Weak Interaction Rates in Mergers

The Nucleon Width Approximation and Applications to Magnetized Neutron Stars

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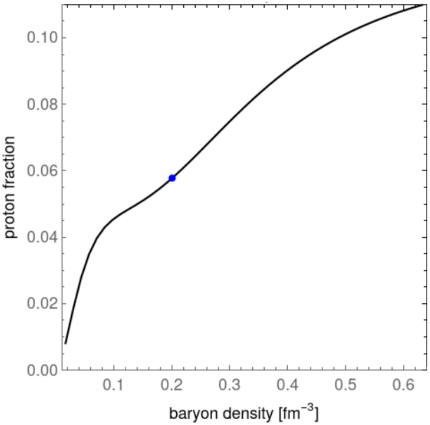
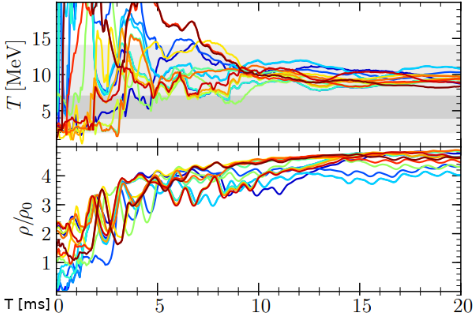
Chemical equilibrium: neutron decay and electron capture balance

$$n + \cdots \to p + e^- + \ldots \qquad p + e^- + \cdots \to n + \cdots$$

- Only weak interactions can change particle content
- In equilibrium: rates balance



Neutron Star Mergers



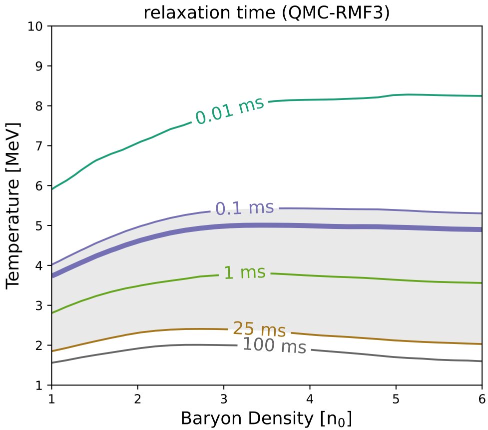
Alford, Bovard et.al., PRL 120 (2018)

- Oscillations in late inspiral and merger drive matter out of equilibrium
- Weak interactions try to drive matter back to equilibrium
- Chemical equilibration also modifies normal modes in inspiral (g-modes: Hyperonic matter: Ghosh, A.H., Andersson, Counsell 2608.07311. Nuclear matter: Zhao, Rau, A.H. et al. 2504.12230)
- Impact on mergers depends on the timescale of oscillations and equilibration times

Chemical Equilibration Time in Nuclear Matter

Alford, Haber, Zhang, 2306.06180

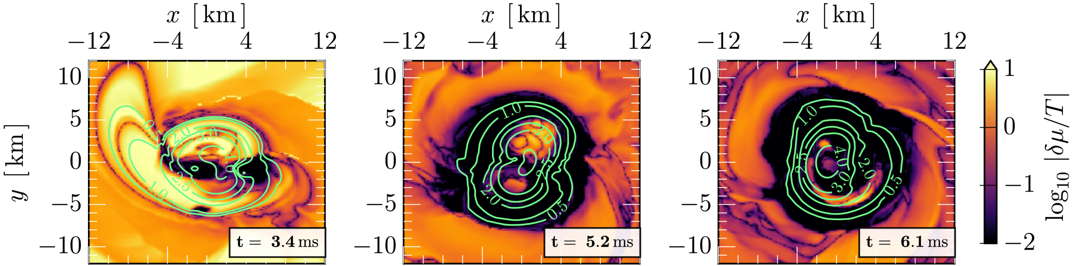
- kHz oscillation $\sim$ ms timescale
- Three distinct regimes: frozen - resonant - instantaneous equilibration
- QMC-RMF models: Alford, Brodie, Haber, Tews: 2205.10283 (Compose)



Effects of Chemical Equilibration

- **Flavor equilibration** (SN, PNS, NS asteroseismology, BNS inspiral and merger)
- **Bulk viscosity** via flavor equilibration: in nuclear matter at $T \sim 5$ MeV
- **Neutrino mean free path** (mfp): kilonova, r-process nucleosynthesis, shear viscosity, ...
- **Cooling** of (proto-) neutron stars

$$\delta\mu = \mu_n - (\mu_p + \mu_e)$$



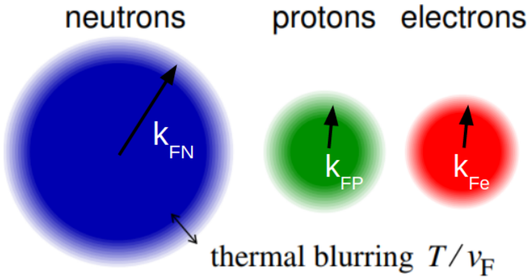
Most, A.H., Harris, Zhang, Alford, Noronha; arXiv:2207.00442

Urca processes

direct Urca (dU)

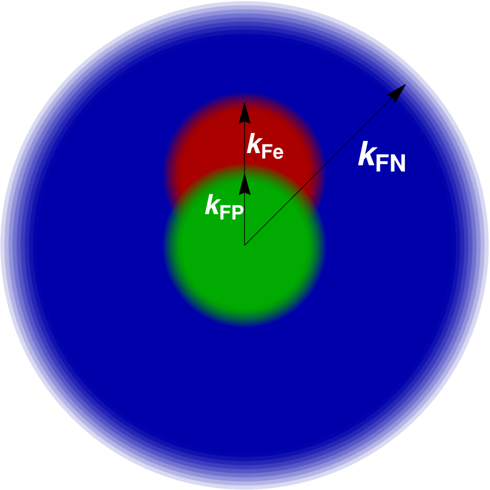
neutron decay: $n \to p + e^- + \bar{\nu}_e$ electron capture: $p + e^- \to n + \nu_e$

- Strongly degenerate npe-matter:
  dominated by particles on their Fermi surface (FS)



Direct Urca Threshold

Below threshold: proton fraction too low



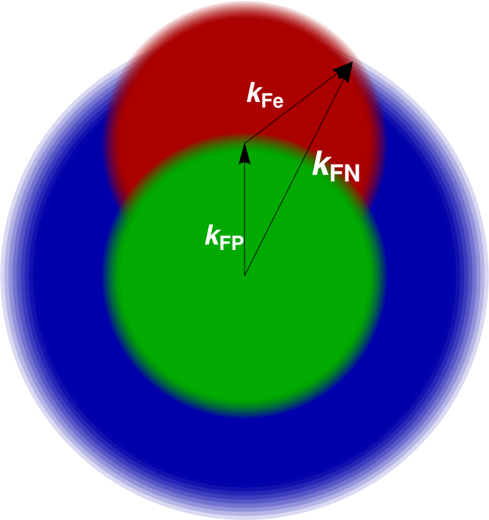
direct Urca (dU)

electron capture: $p + e^- \rightarrow n + \nu_e$

- Momentum conservation on FS demands $\vec{k}_{Fn} \leq \vec{k}_{Fp} + \vec{k}_{Fe}$
- If momentum cons. on FS not possible: rate heavily suppressed

Direct Urca Threshold

Above threshold: proton fraction $\geq 11\%$



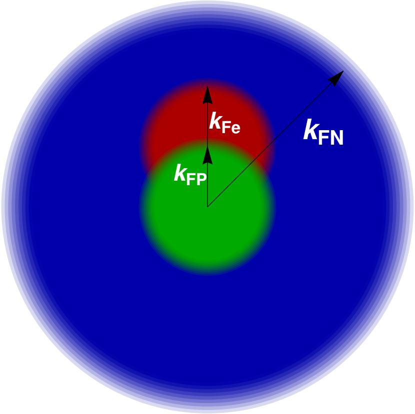
direct Urca (dU)

electron capture: $p + e^- \rightarrow n + \nu_e$

- Momentum conservation on FS demands $\vec{k}_{Fn} \leq \vec{k}_{Fp} + \vec{k}_{Fe}$
- If momentum cons. on FS possible: rate dominated by direct Urca

modified Urca (mU): dU with spectator

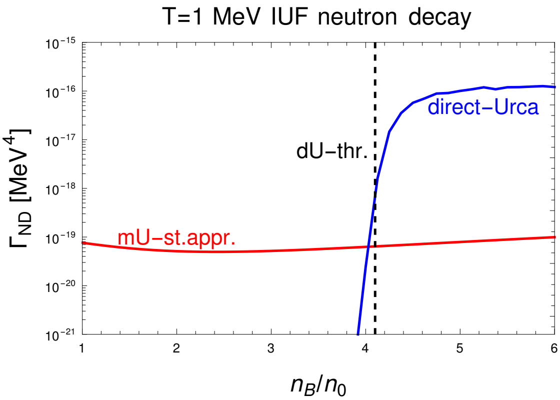
neutron decay: $n + N \to p + e^- + \bar{\nu}_e + N$ electron capture $p + e^- + N \to n + \nu_e + N$



- Add single collision with spectator nucleon

Direct Urca and Modified Urca

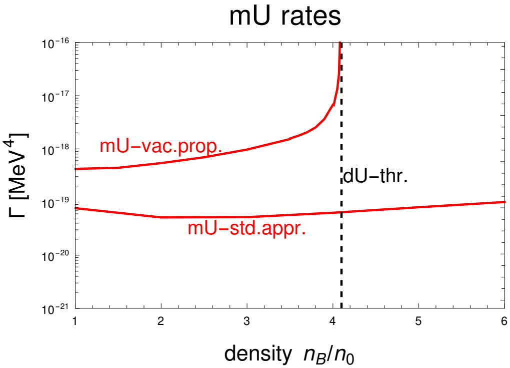
$T = 1$ MeV - neutrino transparent, IUF-EOS



- standard approximation for mU: ultra-non relativistic / momentum independent propagator
- Full phase space calculation for direct Urca: **arXiv:2306.06180, arXiv:2108.03324**

Improved modified Urca is worse?

Divergent rate in Shternin et al. 2018



- ✓ Improved treatment: non-relativistic momentum dependent propagator
- ✗ Divergence at dU threshold

The Solution

- Sum over an infinite number of collisions

→ Regulation of divergence by adding complex nucleon width

- Perform mass-spectral decomposition

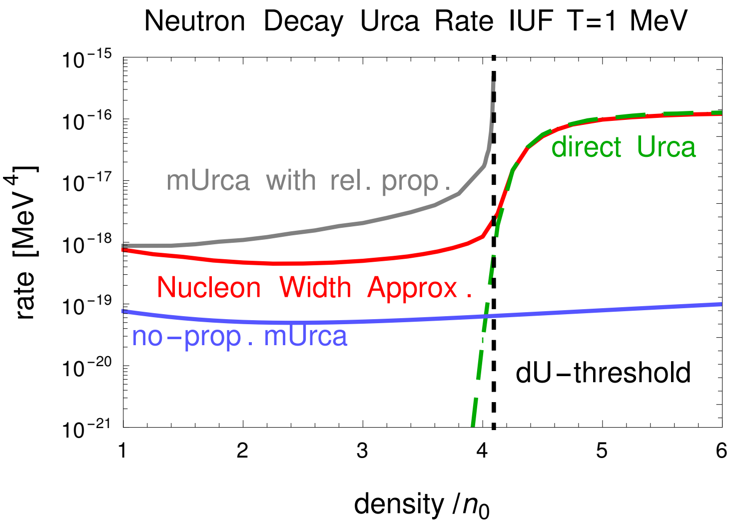
Nucleon Width Approximation - NWA

$$\Gamma^{\mathrm{NWA}} = \int_{-\infty}^{\infty} dm_{\mathrm{n}} dm_{\mathrm{p}} \, \Gamma^{\mathrm{dUrca}}(m_{\mathrm{n}}, m_{\mathrm{p}}) \, R_{\mathrm{n}}(m_{\mathrm{n}}) R_{\mathrm{p}}(m_{\mathrm{p}}) \, .$$

with the Breit-Wigner spectral function $R_a(m_a)$

Recipe: take your standard direct Urca calculation and integrate over masses (weighted with B-W)

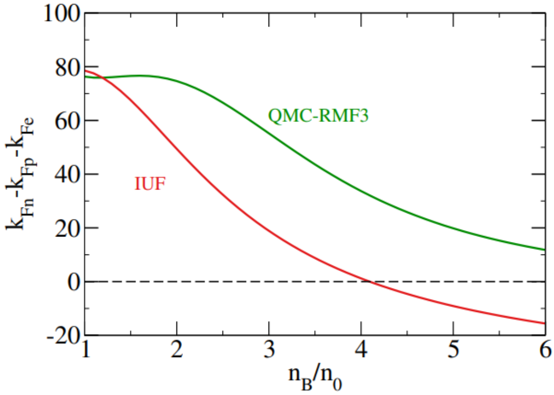
NWA rates


Neutron Decay Urca Rate IUF T=1 MeV
rate [MeV^4]
10^−15
10^−16
10^−17
10^−18
10^−19
10^−20
10^−21
1
2
3
4
5
6
density / n_0
mUrca with rel. prop.
direct Urca
Nucleon Width Approx.
no−prop. mUrca
dU−threshold

Urca at finite magnetic field

Tambe, Chatterjee, Alford, Haber 2409.09423

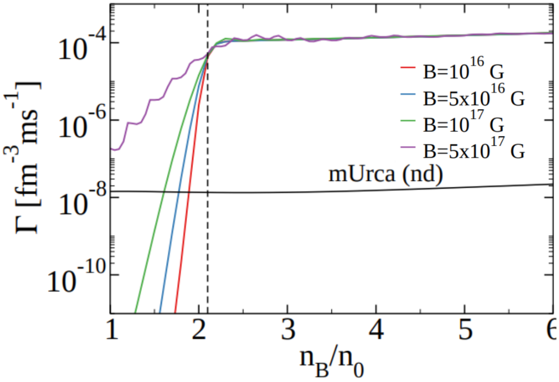
- Magnetic field effects on EOS very small
- Scales are far apart: $\Lambda_{QCD}^2 \approx (150\,\text{MeV})^2 \approx 10^{18}$ Gauss
- Direct Urca threshold introduces new small scale
- We expect changes of the rate close to the threshold



Urca at finite magnetic field

Tambe, Chatterjee, Alford, Haber 2409.09423

- We see an enhancement close to threshold
- Strong B: De Haas–Van Alphen due to Landau levels
- B-field opens up phase space just like temperature
- What about the modified Urca contribution?

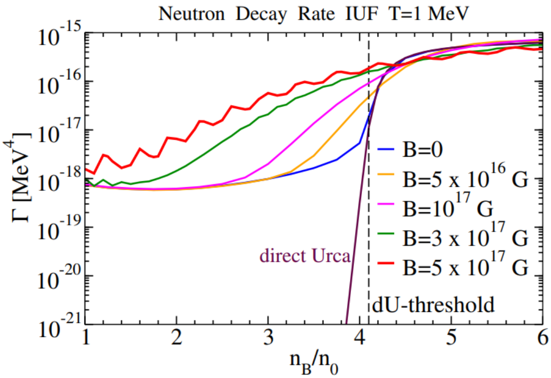


NWA at finite B

Tambe, Chatterjee, Alford, Haber 2510.09104

The power of the recipe

- We see an enhancement close to threshold even without magnetic field
- Strong B: De Haas–Van Alphen due to Landau levels
- Full temperature and magnetic field dependence of complete Urca rates ("dU+mU+mmU+...")

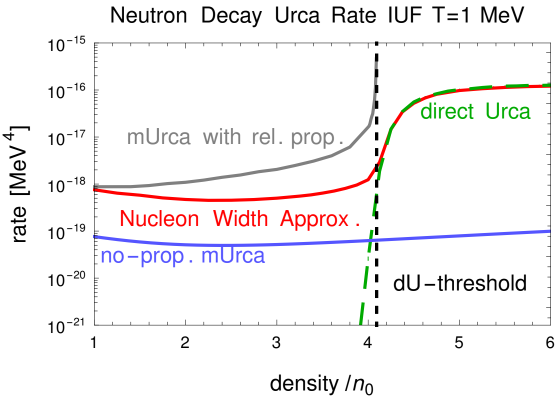


Summary & Outlook

- weak interactions play an important role in neutron star physics
- mU is crude, inconsistent, hard to improve, and wrong
- NWA is easy to implement alternative that gives more accurate results
- Strong magnetic fields can influence transport properties more strongly than the EOS

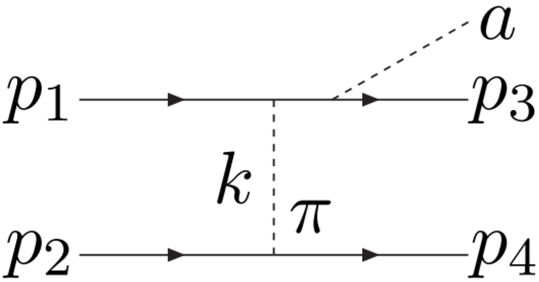
NWA - results

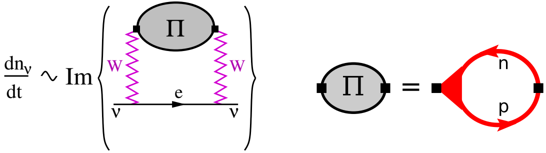
- smooth transition from "mU-regime" to "dU-regime"
- constant nucleon width ($W_n = W_p = T^2/(5\,\text{MeV})$ obtained from Brueckner theory calculation for pure neutron matter using the Paris NN potential by Sedrakian astro-ph/0002228
- Allows us to go beyond Fermi-surface for mU
- Enhancement of low density rate by order of magnitude (see Shternin et al.)
- No divergence
- Matches dU calculation above threshold



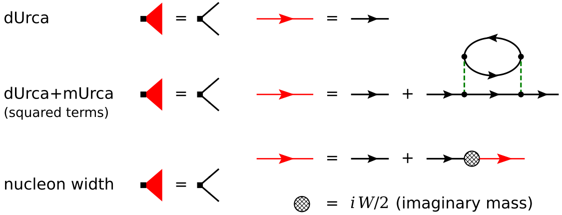
NWA - advantages

- Consistent, simple approach to Urca
- Systematically improvable scheme
- Allows us to go beyond Fermi-surface for mU
- NWA can be applied in any context where dUrca rates can be calculated
  - finite temperature
  - matter with non-equilibrium neutrino distributions
  - strong magnetic fields
  - decays in some models of dark matter, **axion-production**, hyperonic matter, quark matter . . .



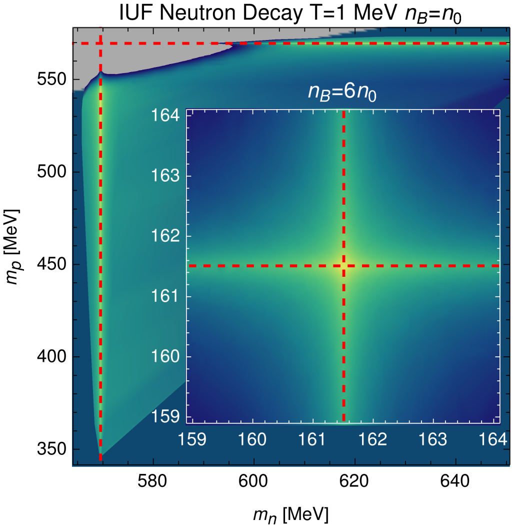


dn_ν / dt ~ Im
Π
W
W
e
ν
ν
Π
=
n
p



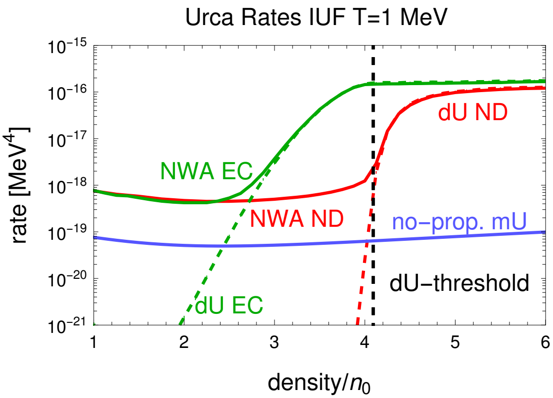
dUrca
dUrca+mUrca
(squared terms)
nucleon width
=
+
i W/2 (imaginary mass)

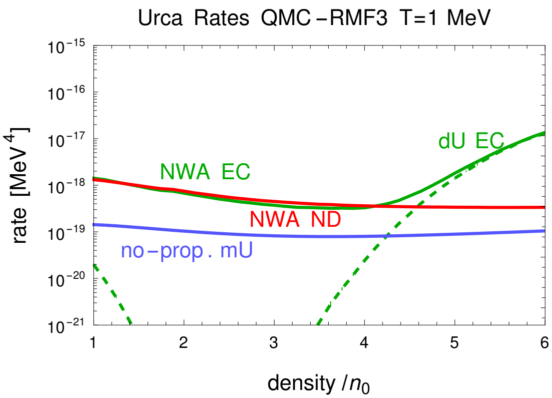
Mass Integral



NWA results

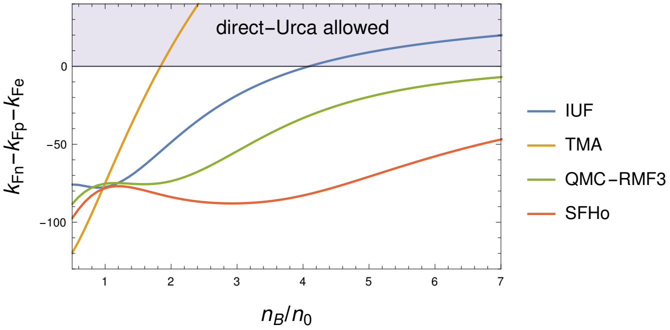
QMC-RMF3 Alford, Brodie, Haber, Tews 2205.10283





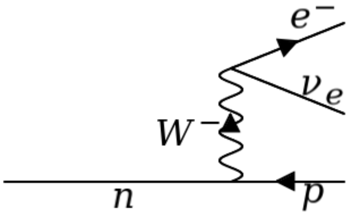
Direct Urca Threshold

strongly EOS depended

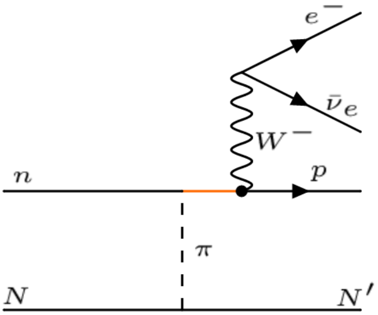


- Momentum conservation on FS demands $\vec{k}_{Fn} \leq \vec{k}_{Fp} + \vec{k}_{Fe}$
- Proton fraction x_p is monotonic with density
- Need $x_p \approx 11\%$ for $k_{Fn} = k_{Fp} + k_{Fe}$
- Threshold density = direct Urca threshold
- Impact on cooling, bulk viscosity, ...

direct Urca



modified Urca



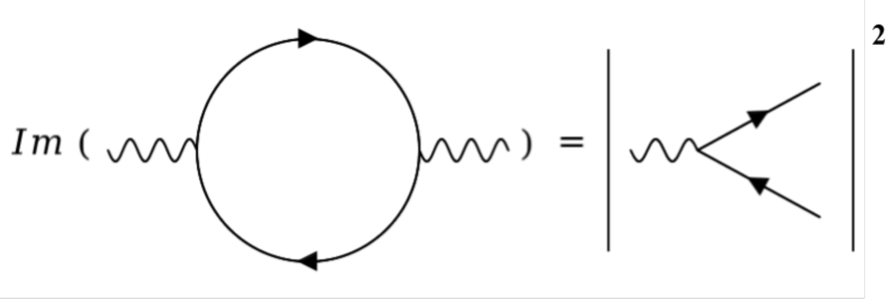
Propgator for off-shell nucleon

How to deal with propagator G_n for internal, off-shell nucleon?

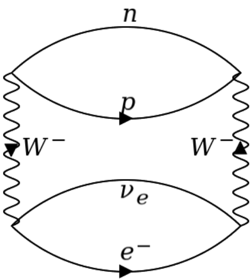
Cutkosky Cutting Rules

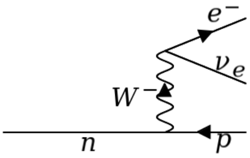
QFT version of optical theorem

Im(diagram) $\propto$ decay rate

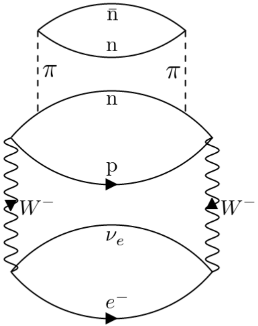


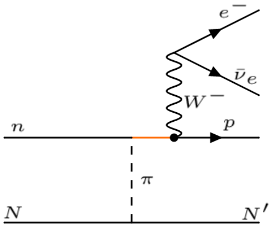
direct Urca from bubble diagram





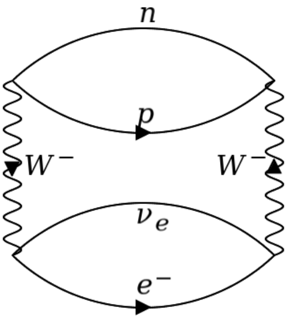
modified Urca from bubble diagram



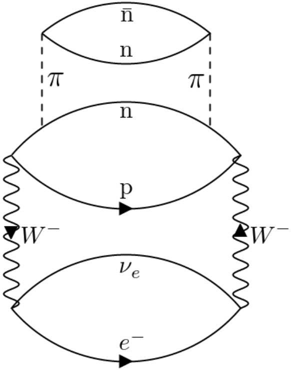


Total Rate

$\Gamma_\nu =$



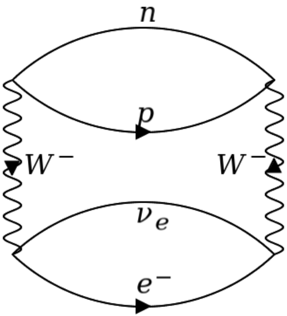
$+$



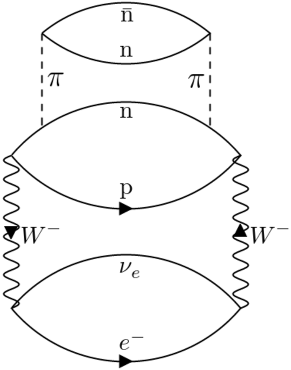
$+ \quad \dots$

Dyson Schwinger Equation

$\Gamma_\nu =$



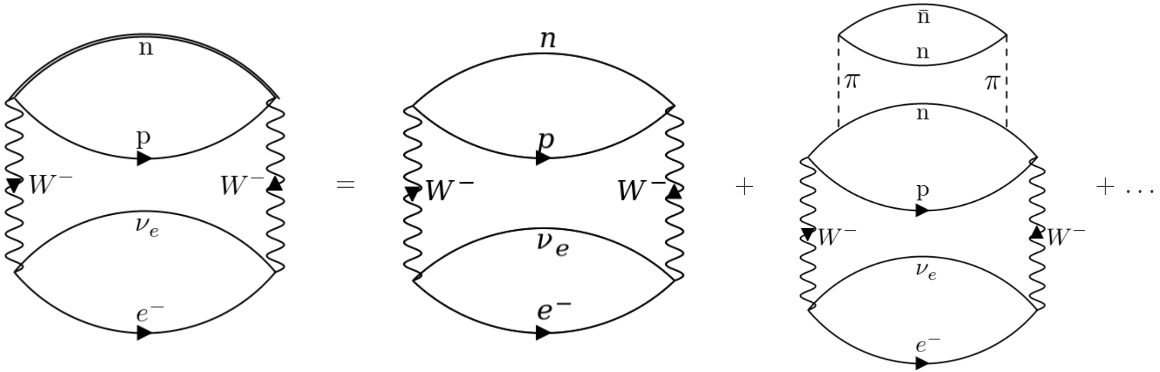
$+$



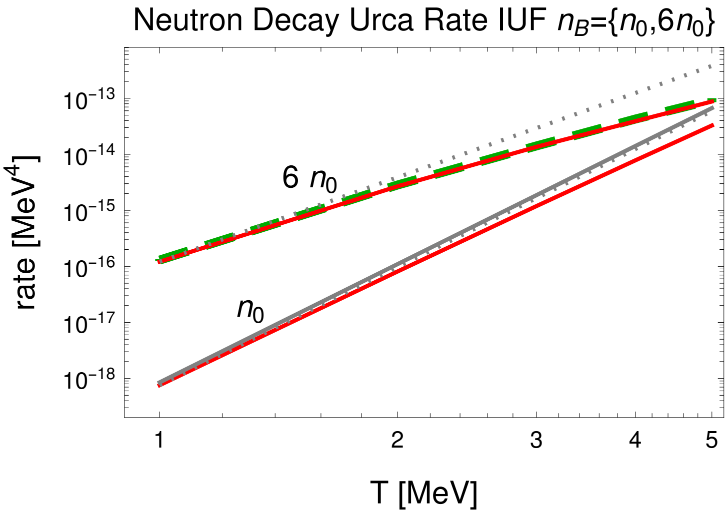
$+ \quad \cdots$

Nucleon Width Approximation NWA

Alford, Haber, Zhang, 2406.13717



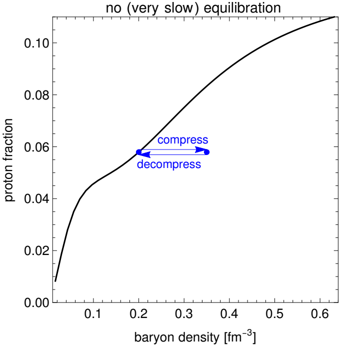
NWA temperature dependence



Bulk Viscosity from Beta Equilibration

Path of fluid element as it is compressed and decompressed

Density Oscillations in merger drive matter out of equilibrium



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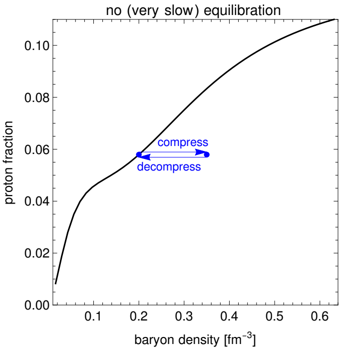

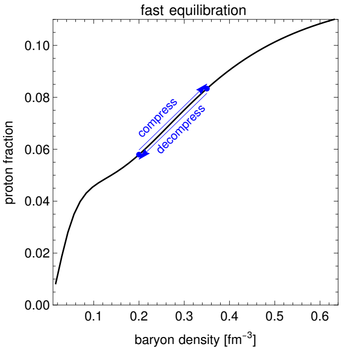

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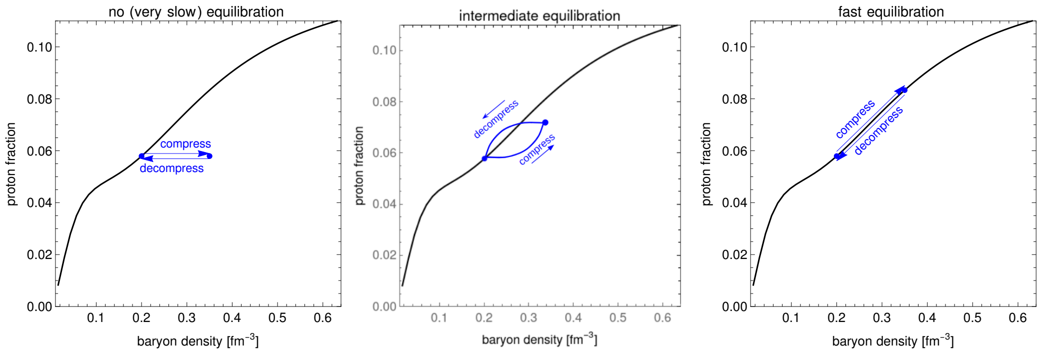

$\{x_P, n_B\}$ plane equiv. to $\{P, V\}$ plane $\rightarrow$ traversing a path in P-V plane leads to $\int PdV$ - **work**