INVESTIGATION OF SUPER-EDDINGTON DISKS AROUND TIDAL DISRUPTION EVENTS

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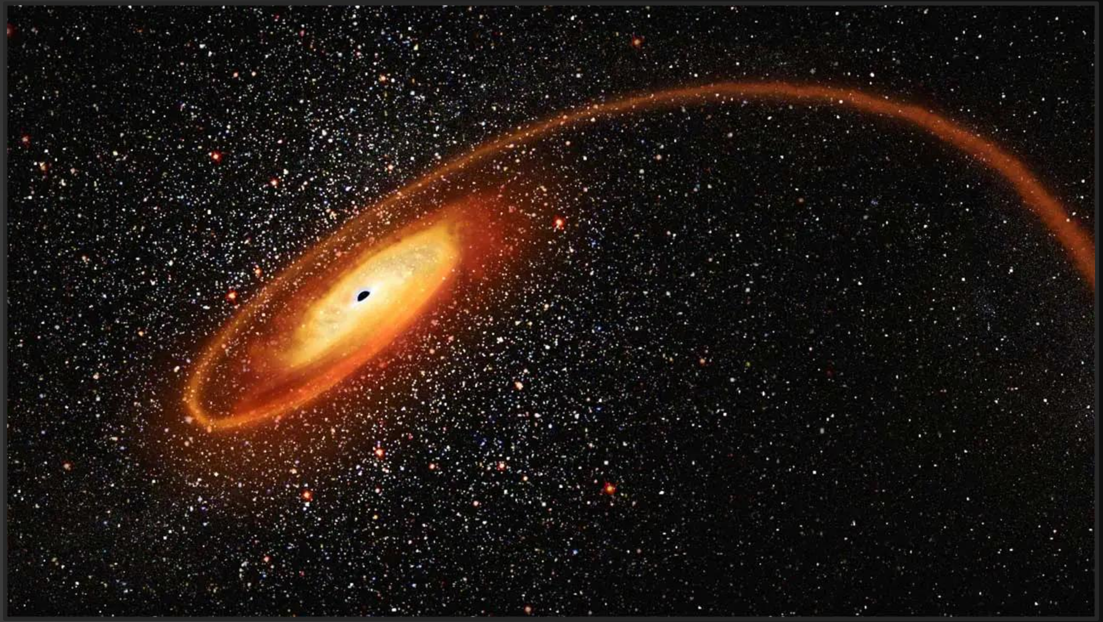
What is a TDE?

- Tidal disruption events (TDE)
  - Black hole: disrupts a star
  - Super-Eddington accretion disk
  - Super-Eddington wind

- Great intensity
  - Detect black holes
  - Measure black hole’s mass



https://vajiramandravi.com/upsc-daily-current-affairs/prelims-pointers/tde/

Scientific background

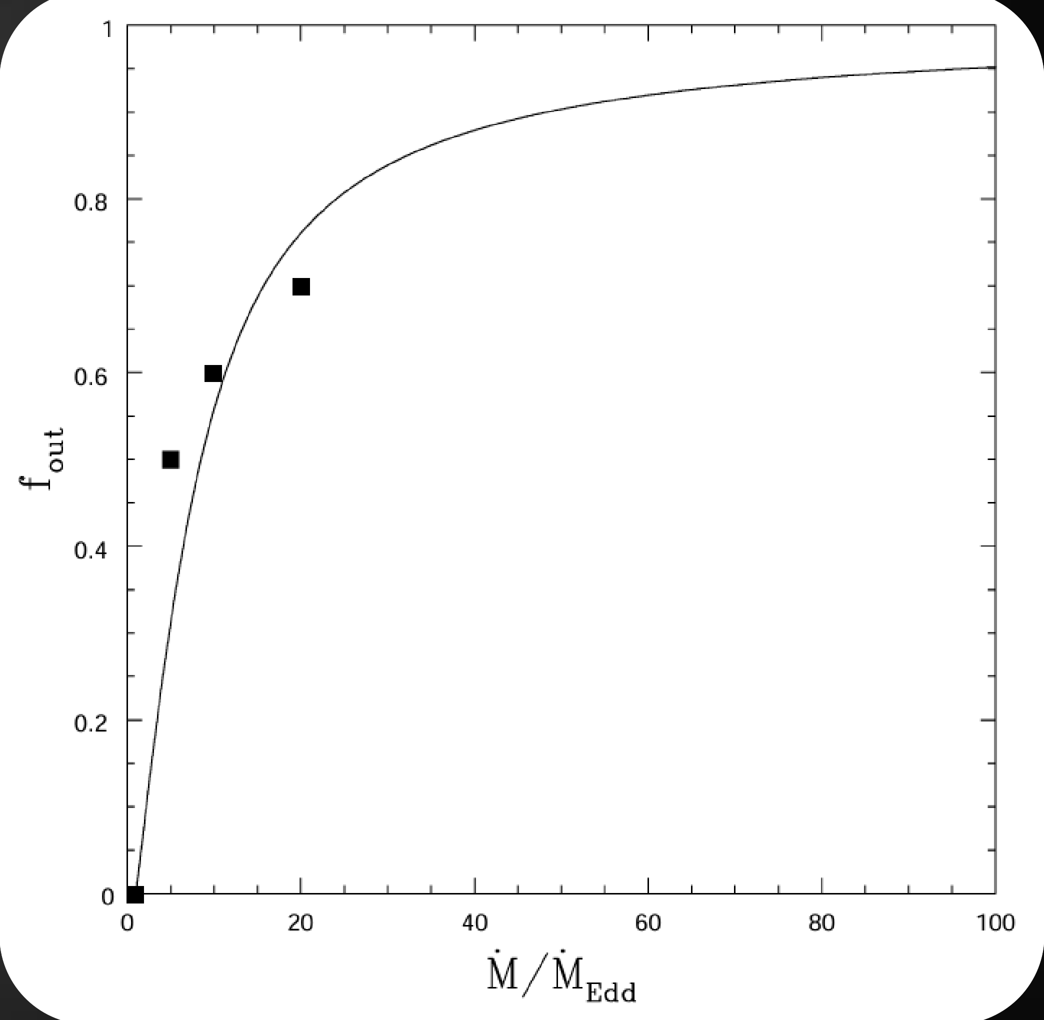
- **TiDE: light curve simulation**

Kovács-Stermeczky & Vinkó (2023)

- **f_{out} parameter:**

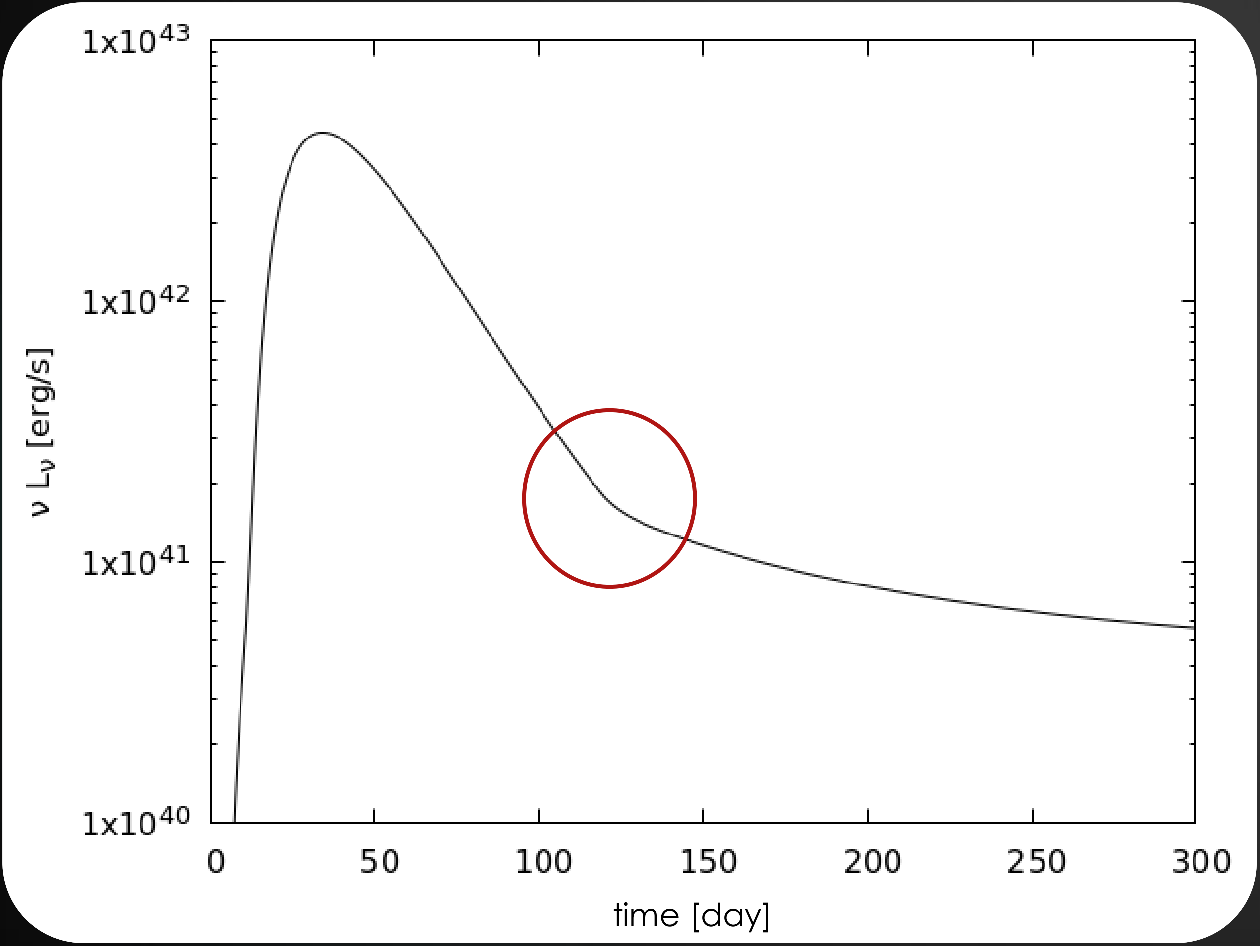
$$\frac{2}{\pi}\,\mathrm{arctg}\left[\frac{1}{7{,}5}\left(\frac{\dot{M}_{fb}}{\dot{M}_{Edd}}-1\right)\right]$$

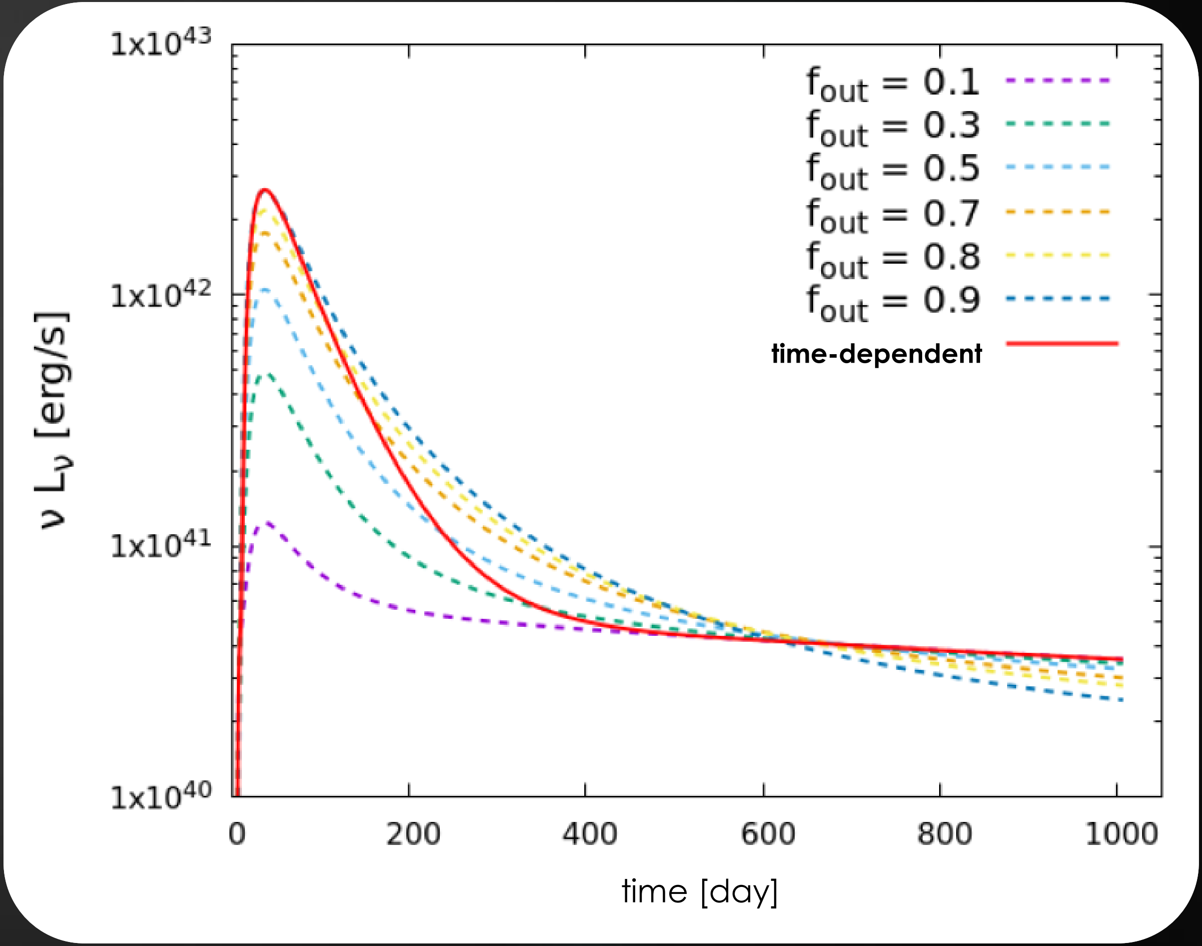
Lodato & Rossi (2011), Dotan & Shaviv (2011)



https://doi.org/10.1111/j.1365-2966.2010.17448.x

Scientific background



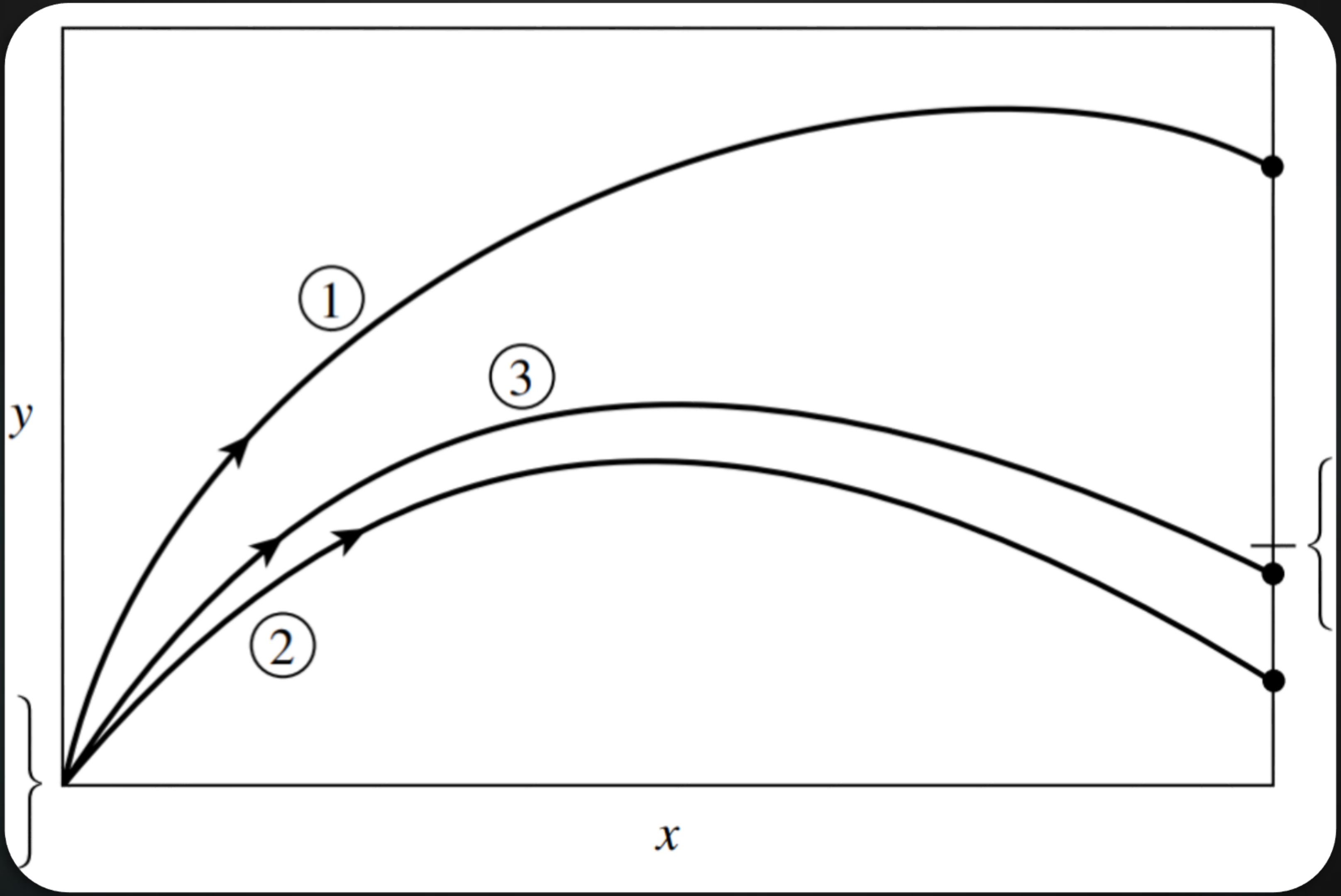


Szabó, 2024

Goal of my work

- Implementation of the Dotan slim disk model
  - Define f_{out} parameter more accurately
- Own numerical solver in C
- Implementation of the Shakura & Sunyaev thin disk model
  - Analogue problem
  - Validating my code

Shooting method



Inner boundary value

Outer boundary value

Fourth order Runge-Kutta method

$$\frac{dy}{dx} = f(x, y)$$

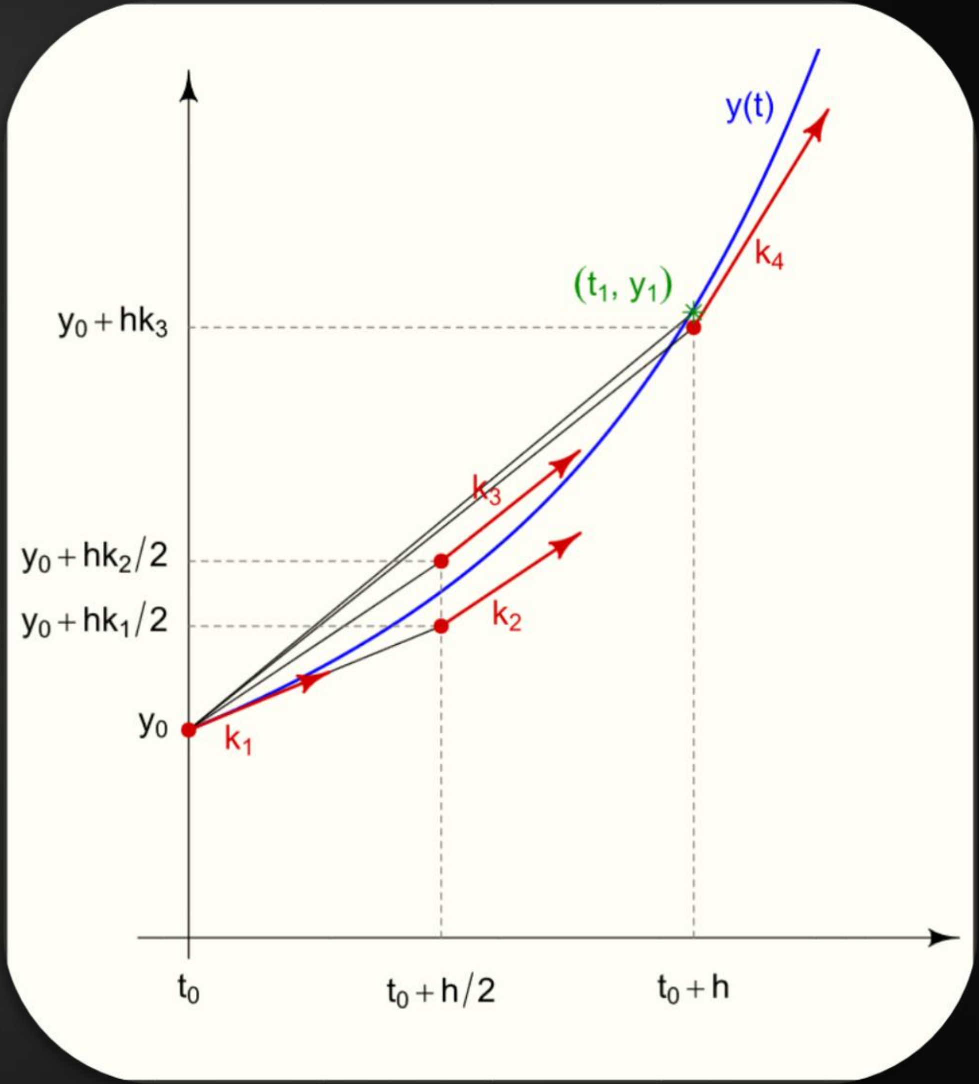
$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$$

$$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$$

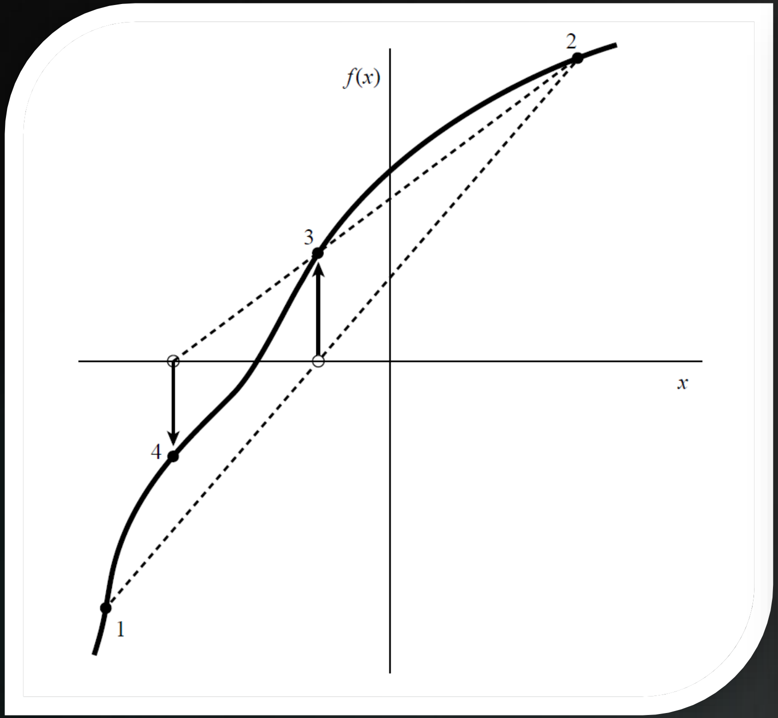
$$k_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4 + O(h^5)$$

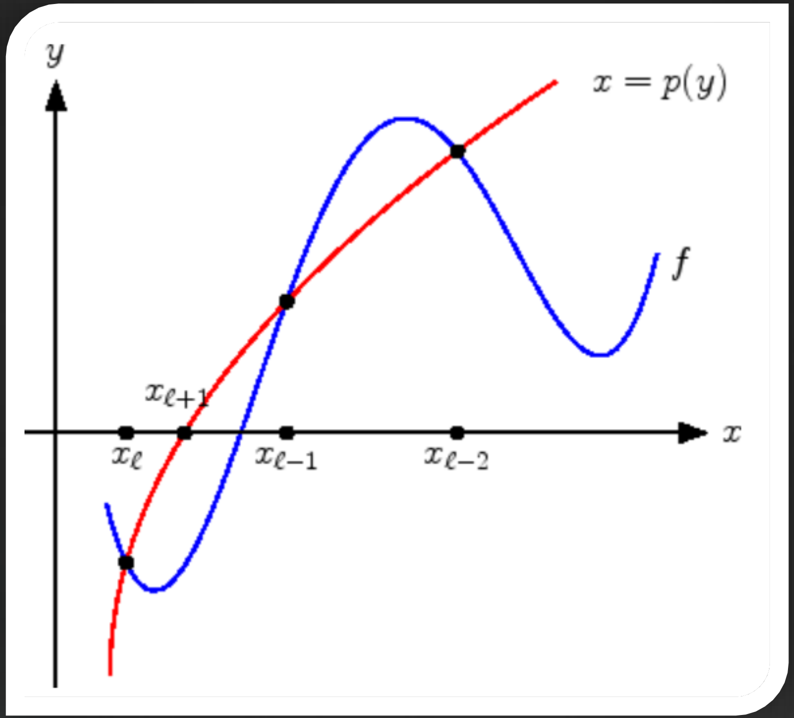


https://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_methods

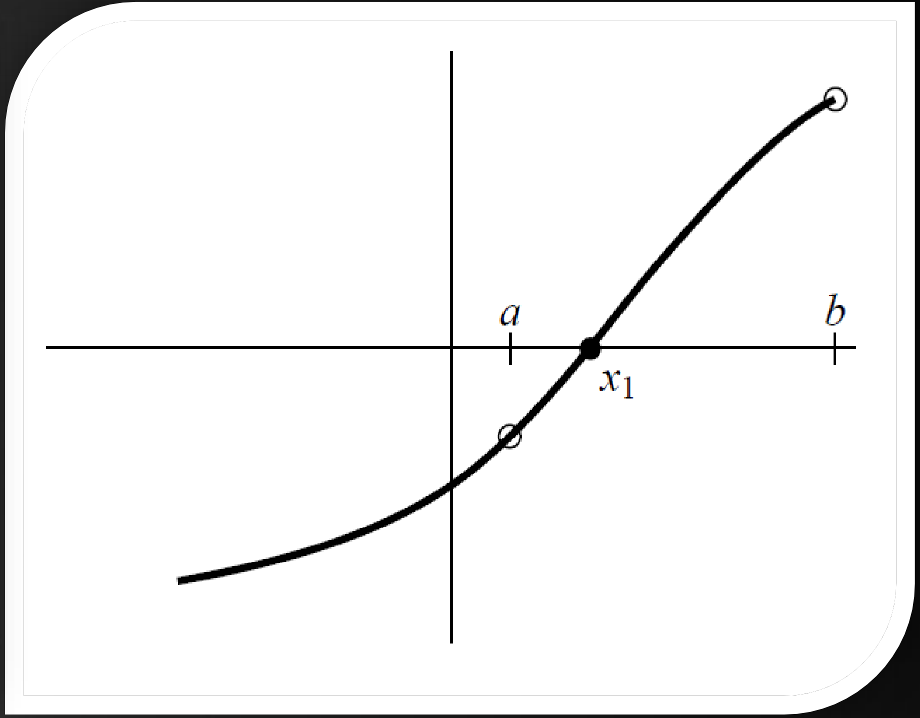
Brent's method



Secant method



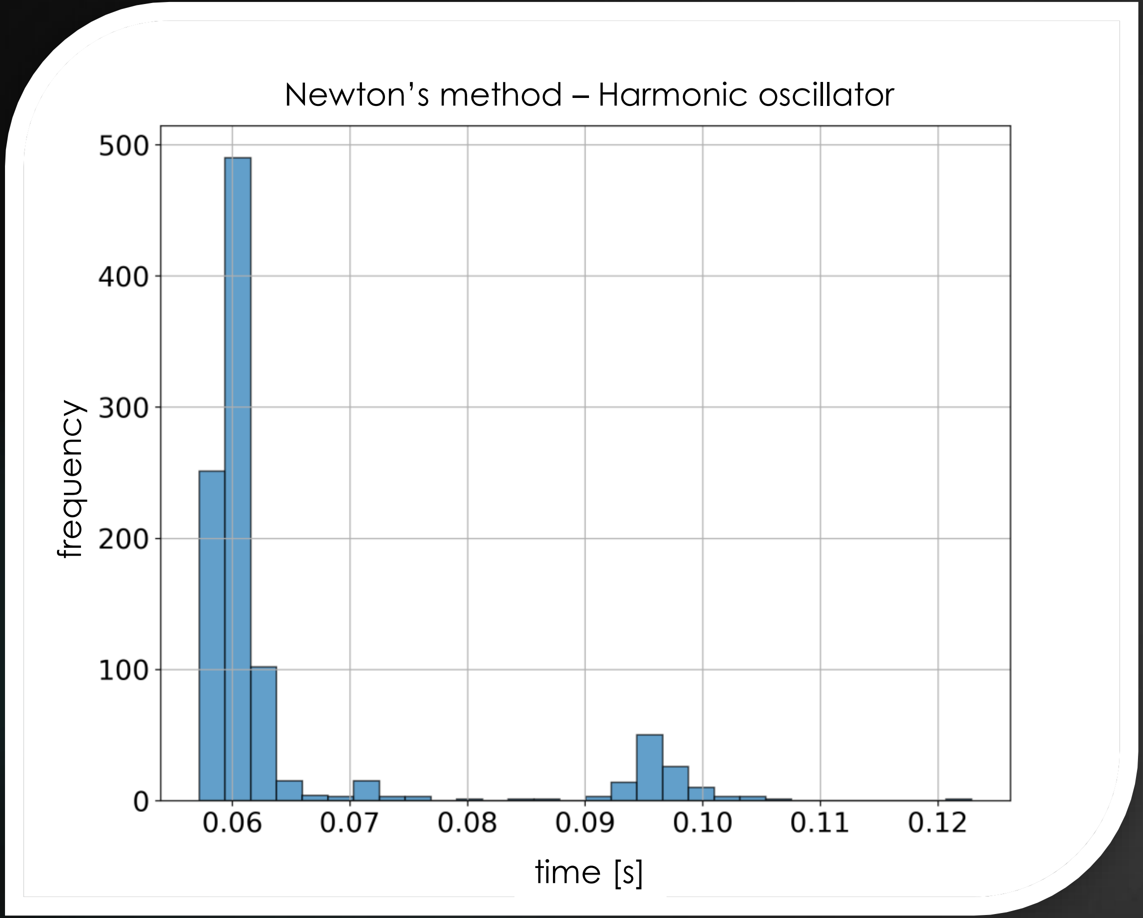
Inverse quadratic interpolation

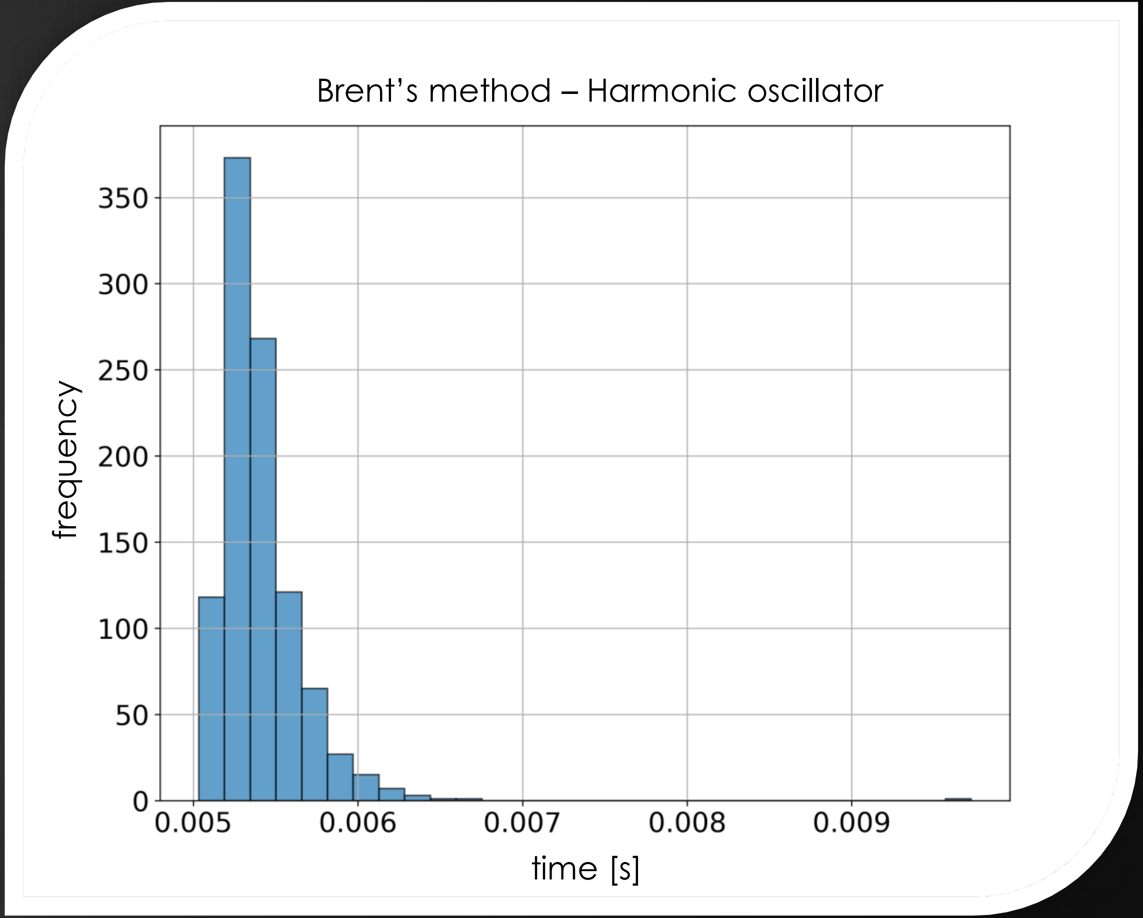


Bisection method

- Hybrid method
- Superlinear convergence
- No numerical derivative

Brent's method





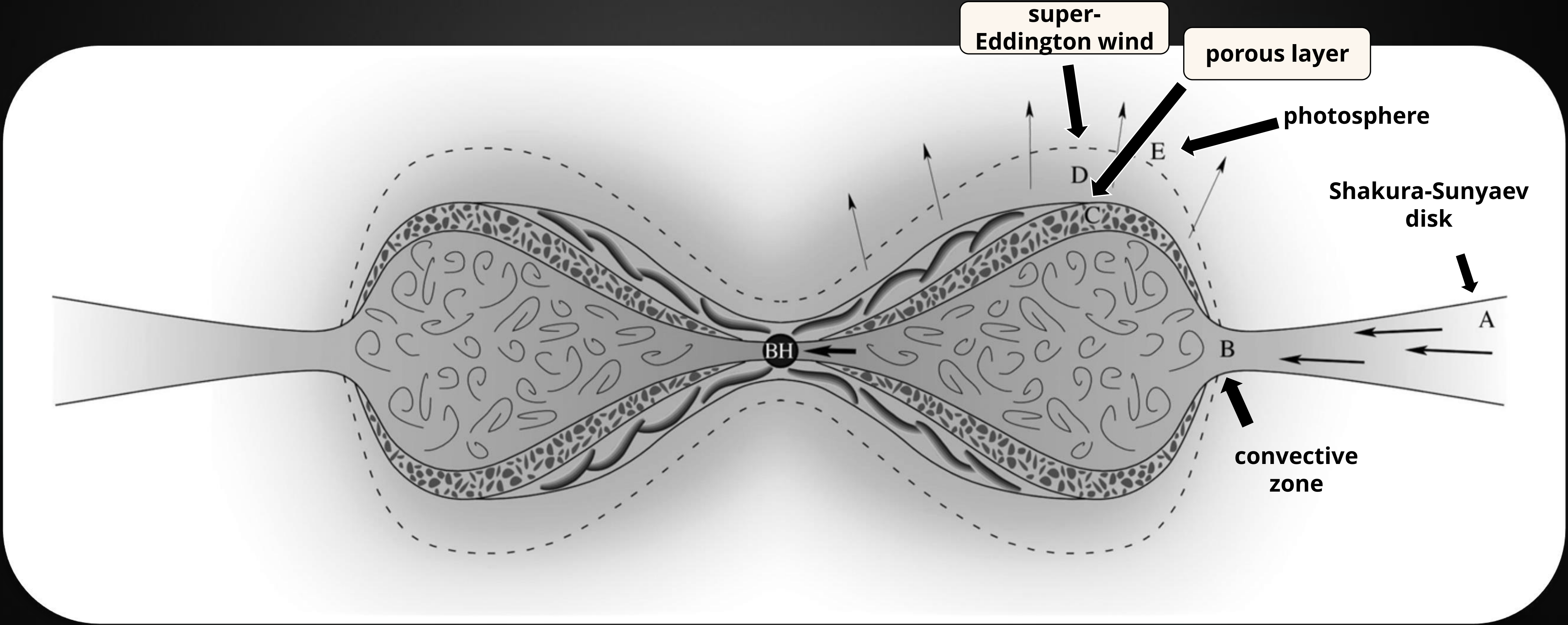
- **1000 iterations**
- **Brent: 0.005[s], Newton: 0.06[s]**

$$x(t) = A \cdot \cos(\omega t) + B \cdot \sin(\omega t)$$

$$\ddot{x} = -\frac{D}{m} x$$

$$v(t) = -A \cdot \omega \cdot \cos(\omega t) + B \cdot \omega \cdot \sin(\omega t)$$

Dotan & Shaviv slim disk model



Dotan & Shaviv slim disk model

- Half-analytical model, can be solved numerically
- Horizontally:
  - Outer boundary value: Shakura & Sunyaev disk solution
  - Inner boundary value : sonic point ⟶ $2 - 3R_g$
- Vertically:
  - Outer boundary value : Flux of photosphere
- Nested univariate boundary value problem

Shakura & Sunyaev thin disk model

- Can be solved analytically
- Horizontally:
  - Inner boundary value : $\tau_{r\varphi} = 0 \longrightarrow 3R_g$
- Vertically:
  - Outer boundary value : Flux of photosphere
- Separated univariate boundary value problem

Shakura & Sunyaev thin disk model

Horizontal structure

Angular momentum transfer

$$\Sigma\frac{d\omega R^2}{dt} = -\Sigma v_r\frac{d\omega R^2}{dR} = \frac{1}{R}\frac{d}{dR}\left(\tau_{r\varphi}R^2\right)$$

Radial velocity

Shear stress

$$\tau_{r\varphi} = 2\int_0^{z_0} w_{r\varphi} = -\alpha\Sigma v_s^2$$

Speed of sound

Alpha parameter

Radiative flux

$$F = \frac{1}{2}\tau_{r\varphi}R\frac{d\omega}{dR} = \frac{3}{8\pi}\dot{M}\frac{GM}{R^3}\left[1-\left(\frac{R_0}{R}\right)^{1/2}\right]$$

Accretion rate

$$\dot{M} = 2\pi\Sigma v_r R = \text{const.}$$

Keplerian angular speed

Gravitational constant

Black hole mass

$$\omega = \sqrt{\frac{GM}{R^3}}$$

Surface density

Half-thickness of the disk

$$\Sigma = 2\int_0^{z_0}\varrho dz$$

Equation of state

Energy density

Radiative energy density

$$\varepsilon = \frac{3}{2}\varrho\frac{kT}{m_p} + bT^4 \sim \varrho\frac{v_s^2}{2}$$

Vertical structure

Hydrostatic equilibrium

Density

$$\frac{1}{\varrho}\frac{dP}{dz} = -\frac{GM}{R^3}z.$$

Vertical energy transport

$$\frac{1}{\varrho}\frac{dq}{dz} = \frac{3}{4\pi}\frac{GM}{R^3}\frac{\dot{M}}{\Sigma}\left[1-\left(\frac{R_0}{R}\right)^{1/2}\right]$$

Radiative transfer

Flux

$$\frac{c}{3\sigma\varrho}\frac{d\varepsilon_r}{dz} = -q(z)$$

Horizontal integration

- **Initial value problem:**
  - $R_0 = 3R_g = 6GM/c^2$
  - $\dot{M} = 3 \times 10^{-9} \frac{M_\odot}{\text{yr}}$
  - $\tau_{r\varphi} = 0$
- **Fourth order Runge-Kutta method**

$$\Sigma \frac{d\omega R^2}{dt} = -\Sigma v_r \frac{d\omega R^2}{dR} = \frac{1}{R}\frac{d}{dR}\left(\tau_{r\varphi} R^2\right)$$

$$\tau_{r\varphi} = 2\int_0^{z_0} w_{r\varphi} = -\alpha \Sigma v_s^2$$

$$\dot{M} = 2\pi \Sigma v_r R = \text{const.}$$

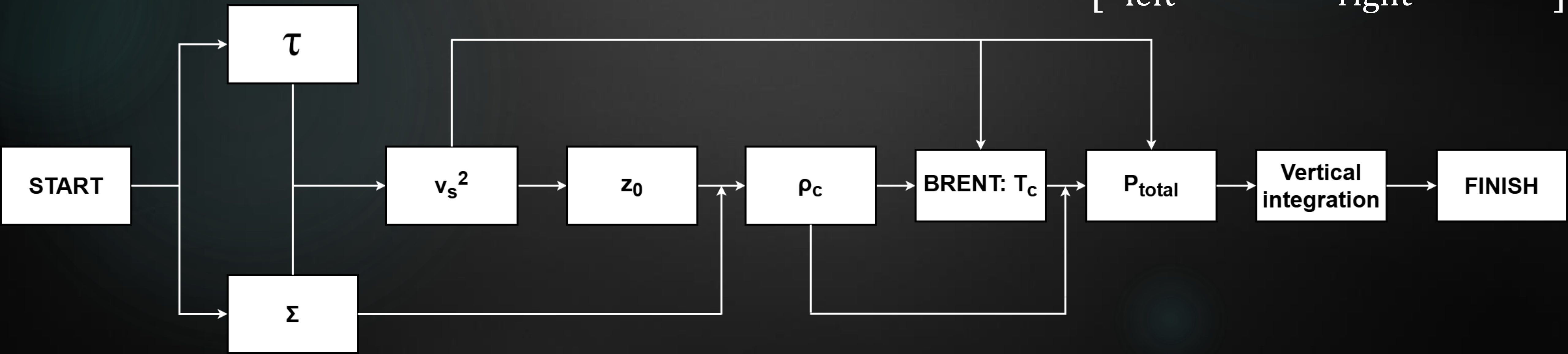
$$F = \frac{1}{2}\tau_{r\varphi} R \frac{d\omega}{dR} = \frac{3}{8\pi}\dot{M}\frac{GM}{R^3}\left[1 - \left(\frac{R_0}{R}\right)^{1/2}\right]$$

Calculation of initial state

- **Horizontal integration** ⟶ $\tau_{r\varphi}$
- **Σ guess**

$$T_c = \frac{v_s^2 m_p}{6k} - \frac{bT_c^4 m_p}{3\varrho k}$$

- **Brent's method:**
  - $f(T_c) = 0$
  - $\left[T_{\text{left}} = 10^2\text{K}, T_{\text{right}} = 10^{10}\text{K}\right]$



Vertical integration

- **Boundary value problem** ⟶ **shooting method**
  - $\left[\Sigma_{\mathrm{left}} = 0.01\ \mathrm{kg/m^2}, \Sigma_{\mathrm{right}} = 2\ \mathrm{kg/m^2}\right]$
  - $T_\mathrm{c} \longrightarrow P_\mathrm{c} \longrightarrow \varrho_\mathrm{c}$
  - $F \longrightarrow T_\mathrm{s}$
  - **Brent:** $f = T_{\mathrm{final}} - T_\mathrm{s} = 0$

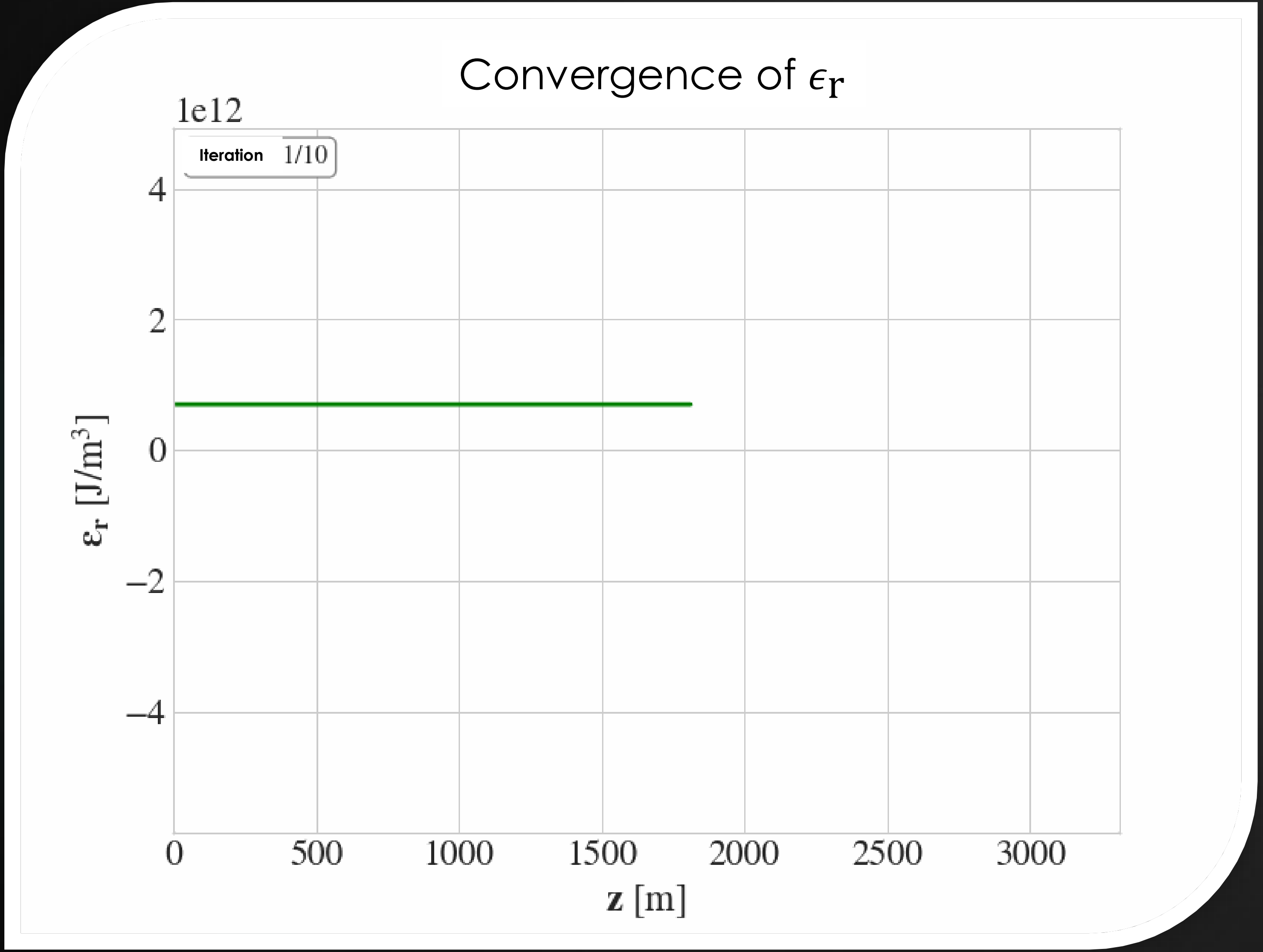
$$\frac{dP}{dz} = -\frac{GM}{R^3} z\varrho$$

$$\frac{dq}{dz} = \frac{3}{4\pi}\frac{GM}{R^3}\frac{\dot{M}}{\Sigma}\left[1-\left(\frac{R_0}{R}\right)^{1/2}\right]\varrho$$

$$\frac{d\varepsilon_r}{dz} = -q(z)\frac{3\sigma\varrho(z)}{c}$$

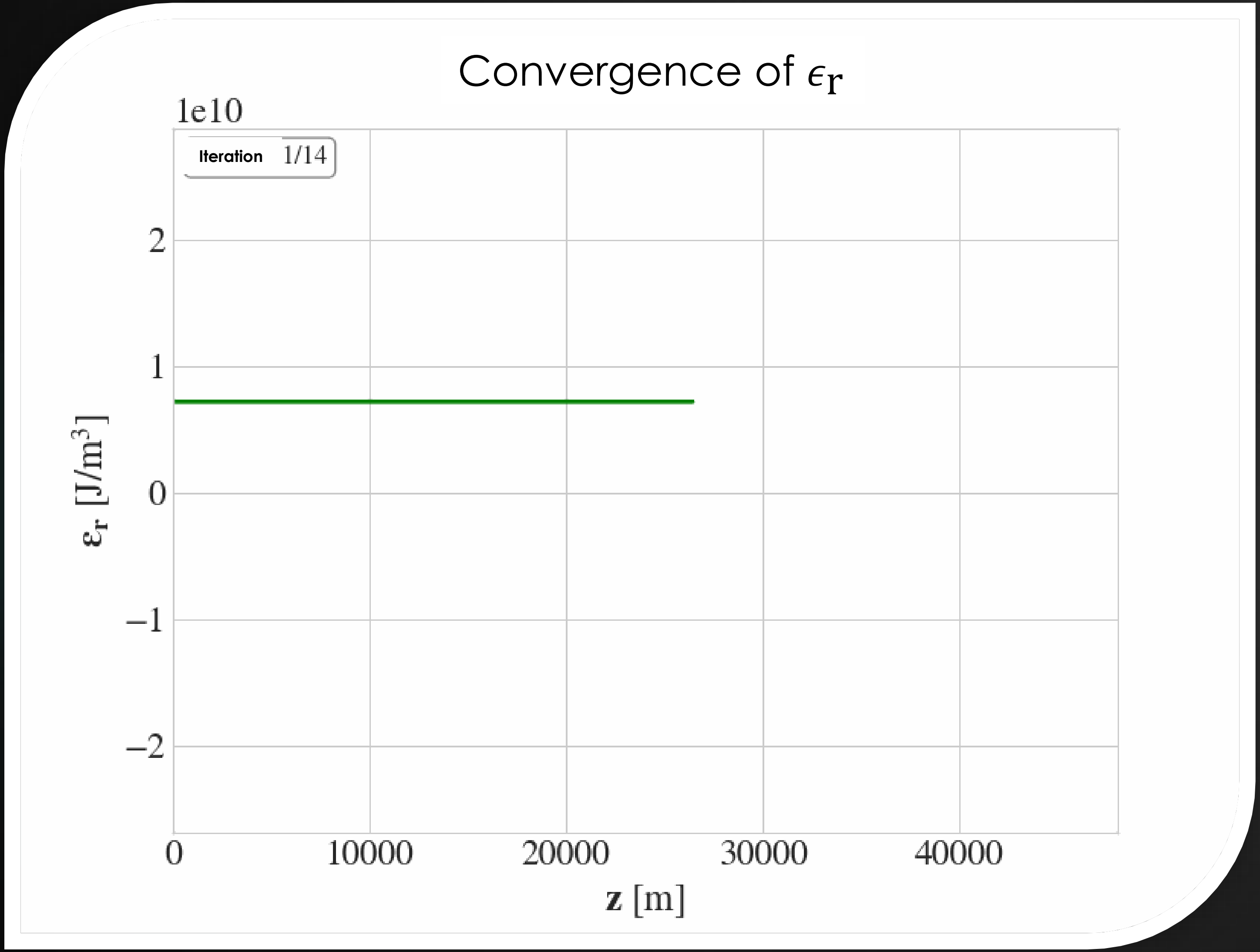
$$P_\mathrm{g} = P_{\mathrm{total}} - \frac{\varepsilon_\mathrm{r}}{3}, \quad \varrho = P_\mathrm{g}\frac{m_\mathrm{p}\mu b^{1/4}}{k\varepsilon_\mathrm{r}}$$

Results: Shakura & Sunyaev solution



- $\tau_{r\varphi} = -4.356642 \times 10^{15} N\backslash m^2$
- $R = 5R_g$
- $P_{\text{radiation}} \gg P_{\text{gas}}$
- $\sigma_{\text{T}} \gg \sigma_{\text{ff}} = 0,04\ kg/\text{m}^2$

Results: Shakura & Sunyaev solution



- $\tau_{r\varphi} = -4.356642 \times 10^{14} N\backslash m^2$
- $R = 50R_g$
- $P_{\text{gas}} \gg P_{\text{radiation}}$
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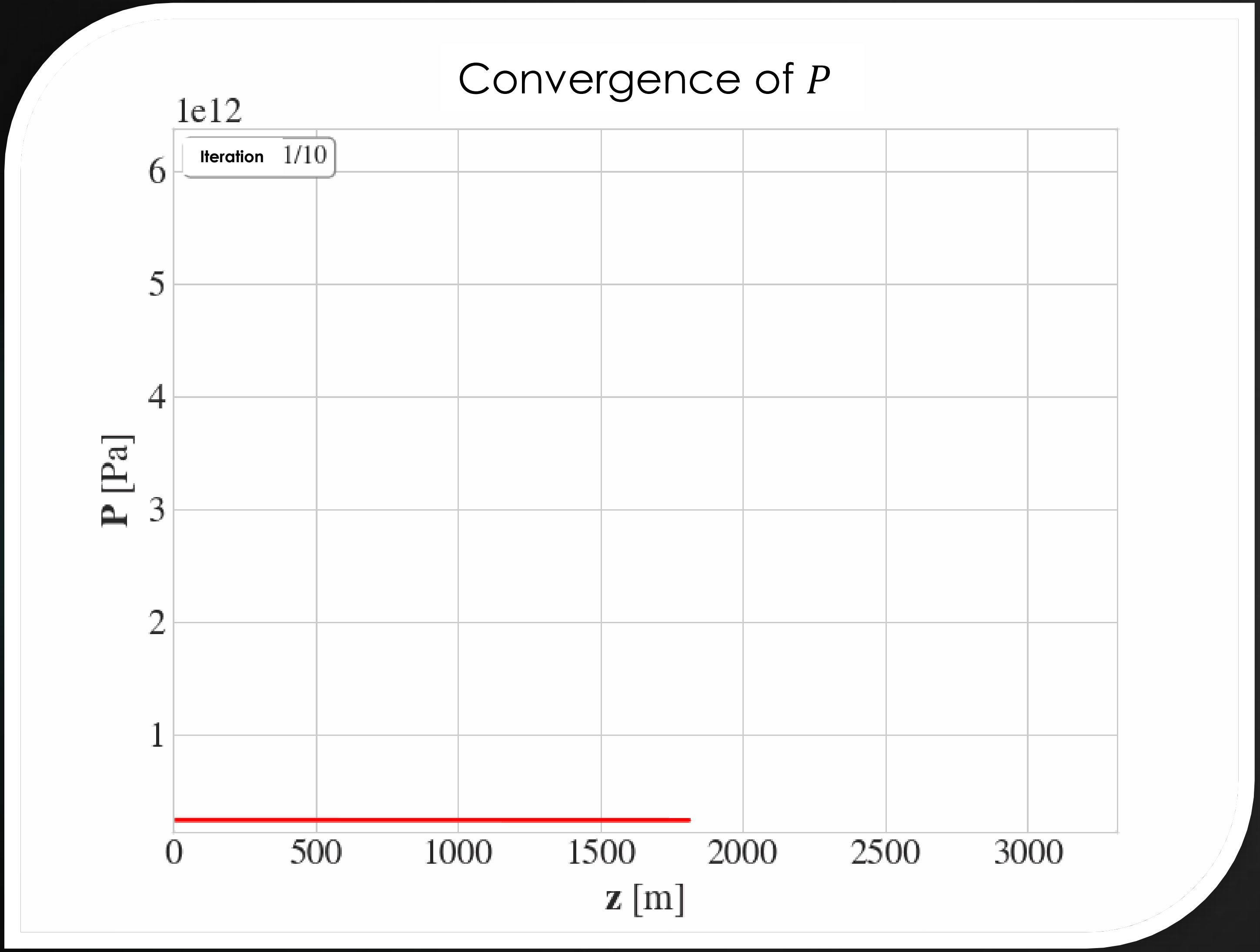
Results: Shakura & Sunyaev solution



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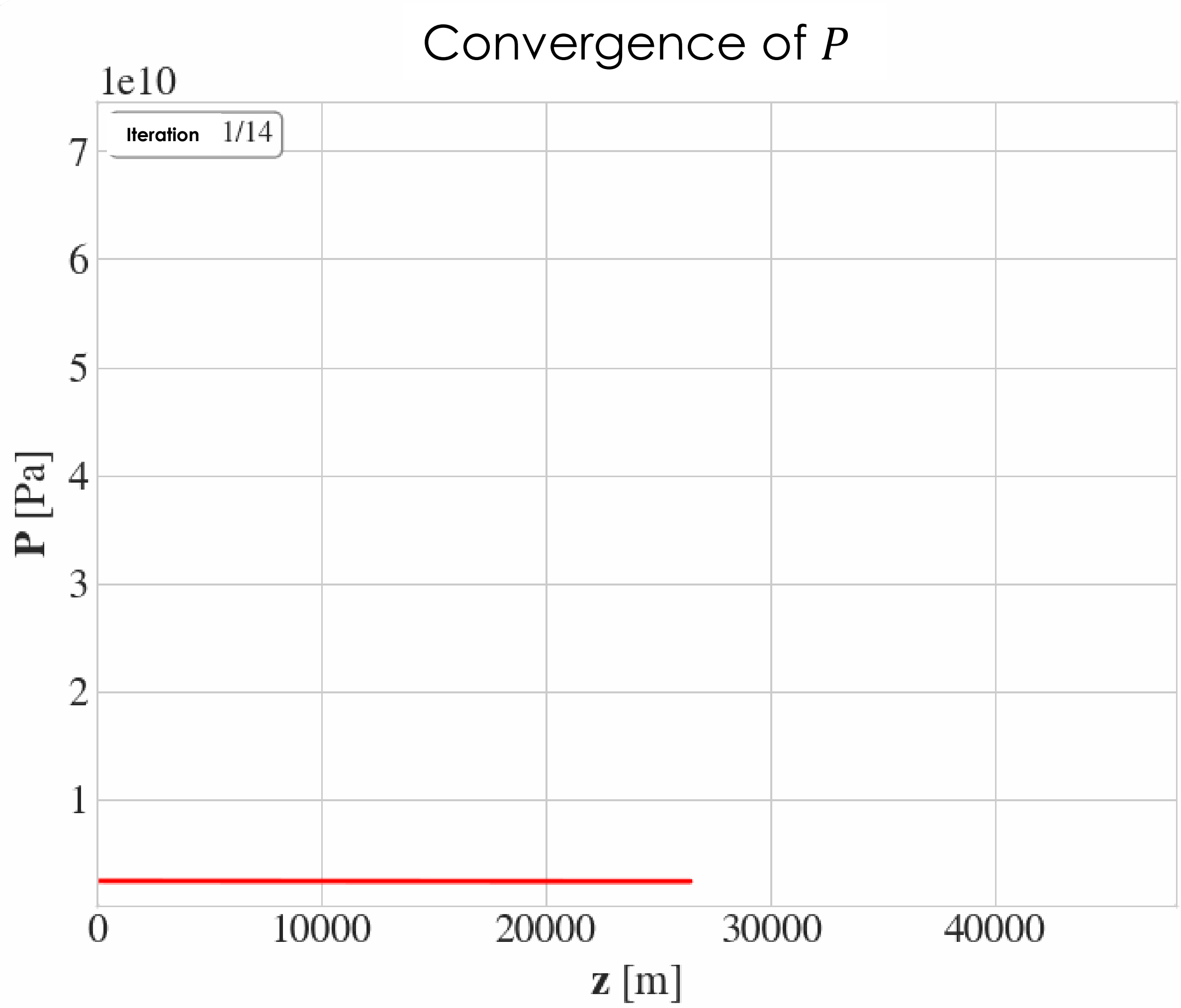

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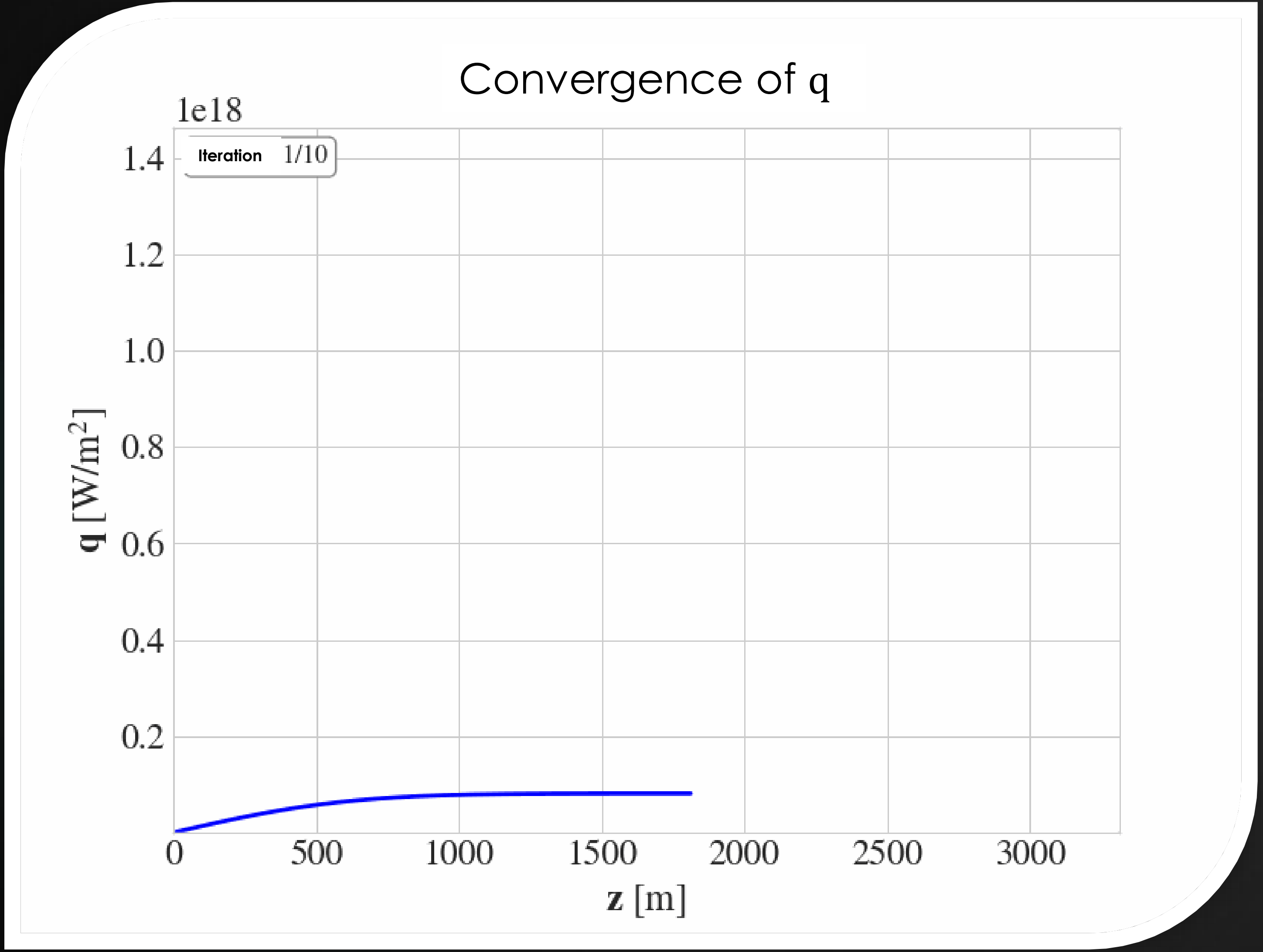

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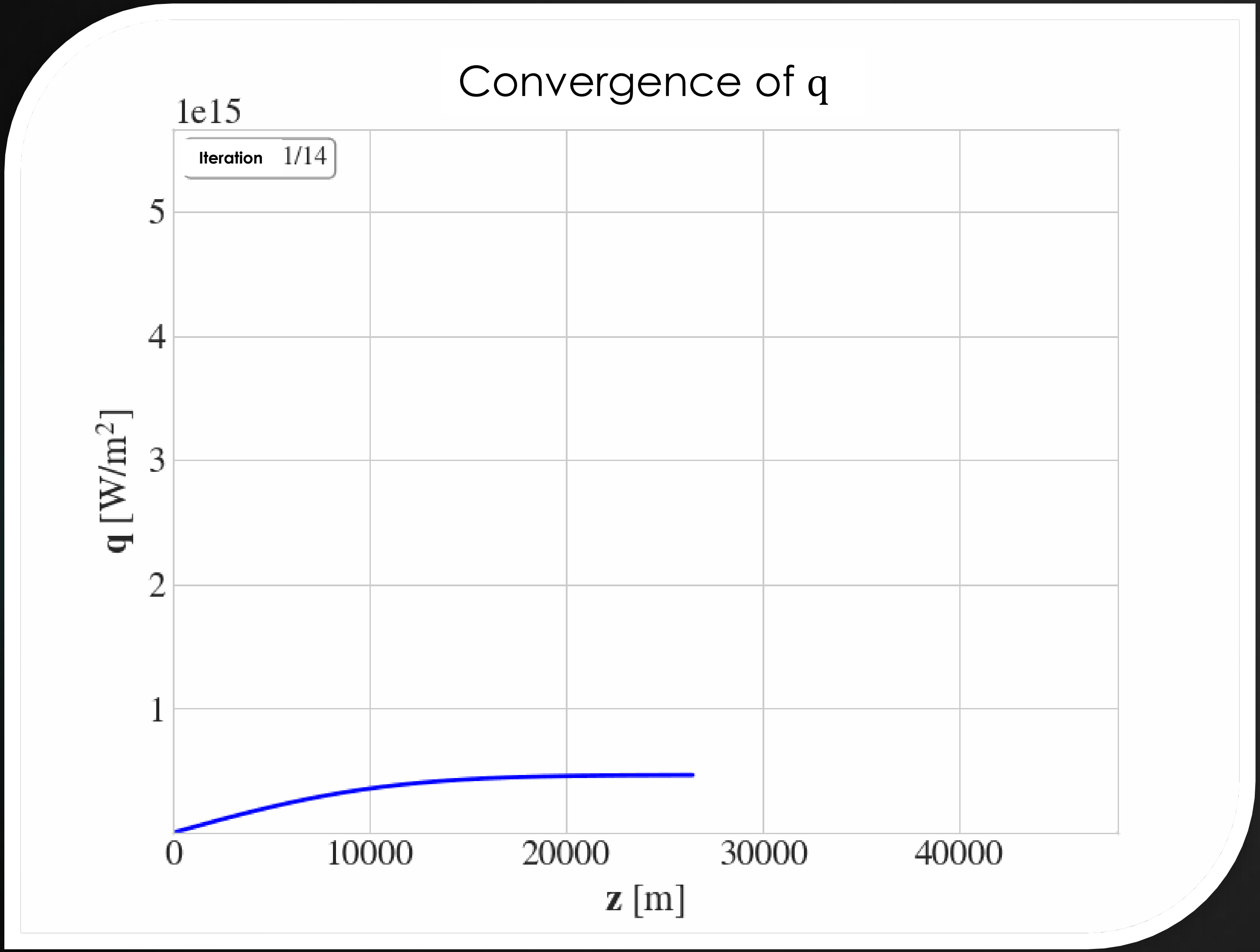

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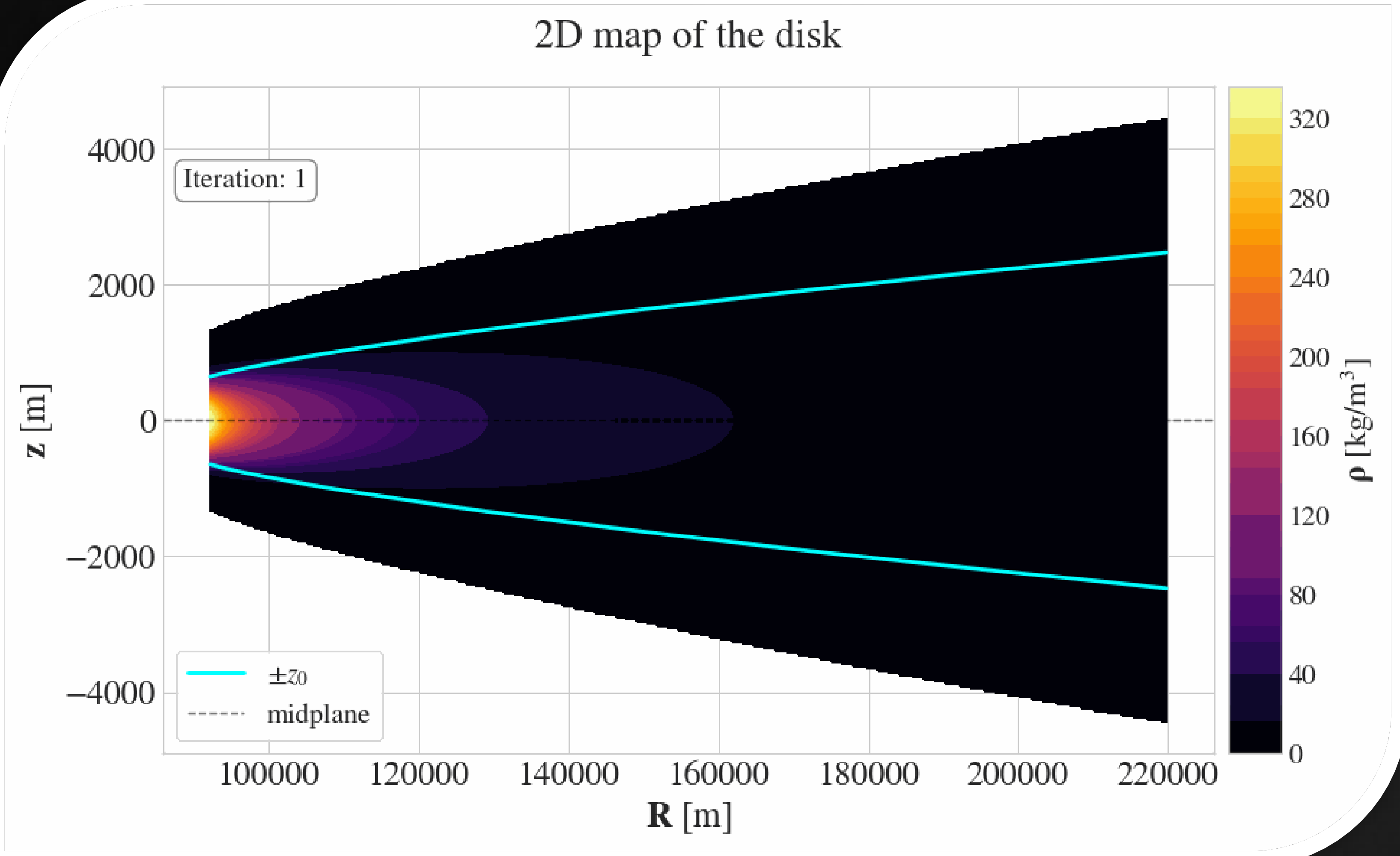
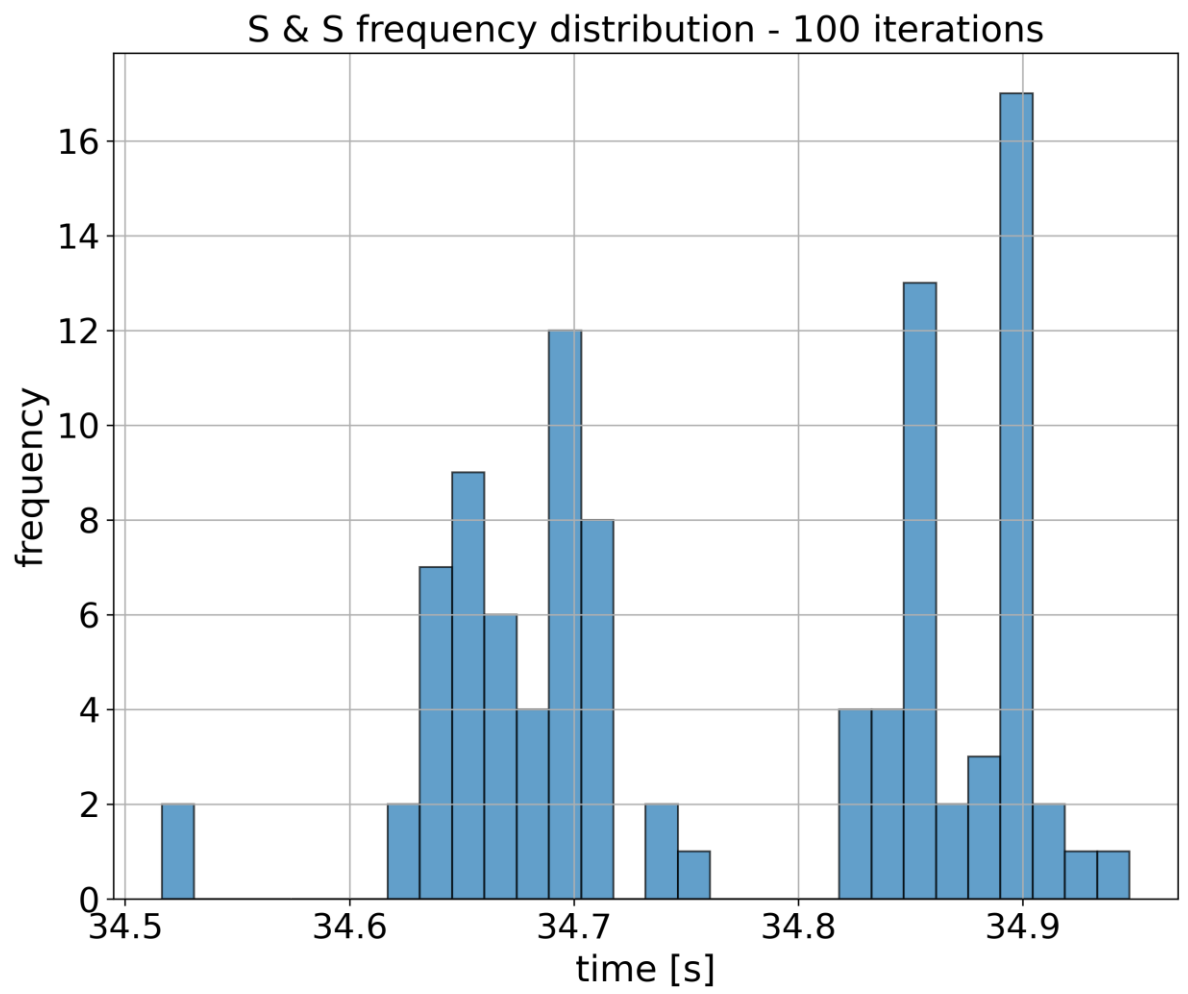
Results: Shakura & Sunyaev solution



- Boundary value problem
- $R_{\text{start}} = 10R_g$

Results: Shakura & Sunyaev solution



- $R_{\text{start}} = 5R_g$
- Median: $\mathbf{34.72138}[s]$

Future steps

1. Own C algorithm ✓
2. Shakura-Sunyaev disk ✓
3. Dotan disk solution
4. f_{out} investigation

Summary

- TDEs
- Dotan disk model:
  - Super-Eddington disk
  - Super-Eddington wind
  - Nested initial boundary value problem
- Shakura & Sunyaev disk model
  - Horizontal structure ✓
  - Vertical structure ✓
- Brent's method ✓
- Own C implementation ✓

Since the Shakura & Sunyaev disk model is a similar problem to the Dotan disk model, the implementation can be done, after which f_{out} can be corrected.