Numerical simulations of neutron star oscillations with thermal effects

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based on preliminary work by Santiago Jaraba, Gaël Servignat, Micaela Oertel, Isabel Cordero-Carrión *et al.*

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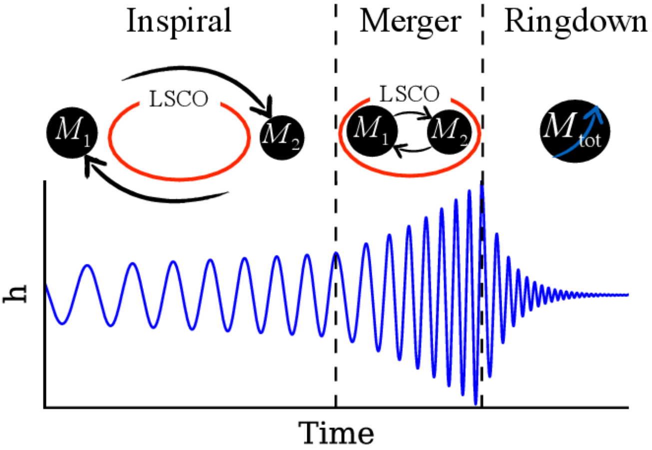
Binary neutron star mergers

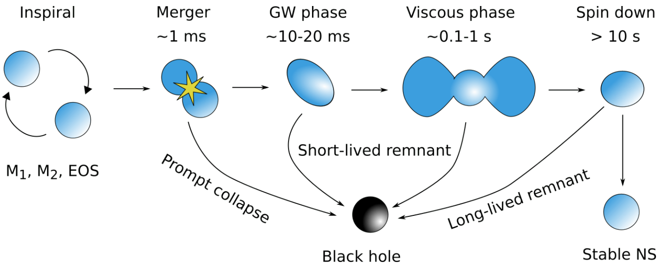


Masses & tidal deformabilities

Oscillation frequencies

Credit: Salcedo et al.(2016)

Post-merger phase



Credit: Radice *et al.* (2020)

After the merger, a super- or hyper- massive neutron star can be formed, oscillating for many hydro and rotation times.

- Contrary to inspiral, matter is hot and out of β-equilibrium.
- The resulting object emits gravitational waves which frequency depends on the EoS.

$\Rightarrow$ Need of a fast numerical tool to compute the modes and evolution for many candidate EoSs.

Conformal Flatness Condition

Isenberg (2008 *and before*), Cordero-Carrión *et al.* (2009)

In General Relativity, metric can be decomposed within the $3+1$ formalism:

$$g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right).$$

CFC is an approximation in which

$$\gamma_{ij} = \psi^4 f_{ij}$$

with f_{ij} being a flat 3-metric.

- no gravitational waves (no dynamical degree of freedom)
- Einstein equation turns into a system of 5 coupled elliptic-type PDEs
- exact in spherical symmetry, but e.g. rotating BH/NS metrics cannot be exactly written in CFC.

Primitive variable GR hydrodynamics

Servignat *et al.* (2023)

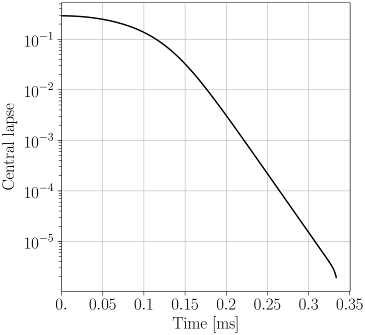
Assuming matter is a perfect fluid, from the conservation of energy-momentum tensor one can derive for $H = \ln\left(\dfrac{e+p}{m_B n_B}\right)$:

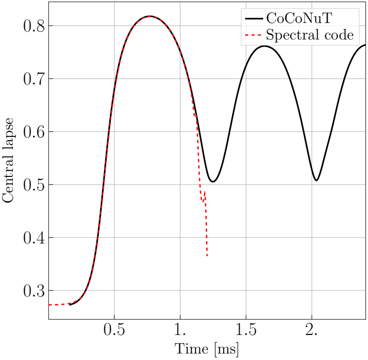
$$
\begin{aligned}
\partial_t H &= -v^i D_i H - c_s^2 N \frac{\Gamma^2}{\Gamma^2 - c_s^2(\Gamma^2-1)} \left(U^i U^j K_{ij} - \frac{U^i}{\Gamma^2} D_i H + D_i U^i \right), \\
\partial_t U_i &= -v^j D_j U_i - D_i N - \frac{N}{\Gamma^2} \left(D_i H - \frac{\Gamma^2(1-c_s^2)}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i U^j D_j H \right) \\
&+ U_j D_i \beta^j + U_i U^j D_j N \\
&+ \frac{N c_s^2}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i D_j U^j + \frac{N\Gamma^2(c_s^2-1)}{\Gamma^2 - c_s^2(\Gamma^2-1)} U_i U^j U^l K_{jl}.
\end{aligned}
$$

$\Rightarrow$ the (cold and chemically equilibrated) EoS appears explicitly and only in $c_s(H)$ (no recovery of any variable)
$\Rightarrow$ not able to treat shocks.

Numerical code and tests

Based on the LORENE library using multi-domain spectral methods in space (spherical coordinates), full non-linear hydro code in CFC, with explicit Adams-Bashforth time-marching scheme.

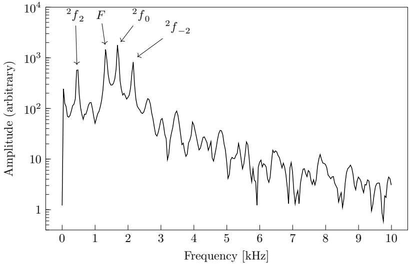




Tests with the spherically symmetric version of the code: collapse of an unstable NS to a BH (left) and migration of an unstable NS to toward the stable branch (right).

ROXAS: SIMULATING STARS IN 3D

SERVIGNAT & NOVAK (2025)



- In 2D/3D the hydro grid is adapted to the star's surface
- GW extracted using the modified quadrupole formula
- Initial data are perturbed sequences of uniformly rotating polytropes with increasing rotation frequencies.

Model	F		H_1		$^2f_0$		$^2f_2$		$^2f_{-2}$	
	Cowling	CFC	Cowling	CFC	Cowling	CFC	Cowling	Full GR	Cowling	Full GR
BU0	0.15	0.83	0.00	0.38	0.48	0.63	0.00	0.83	0.00	0.83
BU1	0.11	0.70	0.20	0.82	0.27	0.12	0.51	0.09	0.09	0.56
BU2	0.12	1.15	0.54	0.59	0.90	0.43	0.98	0.48	0.09	0.10
BU3	0.35	1.25	0.14	0.15	0.63	0.65	0.09	0.63	0.61	0.24
BU4	0.67	1.36	0.20	0.40	0.64	0.99	1.66	1.11	0.57	0.09
BU5	0.64	0.08	0.07	0.15	0.38	0.52	2.69	4.64	0.44	1.44
BU6	0.93	3.10	0.20	0.30	1.82	0.86	2.01	12.3	0.40	1.82
BU7	0.70	0.42	0.02	0.57	1.31	1.77	11.0	×	1.88	3.94

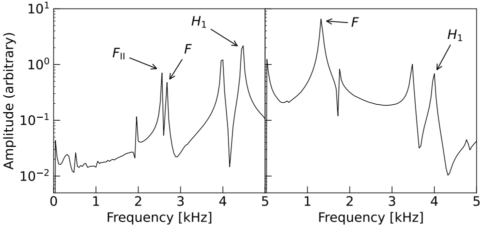
Comparison of mode frequencies with Krüger *et al.* (2010), Krüger & Kokkotas (2020), … in %

ROXAS: differential rotation

Jaraba *et al.* (2026)

Initial data with Komatsu *et al.* 1989 – KEH profiles.

Existence of an "additional" mode $F_{\rm II}$ in Cowling (Stergioulas *et al.* 2004), not present with dynamical spacetime.



Spectra of $\ell = m = 0$ harmonic of B4 initial data in Cowling (left) and CFC (right).

Model	F	H_1	${}^2f_0$	${}^2f_2$	${}^2f_{-2}$	δF (%)	δH_1 (%)	$\delta\, {}^2f_0$ (%)
B0	1.443	3.953	1.595	1.564	1.564	1.06	0.46	0.58
B1	1.429	3.915	1.634	1.118	1.945	1.57	0.32	0.34
B2	1.392	3.914	1.674	0.881	2.072	1.37	0.33	0.27
B3	1.359	3.953	1.718	0.681	2.159	1.99	0.27	0.51
B4	1.327	4.003	1.759	0.514	2.216	3.12	0.27	0.7
B5	1.287	4.073	1.800	0.320	2.276	4.03	0.02	0.63
B6	1.246	4.122	1.840	0.138	2.312	5.74	0.10	1.18
B7	1.210	4.193	1.881	−0.055	2.353	12.36	0.33	1.48
B8	1.172	4.224	1.921	−0.235	2.392	8.54	0.28	1.1
B9	1.130	4.177	1.960	−0.471	2.459	19.60	6.54	2.23

Mode frequencies (kHz) and their comparison with Dimmelmeier *et al.* (2006)

Modeling temperature effects

Initial data & additional equation

- Rigid rotation (as a start)
- Rotating equilibrium sequences computed with constant entropy/baryon profile (e.g. Goussard *et al.* 1997, Marques *et al.* 2017)
- Entropy-dependent EoS $c_s(H, S)$, but no composition information
- EoS: ideal gas (too simple) or a polytrope + analytic thermal contribution of a Fermi gas as in Raithel *et al.* (2019)

Addition of an evolution equation for the entropy per baryon S:

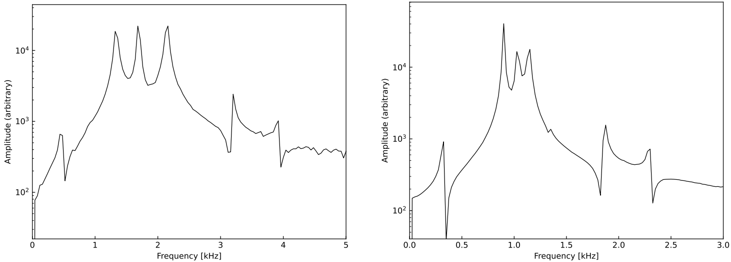
$$\partial_t S = -v^i D_i S,$$

very simple when no thermal sources/sinks are present.

Modeling temperature effects

Some very preliminary results

Runs from initial star with $S = 0$ (cold) and $S = 2$, corresponding to the gravitational mass and axis ratio of BU4 ($M_g = 1.54\,M_\odot, r_p/r_e = 0.8$).
Perturbed either in enthalpy or in entropy.



Spectrograms for $S = 0$ (left) and $S = 2$ (right). CFC dynamics for both.

$\Rightarrow$ Same mode frequencies for both types of perturbations
$\Rightarrow$ Recovery of “cold” modes in the case $S = 0$
$\Rightarrow$ Lower frequencies, as expected (larger radii).

Summary – Outlook

- ROXAS is a full non-linear hydro code in CFC, based on spectral methods and primitive variable hydro formulation.
- Shown to work/retrieve oscillation frequencies of rotating neutron stars in the literature.
- Extended to differential rotation (first time 3D with dynamical spacetime)
- Latest upgrade to include thermal effects: first runs successful, but more testing is needed.
- Combine both to simulate BNS remnant modes.
- ROXAS (without entropy evolution) is publicly available on Zenodo: https://zenodo.org/records/14849547

Thank you!