Geon Propagator in Causal Dynamical Triangulations

Johannes Anzengruber

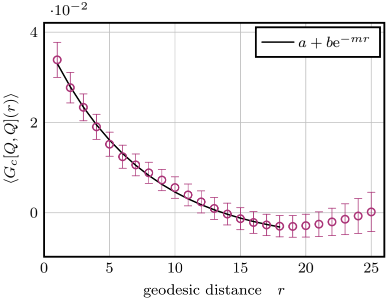
21. September 2026 , ÖPG-CMD Joint Meeting 2026

Supervisors: *Axel Maas*, *Simon Plätzer*

Dark matter from pure gravity

Geons as candidates for dark matter

- Self-bound state of gravitational waves / gravitons
  [Wheeler, Phys. Rev. **97** (1955)]
- Hints for a massive state from curvature-curvature correlation functions $G_c[Q,Q](r)$ in lattice quantum gravity



[Maas et al., Phys. Lett. B **879** (2026), arXiv: 2504.11047 (hep-lat)]

Geons may form (primordial) black holes

- Next generation gravitational wave detectors allow to probe black holes of broad mass range
- Detection of quantum signatures in gravitational wave signals (equation of state for astrophysical objects formed from geons?)

[Bagui et al., Living Rev. Rel. **28** (2025), arXiv: 2310.19857 (astro-ph.CO)]

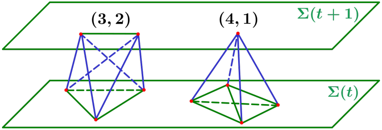
Taking a conservative approach to quantum gravity

Functional integral quantization of pure gravity

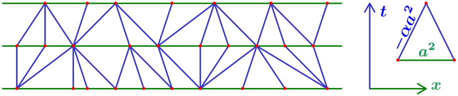
$$Z(G,\Lambda) = \int \mathrm{D}[g]\ \mathrm{e}^{\mathrm{i}S^{\mathrm{EH}}[g_{\mu\nu}]}, \quad S^{\mathrm{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}(R-2\Lambda)$$

- Approximation of a manifold with piecewise flat building blocks – lattice regulator with UV cut-off $a > 0$
  [Regge, Nuovo Cim. **19** (1961)]

$$Z_{a,N}^{\mathrm{CDT}} = \sum_T \frac{1}{C_T}\mathrm{e}^{\mathrm{i}S^{\mathrm{Regge}}[T]}$$



- Summation only over foliated spacetimes with fixed spatial topology (spherical)
- Connection with the continuum theory via scaling limit



[Ambjørn et al., Contemp. Phys. **47** (2006), arXiv: hep-th/0509010] [Ambjørn et al., Phys. Rept. **519** (2012), arXiv: 1203.3591 (hep-th)] [Loll, Class. Quant. Grav. **37** (2020), arXiv: 1905.08669 (hep-th)]

Monte Carlo simulation and phase structure

Numerical simulations of CDT quantum gravity

- Change to Euclidean time (Wick rotation) $t \to \mathrm{i}\tau$ allows to use Monte Carlo techniques

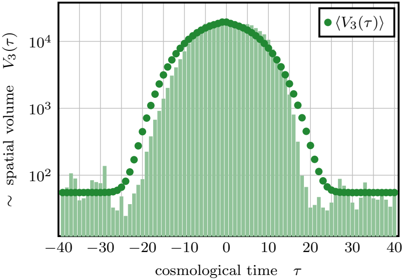
$$(-,+,+,+) \;\to\; (+,+,+,+)\,, \quad \mathrm{e}^{\mathrm{i}S} \;\to\; \mathrm{e}^{-S_E} \quad \sim \quad \text{Boltzmann weights}$$

- Causal information remains although space and time appear with equal sign

Phase structure of CDT

- Determined by Newton constant κ and asymmetry parameter α
- de Sitter phase: spatial volume profile of classical Euclidean de Sitter space
- Allows for connection with classical general relativity and cosmology

[Ambjørn et al., Phys. Rev. Lett. **100** (2008), arXiv: 0712.2485 (hep-th)] [Ambjørn et al., Phys. Rev. D **78** (2008), arXiv: 0807.4481 (hep-th)]



Curvature on simplicial manifolds

Description of a geon via an invariant scalar operator

- Usage of Riemann curvature tensor only on smooth (pseudo) Riemannian manifolds

Quantum Ricci curvature

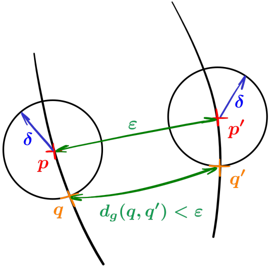
[Klitgaard et al., Phys. Rev. D **97** (2018), arXiv: 1712.08847 (hep-th)]

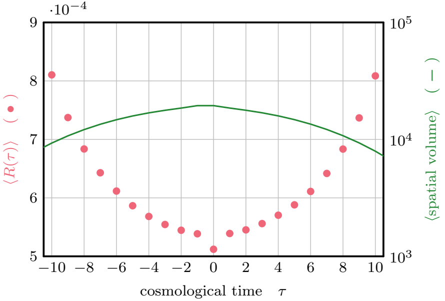
- Measure geodesic distance $d_g(q,q')$ between point pairs (q,q') on the $(d-1)$-spheres with centres p, p'
- Scalar quantity can be obtained from limit $\varepsilon \to 0$

$$Q = \frac{1}{[N_0(S_p^\delta)]^2} \sum_{q,q' \in S_p^\delta} \frac{d(q,q')}{\delta}$$

$$= c_q\{1 - c_2 R\delta^2 + \mathcal{O}(\delta^3)\}$$

- Extract curvature scalar R from a quadratic fit of Q





Correlators in quantum gravity

Correlation between the values of an observable R at a geodesic distance r apart

- Formulation in a diffeomorphism invariant way
- Not possible to track events across different configurations

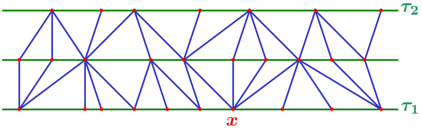
$$
\begin{aligned}
G_g[R,R](r) &= \int \mathrm{d}^4x\sqrt{g(x)}\int \mathrm{d}^4y\sqrt{g(y)}R(x)R(y)\delta(d_g(x,y)-r) \\
&\to \sum_{p,p'\in T} R(p)R(p')\delta_{d(p,p'),r} \\
\langle G[R,R](r)\rangle &= \int \mathrm{D}[g]\ \mathrm{e}^{-S_{\mathrm{E}}[g_{\mu\nu}]}G_g[R,R](r)
\end{aligned}
$$

- Normalization of the correlator with $\langle G[1,1](r)\rangle$ – also quantum observable
- Replace $R \to \Delta R = R - \langle\overline{R}\rangle$ to obtain connected correlator

$$
\langle\tilde{G}_c[R,R](r)\rangle \equiv \frac{\langle G_c[R,R](r)\rangle}{\langle G[1,1](r)\rangle} = \langle\tilde{G}[R,R](r)\rangle - 2\,\langle\tilde{G}[R,1](r)\rangle\ \langle\overline{R}\rangle + \langle\overline{R}\rangle^2
$$

[Ambjørn et al., JHEP **02** (1999), arXiv: hep-lat/9812015] [van der Duin et al., Eur. Phys. J. C **84** (2024), arXiv: 2404.17556 (hep-th)]

Constructing a causal observable

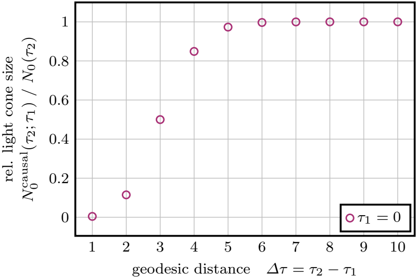


Determination of causally connected points

- Sum over all points y at τ_2

$$N^{\mathrm{causal}}_{0,x(\tau_1)}(\tau_2) \leq N_0(\tau_2)$$

within the light cone of $x(\tau_1)$ and over all points x at τ_1



Correlator of curvature scalars with a geodesic separation of $\Delta\tau$

$$\sum_{\substack{x(\tau_1),y(\tau_2)\\ \text{caus. conn.}}} \Delta R(x(\tau_1))\Delta R(y(\tau_2))$$

$$D_{RR}(\tau_1,\tau_2) = \langle \tilde{G}[R,R]\rangle - \langle \tilde{G}[R,1]\rangle\ \langle \overline{R}(\tau_2)\rangle - \\ - \langle \overline{R}(\tau_1)\rangle\ \langle \tilde{G}[1,R]\rangle + \langle \overline{R}(\tau_1)\rangle\ \langle \overline{R}(\tau_2)\rangle$$

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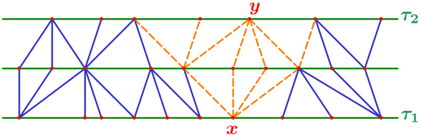

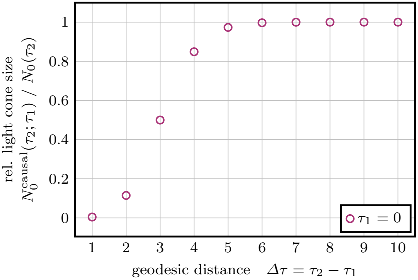

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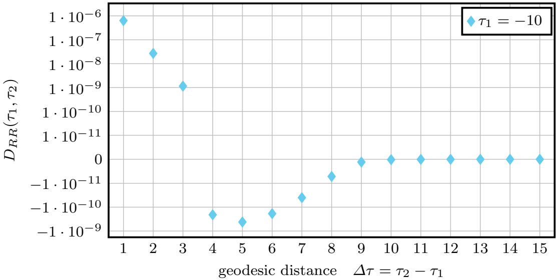
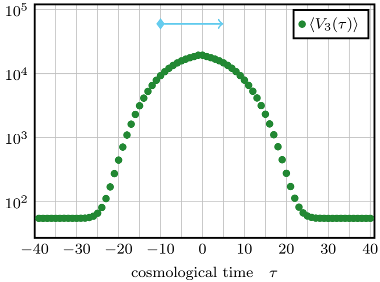
Geon propagator





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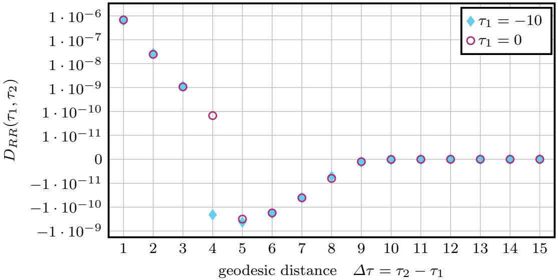

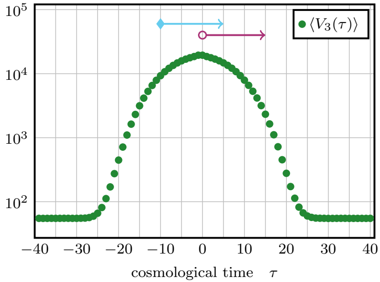

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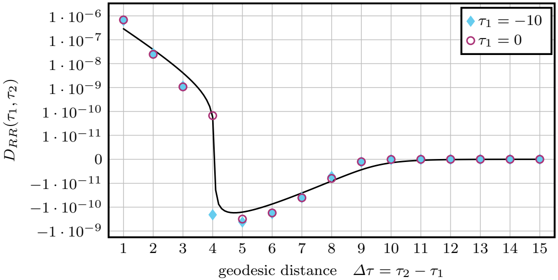


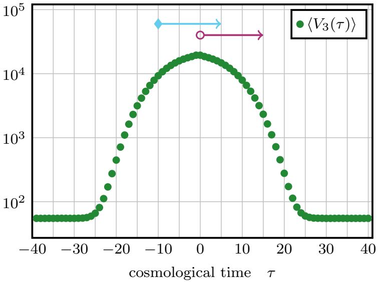
Exponential fit of the propagator to extract mass spectrum

$$\begin{aligned} D_{RR}(\Delta\tau) &= c_1 \cdot \mathrm{e}^{-m_1 \cdot \Delta\tau} - c_2 \cdot \mathrm{e}^{-m_2 \cdot \Delta\tau} \\ &= c_1 \cdot \mathrm{e}^{-m_1 \cdot \Delta\tau} (1 - \tilde{c}_2 \cdot \mathrm{e}^{\Delta m \cdot \Delta\tau}) \end{aligned}$$

[Ambjørn et al., Phys. Rev. D **78** (2008), arXiv: 0807.4481 (hep-th)]

$$m_1 = 1.8 \pm 0.6 \; , \quad m_2 = 1.3 \pm 0.5$$

$$\rightarrow \quad m_i = \mathcal{O}(M_{\text{Planck}})$$

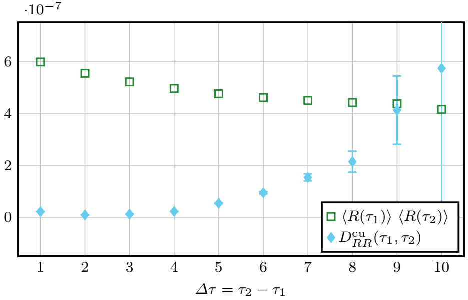


Two-point function for causally unconnected points

- Summation only over points outside the light cone

$$G_c[R(\tau_1), R(\tau_2)] = \sum_{\substack{x(\tau_1), y(\tau_2) \\ \text{caus. unconn.}}} \Delta R(x(\tau_1)) \Delta R(y(\tau_2))$$

- Long distance behaviour dominated by mean curvature – hint for cluster decomposition



Summary and outlook

Curvature correlator for causally connected points

- Particle like behaviour of the propagator
- Mass parameters independent of cosmological time
- Geon mass $m_i \approx \mathcal{O}(M_{\text{Planck}})$ – detection possible when clumping to more massive objects?

Future work and possible modifications

- Reduction of systematic errors – construct propagator only from dual lattice
- Extract gauge field (e.g. $g_{\mu\nu}$) from triangulation and build graviton propagator
- Build correlators from other scalar operators
- Extension to higher order correlation functions

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Causal structure and Wick rotation

Summation (integration) of curved Euclidean $(+,+,+,+)$ spaces and of Lorentzian $(-,+,+,+)$ spaces leads to inequivalent results

- Global timeslicing of universes in the summation (introduction of a cosmological time)
- Fix spatial topology – no branching points are allowed

Wick rotation of curved spacetimes

- Not possible in general Lorentzian manifolds, only due to global hyperbolic nature of CDT spacetimes
- Euclidean spaces obtained from Wick rotation form only subset of general curved Euclidean spaces

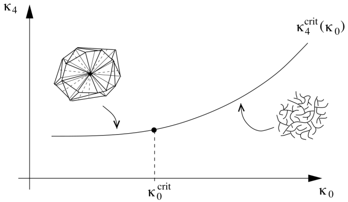
Phase diagram of (C)DT

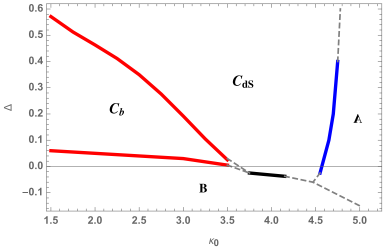
Phases of Euclidean DT

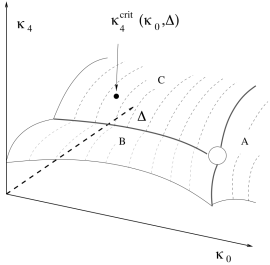
- Crumpled and branched-polymer phase have both not a good classical limit (Hausdorff dimension ∞ and 2)

Phases of Lorentzian DT

- Phases A and B are similar to Euclidean version
- Extended, 4D universes emerge from phase C – connection with classical GR







[Ambjørn et al., (2010), arXiv: 1004.0352 (hep-th)] [Ambjørn et al., Universe **7** (2021), arXiv: 2103.15610 (gr-qc)]

Dual lattice and fractional time

Timelike links on the regular lattice

- Connection of timeslices formed from spatial simplices
- Only one step required to connect a vertex at time t to a vertex at time $t+1$

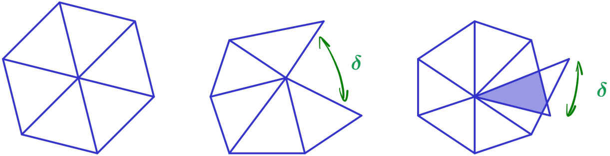
Timelike links on the dual lattice

- Not possible to connect a (m,n)-simplex at dual timeslice t to a (m,n)-simplex at dual timeslice $t+1$ with only one step
- Shortest possible timelike connecting between simplices of equal type $(4,1) \to (3,2) \to (2,3) \to (1,4) \to (4,1)$
- Divide dual timeslice into four slices of fractional time $(4,1) \to t$, $(3,2) \to t+\frac{1}{4}$, $(2,3) \to t+\frac{1}{2}$, $(4,1) \to t+\frac{3}{4}$

[Klitgaard et al., Eur. Phys. J. C **80** (2020), arXiv: 2006.06263 (hep-th)]

Measure curvature from deficit angle

- Flat space: regular pattern of six triangles meet at one vertex
- Positively (negatively) curved surface: less (more) than six triangles meet at one vertex
- Problem: divergent in infinite-volume limit



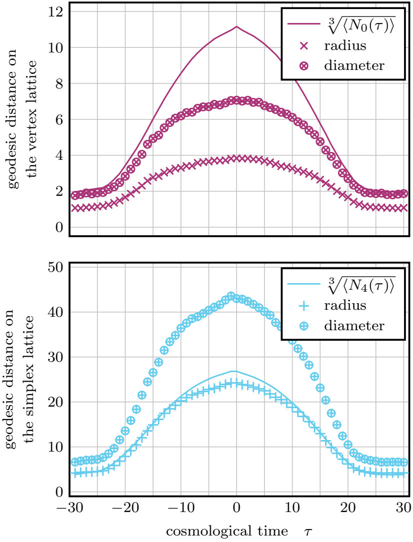
Linear extension of the timeslices

Measures for the linear extension

- Spatial volume increases approximately with 3rd power of linear size
- Eccentricity of a vertex v – largest geodesic distance between v and any other vertex v' of a connected graph
- largest eccentricity = diameter, smallest eccentricity = radius

Physical size of the piecewise flat building blocks $a \approx 2\ell_{\text{Planck}} \to$ linear size of the CDT universes $\approx 20\ell_{\text{Planck}}$

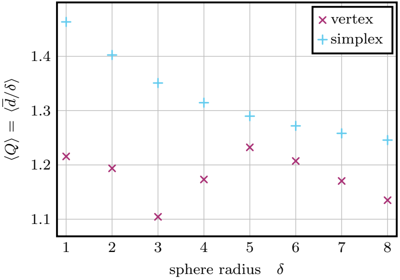
[Ambjørn et al., Phys. Rev. D **78** (2008), arXiv: `0807.4481` (hep-th)]



Quantum Ricci curvature on the vertex and simplex (dual) lattice

Measuring curvature on the vertex (regular) lattice

- Anisotropic shape of sphere in spatial and temporal direction: most points of δ–sphere reside on timeslices $\tau \pm \delta$ for large δ
- Occurrence of vertices where hundreds of links meet, leads to small extension in terms of the vertex content → finite size effects already for small δ



Curvature on the simplex (dual) lattice

- Larger extension of the configurations in terms of the simplex content (exactly 5 neighbours for each simplex)
- Better behaviour of the average sphere distance as a function of δ – allows for an extraction of the Ricci curvature

[Klitgaard et al., Eur. Phys. J. C **80** (2020), arXiv: 2006.06263 (hep-th)]

Geon propagator – quantum Ricci curvature