Forming extremally charged black holes with a null coordinates code

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3rd law of BH thermodynamic, reformulation by Israel, 1986

> *A nonextremal black hole cannot become extremal [...] at a finite advanced time in any continuous process in which the stress-energy tensor of accreted matter stays bounded and satisfies the weak energy condition in a neighborhood of the outer apparent horizon.*

But what if we could create extremal black holes ?

Equations

System of equations

- Einstein equation $$R_{ab} - \frac{1}{2} R_c{}^c g_{ab} = 8\pi \left(T^F_{ab} + T^\phi_{ab} \right)$$
- Maxwell equation $$\nabla^a F_{ab} = J_b$$
- Wave equation $$D^a D_a \phi = 0$$

Complex charged scalar field $\phi = \psi + i\chi$, charged covariant derivative $D_a = \nabla_a + iqA_a$

Initial data: $\phi(0, x) = p \text{ Gaussian}(x)\ e^{i\omega x}$

Gundlach and Martel, Phys. Rev. D 113, 044069 (2026), arXiv:2511.14874

Null coordinate metric

$$ds^2 = -2G\left(dudx + Bdu^2\right) + R^2 d^2\Omega$$

- Cones of constant u are null
- B parametrises the radial gauge $x(u, v)$
- Electromagnetic gauge choice $A_x = 0$
- ψ, χ and G are evolved, R is computed

Diagnostics

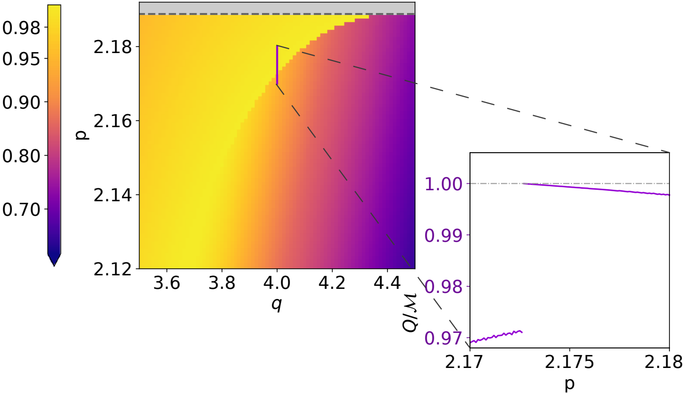
Spacetime local variables

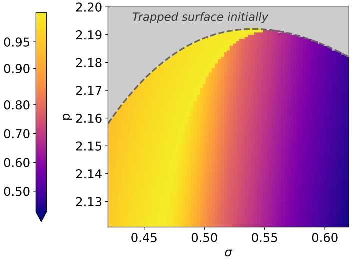
- Charge Q inside a sphere around the regular centre
- Hawking mass $M = \frac{1}{2}R(1 - |\nabla R|^2)$
- Augmented mass $\mathcal{M} := M + \frac{Q^2}{2R}$
- Computed null coordinate v, affine parameter λ

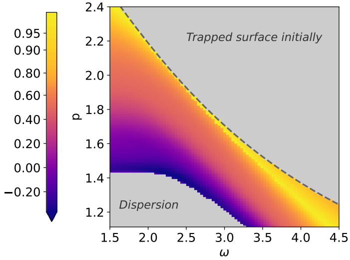
Evolution characteristics

- u_{FMOTS} time of the first marginally outer trapped surface
- Black hole mass $\mathcal{M}_{\text{FMOTS}}$ and charge Q_{FMOTS} estimated at the first marginally outer trapped surface

Parameter space exploration

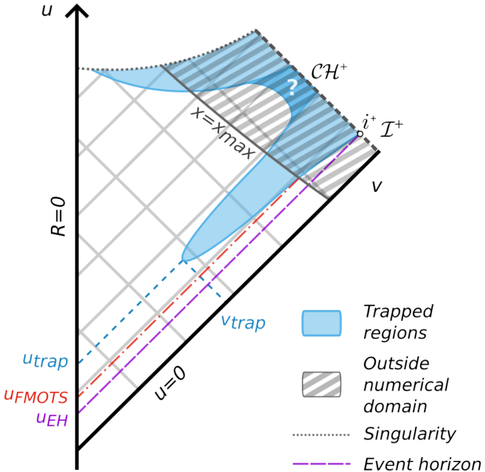


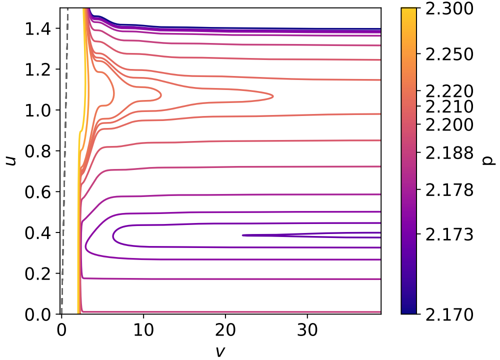




4 2-parameter families as evidence of codimension-1
(charge-to-mass ratio for 3 of them shown here)

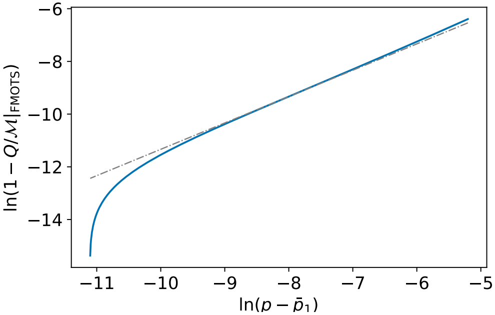
Trapped regions and spacetime picture

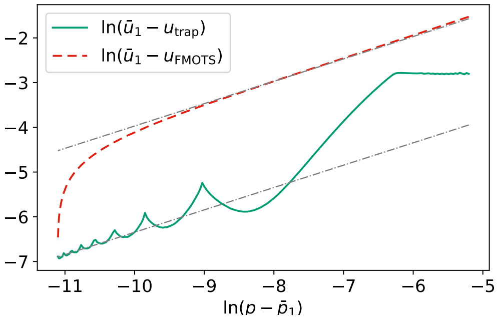




Contours of the trapped regions for a 1-parameter family of evolutions

Scaling for evolutions approaching extremality

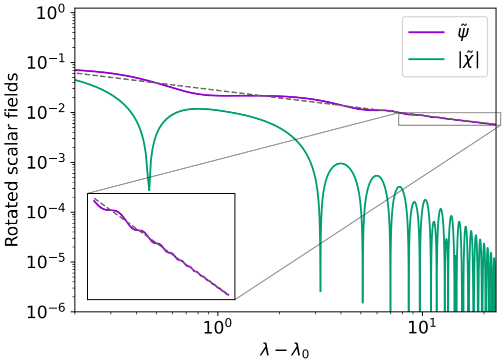


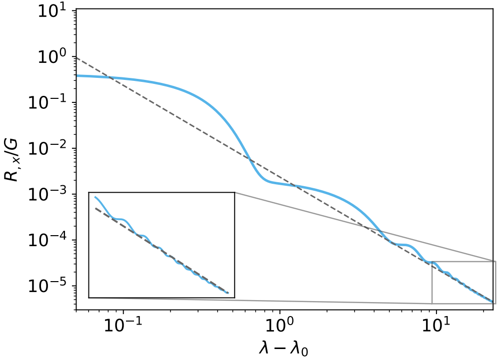


$$1 - (Q/\mathcal{M})_{\mathrm{FMTOS}} \propto (p - p_1)$$

$$(u_1 - u_{\mathrm{FMOTS}}) \propto (p - p_1)^{1/2}$$
$$(u_1 - u_{\mathrm{trap}}) \propto (p - p_1)^{1/2}$$

Scaling for one near-extremal evolution





$$\tilde{\psi}(u_{\mathrm{EH}}, \lambda) \propto (\lambda - \lambda_0)^{-1/2}$$

$|\tilde{\chi}(u_{\mathrm{EH}}, \lambda)|$ decaying

$$R_{,\lambda}(u_{\mathrm{EH}}, \lambda) \propto (\lambda - \lambda_0)^{-2}$$

Conclusion

- Observed evidence of asymptotically extremal black holes formed in a finite time on an internal codimension-1 threshold
- The lower trapped region moves to $v \to +\infty$ (at the Cauchy horizon) and potentially disconnects from the upper trapped region
- Scaling of evolution quantities when approaching extremality and of variables on near-extremal evolutions

Stress-energy tensor and null vector fields

$$T_{ab} : \begin{cases} T^F_{ab} \doteq \frac{1}{4\pi}\left(g^{cd}F_{ca}F_{db} - \frac{1}{4}F^{cd}F_{cd}g_{ab}\right) \\ T^\phi_{ab} \doteq \mathrm{Re}\left(D_a\phi\overline{D_b\phi}\right) - \frac{1}{2}D^c\phi\overline{D_c\phi}g_{ab} \end{cases}$$

$$J_a = -\frac{iq}{2}\left(\bar{\phi}D_a\phi - \phi\overline{D_a\phi}\right)$$

Outgoing null vector field

$$U := \frac{1}{G}\partial_x$$

Ingoing null vector field

$$\Xi := \partial_u - B\partial_x$$

We define $\hat{\Xi}\phi := \Xi\phi + iq\left(A_u - BA_x\right)\phi = D_u\phi - BD_x\phi$, and $\phi = \psi + i\chi$

$$C := 1 - |\nabla R|^2 = 1 + 2\Xi R\ R_{,x}/G$$

$$T = \frac{2}{G}\left[\hat{\Xi}\psi\ \psi_{,x} + \hat{\Xi}\chi\ \chi_{,x}\right]$$

Hierarchy equations

- **Shifted-doublenull gauge**

$$B_{sdn} = \frac{1}{2R_{,x}(u,0)}\left(1 - \frac{x}{x_0}\right)$$

- **Equations for R**

$$R_{,xx} - \frac{G_{,x}}{G} R_{,x} + 4\pi\left(\psi_{,x}^2 + \chi_{,x}^2\right) R = 0 \quad (1)$$

$$\left(\ln \frac{G}{R_{,x}}\right)_{,x} = \frac{4\pi R}{R_{,x}}\left(\psi_{,x}^2 + \chi_{,x}^2\right) \quad (2)$$

- **Other hierarchy equations**

$$Q_{,x} = 4\pi q R^2\left[\psi\chi_{,x} - \chi\psi_{,x}\right] \quad (3)$$

$$A_{u,x} = \frac{GQ}{R^2} \quad (4)$$

$$(R\Xi R)_{,x} = -\frac{G}{2} + \frac{GQ^2}{2R^2} \quad (5)$$

$$\left(R\hat{\Xi}\psi\right)_{,x} = -\psi_{,x}\,\Xi R - \frac{qGQ}{2R}\chi \quad (6)$$

$$\left(R\hat{\Xi}\chi\right)_{,x} = -\chi_{,x}\,\Xi R + \frac{qGQ}{2R}\psi \quad (7)$$

$$\left(B_{,x} - \Xi ln G\right)_{,x} = S_{\mathcal{H}}(R, G, \psi, \chi, Q) \quad (8)$$