Investigating magnetic field burial in neutron stars

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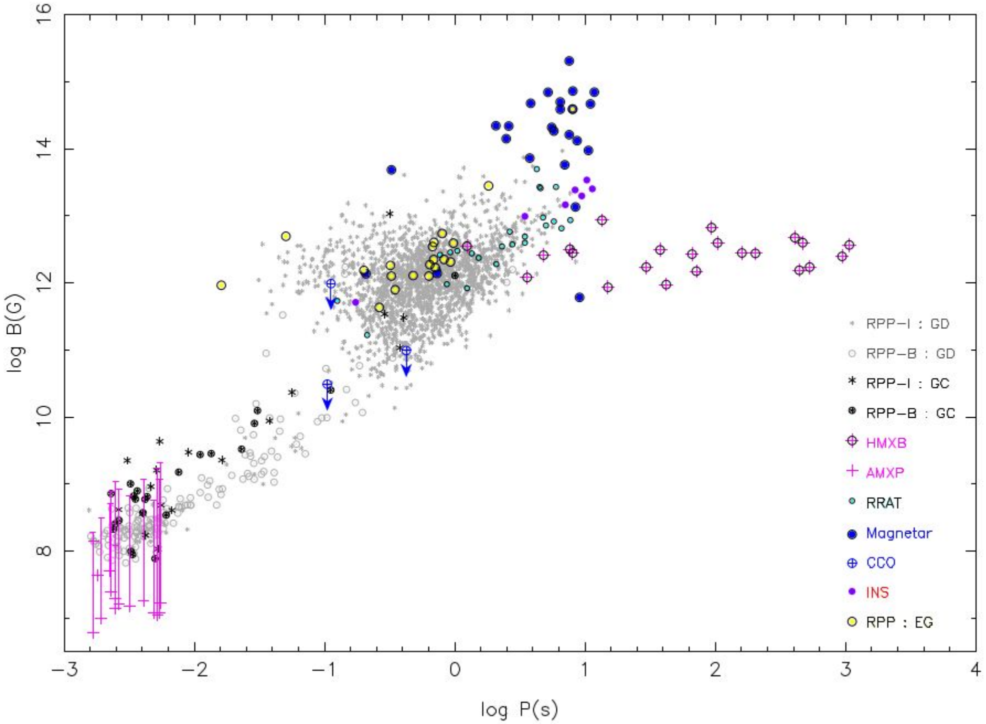
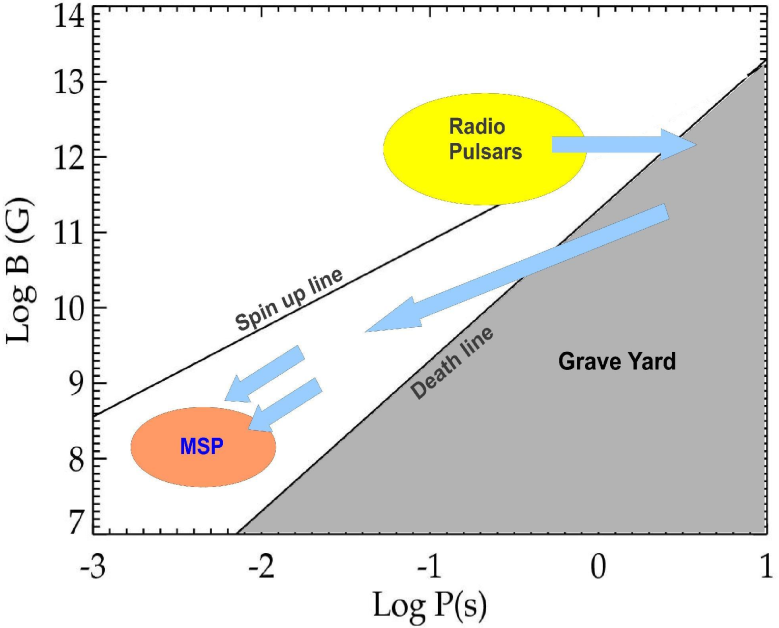
B - P diagram

- Two distinct neutron star populations - millisecond pulsars and bulk of neutron star population
- Low spin period - due to accretion





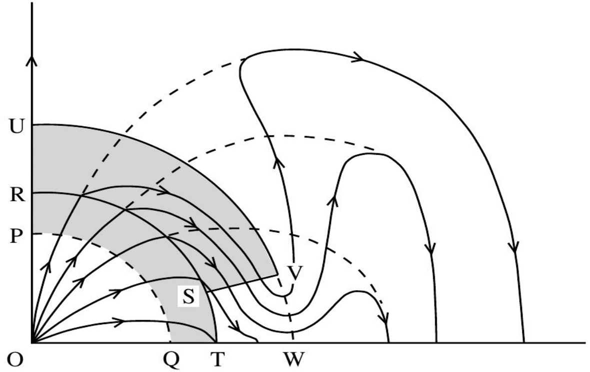
Credits : Konar 2017

The puzzle of low magnetic field of MSPs

Possible mechanisms proposed to explain the low surface field strength in LMXB systems :

1. **Ohmic decay because of high crust resistivity due to accretion induced heating** (Urpin & Geppert 1995; Konar & Bhattacharya 1997; Urpin & Konenkov 1997; Konar & Bhattacharya 1999a)
2. **Fluxoid-vortex migration to the crust** (Muslimov & Tsygan 1985; Srinivasan et al. 1990; Jahan Miri & Bhattacharya 1994; Konar & Bhattacharya 1999b)
3. **Crustal magnetothermal evolution** (Blondin & Freese 1986)
4. **Crustal tectonic motions** (Ruderman 1991a, b)

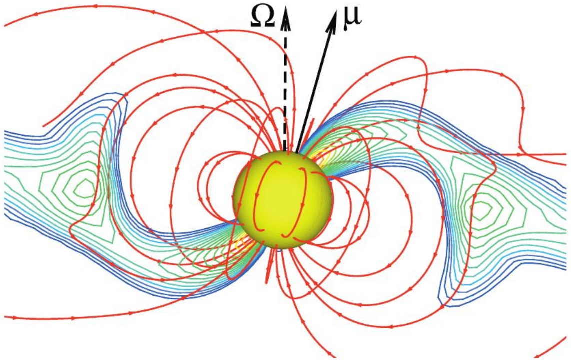
5. **Burial of magnetic field** (Romani 1990, Melatos and Phinney 2001, Cumming et. al 2001, Payne & Melatos 2004, 2007, Vigelius & Melatos 2008, Priymak & Melatos 2011, Mukherjee et al. 2012 a & b, Mukherjee 2017)



Credits : Melatos & Phinney 2001

Overview

- **PROBLEMS AND APPROACH :**
  - Magnetic field geometry and density profile of neutron star before accretion and Matter accreted per magnetic field line? —--- Assume
  - Magnetic field geometry and density profile of mound after accretion ? Accretion timescales - hours to years ; Alfven timescales - millisecs ; —--- Model 2D magnetostatic mounds.
  - Reduction in dipole moment ? —---- Current free boundary condition at the outer radius
  - Stability of the mounds ? —----- 3D MHD simulations
  - Relativistic alfven speeds in the magnetosphere ? —------ Use SemiRMHD equations and FFDE like magnetosphere model.



Credits : Romanova et. al. 2004

Literature review

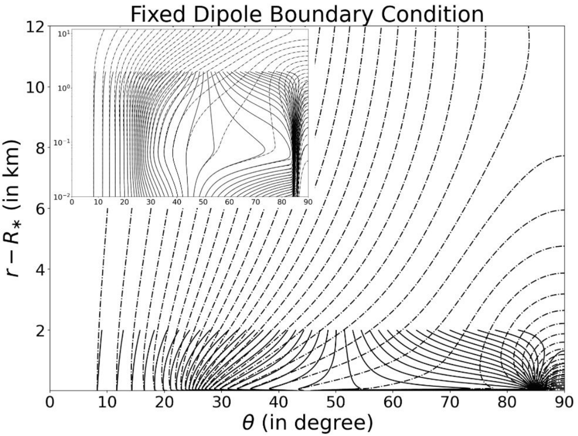
Paper	EOS ($P = P(\rho)$)	Outer radial BC	Max mass ($M_\odot$)	Matter population	Coordinates	Polar magnetic field (G)
Payne, Melatos (2004), Priymak et al. (2011)	Isothermal, Polytropic	Outflow	10^{-4} , 10^{-8}	Polar cap to equator	Stretched spherical	10^{12}, $10^{12.5}$
Mukherjee, Bhattacharya (2012) , Mukherjee (2017)	Polytropic, Paczynski	Fixed	10^{-12}	Confined near polar cap, Polar cap to truncation angle	Cylindrical, Spherical	10^{12}
This work - Yeole et al. (2025)	Paczynski	Current free	10^{-11}-10^{-10}	Polar cap to truncation angle (large range)	logarithmic radial and cos Θ coordinates	10^{9}-10^{12}

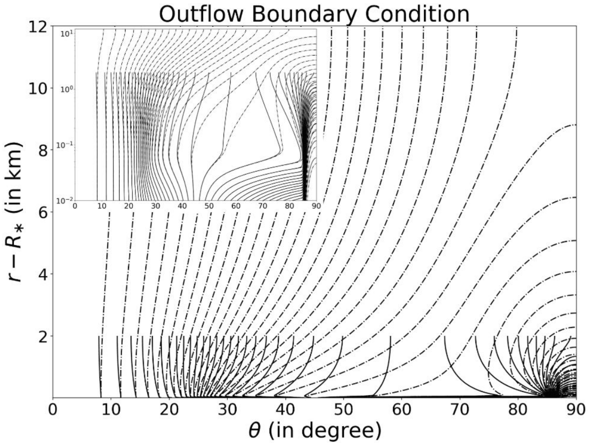
- Parallel architecture implemented for the new code.
- New high resolution solutions (5000 x 5000) and extended domain (0^0-90^0 ; 10-22 km).

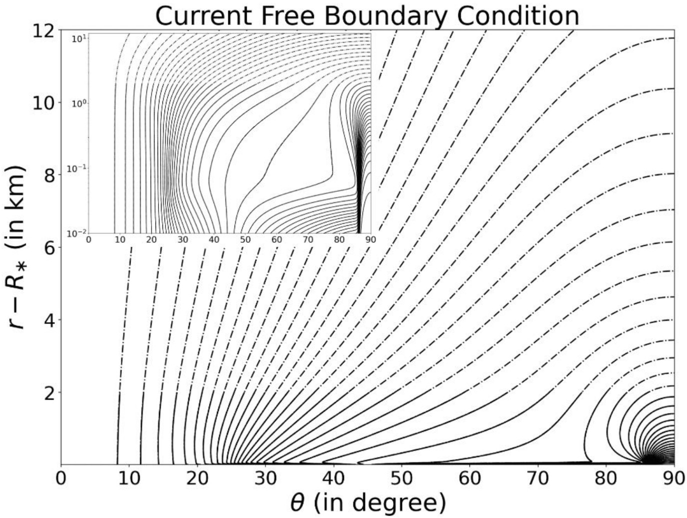
Current Free Boundary

- **INNER BOUNDARY -** Fixed magnetic fields (surface or crust) i.e. fixed ψ

- **θ = 0 -** $\psi = 0$; **θ = 90 - equatorially symmetric**
- **OUTER BOUNDARY - Current free boundary**







Grad Shafranov (GS) eqn

Balancing the forces, assuming axisymmetry and zero toroidal magnetic field, we get

$$\frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi} = \nabla p + \rho \nabla \phi_g \qquad \text{where} \qquad \boldsymbol{B} = \frac{\nabla \psi(r,\theta) \times \hat{\phi}}{r \sin\theta}$$

Calculating dG = dp / ρ from p(ρ), assuming constant gravitational acceleration, assuming an $r_0(\psi)$ (radius of the mound per magnetic field line) , rewrite the RHS

Using ψ and the new RHS, rewrite the force balance equation as the GS equation :

$$\Delta^2 \psi = K(\psi, r, \theta)$$

where

$$\Delta^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) \qquad K(\psi, r, \theta) = -4\pi r^2 \sin^2\theta \rho g \frac{\mathrm{d}r_0(\psi)}{\mathrm{d}\psi}$$

P = P(ρ)

Paczynskii 1983 -

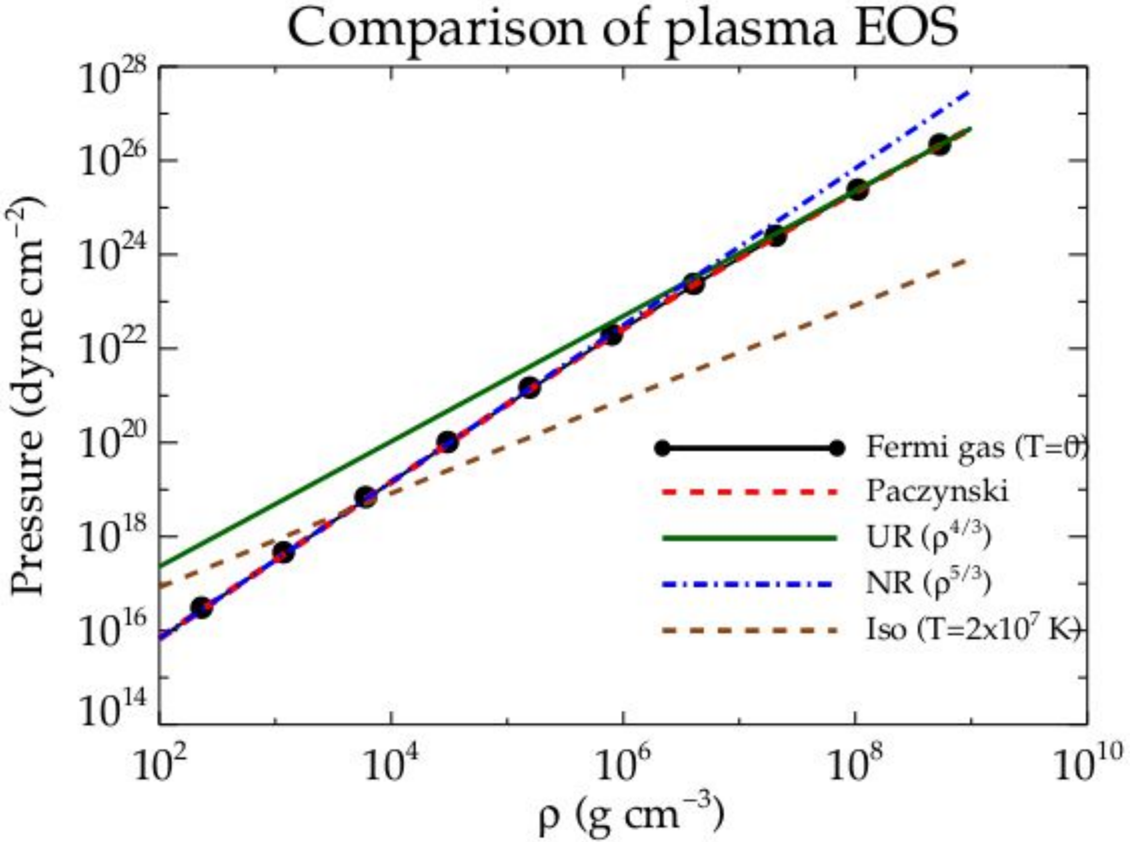
$$P_{ed} = \left(P_{ednr}{}^{-2} + P_{edr}{}^{-2}\right)^{-1/2}$$

Final Pressure Formula -

$$p = \frac{\pi}{3}\frac{m_e^4 c^5}{h^3}\frac{(8/5)x_F^5}{\sqrt{1+(16/25)x_F^2}}$$

where

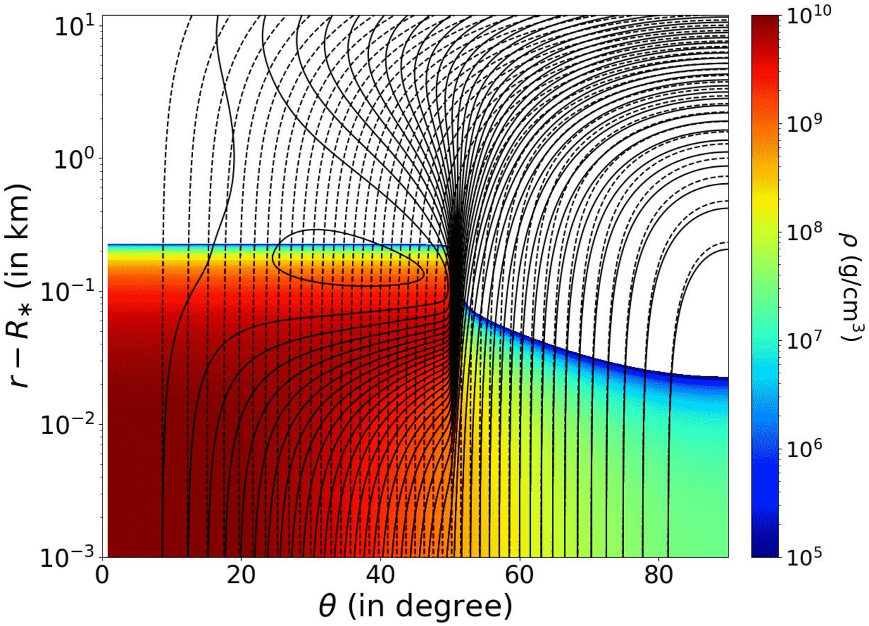
$$x_F = \frac{1}{m_e c}\left(\frac{3h^3}{8\pi\mu_e m_p}\right)^{1/3}\rho^{1/3}.$$



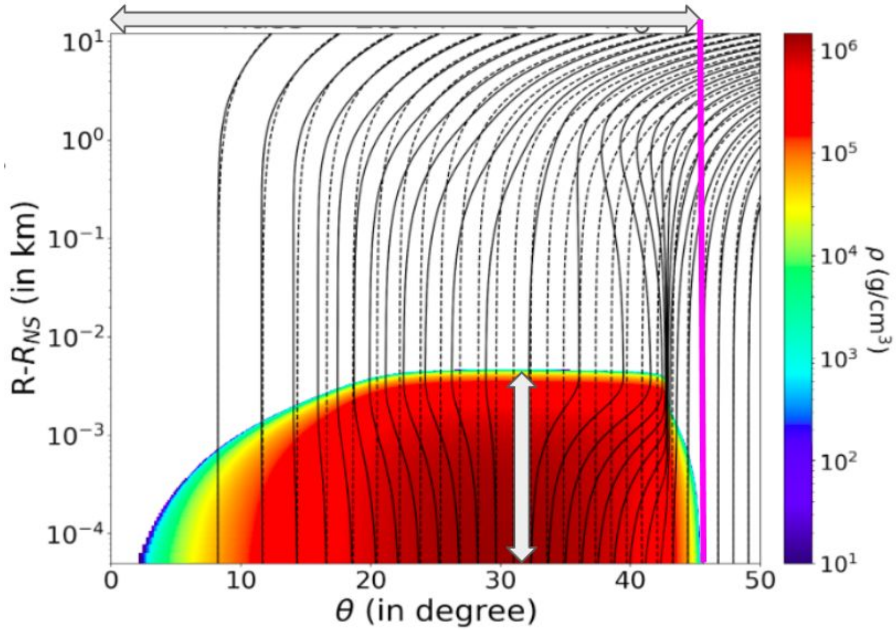
Credits : Mukherjee 2017

Two mound profiles

Filled mound with exponential decay beyond truncation angle

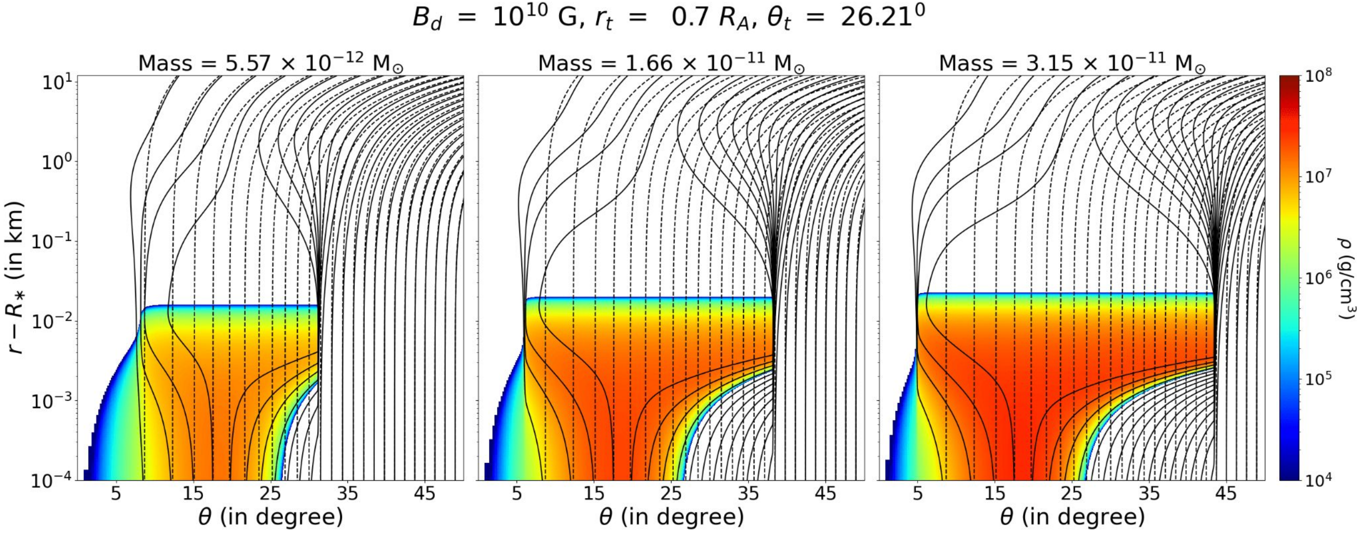


Ring-shaped mound



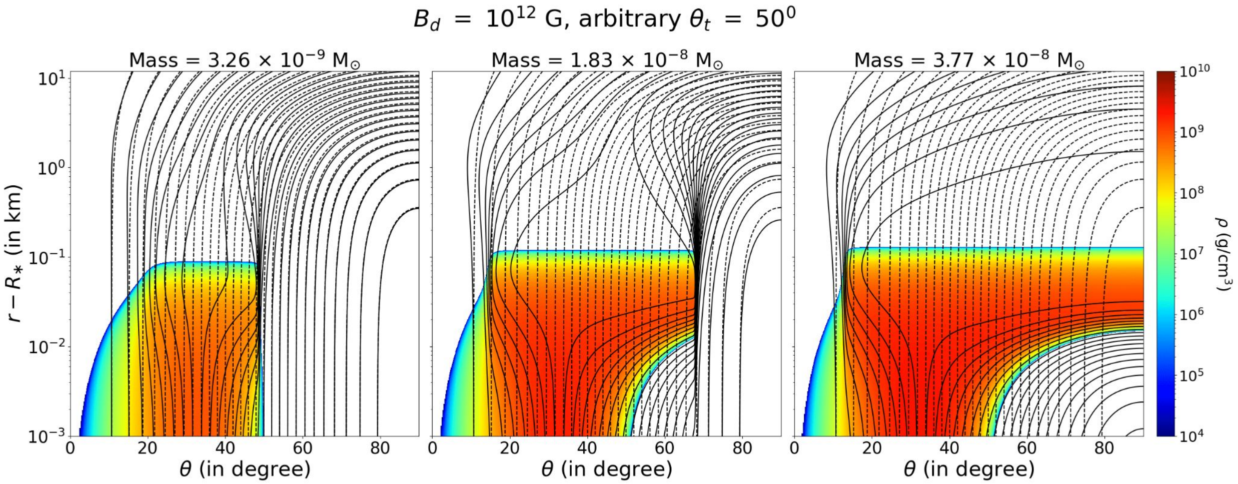
Spreading ring-shaped mounds

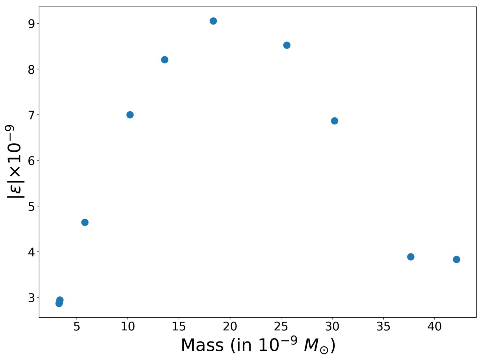
- Truncation angle $\theta_t = f(r_t)$ and Accretion disk radius $r_t = 0.6\text{-}1.0\ R_A$.
- Fix θ_t and B_d increase maximum height to increase mass of the mound.

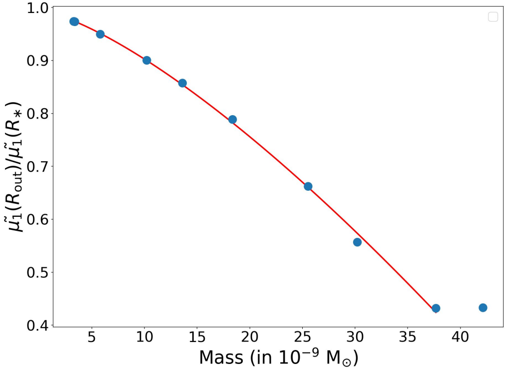


- Large (Small) magnetic fields - Supports relatively larger (smaller) mass and has smaller (larger) angular extent.
- Max. ellipticity - 3.75×10^{-10} (10^{12} G)
- Dipole moment reduction - 0.627 (10^9 G)
- Short term field burial possible - supported by Patruno et al. (2012)
- Also modelled for quadrudipolar fields.

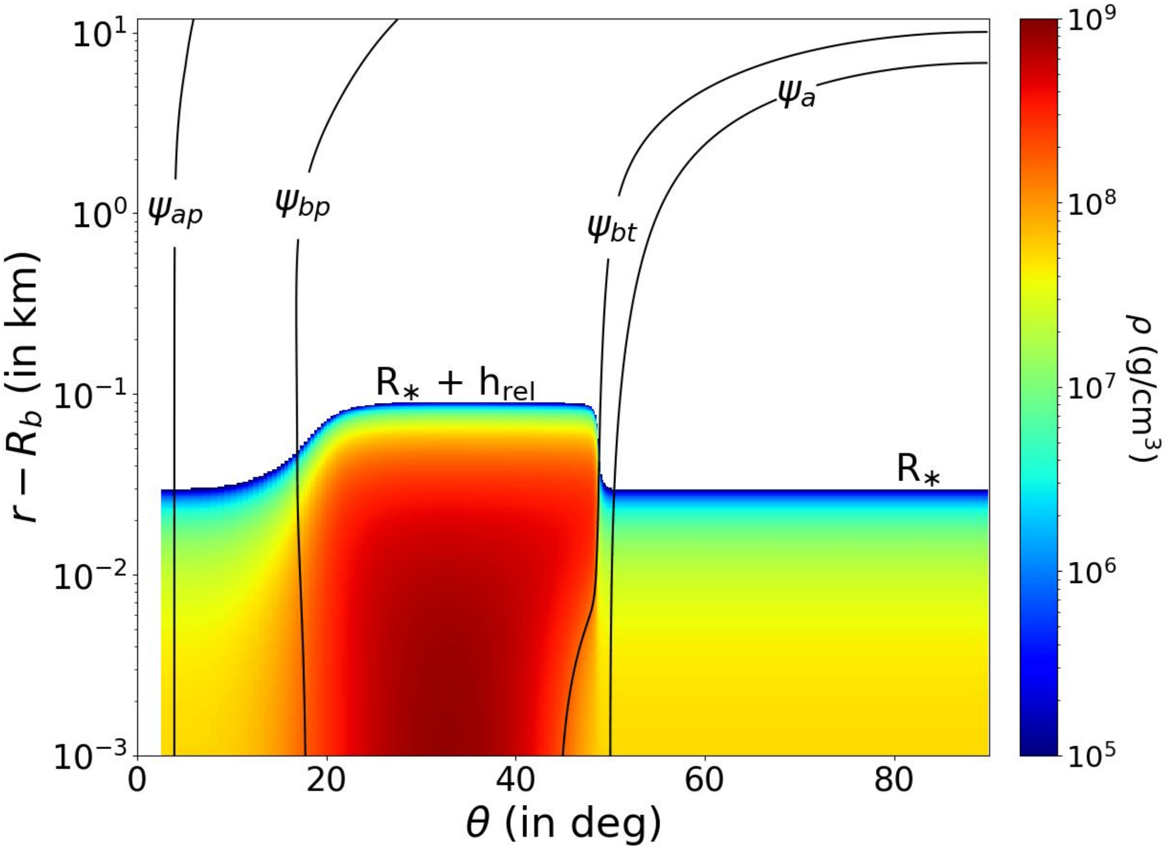
Arbitrary truncation angle







Ring-shaped mound on ocean



- Assuming sinking - reduces absolute mass ellipticity and dipole moments.
- Modelled only for arbitrary truncation angles.
- Fixing the inner boundary -
    - Crystallization density

SemiRMHD equations in PLUTO

$$\frac{\partial \mathbf{U}}{\partial t} + \sum_k \frac{\partial \mathbf{F^k}(\mathbf{U})}{\partial x^k} = \mathbf{Q}$$

$$F^x(U) = \begin{pmatrix} \rho v_x \\ m_x v_x + P_{\rm tot} - B_x^2(1 - \frac{v^2}{c^2}) - \frac{1}{c^2}(\mathbf{v}.\mathbf{B})B_x v_x \\ m_y v_x - B_x B_y(1 - \frac{v^2}{c^2}) - \frac{1}{c^2}(\mathbf{v}.\mathbf{B})B_x v_y \\ m_z v_x - B_x B_z(1 - \frac{v^2}{c^2}) - \frac{1}{c^2}(\mathbf{v}.\mathbf{B})B_x v_z \\ 0 \\ v_x B_y - B_x v_y \\ v_x B_z - B_x v_z \\ (e + P_{\rm tot})v_x - (\mathbf{v}.\mathbf{B})B_x \end{pmatrix}$$

$$\mathbf{U} = (\rho, m_x, m_y, m_z, B_x, B_y, B_z, e)$$

$$\mathbf{m} = \rho\mathbf{v} + \frac{1}{c^2}(B^2\mathbf{v} - (\mathbf{v}.\mathbf{B})\mathbf{B}),$$

$$P_{\rm tot} = P + \frac{1}{2}\left(B^2 - \frac{1}{c^2}(v^2B^2 - (\mathbf{v}.\mathbf{B})^2)\right),$$

$$e = \frac{1}{2}\rho v^2 + \frac{P}{\Gamma - 1} + \frac{1}{2}\left(B^2 + \frac{1}{c^2}(v^2B^2 - (\mathbf{v}.\mathbf{B})^2)\right),$$

$$\mathbf{Q} = \left(1 - \frac{1}{c^2}\right)\mathbf{E}(\nabla.\mathbf{E})$$

and

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}.$$

- **Gombosi et al. (2001)**.
- Since, ConstoPrim() is analytical, SemiRMHD faster than RMHD.
- New SemiRMHD HLLC solver derived for ideal EOS.

Magnetosphere model

$$\rho = \rho_{mnd} + \rho_{mag} \qquad \rho_{mnd} = trc * \rho$$

After each RK2/RK3 step, we find

$$\mathbf{m}_{\|} = \frac{(\mathbf{m}.\mathbf{B})\mathbf{B}}{B^2} \qquad \mathbf{m}_{\perp} = \mathbf{m} - \mathbf{m}_{\|}$$

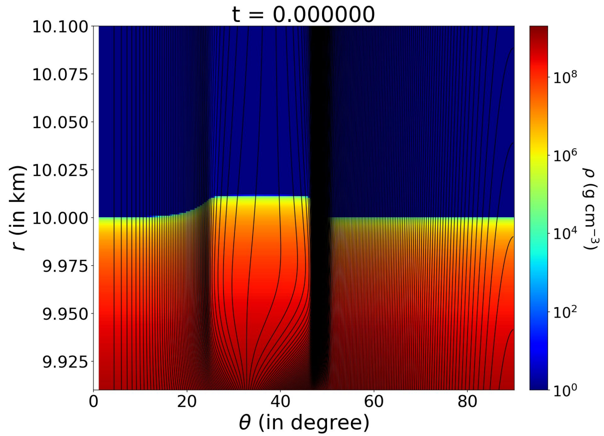
Redefine the momentum

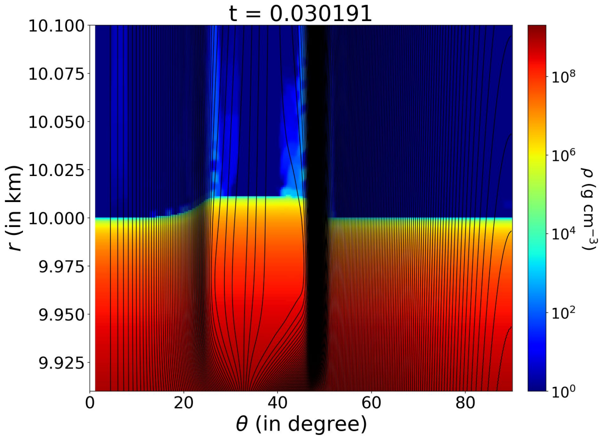
$$\mathbf{m} = \mathbf{m}_{\perp} + trc * \mathbf{m}_{\|}$$

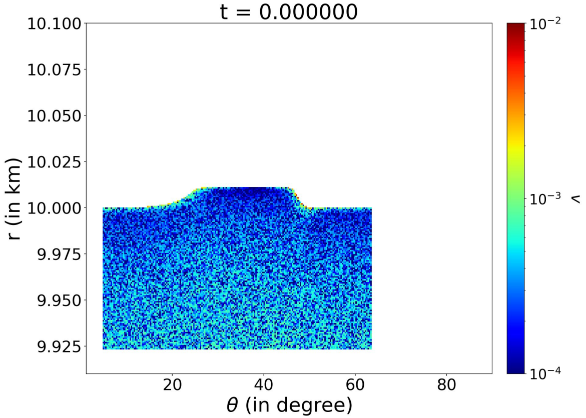
- **Motivated by Parfrey et al. (2024)**
- **B** and **v** approach the FFDE regime provided v_A is highly relativistic.
- Initial density set to v_A = 10c and limited to this v_A.
- trc redefined for new magnetospheric density.
- No gravity for magnetospheric density.

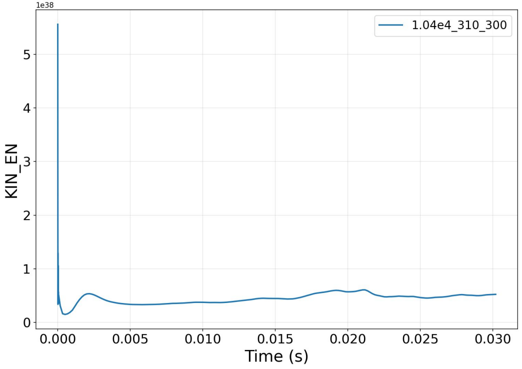
2D Simulation

Trigger random velocity perturbations inside the mound away from the boundaries and evolve.

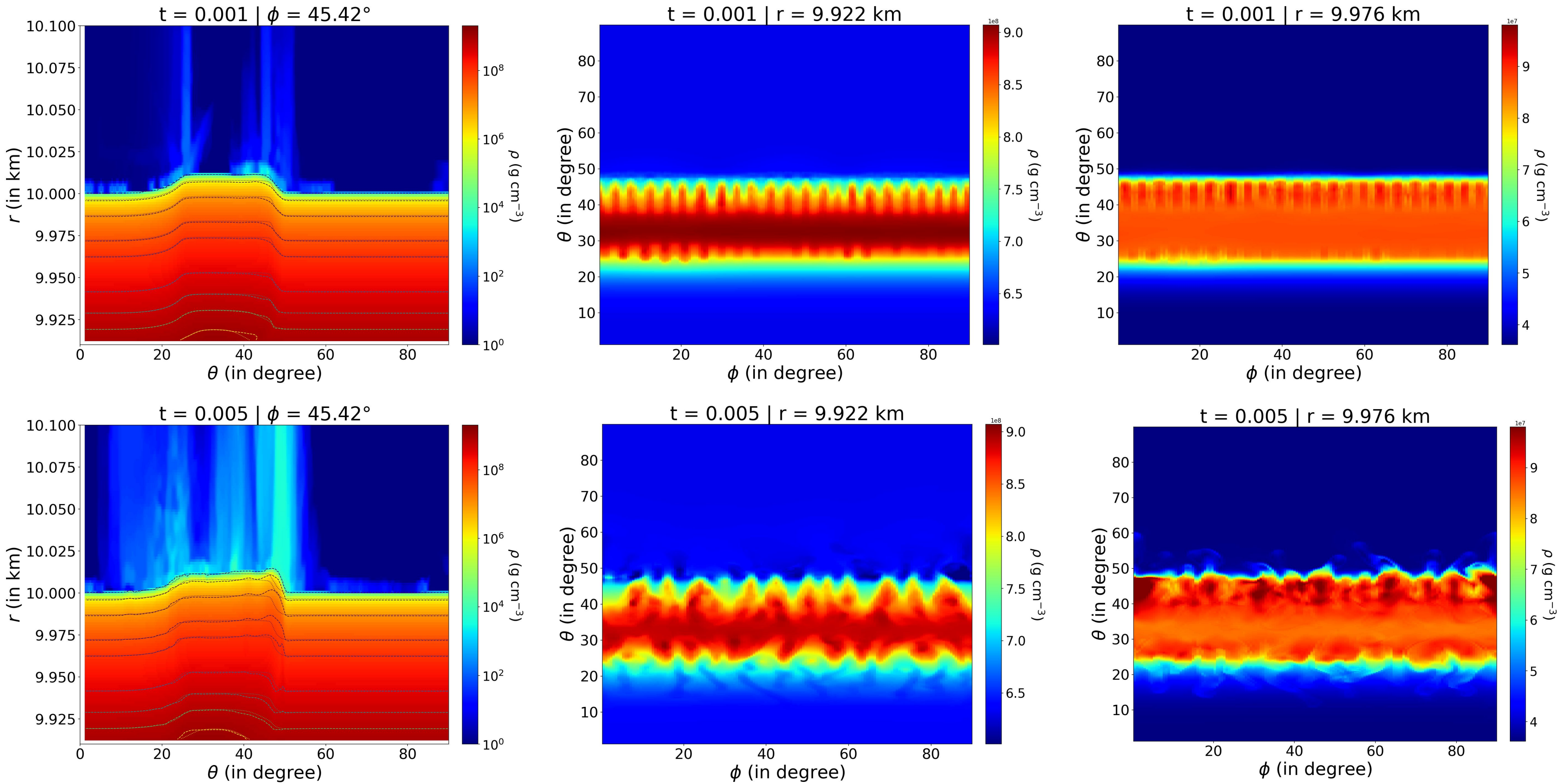




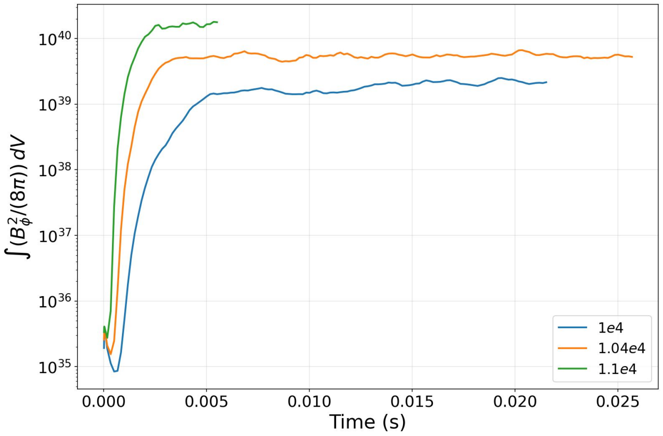


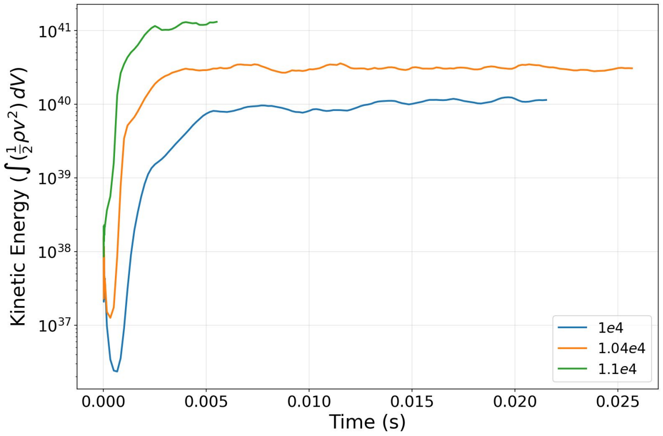


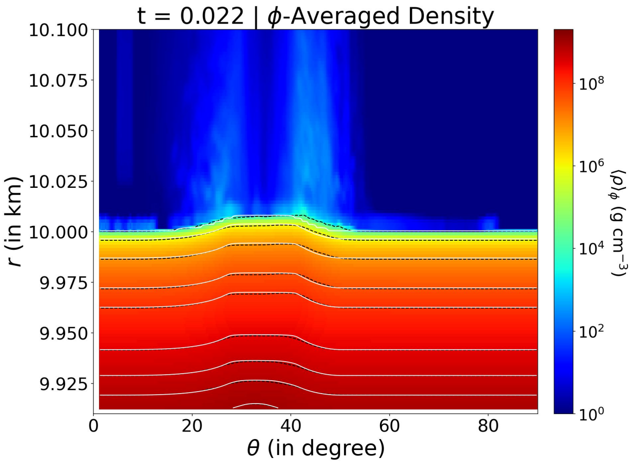
3D Simulation

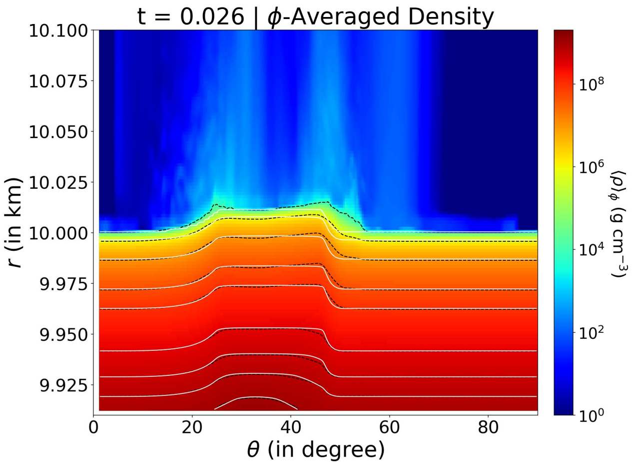


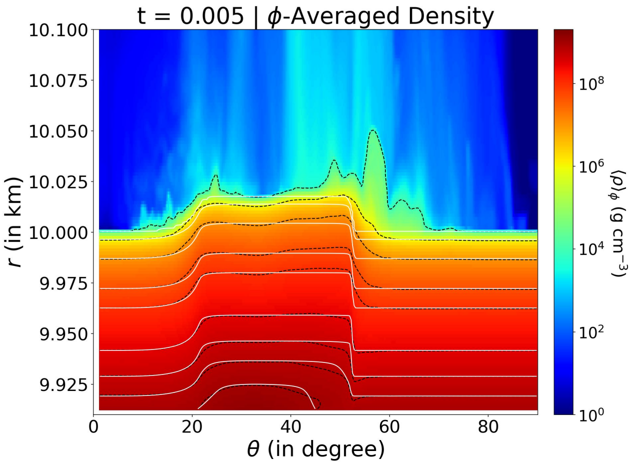
Spreading ?


$\int (B_\phi^2/(8\pi))\, dV$
Time (s)
1e4
1.04e4
1.1e4


Kinetic Energy ($\int (\frac{1}{2}\rho v^2)\, dV$)
Time (s)
1e4
1.04e4
1.1e4

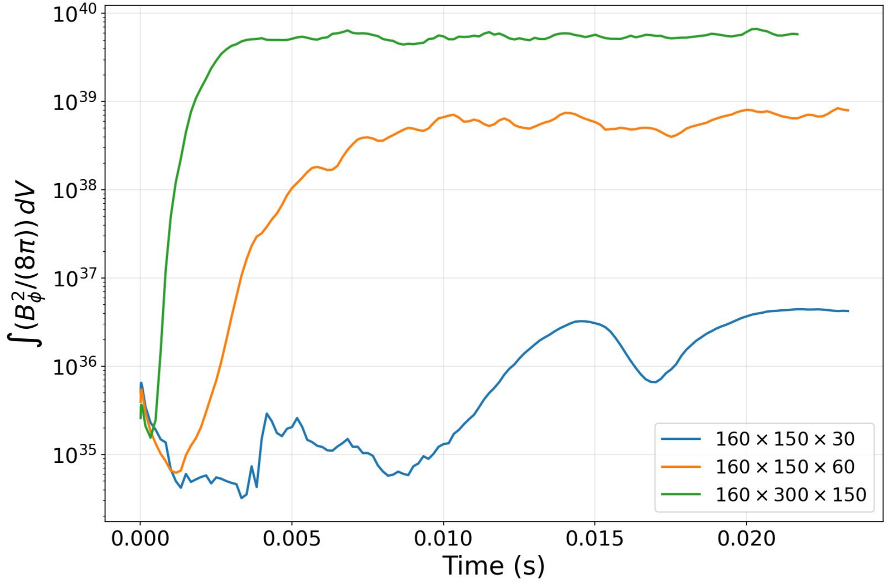

t = 0.022 | ϕ-Averaged Density
r (in km)
θ (in degree)
$\langle\rho\rangle_\phi$ (g cm^{-3})

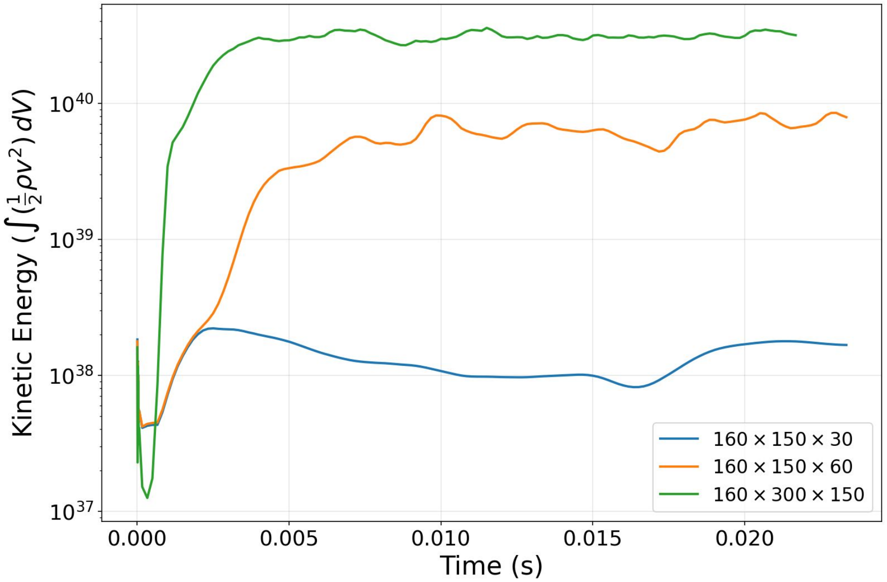

t = 0.026 | ϕ-Averaged Density
r (in km)
θ (in degree)
$\langle\rho\rangle_\phi$ (g cm^{-3})


t = 0.005 | ϕ-Averaged Density
r (in km)
θ (in degree)
$\langle\rho\rangle_\phi$ (g cm^{-3})

Effects of resolution

Saturated Bϕ and kinetic energy depends on resolution in the ϕ direction (r x θ x ϕ)





Future prospects and Summary

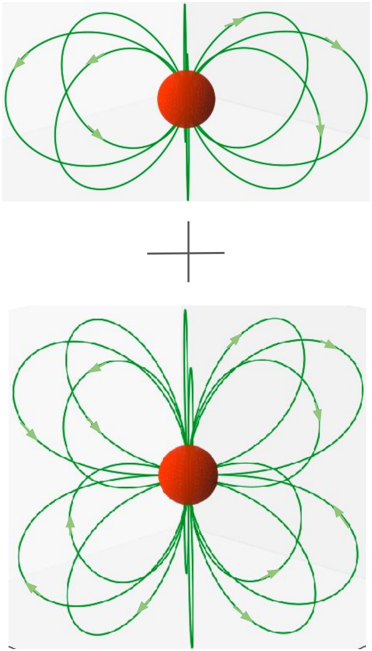
- Future prospects
  - Observational implications - X-ray emission and CGW.
  - Initial poloidal+toroidal field
  - Rotation
- Summary
  - Ring shaped mound - No magnetic loops.
  - 2D Ring shaped mounds with a fixed surface boundary spread on the NS surface as mass increases.
  - Ring shaped mound on ocean - 3D sims show that instabilities form non axisymmetric structures and sudden increase in Bphi followed by saturation.
  - Saturated $B\phi$ and kinetic energy values depends on ϕ resolution.

Publications

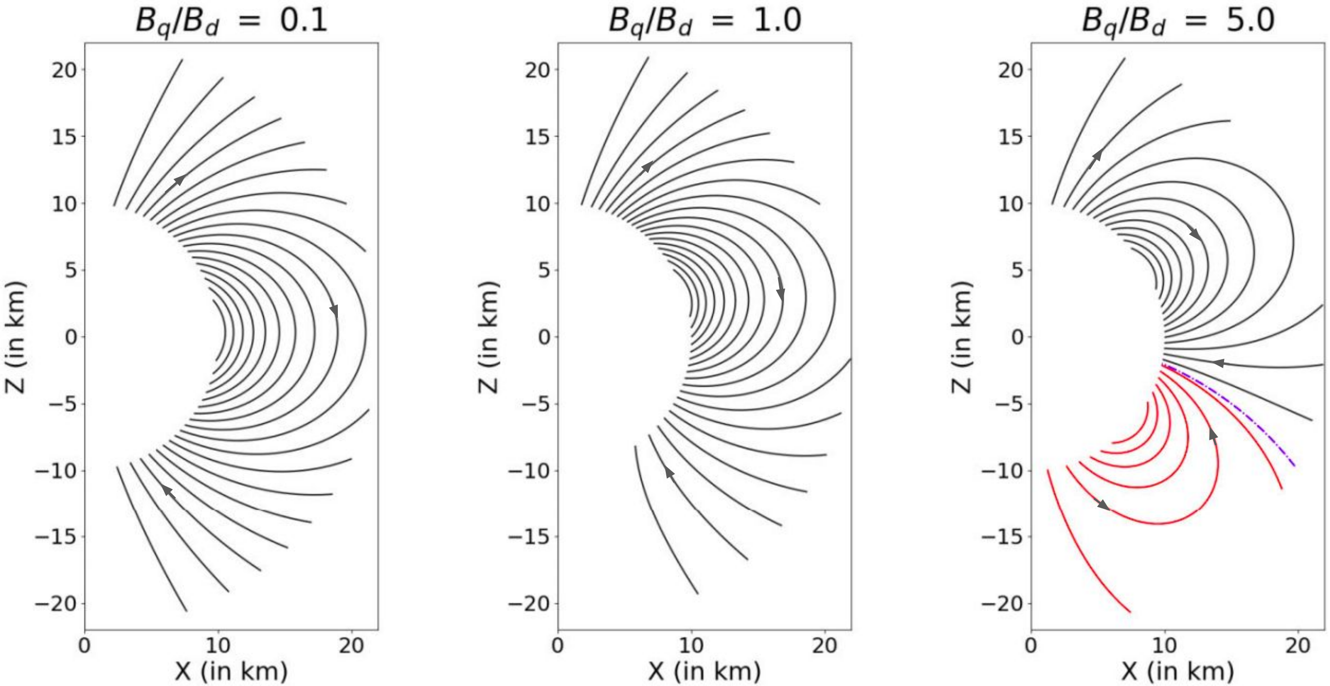
1. Investigating field burial by magnetically confined accretion mounds on Neutron Stars by Saurabh Yeole, Dipanjan Mukherjee, Ankush Mandal (Yeole et al., 2025) (submitted on April 3, 2025; published on July 12, 2025).
2. SemiRMHD solver in PLUTO (in prep)
3. 2D and 3D simulations of ring shaped accretion mound with ocean (in prep)

Quadru-dipolar magnetic fields

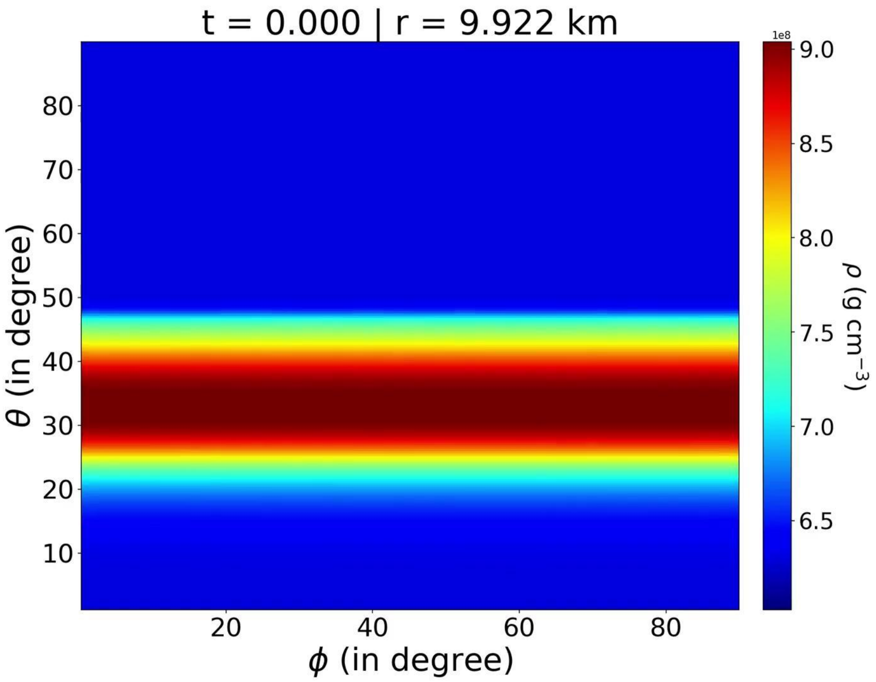
Dipolar

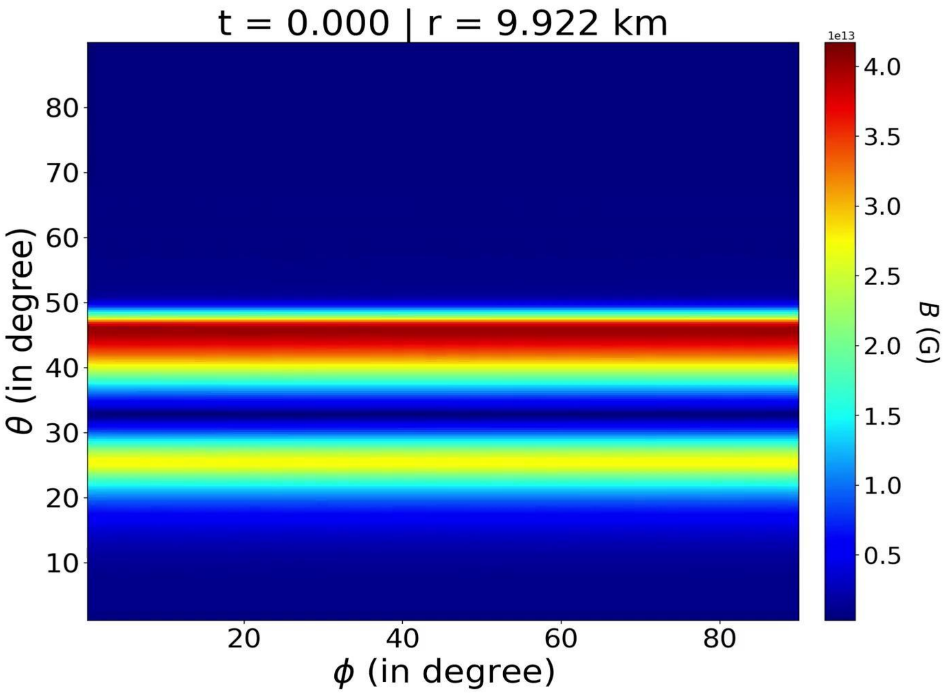


Quadrupolar

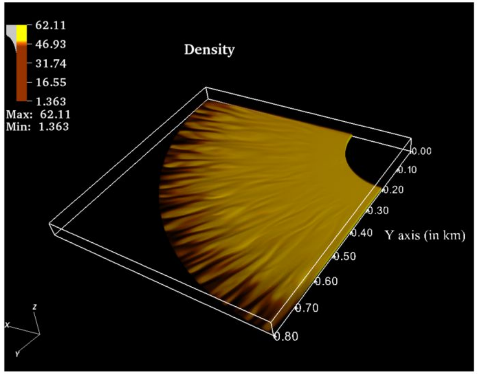


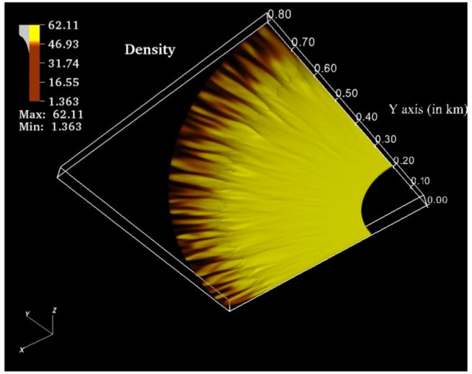
3D Simulation videos



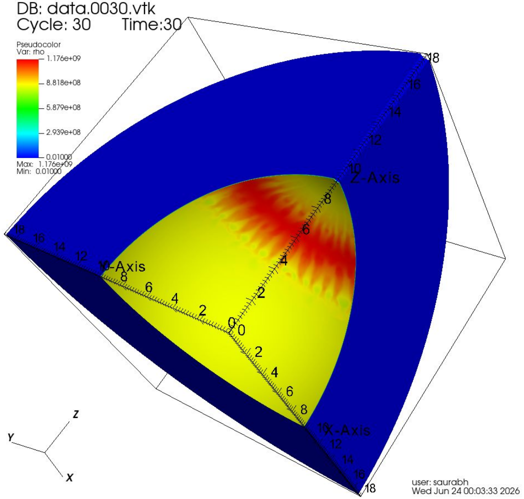


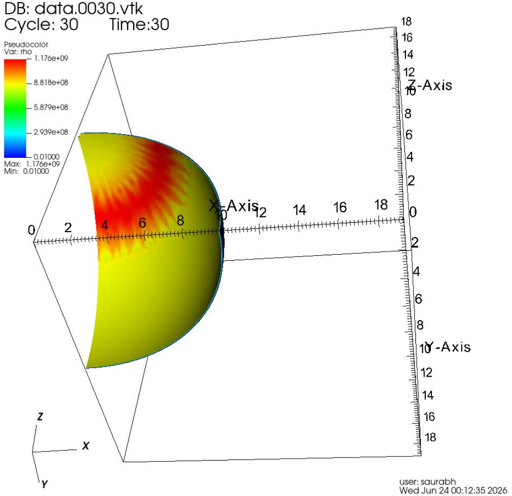
3D Simulations





Mukherjee et al. 2013b





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- To avoid ConstoPrim() errors due to low density, density limited to v_A = 10c
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