FMS on the lattice

The structure of asymptotic states in weak physics

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Brout-Englert-Higgs (BEH) effect

- Weak sector of the Standard Model

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} + \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) - \lambda \left(\phi^\dagger \phi - f^2\right)^2$$

- Perturbative approach to Higgs physics

1. $V(\phi)$ allows for a non-trivial degenerate minimum at $\phi \neq 0$ $\longrightarrow$ BEH effect active
2. Shift $\phi(x) = vN + \varphi(x)$ with $\langle \phi \rangle = vN$ $\longrightarrow$ mass for gauge bosons & Goldstone fields
3. Quantize theory using the path integral formalism $\longrightarrow$ 't Hooft gauge & ghost fields
4. BRST construction $\longrightarrow$ ghosts/Goldstones cancel & gauge bosons/Higgs are singlets

Böhm, Denner, Joos, *Gauge theories of the strong and electroweak interaction* (Teubner, Stuttgart, 2001)
Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721

Weak physics and its challenges

- The standard perturbative approach to Higgs physics has some problems
  - **Elitzur's theorem:** gauge symmetry can never be spontaneously broken
  - **Gribov copies:** explicitly breaks the perturbative BRST construction
  - **Gauges with zero vev possible:** massless gauge boson propagator at tree-level

- Consequences
  - Asymptotic states must be described by **gauge-invariant objects**
  - **Composite operators** with the same global quantum numbers as the elementary fields

$\longrightarrow$ **Gauge-invariant approach to weak physics**

Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721

Fröhlich-Morchio-Strocchi (FMS) mechanism

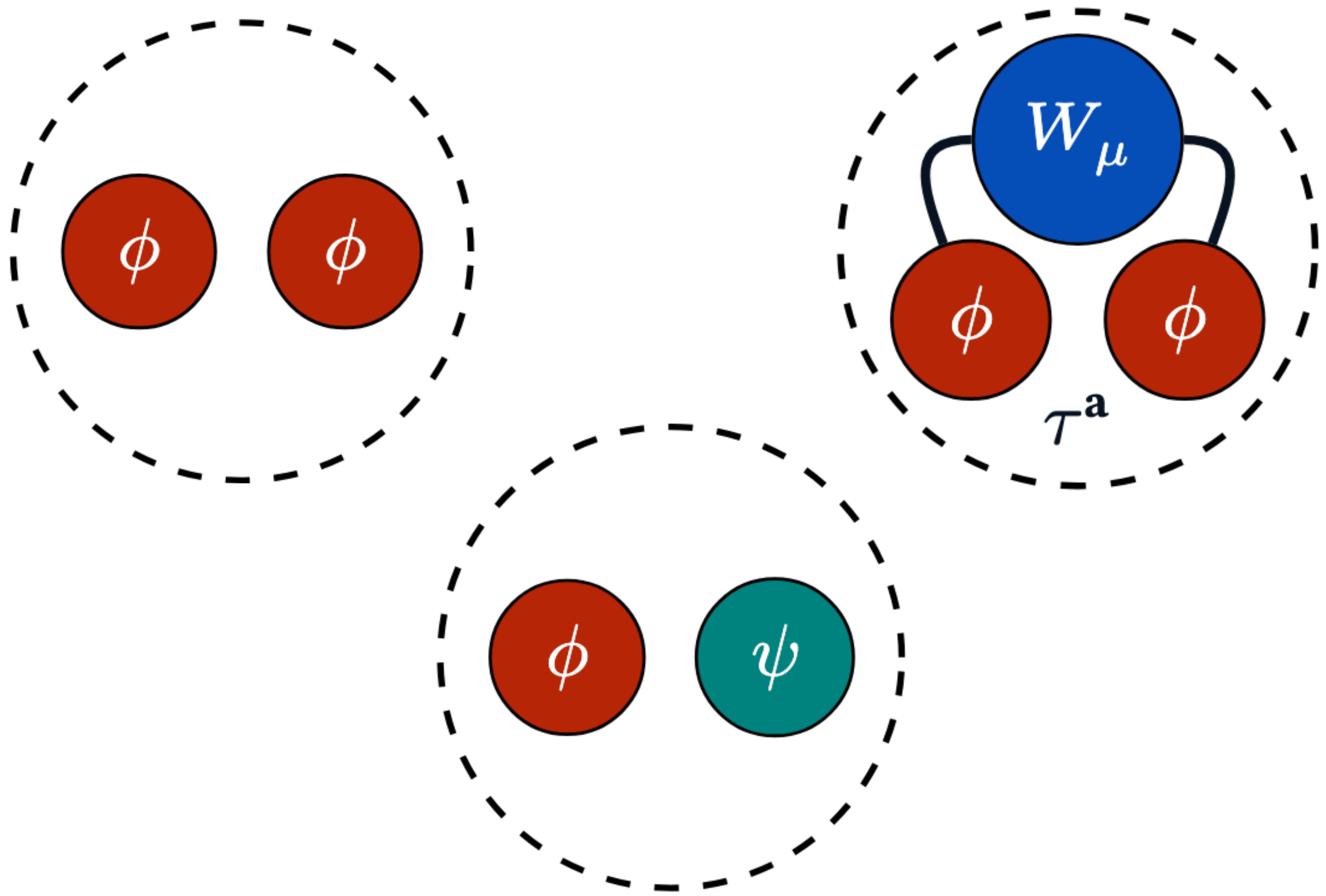
- **Weak SM:** FMS mechanism predicts a **one-to-one mapping** between gauge-dependent and gauge-invariant states and explains why perturbation theory is successful

- **Mechanism:** Representations of the gauge group are mapped onto representations of the global custodial group

- Physical Higgs (scalar singlet): $\phi(x) \rightarrow \left(\phi^{\dagger}\phi\right)(x)$

- Physical W (vector triplet): $W_{\mu}^{a}(x) \rightarrow \left(\tau^{\mathbf{a}}\phi^{\dagger}D_{\mu}\phi\right)(x)$

- Physical Lepton (spin-1/2 doublet): $\psi(x) \rightarrow \left(\phi^{\dagger}\psi\right)(x)$



Fröhlich, Morchio, Strocchi, Phys. Lett. B97 (1980)
Fröhlich, Morchio, Strocchi, Nucl. Phys. B190 (1981)
Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721

Augmented perturbation theory (APT)

- Masses of bound states are **non-perturbative quantities**, but as long as the BEH effect is active, analytical calculations are possible

- **Example:** Scalar bound state $\mathcal{O}_{0^+}(x) = \phi^\dagger(x)\phi(x)$
  - Rewrite the propagator using $\phi(x) = vN + \varphi(x)$ and $h(x) = \mathrm{Re}\{N^\dagger\varphi(x)\}$
  - Perform a perturbative tree-level expansion

$$\langle \mathcal{O}_{0^+}(x)\mathcal{O}^\dagger_{0^+}(y)\rangle = 4v^2\langle h(x)h(y)\rangle_{\mathrm{tl}} + \langle h(x)h(y)\rangle^2_{\mathrm{tl}} + \mathcal{O}(g^2, \ldots)$$

  - Mass pole of bound state coincides with the mass pole of the elementary correlator

Fröhlich, Morchio, Strocchi, Phys. Lett. B97 (1980)
Fröhlich, Morchio, Strocchi, Nucl. Phys. B190 (1981)
Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721

Why should we care?

- **Deviations at higher orders** between perturbation theory and a gauge-invariant approach, e.g., vector boson scattering Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756

- **Deepen understanding of QFT:** Learn more about the fundamental field-theoretical effects related to gauge invariance → guaranteed discovery

- **Implications for future experiments:** Set baseline for future measurements & avoid false positive regarding new physics

- **Phenomenological implications:** Model building should focus on global custodial group
  → Explicit example: SU(N) for N > 2 with one fundamental scalar
  Maas, Sondenheimer, Török, Ann. Phys. 402 (2019), 1709.07477
  Maas, Török, Phys. Rev. D 95 (2017), 1607.05860
  Maas, Török, Ann. Phys. 397 (2018), 1804.04453

SU(2) scalar-fermion-gauge system

- Two generations of leptons & matrix-valued scalar field

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} + \frac{1}{2}\mathrm{tr}\left[\left(D_\mu X\right)^\dagger\left(D^\mu X\right)\right] - \lambda\left(\frac{1}{2}\mathrm{tr}\left[X^\dagger X\right] - f^2\right)^2$$
$$+ \sum_g \bar{\psi}^L_g i\not{D}\psi^L_g + \sum_f \bar{\chi}^R_f i\not{\partial}\chi^R_f - \sum_{\bar{f}f} Y_{\bar{f}f}\left[\left(\bar{\psi}^L X\right)_{\bar{f}}\chi^R_f + \bar{\chi}^R_f\left(X^\dagger\psi^L\right)_{\bar{f}}\right]$$

$$X := X_i^{\mathbf{i}} = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}$$

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$$+ \sum_g \bar{\psi}_g \left(i\not{D} - m_{\psi_g}\right)\psi_g + \sum_f \bar{\chi}_f \left(i\not{\partial} - m_{\chi_f}\right)\chi_f - \sum_{\bar{f}f} Y_{\bar{f}f}\left[\left(\bar{\psi}X\right)_{\bar{f}}\chi_f + \bar{\chi}_f\left(X^\dagger\psi\right)_{\bar{f}}\right]$$

- Chiral nature poses conceptual problems on the lattice → **vectorial leptons** (Dirac spinors)

- No *L* & *R*; **parity is restored** → not important for testing the FMS mechanism

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$$+ \sum_g \bar{\psi}_g \left(i\not{D} - m_{\psi_g} \right) \psi_g + \cancel{\sum_f \bar{\chi}_f \left(i\not{\partial} - m_{\chi_f} \right) \chi_f} - \cancel{\sum_{\bar{f} f} Y_{\bar{f} f} \left[\left(\bar{\psi} X \right)_{\bar{f}} \chi_f + \bar{\chi}_f \left(X^\dagger \psi \right)_{\bar{f}} \right]}$$

N_g

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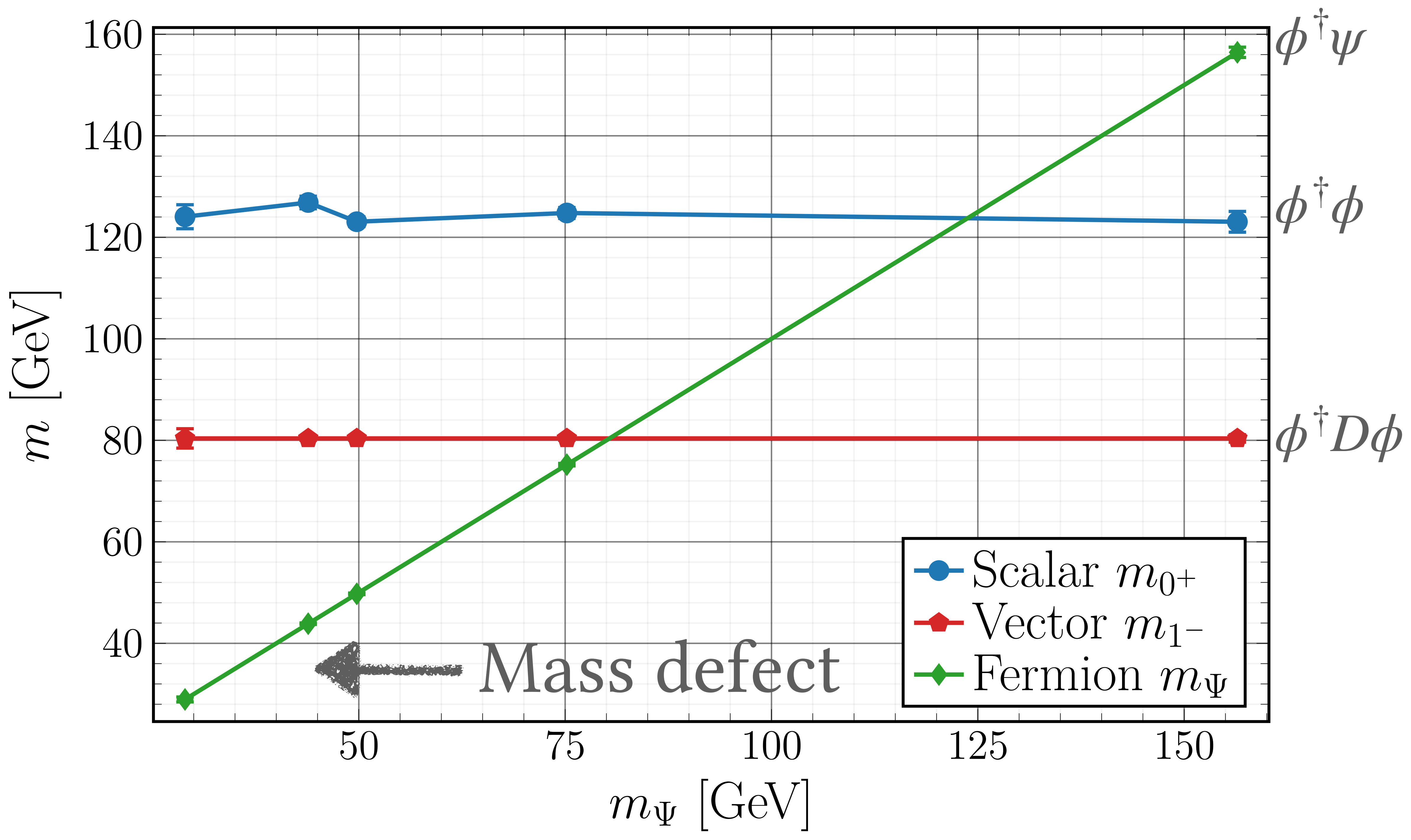
First simulations

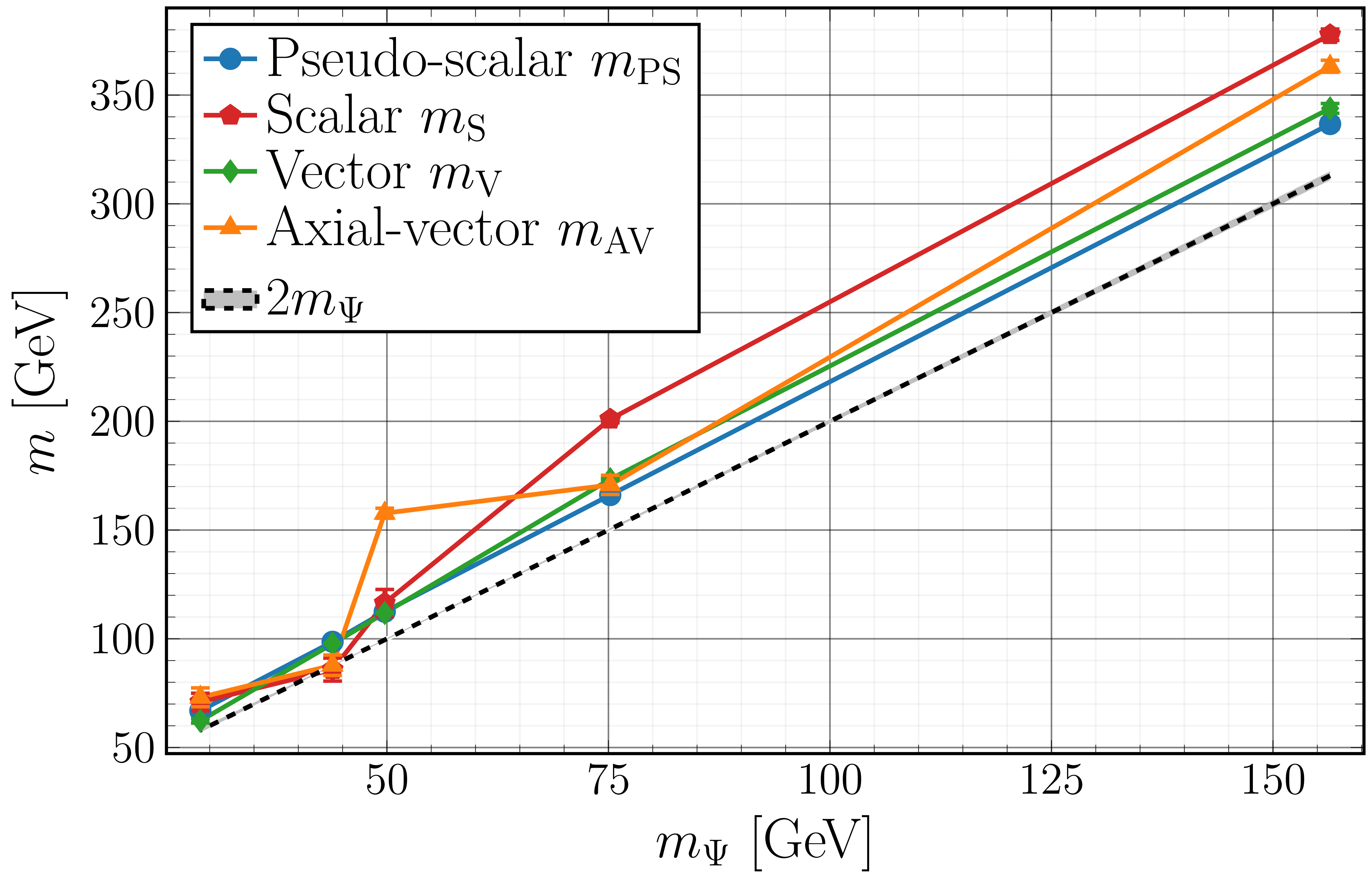
- $Y = 0$ (ungauged fermions decouple) & two degenerate generations of gauged fermions
- Publicly available **HiRep** simulation code

Del Debbio, Patella, Pica, Phys. Rev. D 81 (2010), 0805.2058
Hansen *et al.*, EPJ Web Conf. 175 (2018), 1710.10831

Fermion mass as a control parameter

Composite FMS operators & leptonium channels

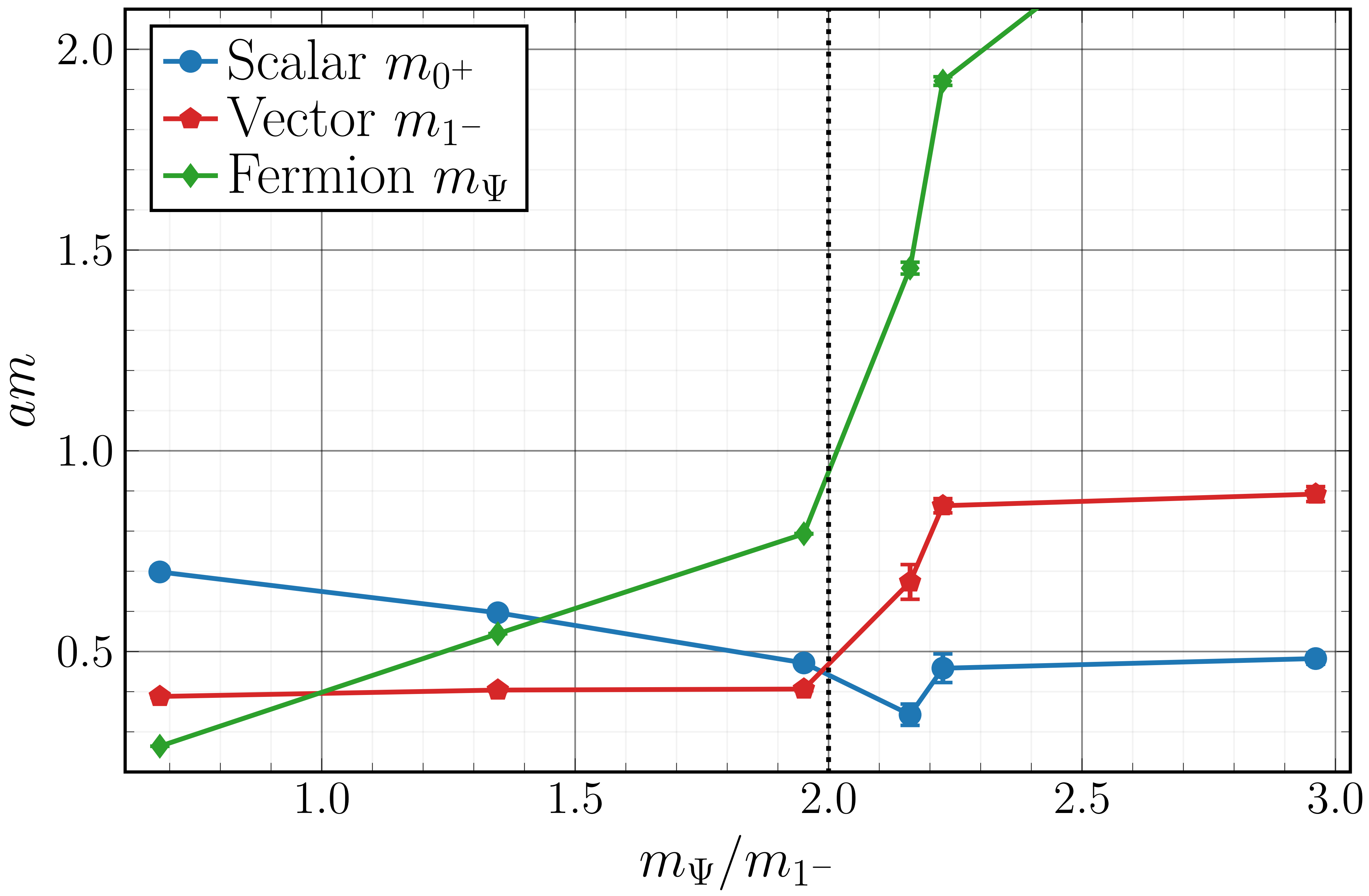


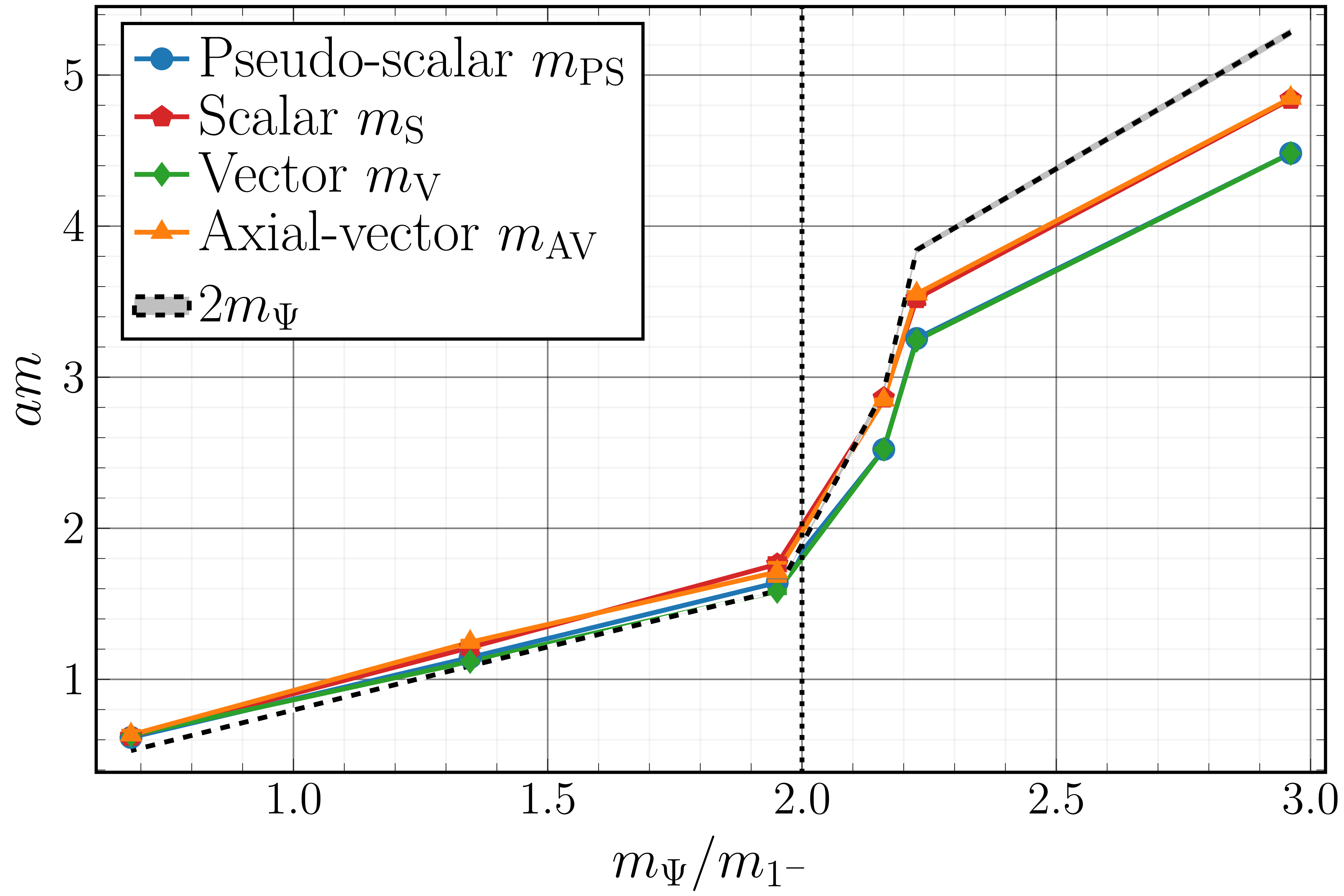


- System can be controlled via the fermion mass without changing the dynamics of the gauge-scalar subsystem
- Leptonium channels above the $2m_\Psi$ threshold → consistent with scattering states of two physical leptons

Light fermions trigger the BEH effect

Composite FMS operators & leptonium channels

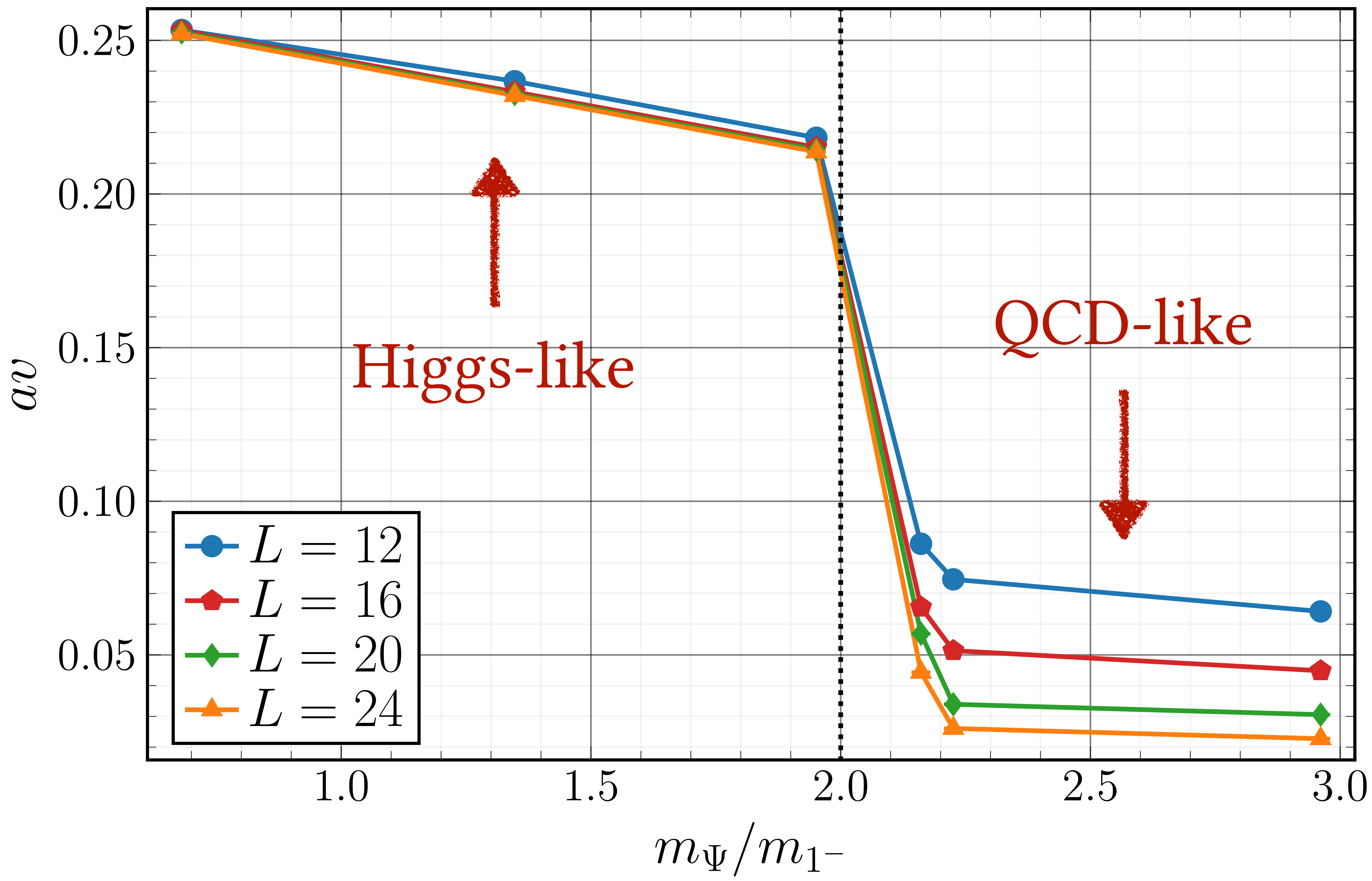




- Starting in the QCD-like domain shows qualitative differences: changes in the mass hierarchy at $m_\Psi = 2m_{1^-}$
- Transition in all leptonium channels: below vs. above the $2m_\Psi$ threshold

Light fermions trigger the BEH effect

Transition from QCD-like to Higgs-like



- Volume dependence in the confinement-like domain hints towards $v \to 0$ for $L \to \infty$
- Only small finite size effects in the Higgs-like domain

Quasi-PDFs from the lattice

Substructure of composite objects

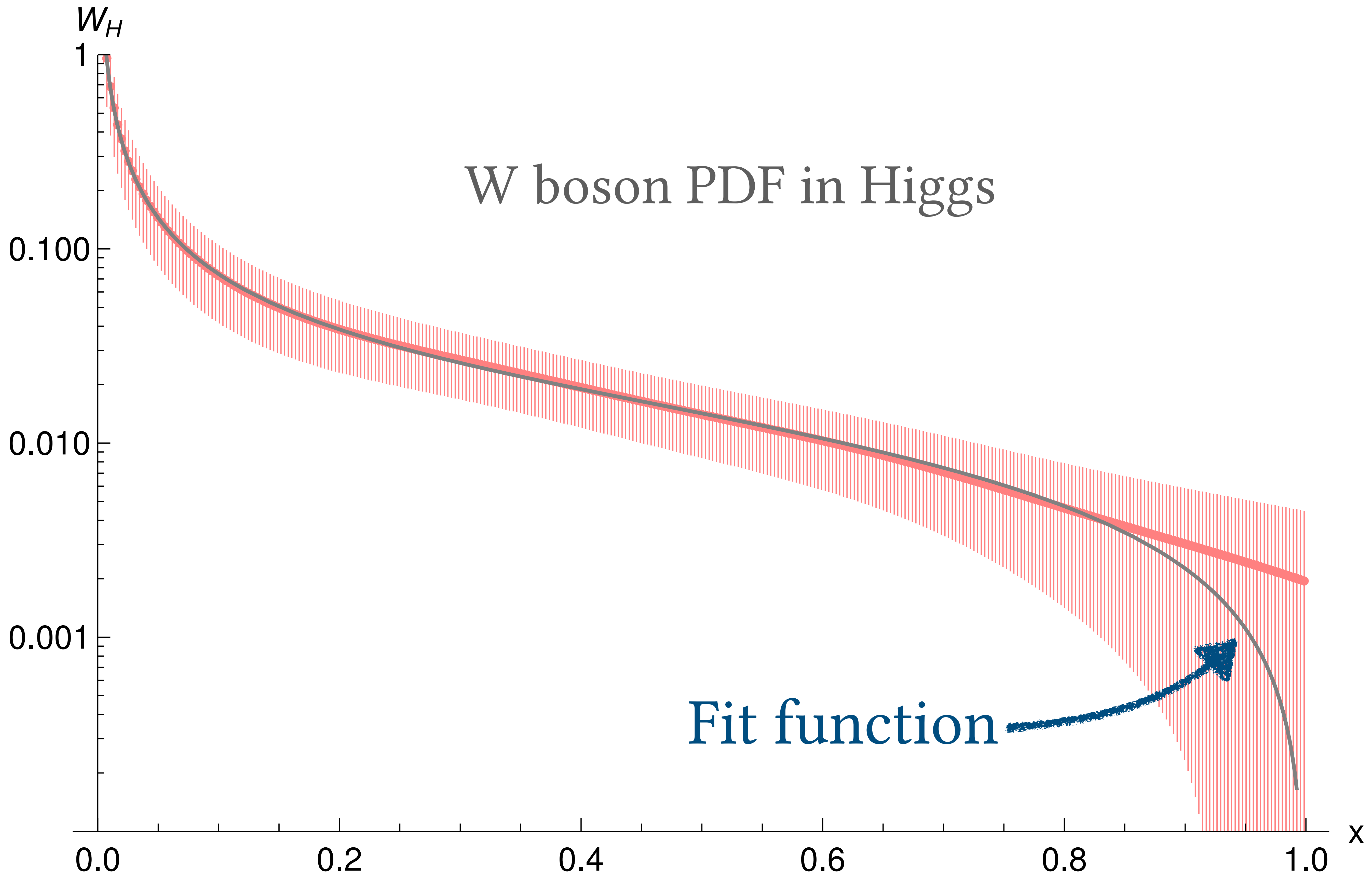
- PDFs obtained from lattice calculations for the gauge-scalar theory (large P_3)

$$\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle p|F(z)|p\rangle$$

- Complex structure of asymptotic states could be visible in HL-LHC data

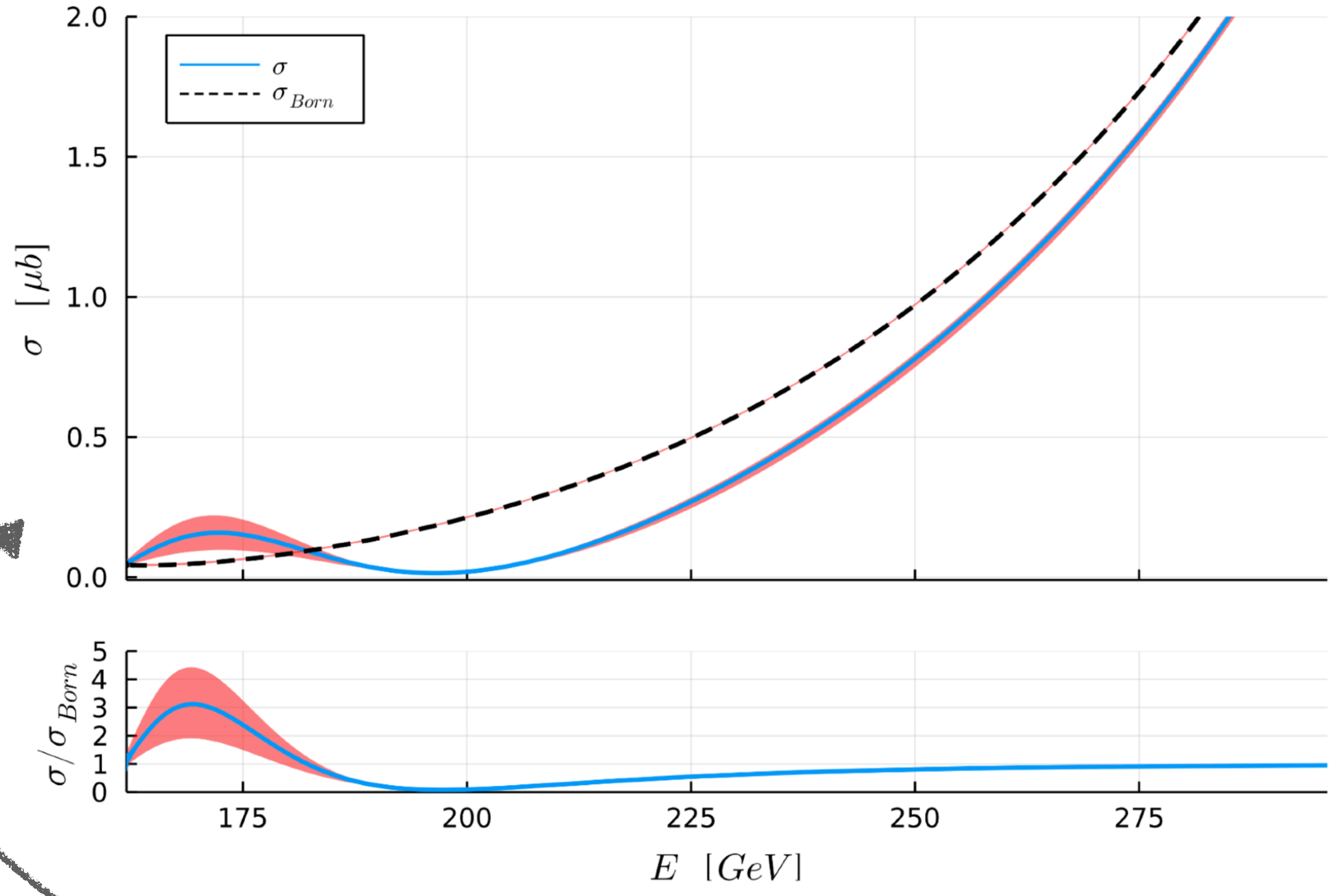
- Generic fit form is compatible with results from phenomenology

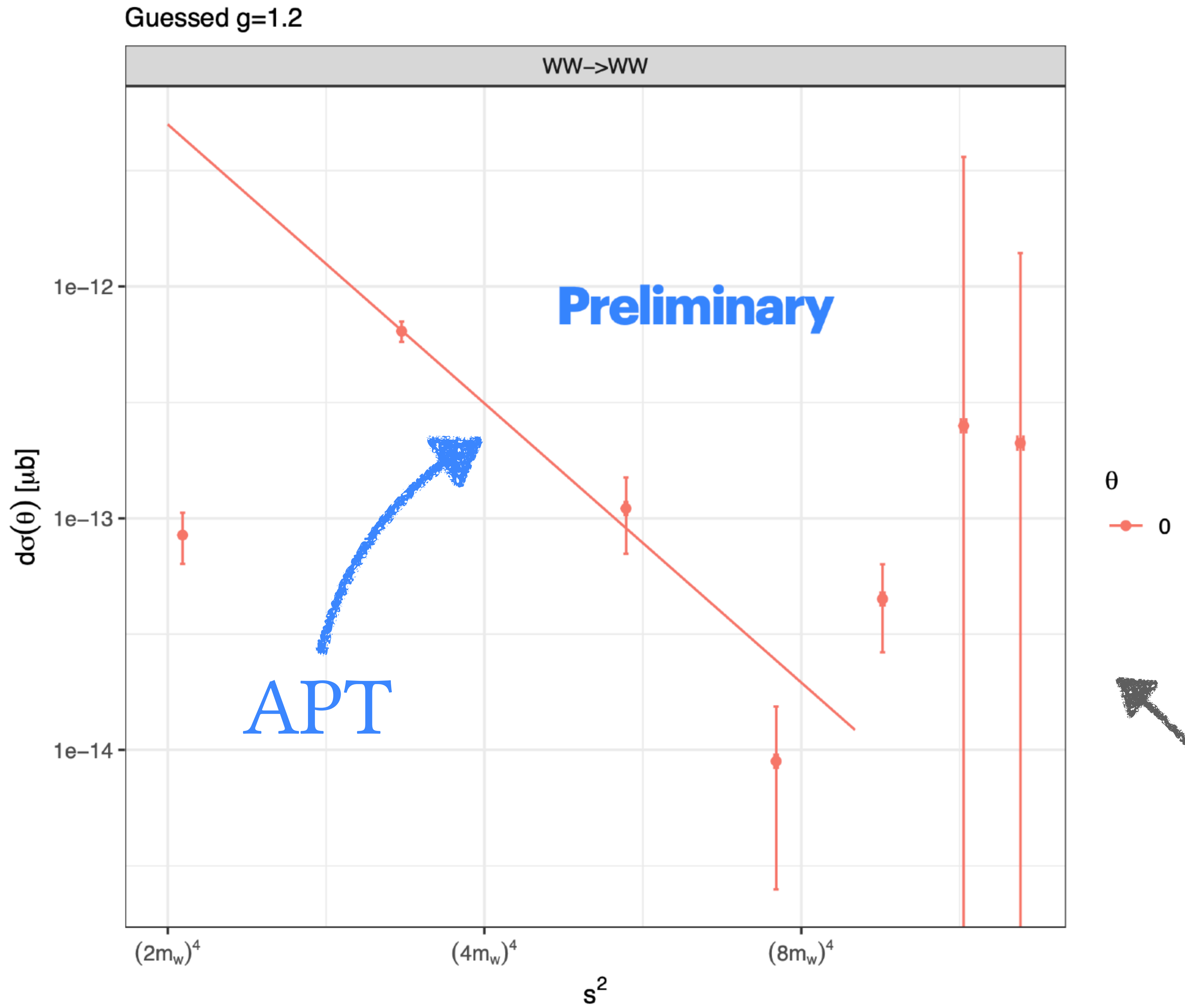
Fernbach et al., Phys. Rev. D 101 (2020), 2002.01688



Cross sections

First steps towards FMS phenomenology





- **Gauge-scalar system:** Cross-section for vector boson scattering
  Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756
- **Including gauged fermions dynamically:** Noisy but above the inelastic threshold, matching APT

Summary & outlook

- Simulation of a proxy theory to the weak sector of the SM in a fully gauge-invariant setup
- The physical spectrum of the theory in the Higgs-like domain supports an interpretation of weak physics based on the FMS mechanism
- Quasi-PDFs for H/W & first results for cross sections from lattice calculations with dynamical fermions → first steps towards FMS phenomenology for HL-LHC & FCC-ee
- **Outlook**

  - Scattering analysis: $e^+e^- \to H/W \to \mu^+\mu^-$ & excited states of e/μ

  - Compare simulations with analytical results from APT

  - Dynamic simulations with Yukawa couplings & ungauged fermions

Backup

Composite operators on the lattice

- Scalar singlet: $\mathcal{O}_{0^+}(x) = \frac{1}{2}\mathrm{tr}\left[X^\dagger(x)X(x)\right]$

- Vector triplet: $\mathcal{O}^{\mathbf{a},\mu}_{1^-}(x) = -\frac{i}{2}\mathrm{tr}\left[\tau^{\mathbf{a}}X^\dagger(x)U^\mu(x)X(x+e_\mu)\right]$

  Evertz *et al.*, Phys. Lett. B 171 (1986)

- Fermion bound state: $\Psi^L_{\alpha,\mathbf{i}}(x) = \left(X^\dagger\right)^i_{\mathbf{i}}(x)\,\psi^L_{\alpha,i}(x)$

  \- Correlator constructed by Wick contraction and a trace in the Dirac structure

$$M_{\mathbf{ij}}(x|y) = \left(X^\dagger\right)^i_{\mathbf{i}}(x)\ \left(D^{-1}\right)_{ij}(x|y)\ \left(X\right)^j_{\mathbf{j}}(y)$$

  Afferrante *et al.*, SciPost Phys. 10 (2021), 2011.02301

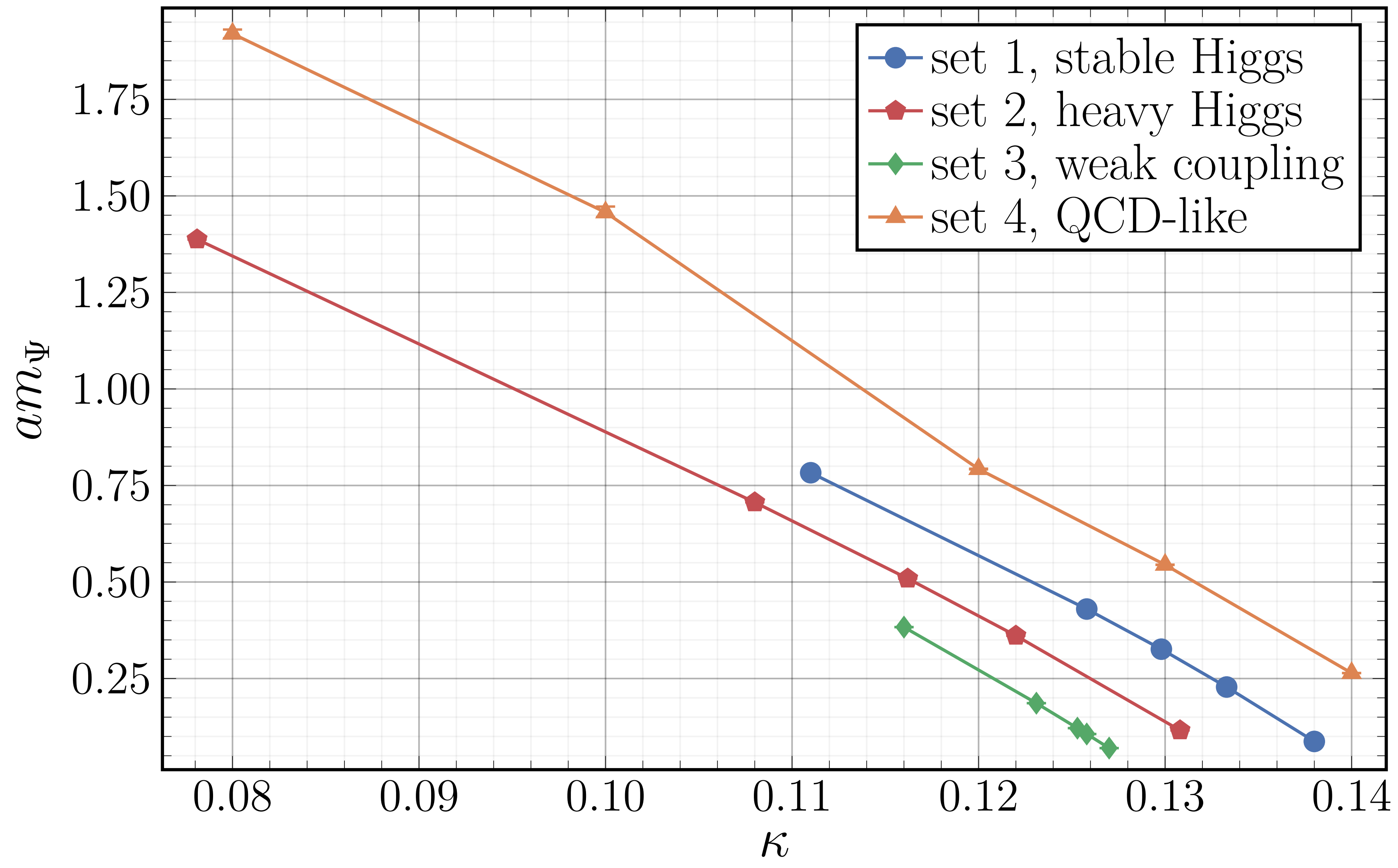
- Leptonium operator: $\mathcal{O}_\Gamma(x) = \bar{\psi}_1\Gamma\psi_2$ with 1 and 2 generational indices

Γ	γ_5	$\mathbb{1}$	γ_k	$\gamma_5\gamma_k$
J^P	0^-	0^+	1^-	1^+

Data sets

- 4 parameter sets from gauge-scalar simulations

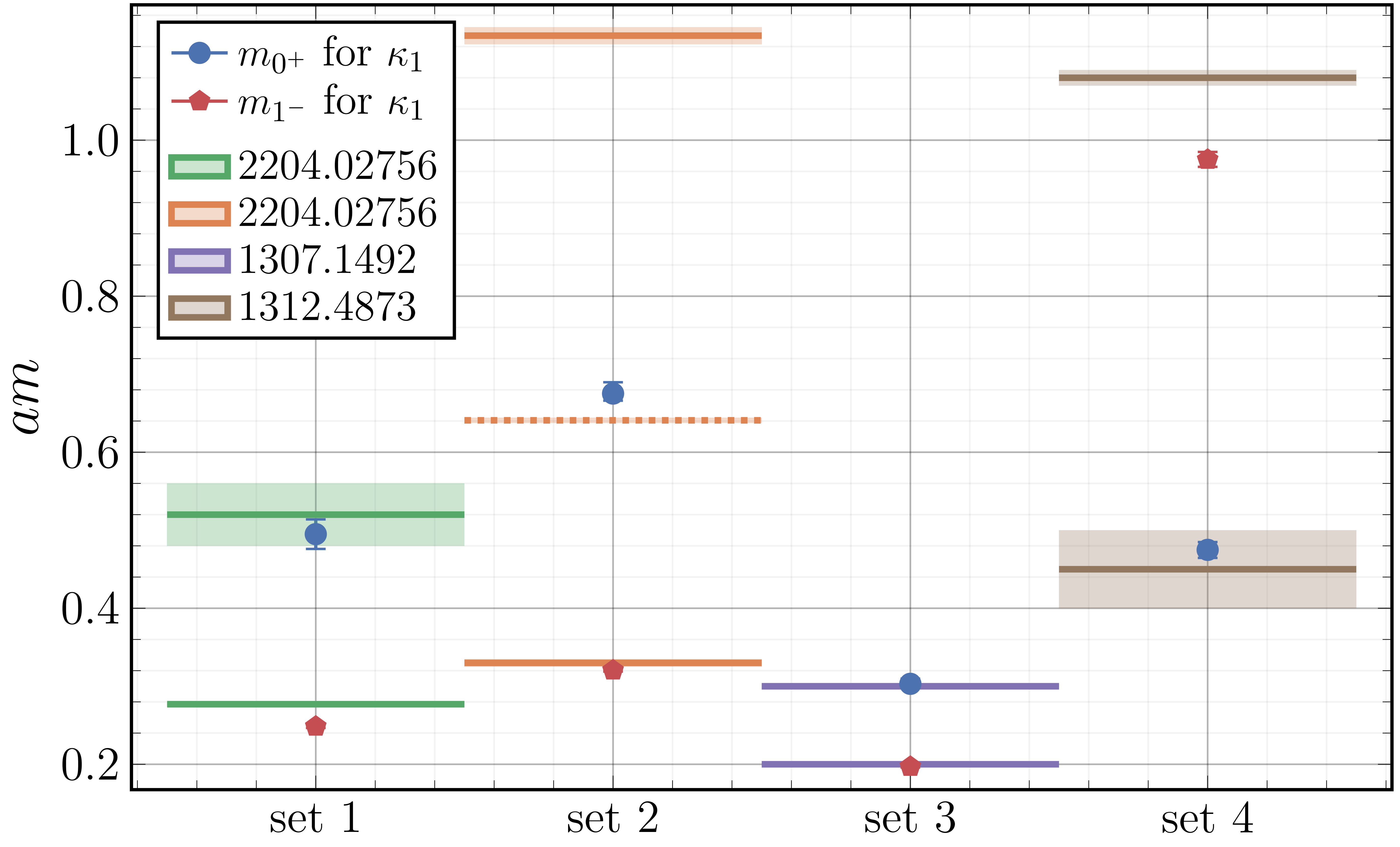
  - **Set 1:** stable Higgs
    Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756
  - **Set 2:** Higgs-like resonance
    Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756
  - **Set 3:** weak coupling
    Wurtz, Lewis, Phys. Rev. D 88 (2013), 1307.1492
  - **Set 4:** QCD-like domain
    Maas, Mufti, JHEP 1404 (2014), 1312.4873



- **Idea:** start with a parameter set from gauge-scalar simulations and slowly reduce the fermion mass by increasing the hopping parameter $\kappa = \left[2\left(am_F + 4\right)\right]^{-1}$

- Search for points of interest to simulate various scenarios (scattering analysis)

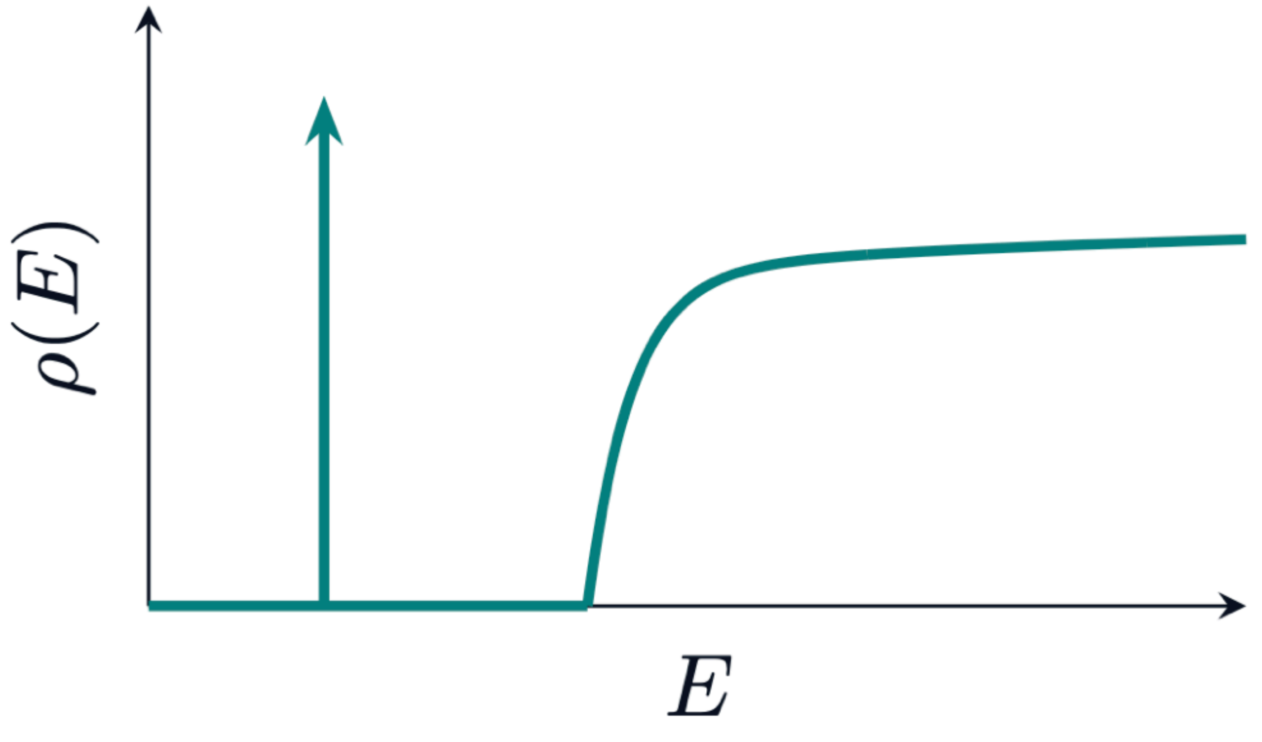
Comparison to gauge-scalar simulations



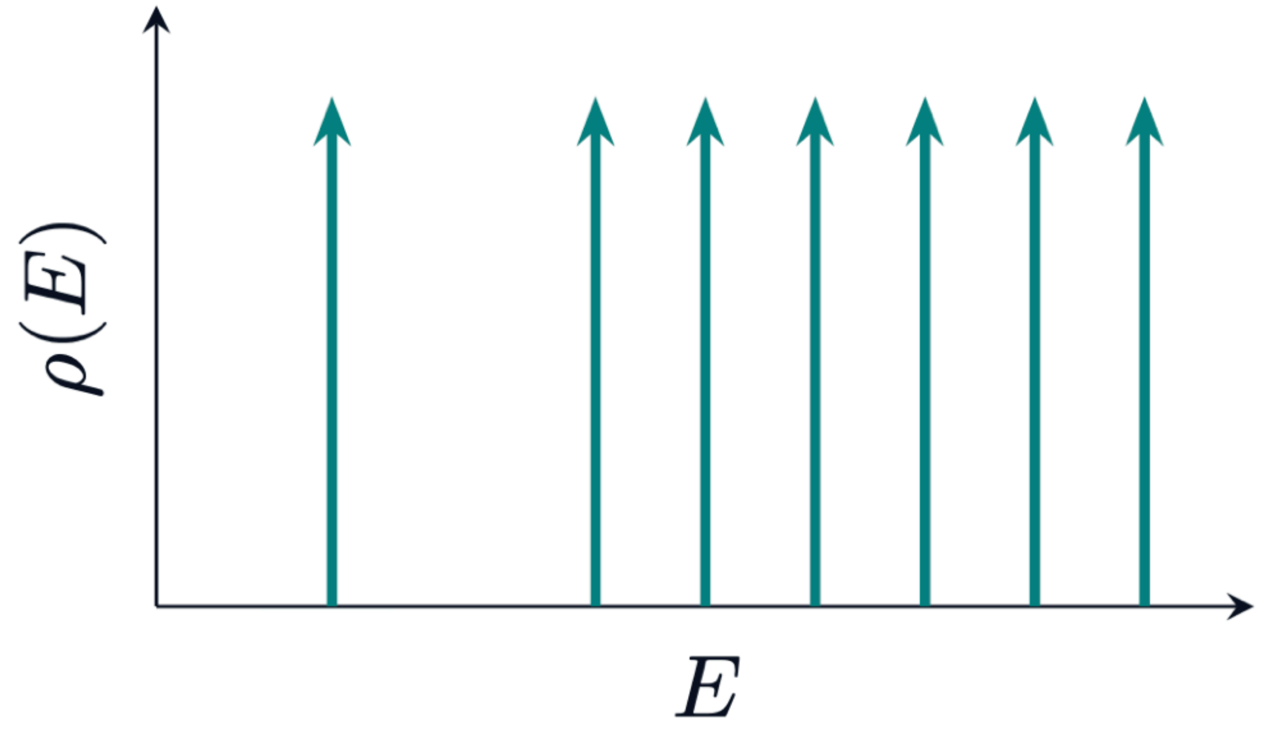
- Choose lowest κ (largest fermion mass)
- Discrepancies due to different bases and fitting methods

Spectral densities from lattice calculations

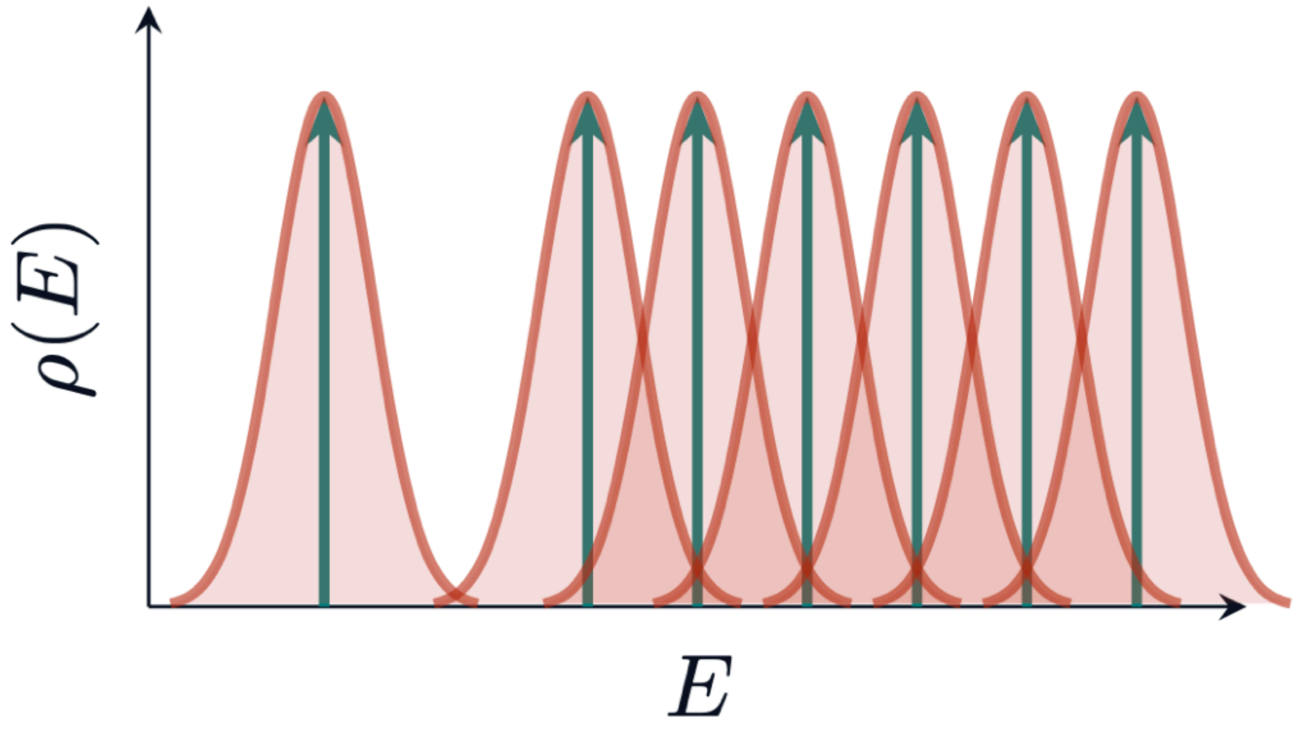
Continuum



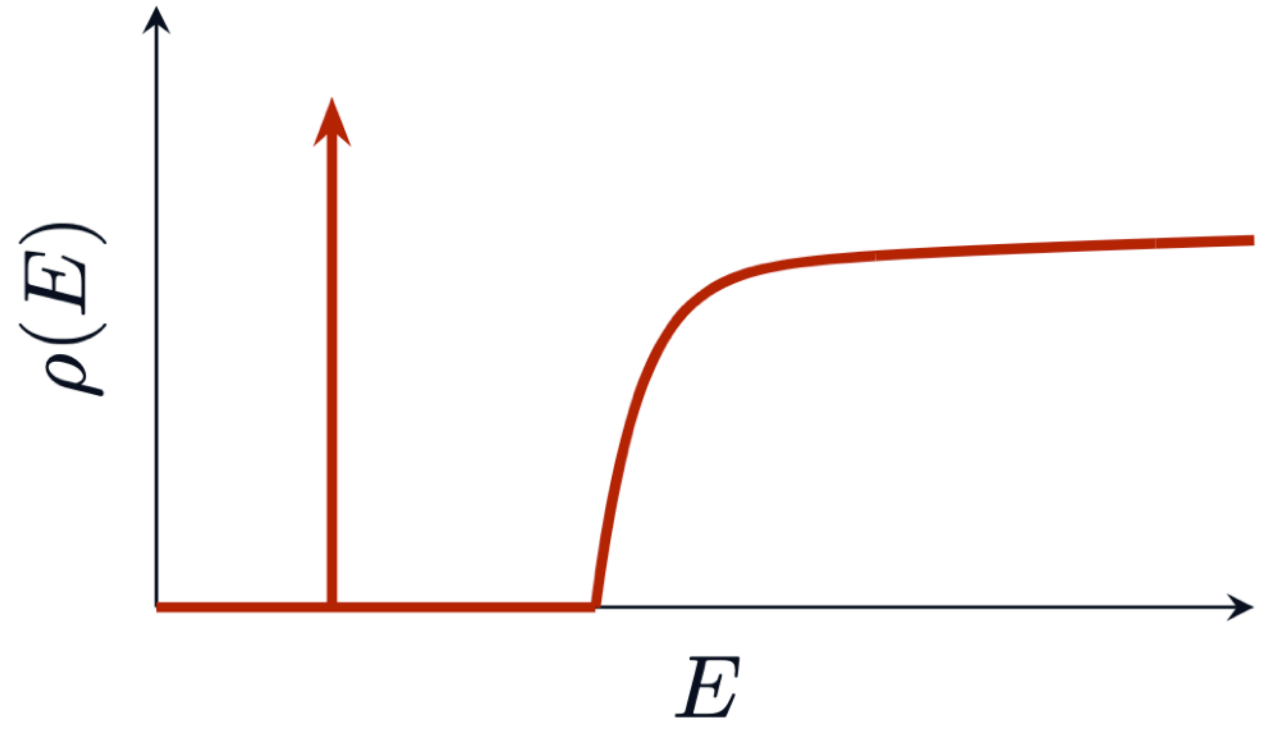
Lattice / Finite Volume



Smearing procedure with width σ



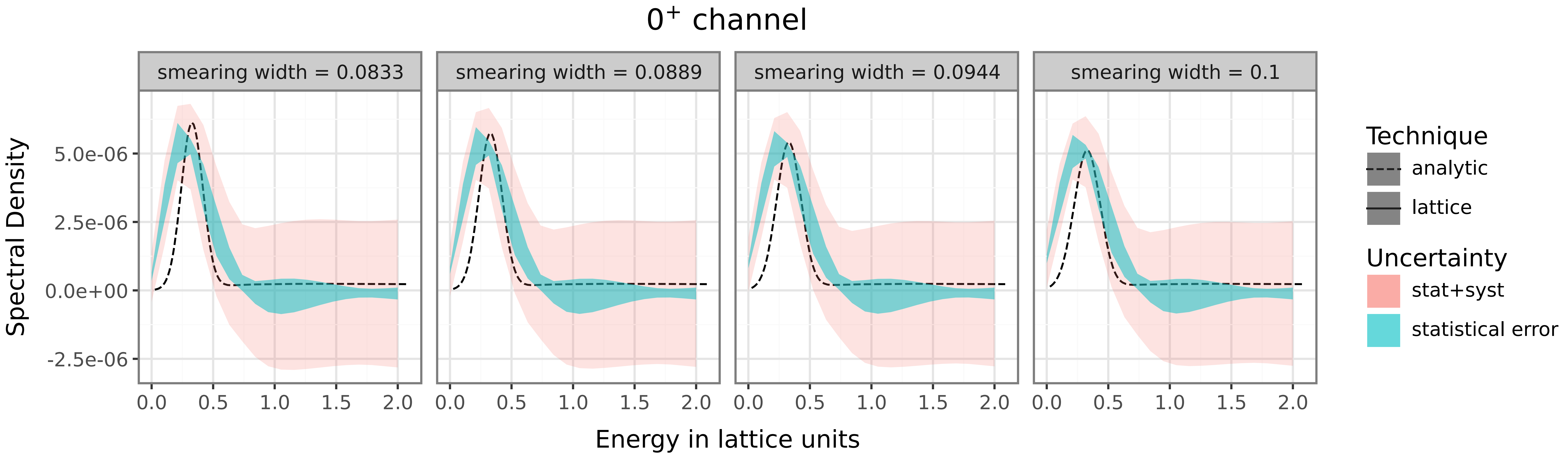
Take limits $L \to \infty$ and $\sigma \to 0$



Backus, Gilbert, Geophys. J. R. Astron. Soc. 16 (1968)
Backus, Gilbert, Phil. Trans. R. Soc. A266 (1970)

Spectral densities from lattice calculations

- Higgs spectral densities obtained via the Hansen-Lupo-Tantalo method with $1/L \ll \sigma \ll m_{1^-}$
  Hansen, Lupo, Tantalo, Phys. Rev. D 99 (2019), 1903.06476
- Compared with an analytic computation smeared with the same smearing parameter
  Maas, Sondenheimer, Phys. Rev. D 102 (2020), 2009.06671



Deviations at higher orders

- **Example:** Differences between PT and a gauge-invariant approach in vector boson scattering
  Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756

- Generic behavior of interactions between gauge-invariant composite states
  - **Low energies:** Bound states probed as a whole
  - **Intermediate energies:** Only FMS-dominant contributions matter, close to PT
  - **High energies:** Other valence particles and sea particles probed as well

  Maas, *The Fröhlich-Morchio-Strocchi mechanism: an underestimated legacy* (Springer, Cham, 2023), 2305.01960