P \rightarrow \infty$.

Hamiltonian formulation

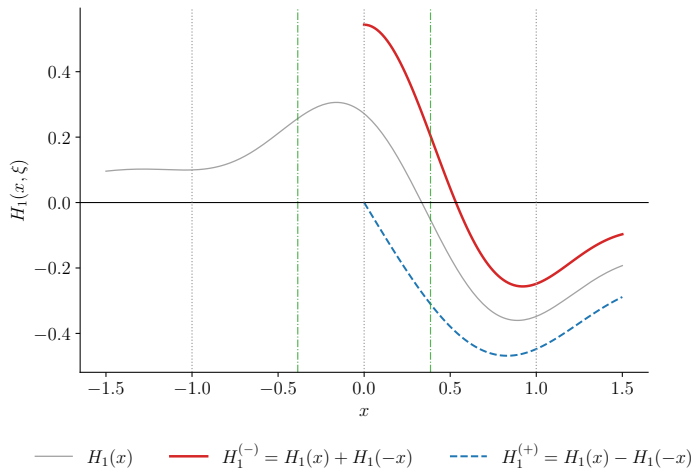
- The equal-time matrix element is evaluated directly.
- The same Hamiltonian and bilocal define the tensor-network and quantum calculations.

Quasi-PDFs



$$\int dx q_0(x) = N_c, \quad \int dx q_1(x) = v N_c$$

Quasi-GPD



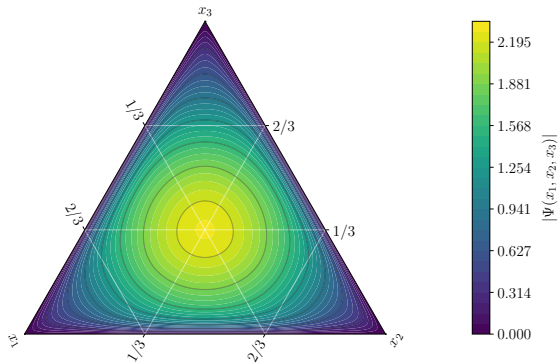
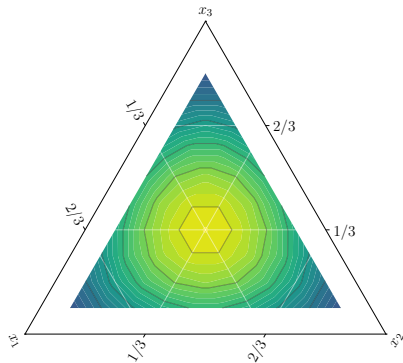
Kinematics

- $\bar{P} = 2.1$
- $\Delta = 1.62$
- $\xi = 0.38$

- Convergence to LC support of $|x| \leq 1$ only for $P \rightarrow \infty$.
- Worse here than for quasi-PDF since we require momentum transfer.

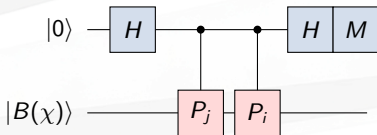
Quasi-Light-Cone Wave Function

$$\Psi(x_1, x_2, x_3) \sim \int \prod_{i=1}^3 \frac{dz_i}{2\pi} (2\pi) \delta(z_1 + z_2 + z_3) \exp \left[-iP \sum_{i=1}^3 x_i z_i \right] \langle 0 | \epsilon_{abc} \psi_a^1 \psi_b^2 \psi_c^3 | B(\chi) \rangle$$



Quantum Simulation Workflow

- 1 Prepare $|B(0)\rangle$ with symmetry-preserving VQE or adiabatic state preparation.
- 2 Apply $e^{-i\chi\mathbb{K}}$ with Suzuki–Trotter real-time evolution. Two rapidities give $\xi \neq 0$.
- 3 Measure the bilocal with a Hadamard test [Griener et al., 2024].
- 4 Fourier transform the bilocal and apply LaMET matching.



Natural register

One $d = 8$ qudit per staggered site, or three qubits per site.

Dominant cost

- Non-Abelian Coulomb term requires long-range connectivity
- Large circuit depth from boost

Summary

Verified LaMET computations for baryon distributions in QCD_2 :

- The reconstructed quasi-PDFs satisfy charge and momentum sum-rule checks
- Quasi-GPDs accessible for small values of skewness ξ
- The full quasi-LCWF is reconstructed from simulated correlators using simplex polynomials

Thank you!

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Present limitations

- LaMET requires a large baryon momentum P , but at the same time $aP \ll 1$.
- Tensor-network benchmark (currently) only practicable in 2d.

Reference calculation

- Quantum-number-conserving tensor networks benchmark the same Hamiltonian and correlators proposed for quantum simulation.
- The workflow in $d \geq 2$ is equivalent.

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Density matrix formulation for quantum renormalization groups.

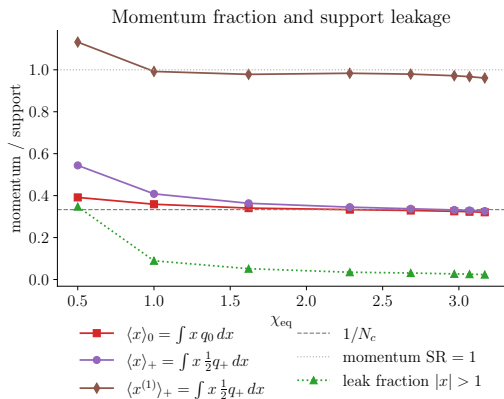
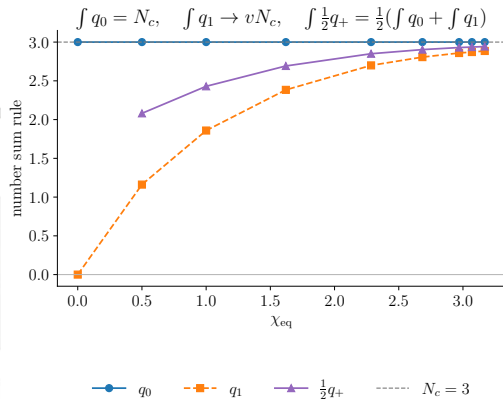
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Backup Slides

Boost and Momentum Operators

$$\begin{aligned}\mathbb{H} &= \int dx \, T^{00}, & \mathbb{K} &= - \int dx \, x T^{00}, \\ \mathbb{P} &= \int dx \, T^{01}, & |B(\chi)\rangle &= e^{-i\chi\mathbb{K}} |B(0)\rangle\end{aligned}$$

Quasi-PDF Sum Rules



Equal-Time Bilocal Correlator

The connected, color-summed one-body matrix element is

$$G[n_1, n_2] = \langle B' | \sum_c \psi_{n_1, c}^\dagger \psi_{n_2, c} | B \rangle_{\text{conn}}.$$

Straight Wilson line is the identity in axial gauge. We use

$$X = \frac{n_1 + n_2}{2}, \quad s = n_2 - n_1, \quad \zeta = as$$

and project the pair center onto the momentum transfer

$$S(s; \Delta^1) = \sum_X e^{+i\Delta^1 a X} G[X, s]_{\text{conn}}, \quad \Delta^1 = P_{\text{bra}}^1 - P_{\text{ket}}^1.$$

The forward limit has $\Delta^1 = 0$, while

$$S(0; 0) = \sum_n G[n, n]_{\text{conn}} = N_c$$

fixes the charge normalization exactly.

Unstaggering the Bilocal

Even separations populate the same-parity channels, while odd separations populate the cross channels

$$\begin{aligned} [\mathcal{C}_{ee} + \mathcal{C}_{oo}](2\ell) &= (-1)^\ell M_0(\ell), \\ [\mathcal{C}_{eo} + \mathcal{C}_{oe}](2\ell + 1) &= -i(-1)^\ell M_1(\ell). \end{aligned}$$

With $S(s)$ the center-summed bilocal, the two smooth amplitudes are

$$M_0(\ell) = (-1)^\ell S(2\ell), \quad M_1(\ell) = i(-1)^\ell S(2\ell + 1).$$

They sample the two loffe-time distributions on different grids

$$\mathcal{M}_{\gamma^0}(\nu_\ell) = M_0(\ell), \quad \nu_\ell = 2aP\ell, \quad \mathcal{M}_{\gamma^1}(\nu_\ell) = M_1(\ell), \quad \nu_\ell = aP(2\ell + 1).$$

Fourier Reconstruction

The even and odd separation grids give

$$q_0(x) = \frac{aP}{\pi} \left[M_0(0) + 2 \operatorname{Re} \sum_{\ell \geq 1} e^{-i2aP\ell x} M_0(\ell) \right],$$

$$q_1(x) = -\frac{aP}{\pi} 2 \operatorname{Re} \sum_{\ell \geq 0} e^{-iaP(2\ell+1)x} M_1(\ell).$$

Both have the Nyquist half-window

$$x_N = \frac{\pi}{2aP}, \quad q_0(x + 2x_N) = q_0(x), \quad q_1(x + 2x_N) = -q_1(x).$$

Their light-cone combination and the two normalization conditions are

$$q_+(x) = q_0(x) + q_1(x), \quad \int_{-x_N}^{x_N} q_0(x) dx = N_c, \quad \int_{-x_N}^{x_N} q_1(x) dx \longrightarrow N_c v.$$

Trilinear Quark Correlator

The staggered color-singlet annihilation operator is

$$\mathcal{T}_B(n_1, n_2, n_3) = \varepsilon_{c_1 c_2 c_3} (\psi_{n_1, c_1} + \psi_{n_1+1, c_1}) (\psi_{n_2, c_2} + \psi_{n_2+1, c_2}) \\ \times (\psi_{n_3, c_3} + \psi_{n_3+1, c_3}).$$

Its centroid-projected matrix element is

$$A_B(n_1, n_2, n_3; \chi) = \sum_X e^{-iaPX} \langle 0 | \mathcal{T}_B(X + n_1, X + n_2, X + n_3) | B(\chi) \rangle, \quad n_1 + n_2 + n_3 = 0.$$

The phase carries the momentum transfer $\Delta^1 = -P$

Quasi-LCWF Reconstruction

Two independent separations reconstruct the quasi-light-cone wave function

$$\Psi_B^{(\chi)}(x_1, x_2, x_3) = \sum_{n_1, n_2} e^{-iaP(x_1 n_1 + x_2 n_2 + x_3 n_3)} A_B(n_1, n_2, n_3; \chi), \quad n_3 = -n_1 - n_2.$$

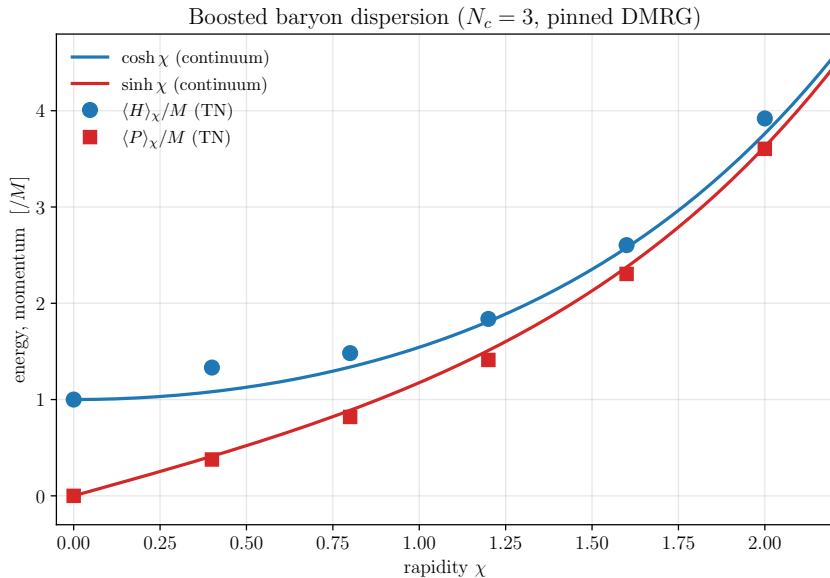
The momentum fractions lie on the two-simplex

$$x_1 + x_2 + x_3 = 1, \quad x_i \geq 0.$$

Color antisymmetry makes $\Psi_B^{(\chi)}$ symmetric under permutations of (x_1, x_2, x_3) . Normalized via

$$q_B(x) = N_c \frac{\int [dx]_2 \left| \Psi_B^{(\chi)}(x, x_2, x_3) \right|^2}{\int [dx]_3 \left| \Psi_B^{(\chi)} \right|^2}.$$

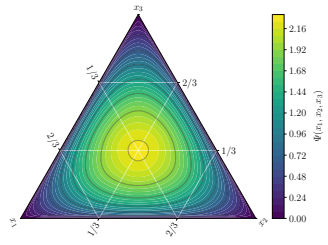
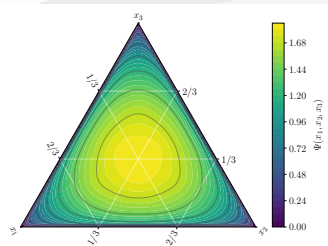
Pinned-DMRG Dispersion Scan



Large- N_c Light-Cone Wave Functions

Projecting the 't Hooft-model Hamiltonian onto the three-quark Fock sector gives a qualitative light-cone benchmark

$$M_B^2 \Psi = \bar{m}^2 \sum_i \frac{\Psi}{x_i} - \frac{g^2}{\pi} \sum_{i < j} \text{PP} \int dy \frac{\Psi(\dots) - \Psi}{(y - x_i)^2}.$$



Large- N_c Valence PDF

Integrating over the spectators gives

$$q_B(x) = 3 \int [dx]_2 |\Psi(x, x_2, x_3)|^2.$$

The normalization is imposed as $\int_0^1 q_B(x) dx = 3$, while permutation symmetry gives $\langle x \rangle = 1/3$. This remains a qualitative large- N_c benchmark.

