CoCoNuT Meeting 2026 – Southampton

10th September 2026

CONVEXITY OF THE EQUATION OF STATE IN MERGER SIMULATIONS AND GRAVITATIONAL WAVES

G. RIVIECCIO, A. RIOS, P. CERDÀ-DURÀN, M. RUIZ, J. A. FONT

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CONVEXITY OF THE **EQUATION OF STATE** IN **MERGER SIMULATIONS** AND **GRAVITATIONAL WAVES**

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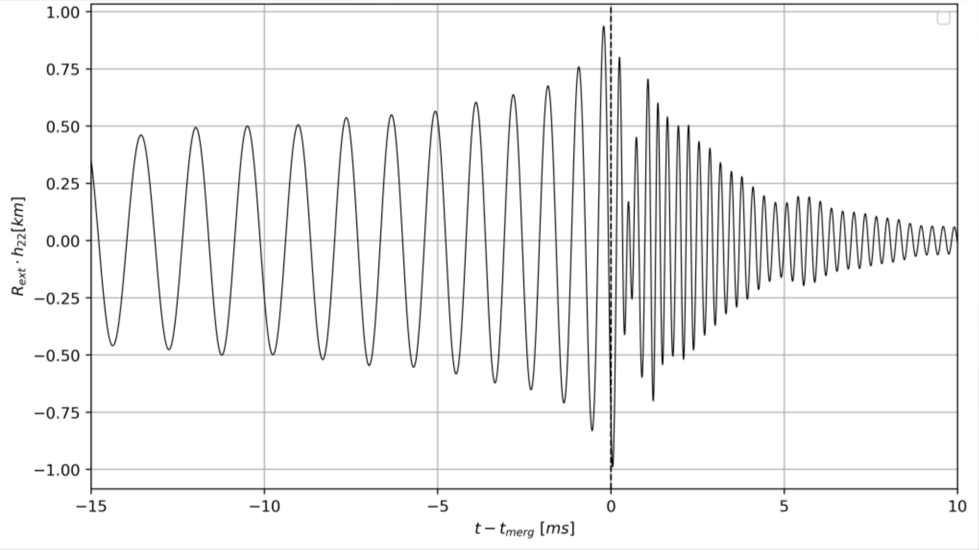
Focus of the talk

GR, Guerra, Ruiz & Font (2024)
Gravitational wave imprints of non-convex dynamics in binary neutron star mergers
Phys. Rev. D 109, 064032

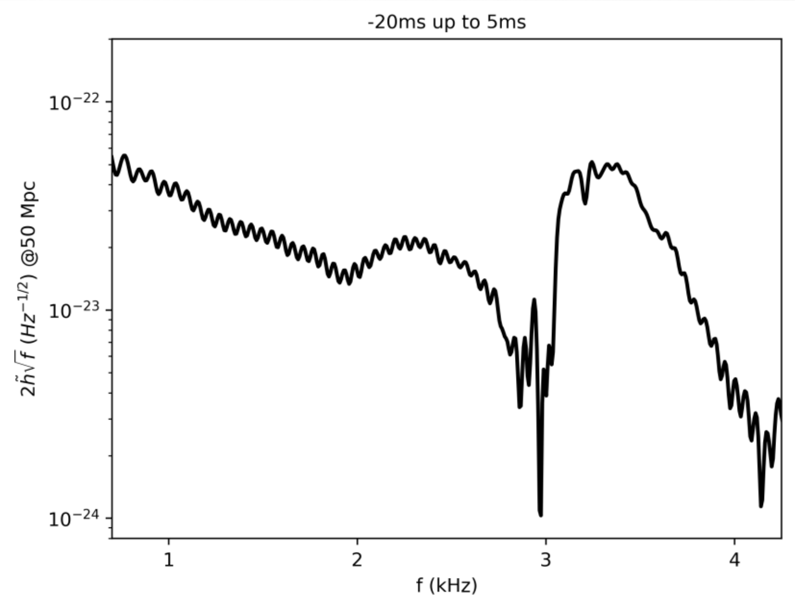
GR, Rios, Cerdà-Duràn, Ruiz & Font (2026)
Convexity of parametrized phase transition and gravitational wave signature
Still loading ….

BINARY NEUTRON STARS (BNS) MERGER

Gravitational Wave



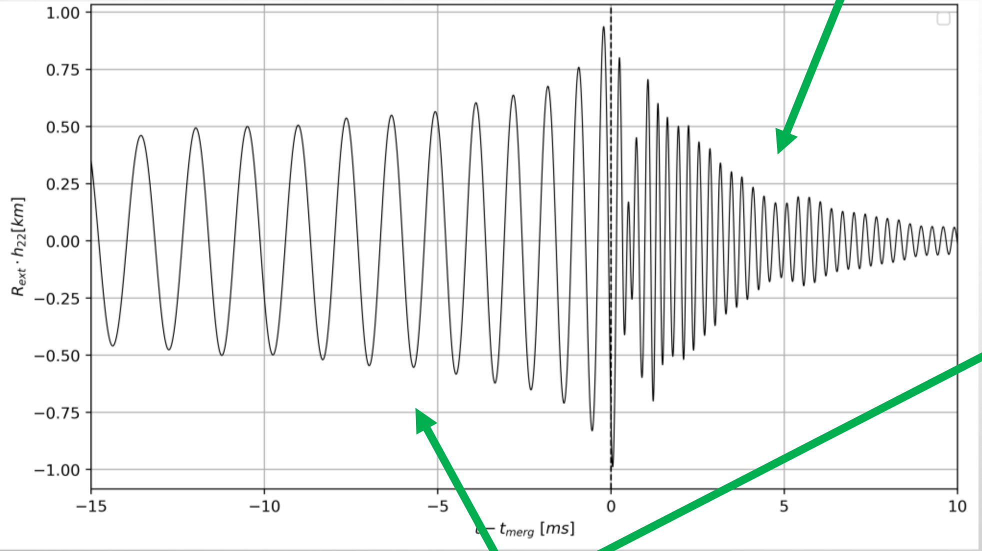
Fourier Transform of Gravitational Wave



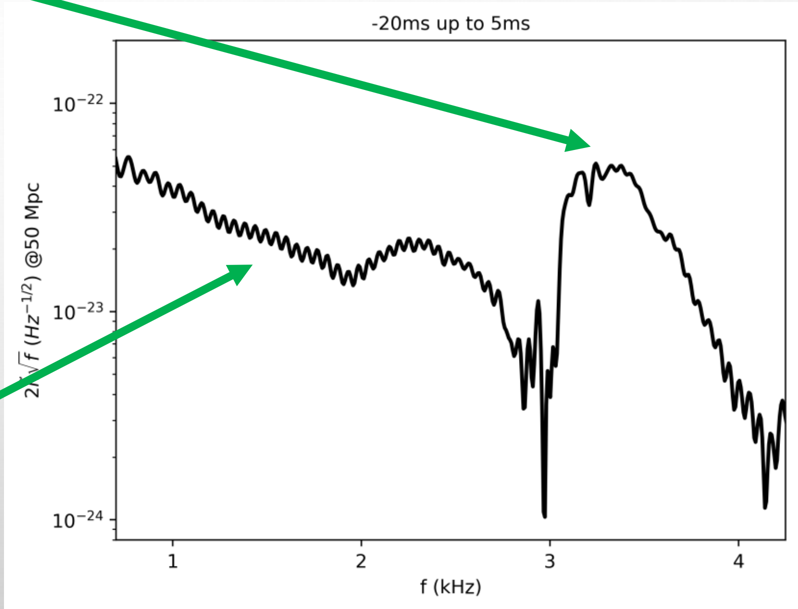
BINARY NEUTRON STARS (BNS) MERGER

Post-Merger

Gravitational Wave

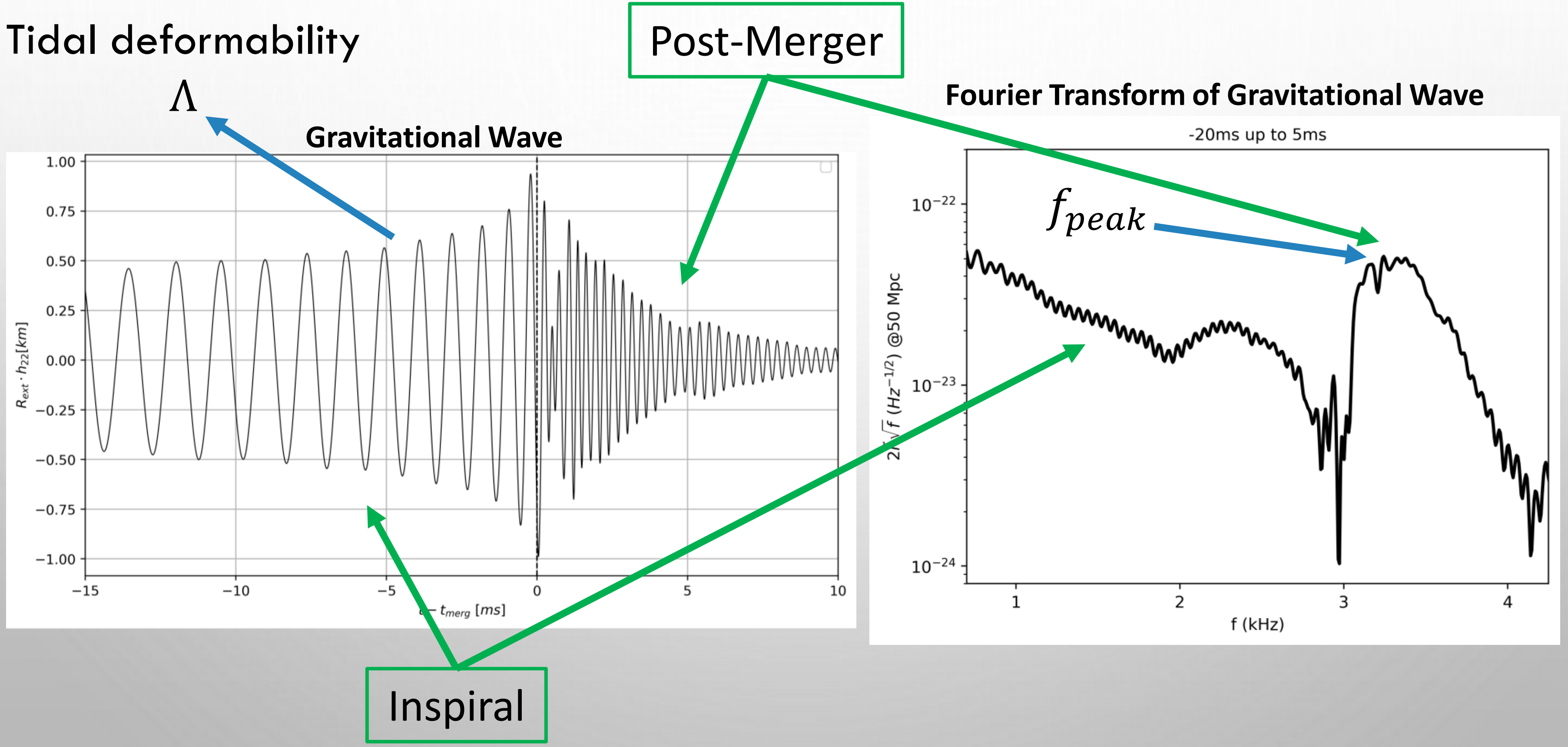


Fourier Transform of Gravitational Wave



Inspiral

BINARY NEUTRON STARS (BNS) MERGER



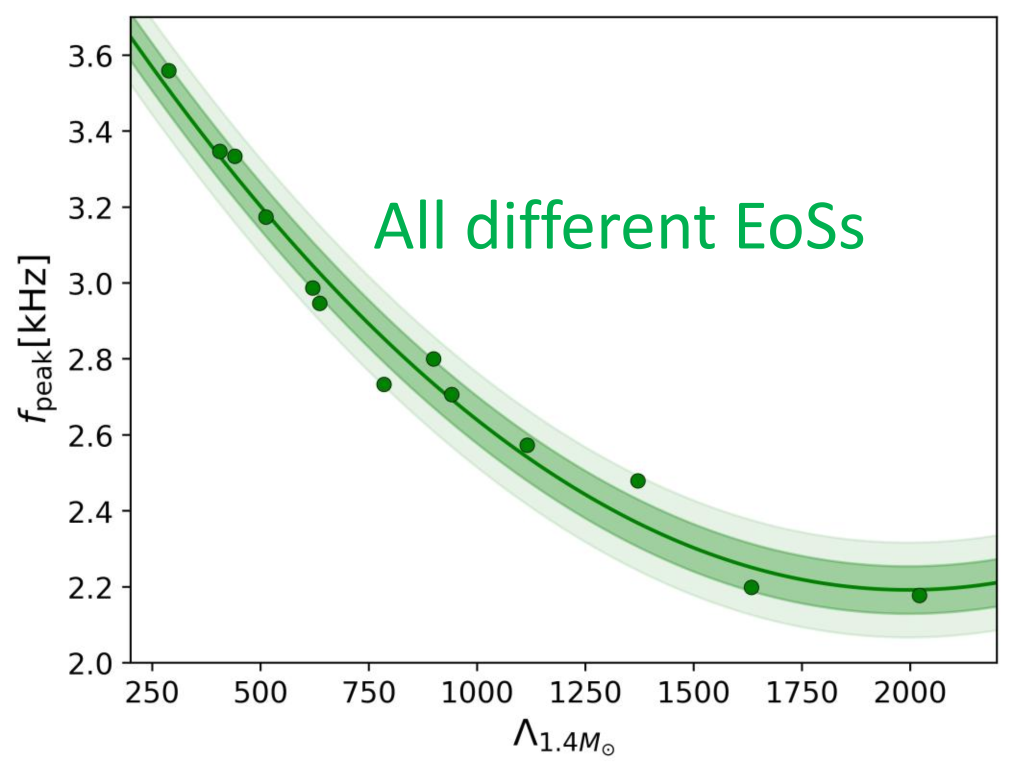
Quasi-Universal Relation

$\Lambda \coloneqq$ Tidal deformability
(Measured pre-merger)

f_{peak} :=Characteristic Frequency post-merger

We can find a relation that is EoS-independent (almost)

Useful to constrain the EoS

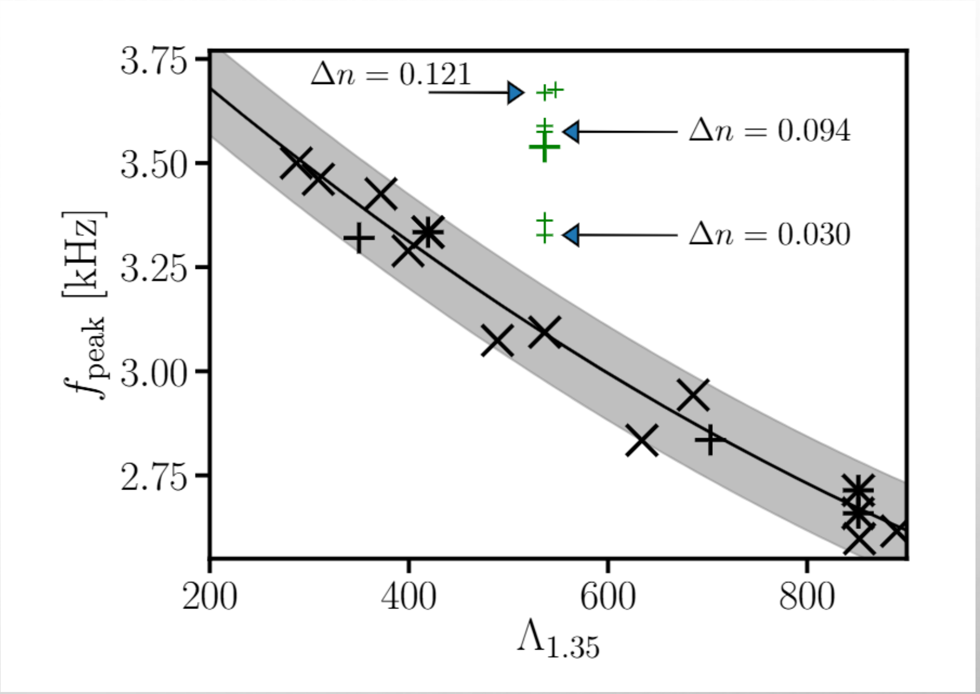


G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

Quasi-Universal Relation including Phase Transition

First order Phase transition softens the EoS during the merger

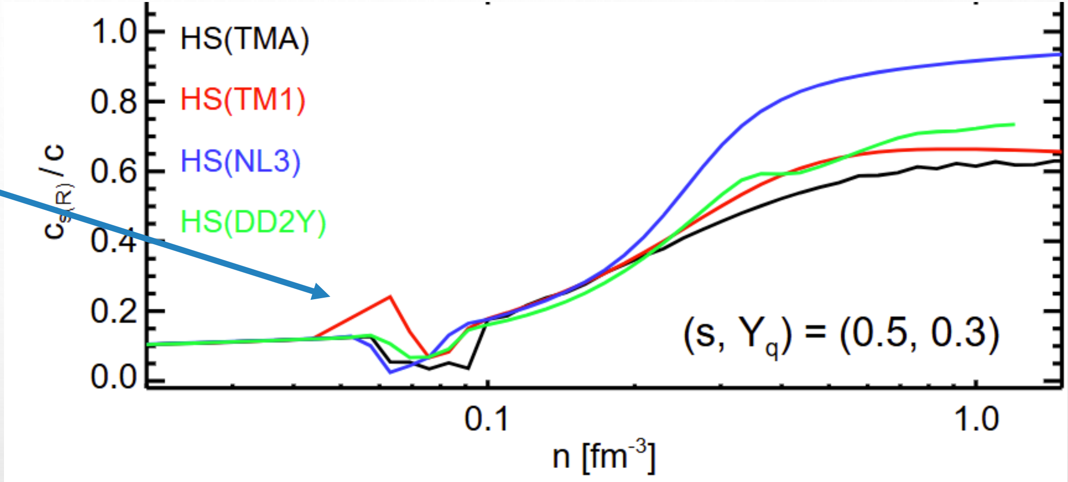
<u>Shift in f_{peak} is informative of a possible phase transition</u>



Bauswein et al. 2019
Blacker et al. 2023,2025

Sound speed is non-monotonic

$c_s^2 = \frac{\partial P}{\partial \epsilon}\Big|_s$ is non-monotonic



Aloy et al. (2019)

Because of the drop
of the sound speed:

Phase Transition are in general
Nonconvex

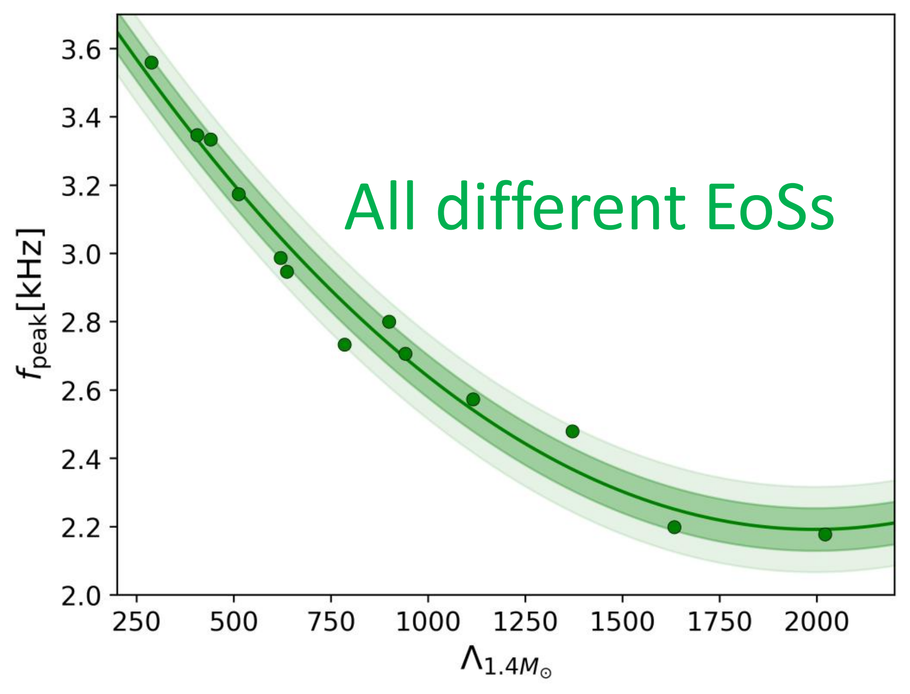
R Menikoff and B. Plohr Rev. Mod. Phys. 61, 75

Quasi-Universal Relation

Λ ≔ Tidal deformability
(Measured pre-merger)

f_{peak} ≔Characteristic Frequency
post-merger

EoS-independent (almost)



G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

How does nonconvexity affect this relation?

Wave structure - Fundamental Derivative

Depends only on EoS

$$G \equiv -\frac{1}{2} V \frac{\left.\frac{\partial^2 P}{\partial V^2}\right|_s}{\left.\frac{\partial P}{\partial V}\right|_s}$$ Fundamental Derivative

Menikoff and Plohr (1986), Ibañez et al. (2013)

Wave structure - Fundamental Derivative

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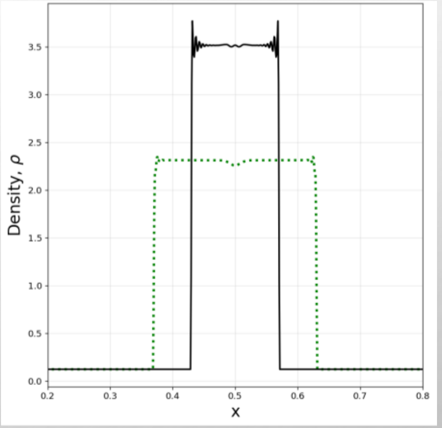
Menikoff and Plohr (1986), Ibañez et al. (2013)

Colliding Slabs

$G > 0$ convex



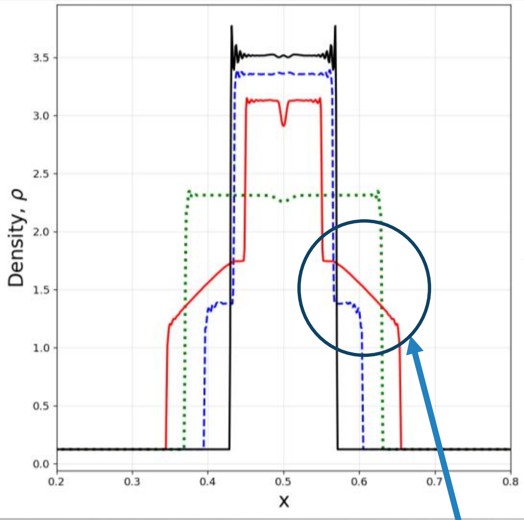
Standard wave propagation

Wave structure - Fundamental Derivative

Depends only on EoS

$$G \equiv -\frac{1}{2} V \frac{\left.\frac{\partial^2 P}{\partial V^2}\right|_s}{\left.\frac{\partial P}{\partial V}\right|_s}$$ Fundamental Derivative

Menikoff and Plohr (1986), Ibañez et al. (2013)

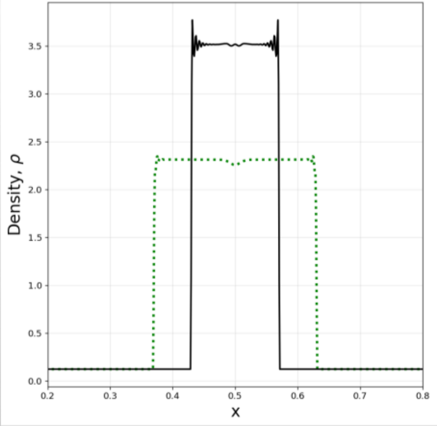


$G > 0$ convex

Standard wave propagation

$G < 0$ nonconvex!

Colliding Slabs



Rarefaction waves
(Lower density but entropy increase)

Composite waves
Rarefaction wave attached to a shock front

Split waves
(e.g. Phase Transition)

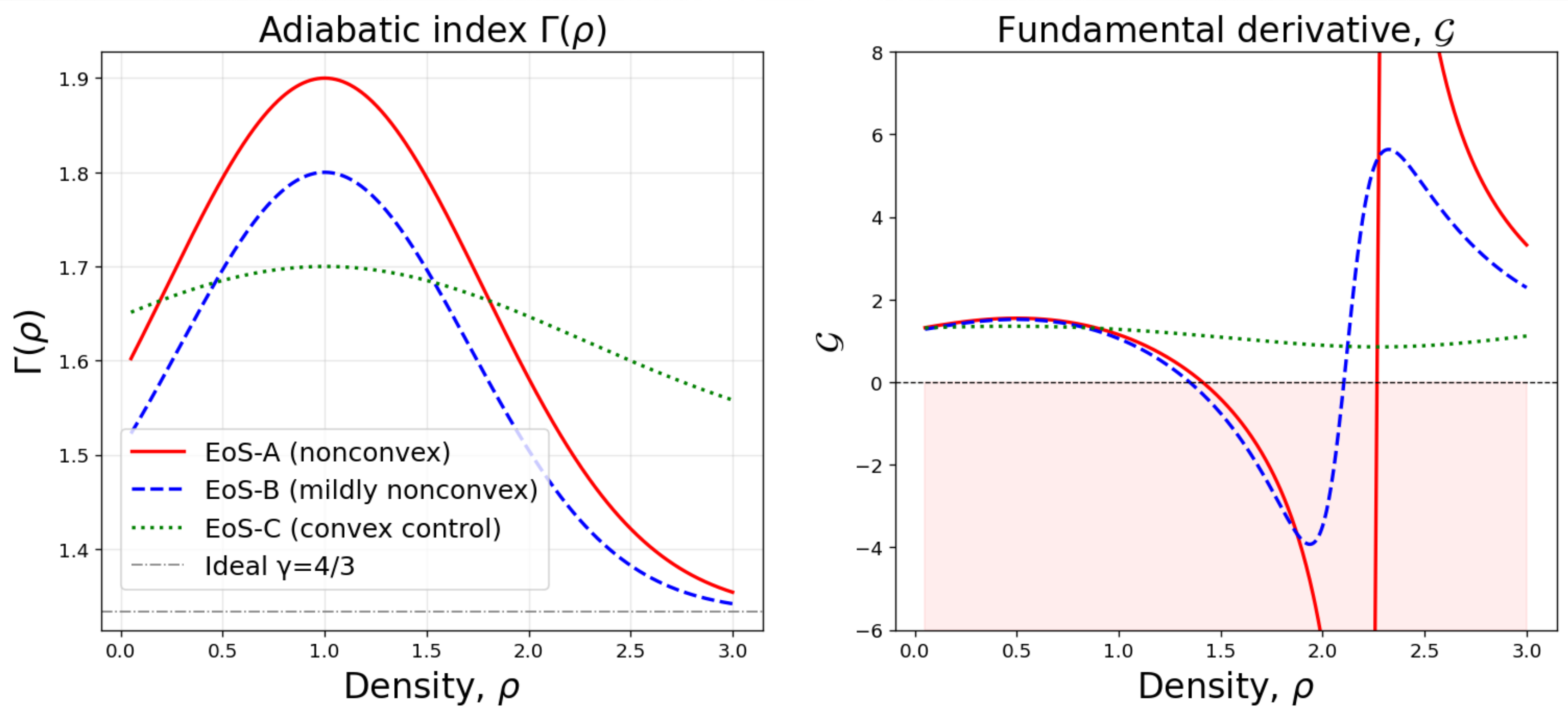
ASSUMING A Γ-LAW EOS

$$P = (\Gamma - 1)\,\rho\epsilon$$

Ideal gas : $\Gamma = const$

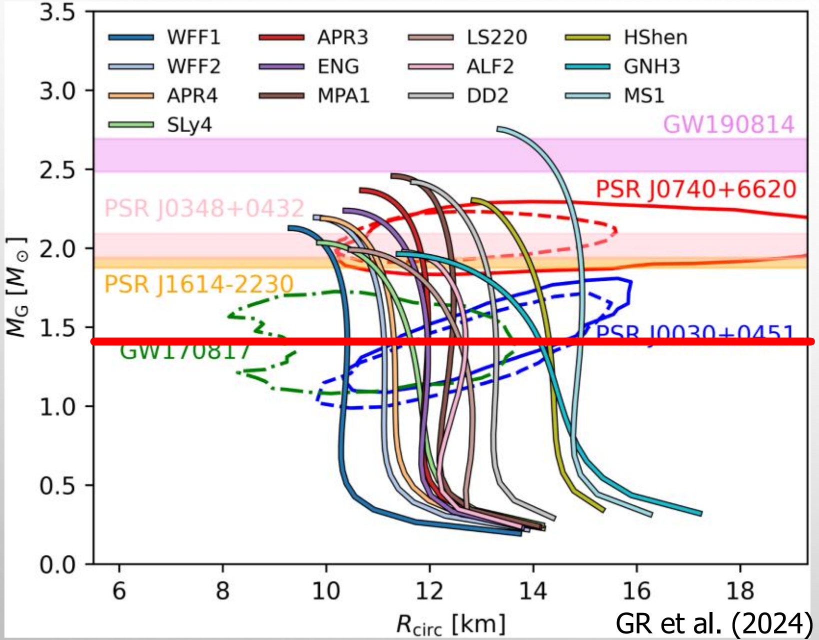
GGL : $\Gamma = \Gamma_0 + (\Gamma_1 - \Gamma_0)e^{\frac{(\rho-\rho_1)^2}{2\sigma^2}}$

(Gaussian Gamma Law)



WHAT ABOUT BNS SIMULATIONS?

Take initial Configurations at $1.4\ M_\odot$



GR et al. (2024)

$1.4\ M_\odot$

LORENE

https://gitlab.in2p3.fr/lorene/Lorene

Run GRHD simulations:

Einstein Toolkit + IllinoisGRMHD

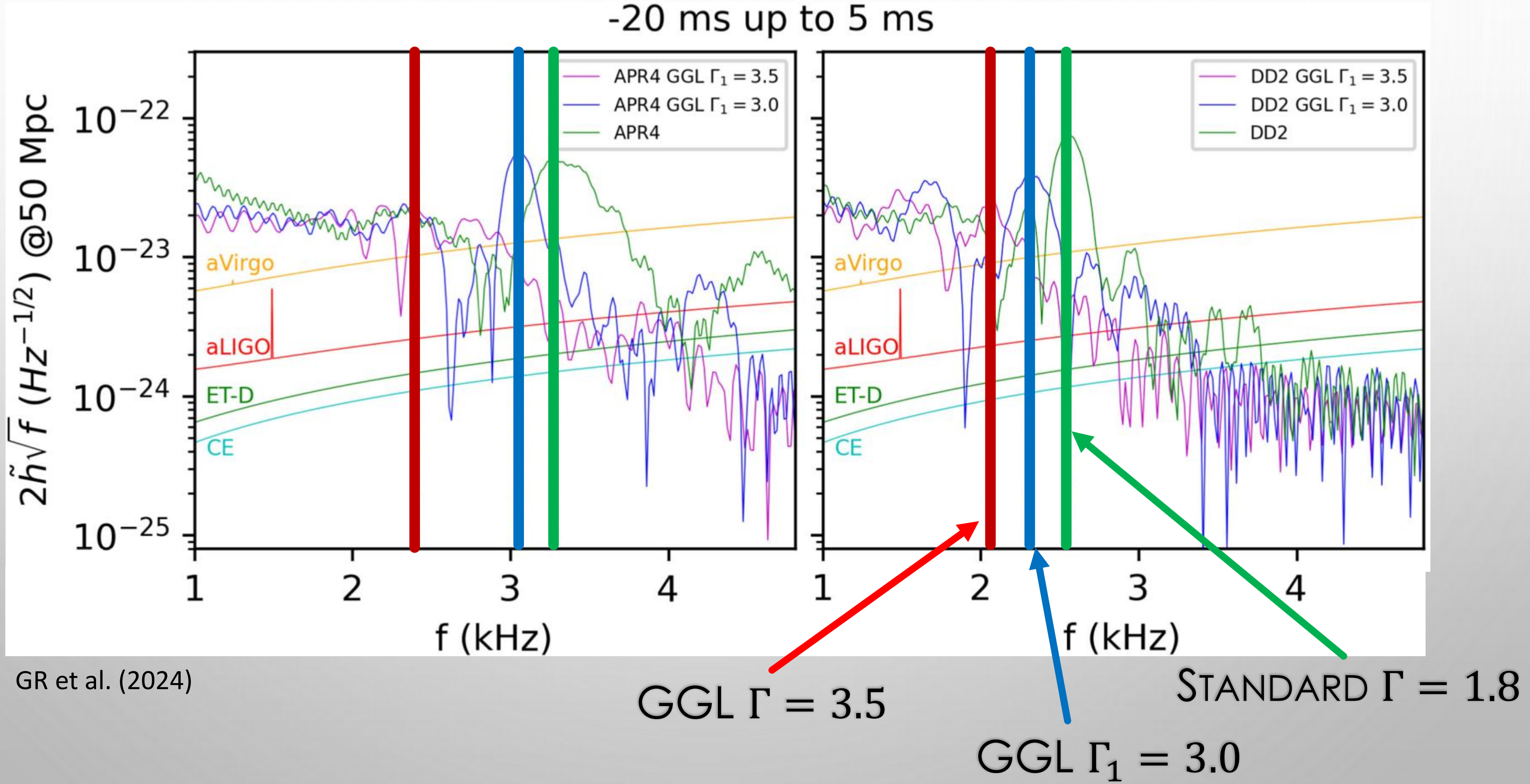
www.einsteintoolkit.org

Piecewise Politrope + Γ-law

$$P = k\rho^n + (\Gamma_{th} - 1)\epsilon n$$

$\Gamma_{th} = 1.8$ or $\Gamma_{th} = GGL$

RESULTS: FREQUENCY PEAK SHIFT WITH STRONGER NONCONVEXITY

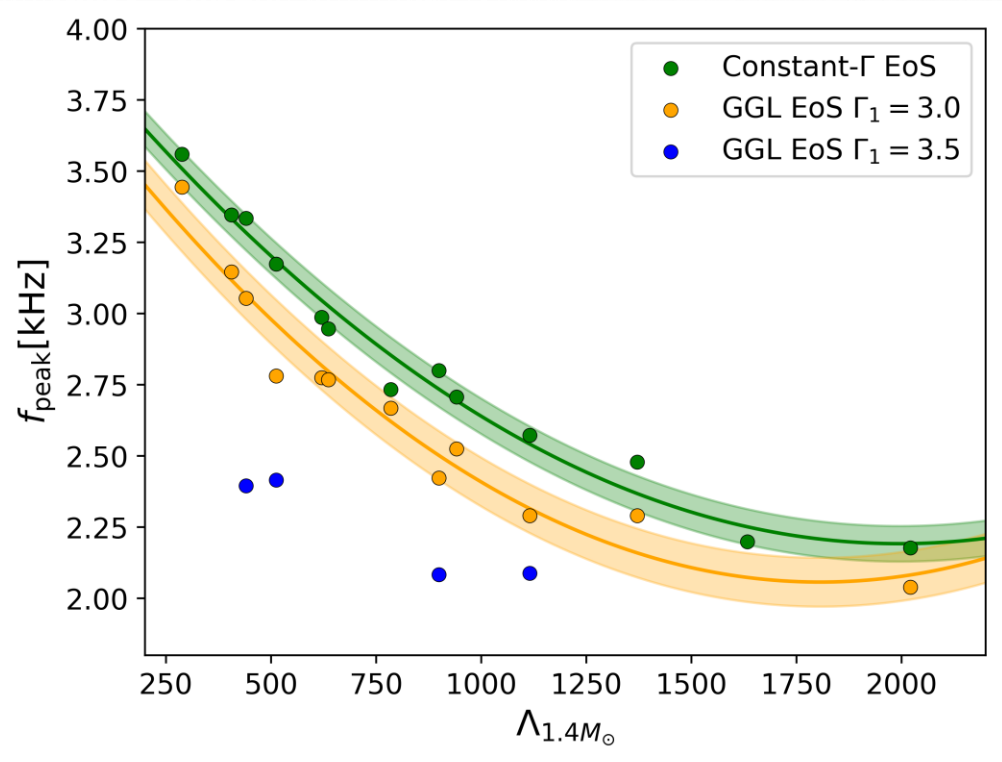


GR et al. (2024)

QUASI-UNIVERSAL RELATION

Shifted Quasi-Universal relation!

Stronger outliers for more nonconvexity



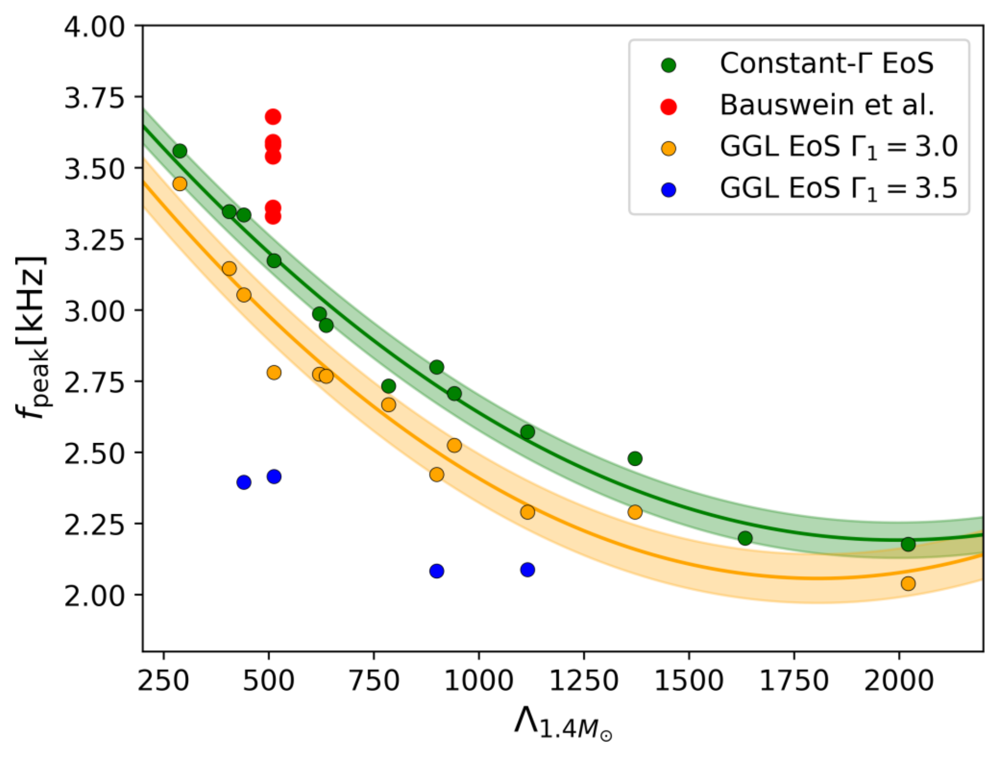
G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

QUASI-UNIVERSAL RELATION

Shifted Quasi-Universal relation!

Stronger outliers for more nonconvexity

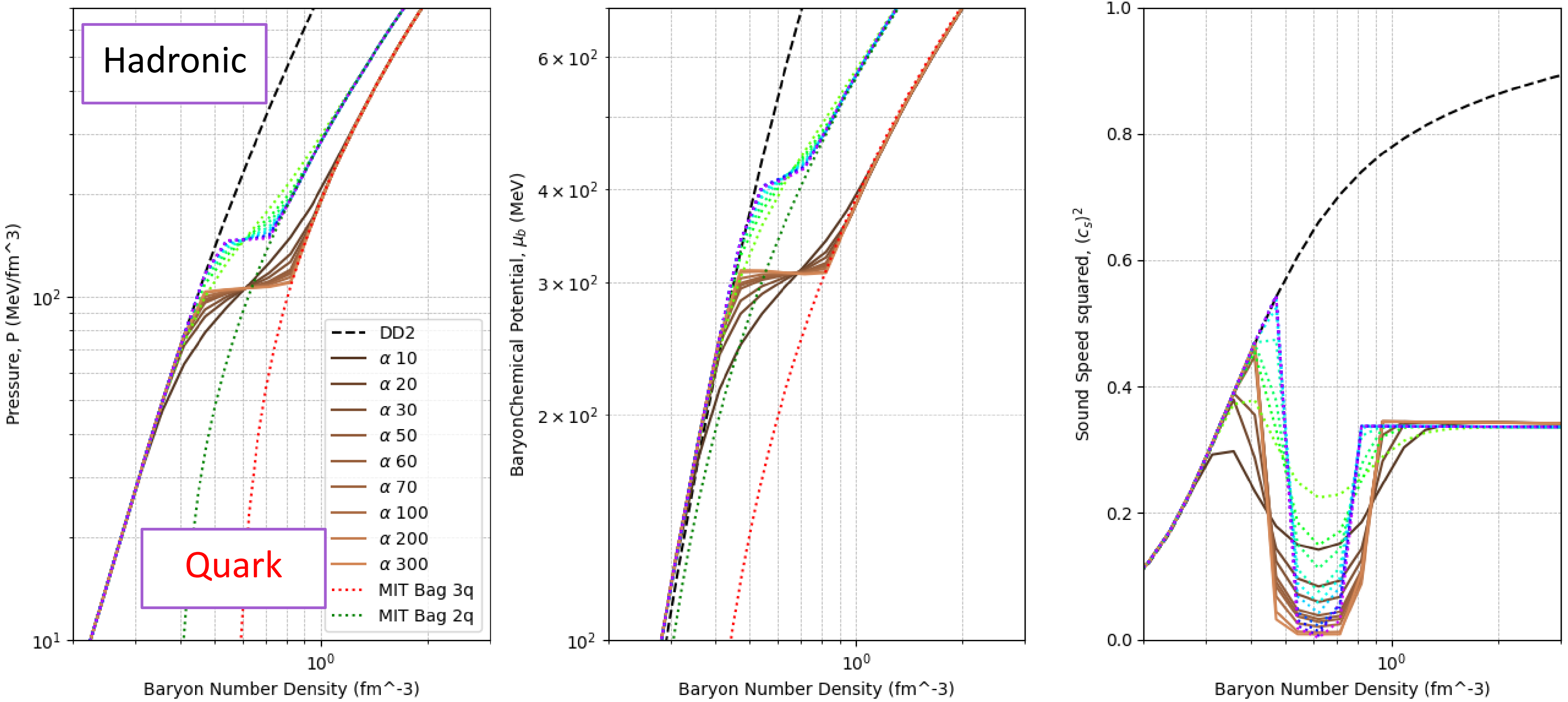
GGL EoS gives **lower frequencies** while phase transition has higher frequencies instead



G. Rivieccio et al. (2024) Phys. Rev. D **109**, 064032

We need to have a phase transition with controlled convexity!

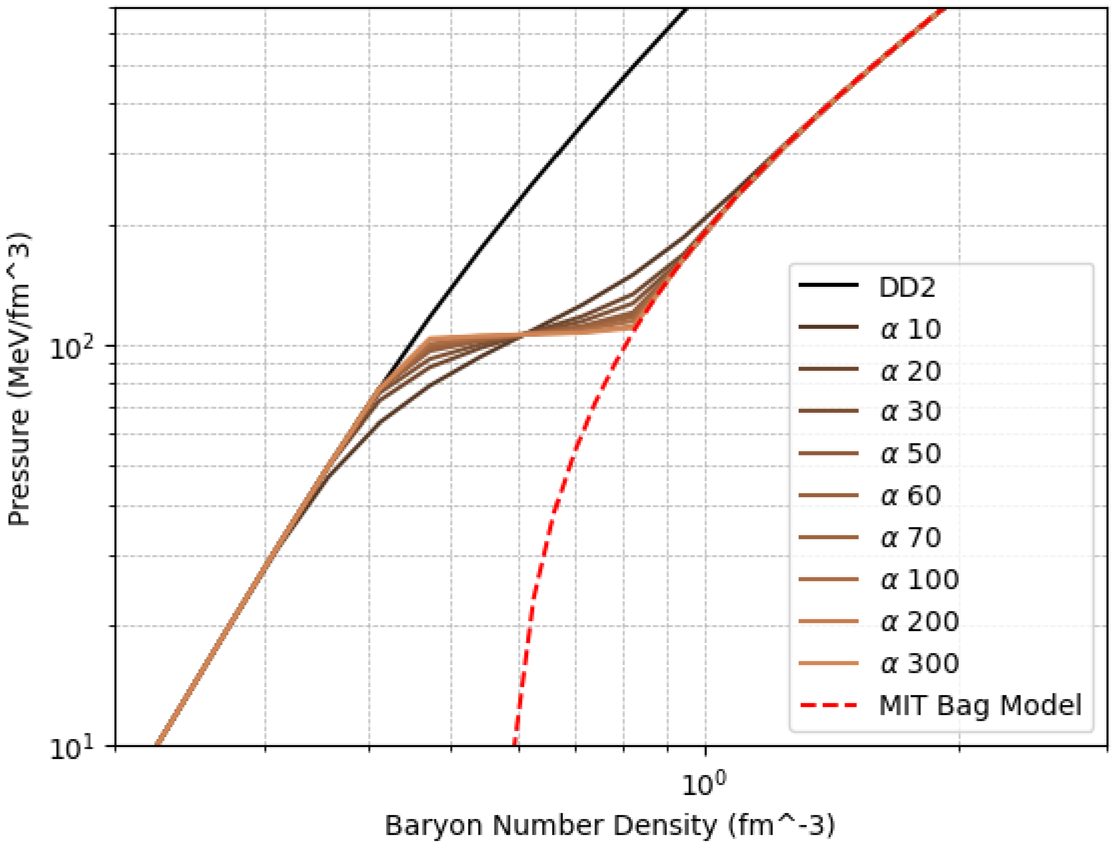
PARAMETRISED PHASE TRANSITION



GR et al. (2026) in prep.

PARAMETRISED PHASE TRANSITION

How do we build it?



GR et al. (2026) in prep.

PARAMETRISED PHASE TRANSITION

How do we build it?

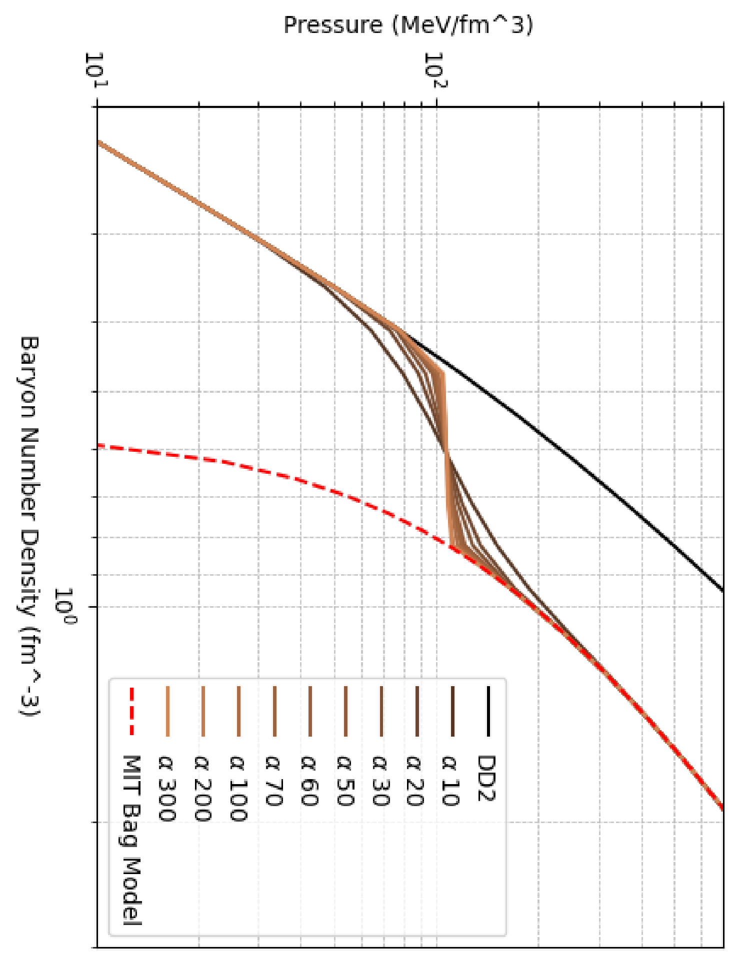
The inverted function is a step!
We apply a sigmoid.

$$n = n_{Nuc}\sigma + n_{Qrk}(1-\sigma)$$

where

$$\sigma = \frac{1}{1+e^{-\alpha(p-p_t)}}$$

$\boldsymbol{\alpha}$ controls the sharpness
$\boldsymbol{p_t}$ is the transition pressure



PARAMETRISED PHASE TRANSITION

How do we build it?

We apply the conditions

Mechanical equilibrium:

$$p_{Nuc}(n_{on}) = p_{Qrk}(n_{off})$$

Chemical equilibrium:

$$\mu_{g,Nuc}(n_{on}) = \mu_{g,Qrk}(n_{off})$$

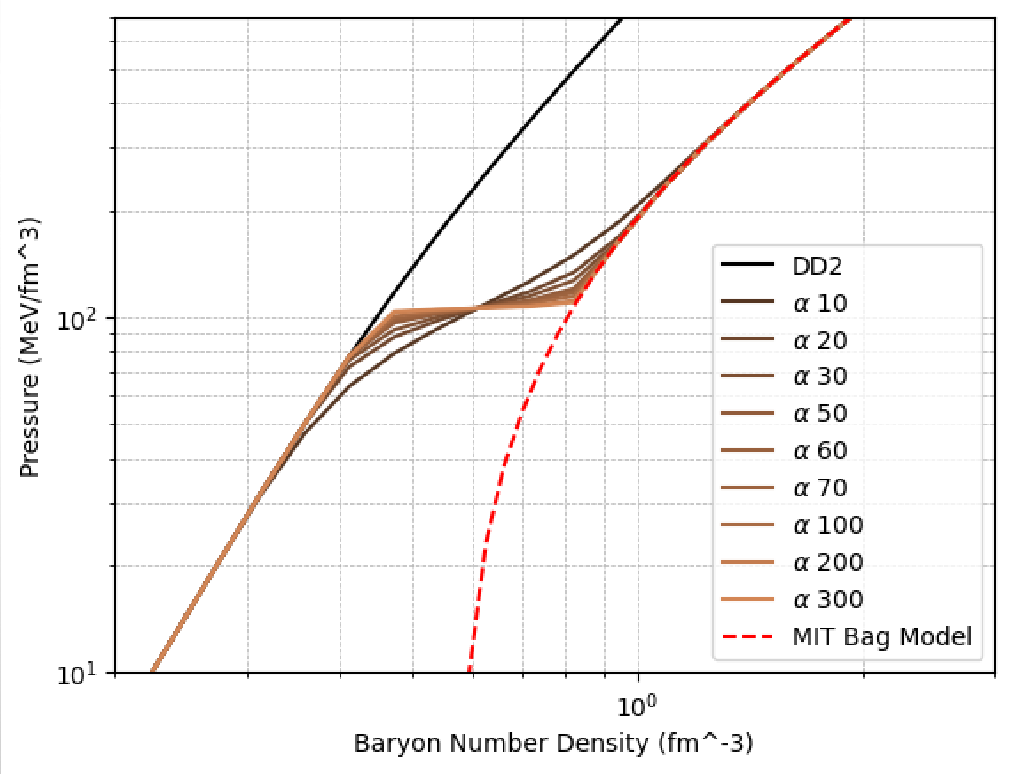
where

$$\mu_g = \mu_b + Y_q * \mu_q$$

or

$$\mu_g = \mu_b + Y_e * \mu_l$$

(if we do the transition with electrons)



GR et al. (2026) in prep.

PARAMETRISED PHASE TRANSITION

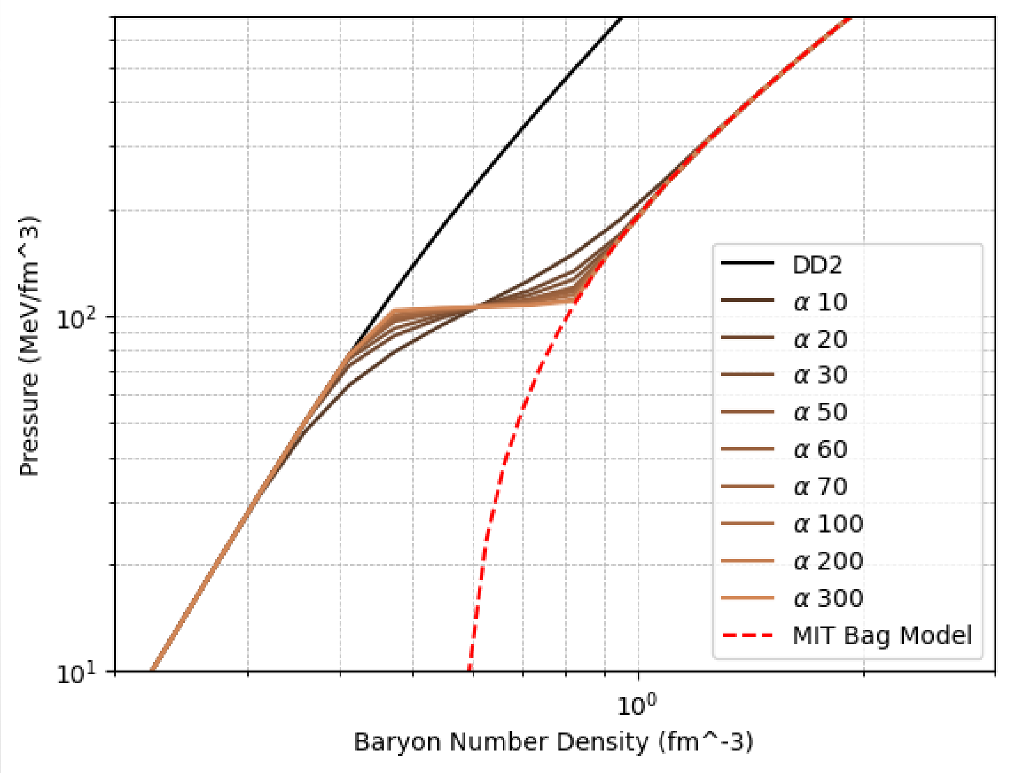
We have the pressure!

How do we get everything else?

- Invert $\mathrm{P} = n^2 \frac{\partial F}{\partial n}$
- From the free energy F we get everything else!

(Typel et al 2015)

To ensure thermodynamical concistency
Piecewise quintic Hermite polynomials



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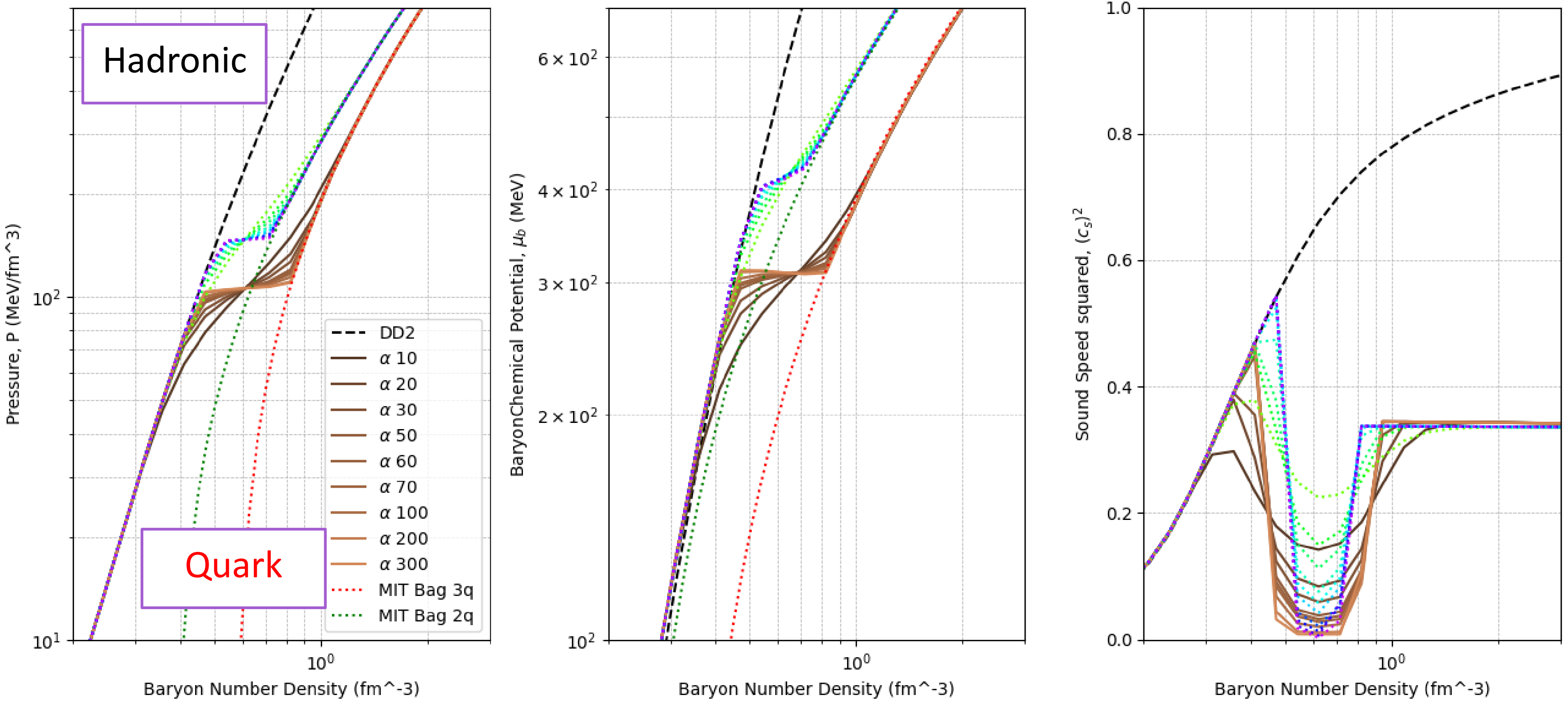
PARAMETRISED PHASE TRANSITION



GR et al. (2026) in prep.

Parametrised Phase Transition

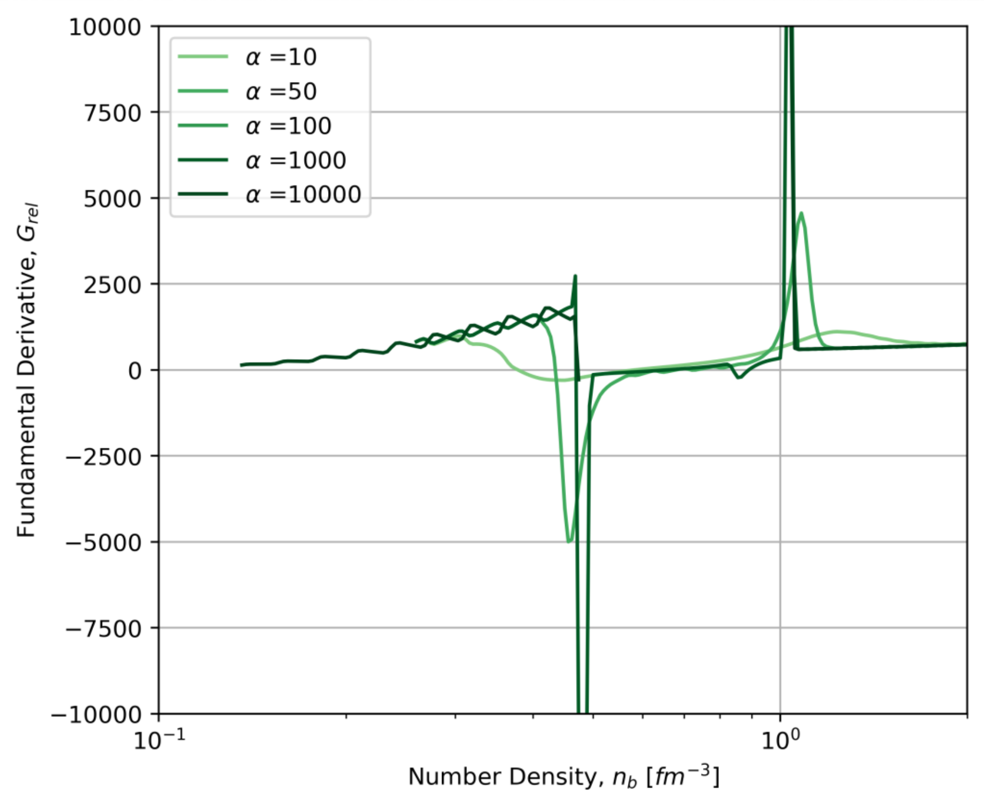
And the fundamental derivative?

We write

$$G = 1 + \frac{n_b}{2\, c_s^2} \frac{\partial c_{s,C}^2}{\partial n_b}\bigg|_s - \frac{3}{2} c_{s,R}^2$$

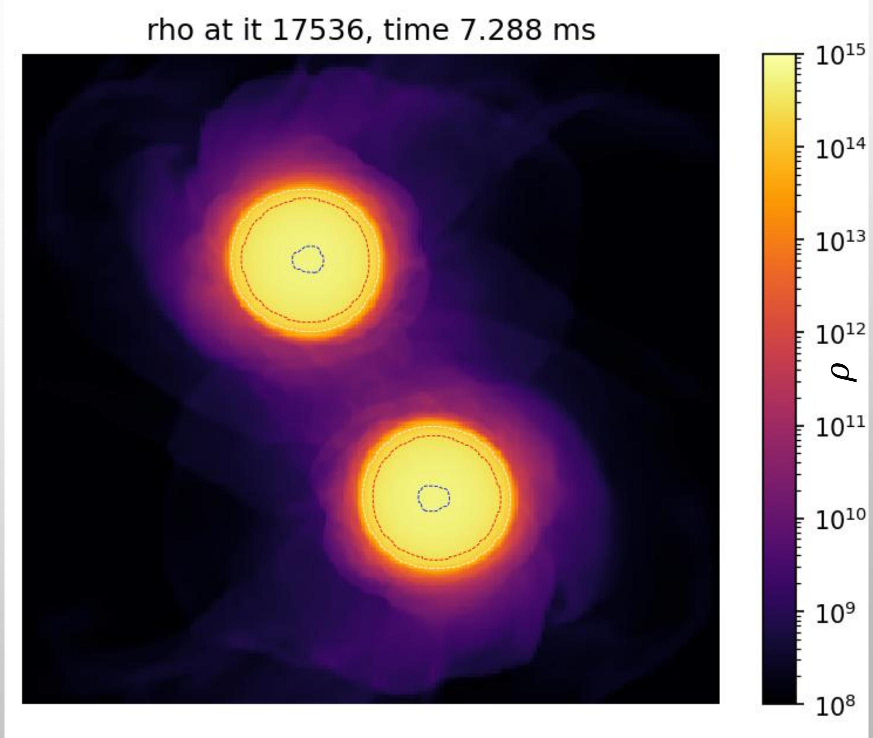
(C is Classical, R is Relativistic)

We will have a continous crossover with still non-convex matter.



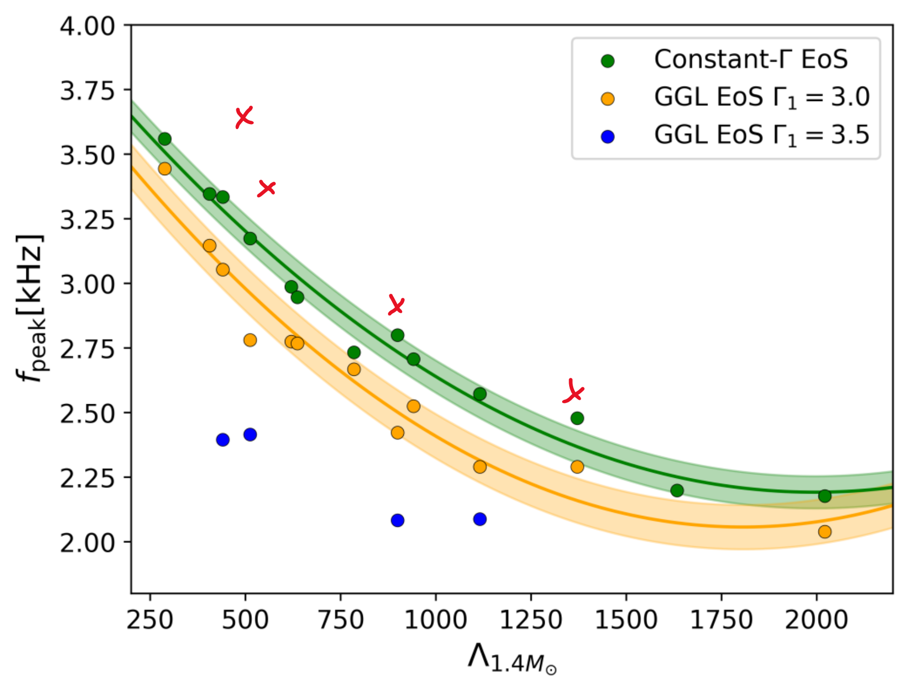
GR et al. (2026) in prep.

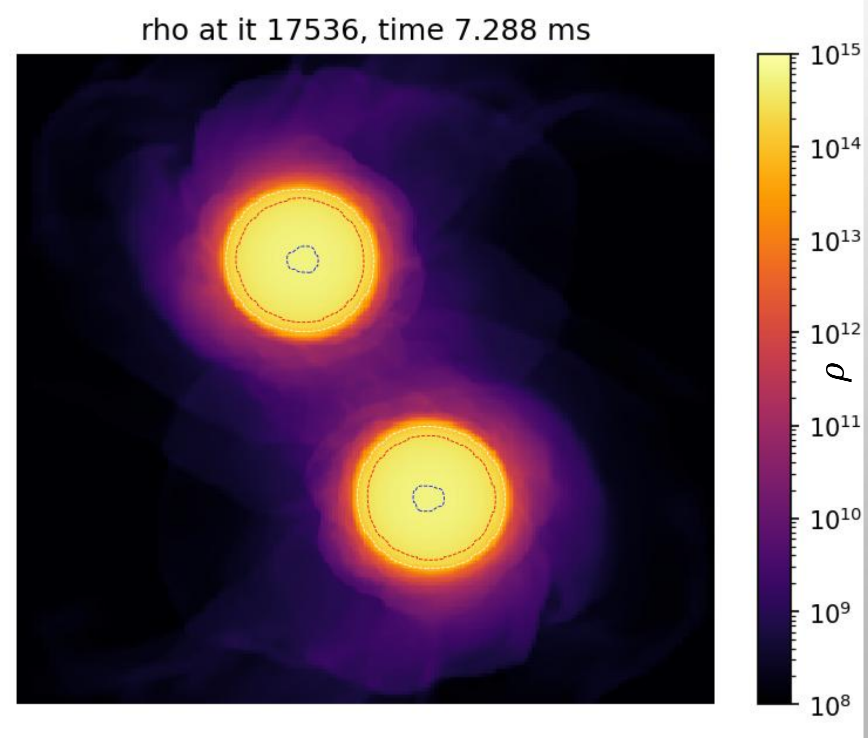
Simulations are running!



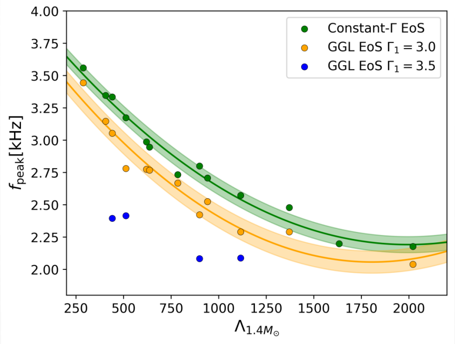
Simulations are running!

We expect to populate the high frequency side with both nonconvex crossover and first order phase transition.

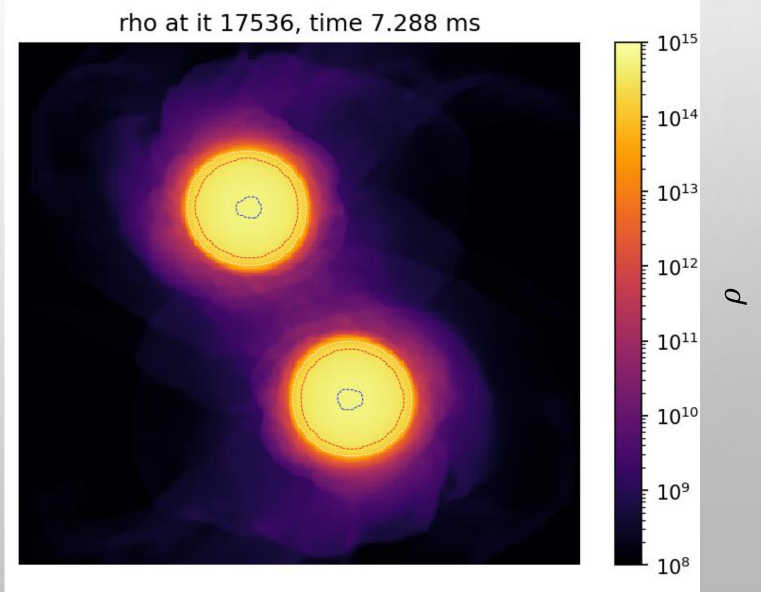




Take away message!



- Convexity determines the wave structure and the evolution of the fluid
- Nonconvexity has an imprint in the gravitational wave!
- Phase transitions and crossovers are nonconvex
- Simulation are running to determine the definitive contribution of nonconvexity in BNS merger



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THANKS!

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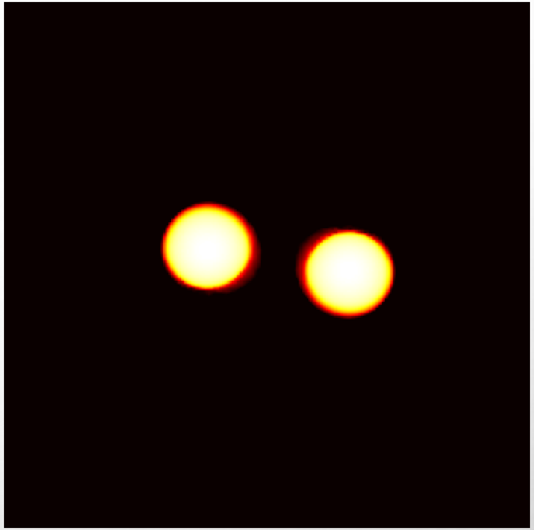

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