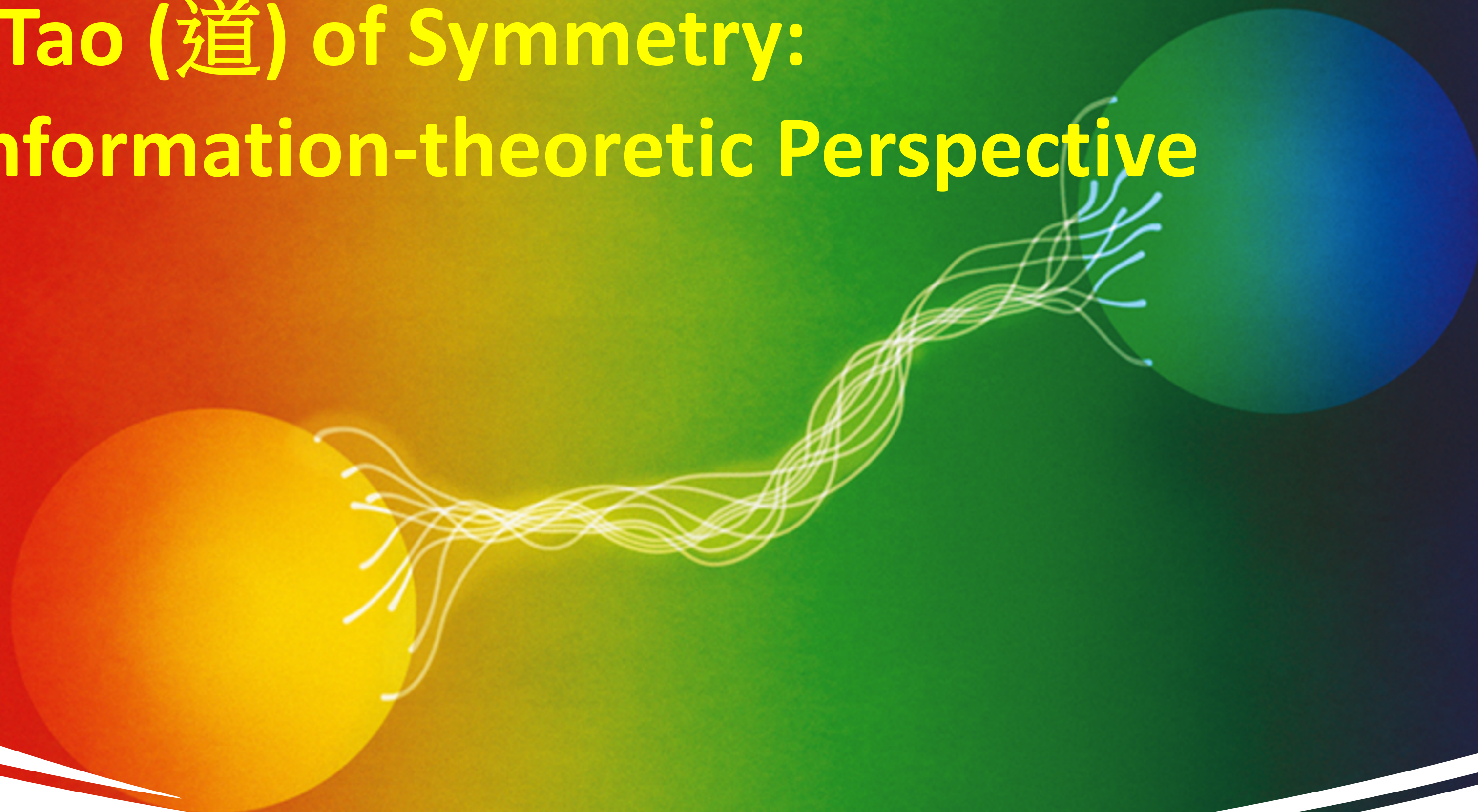
The Tao (道) of Symmetry: an Information-theoretic Perspective



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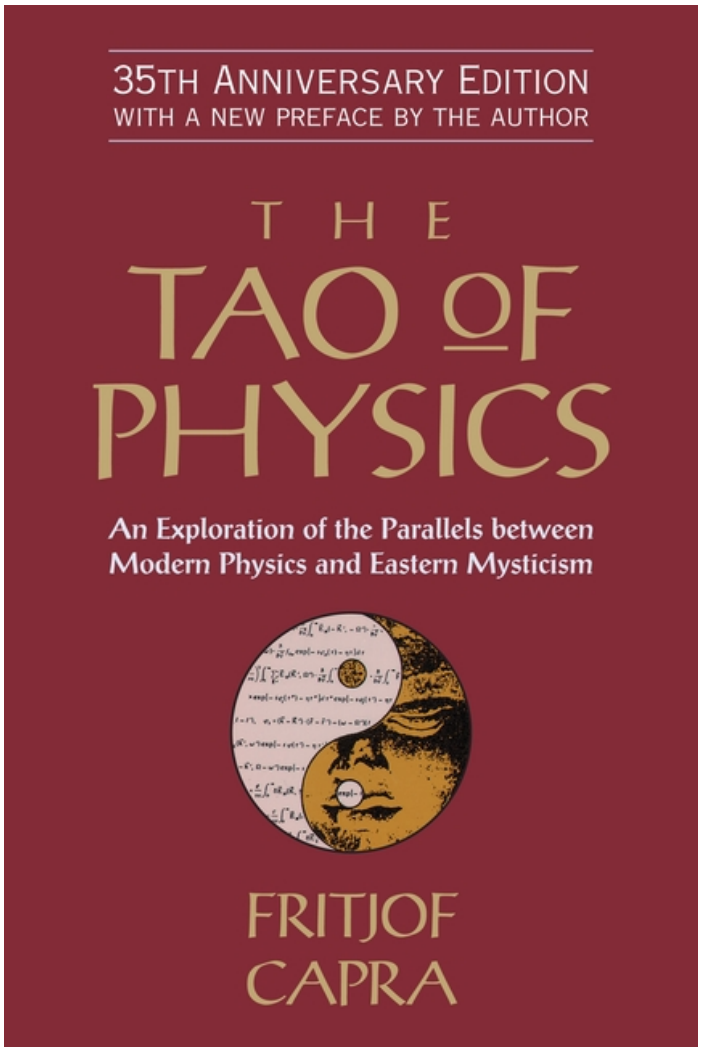
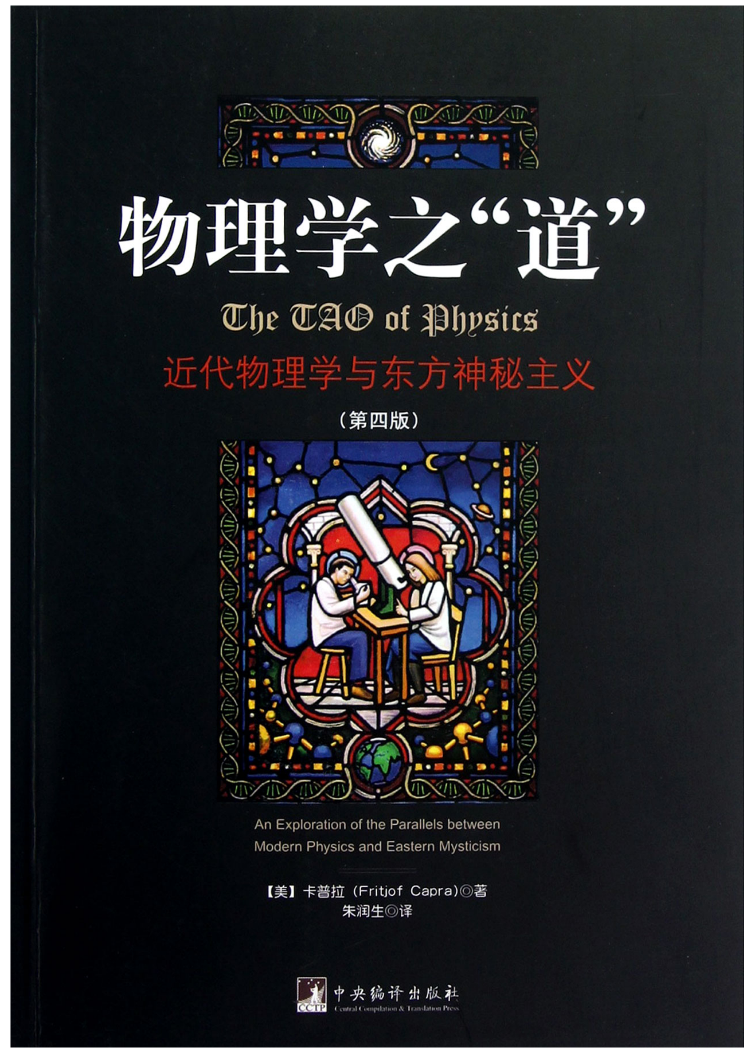
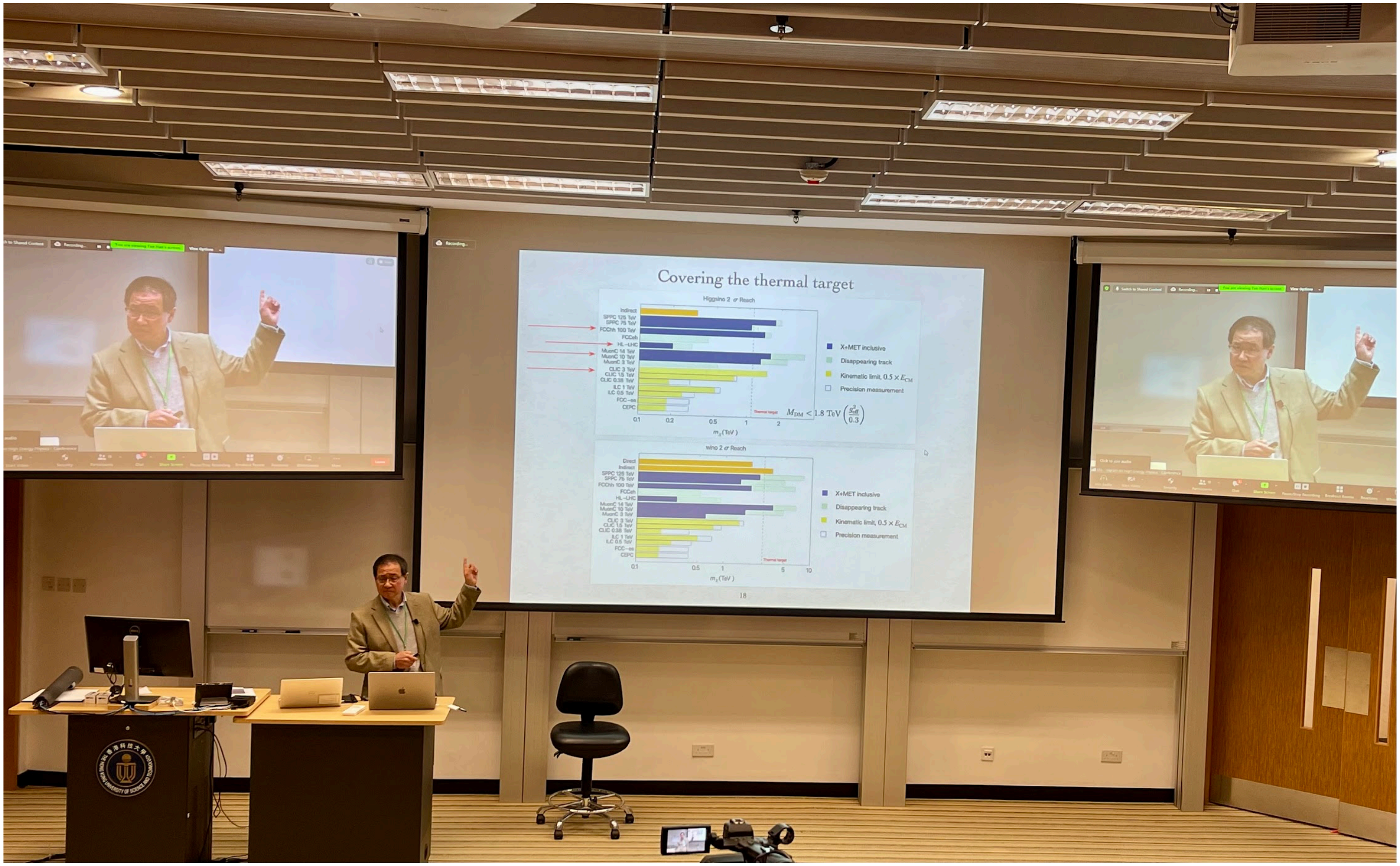
May 15, 2026

Tao(道): “the Way,” but not just a road or method. It refers to the underlying order, source, and natural process of the cosmos. (ChatGPT 4.4)





Tao(濤): “big waves,” or “surging sea waves.”



Making “tao” in physics

Tao(濤): “big waves,” or “surging sea waves.”

Making “tao” in the community

Tao(濤): “big waves,” or “surging sea waves.”

Making “tao” in the glass!

Symmetry is among the most fundamental principles in physics:

Chen-Ning Yang famously coined the phrase --

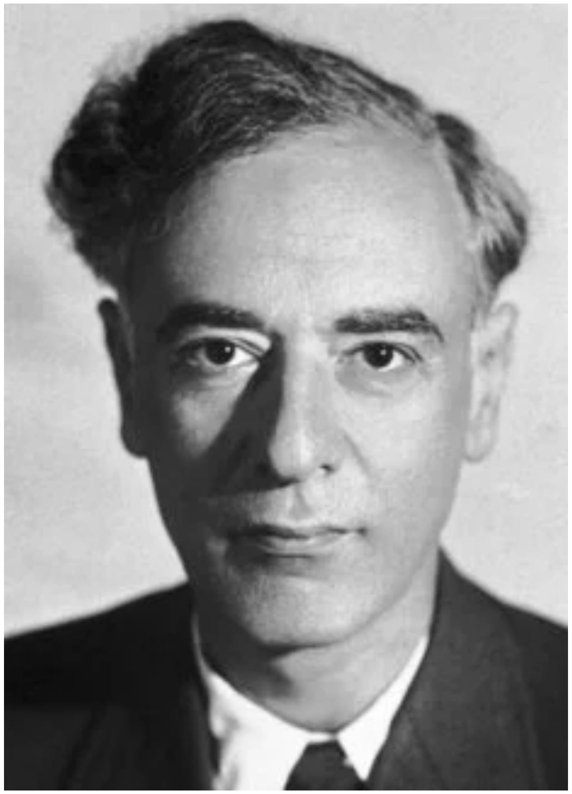
Symmetry dictates Interaction.

- Lorentz invariance →
  Special Relativity
- General coordinate invariance →
  General Relativity
- Gauge invariance →
  QCD and Electroweak theory.

In condensed matter physics, the Landau paradigm:

Phases of matter are represented by their symmetries and whether they are spontaneously broken or not.

- Gapless degrees of freedom → Goldstone modes
- Locus of critical points → Enhanced (emergent) symmetries
- Ginzburg-Landau theory gives a macroscopic description.



But what is the origin of symmetry?

Can symmetry be the outgrowth of more fundamental principles?

John Wheeler famously claimed:

It from bit : "All things physical are information-theoretic in origin"

INFORMATION, PHYSICS, QUANTUM: THE SEARCH FOR LINKS

John Archibald Wheeler * †

Abstract

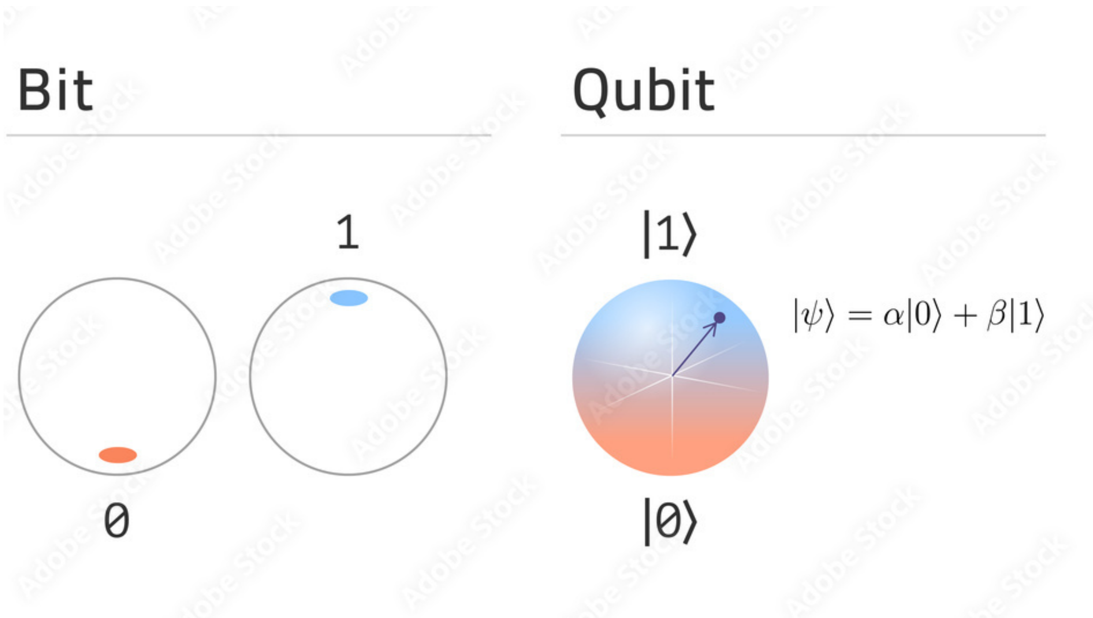
This report reviews what quantum physics and information theory have to tell us about the age-old question, How come existence? No escape is evident from four

winnowing: **It from bit**. Otherwise put, every **it** — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes or no questions, binary choices [52], **bits**.

Indeed, we have seen remarkable connections between fundamental physics and information science in the past decade.

It is natural to ask:

Can symmetry come from qubit?



Recent efforts to understand the origin of symmetry from the QIS perspective have uncovered intriguing insights:

- Extremization (minimization or maximization) of entanglement entropy in particle interactions lead to enhanced symmetries.
- Examples encompass both non-relativistic (low-energy QCD) and fully relativistic (two-Higgs-doublet models) systems.
- The observation applies to qubits (spin-1/2) and qudits (spin-3/2).

Enhanced symmetries in low-energy QCD (not present in the QCD Lagrangian):

- Schrodinger symmetry (non-relativistic conformal invariance):

$$\text{boosts:} \qquad \vec{x}\,' = \vec{x} + \vec{v}\,t \ , \quad t' = t \ ,$$

$$\text{scale:} \qquad \vec{x}\,' = \vec{x} + s\vec{x} \ , \quad t' = t + 2st \ ,$$

$$\text{conformal:} \qquad \vec{x}\,' = \vec{x} - ct\vec{x} \ , \quad t' = t - ct^2 \ ,$$

Hagen and Niederer, 1972

- Wigner's SU(4) Spin-flavor symmetries for protons and neutrons

$$N = \begin{pmatrix} p_\uparrow \\ p_\downarrow \\ n_\uparrow \\ n_\downarrow \end{pmatrix} \qquad N \rightarrow \mathcal{U}N \ , \quad \mathcal{U} \in SU(4)$$

E. P. Wigner (1934)

Let's consider non-relativistic, S-wave scattering of a neutron and a proton:

- Treat them as two qubits -- Alice (neutron) and Bob (proton)

- The S-matrix can be decomposed into ${}^1S_0$ and ${}^3S_1$ channels
  → there are two phase shifts: δ_0 and δ_1 , respectively.

- Rotational invariance and Unitarity then uniquely fix the S-matrix:

$$S = e^{2i\delta_0}\frac{(1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})}{4} + e^{2i\delta_1}\frac{(3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})}{4}$$

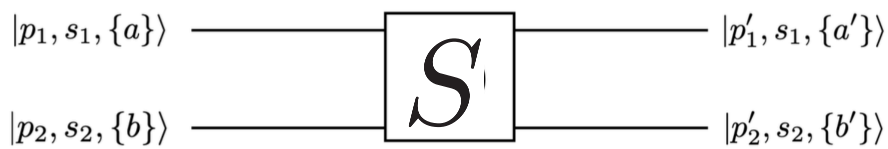
Spin-projector into ${}^1S_0$ channel

Spin-projector into ${}^3S_1$ channel

- In the scattering process the S-matrix acts on the IN-state:

$$|\text{out}\rangle = S\ |\text{in}\rangle$$

- For 2-to-2 scattering of spin-1/2 fermions, the S-matrix can be viewed as a two-qubit quantum logic gate acting on the spin-space:



- Can characterize the ability of the S-matrix to generate entanglement from unentangled initial states.

- Entanglement is a property of the quantum state. But we are more interested in the ability of a *quantum-mechanical operator* (i.e. the S-matrix) to entangle.

- However, there is a subtlety here, as the amount of entanglement generated by an operator could depend on the initial state.

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- The “entanglement power” deals with this issue is by averaging over the initial states:

$$E(U) = \overline{E(U\,|\psi_1\rangle \otimes |\psi_2\rangle)},$$

For quibts, the average is over the Bloch sphere.

It is a measure of the ability of an operator *U* to generate entanglement on product states.

- A minimally entangling operator has *E*(*U*) = 0, i.e.,

$$|\quad\rangle \otimes |\quad\rangle \xrightarrow{U} |\quad\rangle \otimes |\quad\rangle$$

It turns out there are two and only two operators with vanishing entanglement power, which in the computational basis, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

$$\mathrm{SWAP} \sim -1 \quad \text{as} \quad [\mathrm{SWAP}]^2 = 1$$

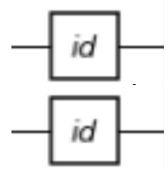
In terms of Pauli matrices,

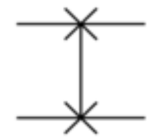
$$\mathrm{SWAP} = (1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma})/2 \ , \qquad \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \equiv \sum_a \boldsymbol{\sigma}^a \otimes \boldsymbol{\sigma}^a \ .$$

Low, Mehen: 2104.10835

Re-write the S-matrix in terms of quantum logic gates,

$$S = \frac{1}{2}\left(e^{2i\delta_1} + e^{2i\delta_0}\right)\mathbf{1} + \frac{1}{2}\left(e^{2i\delta_1} - e^{2i\delta_0}\right)\ \mathrm{SWAP},$$

$$S = \frac{1}{2}\left(e^{2i\delta_1} + e^{2i\delta_0}\right)$$ 

$$+ \ \frac{1}{2}\left(e^{2i\delta_1} - e^{2i\delta_0}\right)$$ 

Conditions for the S-matrix to minimize entanglement:

1. $S = 1$ if $\delta_0 = \delta_1$ → SU(4) spin-flavor symmetry

2. S = SWAP if $|\delta_0 - \delta_1| = \pi/2$ → Schrodinger symmtery

Beane et. al.: 1812.03138

Low, Mehen: 2104.10835

We have observed similar correlations between entanglement minimization and the appearance of enhanced symmetries in several other systems:

- 2-to-2 scattering of spin-1/2 octet baryons. (Liu, Low, Mehen: 2210.12085)
- 2-to-2 scattering of spin 3/2 decuplet baryons. (Hu, Sone, Guo, Hyodo, Low: 2506.08960)
- Exotic mesons (four-quark bound states) in X(3872) and $T_{cc}(3875)^{+}$. (Hu, Chen, Guo: 2404.05958.)
- 2-to-2 scattering of Higgs bosons in two-Higgs-doublet models. (Carena, Low, Wagner and Xiao: 2307.08112)

There is also an example of entanglement *maximization* and enhanced symmetries. (Carena, Coloreti, Liu, Littmann, Low, and Wagner: 2505.00873)

A most outstanding question:

What is the mechanism underlying entanglement extremization and enhanced symmetry?

The correlation is observed in the QFT setting — but the mechanism is mysterious. QFT carries infinitely many degrees of freedom, and the interplay between symmetry and entanglement is hard to isolate.

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Strategy:

Study finite systems – 1-dimensional systems with finite number of qubits/qutrits.

In particle physics, these are “1-D lattice field theory”.

In condensed matter physics, these are 1-D spin-chains.

As a warm up, let’s consider an anisotropic two-site spin-1/2 Hamiltonian (a two-qubit model):

$$H \;=\; a_x\, S_x \otimes S_x \;+\; a_y\, S_y \otimes S_y \;+\; a_z\, S_z \otimes S_z$$

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The Hamiltonian itself is not unitary, but the time-evolution operator U(t) is:

$$U(t) \;=\; e^{\,iHt}$$

But then the entanglement power of U(t) is time-dependent and is in general oscillatory.

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This motivates looking at the (infinitely) time-averaged entanglement power:

$$\overline{\mathcal{E}_p} \;\equiv\; \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathcal{E}_p\big[U(t)\big]\, dt$$

This is time-independent (just like the S-matrix is.)

For the two-qubit model, **everything** is exactly solvable analytically.

The Hamiltonian

$$H \;=\; a_x\, S_x \otimes S_x \;+\; a_y\, S_y \otimes S_y \;+\; a_z\, S_z \otimes S_z$$

has four eigenvalues

$$E \in \left\{ \frac{-a_x - a_y - a_z}{4},\; \frac{a_x + a_y - a_z}{4},\; \frac{a_x - a_y + a_z}{4},\; \frac{-a_x + a_y + a_z}{4} \right\}$$

The eigenvectors turn out to be the maximally entangled Bell pairs, independent of (a_x , a_y , a_z):

$$|\Phi_\pm\rangle \;=\; \tfrac{1}{\sqrt{2}}\big(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\big), \qquad |\Psi_\pm\rangle \;=\; \tfrac{1}{\sqrt{2}}\big(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\big)$$

The instantaneous ep depends on six "frequencies"

$$\mathrm{ep}(U) = \frac{1}{36}\Big(6 - \cos\big[(a_x - a_y)t\big] - \cos\big[(a_x + a_y)t\big] - \cos\big[(a_x - a_z)t\big] - \cos\big[(a_y - a_z)t\big] - \cos\big[(a_x + a_z)t\big] - \cos\big[(a_y + a_z)t\big]\Big), \qquad (7)$$

The average ep only depends on N_0 = the number of six frequencies which vanish.
(It counts the number of "degenerate-in-absolute-value" eigenvalues!)

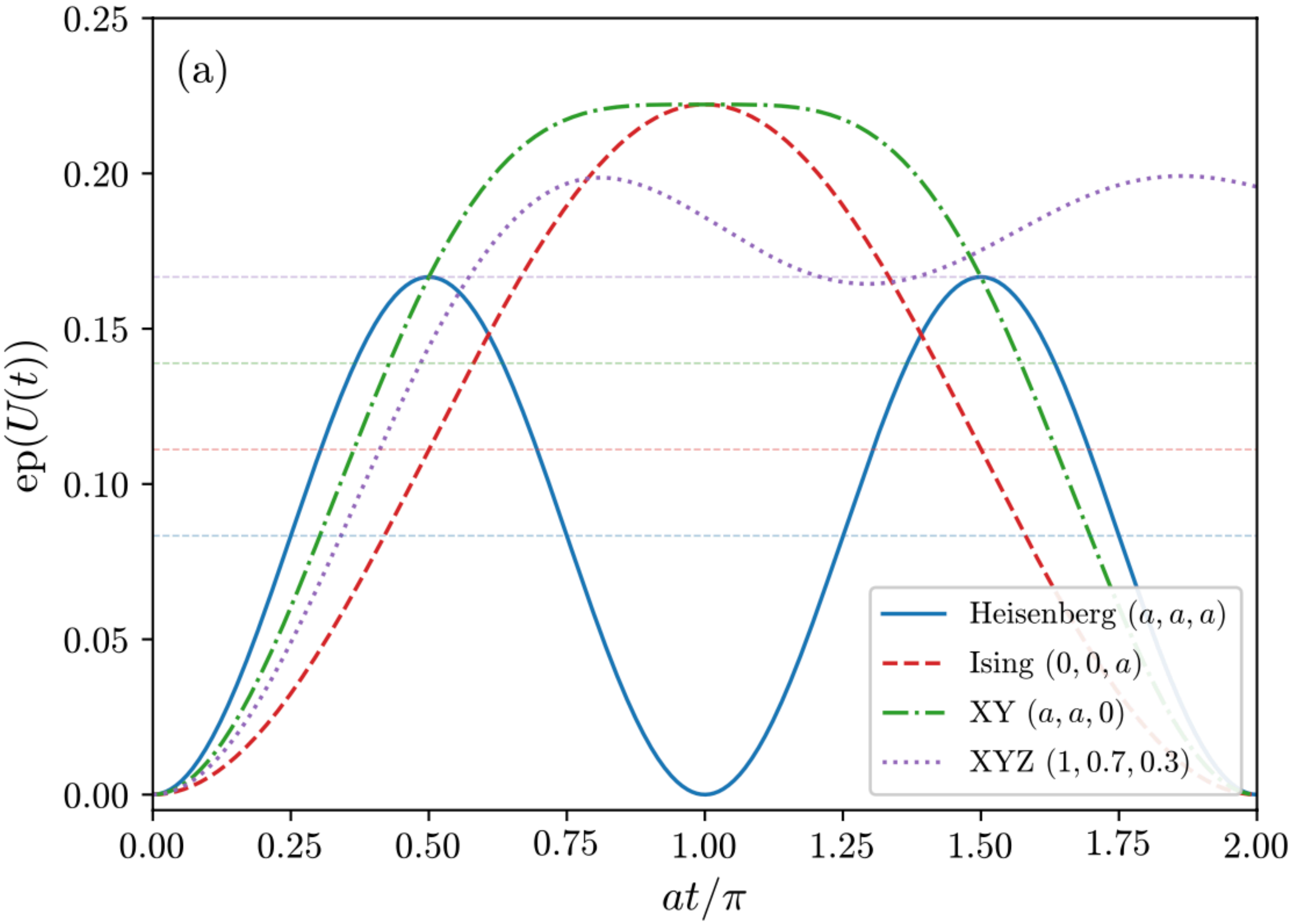
$$\overline{\mathrm{ep}} = \frac{6 - N_0}{36}$$

Model	(a_x, a_y, a_z)	Symmetry	N_0	$\overline{\mathrm{ep}}$	Decimal
XYZ	all distinct	—	0	1/6	0.1667
XY	$(a, a, 0)$	$U(1)$	1	5/36	0.1389
XXZ (generic)	(a, a, Δ)	$U(1)$	1	5/36	0.1389
Ising	$(0, 0, a)$	$U(1)^2$	2	1/9	0.1111
XX	$(a, 0, 0)$	$U(1)^2$	2	1/9	0.1111
Heisenberg	(a, a, a)	$SU(2)$	3	1/12	0.0833

TABLE I: *Time-averaged entanglement power for two-qubit models and their continuous symmetry groups.*

$$\overline{\mathrm{ep}}_{\mathrm{XYZ}} > \overline{\mathrm{ep}}_{\mathrm{XY}} = \overline{\mathrm{ep}}_{\mathrm{XXZ}} > \overline{\mathrm{ep}}_{\mathrm{Ising}} = \overline{\mathrm{ep}}_{\mathrm{XX}} > \overline{\mathrm{ep}}_{\mathrm{Heis}}$$

The instantaneous ep and time-average:



Low and Goswami, to appear

The time-averaged entanglement power counts the ”degeneracies” in the eigenvalues:

Higher Symmetry ⟺ **More Degenerate Spectrum** ⟺ **Lower EP**

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T e^{-i\omega\, t}\, dt = \delta_{\omega,\,0}$$

The SU(2) Heisenberg model has the most degenerate spectrum:

$$H_{SU(2)} = J\left(S_x \otimes S_x + S_y \otimes S_y + S_z \otimes S_z\right) \qquad \{-3J/4, J/4, J/4, J/4\}$$

The spectrum is organized into total S=0 and S=1 blocks.

Since eigenstates of this two-qubit model are independent of (a_x , a_y , a_z), one may wonder if eigenstates modify the entanglement power?

Let's construct the most general Hamiltonian with the same "eigenvalues" as the Heisenberg model:

$$H = (J/4)P_{\mathbf{3}} + (-3J/4)P_{\mathbf{1}} = (J/4)\mathbf{1} - JP_{\mathbf{1}}$$

P_3 is the projector into the 3-D eigenspace for J/4 and P_1 the projector into the 1-D eigenspace for (-3J/4). Moreover, $P_3 + P_1 = 1$.

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Since P_1 is 1-D, we can parameterize it by a normalized state vector:

$$P_{\mathbf{1}} = |z\rangle\langle z| \qquad |z\rangle = (z_1, z_2, z_3, z_4)\ , \quad \langle z|z\rangle = 1$$

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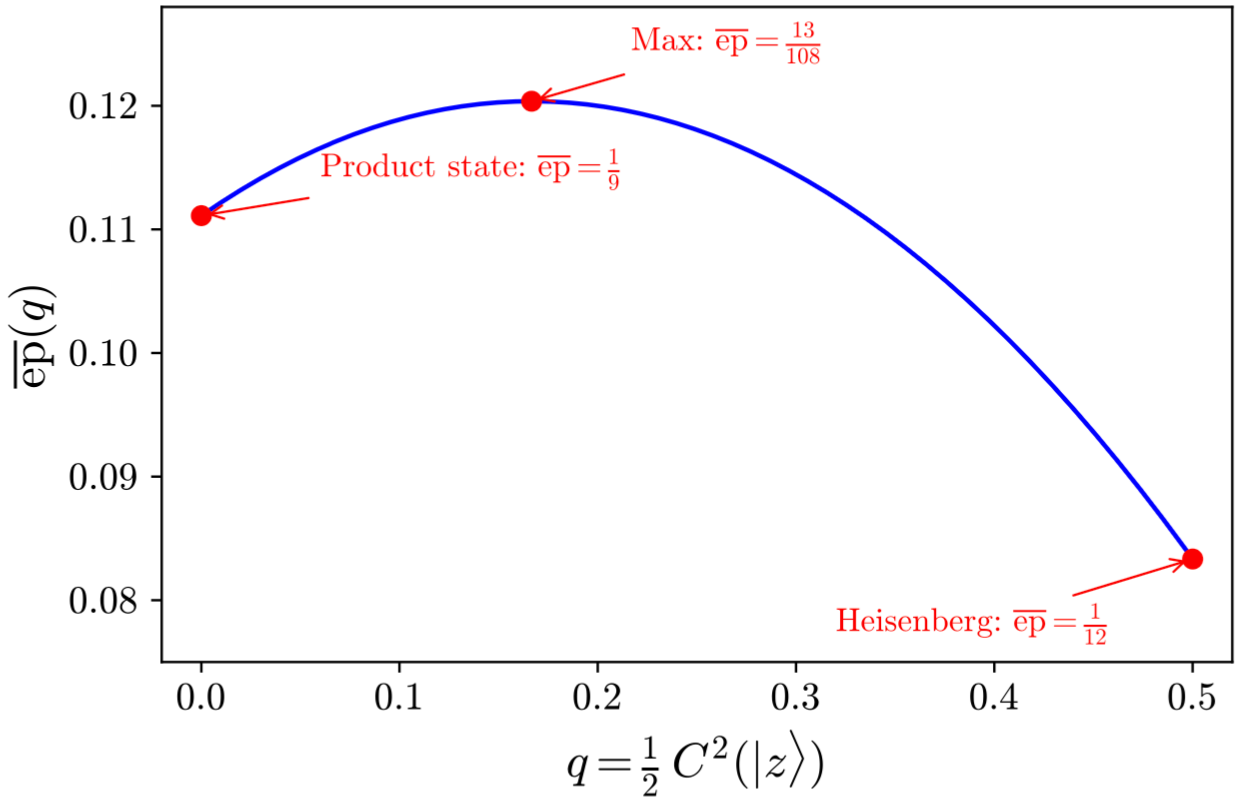
The time-averaged EP for this family of Hamiltonian turns out to only depend on the concurrence "C" of |z>:

$$\overline{\text{ep}}(q) = \frac{1 + q - 3q^2}{9} \qquad\qquad q = \tfrac{1}{2}\, C^2(|z\rangle)$$

- In this family of Hamiltonian, the most symmetric point again has the minimum entanglement power:

Spin-chain with a transverse magnetic field:

$$H = -\frac{J}{2}\sum_i \Big(\langle\sigma_i\rangle_1\, S_{i,1} + \langle\sigma_i\rangle_2\, S_{i,2}\Big) - J\sum_{i,j}\langle\sigma_i\otimes\sigma_j\rangle\, S_{i,1}\, S_{j,2}\,.$$

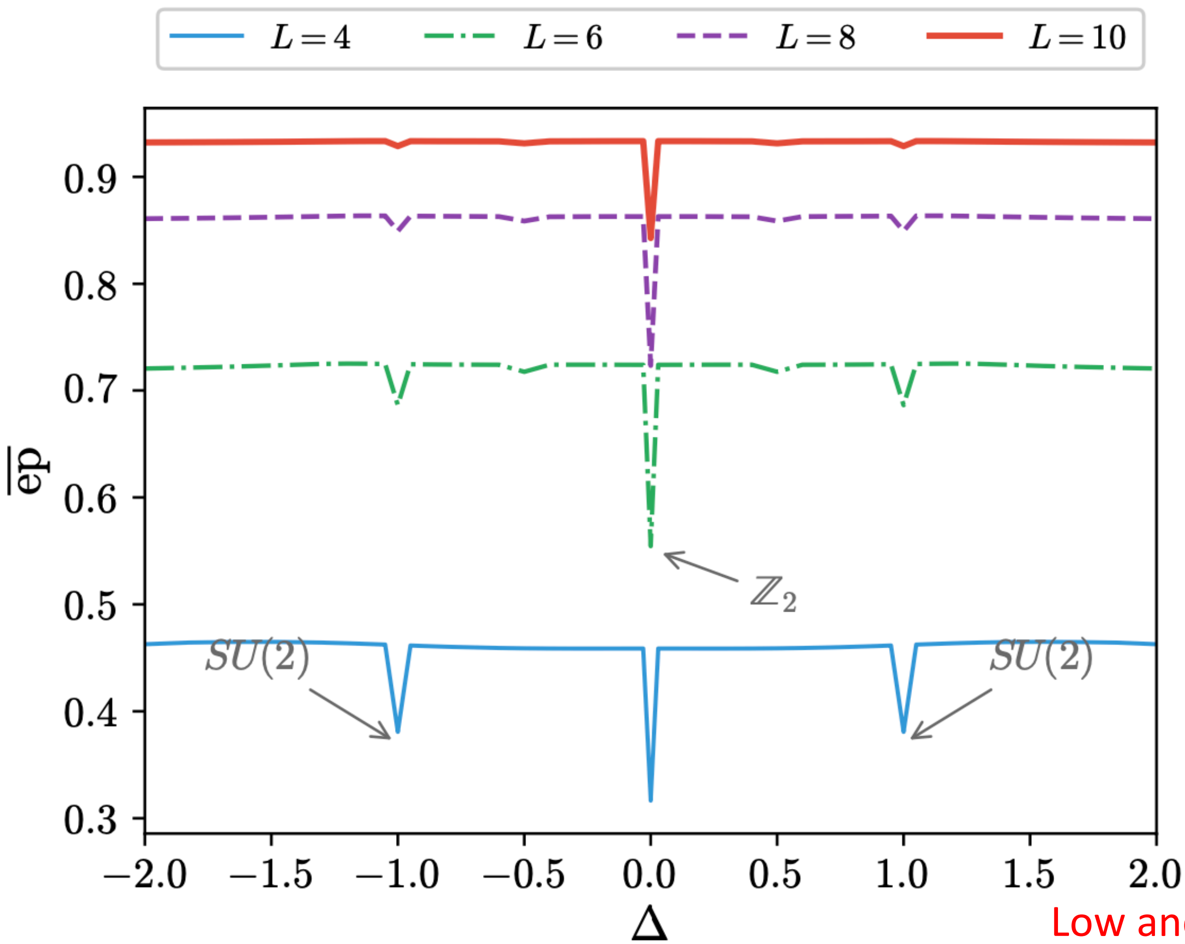


More "symmetric" eigensystems reduce EP.

It is quite informative to consider the XXZ spin-chain:

$$H_{\mathrm{XXZ}} = \sum_{j=1}^{L-1} \left(S_x^{(j)} S_x^{(j+1)} + S_y^{(j)} S_y^{(j+1)} + \Delta\, S_z^{(j)} S_z^{(j+1)} \right)$$

with equal bipartition: $\{1, \ldots, L/2\} | \{L/2+1, \ldots, L\}$

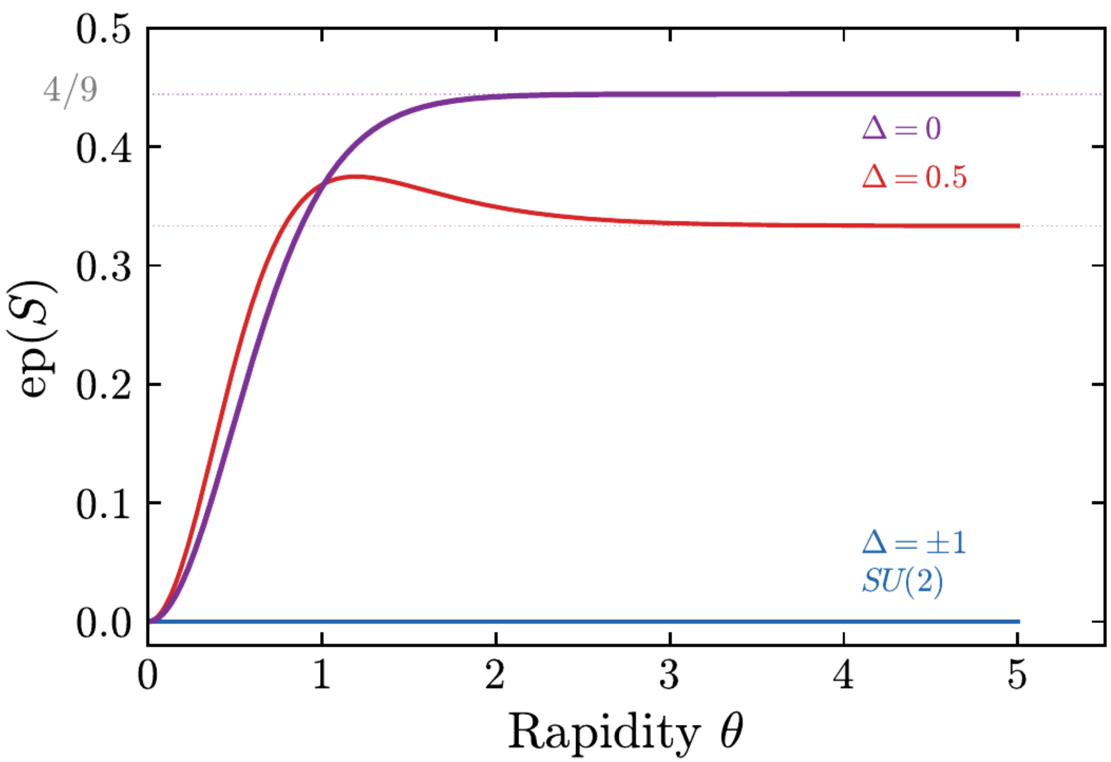


Low and Goswami, to appear

In the thermodynamic limit, L → infinity, two-magnon scattering decomposes into three quantum logic gates:

$$S \;=\; \tfrac{1}{2}(a+b-c)\,\mathbb{I} \;+\; c\,\mathrm{SWAP} \;+\; \tfrac{1}{2}(a-b-c)\,\sigma_z\otimes\sigma_z$$

This is the generalization of S ~ 1 + SWAP in neutron-proton scattering



Δ = ±1 (SU(2)): Ep = 0 for all rapidities (S-matrix in Identity class)

Δ = 0 (free fermion): Ep maximal (largest spin-exchange amplitude)

Low and Goswami, to appear

Concluding Remarks:

- In 2-to-2 scatterings, the correlation between entanglement extremization and enhanced symmetries is well-established.

- Finite-size spin-chains offer insights into the mechanism: symmetry produces degeneracies in eigenvalues and constrains eigenvectors, which in turn reduces EP.

- Entanglement power provides an *operator* witness of symmetry and integrability — from fundamental interactions to quantum many-body systems.

- The physical Universe is a system of quantum information processing.

  The S-matrix is a quantum logic gate. As such one can and should analyze the information processing properties of fundamental interactions.