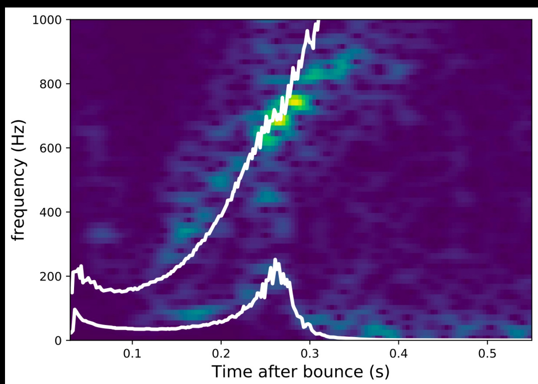
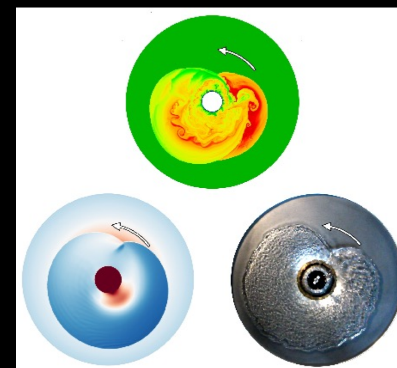
# Inferring the progenitor properties from the gravitational waves and neutrino signals of a CCSN

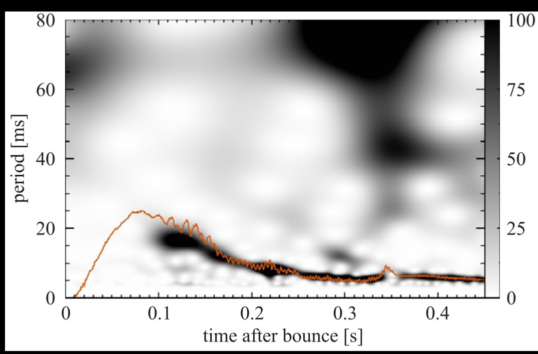



Powell+21


Thierry Foglizzo
**Alphonse Moreau**
Eric Chassande-Mottin
Bernhard Müller, Jade Powell
G. Durand, J. Guilet, M. Bugli

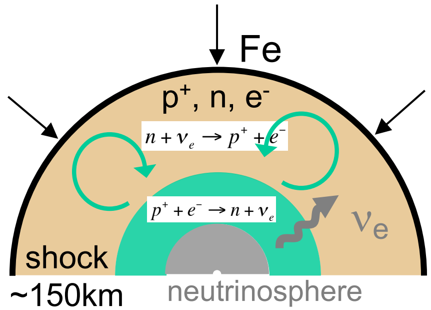
CEA Saclay
APC
Monash U.
Swinburne U.

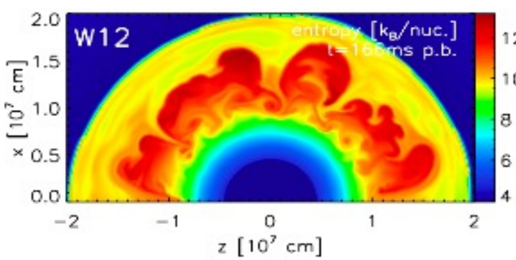





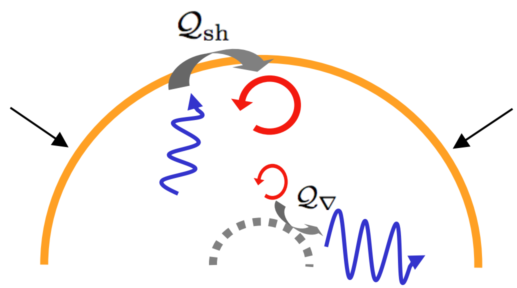

Müller & Janka+14

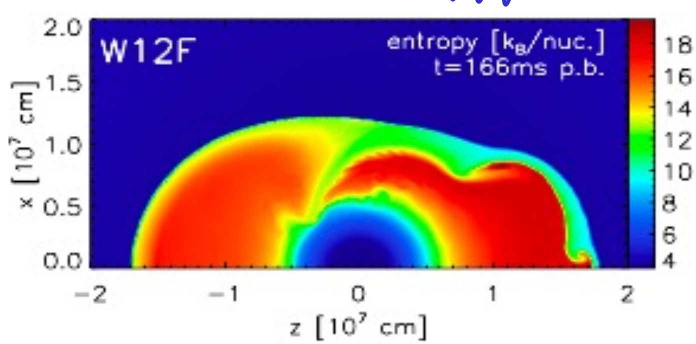
# Hydrodynamical instabilities during the phase of stalled accretion shock

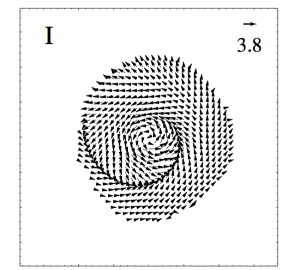


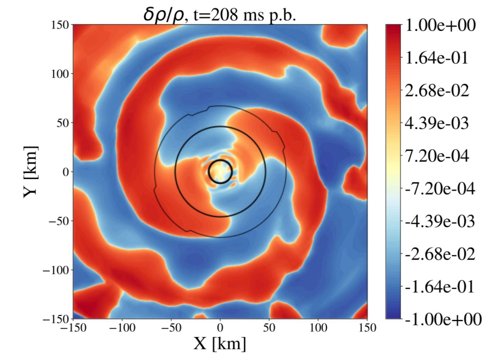

Foglizzo+06




Saijo & Yoshida 06




Bugli+23

## Neutrino-driven convection
(Herant+92)

- entropy gradient
- angular scale l=5,6

## "SASI"
Standing Accretion Shock Instability
(Blondin+03)

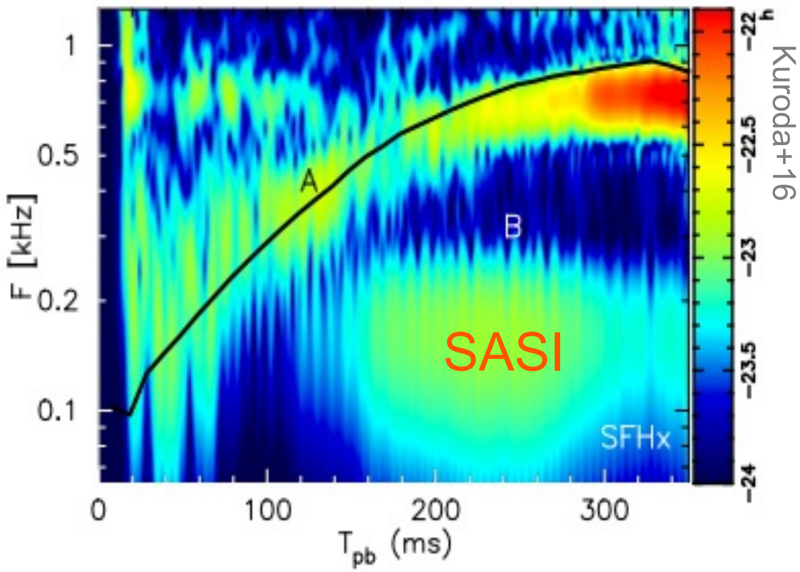
- advective-acoustic cycle
- oscillatory, large angular scale
  l=1,2

## "Low T/|W|"
corotation instability
(Shibata+02)

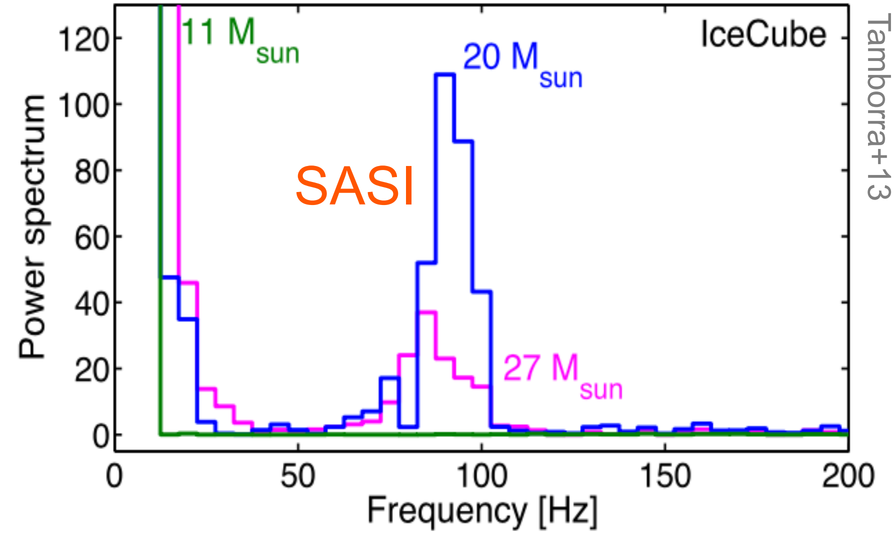
- differential rotation, corotation radius
- oscillatory, large angular scale
  m=1,2

SASI oscillations can leave a direct imprint
on the gravitational wave and neutrino signals: reverse engineering?

Gravitational Waves




Oscillations of the neutrino flux




**stellar parameters:**
progenitor mass
compactness
angular momentum
inhomogeneities

+

**3D dynamics**:
SASI
ν-driven convection
low T/|W|
PNS dynamo

+

**uncertain physics**:
reaction rates
EOS
neutrino interactions
magnetic fields

+

**numerical approximations:**
neutrino transport
2D vs 3D
turbulence
numerical viscosity

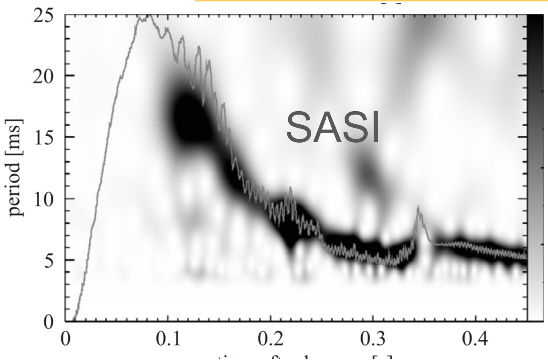
+

**detection challenge**:
neutrinos: 1987A
3rd generation detectors

Can gravitational wave and neutrino signatures disentangle so many processes?

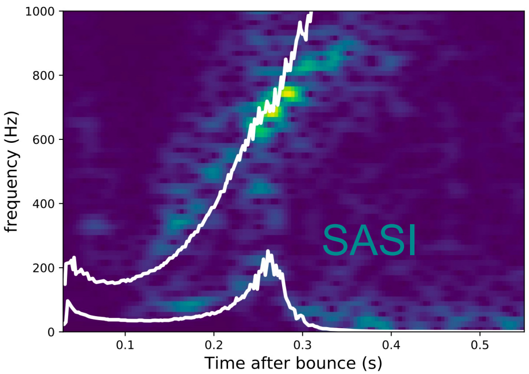
# Multi-messenger reverse engineering: empirical fits

Neutrino modulation



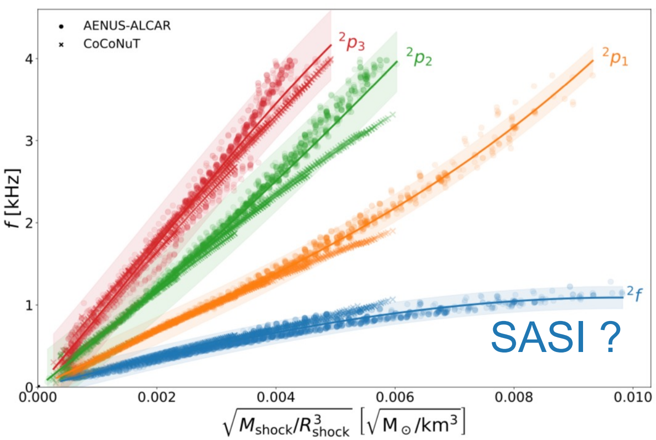

Müller & Janka+14

GW




Powell+21

GW




Torres-Forné+19

## Modulation of the neutrino flux

empirical formula (Müller & Janka+14)

obtained from a 2D axisymmetric simulation with $M_{ZAMS}$=25$M_{sol}$

$$\boxed{{}^{1}f_{\rm M14} \equiv \frac{52.6\,\mathrm{Hz}}{\log(r_{\rm sh}/r_{\rm ns})}\left(\frac{100\,\mathrm{km}}{r_{\rm sh}}\right)^{\frac{3}{2}}}$$

accuracy?

## Frequency of gravitational waves

2x($^{1}f_{SASI}$)

compared to GW from a 3D simulation with $M_{ZAMS}$= 85$M_{sol}$ (Powell+21)

empirical relation (Torres-Forné+19)

obtained from 2D axisymmetric simulations with equatorial symmetry (l=2) and $M_{ZAMS}$=20$M_{sol}$

$${}^{2}f_{\rm T21} = 100\,\mathrm{Hz}\left(\frac{M}{M_\odot}\right)^{\frac12}\left(\frac{100\,\mathrm{km}}{r_{\rm sh}}\right)^{\frac32}\times\left\{1.41-0.0423\left(\frac{M}{M_\odot}\right)^{\frac12}\left(\frac{100\,\mathrm{km}}{r_{\rm sh}}\right)^{\frac32}\right\}$$

-connection to SASI?
-frequency is independent of $r_{PNS}$?
-accuracy?

Main physical parameters: shock radius $r_{sh}$, accretor mass $M_{sh}$~$M_{ns}$, accretor radius $r_{PNS}$

# Perturbative study of SASI

- Stationary accretion with non adiabatic cooling ($\nu$-emission)

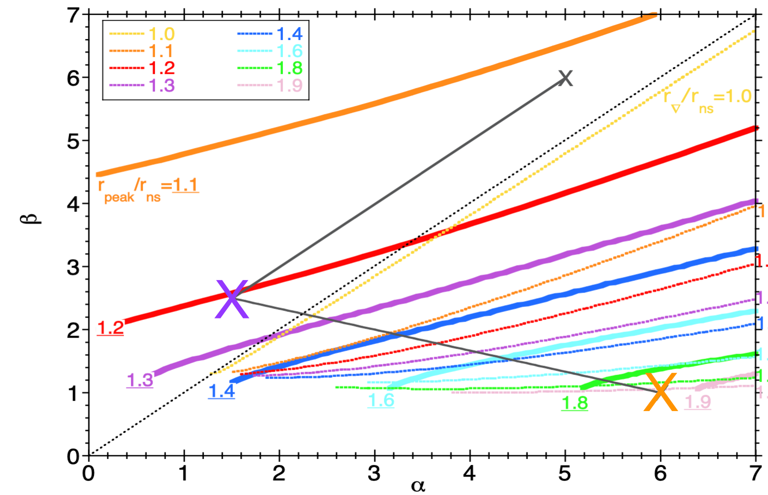
$$\frac{\partial S}{\partial r} = \frac{\mathcal{L}}{pv}\,,$$

$$\frac{\partial}{\partial r}\left(\frac{v^2}{2} + \frac{c^2}{\gamma - 1} + \Phi\right) = \frac{\mathcal{L}}{\rho v}\,,$$

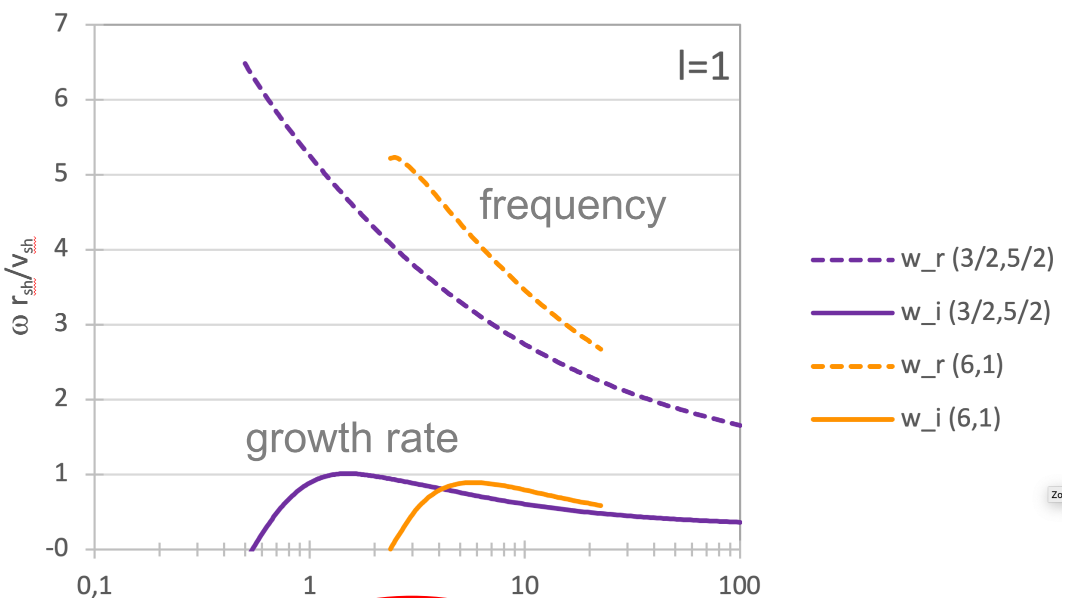
$$\mathcal{L} = A_{\rm cool}\rho^{\beta-\alpha}p^{\alpha}$$

temperature peak $r_{peak}$

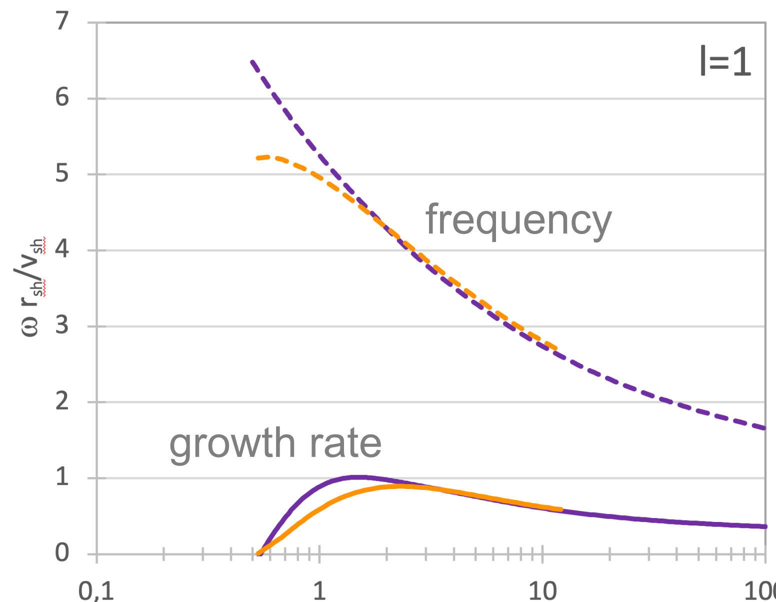
deceleration radius $r_{\nabla}$




- Perturbations: eigenfrequency ($\omega_r$,$\omega_i$)




Foglizzo+07




$r_{sh}/r_{ns}$ -1 → $r_{sh}/r_{\nabla}$ -1

SASI frequency is better described by $r_{\nabla}$ than $r_{peak}$ or $r_{ns}$ (Scheck+08)

# Analytical estimate of the l=1&2 SASI frequencies including rotation



$$^1 f_{M14} \equiv \frac{52.6\mathrm{Hz}}{\log(r_{sh}/r_{ns})}\left(\frac{100\mathrm{km}}{r_{sh}}\right)^{\frac{3}{2}}$$

physical formula:

obtained from a perturbative analysis of l=1,2 SASI for any progenitor

$$^1 f = \frac{7v_{sh}}{v_1}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}\left(\frac{100\mathrm{km}}{r_{sh}}\right)^{\frac{3}{2}} \times \left[\frac{61.6\mathrm{Hz}}{\left(\frac{r_{sh}}{r_\nabla}-1\right)^{0.29}} + \frac{7.34\mathrm{Hz}}{\left(\frac{r_{sh}}{r_\nabla}-1\right)^{0.38}}\frac{L}{L_K^\nabla}\left(\frac{r_{sh}}{2r_\nabla}\right)^{\frac{1}{2}}\right]$$

→. neutrinos ($^1$f), GW (2x$^1$f)

$$^2 f = \frac{7v_{sh}}{v_1}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}\left(\frac{100\mathrm{km}}{r_{sh}}\right)^{\frac{3}{2}} \times \left[\frac{76.8\mathrm{Hz}}{\left(\frac{r_{sh}}{r_\nabla}-1\right)^{0.33}} + \frac{15.1\mathrm{Hz}}{\left(\frac{r_{sh}}{r_\nabla}-1\right)^{0.28}}\frac{L}{L_K^\nabla}\left(\frac{r_{sh}}{2r_\nabla}\right)^{\frac{1}{2}}\right]$$

→ neutrinos ($^2$f), GW ($^2$f)

$^2$f/$^1$f ~1.25 without rotation

talk by Alphonse Moreau

The SASI frequency depends on ($\alpha$,$\beta$) through $r_\nabla$

3 physical parameters:

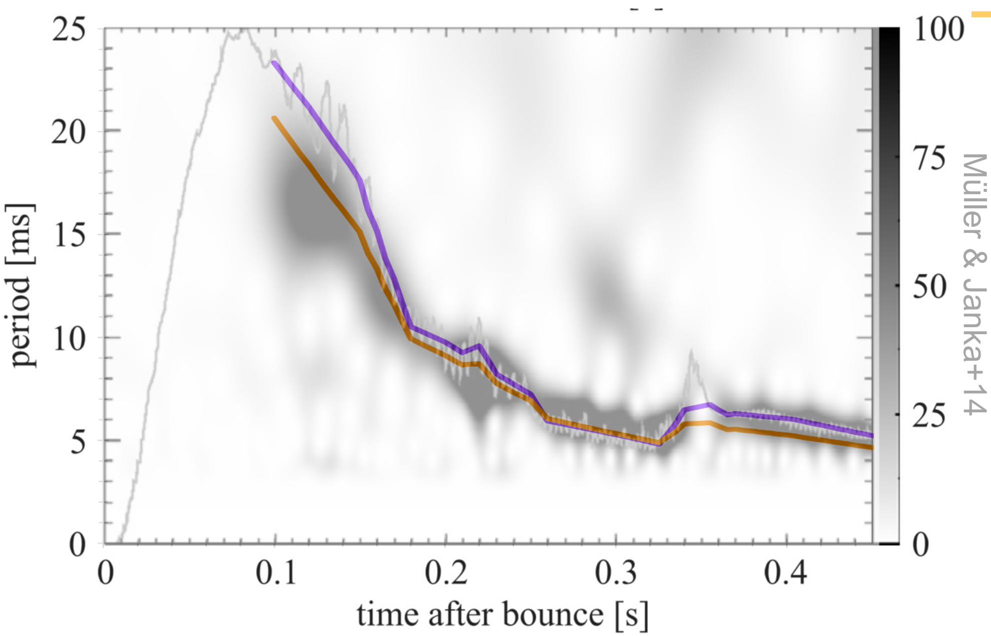
- -shock radius $r_{sh}$,
- -radius of maximum deceleration $r_\nabla$,
- -proto-neutron star mass M
- -specific angular momentum L

To be tested with rotation

Further improvement: $\nu$-heating, $\alpha$ dissociation, GR

# Neutrino test of the analytical estimate of the l=1 SASI frequency



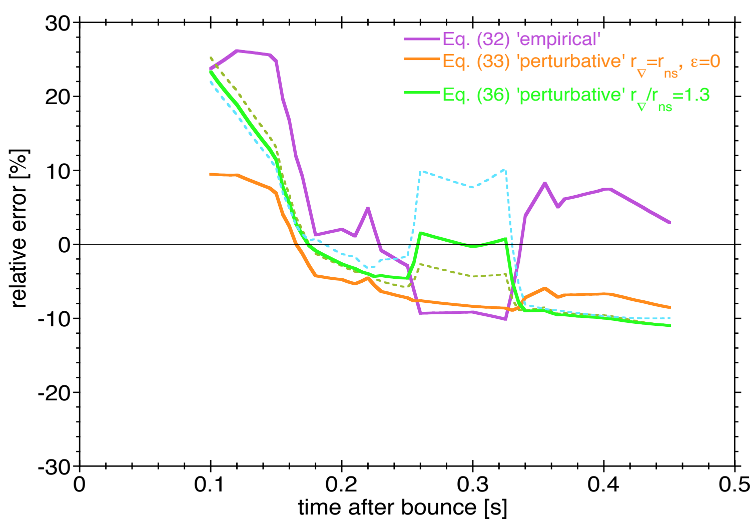



**empirical formula:** accuracy 26%

obtained from the neutrino signal with $M_{ZAMS}=25M_{sol}$

$$\boxed{{}^{1}f_{\rm M14} \equiv \frac{52.6\rm Hz}{\log(r_{\rm sh}/r_{\rm ns})}\left(\frac{100\rm km}{r_{\rm sh}}\right)^{\frac{3}{2}}}$$

Müller & Janka+14




**physical formula:** improved accuracy 10%

obtained from a perturbative analysis of l=1 SASI for any progenitor

$$\boxed{{}^{1}f = \frac{61.6\rm Hz}{\left(\frac{r_{\rm sh}}{r_\nabla}-1\right)^{0.29}}\frac{7v_{\rm sh}}{v_1}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}\left(\frac{100\rm km}{r_{\rm sh}}\right)^{\frac{3}{2}}}$$

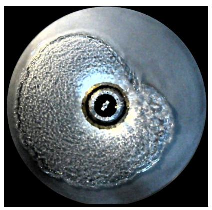
# Turbulent stabilization ?

$$H_\Phi \equiv -\frac{R_{45}^2}{r}$$

without rotation,
turbulent SASI @ 100L/s
seems less unstable than
laminar SASI @1L/s

$R_{45} = 5.6cm$

66cm



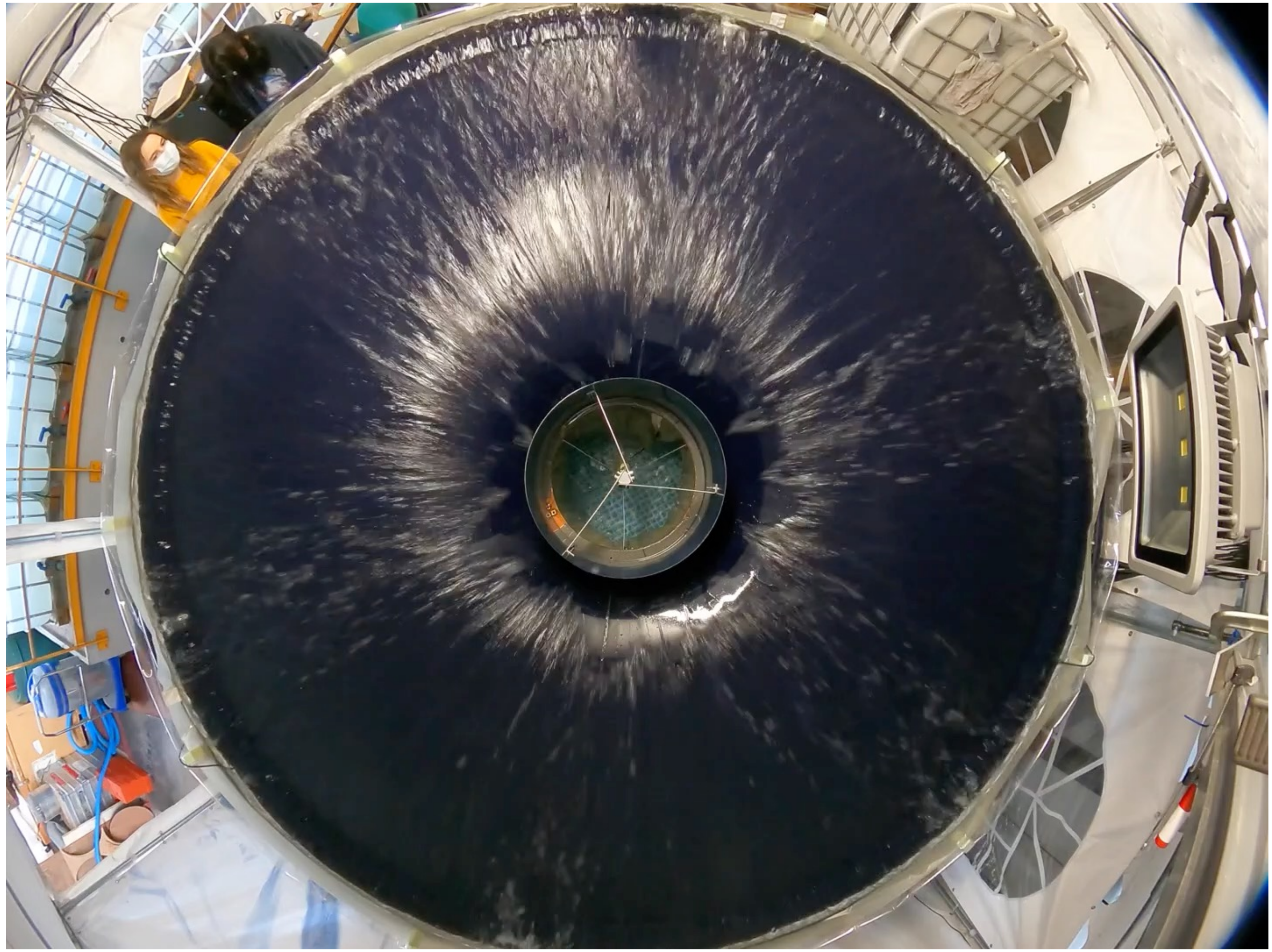
$$\lambda \equiv \frac{R'_{45}}{R_{45}} = 6.25$$

Reynolds x $6.25^{\frac{3}{2}} \sim 15.6$

Flow rate x $6.25^{\frac{5}{2}} \sim 98$

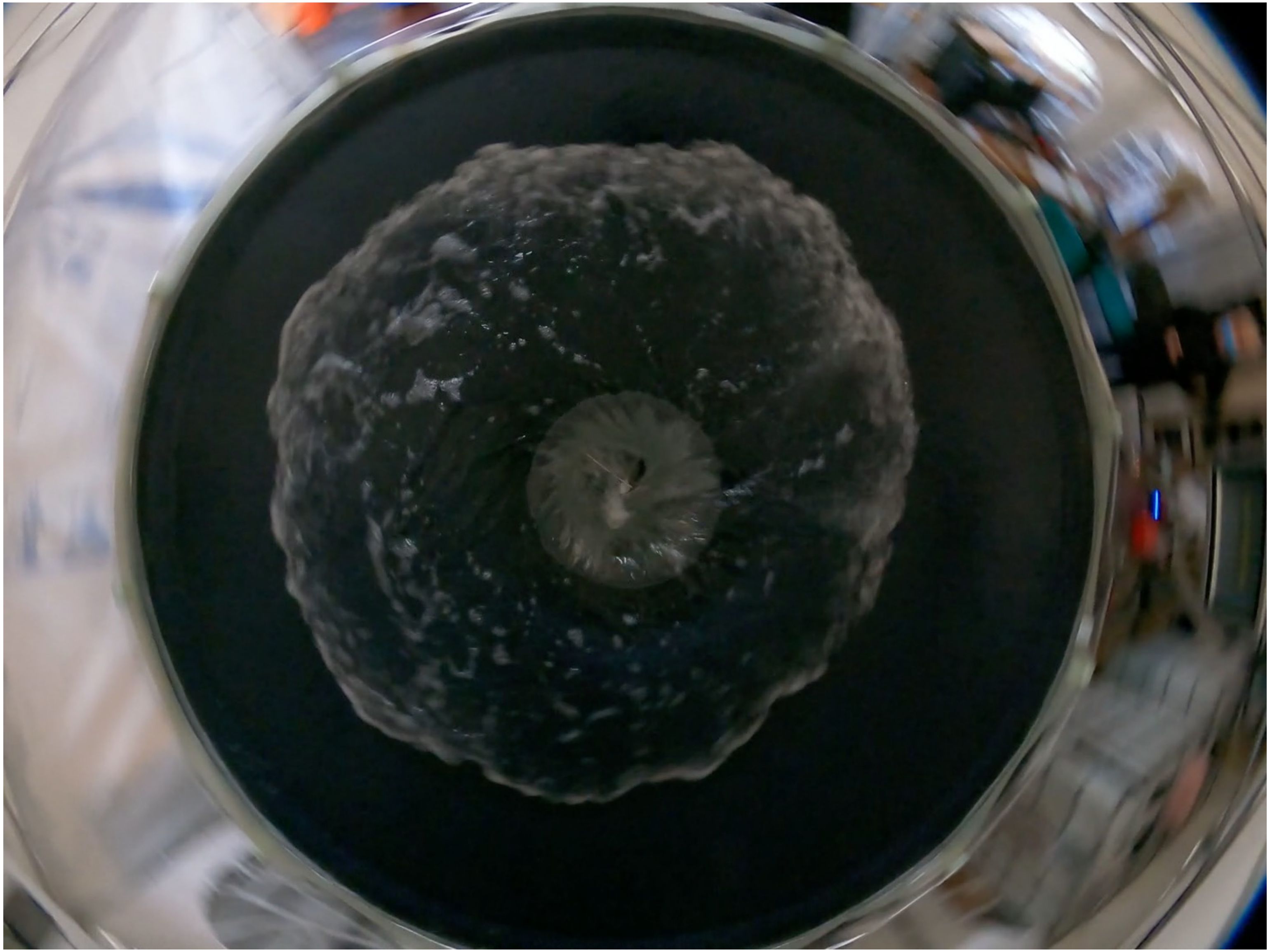
$R'_{45} = 35cm$

350cm



turbulent hydraulic jump

# A small amount of rotation is sufficient to destabilize the flow



# SASI as a self-forced acoustic oscillator



## Simplified perturbative studies:

- non adiabatic cooling/heating (ν-processes)

$$\mathcal{L} = A_{\rm cool}\rho^{\beta-\alpha}p^{\alpha}$$

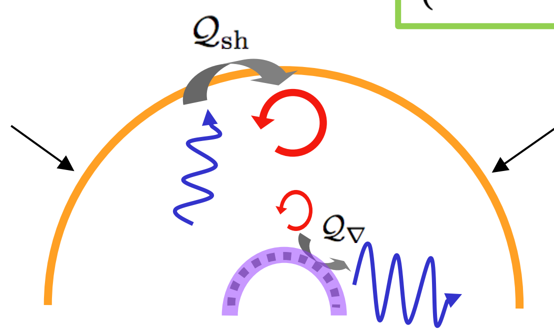
- $4^{\rm th}$ order differential system

$$\left\{\begin{aligned}
\left(\frac{1-\mathcal{M}^2}{v_r}\frac{\partial}{\partial r}+\frac{i\omega'}{c^2}\right)\delta h &= -\frac{\omega'^2-\omega_{\rm Lamb}^2}{v_r^2c^2}\frac{r\delta v_\varphi}{im}+\frac{i\omega'}{v_r^2}\left(\frac{\delta k_\theta}{imc^2}-\delta S\right),\\
\left(\frac{1-\mathcal{M}^2}{v_r}\frac{\partial}{\partial r}+\frac{i\omega'}{c^2}\right)\frac{r\delta v_\varphi}{im} &= \delta h-\frac{\delta k_\theta}{imv_r^2}+\left(\gamma-1+\frac{1}{\mathcal{M}^2}\right)\frac{\delta S}{\gamma},\\
\left(\frac{\partial}{\partial r}-\frac{i\omega'}{v_r}\right)\frac{\delta k_\theta}{im} &= \delta\left(\frac{\mathcal{L}}{\rho v_r}\right),\\
\left(\frac{\partial}{\partial r}-\frac{i\omega'}{v_r}\right)\delta S &= \delta\left(\frac{\mathcal{L}}{p v_r}\right).
\end{aligned}\right.$$

$$\delta h \equiv \frac{\delta v_r}{v_r}+\frac{\delta\rho}{\rho}$$

$$\frac{\delta k_\theta}{im} \equiv \frac{c^2}{\gamma}\delta S+\frac{rv_r}{im}\delta w_\theta$$

$$\delta\mathbf{w} \equiv \nabla\times\delta\mathbf{v}$$




## Adiabatic approximation

- linear conservation of entropy δS and baroclinic vorticity $\delta k_\theta$

- $2^{\rm nd}$ order differential system

$$\omega_{\rm Lamb}^2 \equiv \ell(\ell+1)\frac{c^2}{r^2}(1-\mathcal{M}^2)$$

$$\mathrm{dX} \equiv \frac{v}{1-\mathcal{M}^2}\mathrm{d}r$$

$$\boxed{\left\{\left(\frac{\partial}{\partial X}+\frac{i\omega'}{c^2}\right)^2+\frac{\omega'^2-\omega_{\rm Lamb}^2}{v_r^2c^2}\right\}(r\delta v_\varphi) = -\frac{\partial}{\partial X}\left(\frac{r\delta w_\theta}{v_r}\right)}$$

perturbed specific angular momentum

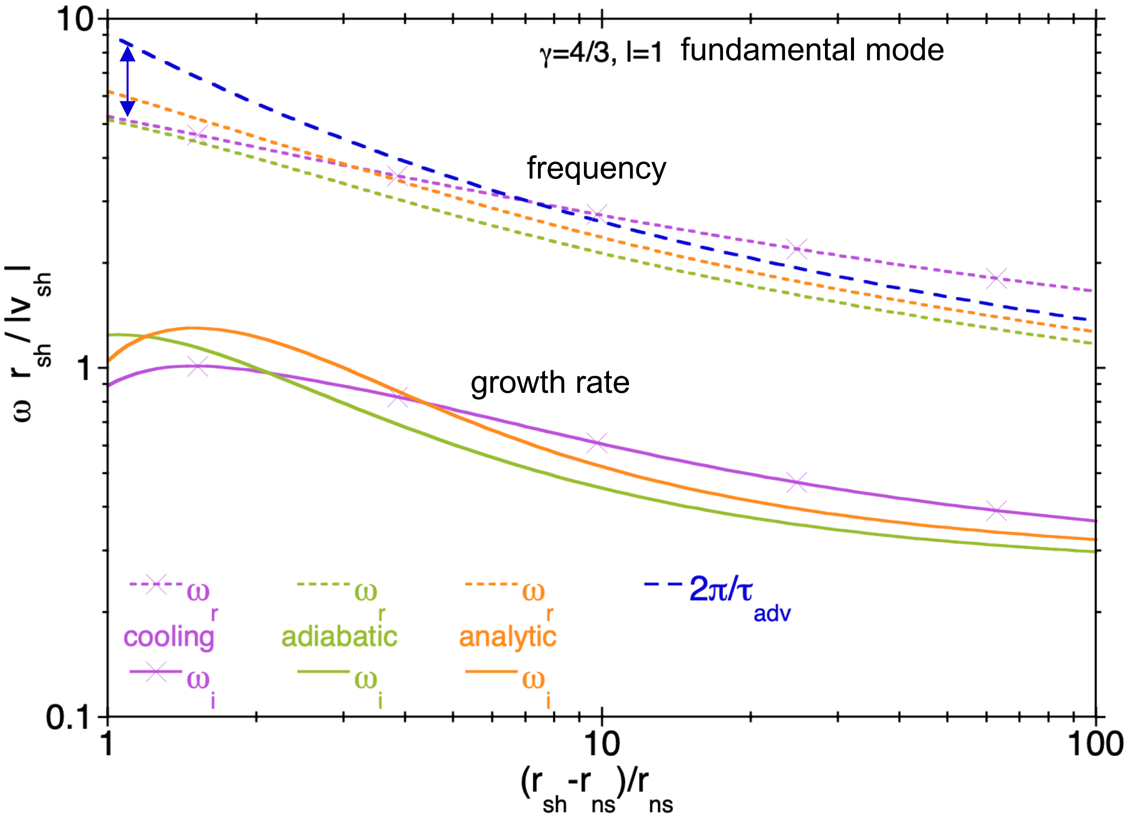
perturbed vorticity

$$\boxed{\omega' \equiv \omega-\frac{mL}{r^2}}$$

- acoustic oscillator
  self-forced by the advection of vorticity

# Comparison of SASI eigenfrequencies with/without a cooling function

Foglizzo 24,26




$$\omega^2_{\rm Lamb} \equiv \ell(\ell+1)\frac{c^2}{r^2}(1-\mathcal{M}^2) \qquad {\rm d}X \equiv \frac{v}{1-\mathcal{M}^2}{\rm d}r$$

$$\boxed{\left\{\left(\frac{\partial}{\partial X}+\frac{i\omega'}{c^2}\right)^2+\frac{\omega'^2-\omega^2_{\rm Lamb}}{v_r^2c^2}\right\}(r\delta v_\varphi) = -\frac{\partial}{\partial X}\left(\frac{r\delta w_\theta}{v_r}\right)}$$

perturbed specific angular momentum

perturbed vorticity

Forced oscillator + shock & pns boundary conditions

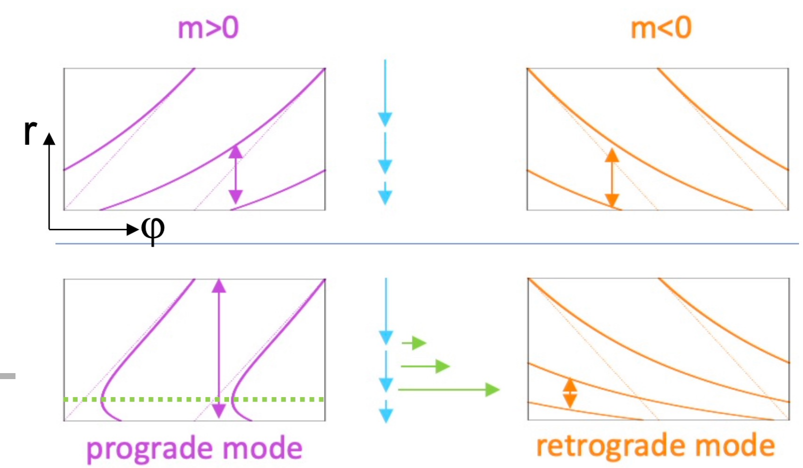
$2\pi/\tau_{adv}$ is ~80% inaccurate

Wronskian Method
→integral equation defining the eigenfrequencies

→analytic approximation

→ the adiabatic approximation captures the SASI eigenfrequency within 30%

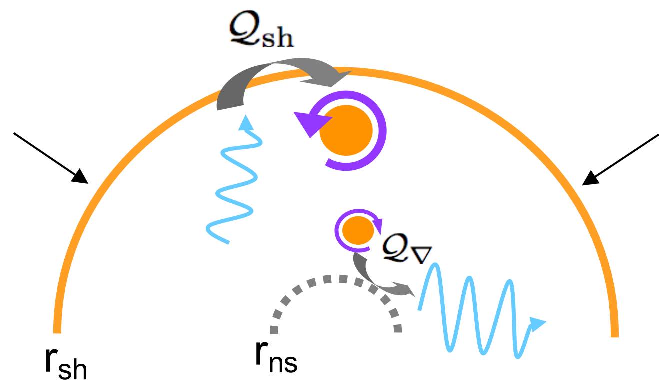
→ best phase match at corotation



# Impact of viscosity ν and thermal diffusivity κ on SASI?

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho v) = 0,$$
$$\frac{\partial v}{\partial t}+(v\cdot\nabla)v+\nabla\Phi = -\frac{\nabla p}{\rho}+\nu\left[\nabla^2 v+\frac{1}{3}\nabla(\nabla\cdot v)\right]$$
$$\frac{\partial S}{\partial t}+(v\cdot\nabla)S = \frac{\gamma\kappa}{\gamma-1}\frac{\nabla^2 c^2}{c^2}+\frac{1}{p}\tau:\nabla v.$$

$$\tau:\nabla v = 2\nu\rho\left[\frac{1}{2}(\partial_j v_i+\partial_i v_j)-\frac{1}{3}(\nabla\cdot v)\delta_{ij}\right]^2$$




In a plane parallel uniform flow with viscosity and thermal diffusion:

- vorticity perturbations are damped by viscosity $\boxed{\omega_i^{\rm visc} = -k^2\nu}$
- entropy perturbations are damped by thermal diffusivity $\boxed{\omega_i^{\rm diff} = -k^2\kappa}$
- acoustic perturbations are damped by both (with a longer wavelength) $\boxed{\omega_i^{\rm ac} = -\left[\frac{2\nu}{3}+\frac{\kappa}{2}(\gamma-1)\right]k^2}$

$$\mathrm{d}X \equiv \frac{v}{1-\mathcal{M}^2}\mathrm{d}r$$

In a spherical accretion flow

- without dissipative processes:

-vorticity
-entropy

are source terms for the acoustic oscillations of

$$\delta\mathbf{L} \equiv \mathbf{r}\times\delta\mathbf{v}$$
$$\delta A \equiv r^2\nabla\cdot\delta v_\perp$$

$$\boxed{\left[\left(\frac{\partial}{\partial X}+\frac{i\omega}{c^2}\right)^2+\frac{\omega^2-\omega_{\rm Lamb}^2}{v^2c^2}\right]\delta\boldsymbol{L} = \frac{\partial}{\partial X}\frac{r\delta\boldsymbol{w}}{v}}$$

$$\boxed{\left[\left(\frac{\partial}{\partial X}+\frac{i\omega}{c^2}\right)^2+\frac{\omega^2-\omega_{\rm Lamb}^2}{v^2c^2}\right]\frac{\delta A}{\ell(\ell+1)} = -\frac{\partial}{\partial X}\frac{\delta S}{\gamma\mathcal{M}^2}}$$

- with viscosity and thermal diffusion:

a global calculation of $(\omega_r,\omega_i)$ from $r_{sh}$ to $r_{ns}$ is necessary
+ it can also shed light on the dominant process driving SASI

Method: Taylor expansion of $\omega_i$ for small $(\nu, \kappa)$ to measure and compare $\frac{\partial\omega_i}{\partial\nu}$ and $\frac{\partial\omega_i}{\partial\kappa}$

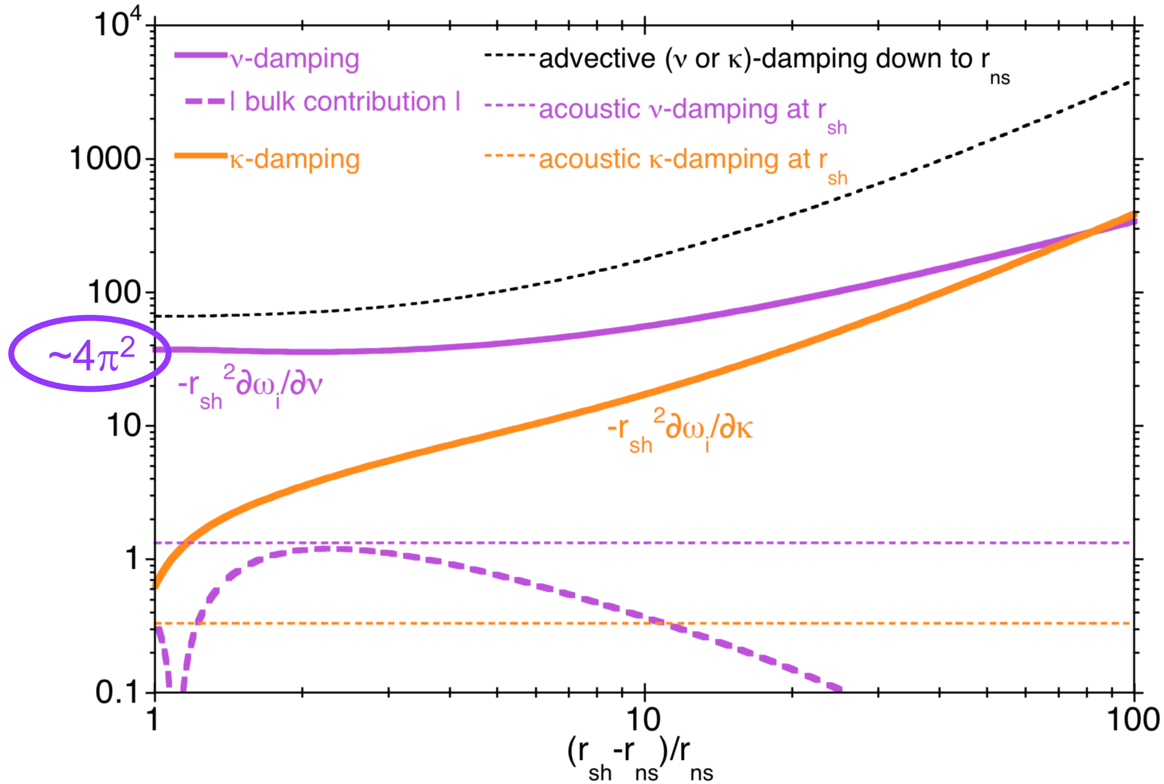
# SASI growth rate is very sensitive to viscosity

perturbative calculation

$$\rightarrow \boxed{\frac{\partial w_i}{\partial \nu} \sim -\frac{4\pi^2}{r_{\rm sh}^2}}$$

→SASI mechanism is mainly governed by the advection of vorticity perturbations

$$\frac{\delta w_i}{\omega_i} \sim -4\pi^2 \frac{\nu}{r_{\rm sh}|v_{\rm sh}|}$$




stabilization by numerical viscosity

$$\nu_{\rm num} \sim \left(\frac{1-C_{\rm CFL}}{2}\right) v\Delta r$$ (e.g. G. Leidi+24)

$$\frac{\delta w_i}{\omega_i} \sim -\frac{4\pi^2}{N_r}\left(\frac{1-C_{\rm CFL}}{2}\right)\frac{r_{\rm sh}-r_{\rm ns}}{r_{\rm sh}}$$

$$\boxed{\frac{\delta w_i}{\omega_i} \sim -26\% \left(\frac{30}{N_{\rm pns}^{\rm sh}}\right)}$$ ($C_{\rm CFL}$~0.4)

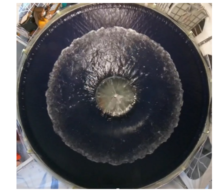
→ 30 grid points from $r_{ns}$=50km to $r_{sh}$=150km are insufficient in 3D simulations

stabilization by turbulence

$$\boxed{\frac{v_{\rm turb}}{v_{\rm sh}} \sim \frac{\nu_{\rm stab}}{r_{\rm sh}|v_{\rm sh}|} \sim \frac{1}{4\pi^2} \sim 3\%}$$

→ the turbulence invoked for SN explosions (Müller & Janka 15, Müller+17) and/or convection in the gain region can stabilize SASI

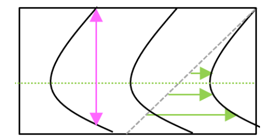
# Conclusions



## The shallow water experiments suggest

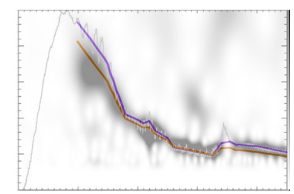
-the stabilizing effect of turbulence on SASI

-an adiabatic framework to understand SASI as a self-forced oscillator



## Rotation effects on SASI are clarified

-a forced oscillator sensitive to phase mixing



## Towards reverse engineering of multi-messenger SASI signatures

- improved accuracy based on perturbative analysis
- SASI l=1&2 frequencies depend on $r_{sh}$, Mns, $r_{\nabla}$ + rotation

## Damping of vorticity-driven SASI by viscosity (without rotation)

-turbulent velocities ≥ 3% $|v_{sh}|$ can stabilize SASI

-warning on the damping of vorticity by numerical viscosity

$$\frac{\partial w_i}{\partial \nu} \sim -\frac{4\pi^2}{r_{\rm sh}^2}$$