# Large Pm, smale-scale kinematic dynamos in protoneutron stars

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# The convective dynamo hypothesis

NEUTRON STAR DYNAMOS AND THE ORIGINS OF PULSAR MAGNETISM

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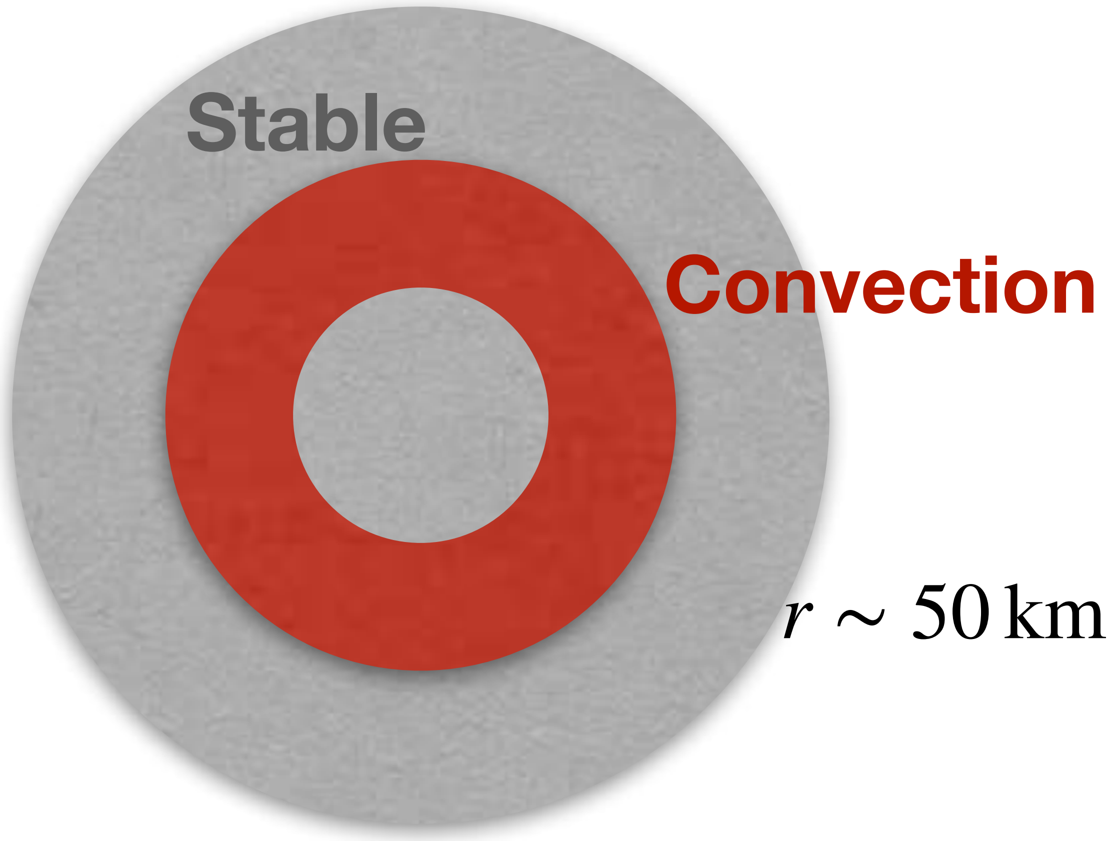
PNS structure




t = 0.2 s post bounce

## PNS convection

**Duration** $\Delta t \sim 10\,\mathrm{s}$ $L \sim 10^5\,\mathrm{cm}$

**Velocity** $U \sim 10^8\,\mathrm{cm\,.\,s^{-1}}$

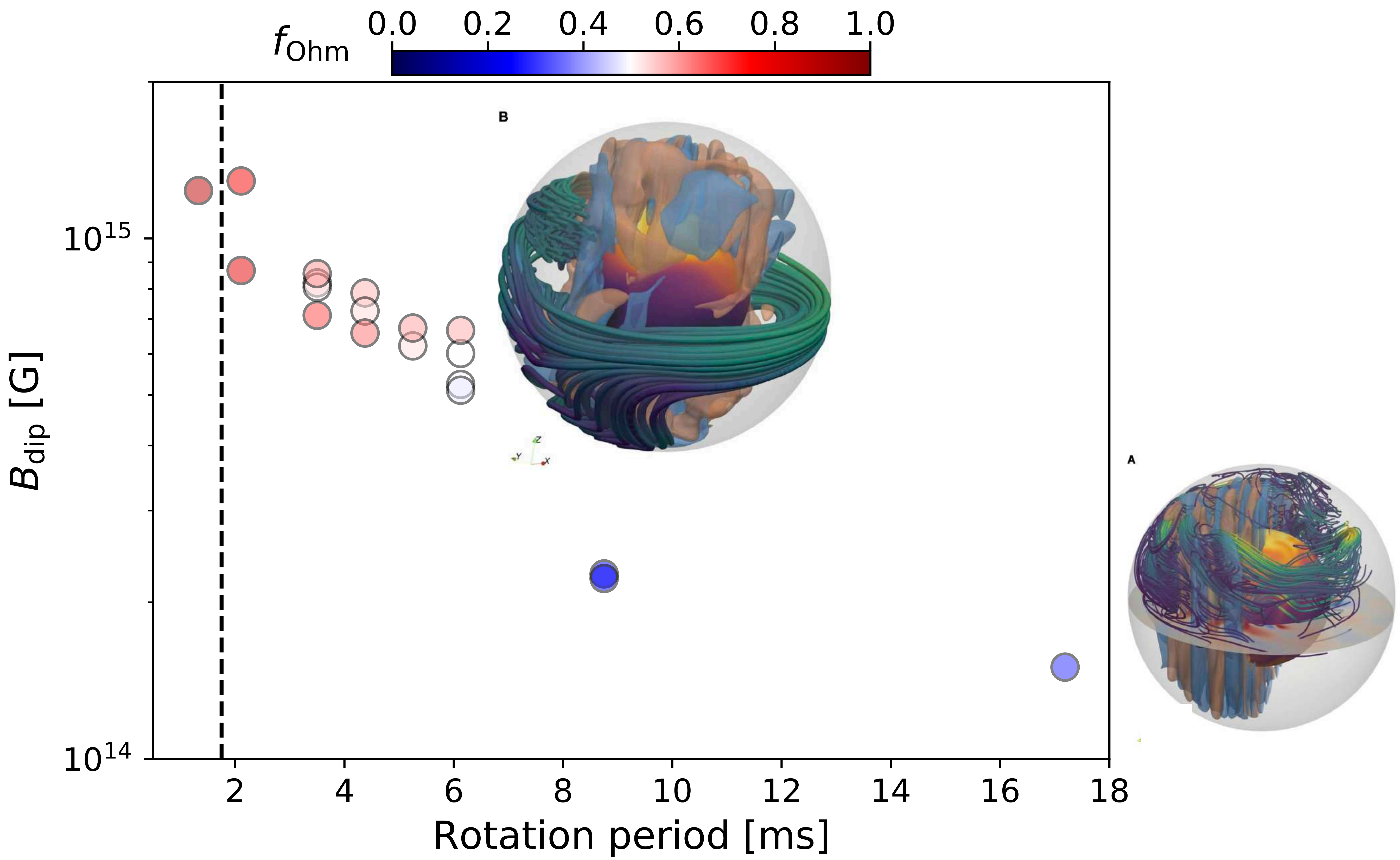
**Neutrino viscosity** $\nu \sim 10^{10}\,\mathrm{cm^2\,.\,s^{-1}}$

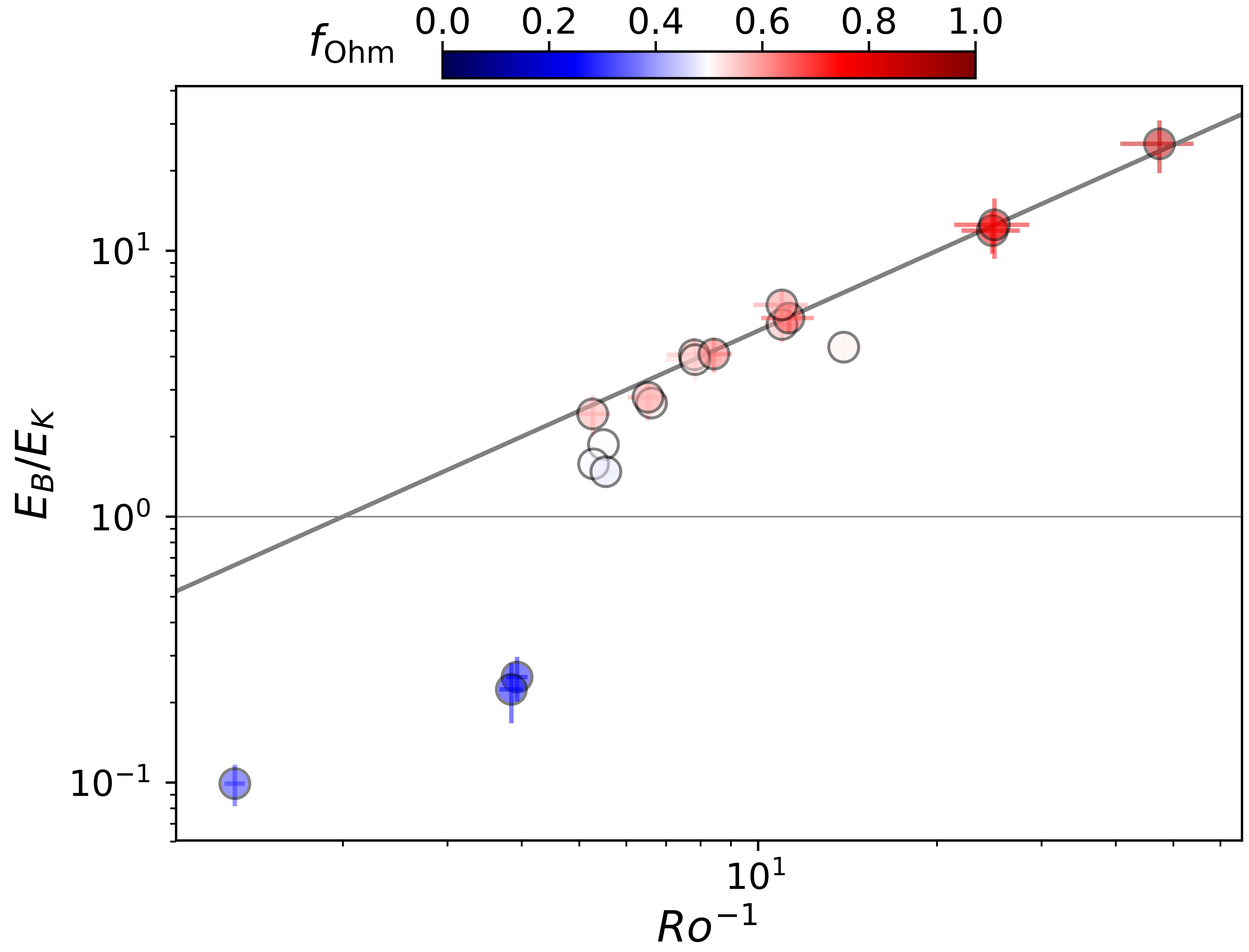
**Magnetic diffusivity** $\eta \sim 10^{-3}\,\mathrm{cm^2\,.\,s^{-1}}$

$$Re \equiv \frac{UL}{\nu} \sim 10^4$$

$$Pm \equiv \frac{\nu}{\eta} \sim 10^{13}$$

$$Rm \equiv \frac{UL}{\eta} \sim 10^{17}$$



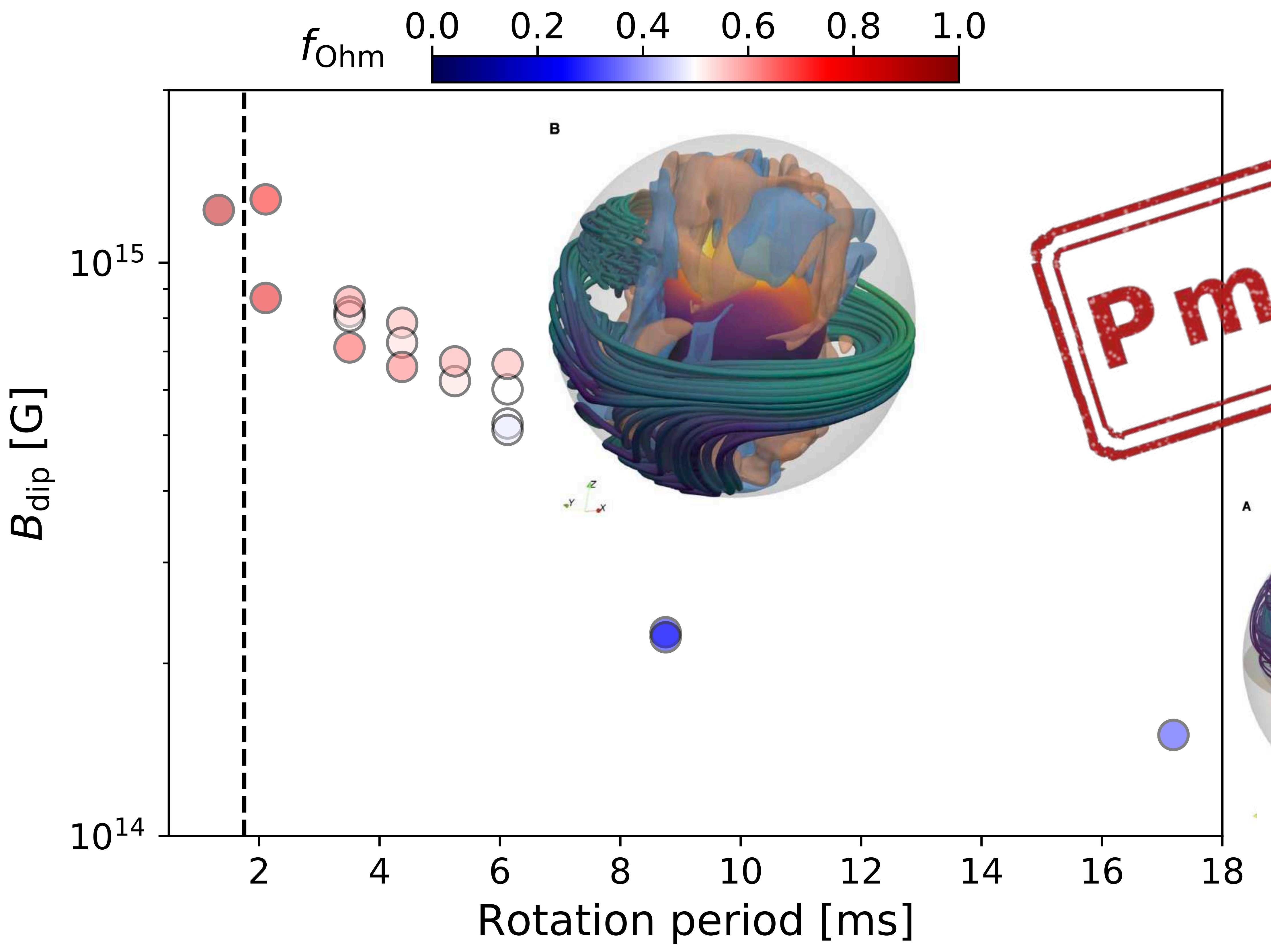

**Magnetostrophic force balance**

$$\text{Lorentz} \sim \text{Coriolis} \implies \frac{E_B}{E_k} \propto Ro^{-1} \equiv \left(\frac{u}{\Omega d}\right)^{-1}$$

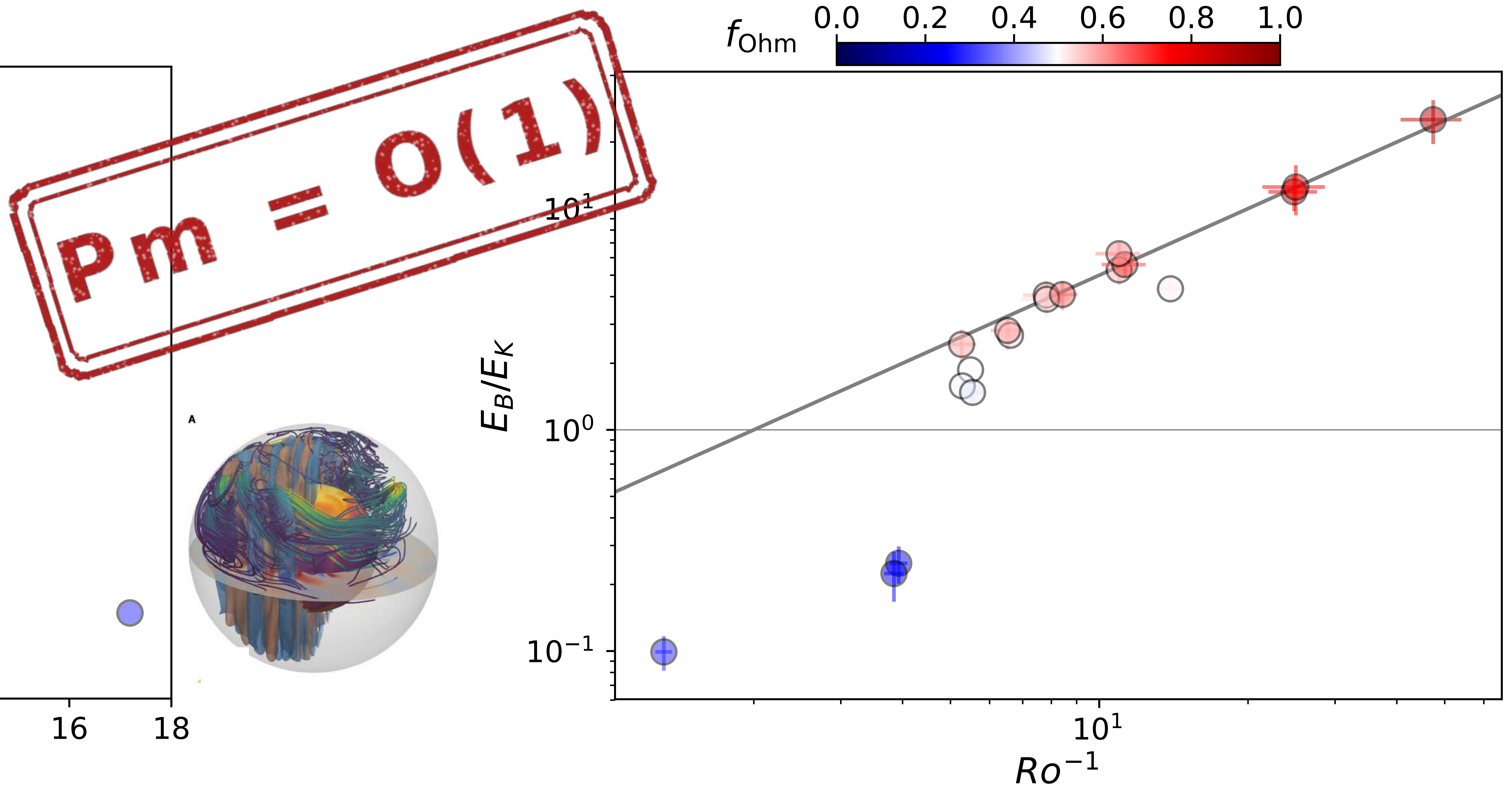
Raynaud+20

# But…

## Dipole field strength




## Strong field scaling



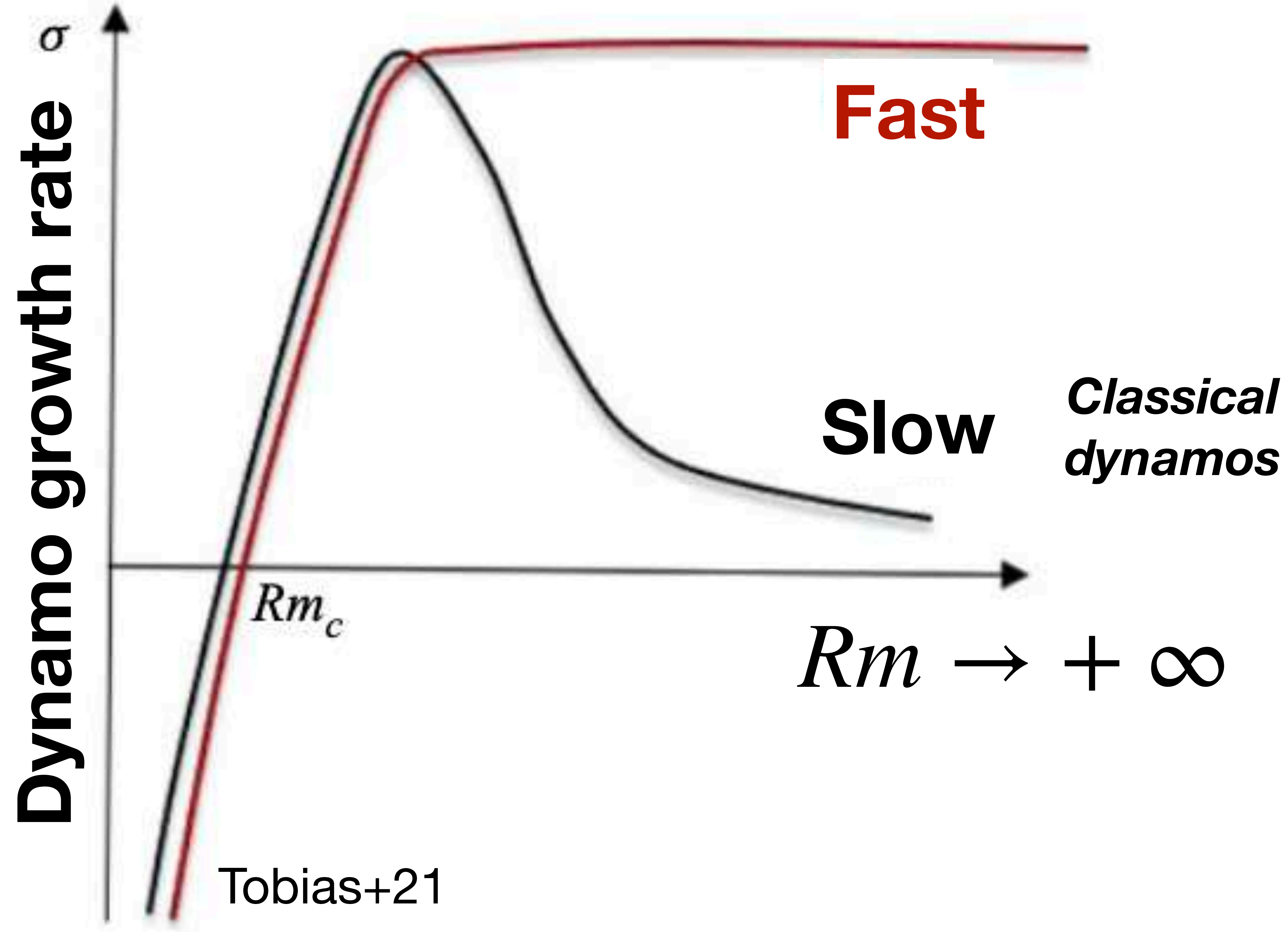

Raynaud+20

## Magnetostrophic force balance

$$\text{Lorentz} \sim \text{Coriolis} \implies \frac{E_B}{E_k} \propto Ro^{-1} \equiv \left(\frac{u}{\Omega d}\right)^{-1}$$

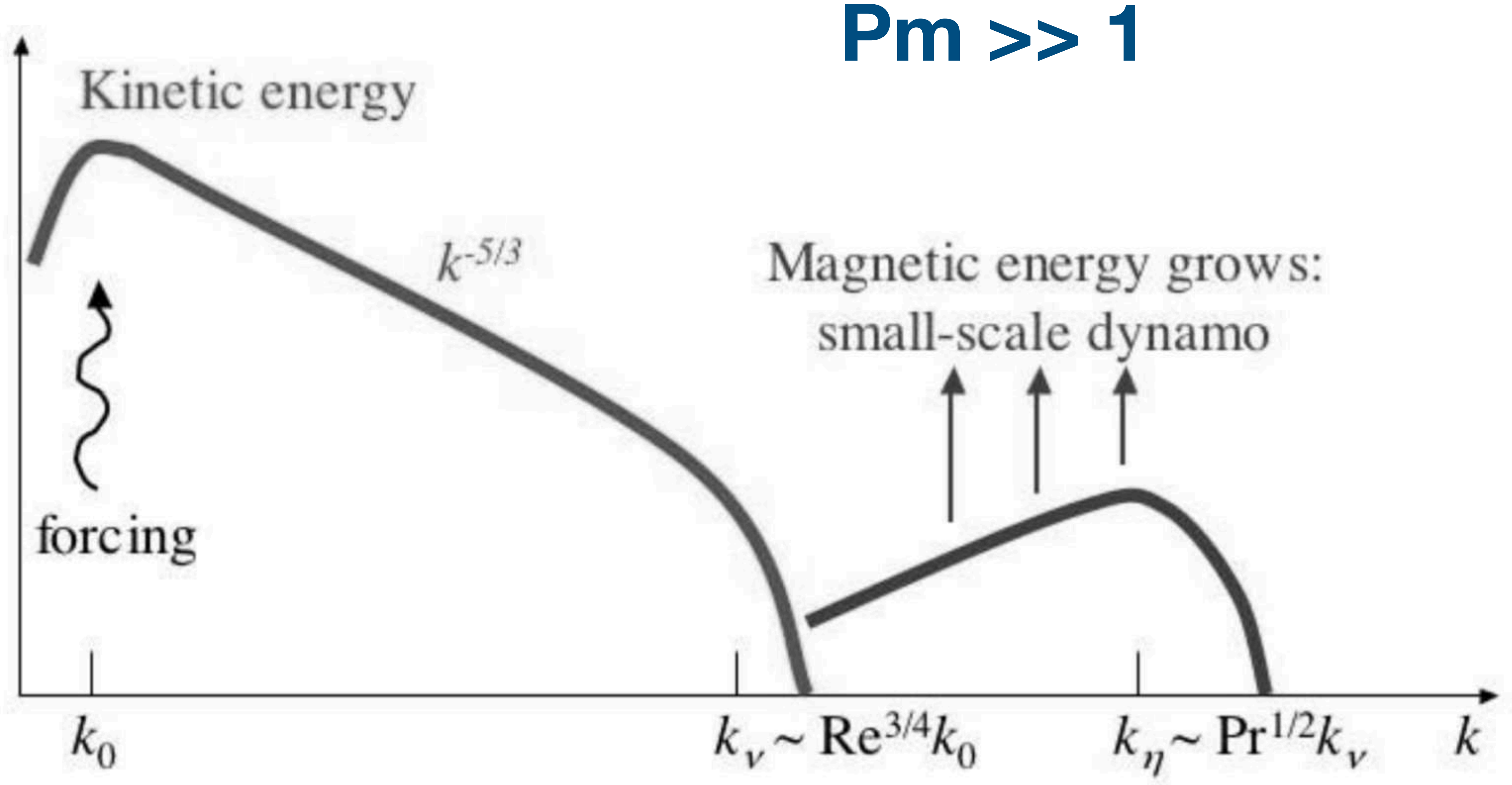
# High *Rm* and high *Pm* difficulties

**Kinematic dynamo problem**




**High Rm helps… up to a certain point**

**Pm >> 1**




Schekochihin+04

**Viscous scale** >> **Resistive scale**

***Q1: Is the amplification process « fast » enough in PNS ?***

***Q2: Can it explain magnetar large-scale dipolar field ?***

# Protoneutron star model 0.2 s after bounce

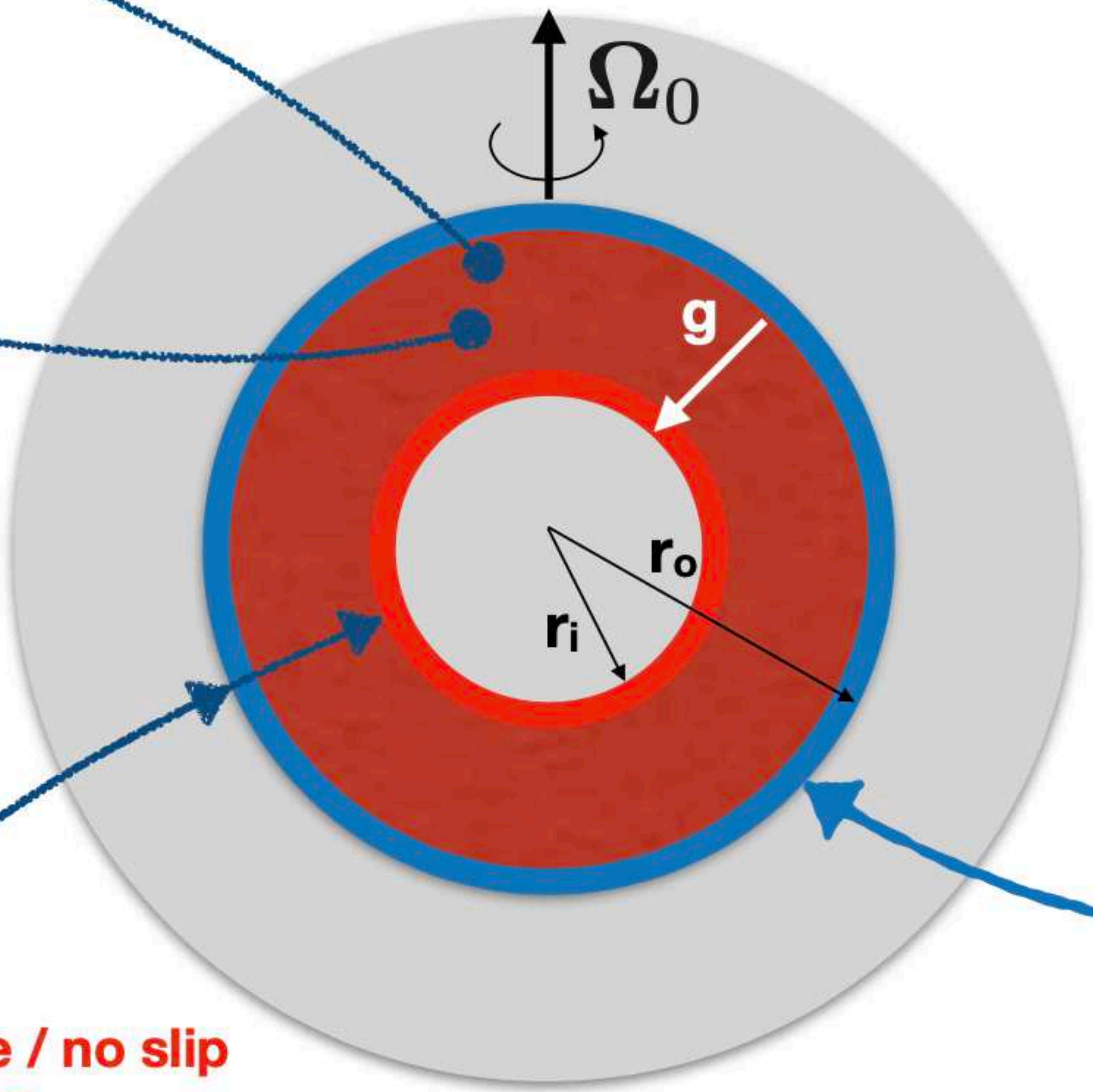
Taken from 1D CCSN

**Input:**

- Temperature profile
- Density profile

**Transport coefficients:**

- Kinematic viscosity $\nu$
- Thermal diffusivity $\kappa$
- Magnetic diffusivity $\eta$




**Hypothesis:**

- Spherical geometry
- Adiabatic stratification
- Low Mach convection
- 2nd order diffusion approximation for the neutrino transport
- Electrical conductivity of degenerate, relativistic electrons
- Orders of magnitude

$$\begin{cases} \Phi_o \sim 10^{52}\,\mathrm{erg/s} \\ r_o \sim 25\,\mathrm{km} \\ T_o \sim 10^{11}\,\mathrm{K} \\ \varrho_o \sim 10^{13}\,\mathrm{g/cm^3} \\ \nu_o \sim 10^{10}\,\mathrm{cm^2/s} \\ \kappa_o \sim 10^{12}\,\mathrm{cm^2/s} \\ \eta_o \sim 10^{-3}\,\mathrm{cm^2/s} \end{cases}$$

**Boundary conditions:**

- **Mechanical:** stress-free / no slip
- **Thermal:** fixed entropy flux
- **Magnetic:** perfect conductor ($B_{//}$) / pseudo-vacuum ($B_\perp$)

# Governing equations

$$[d] = r_o - r_i, \quad [t] = d^2/\nu_o, \quad [S] = d\,\partial S/\partial r|_{r_o}, \quad [p] = \Omega \varrho_o \nu_o, \quad [B] = \sqrt{\Omega \varrho_o \mu_0 \eta_o}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{Induction}} - \underbrace{\frac{1}{Pm} \nabla \times (\eta \nabla \times \mathbf{B})}_{\text{Dissipation}}$$

$$0 = \nabla \cdot (\tilde{\varrho} \mathbf{u})$$

$$\frac{D\mathbf{u}}{Dt} = -\overbrace{\boldsymbol{\nabla}\left(\frac{p}{E\tilde{\varrho}}\right)}^{\text{Pressure}} - \overbrace{\frac{2}{E}\mathbf{e}_z \times \mathbf{u}}^{\text{Coriolis}} - \overbrace{\frac{Ra}{Pr}\frac{d\tilde{T}}{dr} S\mathbf{e}_r}^{\text{Buoyancy}} + \overbrace{\mathbf{F}_\nu}^{\text{Viscosity}} + \overbrace{\frac{1}{EPm}\frac{1}{\tilde{\varrho}}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B}}^{\text{Lorentz}}$$

$$\frac{DS}{Dt} = \frac{1}{Pr\tilde{\varrho}\tilde{T}} \underbrace{\nabla \cdot \left(\kappa \tilde{\varrho} \tilde{T} \boldsymbol{\nabla} S\right)}_{\text{Heat flux}} + \frac{Pr}{Ra\tilde{\varrho}\tilde{T}} \left( \underbrace{\frac{\eta}{Pm^2 E}(\nabla \times \mathbf{B})^2}_{\text{Ohmic heating}} + \underbrace{Q_\nu}_{\text{Viscous heating}} \right)$$

Braginsky+95
Lantz+99

# Systematic parameter study : 82  3D-MHD direct numerical simulations

## Control parameters

$Pm \in [2,...,35]$ ***(11 points)***   $Pr = 0.1$ ***(fixed)***

**Ekman** $E = \dfrac{\nu}{\Omega d^2} \in \{0.73 \times 10^{-3}, 1.66 \times 10^{-3}\}$

**Rayleigh** $\mathrm{Ra} = \dfrac{T_{r_o} d^3 \left(\frac{\partial s}{\partial r}\right)_{r_o}}{\nu_{r_o} \kappa_{r_o}} \in [5.8 \times 10^3, 1.3 \times 10^4, 3 \times 10^4]$

## Output parameters

$Re \sim 42 - 101$

$Rm \sim 80 - 3 \times 10^3$

$Ro = \dfrac{U}{\Omega d} \sim 0.07 - 0.15$

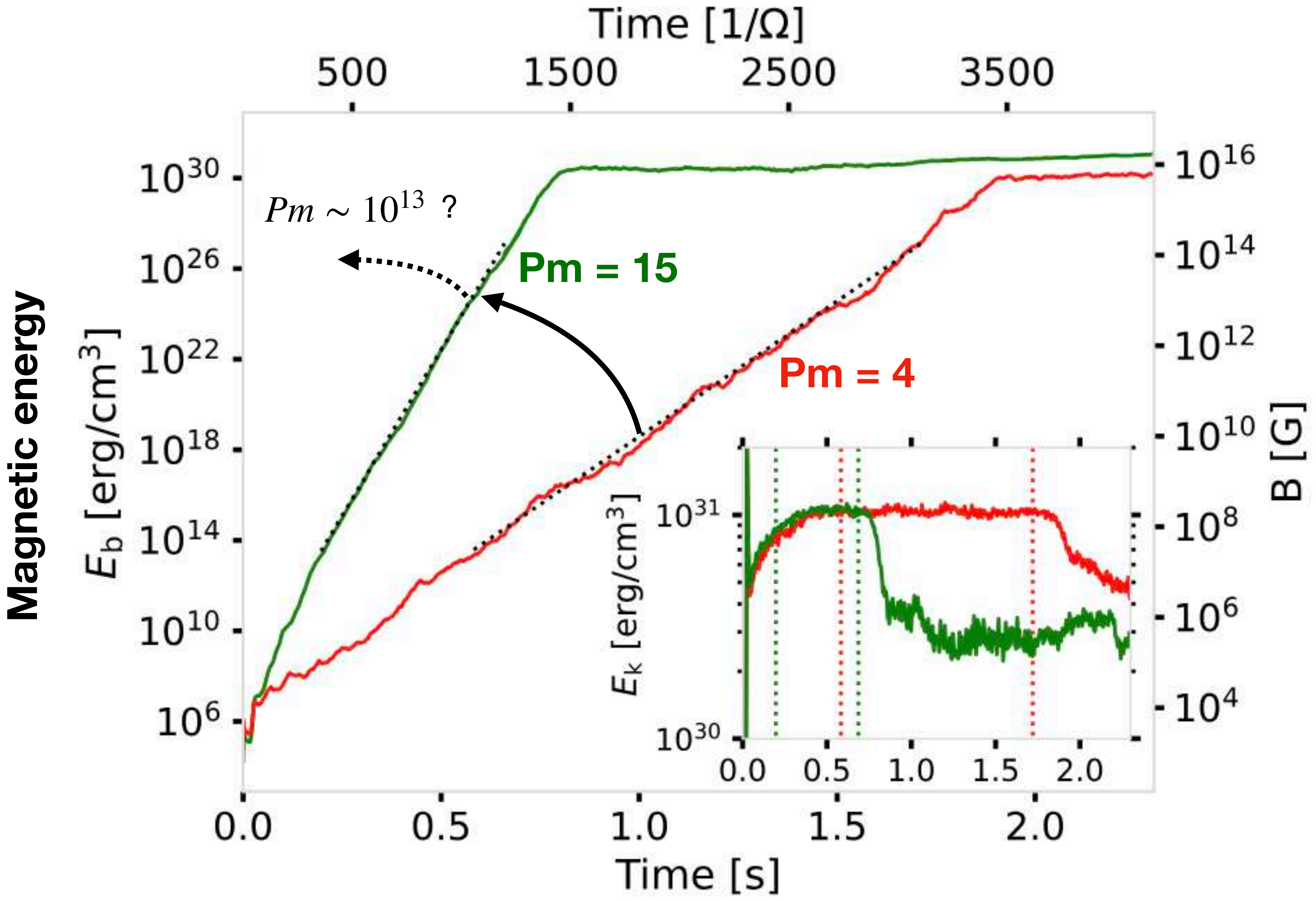
## Exploration strategies (increasing *Re*)

- Increase Ra (buoyancy forcing) + constant E => Rossby number increases
- Increase Ra (buoyancy forcing) + decrease E => ~ constant Rossby number

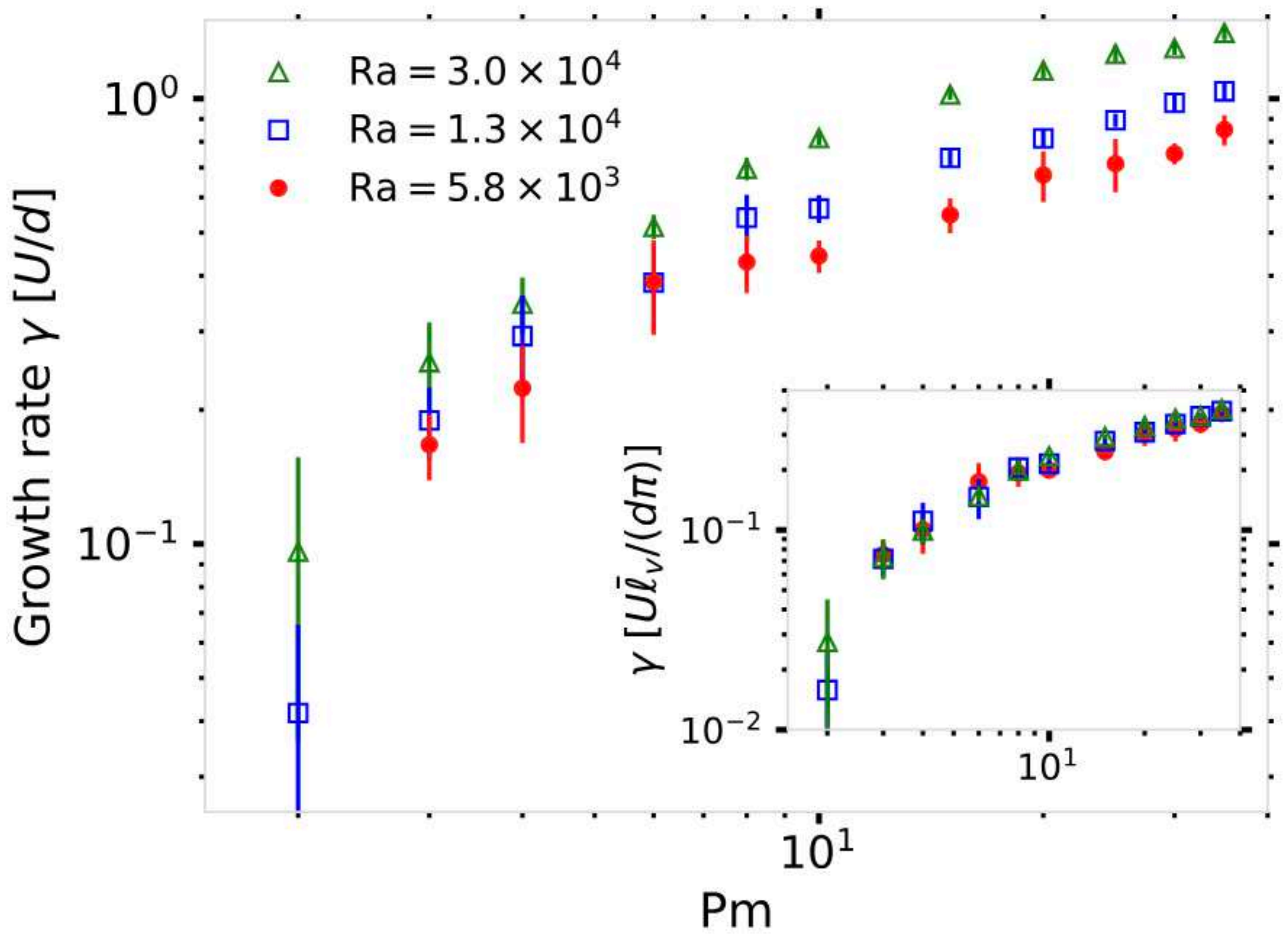
$Ro \ll 1 \implies$ ***fast rotation regime***

**Rotation period** $P \sim 3 - 8\,\mathrm{ms}$

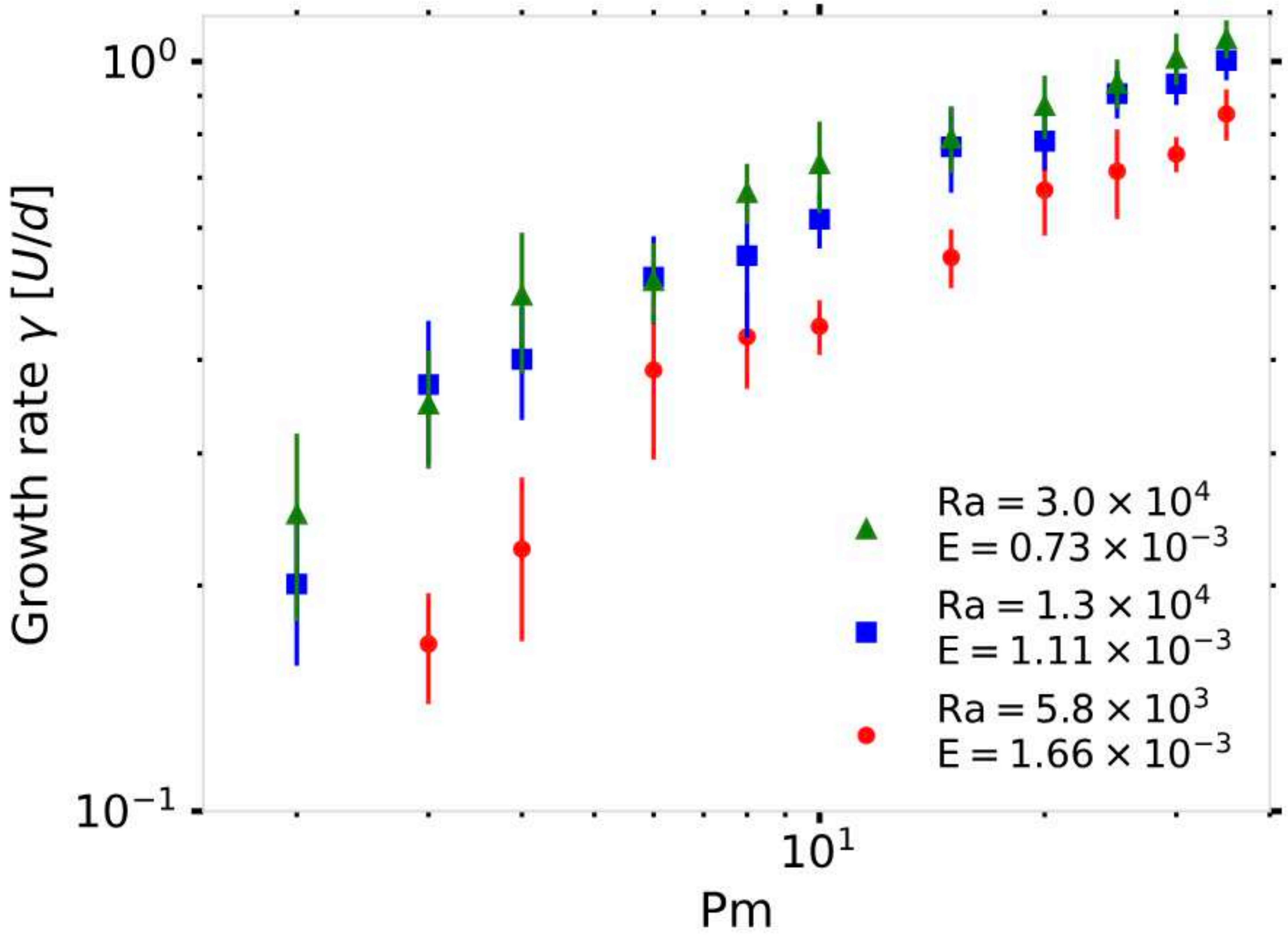
# Typical kinematic growth and saturation



# Growth rate : no sign of plateau at *Pm*=35




**Constant Ekman** $E = 1.66 \times 10^{-3}$

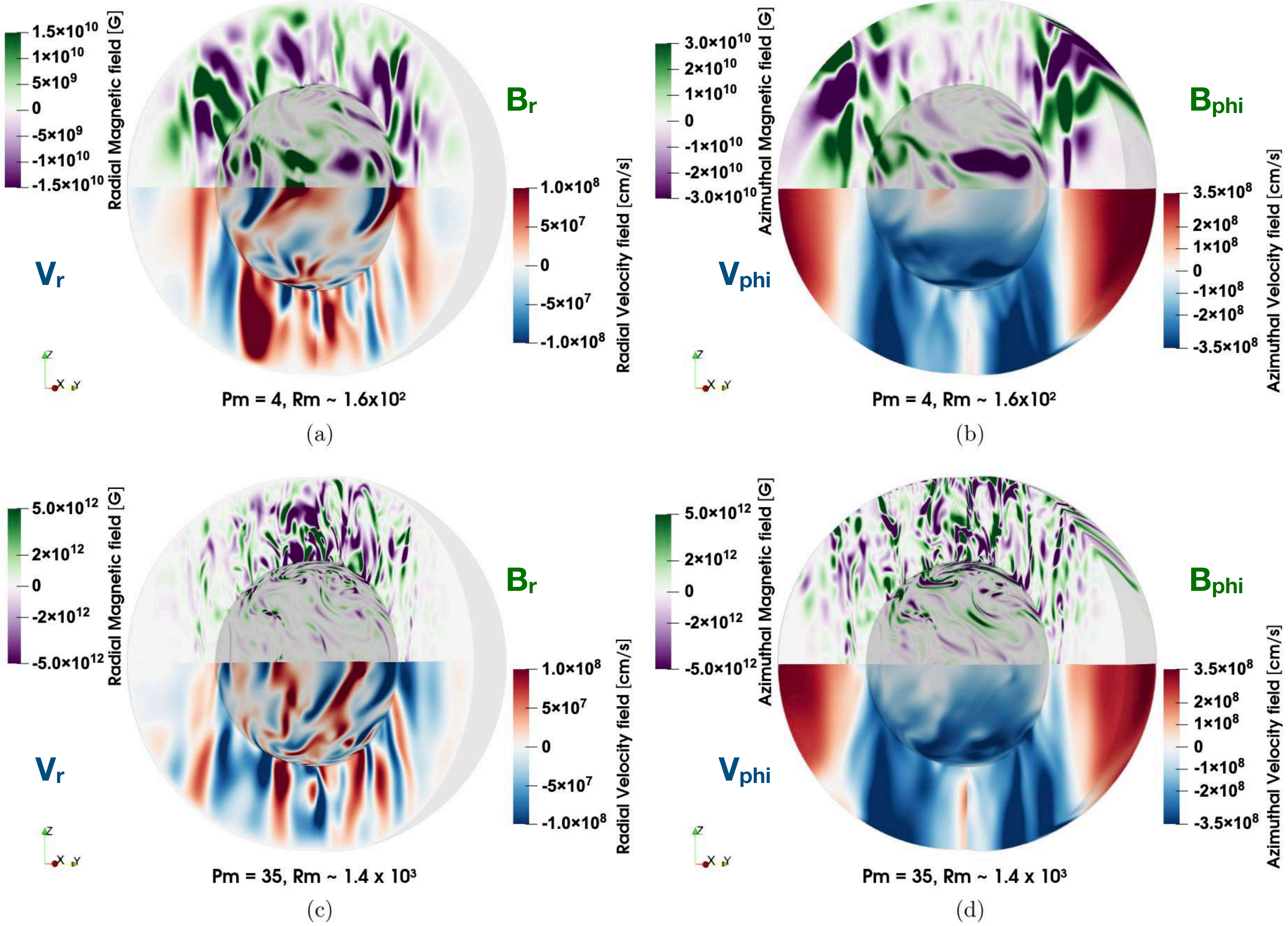



**~ constant Rossby**

# Magnetic field topology

$$Pm = 4$$
$$Rm \sim 10^2$$

$$Pm = 35$$
$$Rm \sim 10^3$$



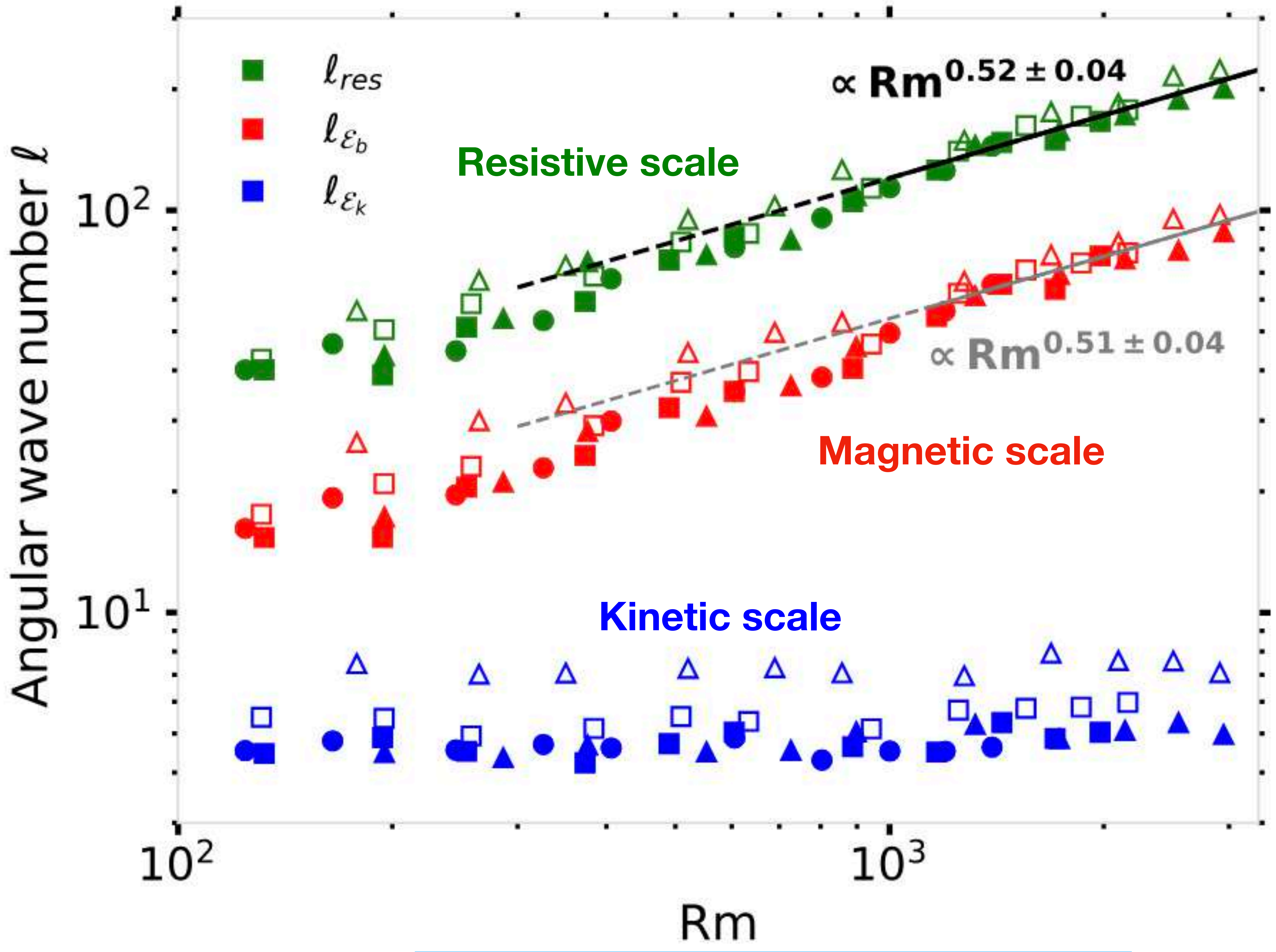

**Magnetic field**

**Velocity field**

**Magnetic field**

**Velocity field**

# Magnetic field topology




**Shear rate** **Resistive decay rate**

$$\frac{U}{d} \sim \frac{\eta}{l_{\mathrm{res}}^2}$$

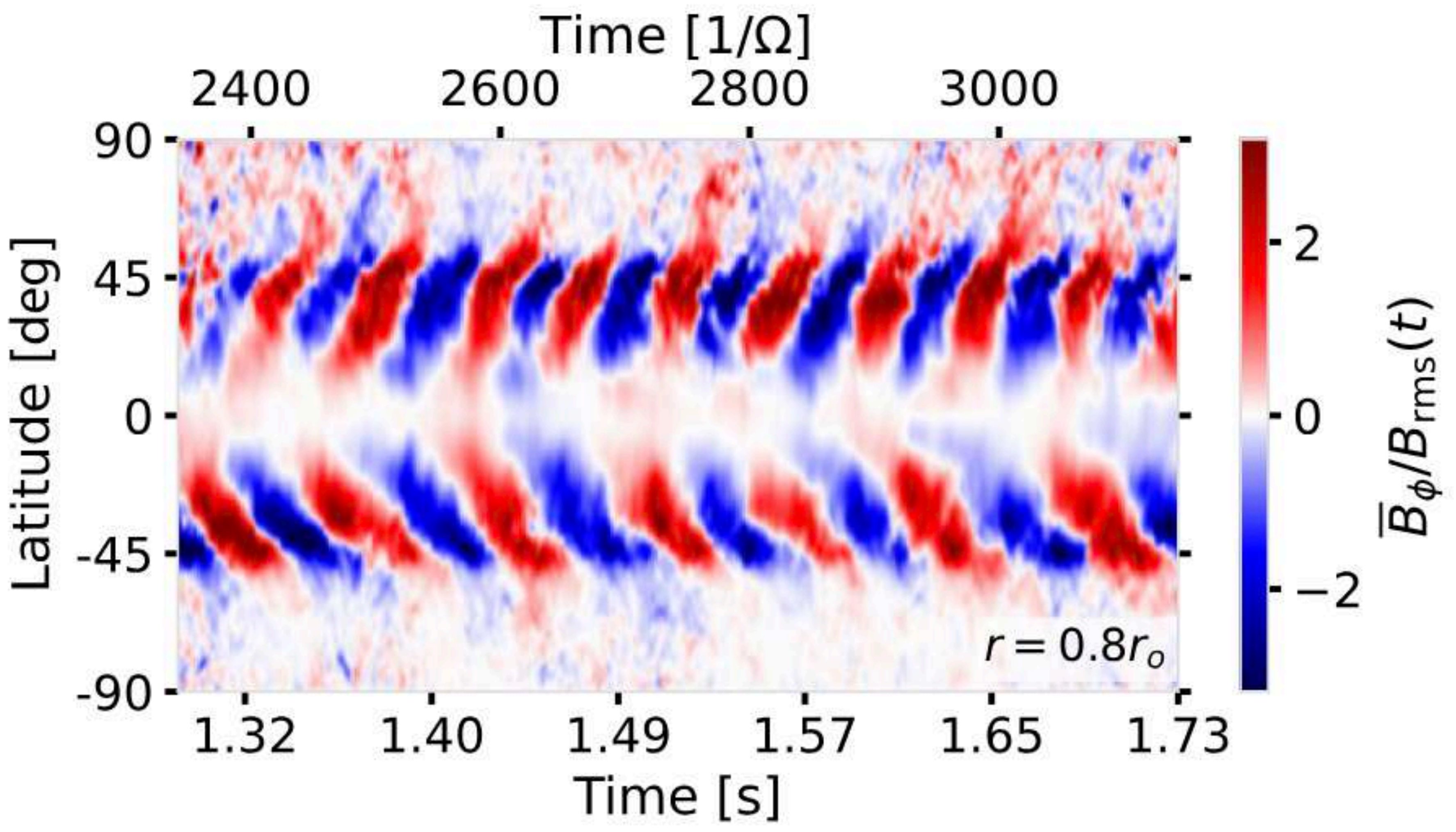
$$\Longrightarrow \frac{l_{res}}{d} \propto Rm^{-1/2}$$

**Extrapolation** $Rm \sim 10^{17} \implies \ell_B \gtrsim 10^8$

**(Prohibitive for « realistic » DNS)**

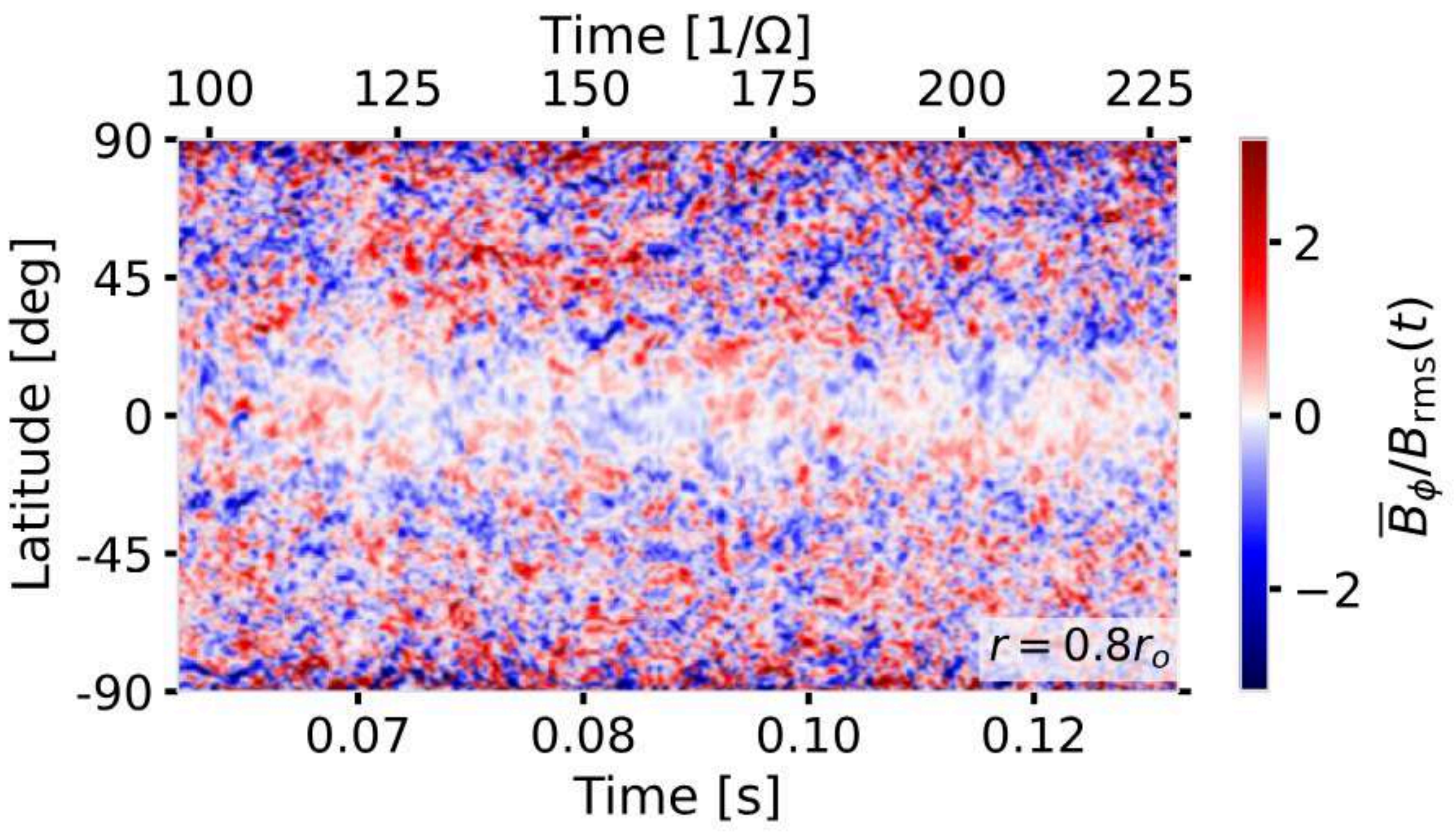
# From large-scale to small-scale dynamo

Pm = 4



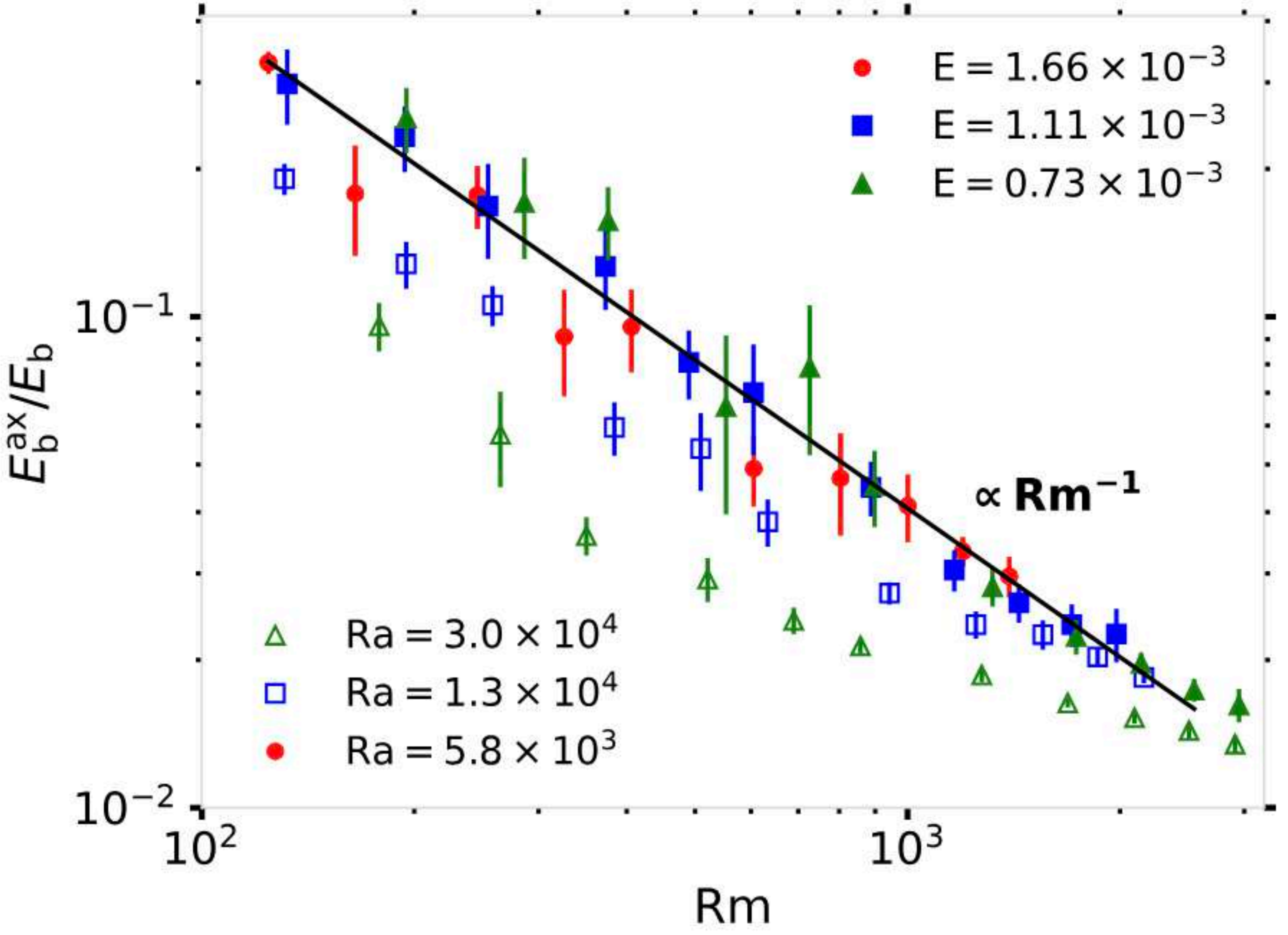

(a) $\mathrm{Ra} = 5.8 \times 10^3$, $\mathrm{Pm} = 4$ ($\mathrm{Rm} \sim 1.6 \times 10^2$).

Pm = 35




(b) $\mathrm{Ra} = 3.0 \times 10^4$, $\mathrm{Pm} = 35$ ($\mathrm{Rm} \sim 2.9 \times 10^3$).

## Axisymmetric to total energy density ratio




**Extrapolation** $B_{rms} \sim 10^{16}\,\mathrm{G} \implies B_{dipole} \sim 10^{8}\,\mathrm{G}$

# Towards higher Pm : from DNS to the Kazantsev dynamo model (1968)

- **Solvable model for the statistics of the magnetic field, assuming a prescribed statistics of a $\delta$-correlated velocity field**

$$-\kappa_{\mathrm{d}}(r)\psi''(r) + V(r)\psi(r) = -\Gamma\psi(r).$$

## Correlation function modelling

$$\langle v_i(\vec{r}_1, t)\, v_j(\vec{r}_2, t')\rangle = T_{ij}(r)\,\delta(t - t'), \qquad T_{ij}(r) = \frac{r_i r_j}{r^2} T_L(r) + \left(\delta_{ij} - \frac{r_i r_j}{r^2}\right) T_N(r).$$

**Longitudinal component**

$$T_L(r) = \begin{cases} \frac{UD}{3}\left[1 - \mathrm{Re}^{(1-\theta_L)/(1+\theta_L)}\left(\frac{r}{D}\right)^2\right], & 0 < r < \ell_c, \quad \textit{Diffusive range} \\ \frac{UD}{3}\left[1 - \left(\frac{r}{D}\right)^{1+\theta_L}\right], & \ell_c < r < D, \quad \textit{Inertial range} \\ 0, & D < r. \end{cases}$$

**Transverse component**

$$T_N(r) = \begin{cases} \frac{UD}{3}\left[1 - \zeta\,\mathrm{Re}^{(1-\theta_N)/(1+\theta_N)}\left(\frac{r}{D}\right)^2\right], & 0 < r < \ell_c, \\ \frac{UD}{3}\left[1 - \zeta\left(\frac{r}{D}\right)^{1+\theta_N}\right], & \ell_c < r < D, \\ 0, & D < r. \end{cases}$$

Schober+12

**Velocity increment parameters**

$\theta_L$ $\qquad \Delta v_L(r) \sim r^{\theta_L}$

$\theta_N$ $\qquad \Delta v_N(r) \sim r^{\theta_N}$

$\zeta$ $\qquad \dfrac{\Delta v_N}{\Delta v_L} = \zeta\left(\dfrac{r}{d}\right)^{\theta_N - \theta_L}$

**Kolmogorov turbulence**

$$\theta_L = \theta_N = 1/3$$

$$\zeta = 1.67$$

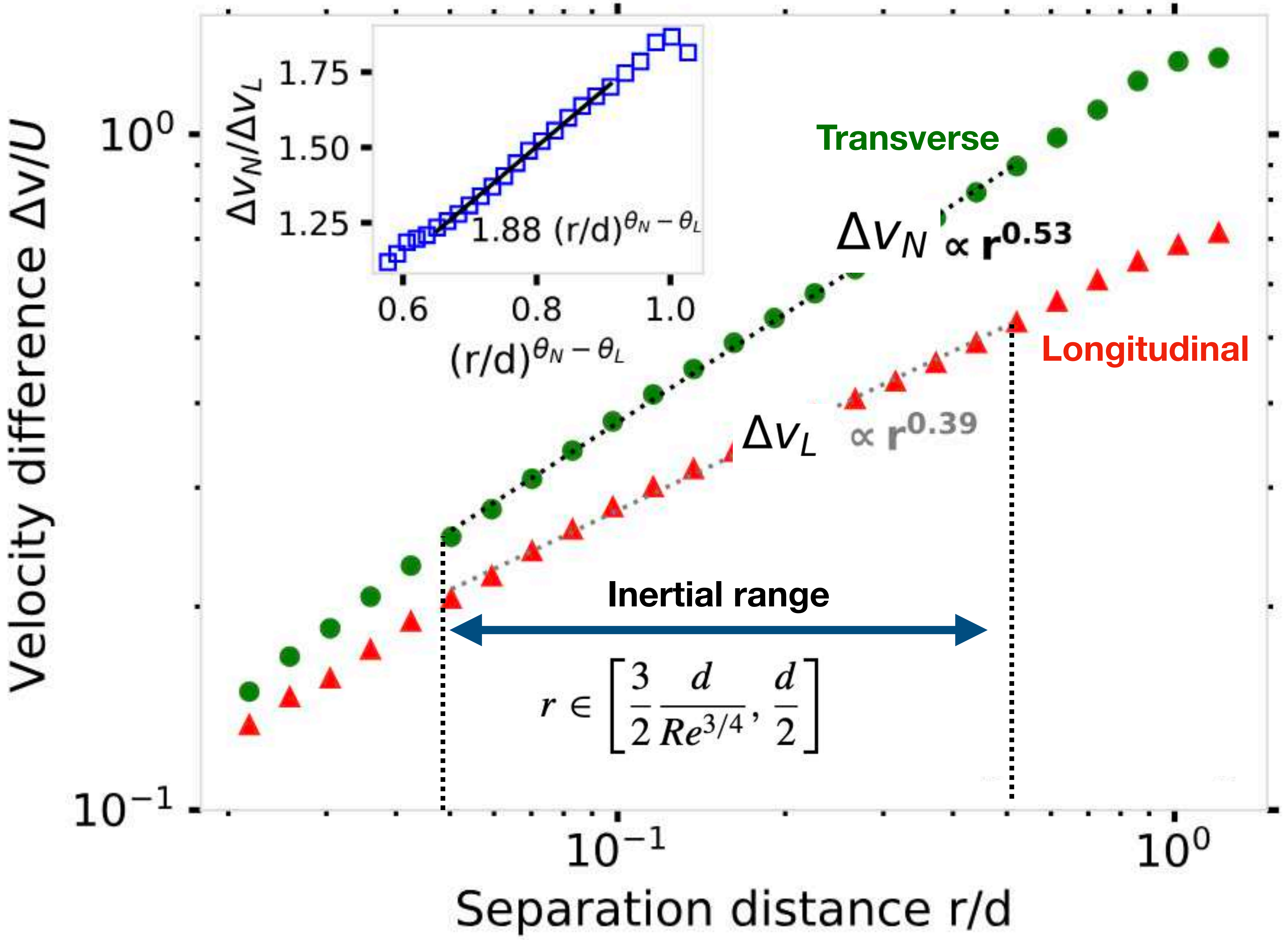
**Rotating anelastic convection**

$$\theta_L \sim 0.39$$

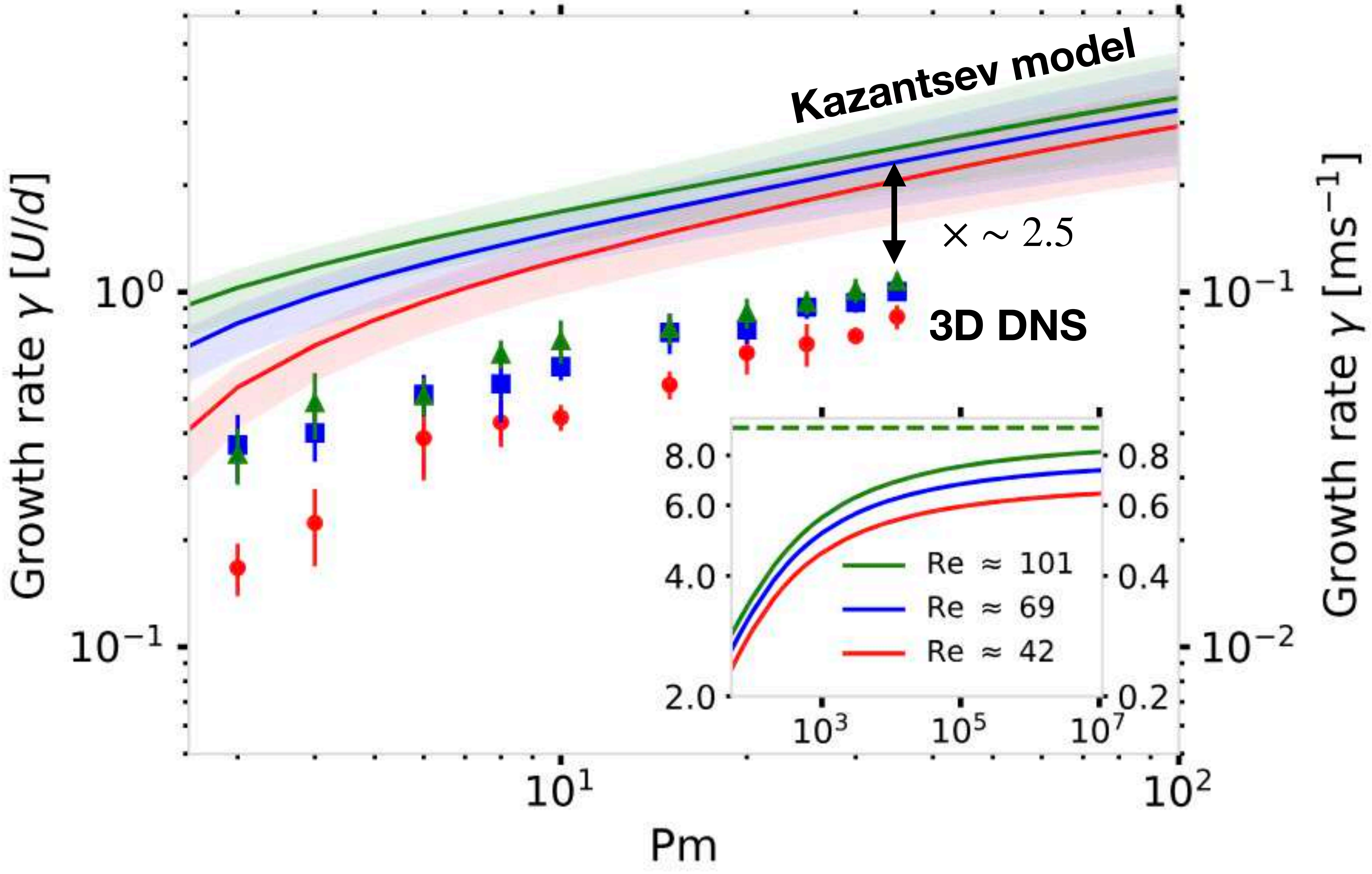
$$\theta_N \sim 0.53$$

$$\zeta \sim 1.88$$

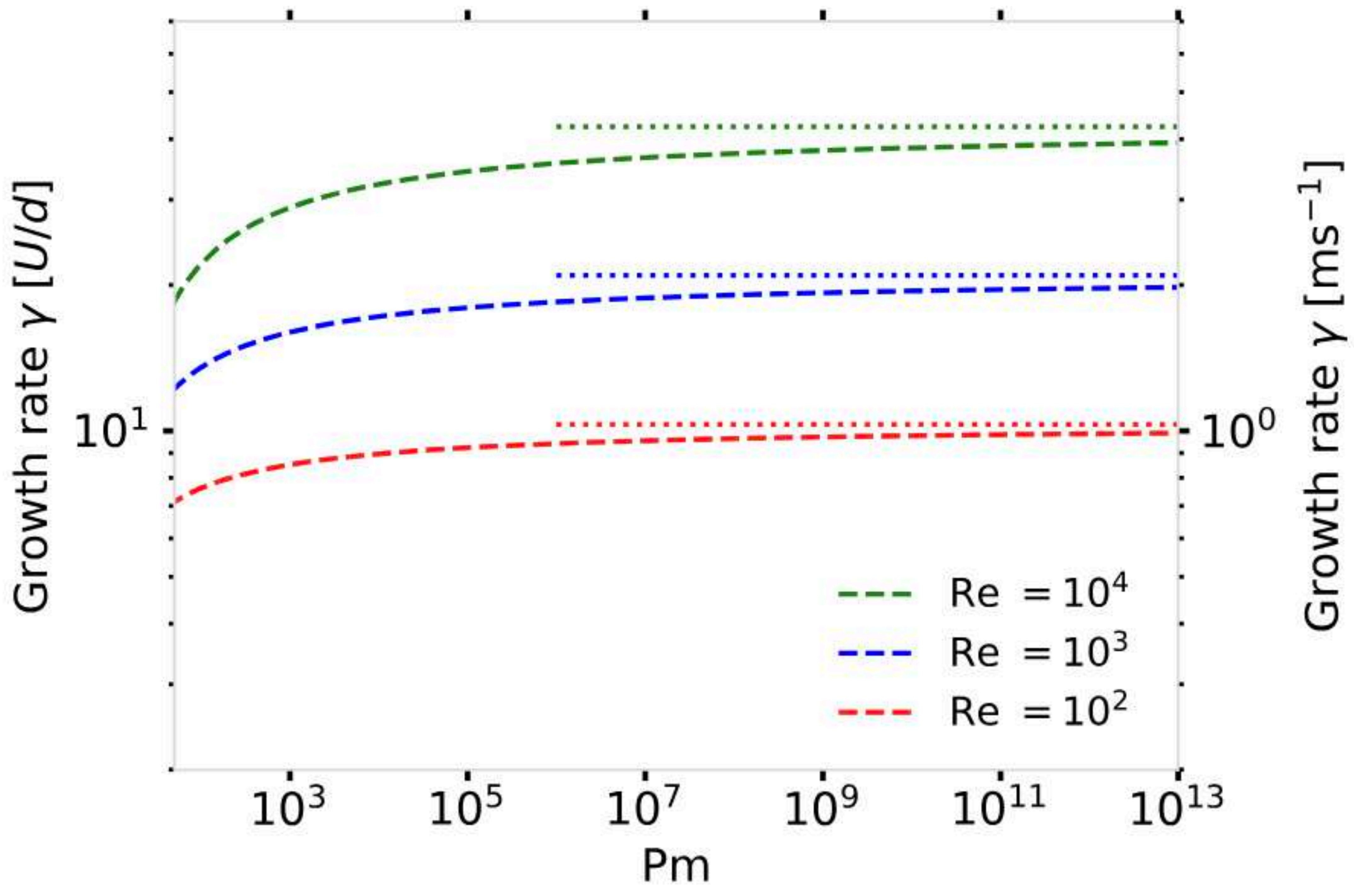
**DNS with highest Reynolds number** $Re \sim 10^2$



# Growth rate in the high Pm limit




**Asymptotic regime**




Extrapolation

$$\begin{cases} Pm \sim 10^{13} \\ Re \sim 10^{4} \end{cases} \Longrightarrow \gamma \sim \mathcal{O}(1\,\mathrm{ms}^{-1})$$

# Conclusion

**Systematic study of the kinematic phase combining**
**direct numerical simulations & (modest) direct « statistical » simulations**

***Q1 : Is the amplification process « fast » enough ?*** ***~ yes***

***Q2 : Can it explain magnetar dipolar field ?*** ***~ no…***

**Fast growth rate controlled by small-scale dynamo action**

# Next step : how does the dynamo saturates ?   … stay tuned !

***Q1 : Is the amplification process « fast » enough ?*** ***~ yes***

***Q2 : Can it explain magnetar dipolar field ?*** ***~ no…***

**Fast growth rate controlled by small-scale dynamo action**

- Verma, Seshasayanan, Raynaud, Guilet, *Large Pm small-scale kinematic dynamo in protoneutron stars,* Phys. Rev. E 114, 015221 (2026)
- Verma, Seshasayanan, Raynaud, Guilet, *Nonlinear evolution of convective dynamo in protoneutron stars,* A&A, in prep

PRELIMINARY