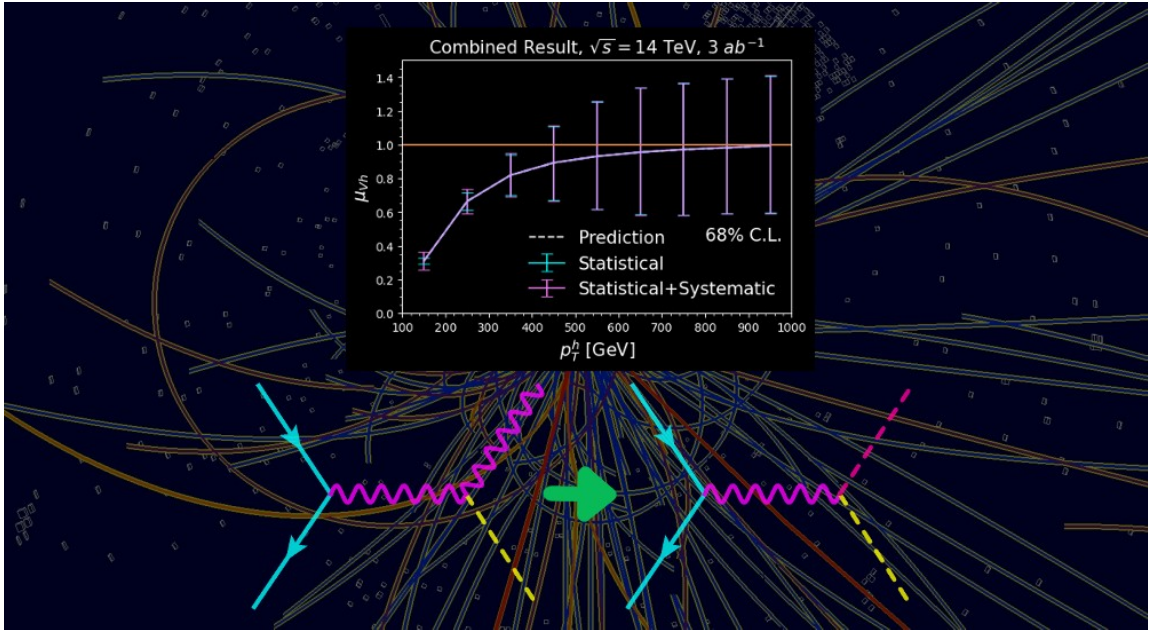
Electroweak Restoration at the LHC and Beyond



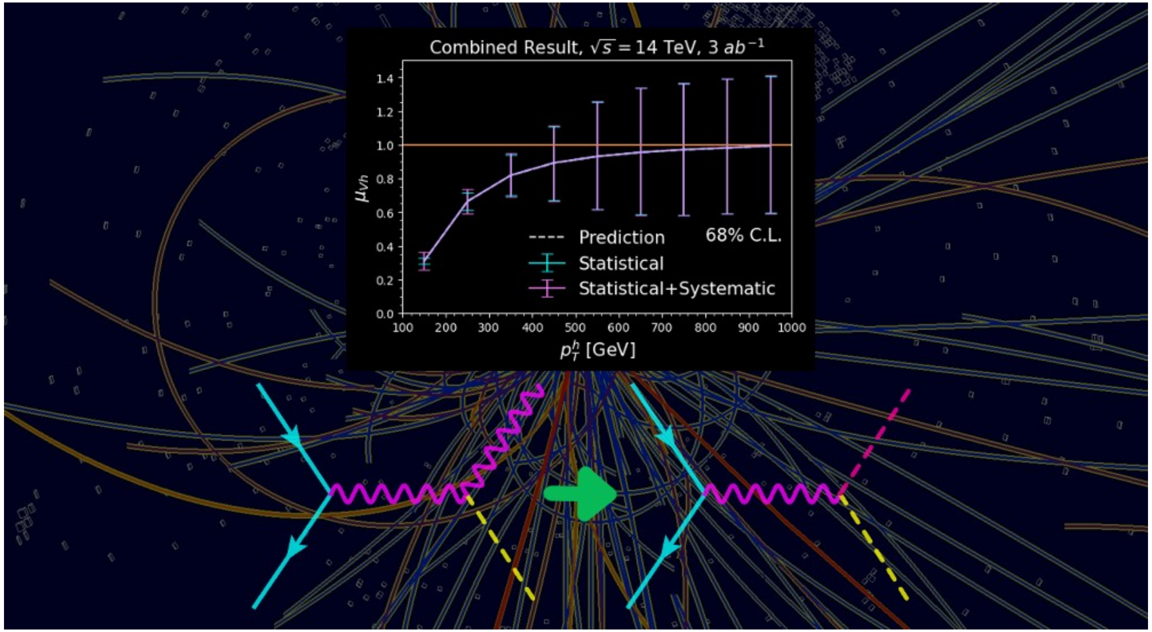
Ian Lewis
(University of Kansas)

Li Huang, Sam Lane, I.M. Lewis, Zhen Liu, PRD103 053007
I.M. Lewis, Zhen Liu, Ishmam Mahbub, arXiv:2605.08433

Personal History

- Went to Madison in 2005 for Ph.D., graduated in 2011 when Tao left for Pittsburgh.
  - Several choices for Ph.D. programs, but undergraduates advisors guided me towards Madison and in particular Tao.
  - Strong reputation of doing great work and taking care of and being supportive of their students.
    - I have worked with excellent undergraduates towards Pittsburgh because I know they will be well taken care of.
  - Learned much about physics (with moments of terror).
    - Use the tools, but understand your results as well as you can physically and analytically.
    - Work on the hot topics and use the newest tools, but always be skeptical (especially about anomalies).
    - Never be afraid of working on new topics or with new people, but don't forget your basics or old friends.
  - Generated many long-term collaborations with through Tao.
    - The sense of community around physics that Tao fosters is phenomenal
  - One of the most important lessons I've learned: be supportive and patient (mostly) with your students.
    - Our students' success is our success, whether they stay in academia or not.
    - They all have their different strengths: work to support their strengths while pushing them in new directions.
    - Important to create situations where younger colleagues can present their results in a supportive environment with their more experienced colleagues.
  - Another important lesson: always have fun and always enjoy good food and drink with friends (I may have learned this lesson too well).
  - I try to emulate many things Tao does, but it's always a pale comparison.
- Final lesson: physics first.

Electroweak Restoration at the LHC and Beyond



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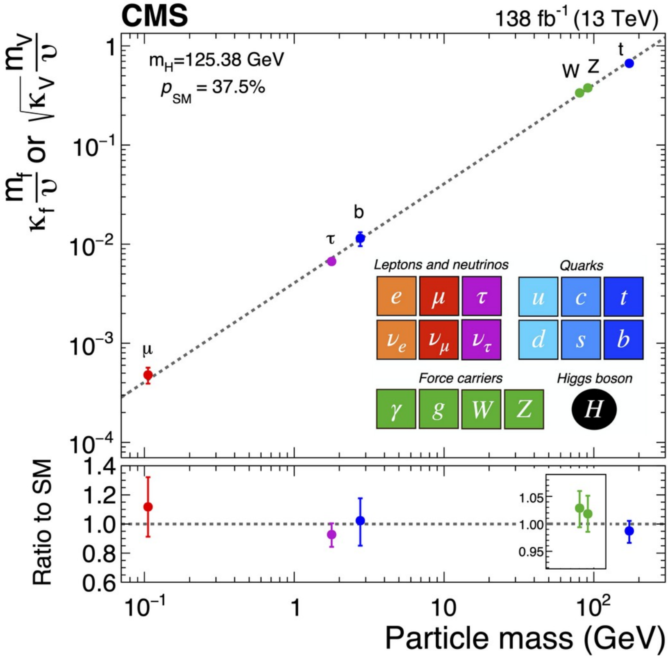
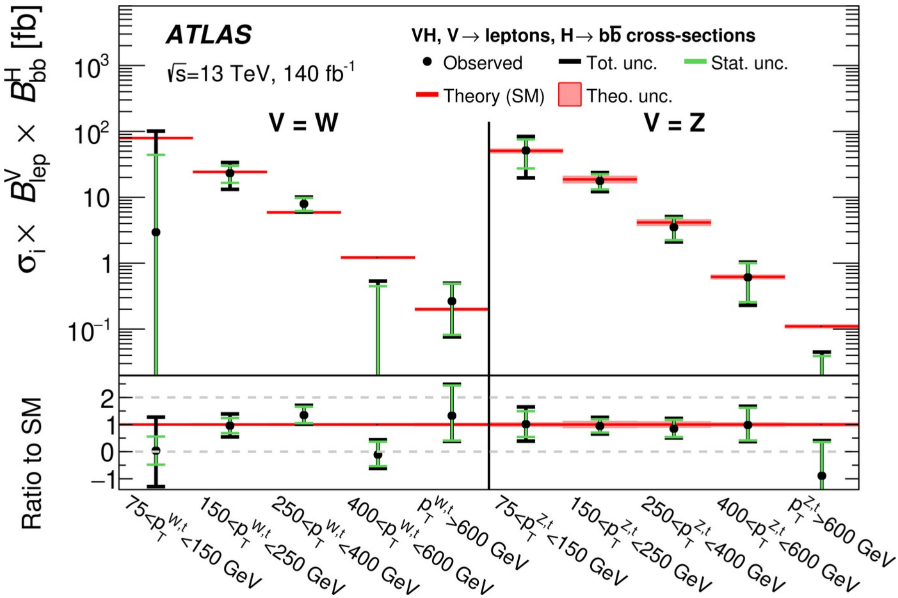
High Energy LHC Measurements

- Remarkable agreement with SM predictions for Higgs.
- LHC going to run for another ~15 years.
- Want to discover new physics.
- What else can we learn from these high energy measurements?
  - At high energy, SM particle masses become more negligible.
- EW restoration
  - As we go to higher and higher energies, expect the SM to asymptotically approach the unbroken EW symmetry.





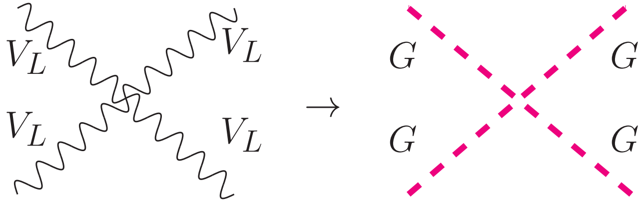
ATLAS JHEP 04 (2025) 075

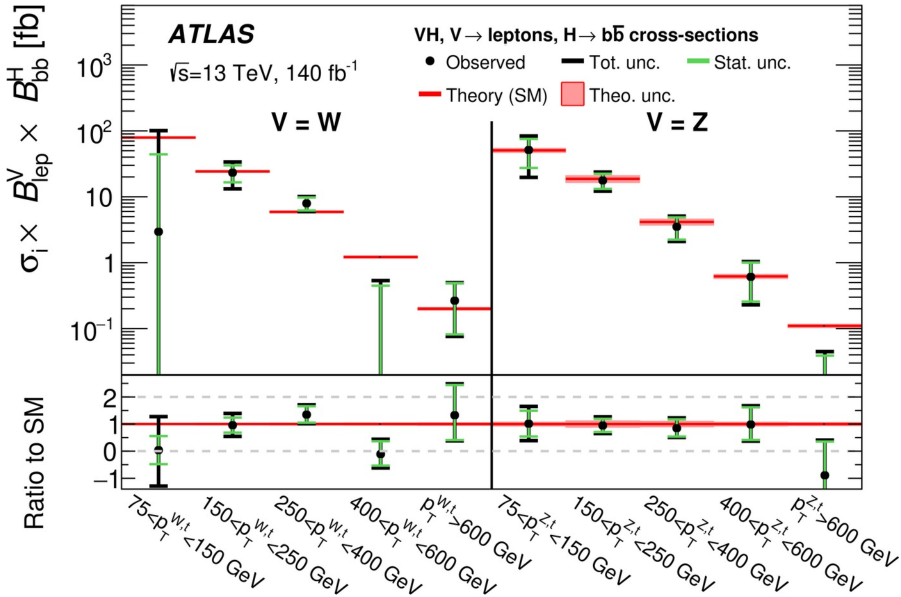
Electroweak Restoration

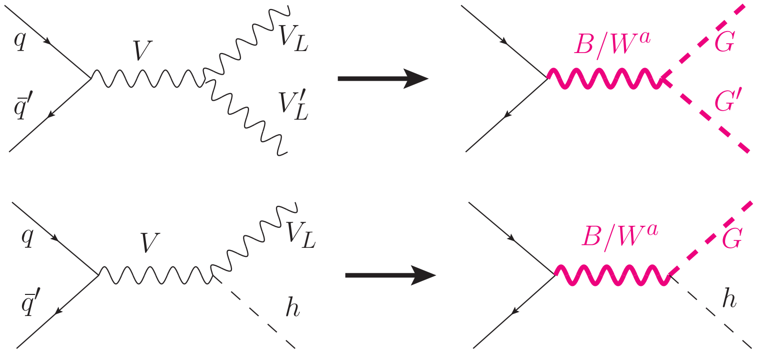
- The LHC is operating at the energies necessary to thoroughly explore the electroweak sector at the scale it is broken.
- As the LHC and future colliders measure the Standard Model to ever higher energies, we should be able to start probing not only the breaking of EW symmetry, but the restoration.
  - At high energies the SM particles are essentially massless.
  - This is equivalent to the Higgs vev going to zero (in the SM):

$$v \to 0$$

- At colliders, EW symmetry is always broken.
  - However, the SM converges to an EW symmetric theory at high energies with corrections of order $\delta \sim v^2/E^2$
  - This convergence should be directly measurable







Signal Strength

- To measure EW restoration, will define a signal strength that measures the deviation between the broken and unbroken phases of EW symmetry:

$$\mu_{Wh} = \frac{d\sigma(pp \to W^\pm h)/dp_T^h}{d\sigma(pp \to G^\pm h)/dp_T^h},$$

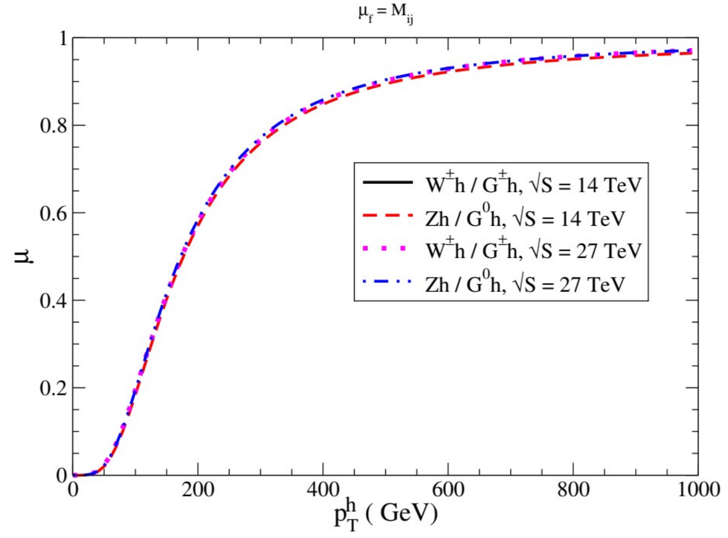
$$\mu_{Zh} = \frac{d\sigma(pp \to Zh)/dp_T^h}{d\sigma(pp \to G^0 h)/dp_T^h}.$$

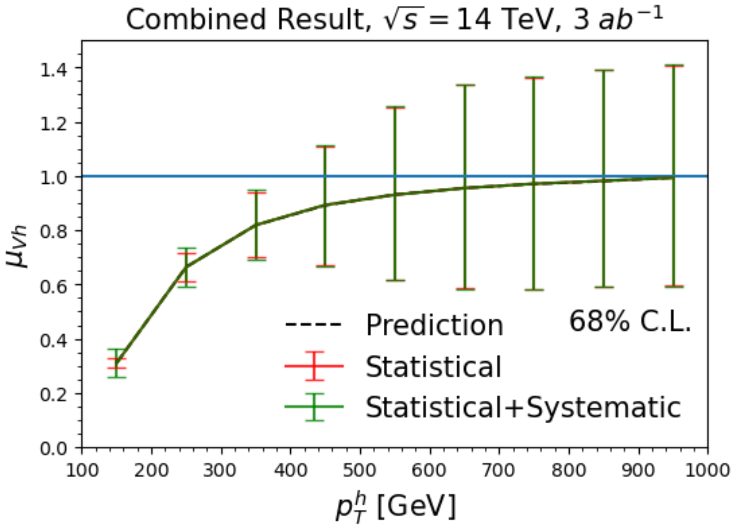
  – Numerator calculated in theory with broken EW symmetry.
  – Denominator calculated in theory of unbroken EW symmetry.
  – Chose Vh because in SM they are longitudinally dominated very quickly.
- Both approach one at high energies.
- Numerically both signal strengths are very similar:
  – Define universal signal strength:

$$\mu_{Vh} = \mu_{Wh} = \mu_{Zh}.$$

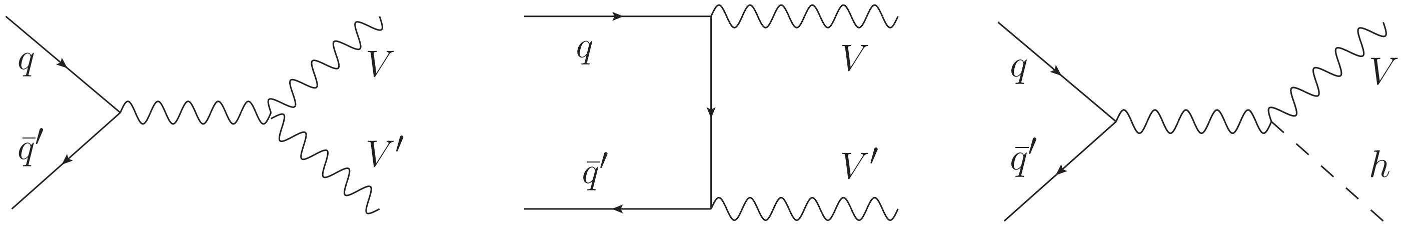
  – Can combine Wh and Zh measurements.

Huang, Lane, **I.M.L.** Liu, PRD103 (2021) 053007





What about BSM?



- Previous analysis fully SM.
- Contributions from BSM could be expected to change the nature of EW symmetry and its restoration.
  - BSM can have different realizations of EW symmetry and new contributions to the breaking of EW symmetry.
- Consider two general classes:
  - Linear realization of EW symmetry: The Higgs boson lives in an complex $SU(2)_L$ doublet along with the Goldstone bosons
  - Non-linear realization of EW symmetry: The Goldstones live in a real $SU(2)_L$ triplet and the Higgs boson is a singlet.
- Will analyze EW symmetry restoration two scenarios:
  - The Standard Model Effective Field Theory (SMEFT): EW symmetry is linearly realized
  - Higgs Effective Field Theory (HEFT): EW symmetry is non-linearly realized.

Linear EW Symmetry: SMEFT

- We work in the dimension-6 Warsaw basis Grzadkowski, Iskrynski, Misiak, Rosiek, JHEP 10 (2010) 085
- The Higgs field is a doublet.
- Consider all operators that contribute to energy growing amplitudes in diboson production at linear order the SMEFT Wilson coefficients:
  - Have checked through complete calculations that these are the only operators contributing to energy growth

$$\mathcal{Q}_{Hq} = (H^\dagger i \overleftrightarrow{D}_\mu H)\,(\bar{q}_R \gamma^\mu q_R)\,,$$
$$\mathcal{Q}^{(1)}_{Hq} = (H^\dagger i \overleftrightarrow{D}_\mu H)\,(\bar{Q}_L \gamma^\mu Q_L)\,,$$
$$\mathcal{Q}^{(3)}_{Hq} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)\,(\bar{Q}_L \sigma^I \gamma^\mu Q_L)\,,$$
$$\mathcal{Q}_{Hud} = (\widetilde{H}^\dagger i \overleftrightarrow{D}_\mu H)\,(\bar{u}_R \gamma^\mu d_R)\,,$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2}\, i\, G^+ \\ v + h + i\, G^0 \end{pmatrix}$$

- Consider one operator that has a sub-leading, custodial symmetry violating contribution:
  - There are others but they are more strongly constrained.

$$\mathcal{Q}_{HD} = (D^\mu H^\dagger H)\,(H^\dagger D_\mu H)\,,$$

Non-linear EW Symmetry: HEFT

- We use the basis of Brivio, Gonzalez-Fraile, Gonzalez-Garcia, Merlo, EPJC 76 (2016) 416
- The Goldstone bosons are a real $SU(2)_L$ triplet
  - Transformation under global $SU(2)_L \times SU(2)_R$

$$U(x) = \exp\left(\frac{i\,\sigma^I \pi^I(x)}{v}\right), \quad U(x) \to L\,U(x)\,R^\dagger$$

- We consider a subset of operators contributing to di-boson production.
  - There are more that we have neglected.
  - Overall point is unaffected by neglecting these operators.
- The arbitrary functions F reflect the singlet nature of the Higgs boson:

$$\mathcal{F}_i(h) = \left(1 + 2\kappa_i \frac{h}{v} + \kappa_i^{(2)} \frac{h^2}{v^2} + \mathcal{O}(h^3)\right)$$

- A parameter point in HEFT can reproduce SMEFT results.
  - For example, to reproduce SMEFT would expect $\kappa_i = 1$ due to the doublet nature of the Higgs field in SMEFT.
  - Do not need $\kappa_i = 1$ in HEFT.

$$\mathcal{N}_1^Q(h) = i\,\bar{Q}_L\gamma^\mu V_\mu Q_L\,\mathcal{F}_1(h),$$

$$\mathcal{N}_2^Q(h) = i\,\bar{Q}_R\gamma^\mu U^\dagger V_\mu U\,Q_R\,\mathcal{F}_2(h),$$

$$\mathcal{N}_4^Q(h) = \bar{Q}_R\gamma_\mu U^\dagger[V^\mu, T]UQ_R F_4(h)$$

$$\mathcal{N}_5^Q(h) = i\,\bar{Q}_L\gamma_\mu\{V^\mu, T\}Q_L\,F_5(h)$$

$$\mathcal{N}_6^Q(h) = i\,\bar{Q}_R\gamma_\mu U^\dagger\{V^\mu, T\}UQ_R F_6(h)$$

$$\mathcal{N}_7^Q(h) = i\,\bar{Q}_L\gamma^\mu T V_\mu T\,Q_L\,\mathcal{F}_7(h),$$

$$\mathcal{N}_8^Q(h) = i\,\bar{Q}_R\gamma^\mu U^\dagger T V_\mu T U\,Q_R\,\mathcal{F}_8(h)$$

$$\mathcal{P}_3(h) = \frac{i}{4\pi}\,\mathrm{Tr}\big(W_{\mu\nu}[V^\mu, V^\nu]\big)\,\mathcal{F}_3(h).$$

$$T = U\sigma^3 U^\dagger$$

$$V_\mu = (D_\mu U)U^\dagger$$

High Energy Ratios of Di-Boson Amplitudes

I.M.L. Liu, Mahbub, 2605.08433

- Explicit calculation of amplitudes.
- Ratios of di-boson rates do not automatically converge to one in SM and SMEFT
  - Depends on initial state fermion couplings an helicity
- Still, can combine shaded amplitudes to get “good” behavior in SMEFT and HEFT.
  - Keep initial state helicities the same.
  - Combine neutral final states.

To $\mathcal{O}(m^2/s)$	SM	SMEFT	HEFT
$\frac{\mathcal{M}(q_-\bar{q}'_+\to W_L^\pm Z_L)}{\mathcal{M}(q_-\bar{q}'_+\to W_L^\pm h)}$	±1	±1	$\mp\frac{c_3 g-8\pi\,(n_1^Q+n_7^Q)}{8\pi\,(\kappa_1 n_1^Q-\kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_-\bar{q}_+\to W_L^+ W_L^-)}{\mathcal{M}(q_-\bar{q}'_+\to W_L^\pm h)}$	$-\sqrt{2}\left(T_3^q+\frac{s_W^2}{c_W^2}Y_L^q\right)$	$-\frac{C_{Hq}^{(1)}+2\,T_3^q C_{Hq}^{(3)}}{\sqrt{2}\,C_{Hq}^{(3)}}$	$\sqrt{2}\frac{8\pi\,n_5^Q+T_3^q\left(c_3 g-8\pi\,(n_1^Q-3n_7^Q)\right)}{8\pi\,(\kappa_1 n_1^Q-\kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_-\bar{q}_+\to Z_L h)}{\mathcal{M}(q_-\bar{q}'_+\to W_L^\pm h)}$	$\sqrt{2}\frac{g_L^{Zq}}{c_W^2}$	$-\frac{C_{Hq}^{(1)}-2\,T_3^q C_{Hq}^{(3)}}{\sqrt{2}\,C_{Hq}^{(3)}}$	$\sqrt{2}\frac{\kappa_5 n_5^Q+T_3^q(\kappa_1 n_1^Q+\kappa_7 n_7^Q)}{\kappa_1 n_1^Q-\kappa_7 n_7^Q}$
$\frac{\mathcal{M}(q_+\bar{q}_-\to W_L^+ W_L^-)}{\mathcal{M}(q_-\bar{q}_+\to W_L^+ W_L^-)}$	$-\frac{s_W^2 Y_R^q}{c_W^2 T_3^q+s_W^2 Y_L^q}$	$-\frac{C_{Hq}}{C_{Hq}^{(1)}+2\,T_3^q C_{Hq}^{(3)}}$	$-\frac{8\pi\,(n_6^Q+T_3^q(n_2^Q+n_8^Q))}{8\pi\,n_5^Q+T_3^q(c_3 g-8\pi\,(n_1^Q-3n_7^Q))}$
$\frac{\mathcal{M}(q_+\bar{q}_-\to Z_L h)}{\mathcal{M}(q_-\bar{q}_+\to Z_L h)}$	$-\frac{g_R^{Zq}}{g_L^{Zq}}$	$-\frac{C_{Hq}}{C_{Hq}^{(1)}-2\,T_3^q C_{Hq}^{(3)}}$	$-\frac{\kappa_6 n_6^Q+T_3^q(\kappa_2 n_2^Q+\kappa_8 n_8^Q)}{\kappa_5 n_5^Q+T_3^q(\kappa_1 n_1^Q+\kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}'_-\to W_L^{-(+)} Z_L)}{\mathcal{M}(q_-\bar{q}'_+\to W_L^{-(+)} Z_L)}$	0^*	$\frac{C_{Hud}^{(*)}}{2C_{Hq}^{(3)}}$	$-8\,\pi\frac{n_2^Q-n_8^Q+(-)2\,i\,n_4^Q}{c_3 g-8\pi(n_1^Q+n_7^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}'_-\to W_L^{-(+)} h)}{\mathcal{M}(q_-\bar{q}'_+\to W_L^{-(+)} h)}$	0^*	$-\frac{C_{Hud}^{(*)}}{2\,C_{Hq}^{(3)}}$	$-\frac{\kappa_2 n_2^Q-\kappa_8 n_8^Q+(-)2\,i\,\kappa_4 n_4^Q}{\kappa_1 n_1^Q-\kappa_7 n_7^Q}$

TABLE I: Ratios of amplitudes in SM, SMEFT, and HEFT in the high energy limit $E\gg m_W, m_Z, m_h$ up to $\mathcal{O}(m^2/s)$. 0^* notes that these quantities are zero in the massless quark limit.

High Energy Ratios of Di-Boson Amplitudes

I.M.L. Liu, Mahbub, 2605.08433

- Explicit calculations
- Set of ratios in the SM and SMEFT converge to one.
  - Need to tune parameters in HEFT to reproduce that behavior.
- Use EW restoration and the Goldstone boson equivalence theorem to understand SM and SMEFT.

To $\mathcal{O}(m^2/s)$	SM	SMEFT	HEFT
$\frac{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm Z_L)}{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	± 1	± 1	$\mp \frac{c_3 g - 8\pi\,(n_1^Q + n_7^Q)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\sqrt{2}\,\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	$2\,T_3^q$	$2\,T_3^q$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7 - 3)n_7^Q\right]\right)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}_- \to W_L^+ W_L^-)}{\mathcal{M}(q_+\bar{q}_- \to Z_L h)}$	1	1	$\frac{n_6^Q + T_3^q(n_2^Q + n_8^Q)}{\kappa_6 n_6^Q + T_3^q(\kappa_2 n_2^Q + \kappa_8 n_8^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm Z_L)}{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm h)}$	$-$	∓ 1	$\mp \frac{n_2^Q - n_8^Q \mp 2\,i\,n_4^Q}{\kappa_2 n_2^Q - \kappa_8 n_8^Q \mp 2\,i\,\kappa_4 n_4^Q}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) + \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}$	$-\frac{g^2 T_3^q}{g'^2 Y_L^q}$	$-2\,T_3^q \frac{C_{Hq}^{(3)}}{C_{Hq}^{(1)}}$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7 - 3)n_7^Q\right]\right)}{8\pi(1+\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1-\kappa_1)n_1^Q - (\kappa_7 + 3)n_7^Q\right]\right)}$

TABLE II: Selected ratios of amplitudes in the SM, SMEFT, and HEFT in the high energy limit, shown up to $\mathcal{O}(m^2/s)$. The dash $-$ indicates that in the massless quark limit the numerator and denominator are both zero.

High Energy Ratios of Di-Boson Amplitudes: SM

- EW restoration in SM:
  - Longitiduinal vector bosons replaced by Goldstone bosons.
  - Double Goldstone/Higgs boson production through hypercharge and $SU(2)_L$ gauge bosons.
- Hypercharge ($SU(2)_L$ singlet currents): neutral currents with left and right-handed quarks:

$$J_Q^\mu J_{H,\mu} \quad J_q^\mu J_{H,\mu} \qquad J_{H,\mu} = H^\dagger i \overleftrightarrow{\partial}_\mu H$$

$$J_{Q,\mu} = \bar{Q}_L \gamma^\mu Q_L \qquad J_{q,\mu} = \bar{q}_R \gamma^\mu q_R$$

- $SU(2)_L$ triplet currents: neutral and charged currents with left-handed quarks from triplet currents.

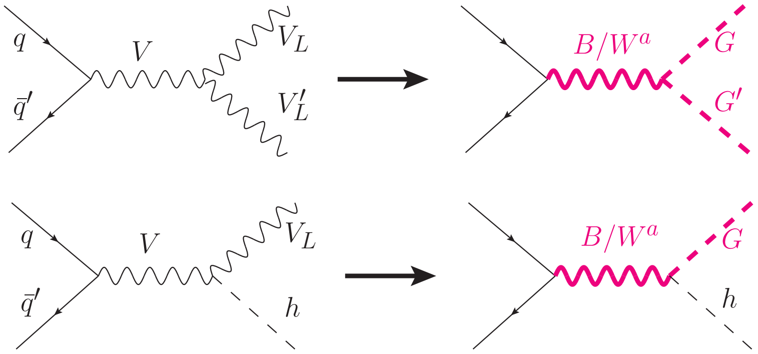
$$J_Q^{I,\mu} J_{H,\mu}^I \quad J_{Q,\mu}^I = \bar{Q}_L \gamma_\mu \sigma^I Q_L. \quad J_{H,\mu}^I = H^\dagger \sigma^I \, i \overleftrightarrow{\partial}_\mu H,$$

$$J_Q^{I,\mu} J_{H,\mu}^I = J_Q^{+,\mu} J_{H,\mu}^- + J_Q^{-,\mu} J_{H,\mu}^+ + J_Q^{0,\mu} J_{H,\mu}^0$$

- High energy ratios of amplitudes approach one:
  - Only hypercharge contributes to neutral final state with right handed quarks.
  - Only $SU(2)_L$ contributes to charged final states
  - Both hypercharge and $SU(2)_L$ contribute to neutral final states with left-handed quarks:
    - Project out $U(1)_Y$ and $SU(2)_L$.
    - Ratio depends on couplings and charges.

To $\mathcal{O}(m^2/s)$	SM
$\frac{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm Z_L)}{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	± 1
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\sqrt{2}\,\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	$2\,T_3^q$
$\frac{\mathcal{M}(q_+\bar{q}_- \to W_L^+ W_L^-)}{\mathcal{M}(q_+\bar{q}_- \to Z_L h)}$	1
$\frac{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm Z_L)}{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm h)}$	–
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) + \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}$	$-\frac{g^2 T_3^q}{g'^2 Y_L^q}$

I.M.L. Liu, Mahbub, 2605.08433



High Energy Di-Boson Ratios: SMEFT

I.M.L. Liu, Mahbub, 2605.08433

- In Warsaw basis it's clear: energy growing SMEFT contributions come from products of currents:

$$\mathcal{Q}_{Hq} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_R \gamma^\mu q_R)\,,$$
$$\mathcal{Q}^{(1)}_{Hq} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{Q}_L \gamma^\mu Q_L)\,,$$
$$\mathcal{Q}^{(3)}_{Hq} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{Q}_L \sigma^I \gamma^\mu Q_L)\,,$$
$$\mathcal{Q}_{Hud} = (\widetilde{H}^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu d_R)\,,$$

- Same arguments for ratios as in the SM.
  - Same ratios of high energy amplitudes approach one.
  - Ratio of singlet and triplet SU(2)L contributions to neutral currents now depends on Wilson coefficients.

- New charged current with right-handed fermions:

$$\mathcal{Q}_{Hud} = (\widetilde{H}^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu d_R)$$

  - Not in SM.
  - Ratio in SMEFT of right-handed fermions charged currents is one.

To $\mathcal{O}(m^2/s)$	SM	SMEFT
$\frac{\mathcal{M}(q_-\bar{q}'_+ \to W^\pm_L Z_L)}{\mathcal{M}(q_-\bar{q}'_+ \to W^\pm_L h)}$	± 1	± 1
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W^+_L W^-_L)}{\sqrt{2}\,\mathcal{M}(q_-\bar{q}'_+ \to W^\pm_L h)}$	$2\,T^q_3$	$2\,T^q_3$
$\frac{\mathcal{M}(q_+\bar{q}_- \to W^+_L W^-_L)}{\mathcal{M}(q_+\bar{q}_- \to Z_L h)}$	1	1
$\frac{\mathcal{M}(q_+\bar{q}'_- \to W^\pm_L Z_L)}{\mathcal{M}(q_+\bar{q}'_- \to W^\pm_L h)}$	–	∓ 1
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W^+_L W^-_L)}{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) + \mathcal{M}(q_-\bar{q}_+ \to W^+_L W^-_L)}$	$-\frac{g^2 T^q_3}{g'^2 Y^q_L}$	$-2\,T^q_3 \frac{C^{(3)}_{Hq}}{C^{(1)}_{Hq}}$

High Energy Ratios of Di-Boson Amplitudes

I.M.L., Liu, Mahbub, 2605.08433

- SM and SMEFT behavior is predicted from EW restoration and the Goldstone boson equivalence theorem.
- HEFT amplitudes depend on many model parameters.
  - HEFT can be tuned to reproduce the SM and SMEFT.
  - SM and dimension-6 SMEFT require no tuning to have these ratios converge to one.
- To fully test EW restoration, would want to measure all these ratios.
- Realistic colliders present (at least) two obstacles:
  - Some ratios include linear combinations of different final states.
  - Some ratios depend on initial state quark couplings and helicities.
- Ratios of WZ to Wh is promising because the ratios do not depend on initial state quark helicities

To $\mathcal{O}(m^2/s)$	SM	SMEFT	HEFT
$\frac{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm Z_L)}{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	± 1	± 1	$\mp \frac{c_3 g - 8\pi\,(n_1^Q + n_7^Q)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\sqrt{2}\,\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	$2\,T_3^q$	$2\,T_3^q$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7-3)n_7^Q\right]\right)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}_- \to W_L^+ W_L^-)}{\mathcal{M}(q_+\bar{q}_- \to Z_L h)}$	1	1	$\frac{n_6^Q + T_3^q(n_2^Q + n_8^Q)}{\kappa_6 n_6^Q + T_3^q(\kappa_2 n_2^Q + \kappa_8 n_8^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm Z_L)}{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm h)}$	$-$	∓ 1	$\mp \frac{n_2^Q - n_8^Q \mp 2\,i\,n_4^Q}{\kappa_2 n_2^Q - \kappa_8 n_8^Q \mp 2\,i\,\kappa_4 n_4^Q}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) + \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}$	$-\frac{g^2 T_3^q}{g'^2 Y_L^q}$	$-2\,T_3^q \frac{C_{Hq}^{(3)}}{C_{Hq}^{(1)}}$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7-3)n_7^Q\right]\right)}{8\pi(1+\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1-\kappa_1)n_1^Q - (\kappa_7+3)n_7^Q\right]\right)}$

TABLE II: Selected ratios of amplitudes in the SM, SMEFT, and HEFT in the high energy limit, shown up to $\mathcal{O}(m^2/s)$. The dash $-$ indicates that in the massless quark limit the numerator and denominator are both zero.

WW/Zh vs. Wh

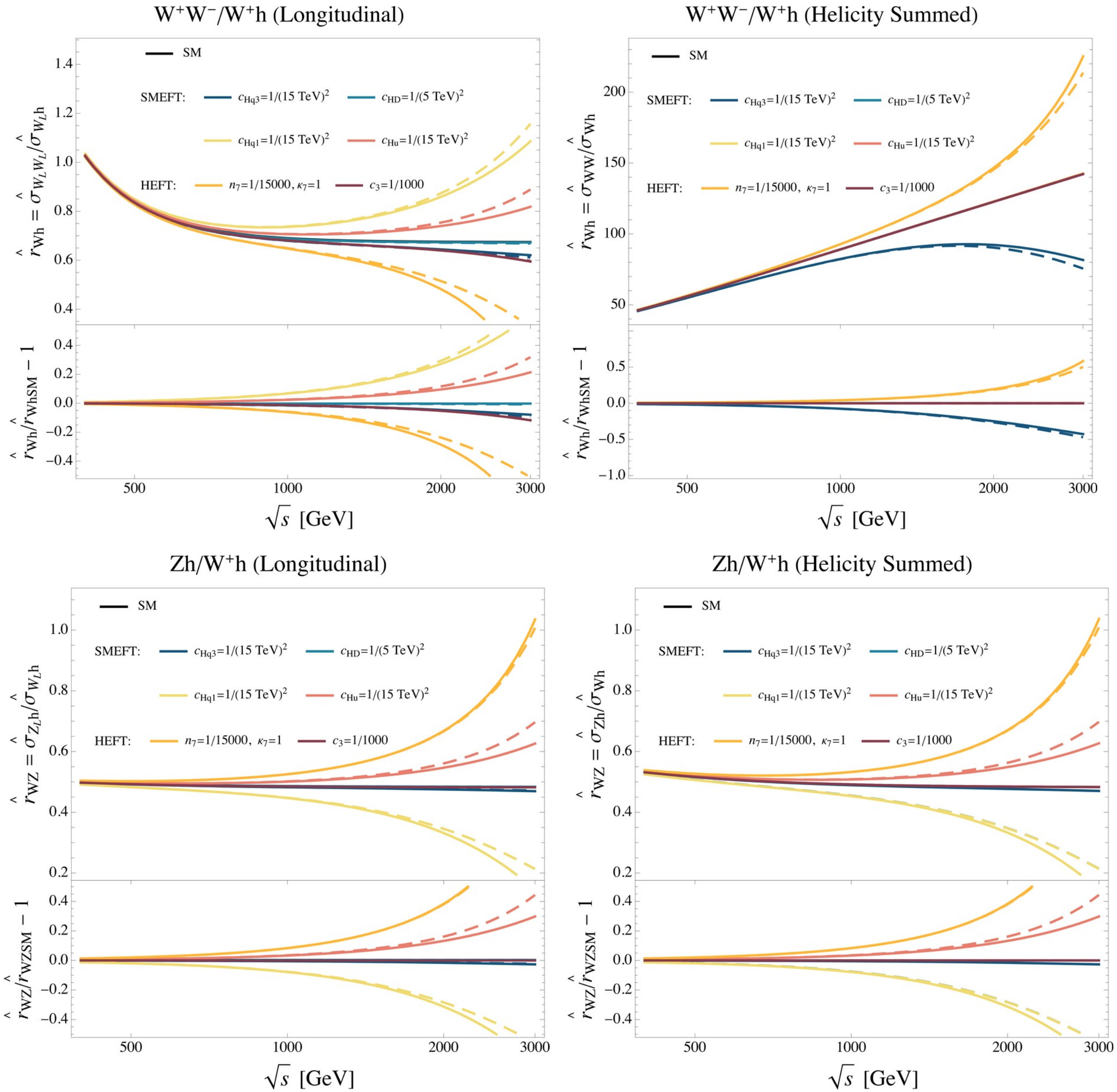
- Partonic cross sections.
- Dashed lines: including quadratic
- Both SMEFT and HEFT deviate from SM with energy growth.
- Blue, yellow, red: SMEFT
- Orange, maroon: HEFT

I.M.L. Liu, Mahbub, 2605.08433

$$\text{SMEFT}: \quad C_{Hq}^{(3)} = C_{Hq}^{(1)} = C_{Hu} = (15\ \text{TeV})^{-2}$$

$$C_{HD} = (5\ \text{TeV})^{-2}$$

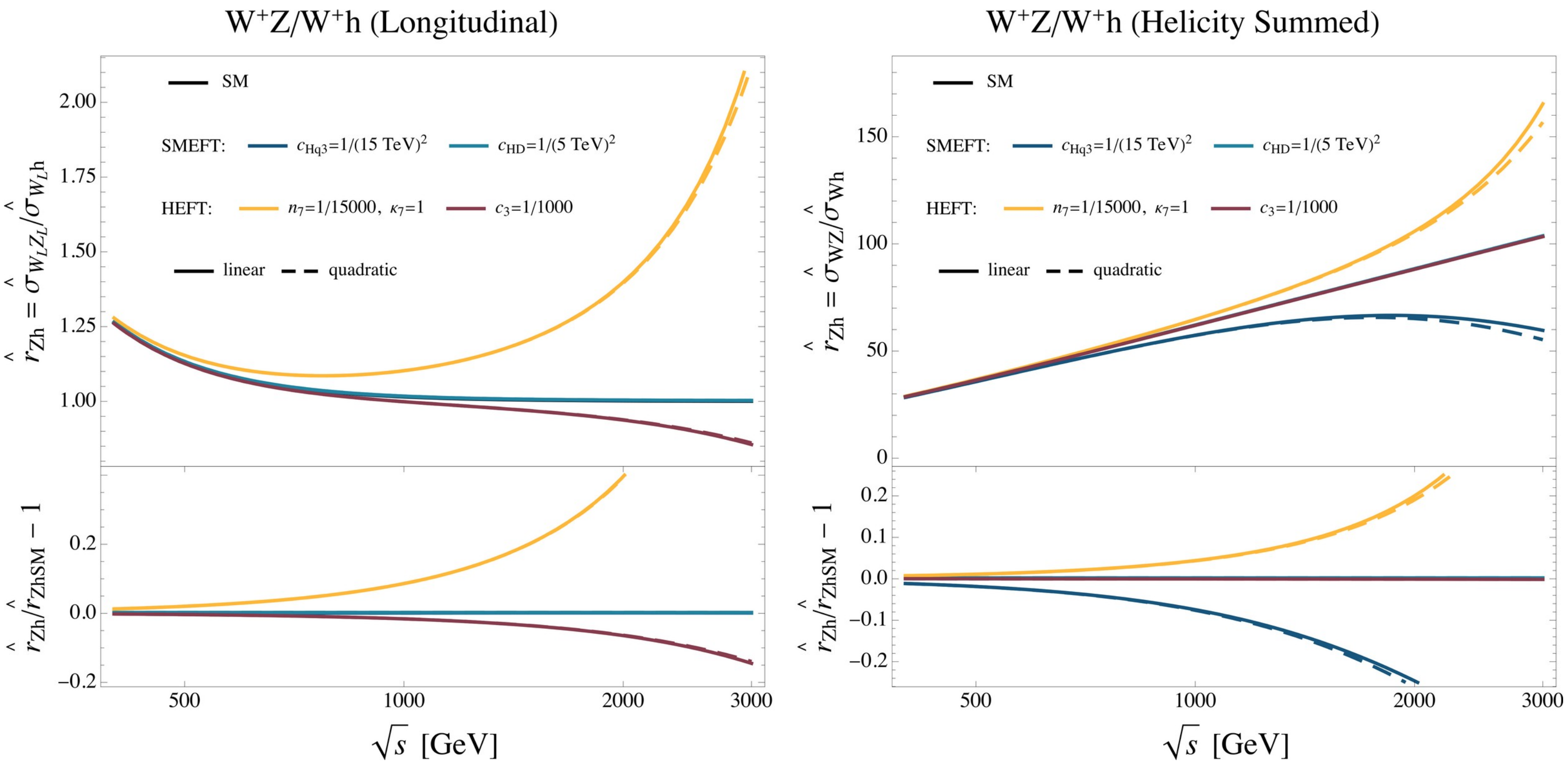
$$\text{HEFT}: \quad c_3 = 10^{-3},\ n_7^Q = \frac{2}{3} \times 10^{-4},\ \kappa_7 = 1$$



WZ vs. Wh

I.M.L. Liu, Mahbub, 2605.08433

- Dashed lines: including quadratic contributions
  - Large deviations, but linear and quadratic orders agree well.
- Both SM and SMEFT converge to one for longitudinal gauge bosons.
- HEFT has large, energy growing deviations from the SM.
- Polarization tagging important: both SMEFT and HEFT have similar deviations with gauge boson polarizations are summed.
- Blue: SMEFT
- Orange, maroon: HEFT

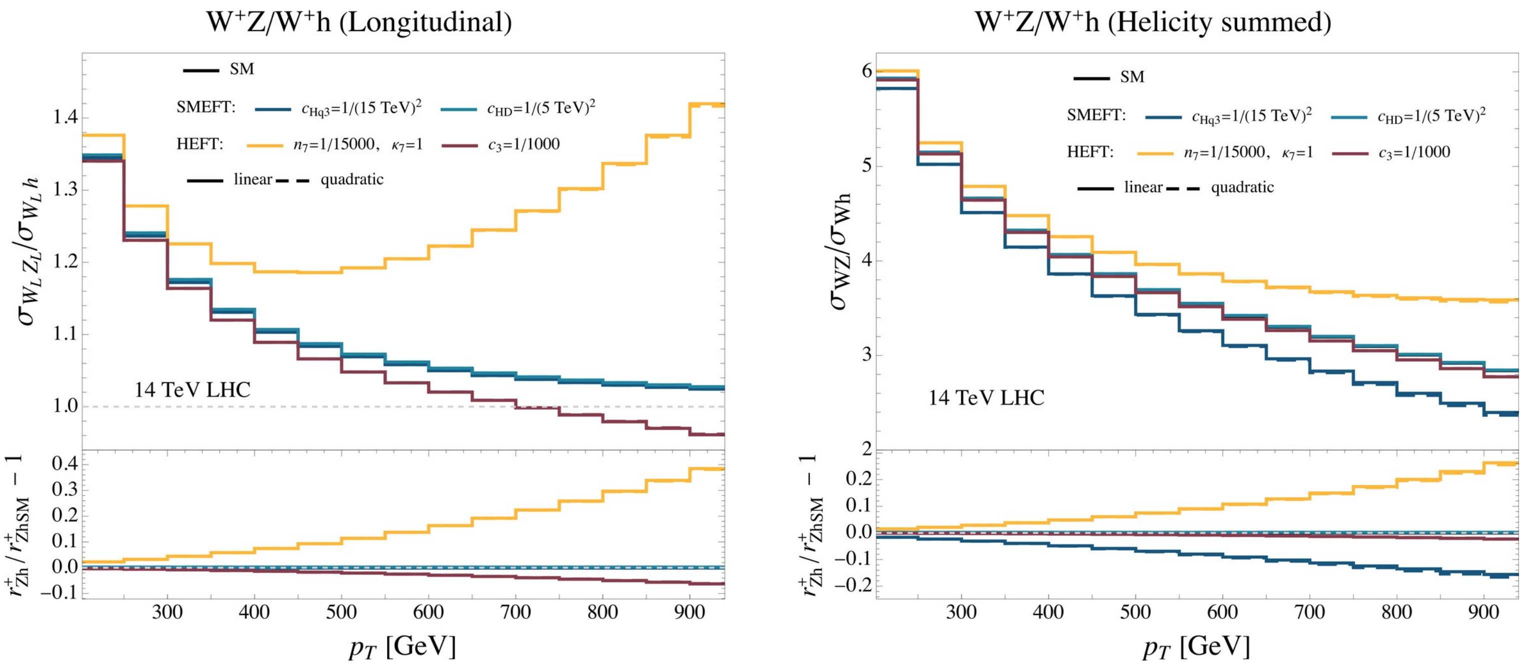


$$\text{SMEFT}: \quad C_{Hq}^{(3)} = C_{Hq}^{(1)} = C_{Hu} = (15\ \text{TeV})^{-2}$$
$$C_{HD} = (5\ \text{TeV})^{-2}$$
$$\text{HEFT}: \quad c_3 = 10^{-3},\ n_7^Q = \frac{2}{3} \times 10^{-4},\ \kappa_7 = 1$$

LHC Cross Section

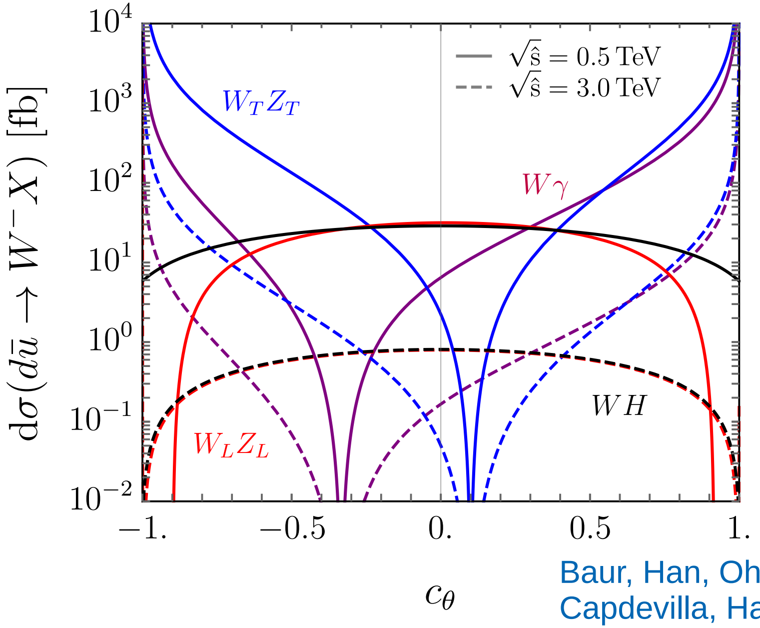
I.M.L. Liu, Mahbub, 2605.08433

- Ratio of rates still agree in the SM and SMEFT still approach one at high energies.
  - Polarization tagging is important.
- HEFT still has large deviations.
- Beyond being theoretically promising, these processes could be measured at the LHC.
- Blue: SMEFT
- Orange, maroon: HEFT

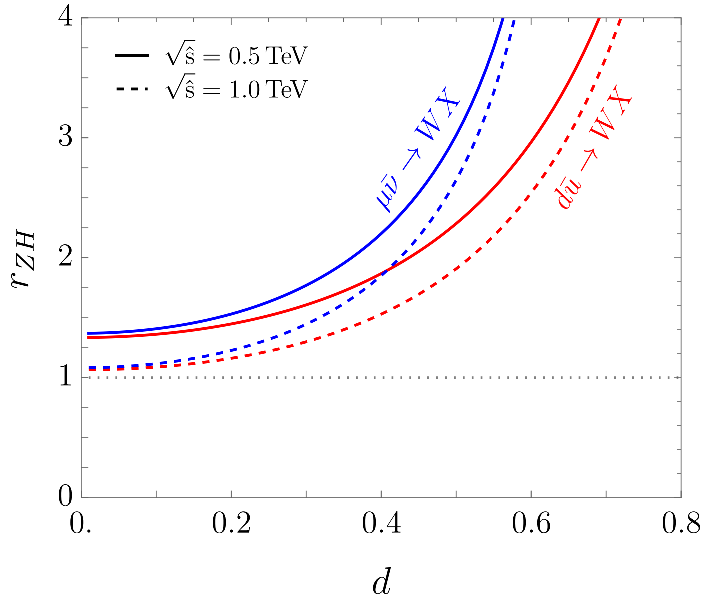


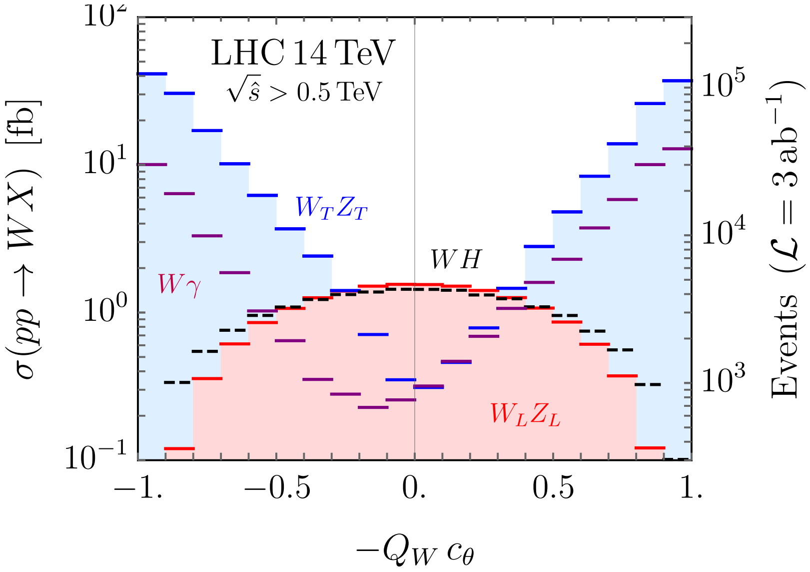
$$\text{SMEFT}: \quad C_{Hq}^{(3)} = C_{Hq}^{(1)} = C_{Hu} = (15\ \text{TeV})^{-2}$$
$$C_{HD} = (5\ \text{TeV})^{-2}$$
$$\text{HEFT}: \quad c_3 = 10^{-3},\ n_7^Q = \frac{2}{3}\times 10^{-4},\ \kappa_7 = 1$$

Radiation Amplitude Zeros



Baur, Han, Ohnemus, PRL 72 (1994) 3941
Capdevilla, Han, PRL 135 (2025) 101801





- Focusing on radiation amplitude zeros could enhance longitudinal fraction of signal.
  - d: a cut around the radiation amplitude zero.

Measurement of Longitudinals at LHC

- ATLAS has observed the polarization fractions in transverse momentum bins:

	Measurement $100 < p_T^Z \leq 200$ **GeV**	$p_T^Z > 200$ **GeV**		**Prediction** $100 < p_T^Z \leq 200$ **GeV**	$p_T^Z > 200$ **GeV**
f_{00}	$0.19 \pm^{0.03}_{0.03}$ (stat) $\pm^{0.02}_{0.02}$ (syst)	$0.13 \pm^{0.09}_{0.08}$ (stat) $\pm^{0.02}_{0.02}$ (syst)	f_{00}	0.152 ± 0.006	0.234 ± 0.007
f_{0T+T0}	$0.18 \pm^{0.07}_{0.08}$ (stat) $\pm^{0.05}_{0.06}$ (syst)	$0.23 \pm^{0.17}_{0.18}$ (stat) $\pm^{0.06}_{0.10}$ (syst)	f_{0T}	0.120 ± 0.002	0.062 ± 0.002
f_{TT}	$0.63 \pm^{0.05}_{0.05}$ (stat) $\pm^{0.04}_{0.04}$ (syst)	$0.64 \pm^{0.12}_{0.12}$ (stat) $\pm^{0.06}_{0.06}$ (syst)	f_{T0}	0.109 ± 0.001	0.058 ± 0.001
f_{00} obs (exp) sig.	5.2 (4.3) σ	1.6 (2.5) σ	f_{TT}	0.619 ± 0.007	0.646 ± 0.008

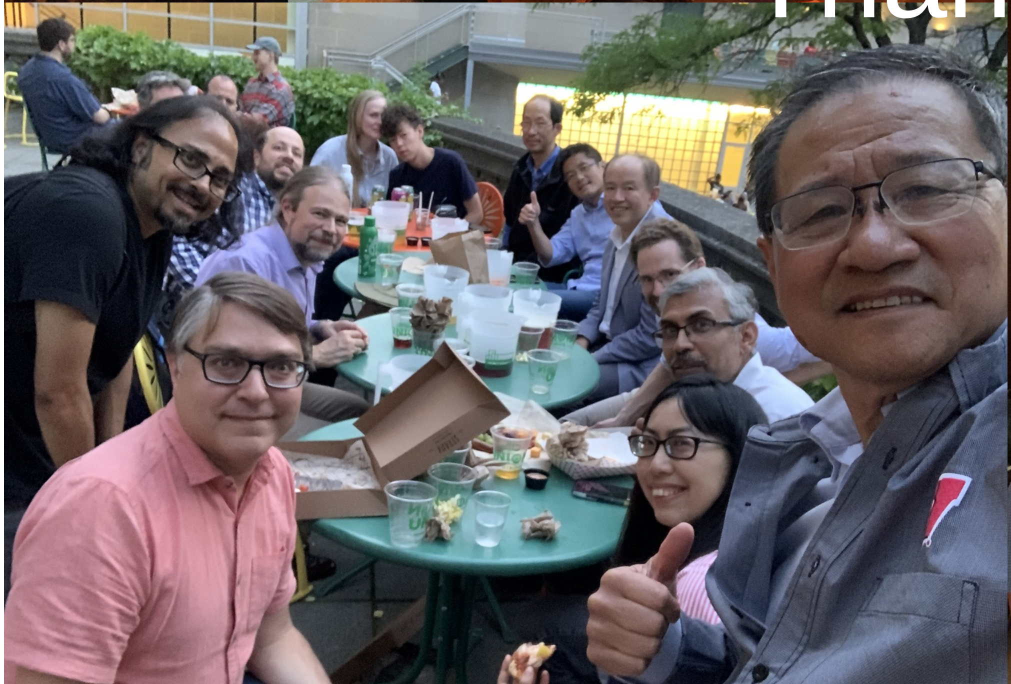
ATLAS PRL (2024) 133

- $W_L h$ is projected to be measured to 10% Colyer, Duda 2506.13002
- Ratio of $W_L Z_L / W_L h$ can be measured at the LHC.

Conclusions

- The LHC is operating at the energies necessary to thoroughly explore the electroweak sector at the scale it is broken.
- As the LHC and future colliders measure the Standard Model to ever higher energies, we should be able to start probing not only the breaking of EW symmetry but also the restoration.
- Studied EW restoration in SMEFT and HEFT.
  - Found ratios of longitudinal di-boson final states predicted to converge to one in SM and dimension-6 SMEFT, but not necessarily HEFT.
  - Found the ratio of $W_L Z_L$ and $W_L h$ production to be particularly promising both theoretically and experimentally.
- Many studies claim to distinguish HEFT and SMEFT need to compare one, two, three, etc. Higgs boson production.
  - Our study shows you can compare zero and one Higgs production to distinguish linear vs. non-linear realizations of EW symmetry.
  - Easier measurements to make and are feasible at LHC.
- Our analysis focus was on initial state quarks, but is more broadly applicable.
  - Polarized beams at lepton colliders could provide more handles on measuring the various ratios of amplitudes, particularly those that depend on the fermion helicities.

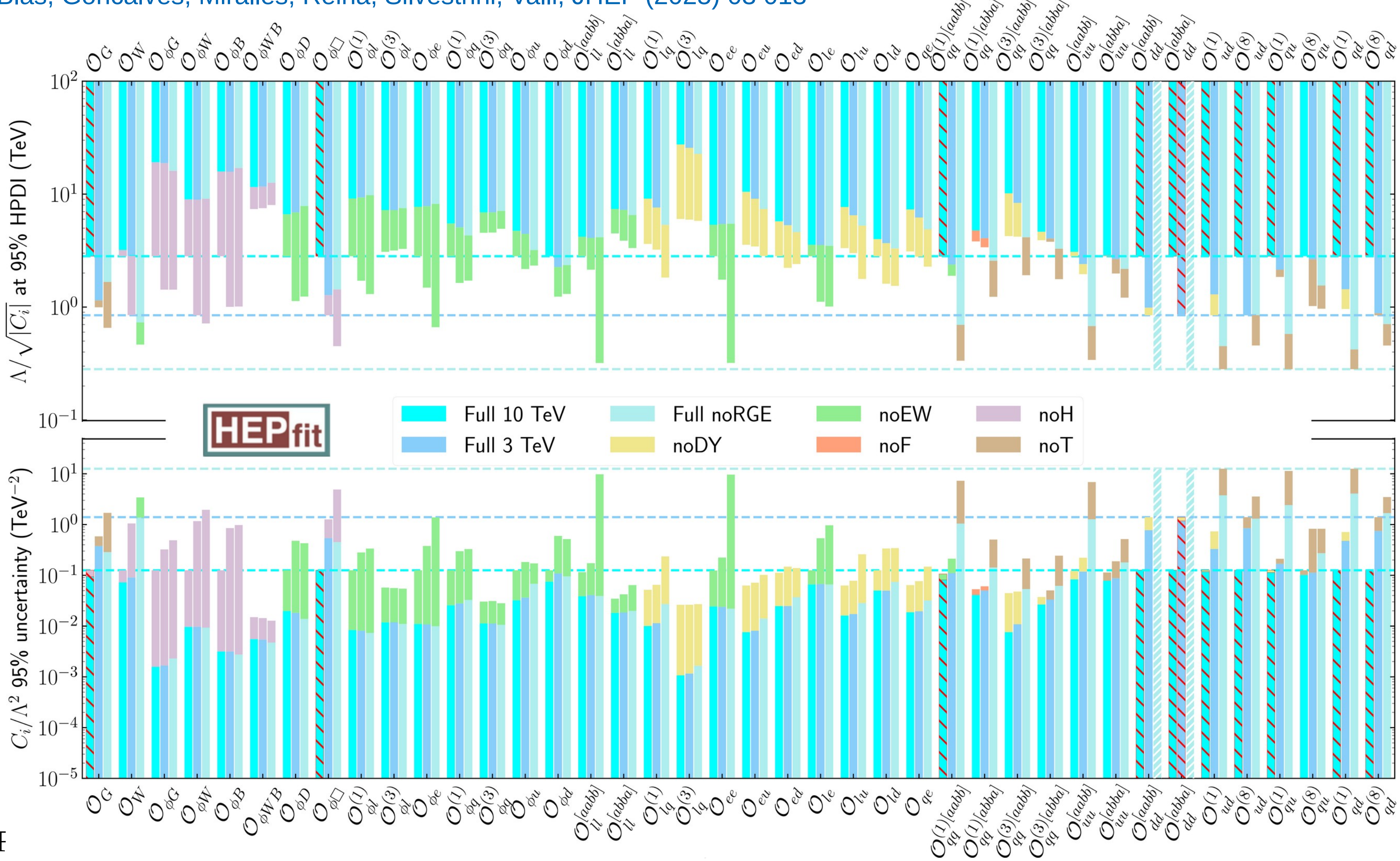
Thank You



Extra Slides

EFT Constraints

De Blas, Goncalves, Miralles, Reina, Silvestrini, Valli, JHEP (2025) 03 013



Electroweak Restoration

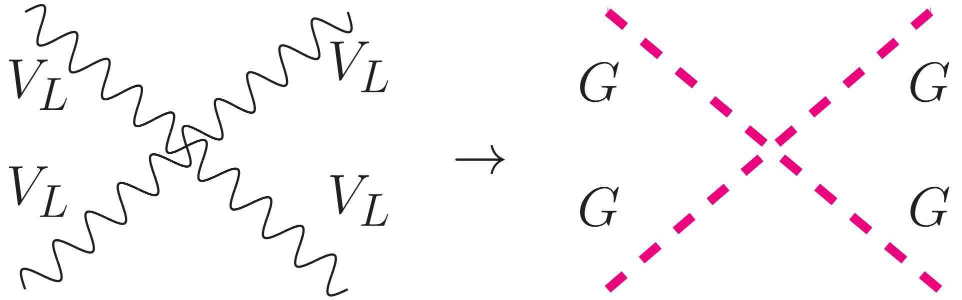
- The LHC is operating at the energies necessary to thoroughly explore the electroweak sector at the scale it is broken.
- As the LHC and future colliders measure the Standard Model to ever higher energies, we should be able to start probing not only the breaking of EW symmetry, but the restoration.
  - At high energies the SM particles are essentially massless.
  - This is equivalent to the Higgs vev going to zero (in the SM):

$$v \to 0$$

  - When the vev is zero, EW symmetry is restored.
- At colliders, EW symmetry is always broken.
  - However, the SM converges to an EW symmetric theory at high energies with corrections of order $\delta \sim v^2/E^2$
  - This convergence should be directly measurable
  - Measuring the convergence further tests our understanding of EW symmetry breaking.
- In the limit of zero vev, the longitudinal gauge bosons are replaced with the Goldstone bosons.
  - Hence, measuring EW symmetry restoration is essentially measuring the convergence of the Goldstone boson equivalence theorem
  - Nice test of our understanding of SM tests and the nature of EW symmetry.

Motivation

- Long history of using this EW restoration. Llewellyn Smith PLB46 (1973) 233; Veltman Acta Phys. Polon. (1977) 475; Lee, Quigg, Thacker PRD16 (1977) 1519; Bagger, Barger, Cheung, Gunion, Han, Ladinksy, PRD49 (1994) 1246; Han, Krohn, Wang, Zhu, JHEP03 (2010) 082; Brehmer Jaeckel, Plehn, PRD90 (2014) 054023; etc.
  - Mostly focused on longitudinal vector boson scattering.
  - By the Goldstone Boson Equivalence Theorem:



- The Goldstone boson quartic coupling arises from the Higgs potential:

$$V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 \qquad H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + i\, G^0\right) \end{pmatrix}$$

  - Hence, in principle, measuring longitudinal vector boson scattering probes the Higgs potential shape.

Kinetic Term

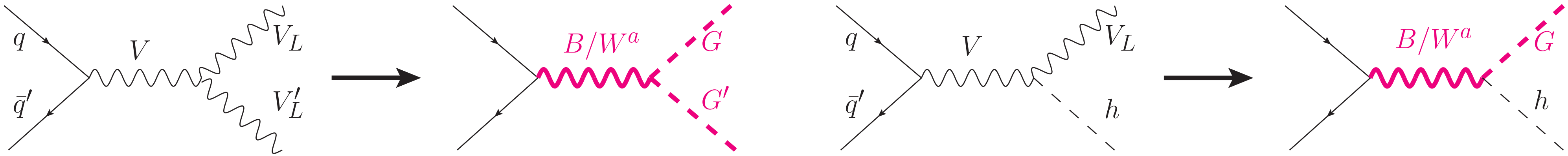
- The kinetic terms gives rise to the interactions with Goldstone bosons.

$$\mathcal{L}_{\rm kin} = |D_\mu H|^2 \qquad\qquad H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + i\,G^0\right) \end{pmatrix}$$

- These can contribute to di-boson production:

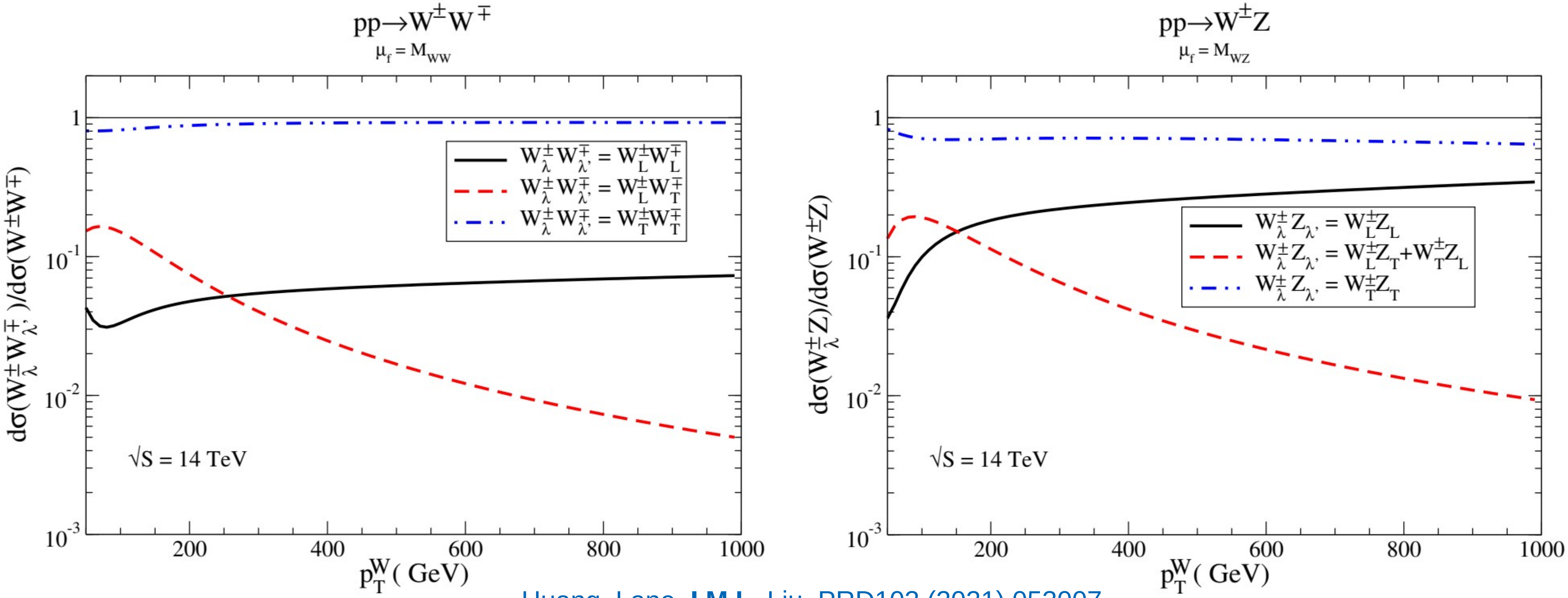
$$Z - G^0 - h,\; W^\pm - G^\mp - h \qquad Z/\gamma - G^+ - G^-,\; W^\pm - G^\mp - G^0.$$

- Can produce Goldstone's in s-channel processes with quark initial states.
  - Can search for them in $q\bar{q}' \to VV'$ and $q\bar{q}' \to Vh$ $\;V = W^\pm, Z$



- Note:
  - Can directly test the Goldstone Boson Equivalence Theorem and observe EW restoration.
  - Gain in rates at the LHC

Polarization Fractions for WW/WZ



Huang, Lane, **I.M.L**. Liu, PRD103 (2021) 053007

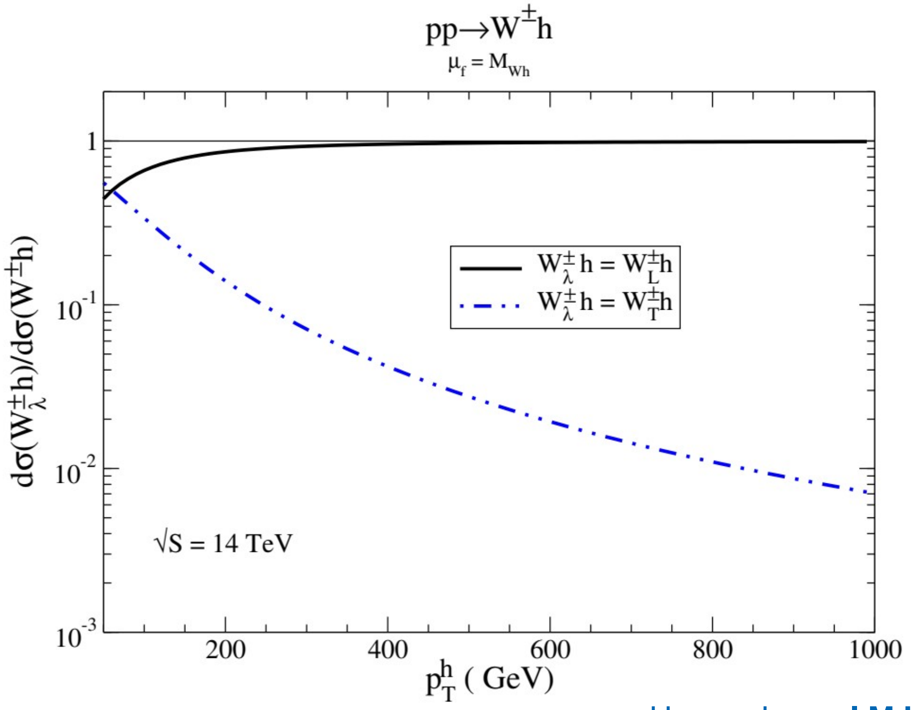
Black: Longitudinal+Longitudinal

Red: Longitudinal+Transverse

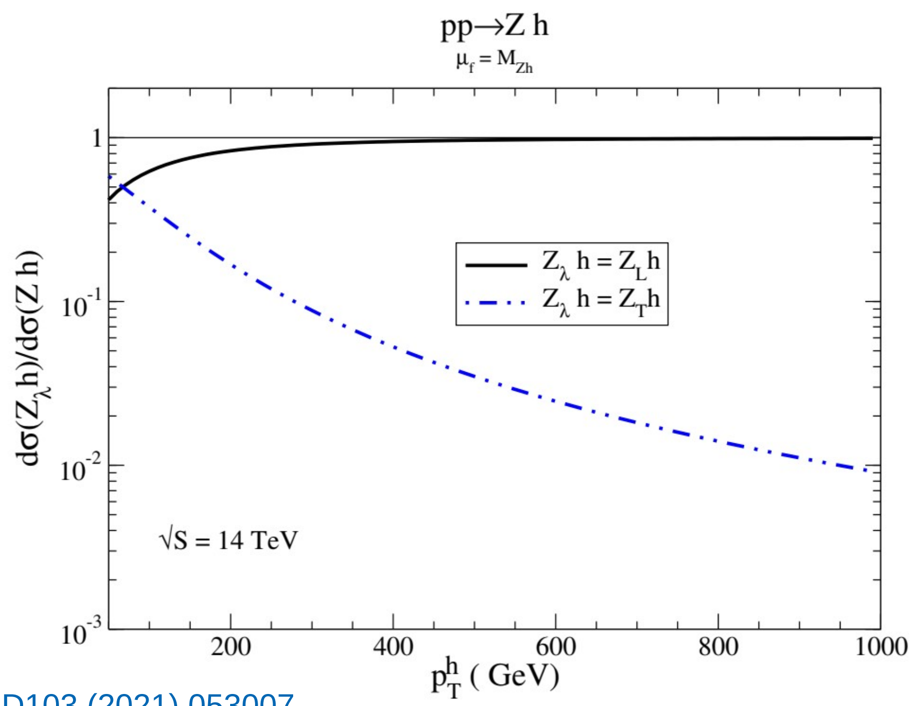
Blue:Transverse+Transverse

Wh/Zh Polarization Fractions

Wh Production



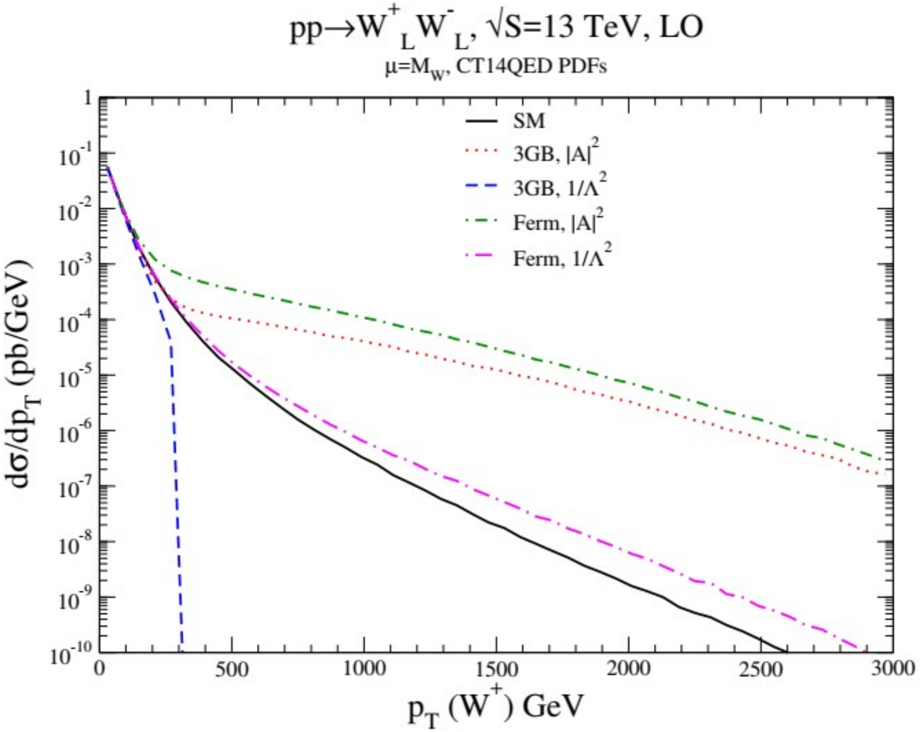
Zh Production

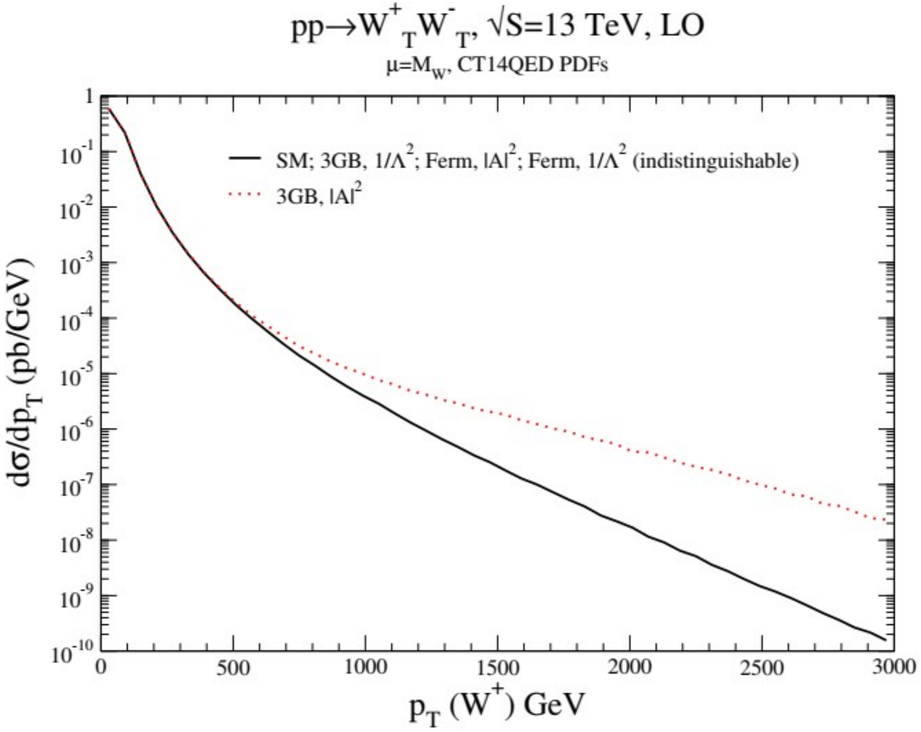


Huang, Lane, **I.M.L**. Liu, PRD103 (2021) 053007

- Black: Longitudinal
- Blue: Transverse

WW/WZ Polarizations in EFT

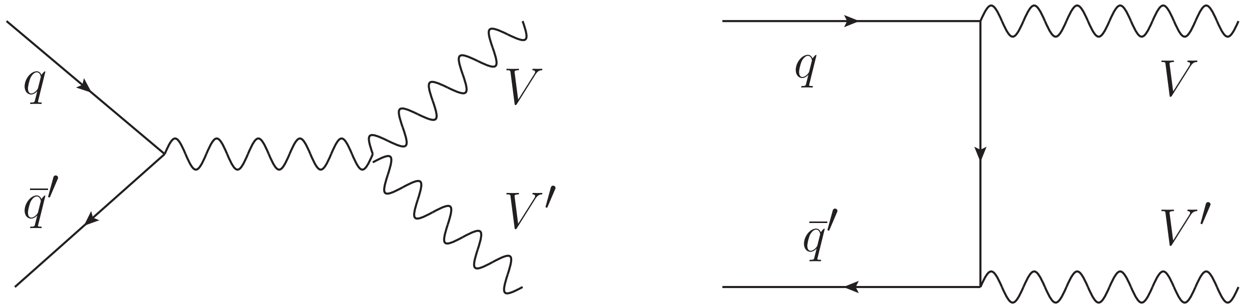




Baglio, Dawson, **I.M.L.**, PRD96 (2017) 073003

- The previous conclusions about longitudinal dominance are for the SM.
  - Picture changes in SM Effective Field Theory.
  - For our purposes, we will focus observing EW restoration in the SM for now.

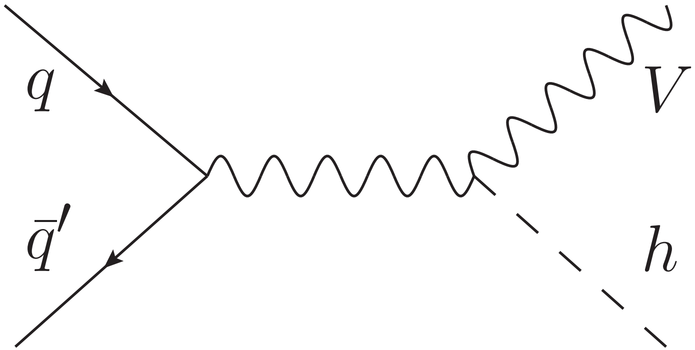
WW/WZ Production



- Issues:
  - Dominated by transverse polarizations even to very high energies. Baglio, Dawson, I.M. Lewis, PRD96 (2017) 073003; Baglio, Le Duc, JHEP04 (2019) 065; Denner Pellicciolli JHEP09 (2020) 164
  - There is no perturbative unitarity violation in these processes in the SM, even without a Higgs.
  - The transverse polarizations exist in the symmetric phase of the SM, so do not decouple.
  - Indeed the problem is that there is a t-channel contribution from opposite helicity case that does not decouple.
- A solution would be to tag polarizations to get a signal rich in longitudinal polarizations.
  - Much work on defining polarizations after decays, calculating higher order corrections, and tagging polarizations Liu, Wang PRD99 (2019) 055001; Panico, Riva, Wulzer PLB776 (2018) 473; Kim, Martin arXiv:2102.05124;; Baglio, Le Duc, JHEP04 (2019) 065; Denner Pellicciolli JHEP09 (2020) 164; etc.
  - Fully longitudinal production of WZ has been observed at 7σ at the LHC ATLAS PRL133 (2024) 101802
  -

Wh/Zh Production

- Simpler solution: find di-boson process naturally longitudinally dominated:
  - There is no h-V-V coupling in unbroken phase.
  - Transverse modes do not survive.



$$\mathcal{A}(q_+\bar{q}_- \to Z_L h) = \pm i\, \frac{e^2\, g_R^{qZ}}{2\, c_W^2\, s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

$$\mathcal{A}(q_-\bar{q}_+ \to Z_L h) = \pm i\, \frac{e^2\, g_L^{qZ}}{2\, c_W^2\, s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

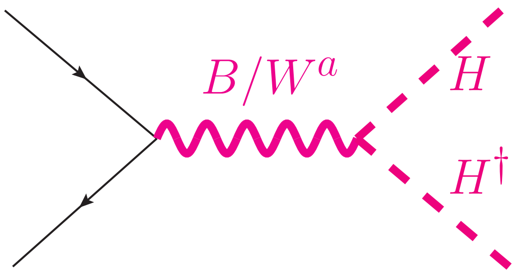
$$\mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm h) = -i\, \frac{e^2}{2\sqrt{2}\, s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

$$\mathcal{A}(q_\pm\bar{q}_\mp \to Z_\pm h) \sim \mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm h) \sim \mathcal{O}(\hat{s}^{-1/2}),$$

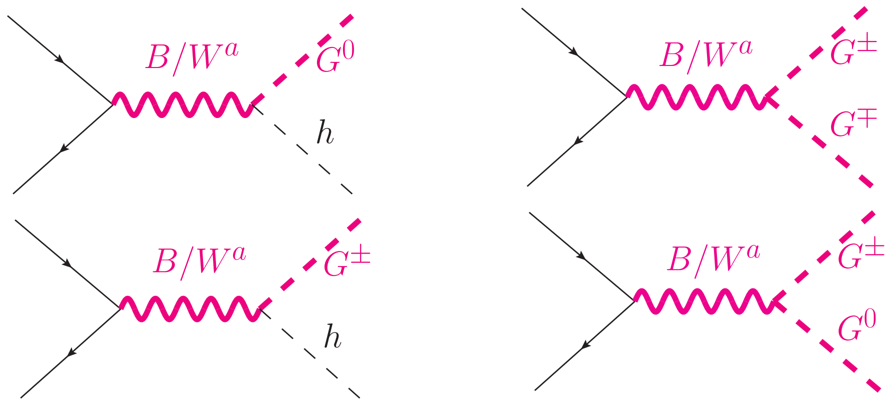
$$\mathcal{A}(q_+\bar{q}'_- \to W_\pm^\pm h) = \mathcal{A}(q_+\bar{q}'_- \to W_\mp^\pm h) = 0.$$

Compare to Goldstone production

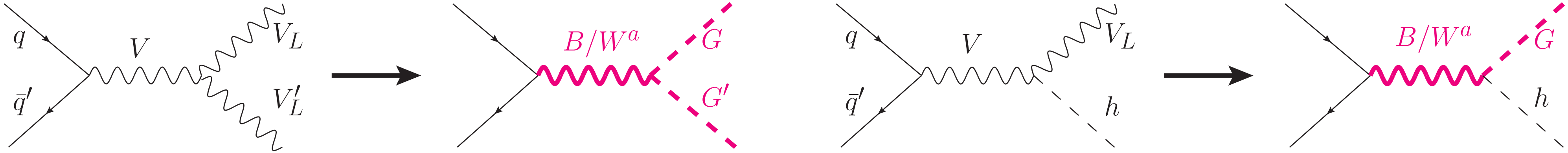
- In principle, should calculate production of Higgs doublet through massless hypercharge and $SU(2)_L$ gauge boson exchange.:



- Once decays are accounted for, detectors can separate electric charge.
  - Gauge and Higgs boson propagate on mass shell where $SU(2)_L xU(1)$ is broken
- So, we calculate component-by-component in the doublet:
  - Start with unbroken theory.
  - For Goldstone calculation, massless $SU(2)_L xU(1)$ gauge bosons used as intermediate particles.
  - Project onto charge eigenstates.



Electroweak Restoration in the SM



- Unbroken phase: Hypercharge and $SU(2)_L$ gauge boson exchange
  - s-channel Hypercharge exchange proceed through the product of two $SU(2)_L$ singlet currents:

$$J^{\mu}_{Q} J_{H,\mu} \qquad J^{\mu}_{q} J_{H,\mu} \qquad J_{Q,\mu} = \bar{Q}_L \gamma^{\mu} Q_L \qquad J_{q,\mu} = \bar{q}_R \gamma^{\mu} q_R \qquad J_{H,\mu} = H^{\dagger} i \overleftrightarrow{\partial}_{\mu} H$$

  - $SU(2)_L$ gauge boson exchange proceeds through product of two SU(2)L triple currents:

$$J^{I,\mu}_{Q} J^{I}_{H,\mu} \qquad J^{I}_{Q,\mu} = \bar{Q}_L \gamma_{\mu} \sigma^{I} Q_L . \qquad J^{I}_{H,\mu} = H^{\dagger} \sigma^{I}\, i \overleftrightarrow{\partial}_{\mu} H,$$

- Broken phase: predictions of of interactions between charge eigenstates
  - Hypercharge neutral current:

$$J^{\mu}_{Q} J_{H,\mu} \supset i\, \bar{q}_L \gamma^{\mu} q_L \left[G^{-} \overleftrightarrow{\partial}_{\mu} G^{+} - i\, G^{0} \overleftrightarrow{\partial}_{\mu} h \right]$$

$$J^{\mu}_{q} J_{H,\mu} \supset i\, \bar{q}_R \gamma^{\mu} q_R \left[G^{-} \overleftrightarrow{\partial}_{\mu} G^{+} - i\, G^{0} \overleftrightarrow{\partial}_{\mu} h \right]$$

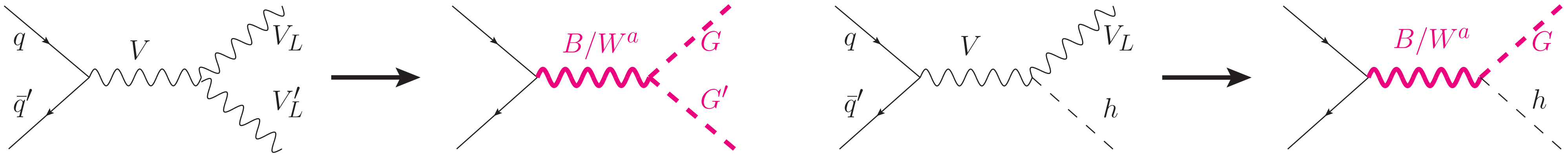
  - $SU(2)_L$ charged and neutral currents:
  - Red: appear in both $U(1)_Y$ and $SU(2)_L$
  - Blue: $SU(2)_L$ only
  - Yellow: $U(1)_Y$ only

$$J^{I,\mu}_{Q} J^{I}_{H,\mu} = J^{+,\mu}_{Q} J^{-}_{H,\mu} + J^{-,\mu}_{Q} J^{+}_{H,\mu} + J^{0,\mu}_{Q} J^{0}_{H,\mu}$$

$$J^{\mp,\mu}_{Q} J^{\pm}_{H,\mu} \supset i \sqrt{2}\, \bar{q}'_L \gamma^{\mu} q_L \left[\mp G^{\mp} \overleftrightarrow{\partial}_{\mu} G^{0} + i\, G^{\mp} \overleftrightarrow{\partial}_{\mu} h \right]$$

$$J^{0,\mu}_{Q} J^{0}_{H,\mu} \supset i\, 2\, T^{q}_{3} \bar{q}_L \gamma^{\mu} q_L \left[G^{-} \overleftrightarrow{\partial}_{\mu} G^{+} + i\, G^{0} \overleftrightarrow{\partial}_{\mu} h \right] .$$

EW Restoration in the SM: Amplitude Relations



- Only $SU(2)_L$ contributed to left-handed initial state quarks with charged final states, and one combination of charged neutral final states:

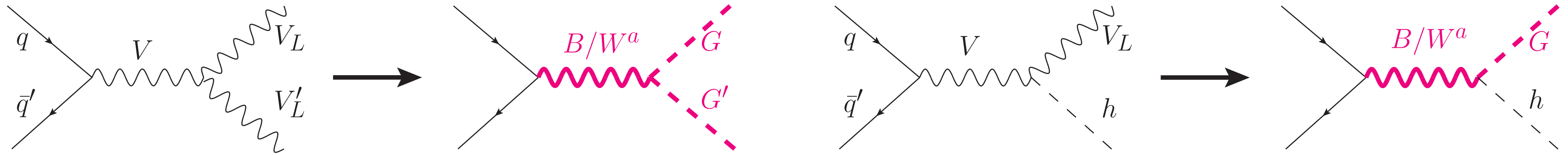
  – $J_Q^{\mp,\mu} J_{H,\mu}^{\pm} \supset i\sqrt{2}\,\bar{q}'_L\gamma^\mu q_L\left[\mp G^\mp \overleftrightarrow{\partial}_\mu G^0 + i\,G^\mp \overleftrightarrow{\partial}_\mu h\right] : \mp\sqrt{2}\mathcal{M}_{\rm SM}(q_-\bar{q}'_+ \to G^\pm(p_1)G^0(p_2)) + i\sqrt{2}\mathcal{M}_{\rm SM}(q_-\bar{q}'_+ \to G^\pm(p_1)h(p_2))$

  – $J_Q^{0,\mu} J_{H,\mu}^{0} \supset i\,2\,T_3^q\bar{q}_L\gamma^\mu q_L\left[G^- \overleftrightarrow{\partial}_\mu G^+ + i\,G^0 \overleftrightarrow{\partial}_\mu h\right] : 2\,T_3^q\left[\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to G^+(p_1)G^-(p_2)) + i\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to G^0(p_1)h(p_2))\right]$

- All have same source, should be proportional:

$$\mp\sqrt{2}\mathcal{M}_{\rm SM}(q_-\bar{q}'_+ \to G^\pm(p_1)G^0(p_2)) = \sqrt{2}i\,\mathcal{M}_{\rm SM}(q_-\bar{q}'_+ \to G^\pm(p_1)h(p_2))\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right]$$
$$= 2\,T_3^q\left[\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to G^+(p_1)G^-(p_2)) + i\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to G^0(p_1)h(p_2)\right]\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right].$$

EW Restoration in the SM: Amplitude Relations



- Only Hypercharge contributes to right-handed initial state quarks, and one combination of charged neutral final states from left handed initial state quarks:

  - $J_q^\mu J_{H,\mu} \supset i\,\bar q_R \gamma^\mu q_R \left[G^- \overleftrightarrow{\partial}_\mu G^+ - i\,G^0 \overleftrightarrow{\partial}_\mu h\right]$: $\mathcal{M}_{\rm SM}(q_+\bar q_- \to G^+(p_1)G^-(p_2)) - i\mathcal{M}_{\rm SM}(q_+\bar q_- \to G^0(p_1)h(p_2))$

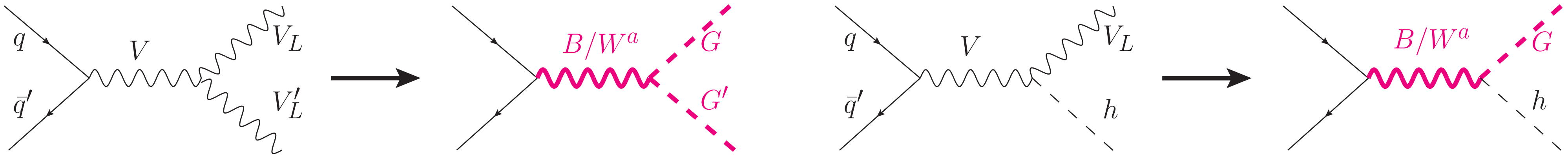
  - $J_Q^\mu J_{H,\mu} \supset i\,\bar q_L \gamma^\mu q_L \left[G^- \overleftrightarrow{\partial}_\mu G^+ - i\,G^0 \overleftrightarrow{\partial}_\mu h\right]$: $\mathcal{M}_{\rm SM}(q_-\bar q_+ \to G^+(p_1)G^-(p_2)) - i\mathcal{M}_{\rm SM}(q_-\bar q_+ \to G^0(p_1)h(p_2))$

- All have same source, should be proportional:

$$\mathcal{M}_{\rm SM}(q_+\bar q_- \to G^+(p_1)G^-(p_2)) = -i\,\mathcal{M}_{\rm SM}(q_+\bar q_- \to G^0(p_1)h(p_2))\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right]$$

- Use Goldstone boson equivalence theorem to relate amplitude proportionality relationships to amplitudes of physical gauge bosons:

EW Restoration in the SM: Amplitude Relations



- Use those proportionality relations for products of currents with Goldstone Boson Equivalence Theorem to find a set of relations between physical fields:

$$\mathcal{M}_{\rm SM}(q_+\bar{q}_- \to W_L^+(p_1)W_L^-(p_2)) = \mathcal{M}_{\rm SM}(q_+\bar{q}_- \to Z_L(p_1)h(p_2))\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right]$$

$$\pm\sqrt{2}\mathcal{M}_{\rm SM}(q_-\bar{q}'_+ \to W_L^\pm(p_1)Z_L(p_2)) = \sqrt{2}\mathcal{M}_{\rm SM}(q_-\bar{q}'_+ \to W_L^\pm(p_1)h(p_2))\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right]$$

$$= 2\,T_3^q\left[\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to Z_L(p_1)h(p_2)) - \mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to W_L^+(p_1)W_L^-(p_2))\right]\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right]$$

- Ratio of two linear combinations of neutral final states should be proportional to charges and coupling constants:

$$\frac{\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to Z_L(p_1)h(p_2)) - \mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to W_L^+(p_1)W_L^-(p_2))}{\mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to Z_L(p_1)h(p_2)) + \mathcal{M}_{\rm SM}(q_-\bar{q}_+ \to W_L^+(p_1)W_L^-(p_2))} = \pm\frac{g^2 T_3^q}{g'^2 Y_L^Q} + \mathcal{O}\left(\frac{m}{\sqrt{s}}\right)$$

High Energy Ratios of Di-Boson Amplitudes

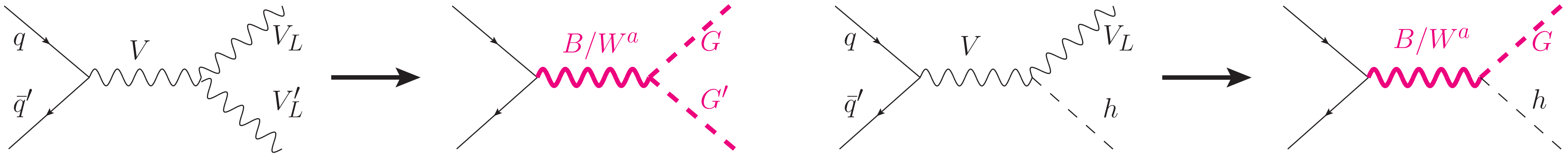
I.M.L. Liu, Mahbub, 2605.xxxxx

- This analysis explains all of the SM ratios.
  - Explicit calculation completely understood by EW symmetry restoration and the Goldstone boson equivalence theorem.
- What about SMEFT?

To $\mathcal{O}(m^2/s)$	SM	SMEFT	HEFT
$\frac{\mathcal{M}(q_-\bar{q}'_+ \to W^\pm_L Z_L)}{\mathcal{M}(q_-\bar{q}'_+ \to W^\pm_L h)}$	± 1	± 1	$\mp \frac{c_3 g - 8\pi\,(n_1^Q + n_7^Q)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W^+_L W^-_L)}{\sqrt{2}\,\mathcal{M}(q_-\bar{q}'_+ \to W^\pm_L h)}$	$2\,T_3^q$	$2\,T_3^q$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7 - 3)n_7^Q\right]\right)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}_- \to W^+_L W^-_L)}{\mathcal{M}(q_+\bar{q}_- \to Z_L h)}$	1	1	$\frac{n_6^Q + T_3^q(n_2^Q + n_8^Q)}{\kappa_6 n_6^Q + T_3^q(\kappa_2 n_2^Q + \kappa_8 n_8^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}'_- \to W^\pm_L Z_L)}{\mathcal{M}(q_+\bar{q}'_- \to W^\pm_L h)}$	$-$	∓ 1	$\mp \frac{n_2^Q - n_8^Q \mp 2\,i\,n_4^Q}{\kappa_2 n_2^Q - \kappa_8 n_8^Q \mp 2\,i\,\kappa_4 n_4^Q}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W^+_L W^-_L)}{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) + \mathcal{M}(q_-\bar{q}_+ \to W^+_L W^-_L)}$	$-\frac{g^2 T_3^q}{g'^2 Y_L^q}$	$-2\,T_3^q \frac{C_{Hq}^{(3)}}{C_{Hq}^{(1)}}$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7 - 3)n_7^Q\right]\right)}{8\pi(1+\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1-\kappa_1)n_1^Q - (\kappa_7 + 3)n_7^Q\right]\right)}$

TABLE II: Selected ratios of amplitudes in the SM, SMEFT, and HEFT in the high energy limit, shown up to $\mathcal{O}(m^2/s)$. The dash $-$ indicates that in the massless quark limit the numerator and denominator are both zero.

EW Restoration in the SMEFT: Amplitude Relations



- SMEFT operators with energy growth are all products of currents
- Two changes from the SM:
  - Ratios of singlet and triplet current with left-handed initial state quarks now depend on Wilson Coefficients:

$$\frac{\mathcal{M}_{\rm SMEFT}(q_-\bar{q}_+\to Z_L(p_1)h(p_2))-\mathcal{M}_{\rm SMEFT}(q_-\bar{q}_+\to W^+_L(p_1)W^-_L(p_2))}{\mathcal{M}_{\rm SMEFT}(q_-\bar{q}_+\to Z_L(p_1)h(p_2))+\mathcal{M}_{\rm SMEFT}(q_-\bar{q}_+\to W^+_L(p_1)W^-_L(p_2))}=\pm 2\,T_3^q\frac{C^{(3)}_{Hq}}{C^{(1)}_{Hq}}+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)$$

  - Have a contribution to the right-handed charged current with a new relationship from Q_{Hud}:

$$\mathcal{M}_{\rm SMEFT}(q_+\bar{q}'_-\to W^\pm_L(p_1)h(p_2))=\mp\mathcal{M}_{\rm SMEFT}(q_+\bar{q}'_-\to W^\pm_L(p_1)Z_L(p_2))\left[1+\mathcal{O}\left(\frac{m}{\sqrt{s}}\right)\right]$$

  - Q_{HD} doesn't play a role in the energy growing behavior.

$$\mathcal{Q}_{Hq}=(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_R\gamma^\mu q_R)\,,$$
$$\mathcal{Q}^{(1)}_{Hq}=(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)\,,$$
$$\mathcal{Q}^{(3)}_{Hq}=(H^\dagger i\overleftrightarrow{D}{}^I_\mu H)(\bar{Q}_L\sigma^I\gamma^\mu Q_L)\,,$$
$$\mathcal{Q}_{Hud}=(\widetilde{H}^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu d_R)\,,$$
$$\mathcal{Q}_{HD}=(D^\mu H^\dagger H)(H^\dagger D_\mu H)\,,$$

High Energy Ratios of Di-Boson Amplitudes

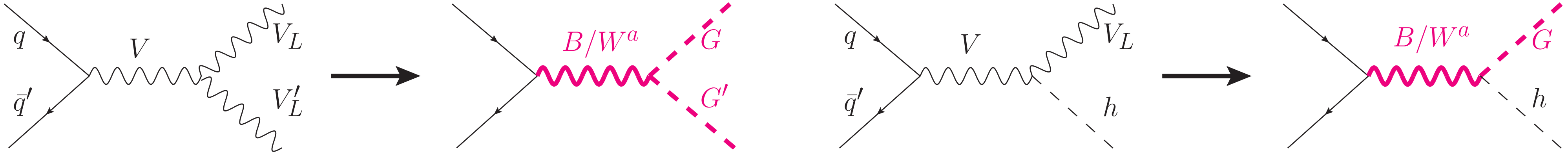
I.M.L. Liu, Mahbub, 2605.xxxxx

- SM and SMEFT behavior is predicted from EW restoration and the Goldstone boson equivalence theorem.
- HEFT can be tuned to reproduce the SM and SMEFT.
  - SM and dimension-6 SMEFT require no tuning to have these ratios converge to one.
- To fully test EW restoration, would want to measure all these ratios.
- Realistic colliders present (at least) two obstacles:
  - Some ratios include linear combinations of different final states.
  - Some ratios depend on initial state quark couplings and helicities.
- Ratios of WZ to Wh is promising because the ratios do not depend on initial state quark helicities

To $\mathcal{O}(m^2/s)$	SM	SMEFT	HEFT
$\frac{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm Z_L)}{\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	± 1	± 1	$\mp \frac{c_3 g - 8\pi\,(n_1^Q + n_7^Q)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\sqrt{2}\,\mathcal{M}(q_-\bar{q}'_+ \to W_L^\pm h)}$	$2\,T_3^q$	$2\,T_3^q$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7 - 3)n_7^Q\right]\right)}{8\pi\,(\kappa_1 n_1^Q - \kappa_7 n_7^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}_- \to W_L^+ W_L^-)}{\mathcal{M}(q_+\bar{q}_- \to Z_L h)}$	1	1	$\frac{n_6^Q + T_3^q(n_2^Q + n_8^Q)}{\kappa_6 n_6^Q + T_3^q(\kappa_2 n_2^Q + \kappa_8 n_8^Q)}$
$\frac{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm Z_L)}{\mathcal{M}(q_+\bar{q}'_- \to W_L^\pm h)}$	$-$	∓ 1	$\mp \frac{n_2^Q - n_8^Q \mp 2\,i\,n_4^Q}{\kappa_2 n_2^Q - \kappa_8 n_8^Q \mp 2\,i\,\kappa_4 n_4^Q}$
$\frac{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) - \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}{\mathcal{M}(q_-\bar{q}_+ \to Z_L h) + \mathcal{M}(q_-\bar{q}_+ \to W_L^+ W_L^-)}$	$-\frac{g^2 T_3^q}{g'^2 Y_L^q}$	$-2\,T_3^q\,\frac{C_{Hq}^{(3)}}{C_{Hq}^{(1)}}$	$-\frac{8\pi(1-\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1+\kappa_1)n_1^Q + (\kappa_7 - 3)n_7^Q\right]\right)}{8\pi(1+\kappa_5)n_5^Q + T_3^q\left(c_3 g - 8\pi\left[(1-\kappa_1)n_1^Q - (\kappa_7 + 3)n_7^Q\right]\right)}$

TABLE II: Selected ratios of amplitudes in the SM, SMEFT, and HEFT in the high energy limit, shown up to $\mathcal{O}(m^2/s)$. The dash $-$ indicates that in the massless quark limit the numerator and denominator are both zero.

EW Restoration in the HEFT: Amplitude Relations



- HEFT is not expected to obey the same relationships between longitudinal gauge boson and Higgs production unless there is a tuning: the Higgs boson is a singlet.
- Would expect relationships among Goldstone bosons.
  - From explicit calculations we derive

$$\mathcal{M}_{\rm HEFT}(q_-\bar{q}'_+ \to W_L^+W_L^-) = \left(\mp\sqrt{2}T_3^q\mathcal{M}_{\rm HEFT}(q_-\bar{q}'_+ \to W_L^\pm Z_L) + 4\sqrt{2}G_F s\left(n_5^Q + 4\,T_3^q\, n_7^Q\right)\,\sin\theta\right) \times\left[1+\mathcal{O}\left(\frac{m^2}{s}\right)\right].$$

$$\mathcal{M}_{\rm HEFT}(q_+\bar{q}'_- \to W_L^+W_L^-) = \left(\mp\sqrt{2}T_3^q\mathcal{M}_{\rm HEFT}(q_+\bar{q}'_- \to W_L^\pm Z_L) - 4\sqrt{2}\,G_F s\left(n_6^Q + 2\,T_3^q\, n_8^Q \pm 2\,i\,T_3^q\, n_4^Q\right)\,\sin\theta\right) \times\left[1+\mathcal{O}\left(\frac{m^2}{s}\right)\right].$$

  - Relationships are broken by operators breaking custodial symmetry and having energy growth.

High Energy Behavior of Partonic Cross Sections

- Specialized to up-type quark initial states.
  - Initial state quark helicity summed.
- Investigate 3 independent ratios:

$$\hat{r}_{Zh} = \frac{\sigma(u\bar{d} \to W^+ Z)}{\sigma(u\bar{d} \to W^+ h)}$$

$$\hat{r}_{Wh} = \frac{\sigma(u\bar{u} \to W^+ W^-)}{\sigma(u\bar{d} \to W^+ h)}$$

$$\hat{r}_{WZ} = \frac{\sigma(u\bar{u} \to Zh)}{\sigma(u\bar{d} \to W^+ h)}.$$

- Cross sections can depend on the square of the Wilson coefficients.

$$\mathcal{A}_{\rm SMEFT} \sim \mathcal{A}_{\rm ren} + \frac{1}{\Lambda^2}\mathcal{A}_{6,{\rm SMEFT}} + \frac{1}{\Lambda^4}\mathcal{A}_{8,{\rm SMEFT}} + \mathcal{O}(\Lambda^{-6})$$

  - Formally the square of dimension-6 occurs at dimension-8. For EFT to be valid, need to check that dimension-6 squared is subleading.

$$|\mathcal{A}_{\rm SMEFT}|^2 \sim |\mathcal{A}_{\rm ren}|^2 + \frac{1}{\Lambda^2}\mathcal{A}_{\rm ren}\mathcal{A}_{6,{\rm SMEFT}} + \frac{1}{\Lambda^4}|\mathcal{A}_{6,{\rm SMEFT}}|^2 + \frac{1}{\Lambda^4}\mathcal{A}_{\rm ren}\mathcal{A}_{8,{\rm SMEFT}} + \mathcal{O}(\Lambda^{-6}).$$

- Choose parameter points are chosen such that the quadratic contribution of Wilson coefficients to cross sections are at most 10% of the linear contribution at 3 TeV. Consider one operator at a time.

$$\begin{aligned}
{\rm SMEFT:}\quad & C_{Hq}^{(3)} = C_{Hq}^{(1)} = C_{Hu} = (15~{\rm TeV})^{-2} \\
& C_{HD} = (5~{\rm TeV})^{-2} \\
{\rm HEFT:}\quad & c_3 = 10^{-3},\ n_7^Q = \frac{2}{3}\times 10^{-4},\ \kappa_7 = 1
\end{aligned}$$

LHC Cross Section

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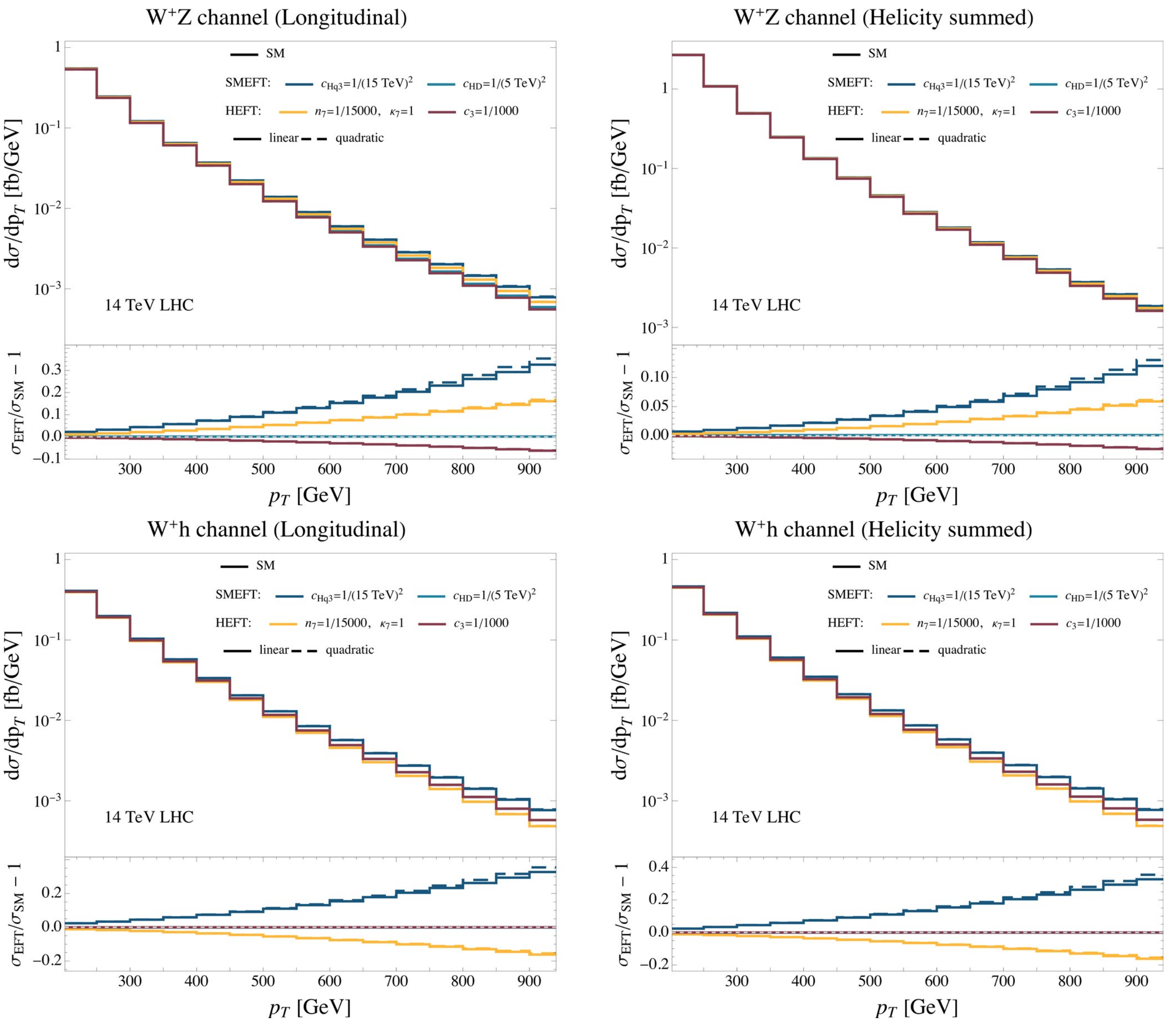
- Convolved with pdfs
- Focus on WZ vs. Wh
- Both HEFT and SMEFT have similar deviations from SM in the individual channels.
- For longitudinal polarizations, those deviations are correlated in SMEFT.
- EFT is well under control for these parameter points.
- Blue: SMEFT
- Orange, maroon: HEFT



May 15, 2026
Tao Fest

Projected Parameter Fits

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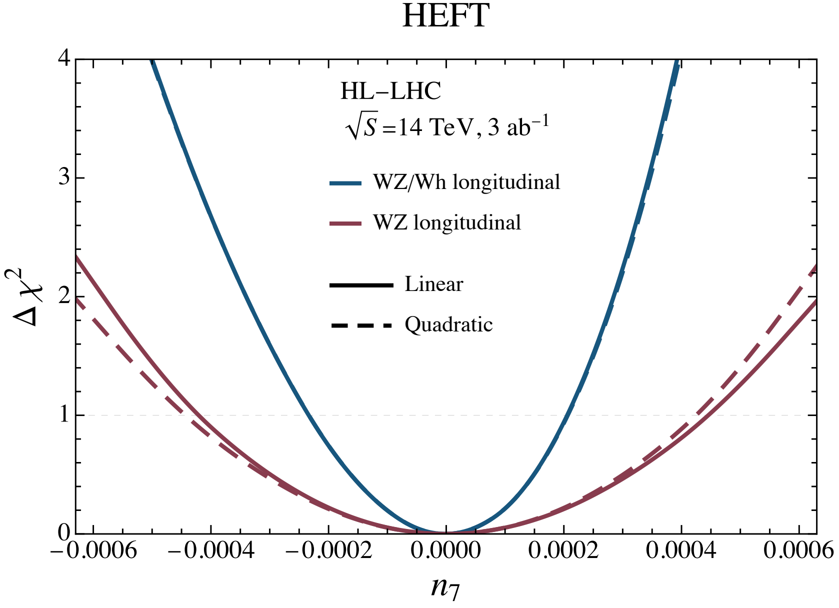
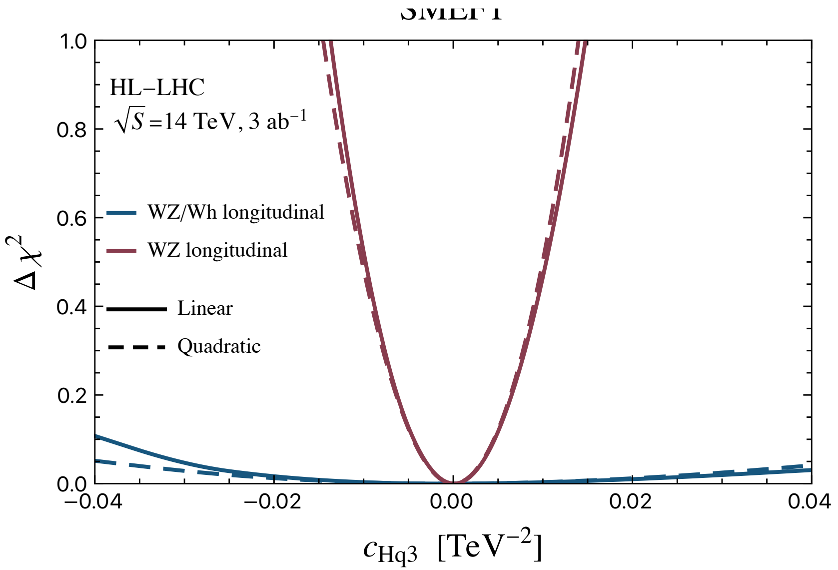
- WZ only: uses on the the $W_L Z_L$ measurements from ATLAS to project to HL-LHC
- WZ/Wh: Projects a ratio measurement
- SMEFT:
  - Lose information from ratio because it converges to one.
- HEFT:
  - Ratio does better because n7 has different interference pattern in $W_L Z_L$ and $W_L h$.





Projected Parameter Fits

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- c_3: Only contributes to W_LZ_L, so ratio add no new information.
- C_{HD}: custodial symmetry breaking.
  - No energy growth, not very good constraints.
- To distinguish linear and non-linear realizations, still want to compare the W_LZ_L and W_Lh rates at high energy.

Observable	EFT treatment	68% CL	95% CL
SMEFT coefficients			
$C_{Hq}^{(3)}$ [TeV^{-2}]			
$W^\pm Z$ only	Linear	$(-1.37,\ 1.47)\times 10^{-2}$	$(-2.66,\ 3.05)\times 10^{-2}$
$W^\pm Z$ only	Quadratic	$(-1.46,\ 1.40)\times 10^{-2}$	$(-3.01,\ 2.76)\times 10^{-2}$
$W^\pm Z/W^\pm h$ ratio	Linear	$(-1.3,\ 4.5)\times 10^{-1}$	$(-2.3,\ 9.3)\times 10^{-1}$
$W^\pm Z/W^\pm h$ ratio	Quadratic	$(-2.7,\ 3.2)\times 10^{-1}$	$(-5.5,\ 6.5)\times 10^{-1}$
C_{HD} [TeV^{-2}]			
$W^\pm Z$ only	Linear	$(-9.18,\ 10.41)$	$(-18.90,\ 20.66)$
$W^\pm Z$ only	Quadratic	$(-9.18,\ 11.89)$	$(-17.36,\ 23.80)$
$W^\pm Z/W^\pm h$ ratio	Linear	$(-2.16,\ 1.98)$	$(-4.34,\ 3.84)$
$W^\pm Z/W^\pm h$ ratio	Quadratic	$(-2.16,\ 1.97)$	$(-4.34,\ 3.86)$
HEFT coefficients			
n_7^Q $(\kappa_7 = 1)$			
$W^\pm Z$ only	Linear	$(-4.19,\ 4.45)\times 10^{-4}$	$(-8.18,\ 9.12)\times 10^{-4}$
$W^\pm Z$ only	Quadratic	$(-4.43,\ 4.23)\times 10^{-4}$	$(-9.09,\ 8.36)\times 10^{-4}$
$W^\pm Z/W^\pm h$ ratio	Linear	$(-3.24,\ 2.76)\times 10^{-4}$	$(-7.11,\ 5.07)\times 10^{-4}$
$W^\pm Z/W^\pm h$ ratio	Quadratic	$(-3.24,\ 2.77)\times 10^{-4}$	$(-7.17,\ 5.15)\times 10^{-4}$
c_3			
$W^\pm Z$ only	Linear	$(-1.6,\ 1.6)\times 10^{-2}$	$(-3.4,\ 3.1)\times 10^{-2}$
$W^\pm Z$ only	Quadratic	$(-1.6,\ 1.6)\times 10^{-2}$	$(-3.2,\ 3.5)\times 10^{-2}$
$W^\pm Z/W^\pm h$ ratio	Linear	$(-2.4,\ 2.4)\times 10^{-2}$	$(-5.0,\ 5.0)\times 10^{-2}$
$W^\pm Z/W^\pm h$ ratio	Quadratic	$(-2.3,\ 2.7)\times 10^{-2}$	$(-4.3,\ 6.4)\times 10^{-2}$

Electroweak Restoration

- These ideas have been important for developing parton showers including EW bosons Cuomo, Vecchi, Wulzer, SciPost 8 (2020) 078; Hook, Katx JHEP09 (2014) 175; Chen, Han, Tweedie JHEP11 (2017) 093; Bauer, Provasoli, Webber JHEP11 (2018) 030, as well as understanding the transition between the broken and unbroken phases Cuomo, Vecchi, Wulzer, SciPost 8 (2020) 078.
- One could make the argument that if the Goldstone Boson Equivalence Theorem was not working, many calculations would break down.
- However, it is interesting to find a process to unequivocally measure the convergence of the Goldstone Boson Equivalence Theorem and the restoration of EW symmetry.
  - Similar argument about Higgs potential.
  - The LHC will measure Higgs trilinear coupling to order 1 accuracy.
  - Hard to believe that new physics could alter the Higgs trilinear that much without appearing some place else.
  - Additionally, as the observed Higgs is confirmed to be ever more SM-like, it may be difficult to change the SM part Higgs potential drastically at EW energies.
  - However, even if the Higgs is SM-like, the Higgs potential is an important piece of our understanding of the SM.
- Measurement of EW restoration would be a nice test of our understanding of SM physics.
  - Note: Throughout this talk, will focus on purely SM physics, although beyond the SM physics may be interesting.

WW/WZ Production

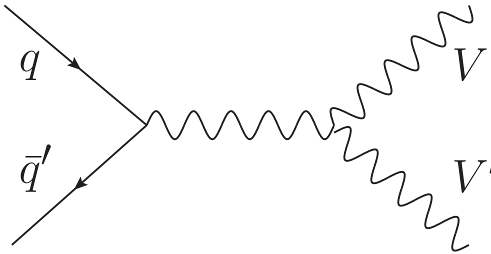
- Both fully longitudinal and fully transverse amplitudes survive to high energy.
- Longitudinal amplitudes survive:

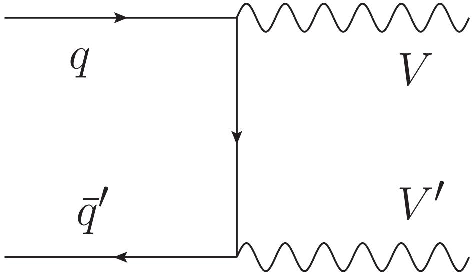
$$\mathcal{A}(q_+\bar{q}_- \to W_L^+W_L^-) \;=\; -i\,\frac{e^2\,Q_q}{2\,c_W^2}\sin\theta + \mathcal{O}(\hat{s}^{-1})\,,$$

$$\mathcal{A}(q_-\bar{q}_+ \to W_L^+W_L^-) \;=\; i\,\frac{e^2\,T_3^q}{6\,c_W^2\,s_W^2}\left(3\,c_W^2 + 2\,T_3^q\,s_W^2\right)\sin\theta + \mathcal{O}(\hat{s}^{-1})\,,$$

$$\mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm Z_L) \;=\; -i\,\frac{e^2\,T_3^q}{\sqrt{2}s_W^2}\sin\theta + \mathcal{O}(\hat{s}^{-1})\,,$$

$$\mathcal{A}(q_\pm\bar{q}'_\mp \to Z_LZ_L) \;=\; \mathcal{O}(\hat{s}^{-1})\,,$$





- Transverse amplitudes survive:

$$\mathcal{A}(q_-\bar{q}_+ \to W_\pm^+W_\mp^-) \;=\; \mp i\frac{e^2}{2\,s_W^2}\frac{1+2\,T_3^q\,\cos\theta}{1\pm\cos\theta}\sin\theta + \mathcal{O}(\hat{s}^{-1})\,,$$

$$\mathcal{A}(q_-\bar{q}'_+ \to W_\pm^\pm Z_\mp) \;=\; \mp i\frac{e^2}{\sqrt{2}\,s_W^2\,c_W}\left(g_L^{q'Z}(1+\cos\theta) + g_L^{qZ}(1-\cos\theta)\right)\frac{\sin\theta}{1\pm\cos\theta} + \mathcal{O}(\hat{s}^{-1})\,,$$

$$\mathcal{A}(q_-\bar{q}_+ \to Z_+Z_-) \;=\; 2\,i\,\frac{e^2}{s_W^2c_W^2}g_L^{qZ^2}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} + \mathcal{O}(\hat{s}^{-1})\,,$$

$$\mathcal{A}(q_+\bar{q}_- \to Z_+Z_-) \;=\; -2\,i\,\frac{e^2}{s_W^2c_W^2}g_R^{qZ^2}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \mathcal{O}(\hat{s}^{-1})\,,$$

Goldstone production

- Comparison of amplitudes in high energy limit:

$$\mathcal{A}(q_+\bar{q}_- \to G^0 h) = -\frac{e^2\, g_R^{qZ}}{2\, c_W^2\, s_W^2} \sin\theta,$$

$$\mathcal{A}(q_-\bar{q}_+ \to G^0 h) = \frac{e^2\, g_L^{qZ}}{2\, c_W^2\, s_W^2} \sin\theta,$$

$$\mathcal{A}(q_-\bar{q}_+ \to G^\pm h) = \mp i \frac{e^2}{2\sqrt{2} s_W^2} \sin\theta,$$

$$\mathcal{A}(q_-\bar{q}_+ \to G^\pm G^0) = \frac{e^2}{2\,\sqrt{2}\, s_W^2} \sin\theta,$$

$$\mathcal{A}(q_+\bar{q}_- \to G^+ G^-) = -i\, \frac{e^2 Q_q}{2\, c_W^2} \sin\theta,$$

$$\mathcal{A}(q_-\bar{q}_+ \to G^+ G^-) = -i\, \frac{e^2\, T_3^q}{6\, c_W^2 s_W^2} \left(3\, c_W^2 + 2\, T_3^q s_W^2\right) \sin\theta.$$

$$\mathcal{A}(q_+\bar{q}_- \to Z_L h) = \pm\, i\, \frac{e^2\, g_R^{qZ}}{2\, c_W^2\, s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

$$\mathcal{A}(q_-\bar{q}_+ \to Z_L h) = \pm\, i\, \frac{e^2\, g_L^{qZ}}{2\, c_W^2\, s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

$$\mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm h) = -i\, \frac{e^2}{2\,\sqrt{2}\, s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

$$\mathcal{A}(q_-\bar{q}'_+ \to W_L^\pm Z_L) = -i\, \frac{e^2\, T_3^q}{\sqrt{2} s_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1})$$

$$\mathcal{A}(q_+\bar{q}_- \to W_L^+ W_L^-) = -i\, \frac{e^2\, Q_q}{2\, c_W^2} \sin\theta + \mathcal{O}(\hat{s}^{-1}),$$

$$\mathcal{A}(q_-\bar{q}_+ \to W_L^+ W_L^-) = i\, \frac{e^2\, T_3^q}{6\, c_W^2\, s_W^2} \left(3\, c_W^2 + 2\, T_3^q\, s_W^2\right) \sin\theta + \mathcal{O}(\hat{s}^{-1})$$