(A motivation to) subleading Higgs effects at lepton colliders

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Introduction

- Physical observables should be gauge-invariant $\rightarrow$ composite in non-abelian gauge theories
- Theories with a Higgs mechanism seem to escape because of Spontaneous Symmetry Breaking
- But Elitzur showed that a local symmetry can never be broken spontaneously
- This talk: give (one of many) motivations for looking into composite gauge-invariant operators for future collider physics

- The Higgs sector of the Standard Model:

$$
\begin{aligned}
\mathcal{L}_\Phi &= (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) - \frac{\lambda}{2}\left(\Phi^\dagger \Phi - \frac{v^2}{2}\right)^2 \\
\mathcal{D}_\mu &= \partial_\mu - ig' B_\mu Y - igW_\mu
\end{aligned}
$$

- The scalar field is an $SU(2)$ doublet $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ \phi_3 + i\phi_4\end{pmatrix}$ transforming as $\Phi \to U_Y(x)\Phi$ & $\Phi \to U_L(x)\Phi$,
- The potential is minimized by $\langle \Phi \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v\end{pmatrix}$, which breaks the local gauge symmetry $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- We define $\Phi = \frac{1}{\sqrt{2}}\left((v+h)1 + i\rho^a\tau^a\right) \cdot \begin{pmatrix}1 \\ 0\end{pmatrix}$, with h the Higgs field and ρ^a the Goldstone bosons, $\langle h \rangle = \langle \rho^a \rangle = 0$.

The propagator for the electroweak gauge boson

$$\langle W^a_\mu(x) W^a_\mu(y) \rangle,$$

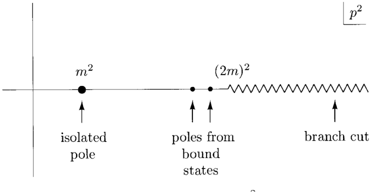
is not gauge-invariant under

$$W^a_\mu \quad \rightarrow \quad U_L(x) W_\mu U_L^{-1}(x) + \frac{i}{g} U_L(x) \partial_\mu U_L^{-1}(x),$$

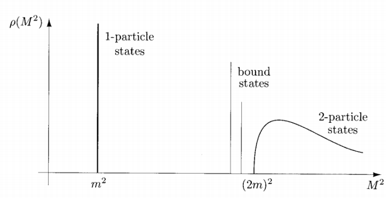
Yet, we find gauge-independent results, such as the W, Z masses. This is garanteed by Nielsen Identities.

Are other quantities that we can derive from the elementary field propagators, in particular the spectral density function, also gauge invariant?

$$G(p^2) = \frac{1}{p^2 + m^2 - \Pi(p^2)} = \int_0^\infty \frac{\rho(t)}{t + p^2} dt,$$



(a) Analytical structure of the two-point function for typical theory



(b) Typical spectral density function

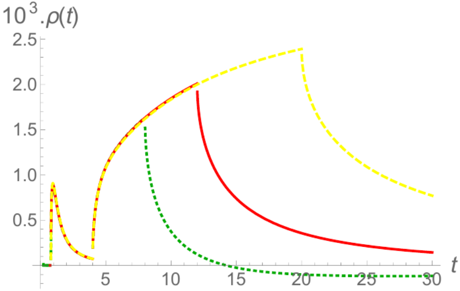


Figure: Spectral function for the reduced Higgs propagator $\langle h(p)h(-p)\rangle$, for gauge parameters $\xi = 2$ (Green, dotted), $\xi = 3$ (Red, Solid), $\xi = 5$ (Yellow, Dashed), with t given in units of μ^2, for the parameter values $e = 1$, $v = 1\,\mu$, $\lambda = \frac{1}{5}$.

Conclusion: we need gauge-invariant operators to access the spectral function!

- Fröhlich-Morchio-Strocchi (FMS) found that the Higgs model provides a natural way to formulate gauge-invariant extensions $\mathcal{O}(x)$ of the elementary fields $\varphi(x) = \{\Phi, A_\mu, \Psi_L\}$

$$\begin{aligned} Z\ boson &: \quad \Phi^\dagger D_\mu \Phi \\ W^\pm\ boson &: \quad \varepsilon_{ij}\Phi_i D_\mu \Phi_j\ \&\ c.c. \\ Higgs\ boson &: \quad \Phi^\dagger \Phi \\ Fermion: &: \quad \Phi^\dagger \Psi_L \end{aligned} \tag{1}$$

- The operators are constructed in such a way that for a constant value of the Higgs field, we regain the elementary field $\varphi(x)$

We can achieve the usual Higgs dynamics by taking Φ around v in the propagator of $\mathcal{O}$, so that

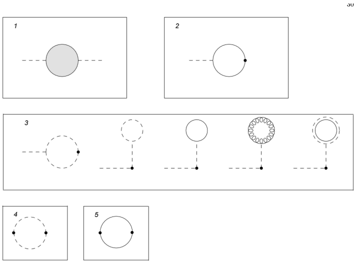
$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \langle \varphi(x)\varphi(y)\rangle + ...,$$

and it was found on the lattice that they share the same pole mass.

A. Maas, Mod.Phys.Lett.A 28 (2013), 1350103

Gauge-invariant spectral description of the $SU(2)$ Higgs model from local composite operators

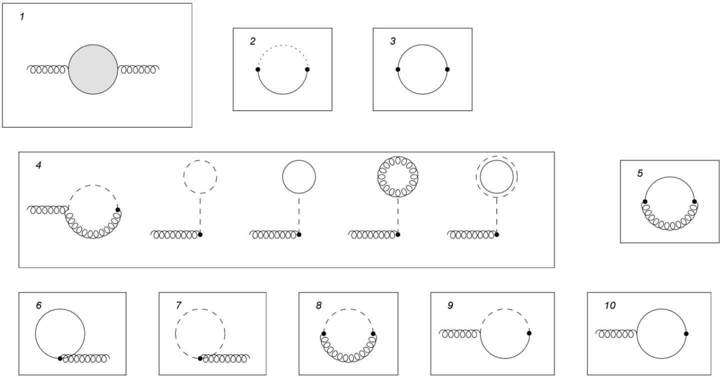
$$
\begin{aligned}
\langle O(x)O(y)\rangle &= v^2\langle h(x)h(y)\rangle + v\left\langle h(x)\rho^b(y)\rho^b(y)\right\rangle + v\left\langle h(x)h(y)^2\right\rangle \\
&+ \frac{1}{4}\left\langle h(x)^2\rho^b(y)\rho^b(y)\right\rangle + \frac{1}{4}\left\langle h(x)^2 h(y)^2\right\rangle \\
&+ \frac{1}{4}\left\langle \rho^a(x)\rho^a(x)\rho^b(y)\rho^b(y)\right\rangle .
\end{aligned}
$$



Gauge-invariant spectral description of the $SU(2)$ Higgs model from local composite operators

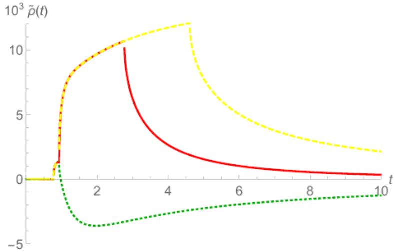
$$
\begin{aligned}
\langle R^a_\mu(p)\, R^a_\nu(-p)\rangle &= \frac{1}{16}g^2v^4\langle A^a_\mu(p)\, A^a_\nu(-p)\rangle - \langle(\rho^a\partial_\mu h)(p)\,(\partial_\nu\rho^a h)(-p)\rangle + \frac{1}{4}p_\mu p_\nu\langle(\rho^a h)(p)\,(\rho^a h)(-p)\rangle \\
&+ \frac{1}{8}p_\mu p_\nu\langle(\rho^a\rho^b)(p)\,(\rho^a\rho^b)(-p)\rangle - \frac{1}{2}\langle(\rho^a\partial_\mu\rho^b)(p)\,(\partial_\nu\rho^a\rho^b)(-p)\rangle \\
&+ \frac{1}{4}g^2v^3\langle A^a_\mu(p)\,(A^a_\nu h)(-p)\rangle - \frac{i}{4}gv^3p_\nu\langle A^a_\mu(p)\,\rho^a(-p)\rangle \\
&+ \frac{1}{4}v^2p_\mu p_\nu\langle\rho^a(p)\rho^a(-p)\rangle + \frac{1}{6}g^2v^2\langle(\rho^a A^b_\mu)(p)\,(\rho^a A^b_\nu)(-p)\rangle \\
&- \frac{1}{24}g^2v^2\langle(\rho^a\rho^a A^b_\mu)(p)\,A^b(-p)\rangle + \frac{1}{8}g^2v^2\langle(h^2A^a_\mu)(p)\,A^a_\nu(-p)\rangle \\
&+ \frac{1}{4}g^2v^2\langle(hA^a_\mu)(p)\,(hA^a_\nu)(-p)\rangle + \frac{i}{4}v^2gp_\mu\langle(h\rho^a)(p)\,A^a_\nu(-p)\rangle \\
&+ \frac{1}{2}gv^2\langle(\partial_\mu h\rho^a)(p)\,A^a_\nu(-p)\rangle + \frac{i}{2}gv^2p_\mu\langle\rho^a(p)\,(hA^a_\nu)(-p)\rangle \\
&- \frac{1}{4}gv^2\varepsilon^{abc}\langle A^a(p)\,(\rho^b\partial_\mu\rho^c)(-p)\rangle - \frac{igv^2}{2}\varepsilon^{abc}p_\mu\langle\rho^a(p)\,(\rho^bA^c_\nu)(-p)\rangle \\
&+ \frac{1}{2}vp_\mu p_\nu\langle(h\rho^a)(p)\,\rho^a(-p)\rangle - ivp_\nu\langle(\partial_\mu h\rho^a)(p)\,\rho^a(-p)\rangle
\end{aligned}
$$

Gauge-invariant spectral description of the $SU(2)$ Higgs model from local composite operators

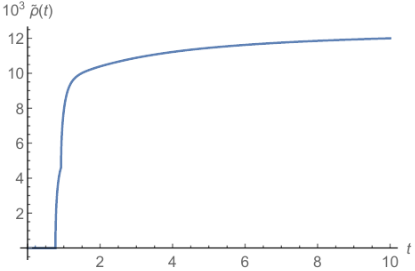


We find that the pole mass of the elementary operators and their gauge invariant extension are the same up to first order in $\hbar$

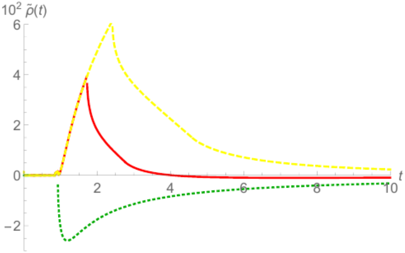
$$\begin{aligned} m_{\text{pole},<\text{hh}>} &= m_{\text{pole},<\text{OO}>} \\ m_{\text{pole},<\text{AA}>} &= m_{\text{pole},<\text{RR}>} \end{aligned}$$



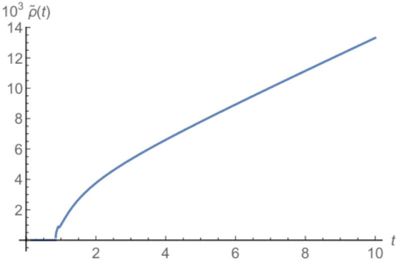
(a) Spectral function for $\langle h(x)h(y)\rangle$,
$\xi = 2$ (Green), $\xi = 3$ (Red), $\xi = 5$ (Yellow)



(b) Spectral function for $\langle O(x)O(y)\rangle$



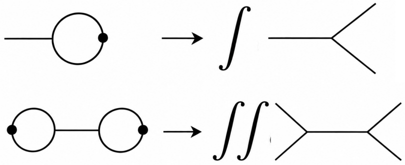
(a) Spectral function for $\langle A^a_\mu(x) A^a_\nu(y)\rangle^T$, $\xi = 2$ (Green), $\xi = 3$ (Red), $\xi = 5$ (Yellow)



(b) Spectral function for $\langle R^a_\mu(x) R^a_\nu(y)\rangle^T$

FMS at future colliders

- The FMS construction is necessary for a complete gauge-invariant picture of the electroweak sector
- Can FMS construction lead to measurable differences? Mass: no. Cross sections?



- All terms have a pole at the relevant mass → contribute in the LSZ-reduced matrix element → cross sections will always deviate
- Augmented Perturbation Theory was tested on the lattice (see next talk)
- NLO and NNLO effect in the SM: detectable at future colliders