

My quantum journey with Tao

Liantao Wang

Pittsburgh, May 15, 2026



Tao clarified

Drink = Postdoc?

Tao clarified

Drink = Postdoc? No true.

Tao clarified

Drink = Postdoc? No true.

Tao did call me before making me a postdoc offer

About physics and whether I would like to come

Tao clarified

Drink = Postdoc? No true.

Tao did call me before making me a postdoc offer

About physics and whether I would like to come

After I arrived, I went to his place first

The first thing he said:

Forgot to ask, do you drink?



Tao clarified

Tao = τ ? Different pronunciation: **tāo** (涛), not tau

Tao clarified

Tao = τ ?

Different pronunciation: **tāo** (涛), not tau

Tao has about 14 papers on τ lepton.
Not many, but quantity \neq quality

Tao: write the best paper, let others follow

Tao clarified

Tao = Taoism? Not quite, should be **dào** 道

Tao clarified

Tao = Taoism? Not quite, should be **dào** 道

But Daoism teaches harmony with nature and universe

Not too different from Taoism.

Tao clarified

The other Taos?

Tao clarified

The other Taos?

True, about 2 million (ranked #7)

Tao clarified

The other Taos?

True, about 2 million (ranked #7)

However, be careful:

Tao clarified

The other Taos?

True, about 2 million (ranked #7)

However, be careful:



Tao Liu. 刘滔

Not the same Tao.

Tao clarified

The other Taos?

True, about 2 million (ranked #7)

However, be careful:



Tao Liu. 刘滔

Not the same Tao.

On the other hand:

Lian-Tao. 连涛

Same Tao!

Tao clarified

The other Taos?

True, about 2 million (ranked #7)

However, be careful:



Tao Liu. 刘滔

Not the same Tao.

On the other hand:

Lian-Tao. 连涛 Same Tao!

Moreover, means “connected to Tao”.

Back to our regular
programming

Tao and I



Tao and I



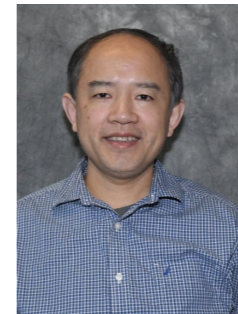
Of course, Tao is strongly entangled with many.

Tao and I



Of course, Tao is strongly entangled with many.

Tao and I



Of course, Tao is strongly entangled with many.
Traced out for the purpose of this talk, sorry.

t=0. 2001

Tao has been leading the pheno study on many new developments, such as extra-dimensions.

t=0. 2001

Tao has been leading the pheno study on many new developments, such as extra-dimensions.

I have been working on SUSY, and something called string-pheno

t=0. 2001

Tao has been leading the pheno study on many new developments, such as extra-dimensions.

I have been working on SUSY, and something called string-pheno

$$\hat{\rho} = \frac{1}{2} \left[\begin{array}{c} \text{[Portrait of Tao] } \rangle \langle \text{ [Portrait of Tao]} \\ \text{[Portrait of Shiu] } \rangle \langle \text{ [Portrait of Shiu]} \end{array} \right]$$

t=0. 2001

- * Pheno 2001

My first conference presentation, on SUSY explanation of $g_\mu - 2$

Certainly an eye-opening moment for me.

t=0. 2001

* Pheno 2001

My first conference presentation, on SUSY explanation of $g_\mu - 2$

Certainly an eye-opening moment for me.

Tao told me he remembered that. I don't know what exactly his impression was.

t=0. 2001

* Pheno 2001

My first conference presentation, on SUSY explanation of $g_\mu - 2$

Certainly an eye-opening moment for me.

Tao told me he remembered that. I don't know what exactly his impression was.

But, he hates ambulance chasing...

t=0. 2001

* Pheno 2001

My first conference presentation, on SUSY explanation of $g_\mu - 2$

Certainly an eye-opening moment for me.

Tao told me he remembered that. I don't know what exactly his impression was.

But, he hates ambulance chasing...

In spite of that:

$e^{-iH(2004 - 2002)}$

I become a postdoc at Madison in 2002

$$e^{-iH(2004 - 2002)}$$

I become a postdoc at Madison in 2002

$$H = H_{\text{physics}}$$

$e^{-iH(2004 - 2002)}$

I become a postdoc at Madison in 2002

$$H = H_{\text{physics}}$$



$e^{-iH(2004 - 2002)}$

I become a postdoc at Madison in 2002

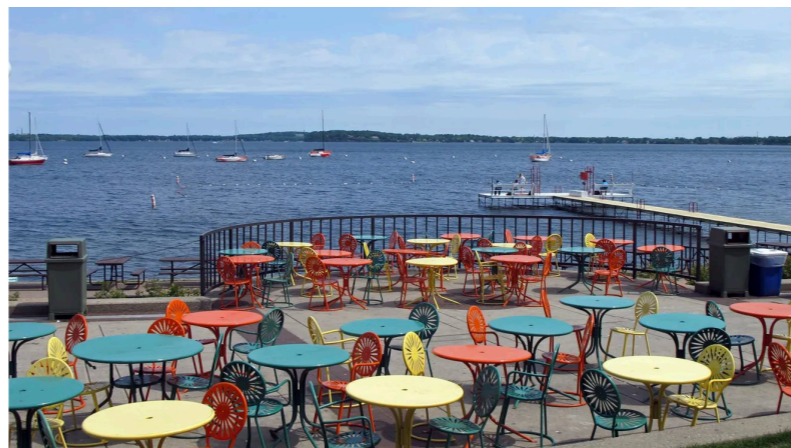
$$H = H_{\text{physics}} + H_{\text{good time}}$$



$e^{-iH(2004 - 2002)}$

I become a postdoc at Madison in 2002

$$H = H_{\text{physics}} + H_{\text{good time}}$$



$$e^{-iH(2004 - 2002)}$$

e^{-iH} (2004 – 2002)

Tao: let's take a look at the Little Higgs model

e^{-iH} (2004 – 2002)

Tao: let's take a look at the Little Higgs model

Me: but, it is not SUSY

e^{-iH} (2004 – 2002)

Tao: let's take a look at the Little Higgs model

Me: but, it is not SUSY

Tao: open your mind.

e^{-iH} (2004 – 2002)

Tao: let's take a look at the Little Higgs model

Me: but, it is not SUSY

Tao: open your mind.

Me: the models are already there, what can we do?

e^{-iH} (2004 – 2002)

Tao: let's take a look at the Little Higgs model

Me: but, it is not SUSY

Tao: open your mind.

Me: the models are already there, what can we do?

Tao: they didn't talk about the physics consequences.
We need to outline, and experiments will tell

e^{-iH} (2004 – 2002)

Tao: let's take a look at the Little Higgs model

Me: but, it is not SUSY

Tao: open your mind.

Me: the models are already there, what can we do?

Tao: they didn't talk about the physics consequences.
We need to outline, and experiments will tell

$$\frac{1}{\sqrt{2}} \left[\begin{array}{c} \text{[Tao's portrait]} \\ \text{[Me's portrait]} \end{array} \right] + \begin{array}{c} \text{[Me's portrait]} \\ \text{[Tao's portrait]} \end{array} \right]$$


My learning curve for collider physics

Phenomenology of the little Higgs model

#11

Tao Han ([Wisconsin U., Madison](#)), Heather E. Logan ([Wisconsin U., Madison](#)), Bob McElrath ([Wisconsin U., Madison](#)), Lian-Tao Wang ([Wisconsin U., Madison](#)) (Jan, 2003)

Published in: *Phys.Rev.D* 67 (2003) 095004 • e-Print: [hep-ph/0301040](#) [hep-ph]


 pdf  DOI  cite  claim  reference search  697 citations

Loop induced decays of the little Higgs: $H \rightarrow gg, \gamma\gamma$

#10

Tao Han ([Wisconsin U., Madison](#)), Heather E. Logan ([Wisconsin U., Madison](#)), Bob McElrath ([Wisconsin U., Madison](#)), Lian-Tao Wang ([Wisconsin U., Madison](#)) (Feb, 2003)

Published in: *Phys.Lett.B* 563 (2003) 191-202, *Phys.Lett.B* 603 (2004) 257-259 (erratum) • e-Print: [hep-ph/0302188](#) [hep-ph]

 pdf  DOI  cite  claim  reference search  160 citations

Smoking-gun signatures of little Higgs models

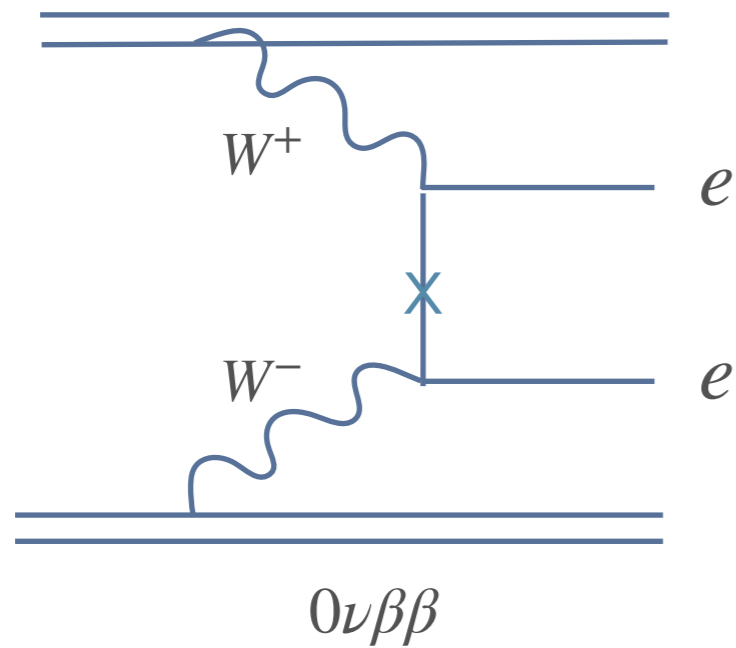
#9

Tao Han ([Wisconsin U., Madison](#) and [Beijing, Inst. Theor. Phys.](#)), Heather E. Logan ([Wisconsin U., Madison](#)), Lian-Tao Wang ([Wisconsin U., Madison](#) and [Harvard U., Phys. Dept.](#)) (Jun, 2005)

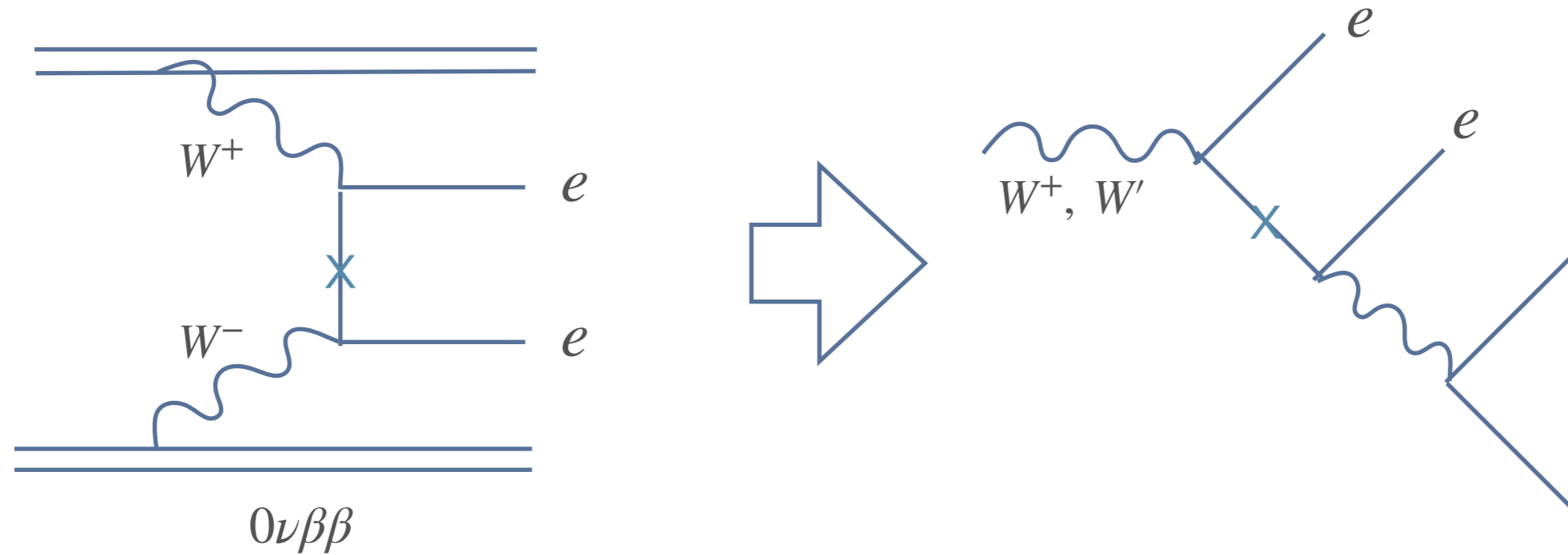
Published in: *JHEP* 01 (2006) 099 • e-Print: [hep-ph/0506313](#) [hep-ph]

 pdf  DOI  cite  claim  reference search  160 citations

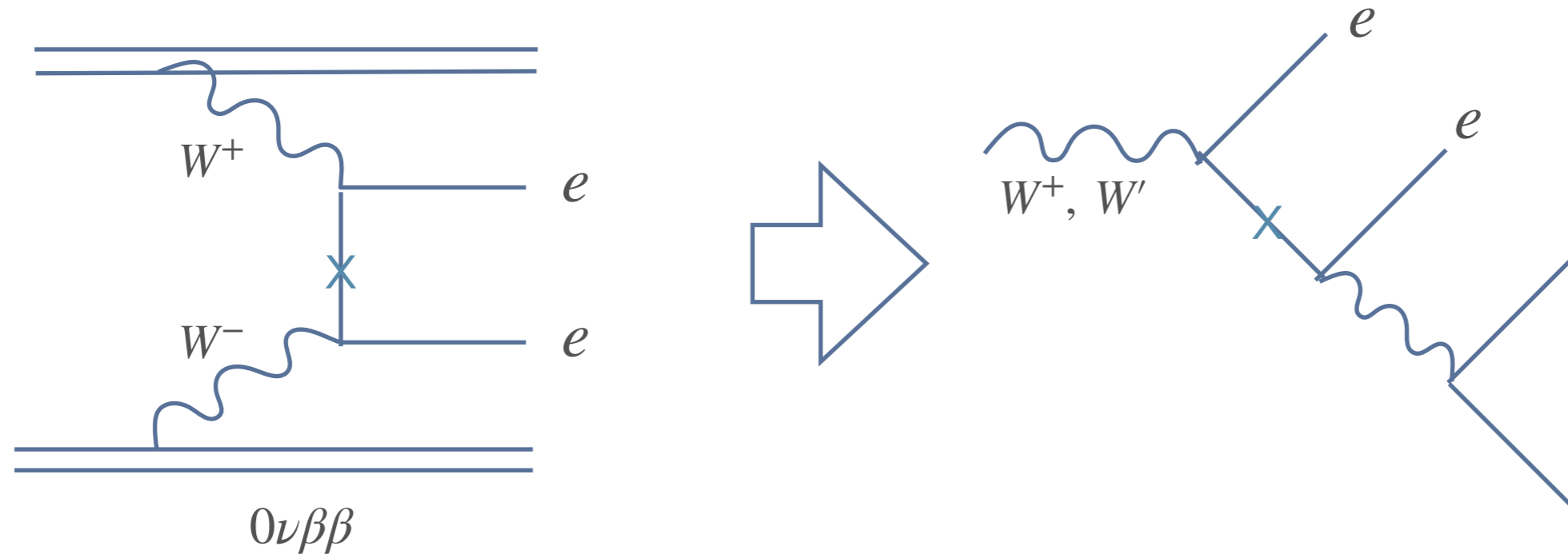
Then Tao went into a twist



Then Tao went into a twist

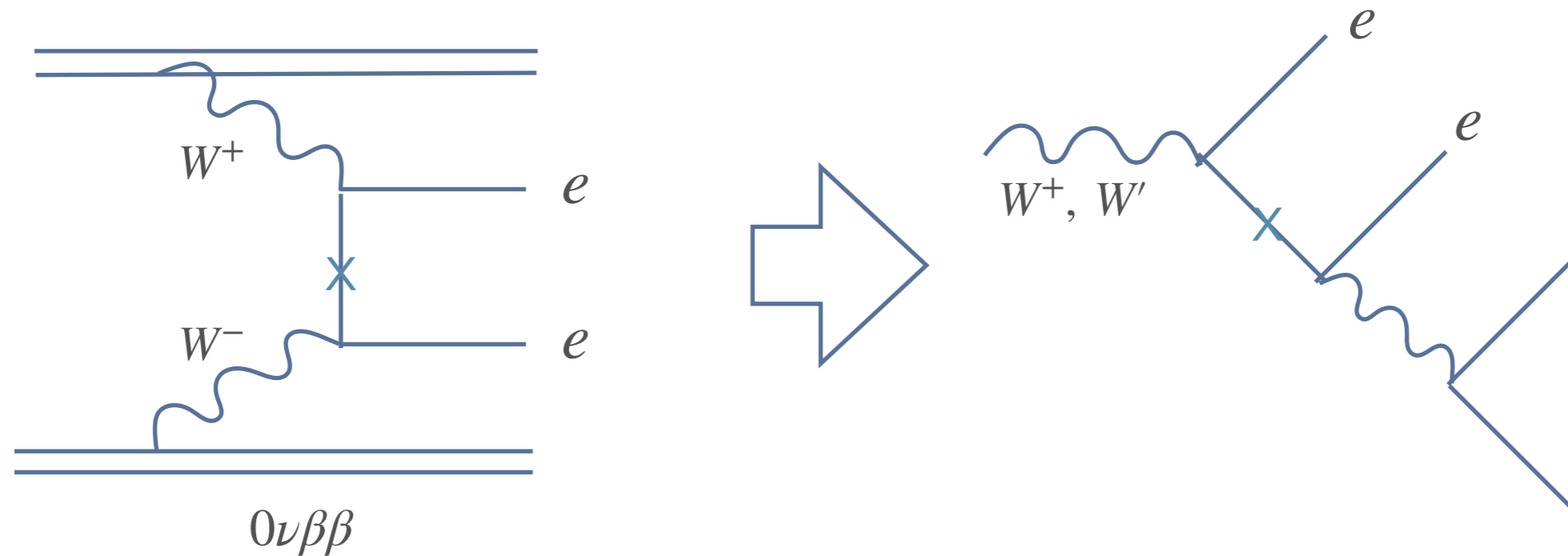


Then Tao went into a twist



Tao: how about neutrinoless double beta decay,
we can twist it around and make it into a collider test!

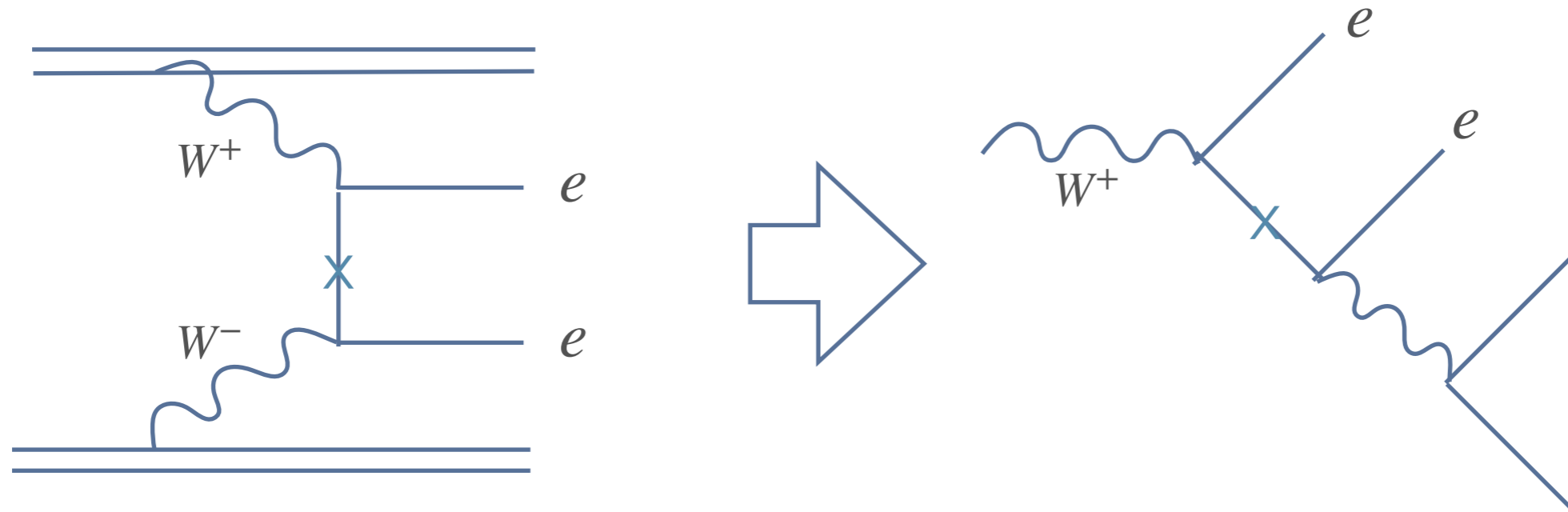
Then Tao went into a twist



Tao: how about neutrinoless double beta decay,
we can twist it around and make it into a collider test!

Me: but, neutrino mass....

Then Tao went into a twist



$$\frac{1}{\sqrt{2}} \left[\left| \begin{array}{c} \text{Portrait of a man in a suit} \\ \text{Portrait of a man in a blue shirt} \end{array} \right\rangle - \left| \begin{array}{c} \text{Portrait of a man in a suit} \\ \text{Portrait of a man in a blue shirt} \end{array} \right\rangle \right]$$

The consequence

后果很严重

The consequence

后果很严重

Tao:

Pair production of doubly-charged scalars: Neutrino mass constraints and signals at the LHC #6

Signatures for Majorana neutrinos at hadron colliders

Tao Han (Wisconsin U., Madison and Tsinghua U., Beijing), Biswarup Mukhopadhyaya (Harish-Chandra Res. Inst.), Zongguo Si (Shandong U.), Kai Wang (Wisconsin U., Madison) (Jun, 2007)

Tao Han (Wisconsin U., Madison and Tsinghua U., Beijing and Beijing, Inst. Zhang (Tsinghua U., Beijing) (Apr, 2006) Published in: *Phys.Rev.D* 76 (2007) 075013 • e-Print: [0706.0441](#) [hep-ph]

Published in: *Phys.Rev.Lett.* 97 (2006) 171804 • e-Print: [hep-ph/0604064](#) pdf DOI cite claim reference search 203 citations

pdf DOI cite claim reference search 290 citations

Neutrino Masses and the CERN LHC: Testing Type II Seesaw Nonstandard neutrino interactions at COHERENT, DUNE, T2HK and LHC #1

Pavel Fileviez Perez (Wisconsin U., Madison), Tao Han (Wisconsin U., Madison), Tao Han (Pittsburgh U.), Jiajun Liao (Zhongshan U.), Hongkai Liu (Pittsburgh U.), Danny Marfatia (U. Gui-yu Huang (Wisconsin U., Madison), Tong Li (Wisconsin U., Madison and Hawaii, Honolulu) (Oct 8, 2019)

Wang (Wisconsin U., Madison) (May, 2008) Published in: *JHEP* 11 (2019) 028 • e-Print: [1910.03272](#) [hep-ph]

Published in: *Phys.Rev.D* 78 (2008) 015018 • e-Print: [0805.3536](#) [hep-ph] pdf DOI cite claim reference search 55 citations

pdf DOI cite claim reference search 427 citations

+ many more

The consequence

后果很严重

Tao:

Pair production of doubly-charged scalars: Neutrino mass constraints and signals at the LHC #6

Tao Han (Wisconsin U., Madison and Tsinghua U., Beijing), Biswarup Mukhopadhyaya (Harish-Chandra Res. Inst.), Zongguo Si (Shandong U.), Kai Wang (Wisconsin U., Madison) (Jun, 2007)

Published in: *Phys.Rev.D* 76 (2007) 075013 • e-Print: [0706.0441](#) [hep-ph]

Tao Han (Wisconsin U., Madison and Tsinghua U., Beijing and Beijing, Inst. Zhang (Tsinghua U., Beijing) (Apr, 2006)

Published in: *Phys.Rev.Lett.* 97 (2006) 171804 • e-Print: [hep-ph/0604064](#) pdf DOI cite claim reference search 203 citations

pdf DOI cite claim reference search 290 citations

Neutrino Masses and the CERN LHC: Testing Type II Seesaw Nonstandard neutrino interactions at COHERENT, DUNE, T2HK and LHC #1

Pavel Fileviez Perez (Wisconsin U., Madison), Tao Han (Wisconsin U., Madison), Tao Han (Pittsburgh U.), Jiajun Liao (Zhongshan U.), Hongkai Liu (Pittsburgh U.), Danny Marfatia (U. Gui-yu Huang (Wisconsin U., Madison), Tong Li (Wisconsin U., Madison and Hawaii, Honolulu) (Oct 8, 2019)

Wang (Wisconsin U., Madison) (May, 2008)

Published in: *JHEP* 11 (2019) 028 • e-Print: [1910.03272](#) [hep-ph]

Published in: *Phys.Rev.D* 78 (2008) 015018 • e-Print: [0805.3536](#) [hep-ph]

pdf DOI cite claim reference search 55 citations

pdf DOI cite claim reference search 427 citations

+ many more

I only came around to this in 2019.

Seeking for sterile neutrinos with displaced leptons at the LHC #2

Jia Liu (Chicago U., EFI), Zhen Liu (Maryland U.), Lian-Tao Wang (Chicago U., EFI and Chicago U., KICP), Xiao-Ping Wang (Argonne) (Apr 1, 2019)

Published in: *JHEP* 07 (2019) 159 • e-Print: [1904.01020](#) [hep-ph]

pdf DOI cite claim reference search 55 citations

e^{-iH} (2023 – 2011)

Around the time of the Higgs discovery, I got more interested in the future beyond the LHC

$e^{-iH(2023 - 2011)}$

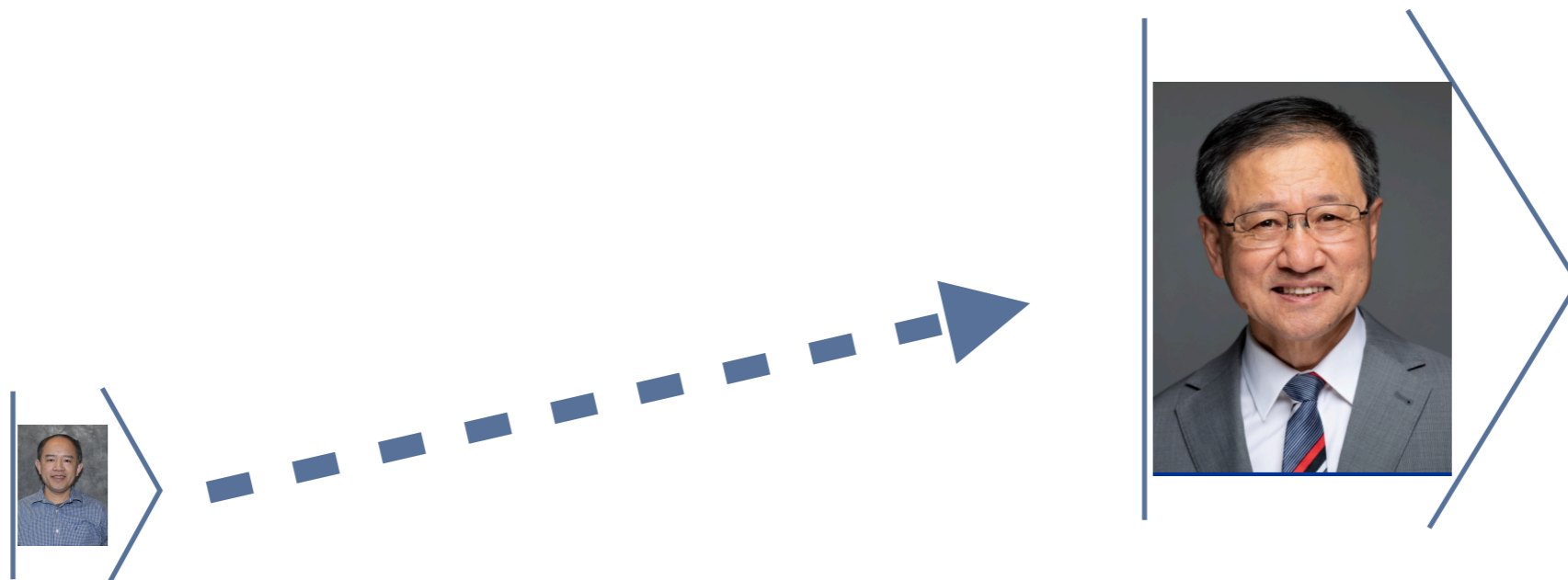
Around the time of the Higgs discovery, I got more interested in the future beyond the LHC

Tao has been on this before I entered grad school.

$$e^{-iH(2023 - 2011)}$$

Around the time of the Higgs discovery, I got more interested in the future beyond the LHC

Tao has been on this before I entered grad school.



Learning and catching up

Physics potential

Physics opportunities of a 100 TeV proton–proton collider

#3

Nima Arkani-Hamed (Princeton, Inst. Advanced Study), Tao Han (Korea Inst. Advanced Study, Seoul and Pittsburgh U. and Tsinghua U., Beijing, CHEP), Michelangelo Mangano (CERN), Lian-Tao Wang (Chicago U., EFI and Chicago U., KICP) (Nov 20, 2015)

Published in: *Phys.Rept.* 652 (2016) 1-49 • e-Print: [1511.06495](#) [hep-ph]

 pdf  DOI  cite  claim


 reference search  294 citations

WIMPs at High Energy Muon Colliders

#2

Tao Han (Pittsburgh U.), Zhen Liu (Maryland U.), Lian-Tao Wang (Chicago U., EFI and Chicago U., KICP), Xing Wang (UC, San Diego) (Sep 23, 2020)

Published in: *Phys.Rev.D* 103 (2021) 7, 075004 • e-Print: [2009.11287](#) [hep-ph]

 pdf  DOI  cite  claim

 reference search  141 citations

Working groups, trips, documents, panels, ...

Snowmass
2012-2013, 2019-2022

CEPC.

Muon collider.

Report of the Topical Group on Physics Beyond the Standard Model at Energy Frontier for Snowmass 2021 #2

Tulika Bose (Wisconsin U., Madison), Antonio Boveia (Ohio State U., CCAPP), Caterina
Doglioni (Manchester U.), Simone Pagan Griso (LBNL, Berkeley), James Hirschauer (Fermilab) et al. (Sep
26, 2022)

Contribution to: [Snowmass 2021](#) • e-Print: [2209.13128](#) [hep-ph]

CEPC Conceptual Design Report: Volume 2 - Physics & Detector #5

CEPC Study Group • João Barreiro Guimarães da Costa (Beijing, Inst. High Energy Phys.) (ed.) et al. (Nov
23, 2018)


e-Print: [1811.10545](#) [hep-ex]

 pdf  links  cite  claim  reference search  1,015 citations

Muon Collider Physics Summary #10

Chiara Aime (Pavia U. and INFN, Pavia), Aram Apyan (Brandeis U.), Mohammed Attia Mahmoud
Mohammed (Fayoum U.), Nazar Bartosik (INFN, Turin), Fabian Batsch (CERN) et al. (Mar 14, 2022)

Contribution to: [Snowmass 2021](#) • e-Print: [2203.07256](#) [hep-ph]

 pdf  links  cite  claim  reference search  173 citations

Working groups, trips, documents, panels, ...

Snowmass
2012-2013, 2019-2022

CEPC.

Muon collider.

Report of the Topical Group on Physics Beyond the Standard Model at Energy Frontier for Snowmass 2021 #2


Tulika Bose (Wisconsin U., Madison), Antonio Boveia (Ohio State U., CCAPP), Caterina Doglioni (Manchester U.), Simone Pagan Griso (LBNL, Berkeley), James Hirschauer (Fermilab) et al. (Sep 26, 2022)

Contribution to: [Snowmass 2021](#) • e-Print: [2209.13128](#) [hep-ph]

CEPC Conceptual Design Report: Volume 2 - Physics & Detector #5

CEPC Study Group • João Barreiro Guimarães da Costa (Beijing, Inst. High Energy Phys.) (ed.) et al. (Nov 23, 2018)

e-Print: [1811.10545](#) [hep-ex]

 pdf  links  cite  claim  reference search  1,015 citations

Muon Collider Physics Summary #10

Chiara Aime (Pavia U. and INFN, Pavia), Aram Apyan (Brandeis U.), Mohammed Attia Mahmoud Mohammed (Fayoum U.), Nazar Bartosik (INFN, Turin), Fabian Batsch (CERN) et al. (Mar 14, 2022)

Contribution to: [Snowmass 2021](#) • e-Print: [2203.07256](#) [hep-ph]

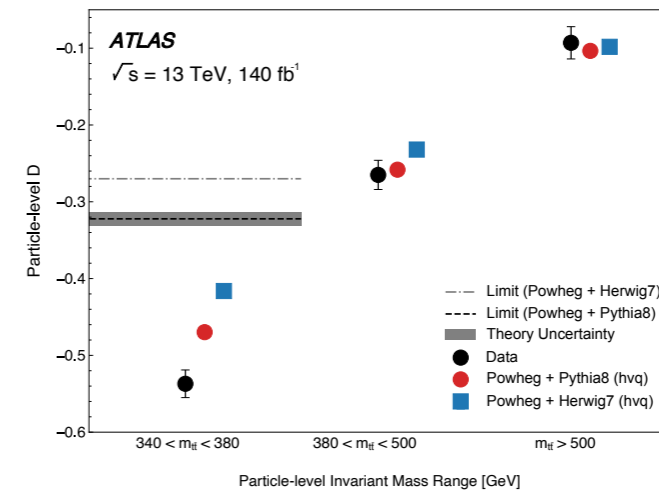
 pdf  links  cite  claim  reference search  173 citations



$$e^{-iH(2025 - 2023)}$$

e^{-iH} (2025 – 2023)

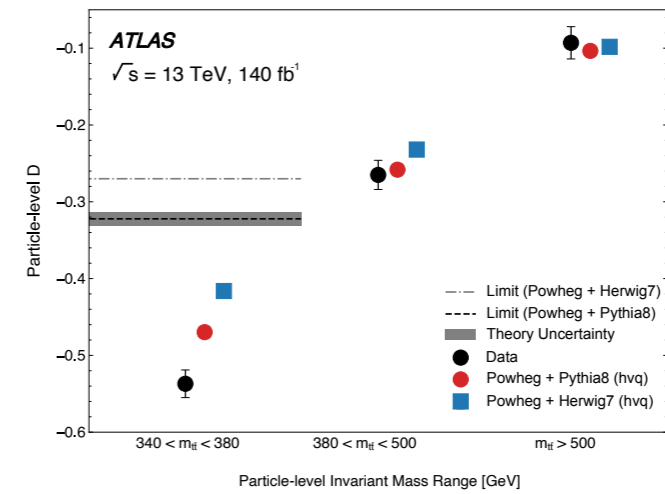
Tao: quantum information at colliders!



e^{-iH} (2025 – 2023)

Tao: quantum information at colliders!

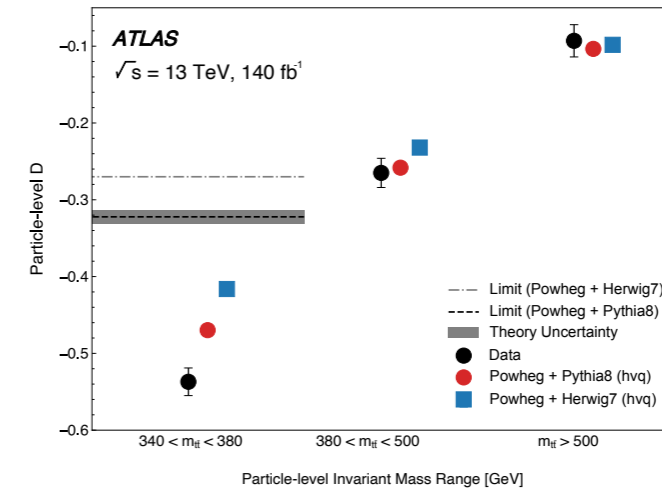
Me: Mmm, loop-holes, useful?



e^{-iH} (2025 – 2023)

Tao: quantum information at colliders!

Me: Mmm, loop-holes, useful?



The consequences

Tao:

Quantum entanglement and Bell inequality violation in semi-leptonic top decays #24
Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.), Tong Arthur Wu (Pittsburgh U.) (Oct 26, 2023)
Published in: *JHEP* 07 (2024) 192 • e-Print: 2310.17696 [hep-ph]
pdf DOI cite claim reference search 89 citations

Optimizing fictitious states for Bell inequality violation in bipartite qubit systems with applications to the $t\bar{t}$ system #23
Kun Cheng (Peking U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.) (Nov 15, 2023)

Optimizing entanglement and Bell inequality violation in top antitop events #17 11 • e-Print: 2311.09166 [hep-ph]
Kun Cheng (Peking U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.) (Jul 1, 2024) aim reference search 50 citations
Published in: *Phys.Rev.D* 111 (2025) 3, 033004 • e-Print: 2407.01672 [hep-ph]
pdf DOI cite claim reference search 52 citations

Quantum Tomography of Fermion Pairs in e^+e^- Collisions: Longitudinal Beam Polarization Effects #2
Yu-Chen Guo (Liaoning Normal U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.), Youle Su (Pittsburgh U.) (Feb 2, 2026)
e-Print: 2602.02719 [hep-ph]
pdf cite claim reference search 13 citations

+ many more

The consequences

Tao:

Quantum entanglement and Bell inequality violation in semi-leptonic top decays #24
Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.), Tong Arthur Wu (Pittsburgh U.) (Oct 26, 2023)
Published in: *JHEP* 07 (2024) 192 • e-Print: [2310.17696](#) [hep-ph]
pdf DOI cite claim reference search 89 citations

Optimizing fictitious states for Bell inequality violation in bipartite qubit systems with applications to the $t\bar{t}$ system #23
Kun Cheng (Peking U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.) (Nov 15, 2023)

Optimizing entanglement and Bell inequality violation in top antitop events #17 11 • e-Print: [2311.09166](#) [hep-ph]
Kun Cheng (Peking U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.) (Jul 1, 2024) aim reference search 50 citations
Published in: *Phys.Rev.D* 111 (2025) 3, 033004 • e-Print: [2407.01672](#) [hep-ph]
pdf DOI cite claim reference search 52 citations

Quantum Tomography of Fermion Pairs in e^+e^- Collisions: Longitudinal Beam Polarization Effects #2
Yu-Chen Guo (Liaoning Normal U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.), Youle Su (Pittsburgh U.) (Feb 2, 2026)
e-Print: [2602.02719](#) [hep-ph]
pdf cite claim reference search 13 citations

+ many more

Decoherence in high energy collisions as renormalization group flow #5
Jiayin Gu (Fudan U. and Fudan U., Shanghai), Shi-Jia Lin (Fudan U. and Chicago U.), Ding Yu Shao (Fudan U. and Fudan U., Shanghai), Lian-Tao Wang (Chicago U. and Chicago U., EFI and Chicago U., KICP), Si-Xiang Yang (Fudan U. and Stanford U., Phys. Dept.) (Oct 15, 2025)
e-Print: [2510.13951](#) [hep-ph]
pdf cite claim reference search 14 citations

This time, I reacted faster

The consequences

Tao:

Quantum entanglement and Bell inequality violation in semi-leptonic top decays #24
Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.), Tong Arthur Wu (Pittsburgh U.) (Oct 26, 2023)
Published in: *JHEP* 07 (2024) 192 • e-Print: 2310.17696 [hep-ph]
pdf DOI cite claim reference search 89 citations

Optimizing fictitious states for Bell inequality violation in bipartite qubit systems with applications to the $t\bar{t}$ system #23
Kun Cheng (Peking U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.) (Nov 15, 2023)

Optimizing entanglement and Bell inequality violation in top antitop events #17 11 • e-Print: 2311.09166 [hep-ph]
Kun Cheng (Peking U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.) (Jul 1, 2024) aim reference search 50 citations
Published in: *Phys.Rev.D* 111 (2025) 3, 033004 • e-Print: 2407.01672 [hep-ph]
pdf DOI cite claim reference search 52 citations

Quantum Tomography of Fermion Pairs in e^+e^- Collisions: Longitudinal Beam Polarization Effects #2
Yu-Chen Guo (Liaoning Normal U. and Pittsburgh U.), Tao Han (Pittsburgh U.), Matthew Low (Pittsburgh U.), Youle Su (Pittsburgh U.) (Feb 2, 2026)
e-Print: 2602.02719 [hep-ph]
pdf cite claim reference search 13 citations

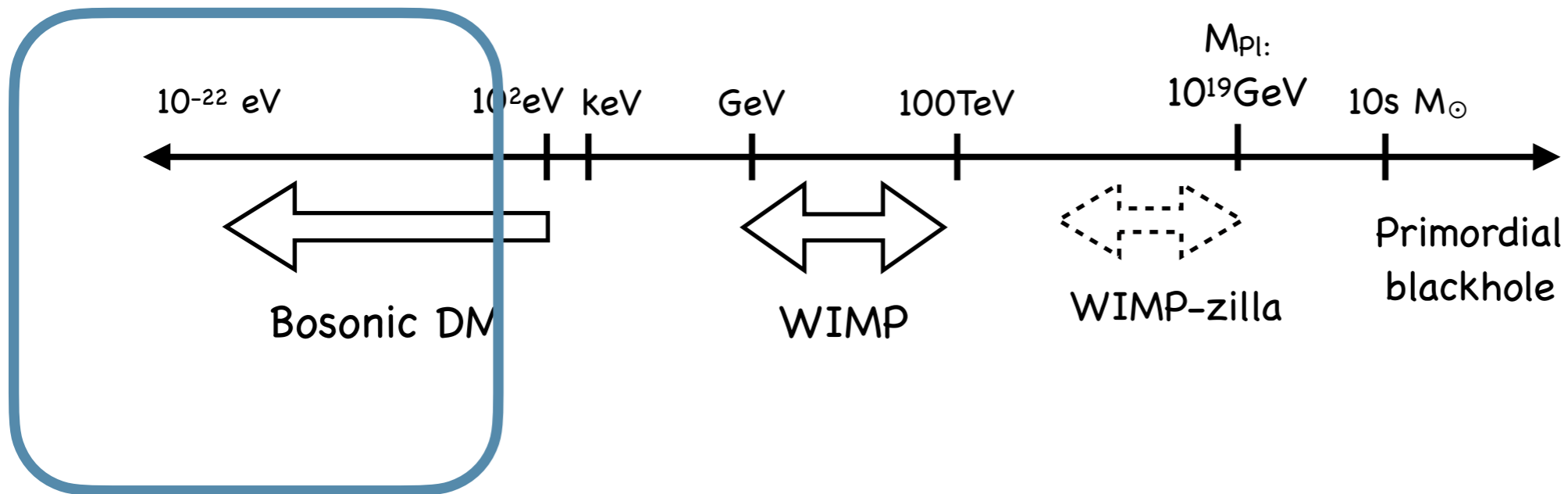
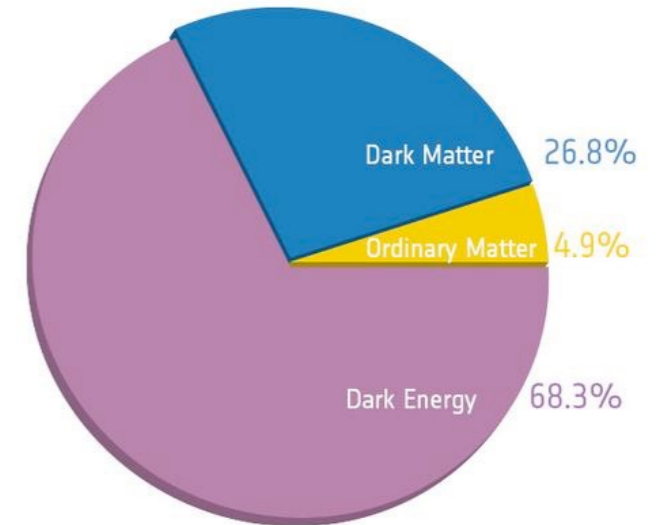
+ many more

Decoherence in high energy collisions as renormalization group flow #5
Jiayin Gu (Fudan U. and Fudan U., Shanghai), Shi-Jia Lin (Fudan U. and Chicago U.), Ding Yu Shao (Fudan U. and Fudan U., Shanghai), Lian-Tao Wang (Chicago U. and Chicago U., EFI and Chicago U., KICP), Si-Xiang Yang (Fudan U. and Stanford U., Phys. Dept.) (Oct 15, 2025)
e-Print: 2510.13951 [hep-ph]
pdf cite claim reference search 14 citations

This time, I reacted faster

But, I also went into a different quantum channel

Wave-like



Wave-like:

DM acts as a classical wave $\phi(t) \sim \frac{\sqrt{2\rho_{\text{DM}}}}{m_{\text{DM}}} \cos(m_{\text{DM}}t + \varphi)$

Quantum questions

- * What is the quantum state of dark matter?
 - * Lore: large occupation number \Rightarrow classical
 - * Coherent state?
 - * Fock state $|N\rangle$, with $N \gg 1$?
- * How “quantum” is it? Quantum vs classical?

Based on

D. Y. Cheong and N. L. Rodd, LTW 2408.08696

Y. Bao, D. Y. Cheong, N. L. Rodd, J. Takach, K. Zhou and LTW, 2510.05198, 2606.xxxx

The state

Density matrix

Using a coherent state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ as a basis

$$\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

Coherent state basis overcomplete, diagonal decomposition of $\hat{\rho}$

[Glauber-Sudharshan P - distribution function]

[Glauber, 1963]
[Sudharshan 1963]

$P(\alpha)$ for wave-like DM

State in the early universe depends on the production mechanism.
For example, could be in coherent state from misalignment.

$P(\alpha)$ for wave-like DM

State in the early universe depends on the production mechanism. For example, could be in coherent state from misalignment.

However, DM today went through a lot late time processing virtualization, galaxy merging, etc.

It is also an open system going through decoherence.

$P(\alpha)$ for wave-like DM

State in the early universe depends on the production mechanism. For example, could be in coherent state from misalignment.

However, DM today went through a lot late time processing virtualization, galaxy merging, etc.

It is also an open system going through decoherence.

Moreover, no experiment can resolve infinite finely, so we are necessarily coarse graining

$P(\alpha)$ for wave-like DM

State in the early universe depends on the production mechanism. For example, could be in coherent state from misalignment.

However, DM today went through a lot late time processing virtualization, galaxy merging, etc.

It is also an open system going through decoherence.

Moreover, no experiment can resolve infinite finely, so we are necessarily coarse graining

Fortunately, there is an universal distribution for “very” random system

$P(\alpha)$ for wave-like DM

Gaussian state:

$$P(\alpha) = \frac{1}{\pi \langle N \rangle} \exp \left[-\frac{|\alpha|^2}{\langle N \rangle} \right]$$

Quantum central limit theorem [Glauber, 1963]:

Superposition of n similar yet independent fields gives this state as $n \rightarrow \infty$

Basically, for something which is “random enough”, or has a lot of processing, it approaches Gaussian.

$P(\alpha)$ for wave-like DM

DM today went through a lot late time processing virialization, galaxy merging, etc. It is also an open system going through decoherence.

That is, random and a lot of processing

This motivates us to consider DM in a Gaussian state

$$P(\alpha) = \frac{1}{\pi \langle N \rangle} \exp \left[-\frac{|\alpha|^2}{\langle N \rangle} \right]$$

Applying this to DM halo

Dark matter field has many momentum modes

$$\hat{\rho} = \int d\{\alpha\} P(\{\alpha\}) |\{\alpha\}\rangle \langle \{\alpha\}| \quad |\{\alpha\}\rangle = \prod_{\mathbf{k}} |\alpha_{\mathbf{k}}\rangle$$

$$P(\{\alpha\}) = \prod_{\mathbf{k}} \frac{1}{\pi N_{\mathbf{k}}} e^{-|\alpha_{\mathbf{k}}|^2 / N_{\mathbf{k}}} \quad \mathbf{k}: \text{momentum of dark matter}$$

Applying this to DM halo

Dark matter field has many momentum modes

$$\hat{\rho} = \int d\{\alpha\} P(\{\alpha\}) |\{\alpha\}\rangle \langle \{\alpha\}| \quad |\{\alpha\}\rangle = \prod_{\mathbf{k}} |\alpha_{\mathbf{k}}\rangle$$

$$P(\{\alpha\}) = \prod_{\mathbf{k}} \frac{1}{\pi N_{\mathbf{k}}} e^{-|\alpha_{\mathbf{k}}|^2 / N_{\mathbf{k}}} \quad \mathbf{k}: \text{momentum of dark matter}$$

$$\langle N_{\mathbf{k}} \rangle = \text{Tr} \left[\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} \hat{\rho}_{\mathbf{k}} \right] \simeq \frac{(2\pi)^3}{g} \bar{n} p(\mathbf{k})$$

\bar{n} : average number density

$p(\mathbf{k})$: momentum space distribution, e.g. from a halo model

Wave-like: $\langle N_{\mathbf{k}} \rangle \gg 1$ Particle-like: $\langle N_{\mathbf{k}} \rangle \ll 1$

More on this later

Quantum and classical

Wave DM field

For $P(\alpha_{\mathbf{k}}) > 0$, $\phi(t, \mathbf{x})$ a classical random field.

For Gaussian $P(\alpha_{\mathbf{k}})$, $\phi(t, \mathbf{x})$ a Gaussian random field.

Wave DM field

For $P(\alpha_{\mathbf{k}}) > 0$, $\phi(t, \mathbf{x})$ a classical random field.

For Gaussian $P(\alpha_{\mathbf{k}})$, $\phi(t, \mathbf{x})$ a Gaussian random field.

This is the typical classical approximation used in the simulations.

We see it emerges in the classical limit of a quantum state.

Wave DM field

For $P(\alpha_{\mathbf{k}}) > 0$, $\phi(t, \mathbf{x})$ a classical random field.

For Gaussian $P(\alpha_{\mathbf{k}})$, $\phi(t, \mathbf{x})$ a Gaussian random field.

This is the typical classical approximation used in the simulations.

We see it emerges in the classical limit of a quantum state.

We can do the computation from the quantum state without classical approximation, and understands its applicability.

Wave DM field

Large occupation number limit: $\langle \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger \rangle = \langle \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} \rangle + 1 \simeq \langle N_{\mathbf{k}} \rangle \quad \langle N_{\mathbf{k}} \rangle \gg 1$

Then in any evaluation of $\langle \phi(t, \mathbf{x})^n \rangle$, we can make the replacement $\hat{a} \rightarrow \alpha, \hat{a} \rightarrow \alpha^*$

Field operator can be replaced by

$$\phi(t, \mathbf{x}) = \sum_{\mathbf{k}} \sqrt{\frac{2}{\mathcal{V}\omega_{\mathbf{k}}}} \operatorname{Re} [\alpha_{\mathbf{k}} e^{-ik \cdot x}]$$

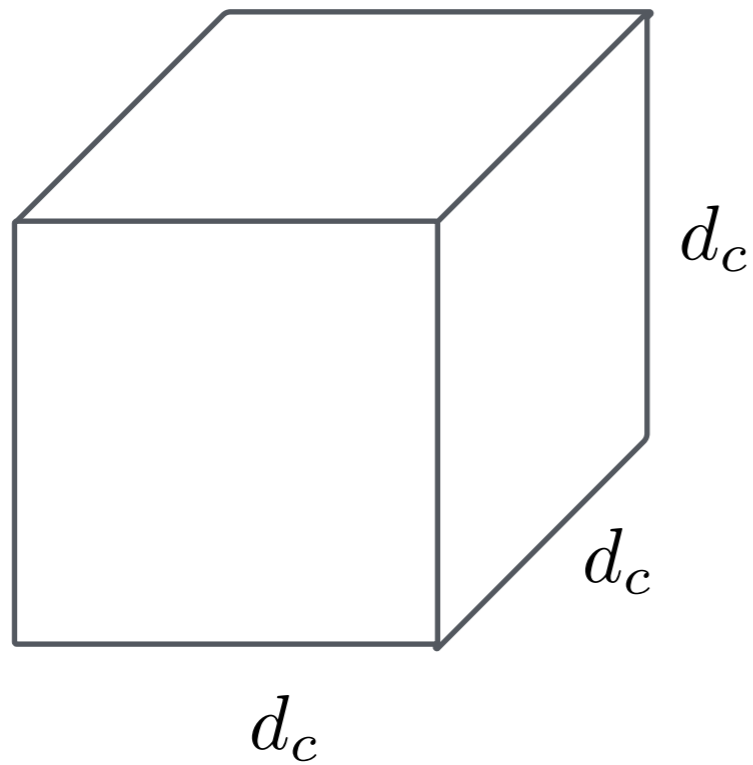
with each $\alpha_{\mathbf{k}}$ having a statistical weight $P(\alpha_{\mathbf{k}})$

Boundary between classical and quantum

Where is this boundary?

What's the transition look like?

Number in a box



Side = coherence length d_c

Volume \equiv coherence volume V_c

Wave-particle boundary

Number statistics

$$\mu_k = \langle k \rangle = \langle N \rangle$$
$$\sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle N \rangle(1 + \langle N \rangle)$$

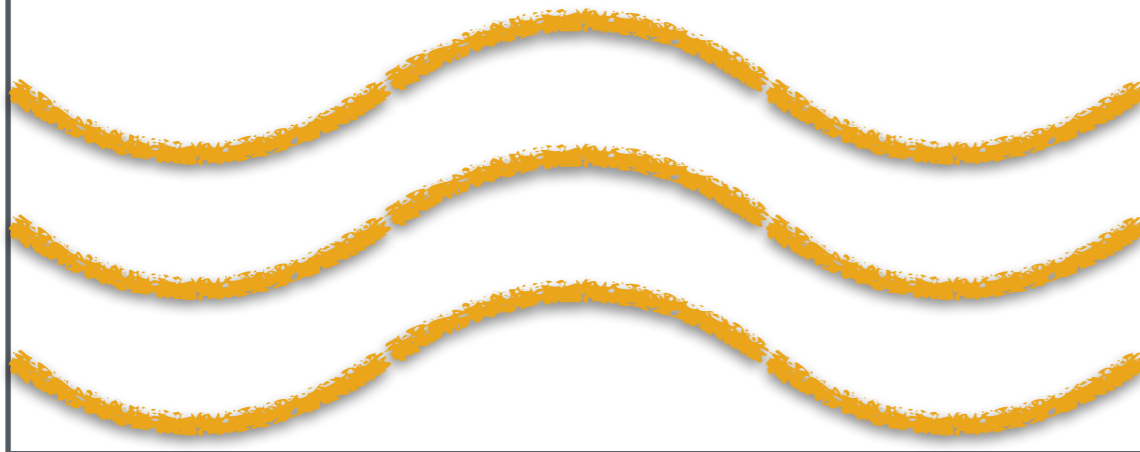
Wave-particle boundary

Number statistics

$$\mu_k = \langle k \rangle = \langle N \rangle$$

$$\sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle N \rangle (1 + \langle N \rangle)$$

$\langle N \rangle \gg 1, \sigma_k^2 \simeq N^2, \text{ exponential}$



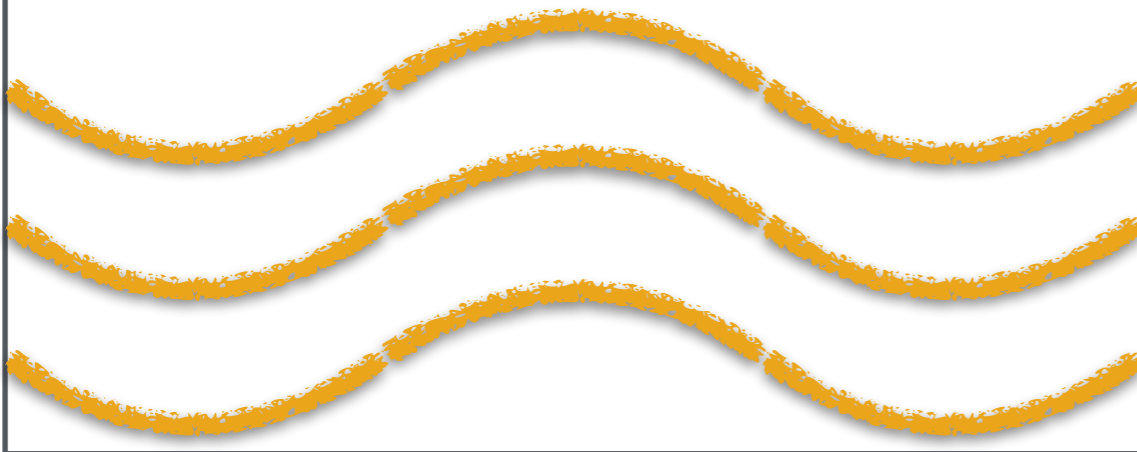
Wave-particle boundary

Number statistics

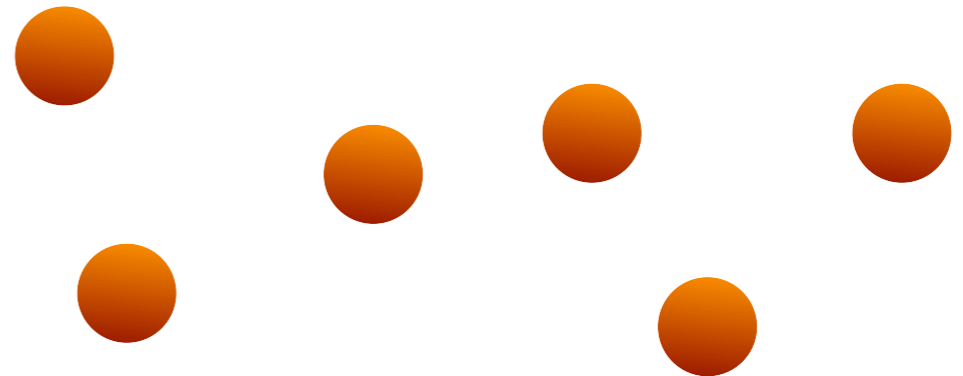
$$\mu_k = \langle k \rangle = \langle N \rangle$$

$$\sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle N \rangle (1 + \langle N \rangle)$$

$\langle N \rangle \gg 1$, $\sigma_k^2 \simeq N^2$, exponential



$\langle N \rangle \ll 1$, $\sigma_k^2 \simeq N = \mu_k$, Poisson



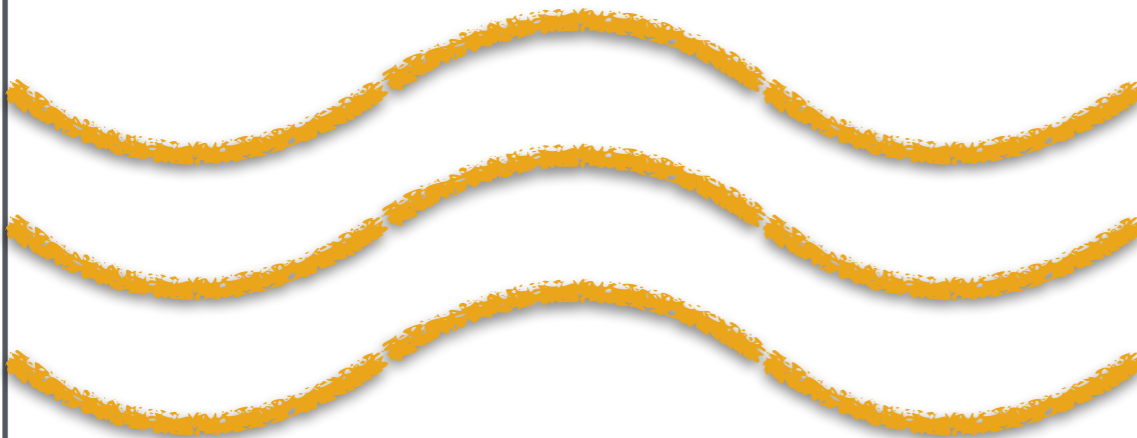
Wave-particle boundary

Number statistics

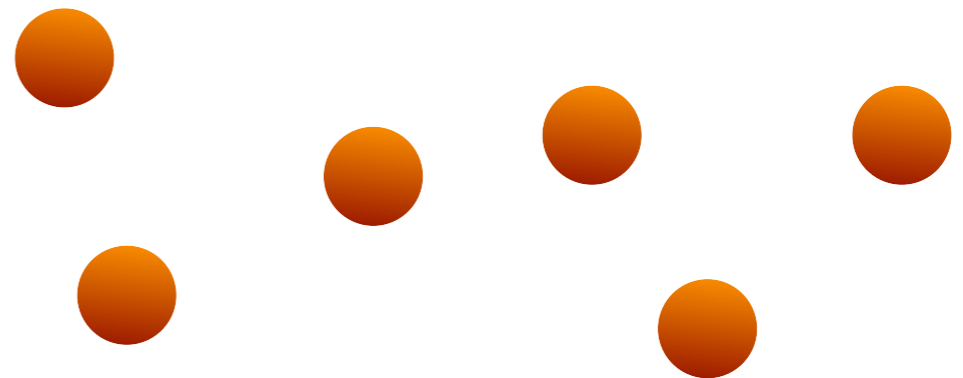
$$\mu_k = \langle k \rangle = \langle N \rangle$$

$$\sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle N \rangle (1 + \langle N \rangle)$$

$\langle N \rangle \gg 1$, $\sigma_k^2 \simeq N^2$, exponential



$\langle N \rangle \ll 1$, $\sigma_k^2 \simeq N = \mu_k$, Poisson



$\langle N \rangle \sim 1$ Neither Poisson or Exponential!

$$m \simeq \frac{18.7 \text{ eV}}{(g_s \langle N \rangle)^{1/4}}$$

Axion $\sim 18.7 \text{ eV}$

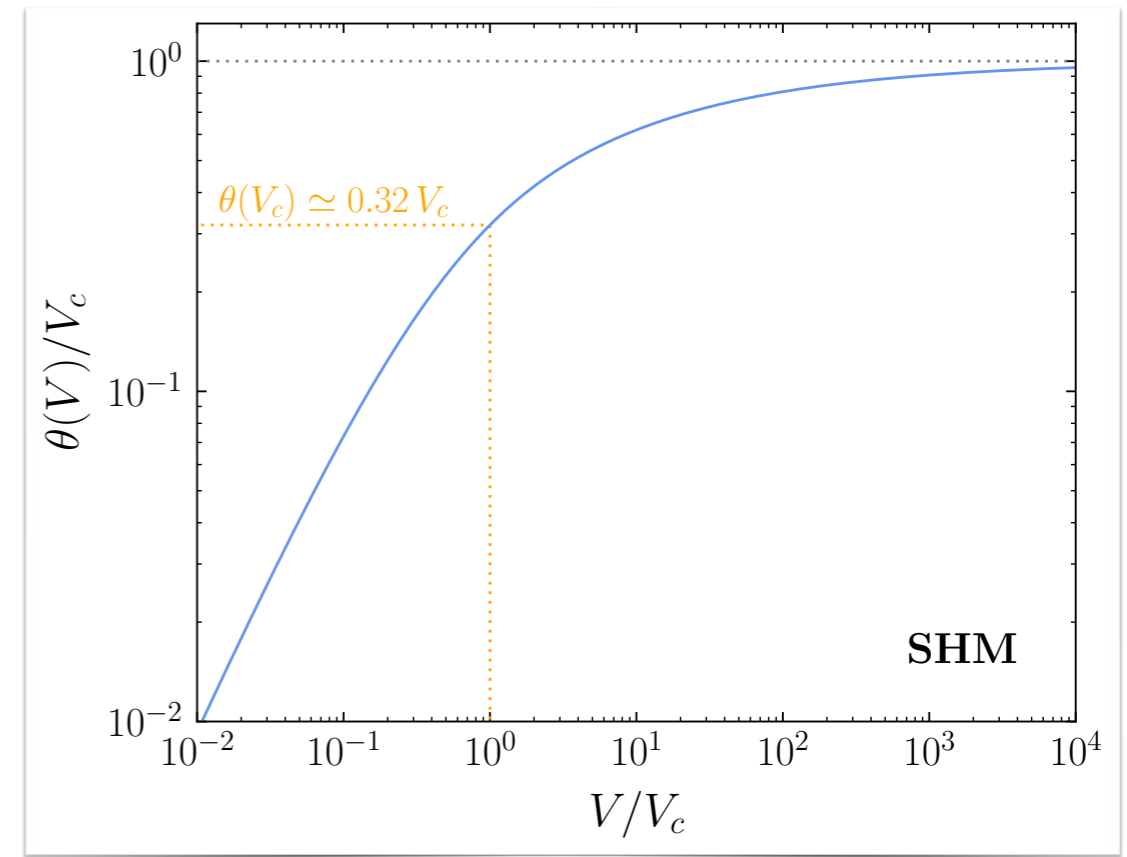
Dark Photon $\sim 14.2 \text{ eV}$

More general volume

$$V = L^3 \neq V_c$$

$$\mu_k = \bar{n}V, \quad \sigma_k^2 = \bar{n}V[\bar{n}\theta(V) + 1]$$

$$\theta(V) \simeq \begin{cases} V & V \ll V_c \\ 0.32V_c & V \sim V_c \\ V_c & V \gg V_c \end{cases}$$



$$\theta(V) = \int d^3\mathbf{d}T(\mathbf{d}) \left| g^{(1)}(0, \mathbf{d}) \right|^2$$

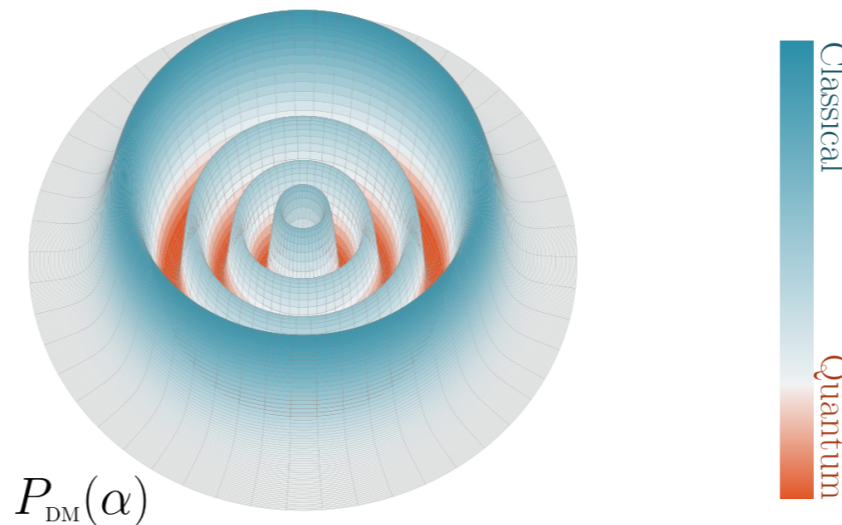
$$T(\mathbf{d}) \equiv V^{-1} (L - |d_x|) (L - |d_y|) (L - |d_z|)$$

Intrinsically quantum?

“quantum”: can not be mimicked by a stochastic classical signal

Need $P(\alpha) < 0$

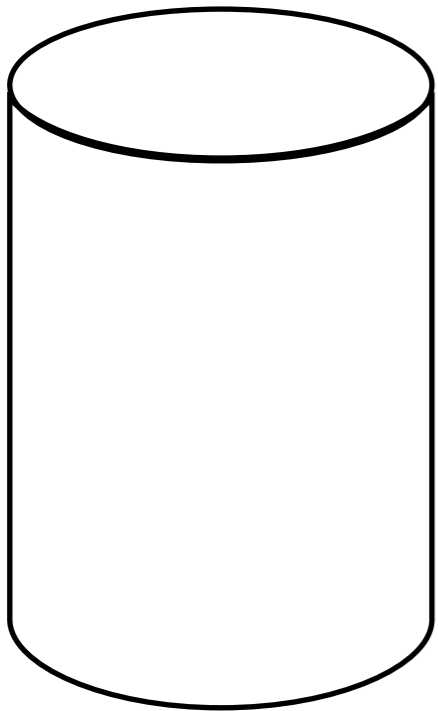
e.g. 7 quanta added
thermal state



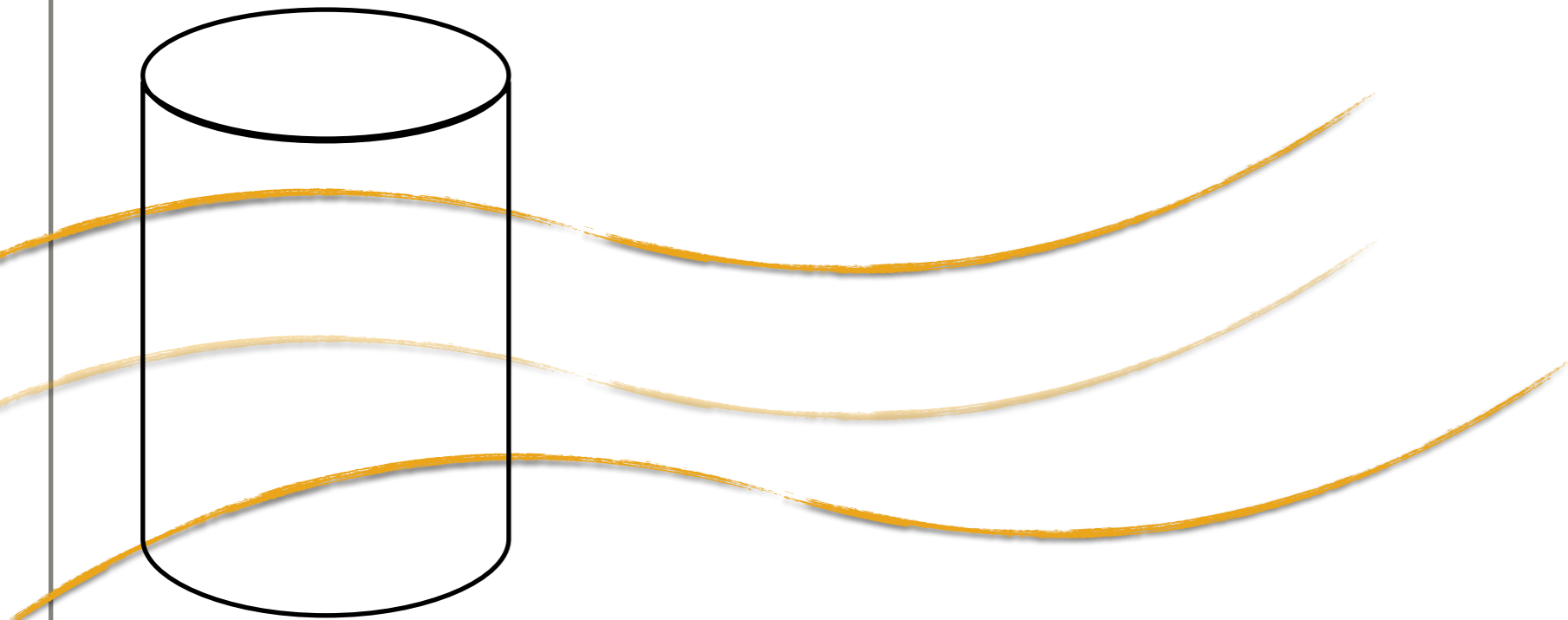
Similar question have been asked in “testing gravity is quantum”

An Axion Haloscope :

An Axion Haloscope :



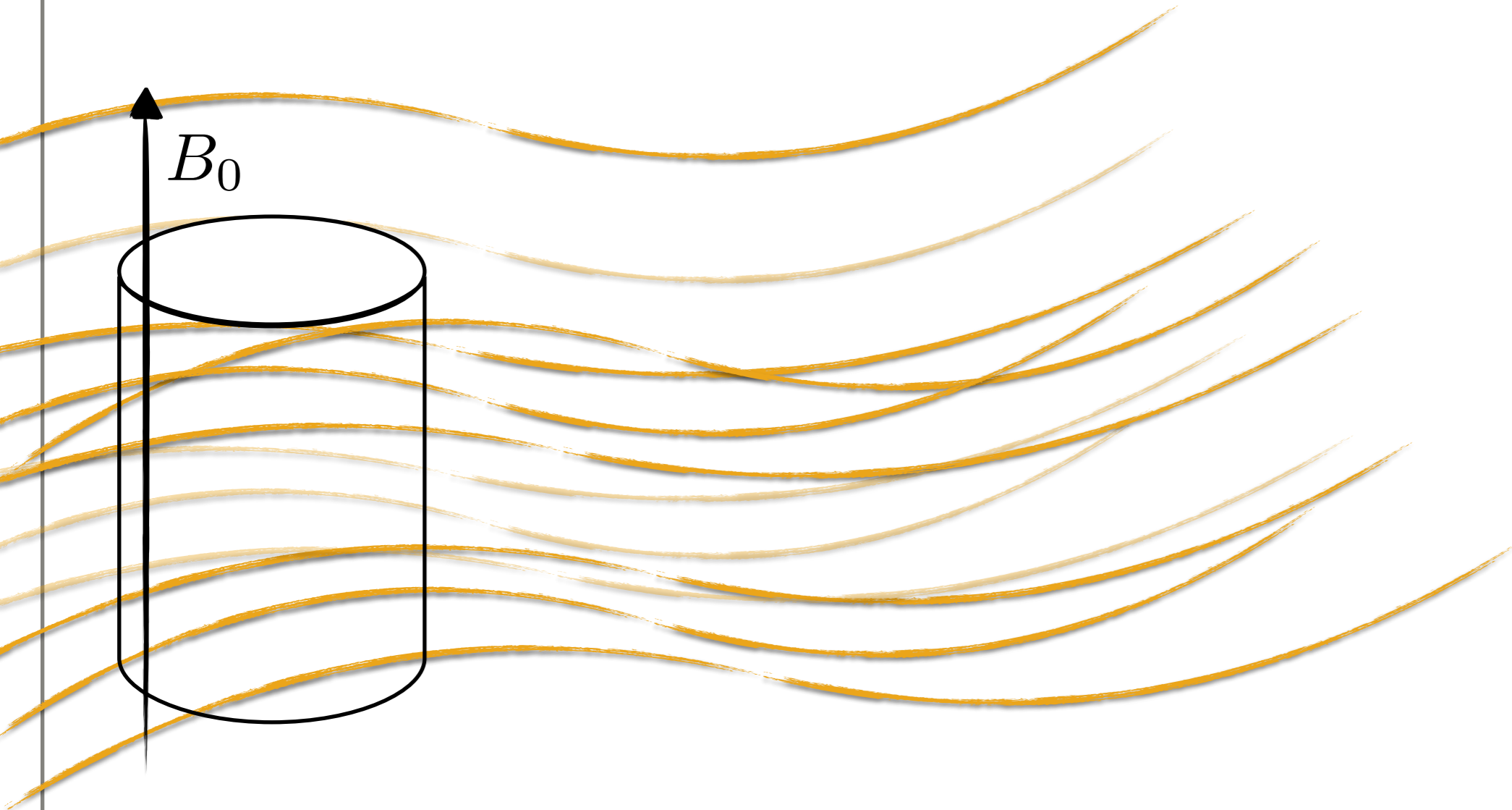
An Axion Haloscope :



An Axion Haloscope :



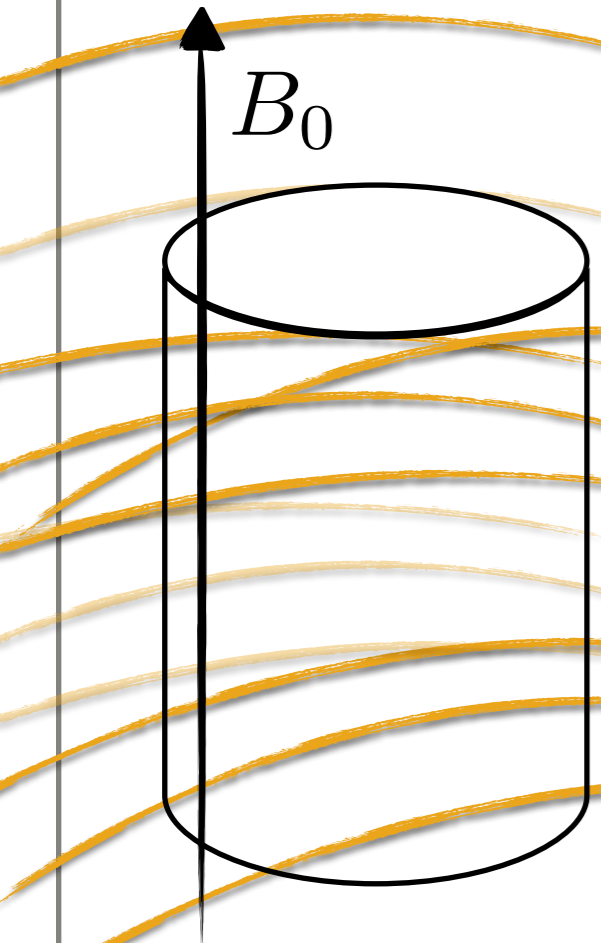
An Axion Haloscope :



An Axion Haloscope :

Axion - Photon Interaction

$$\mathcal{L} = -\frac{1}{4}g_{a\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{a\gamma\gamma}\phi\mathbf{E}\cdot\mathbf{B}$$



An Axion Haloscope :

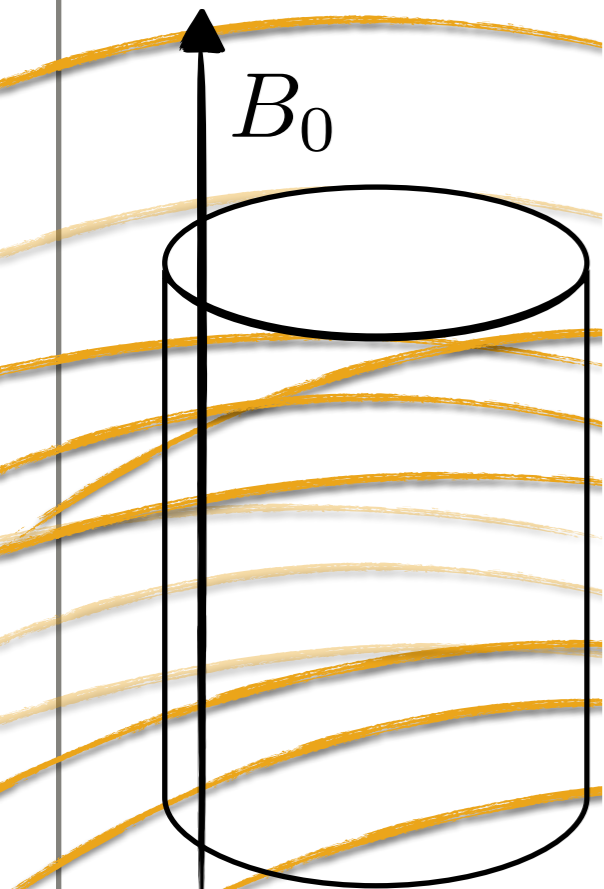
Axion - Photon Interaction

$$\mathcal{L} = -\frac{1}{4}g_{a\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{a\gamma\gamma}\phi\mathbf{E}\cdot\mathbf{B}$$

Imprint on the cavity state:

$$P_{\text{cav}}^f(\alpha) \simeq \int d^2\beta \frac{P_{\text{eff}}(\beta/\sqrt{\eta})}{\eta} P_{\text{cav}}^i(\alpha - \beta)$$

$$\eta \sim 10^{-22} \left(\frac{g_{a\gamma\gamma}}{10^{-15}\text{GeV}^{-1}} \frac{B_0}{10\text{ T}} \frac{Q_c}{10^5} \frac{10^{-5}\text{eV}}{m_{\text{DM}}} \right)^2 \quad \text{Very small detection rate}$$



Non-classical Effects on $P(\alpha)$

$$P_{\text{cav}}^f(\alpha) \simeq \int d^2\beta \frac{P_{\text{eff}}(\beta/\sqrt{\eta})}{\eta} P_{\text{cav}}^i(\alpha - \beta)$$

$$\eta \sim 10^{-22} \left(\frac{g_{a\gamma\gamma}}{10^{-15} \text{GeV}^{-1}} \frac{B_0}{10 \text{ T}} \frac{Q_c}{10^5} \frac{10^{-5} \text{eV}}{m_{\text{DM}}} \right)^2 \quad \text{Very small detection rate}$$

Regions of $P_{\text{cav}}^f(\alpha) < 0$ restricted to $\mathcal{O}(\sqrt{\eta})$ regions in α space.

Typically very small. Need extremely good sensitivity.

Imprints of non-classicality

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}} (b + b^\dagger)$

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}} (b + b^\dagger)$

Number

$$n = b^\dagger b$$

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}}(b + b^\dagger)$

$$S(X) = \text{Var}(X) - \frac{1}{2}$$

Number

$$n = b^\dagger b$$

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}}(b + b^\dagger)$

$$S(X) = \text{Var}(X) - \frac{1}{2}$$

Number $n = b^\dagger b$

$$Q = \frac{\text{Var}(n)}{\langle n \rangle} - 1,$$

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}}(b + b^\dagger)$

Number $n = b^\dagger b$

$$S(X) = \text{Var}(X) - \frac{1}{2}$$

$$Q = \frac{\text{Var}(n)}{\langle n \rangle} - 1,$$

Classical states : $Q, S > 0$,
Quantum states : $Q, S < 0$

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}}(b + b^\dagger)$

Number $n = b^\dagger b$

$$S(X) = \text{Var}(X) - \frac{1}{2}$$

$$Q = \frac{\text{Var}(n)}{\langle n \rangle} - 1,$$

Classical states : $Q, S > 0$,
Quantum states : $Q, S < 0$

$$S \geq -\frac{1}{2}$$

e.g. Infinitely squeezed state

Imprints of non-classicality

Quadrature $X = \frac{1}{\sqrt{2}}(b + b^\dagger)$

Number $n = b^\dagger b$

$$S(X) = \text{Var}(X) - \frac{1}{2}$$

$$Q = \frac{\text{Var}(n)}{\langle n \rangle} - 1,$$

Classical states : $Q, S > 0$,
Quantum states : $Q, S < 0$

$$S \geq -\frac{1}{2}$$

e.g. Infinitely squeezed state

$$Q \geq -1$$

e.g. number state

Suppression of non-classical effects

Quadrature	Number

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$	

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$ $S_{\text{cav}} = \eta S_{\text{DM}} \geq -\frac{\eta}{2}$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$ $Q_{\text{cav}} = \eta Q_{\text{DM}} \geq -\eta$

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$ $S_{\text{cav}} = \eta S_{\text{DM}} \geq -\frac{\eta}{2}$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$ $Q_{\text{cav}} = \eta Q_{\text{DM}} \geq -\eta$

Negativities at most $\mathcal{O}(\eta)$, even in vacuum cavity

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$ $S_{\text{cav}} = \eta S_{\text{DM}} \geq -\frac{\eta}{2}$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$ $Q_{\text{cav}} = \eta Q_{\text{DM}} \geq -\eta$

Negativities at most $\mathcal{O}(\eta)$, even in vacuum cavity

Time required to observe

$$S_{\text{cav}}, Q_{\text{cav}} < 0$$

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$ $S_{\text{cav}} = \eta S_{\text{DM}} \geq -\frac{\eta}{2}$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$ $Q_{\text{cav}} = \eta Q_{\text{DM}} \geq -\eta$

Negativities at most $\mathcal{O}(\eta)$, even in vacuum cavity

Time required to observe
 $S_{\text{cav}}, Q_{\text{cav}} < 0$ $t_{\text{int}} \sim \frac{t_m}{\eta^2}$

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$ $S_{\text{cav}} = \eta S_{\text{DM}} \geq -\frac{\eta}{2}$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$ $Q_{\text{cav}} = \eta Q_{\text{DM}} \geq -\eta$

Negativities at most $\mathcal{O}(\eta)$, even in vacuum cavity

Time required to observe
 $S_{\text{cav}}, Q_{\text{cav}} < 0$

$$t_{\text{int}} \sim \frac{t_m}{\eta^2} \sim 10^{26} \text{ years}$$

$$g_{a\gamma\gamma} = 10^{-14} \text{ GeV}^{-1}$$

$$B_0 = 10 \text{ T}$$

$$Q_c = 10^5$$

$$m_a = 10^{-5} \text{ eV}$$

Suppression of non-classical effects

Quadrature	Number
$p(x) = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle x \sqrt{\eta}\alpha \rangle ^2$ $S_{\text{cav}} = \eta S_{\text{DM}} \geq -\frac{\eta}{2}$	$p_n = \int d\alpha P_{\text{DM}}^{\text{eff}}(\alpha) \langle n \sqrt{\eta}\alpha \rangle ^2$ $Q_{\text{cav}} = \eta Q_{\text{DM}} \geq -\eta$

Negativities at most $\mathcal{O}(\eta)$, even in vacuum cavity

Time required to observe
 $S_{\text{cav}}, Q_{\text{cav}} < 0$

$$t_{\text{int}} \sim \frac{t_m}{\eta^2} \sim 10^{26} \text{ years}$$

$$g_{a\gamma\gamma} = 10^{-14} \text{ GeV}^{-1}$$

$$B_0 = 10 \text{ T}$$

$$Q_c = 10^5$$

$$m_a = 10^{-5} \text{ eV}$$

$$t_m \text{ Measurement interval} \sim \tau_{\text{DM}}$$

Back to Tao and I

t= present, Tao and I

Entangled.

I followed, learned.

Not quite maximally entangled.

Sometimes opposite but not quite.

Beyond description.

t= present, Tao and I

Entangled.

I followed, learned.

Not quite maximally entangled.

Sometimes opposite but not quite.

Beyond description.

Can not be simulated classically.

t=0, Tao and I

But it is magical for me!

$$\frac{1}{\sqrt{2}} \left[\left| \begin{array}{c} \text{[Portrait of John Nirenberg]} \\ \text{[Portrait of Terence Tao]} \end{array} \right\rangle + e^{i\frac{\pi}{4}} \left| \begin{array}{c} \text{[Portrait of Terence Tao]} \\ \text{[Portrait of John Nirenberg]} \end{array} \right\rangle \right]$$

e^{-iH} (... – now)

Hoping for much more to come!

e^{-iH} (... — now)

Hoping for much more to come!

And, congratulations and thank you!



extra

$P(\alpha)$ Distribution properties

$$\hat{\rho} = \int d^2\alpha \underbrace{P(\alpha)} | \alpha \rangle \langle \alpha |$$

System now characterized by a quasi-probability distribution



Properties of $P(\alpha)$

$$P(\alpha) \in \mathbb{R} \quad (\because \hat{\rho}^\dagger = \hat{\rho})$$

$$\int d^2\alpha P(\alpha) = 1 \quad (\because \text{Tr}[\hat{\rho}] = 1)$$

$P(\alpha) < 0$, more singular than a δ -function allowed

$$\text{e.g. } \hat{\rho} = |N\rangle\langle N| \quad P(\alpha) = \frac{e^{|\alpha|^2}}{N!} \partial_\alpha^N \partial_{\alpha^*}^N \delta^{(2)}(\alpha)$$

Density matrix

Density matrix

Using a coherent state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ as a basis

Density matrix

Using a coherent state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ as a basis

$$\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

Coherent state basis overcomplete, diagonal decomposition of $\hat{\rho}$

[Glauber-Sudarshan P - distribution function]

[Glauber, 1963]
[Sudarshan 1963]

Density matrix

Using a coherent state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ as a basis

$$\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

Coherent state basis overcomplete, diagonal decomposition of $\hat{\rho}$

[Glauber-Sudarshan P - distribution function]

[Glauber, 1963]
[Sudarshan 1963]

Unique $P(\alpha)$ for a generic $\hat{\rho}$ in coherent state basis

[Metha, 1967]

$$P(\alpha) = \int \frac{d^2\beta}{\pi^2} \langle -\beta|\hat{\rho}|\beta\rangle e^{|\alpha|^2 + |\beta|^2 + 2i \text{Im}[\alpha\beta^*]}$$

Density matrix

Using a coherent state $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ as a basis

$$\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

Coherent state basis overcomplete, diagonal decomposition of $\hat{\rho}$

[Glauber-Sudarshan P - distribution function]

[Glauber, 1963]
[Sudarshan 1963]

Unique $P(\alpha)$ for a generic $\hat{\rho}$ in coherent state basis

[Metha, 1967]

$$P(\alpha) = \int \frac{d^2\beta}{\pi^2} \langle -\beta|\hat{\rho}|\beta\rangle e^{|\alpha|^2 + |\beta|^2 + 2i \text{Im}[\alpha\beta^*]}$$

Expectation values (ensemble averages now described with $P(\alpha)$)

$$\langle\hat{O}\rangle = \text{Tr} [\hat{O}\hat{\rho}] = \int d^2\alpha P(\alpha) \text{Tr} [\hat{O}|\alpha\rangle\langle\alpha|] = \int d^2\alpha P(\alpha)\langle\alpha|\hat{O}|\alpha\rangle$$

Higher order coherence?

Coherence properties also important in higher order correlations.

Glauber correlation functions for higher order quantum coherence

[Glauber, 1963]

$$\hat{\phi}^+(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}} \hat{a}_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$g^{(1)}(x_1, x_2) = \frac{\langle \hat{\phi}^-(x_1) \hat{\phi}^+(x_2) \rangle}{\sqrt{\langle \hat{\phi}^-(x_1) \hat{\phi}^+(x_1) \rangle \langle \hat{\phi}^-(x_2) \hat{\phi}^+(x_2) \rangle}} \quad g^{(2)}(x_1, x_2) = \frac{\langle \hat{\phi}^-(x_1) \hat{\phi}^-(x_2) \hat{\phi}^+(x_2) \hat{\phi}^+(x_1) \rangle}{\langle \hat{\phi}^-(x_1) \hat{\phi}^+(x_1) \rangle \langle \hat{\phi}^-(x_2) \hat{\phi}^+(x_2) \rangle}$$

Inserting the Gaussian $P(\alpha)$, and the occupation number $\langle N_{\mathbf{k}} \rangle$,

$$g^{(1)}(\tau, \mathbf{d}) = \frac{1}{\langle 1/\omega \rangle} \int d^3\mathbf{k} \frac{p(\mathbf{k})}{\omega_{\mathbf{k}}} e^{-i\mathbf{k}\cdot\mathbf{d}} \quad g^{(2)}(\tau, \mathbf{d}) = 1 + \left| g^{(1)}(\tau, \mathbf{d}) \right|^2$$

Quantum central limit theorem

Glauber, 1963

Number of modes: $\mathcal{N} \sim (m_{\text{DM}} v_{\text{DM}} R)^3$ R: halo radius

Coarse graining Δp : $\mathcal{N} \sim (m_{\text{DM}} v_{\text{DM}} / \Delta p)^3$

Details require numerical simulation.

Wave-particle boundary

Useful to rewrite the Gaussian state in Fock basis

$$\hat{\rho} = \int d^2\alpha \frac{e^{-|\alpha|^2/\langle N \rangle}}{\pi \langle N \rangle} |\alpha\rangle \langle \alpha| = \frac{1}{1 + \langle N \rangle} \sum_{k=0}^{\infty} \left(\frac{\langle N \rangle}{1 + \langle N \rangle} \right)^k |k\rangle \langle k|$$

$|k\rangle$: Fock state with occupation number k , and weight

$$p_k = \frac{\langle N \rangle^k}{(1 + \langle N \rangle)^{k+1}}$$

Number statistics

Mean: $\mu_k = \langle k \rangle = \langle N \rangle$

Variance: $\sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle N \rangle (1 + \langle N \rangle)$

Cavity state evolution

$$H_{\text{int}}(t) \simeq ig (c^\dagger a_{\text{eff}}(t) - c a_{\text{eff}}^\dagger(t))$$

Solve evolution

$$e^{iH_{\text{int}}t} c e^{-iH_{\text{int}}t} = c \cos gt + a_{\text{eff}} \sin gt$$

Cavity state evolution

$$H_{\text{int}}(t) \simeq ig (c^\dagger a_{\text{eff}}(t) - c a_{\text{eff}}^\dagger(t))$$

Solve evolution

$$e^{iH_{\text{int}}t} c e^{-iH_{\text{int}}t} = c \cos gt + a_{\text{eff}} \sin gt$$

Evolve $\rho(t)$



Obtain Effective \mathcal{P}

Cavity state evolution

$$H_{\text{int}}(t) \simeq ig (c^\dagger a_{\text{eff}}(t) - c a_{\text{eff}}^\dagger(t))$$

Solve evolution

$$e^{iH_{\text{int}}t} c e^{-iH_{\text{int}}t} = c \cos gt + a_{\text{eff}} \sin gt$$

Evolve $\rho(t)$



Obtain Effective P

$$P_{\text{cav}}^f(\alpha) \simeq \int d^2\beta \frac{P_{\text{eff}}(\beta/\sqrt{\eta})}{\eta} P_{\text{cav}}^i(\alpha - \beta)$$

Cavity state evolution

$$H_{\text{int}}(t) \simeq ig (c^\dagger a_{\text{eff}}(t) - c a_{\text{eff}}^\dagger(t))$$

Solve evolution

$$e^{iH_{\text{int}}t} c e^{-iH_{\text{int}}t} = c \cos gt + a_{\text{eff}} \sin gt$$

Evolve $\rho(t)$



Obtain Effective P

$$P_{\text{cav}}^f(\alpha) \simeq \int d^2\beta \frac{P_{\text{eff}}(\beta/\sqrt{\eta})}{\eta} P_{\text{cav}}^i(\alpha - \beta)$$

$$\eta \sim 10^{-22} \left(\frac{g_{a\gamma\gamma}}{10^{-15} \text{GeV}^{-1}} \frac{B_0}{10 \text{ T}} \frac{Q_c}{10^5} \frac{10^{-5} \text{eV}}{m_{\text{DM}}} \right)^2$$

Cavity state evolution

$$H_{\text{int}}(t) \simeq ig (c^\dagger a_{\text{eff}}(t) - c a_{\text{eff}}^\dagger(t))$$

Solve evolution

$$e^{iH_{\text{int}}t} c e^{-iH_{\text{int}}t} = c \cos gt + a_{\text{eff}} \sin gt$$

Evolve $\rho(t)$



Obtain Effective P

$$P_{\text{cav}}^f(\alpha) \simeq \int d^2\beta \frac{P_{\text{eff}}(\beta/\sqrt{\eta})}{\eta} P_{\text{cav}}^i(\alpha - \beta)$$

$$\eta \sim 10^{-22} \left(\frac{g_{a\gamma\gamma}}{10^{-15} \text{GeV}^{-1}} \frac{B_0}{10 \text{ T}} \frac{Q_c}{10^5} \frac{10^{-5} \text{eV}}{m_{\text{DM}}} \right)^2$$

Very small
detection rate

Axion haloscope: effective mode

$$H_{\text{int}}(t) \simeq \frac{1}{2} g_{a\gamma\gamma} B_0 \sqrt{\frac{V_c}{\mathcal{V}}} \left(ic^\dagger \sum_{\mathbf{p}} C_{\mathbf{p}} e^{-iK_p t} a_{\mathbf{p}} + \text{h.c.} \right)$$

$$\omega_p = m_{\text{DM}} + K_p$$

$$C_{\mathbf{p}} = \sqrt{\frac{m_{\text{DM}}}{\omega_p V_c}} \int_{V_c} d^3 \mathbf{x} e^{i\mathbf{p} \cdot \mathbf{x}} \tilde{E}_z(\mathbf{x})$$

Effective (single) mode:

$$a_{\text{eff}}(t) \equiv \frac{1}{2} \sqrt{\frac{V_c}{\Omega \mathcal{V}}} \sum_{\mathbf{p}} C_{\mathbf{p}} e^{-iK_p t} a_{\mathbf{p}}$$

Within coherence time $\tau_{\text{DM}} \sim 1/\Delta\omega_{\text{DM}}$, $e^{-iK_p t} \sim \text{constant}$

$$\hat{\rho}_{\text{DM}}^{\text{eff}} = \int d\boldsymbol{\alpha} P_{\text{DM}}(\boldsymbol{\alpha}) |\alpha_{\text{eff}}(t_0)\rangle \langle \alpha_{\text{eff}}(t_0)| \quad a_{\text{eff}}(t_0)|\boldsymbol{\alpha}\rangle = \alpha_{\text{eff}}(t_0)|\boldsymbol{\alpha}\rangle$$

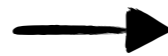
What about distinguishing states?

G : Gaussian C : Coherent

$$S_{\text{DM}}^G \simeq Q_{\text{DM}}^G \simeq N_{\text{eff}}$$

$$S_{\text{cav}}^G \simeq Q_{\text{cav}}^G \simeq \eta N_{\text{eff}} = n_s$$

$$\begin{array}{c} \mathbf{v} \\ S_{\text{DM}}^C = Q_{\text{DM}}^C = 0 \end{array}$$



$$\begin{array}{c} \mathbf{v} \\ S_{\text{cav}}^C = Q_{\text{cav}}^C = 0 \end{array}$$

Time required to distinguish :

$$t_{\text{int}} \sim \frac{t_m}{n_s^2} \sim 0.06 \left(\frac{g_{a\gamma\gamma}}{10^{-14} \text{ GeV}} \right)^{-4} \text{ yrs}$$

Feasible*

Correlation function

How correlated are the field in space and time?

$$\langle \Gamma(\tau, \mathbf{d}) \rangle = \langle \phi(t, \mathbf{x}) \phi(t + \tau, \mathbf{x} + \mathbf{d}) \rangle$$

Correlation function

How correlated are the field in space and time?

$$\langle \Gamma(\tau, \mathbf{d}) \rangle = \langle \phi(t, \mathbf{x}) \phi(t + \tau, \mathbf{x} + \mathbf{d}) \rangle$$

With Gaussian state:

$$\begin{aligned} \langle \Gamma(\tau, \mathbf{d}) \rangle &= \bar{n} \int d^3 \mathbf{k} \frac{p(\mathbf{k})}{\omega_{\mathbf{k}}} \cos(\omega_{\mathbf{k}} \tau - \mathbf{k} \cdot \mathbf{d}) \\ &\simeq \frac{\bar{n}}{m} \int d^3 \mathbf{v} f(\mathbf{v}) \cos\left(m \left(1 + \frac{v^2}{2}\right) \tau - m \mathbf{v} \cdot \mathbf{d}\right) \end{aligned}$$

Correlation function

How correlated are the field in space and time?

$$\langle \Gamma(\tau, \mathbf{d}) \rangle = \langle \phi(t, \mathbf{x}) \phi(t + \tau, \mathbf{x} + \mathbf{d}) \rangle$$

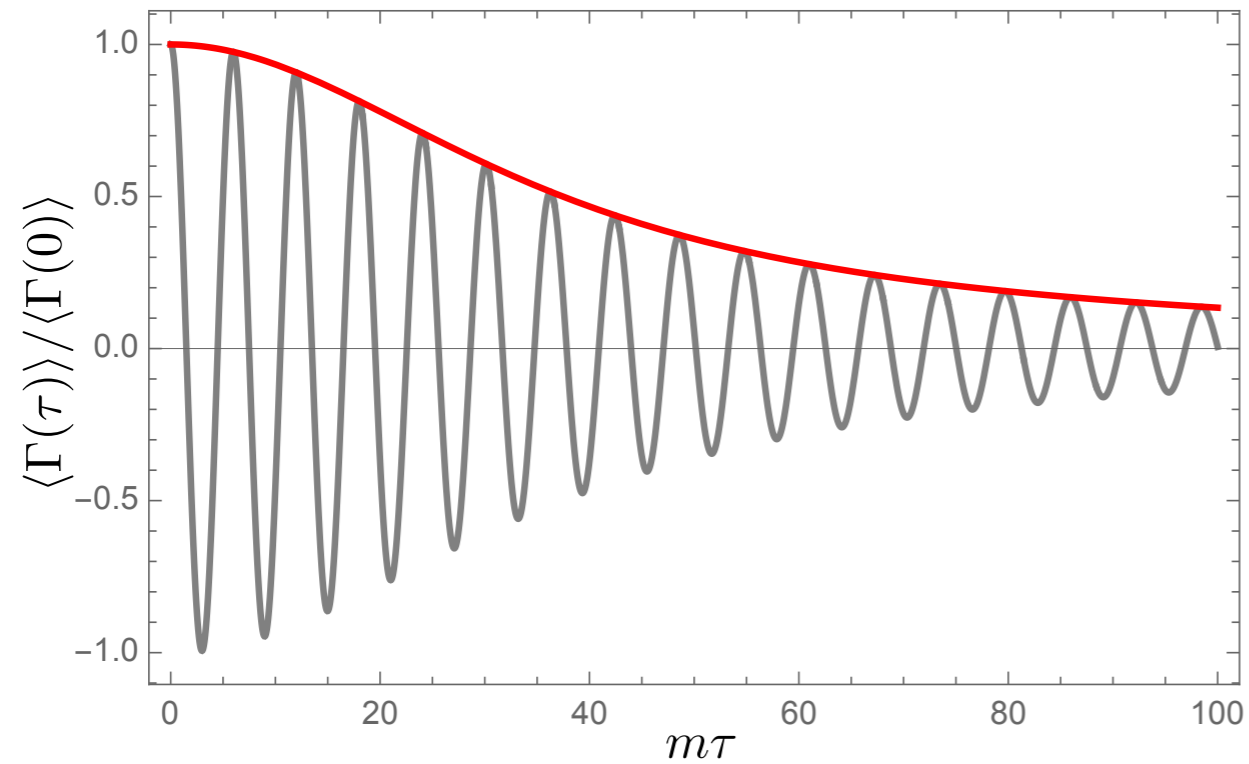
With Gaussian state:

$$\begin{aligned} \langle \Gamma(\tau, \mathbf{d}) \rangle &= \bar{n} \int d^3 \mathbf{k} \frac{p(\mathbf{k})}{\omega_{\mathbf{k}}} \cos(\omega_{\mathbf{k}} \tau - \mathbf{k} \cdot \mathbf{d}) \\ &\simeq \frac{\bar{n}}{m} \int d^3 \mathbf{v} f(\mathbf{v}) \cos\left(m \left(1 + \frac{v^2}{2}\right) \tau - m \mathbf{v} \cdot \mathbf{d}\right) \end{aligned}$$

e.g. The Standard Halo Model (Maxwell-Boltzmann)

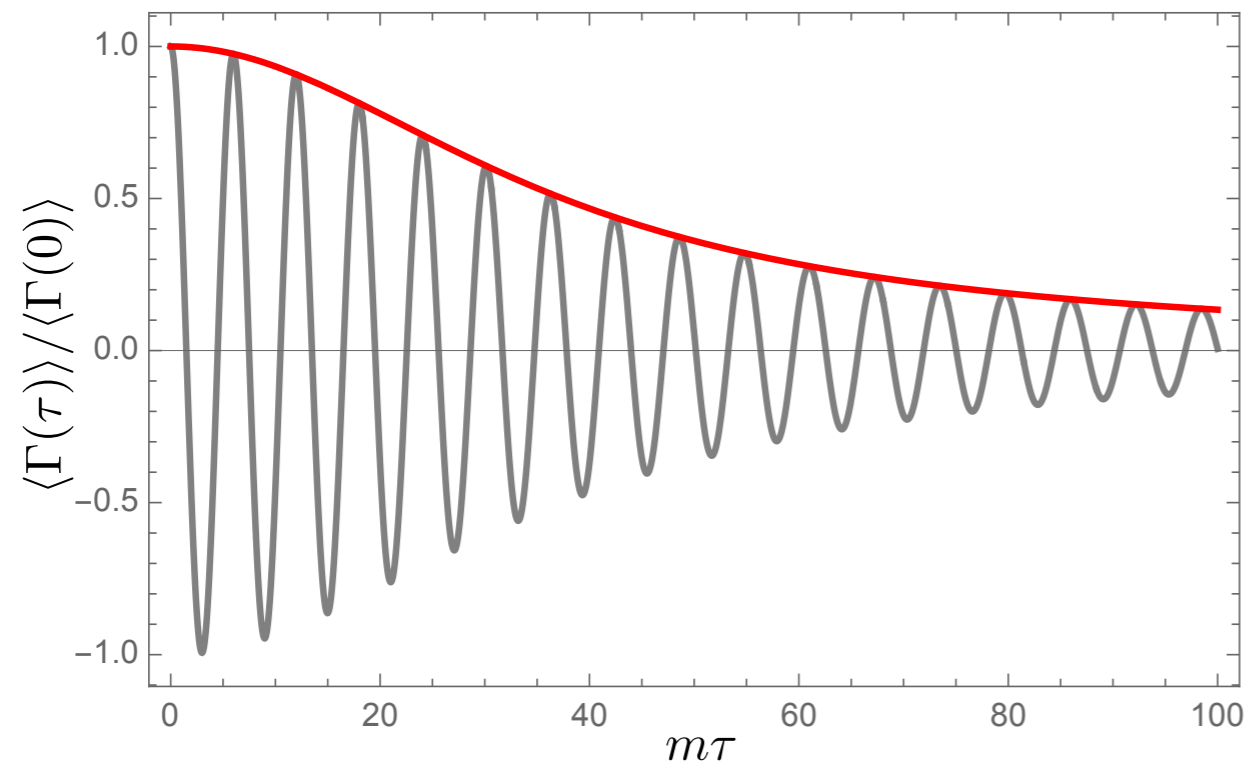
$$f(\mathbf{v}) = \frac{1}{\pi^{3/2} v_0^3} \exp\left[-\frac{(\mathbf{v} + \mathbf{v}_\odot)^2}{v_0^2}\right]$$

Coherence time



Exponential decay:
a time scale at which the field
is “coherent”.

Coherence time



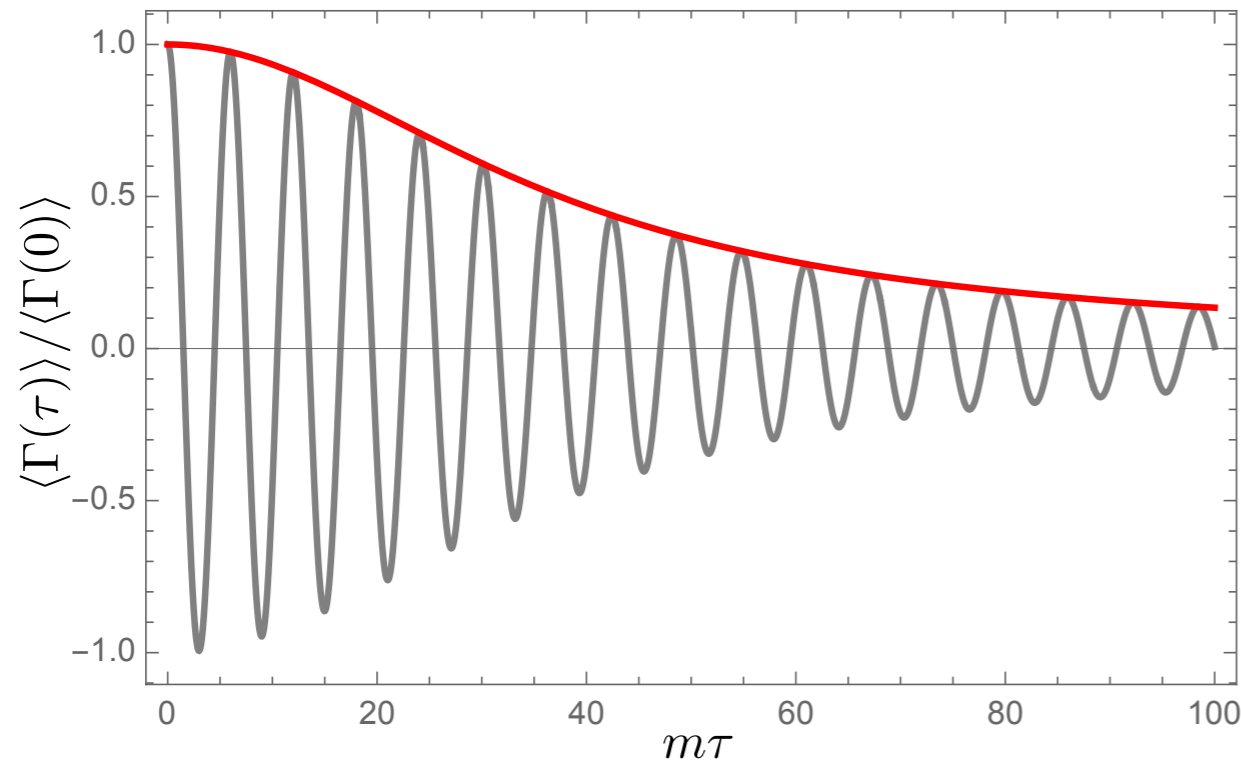
Exponential decay:
a time scale at which the field
is “coherent”.

Within this coherent time, the field can be approximated by

$$\phi(t) \sim \frac{\sqrt{2\rho_{\text{DM}}}}{m_{\text{DM}}} \cos(m_{\text{DM}}t + \varphi)$$

Beyond coherence time, DM field not correlated. Can be approximated by a random phase φ

Coherence time, a rigorous definition



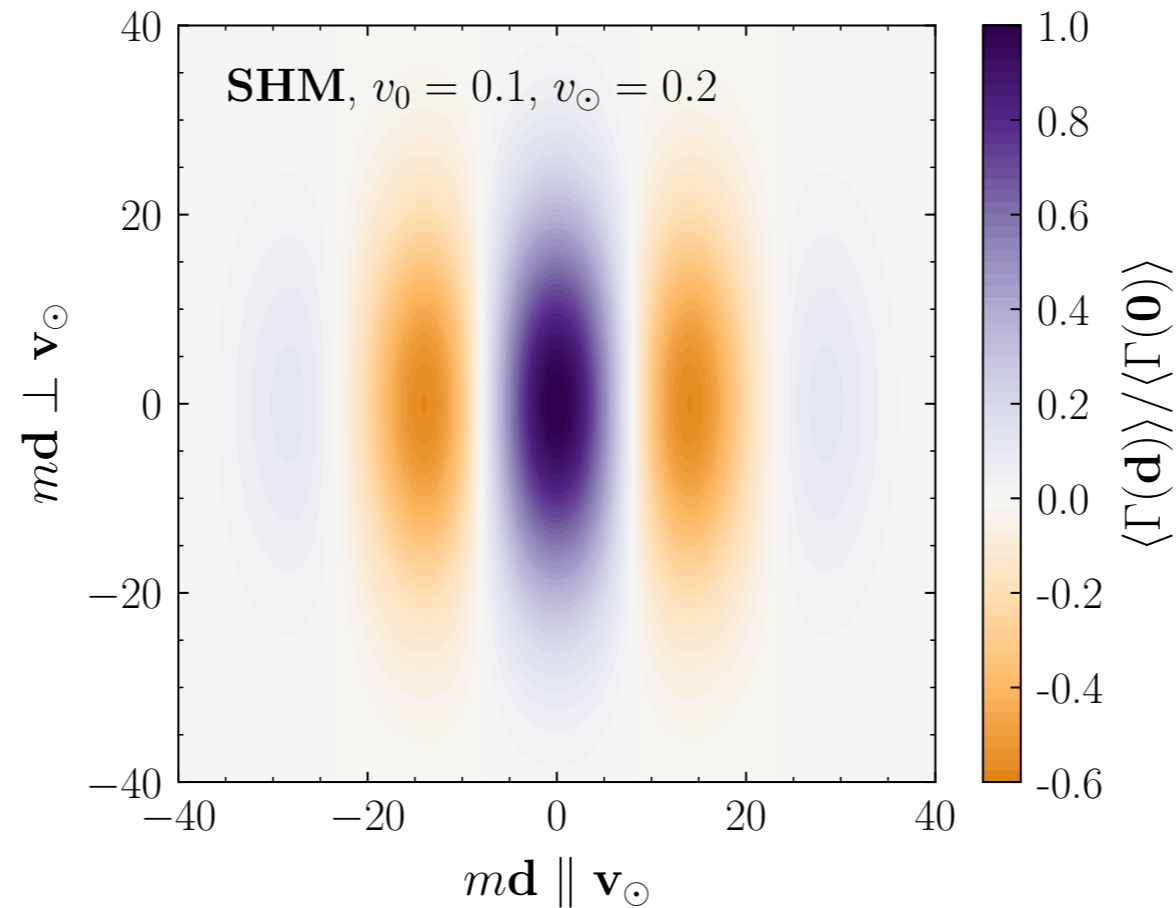
A time scale at which the field is “coherent”.

$$\tau_c = 2 \int d\tau \left[\frac{\langle \Gamma(\tau) \rangle}{\langle \Gamma(0) \rangle} \right]^2 = \frac{2\pi}{\langle 1/\omega \rangle^2} \int \frac{d\omega}{\omega^2} p(\omega)^2 = \frac{2\pi}{\omega_c}$$

DM - SHM

$$\tau_{\text{DM}} \simeq \frac{\sqrt{2\pi} \text{Erf} [\sqrt{2}v_{\odot}/v_0]}{mv_0v_{\odot}} [1 + \mathcal{O}(v^2)] \simeq 2.80\text{ms} \left(\frac{1\mu\text{eV}}{m} \right)$$

Coherence Volume / Length



$$\langle \Gamma(\mathbf{d}) \rangle = \langle \phi(\mathbf{x})\phi(\mathbf{x} + \mathbf{d}) \rangle :$$

$$V_c \simeq 2 \int_{-\infty}^{\infty} d^3\mathbf{d} \left[\frac{\langle \Gamma(\mathbf{d}) \rangle}{\langle \Gamma(\mathbf{0}) \rangle} \right]^2$$

DM - SHM

$$V_c = \frac{(2\pi)^3}{\langle 1/\omega \rangle^2} \int \frac{d^3\mathbf{k}}{\omega_{\mathbf{k}}^2} p(\mathbf{k})^2 \simeq \left(\frac{\sqrt{2\pi}}{mv_0} \right)^3 d_{\text{DM}} \simeq \frac{\sqrt{2\pi}}{mv_0} \simeq 674 \text{ m} \left(\frac{1\mu\text{eV}}{m} \right)$$