A new class of rotating charged black holes in the external Bertotti-Robinson (electro)magnetic field

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arXiv:2604.13202 (2026)

Introduction

Asymptotically flat black holes

	Not charged	*Charged*
Non-rotating	Schwarzschild (1916)	Reissner-Nordstrom (1917)
Rotating	Kerr (1963)	Kerr-Newman (1965)

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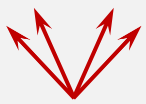
There are up to 4 such directions for Weyl tensor



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  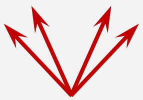

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- For Faraday tensor $\Phi_0 = 0$

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  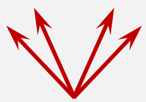

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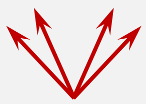

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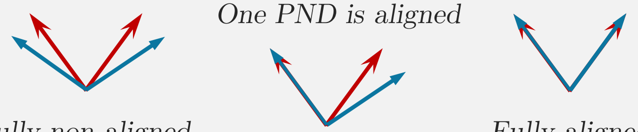
For type D black holes such variants may be



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C-metric (1918)
Taub-NUT (1963)

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Up to what extend are we able to generalize this class?

Aligned type D black holes

The most general solution, satisfying such properties

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- Solution is of algebraic type D
- Satisfies Einstein-Maxwell equations
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⇒ *Debever, Kamran and McLenaghan (1984)*

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→ *Debever, Kamran and McLenaghan (1984)* (first three properties)

→ *Plebanski-Demianski (1976)* (all four properties)

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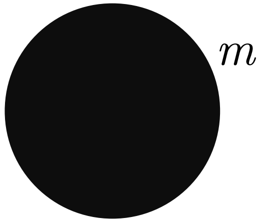
Plebanski-Demianski (1976)

Plebanski-Demianski spacetime contains such parameters:

m

Mass of a BH



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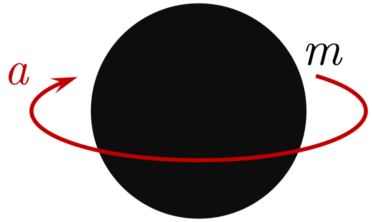
Mass of a BH	Kerr-like parameter, represents angular momentum of BH



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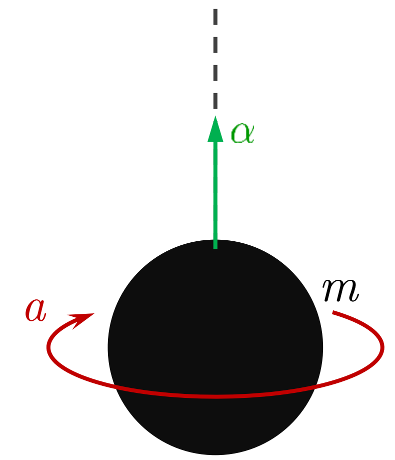

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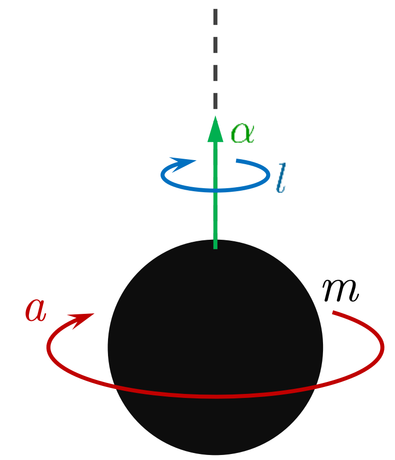

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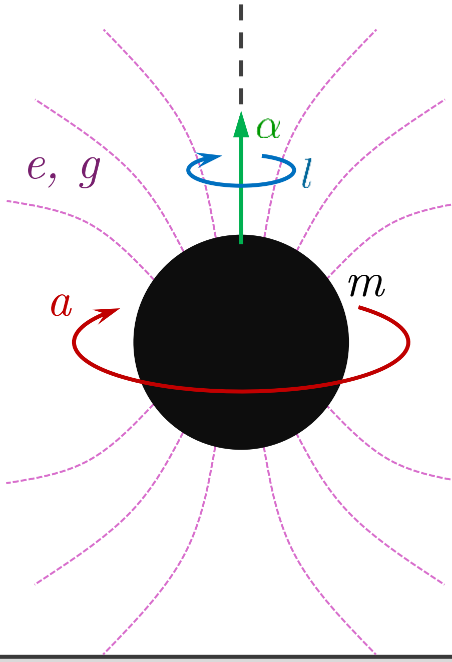

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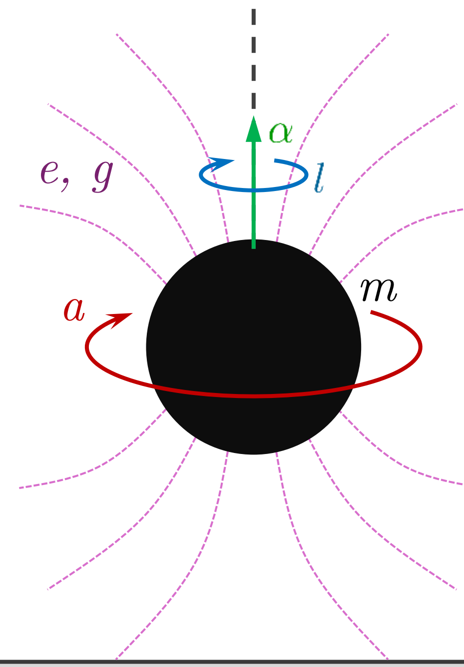

In principle, cosmological constant Λ can be added

Non-aligned type D black holes

Satisfies Einstein-Maxwell equations	Solution is of algebraic type D	PNDs of Faraday and Weyl tensors are aligned

↓

Plebanski-Demianski (1976)

Non-aligned type D black holes

Satisfies Einstein-Maxwell equations

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Non-aligned type D black holes

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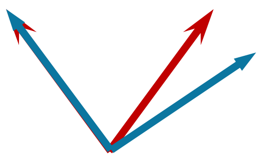
Non-aligned type D black holes

Satisfies Einstein-Maxwell equations | Solution is of algebraic type D | PNDs of Faraday and Weyl tensors are aligned

Non-aligned type D black holes

Plebanski-Demianski (1976)

One PND is aligned



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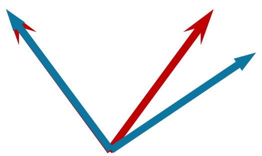

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These solutions reduce to Minkowski in vacuum

Kerr is not in this class

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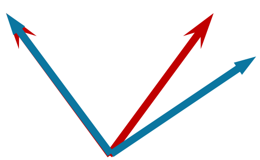

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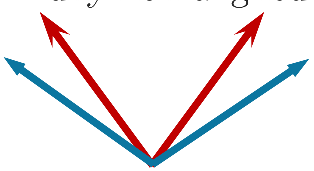
Fully non-aligned



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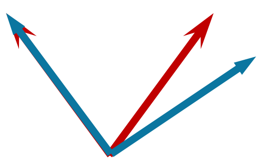

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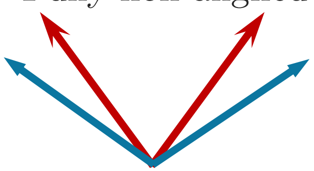

Van den Bergh-Carminati (2020)

Only static case

Physical interpretation is not clear

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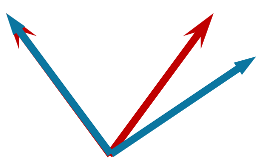

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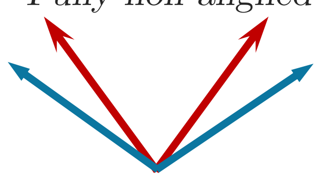

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Our aim: to generalize this class
to add twist and give it a physical interpretation

A new solution

General metric

Metric for the new class is given by (Phys. Rev. D **112** (2025) 6, 064076)

$$ds^2 = \frac{1}{\Omega^2}\Bigg\{ -\frac{\mathcal{Q}}{\varrho^2}\Big[dt - \big(a\sin^2\theta + 2(l+\omega x_0)(1-\cos\theta)\big)d\varphi\Big]^2 + \sin^2\theta\,\frac{\tilde{P}}{\varrho^2}\Big[a dt - \big((r+r_0)^2 + (a+l+\omega x_0)^2\big)d\varphi\Big]^2 + \\ + \frac{\varrho^2}{\mathcal{Q}}dr^2 + \frac{\varrho^2}{\tilde{P}}d\theta^2\Bigg\}$$

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$$\Omega(r,x)^2 = \Big(1 - \alpha r\frac{a\cos\theta + l}{\omega}\Big)^2 + 4\alpha^2(c\bar{c})\Bigg\{k(r^2 - (a\cos\theta+l)^2) - [\epsilon - 4\alpha^2\omega^2(c\bar{c})k^2]r^2\frac{(a\cos\theta+l)^2}{\omega^2}\Bigg\} + 8\frac{\alpha^2}{\omega^2}\frac{(c\bar{c})r(a\cos\theta+l)}{1+4\alpha^2\omega^2(c\bar{c})k}\Big((n - 2\alpha\omega(c\bar{c})km)r + (m + 2\alpha\omega(c\bar{c})kn)(a\cos\theta+l)\Big)$$

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$$+ 8\frac{\alpha^2}{\omega^2}\frac{(c\bar{c})r(a\cos\theta+l)}{1+4\alpha^2\omega^2(c\bar{c})k}\Big((n - 2\alpha\omega(c\bar{c})km)r + (m + 2\alpha\omega(c\bar{c})kn)(a\cos\theta+l)\Big)$$

$$\tilde{P} = 1 - \tilde{a}_3\cos\theta - \tilde{a}_4\cos^2\theta, \qquad \varrho^2 = (r+r_0)^2 + (l+\omega x_0 + a\cos\theta)^2, \qquad \mathcal{Q}(r) = b_0 + b_1 r + b_2 r^2 + b_3 r^3 + b_4 r^4$$

Coefficients $k,\ n,\ \epsilon,\ b_0,\ b_1,\ b_2,\ b_3,\ b_4,\ \tilde{a}_3$ and $\tilde{a}_4$ are not independent and related to $m,\ a,\ l,\ \alpha,\ c$

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Parameters r_0 and x_0 are not independent

$$r_0 = R\cos\beta_s + R\cos\beta$$

$$\omega x_0 = R\sin\beta_s + R\sin\beta$$

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$$\left. + \frac{\varrho^2}{\mathcal{Q}}dr^2 + \frac{\varrho^2}{\tilde{P}}d\theta^2 \right\} \quad \text{where}$$

$$\Omega(r,x)^2 = \left(1 - \alpha r\frac{a\cos\theta + l}{\omega}\right)^2 + 4\alpha^2(c\bar{c})\left\{k(r^2 - (a\cos\theta + l)^2) - [\epsilon - 4\alpha^2\omega^2(c\bar{c})k^2]r^2\frac{(a\cos\theta + l)^2}{\omega^2}\right\}$$
$$+ 8\frac{\alpha^2}{\omega^2}\frac{(c\bar{c})r(a\cos\theta + l)}{1 + 4\alpha^2\omega^2(c\bar{c})k}\left((n - 2\alpha\omega(c\bar{c})km)r + (m + 2\alpha\omega(c\bar{c})kn)(a\cos\theta + l)\right)$$

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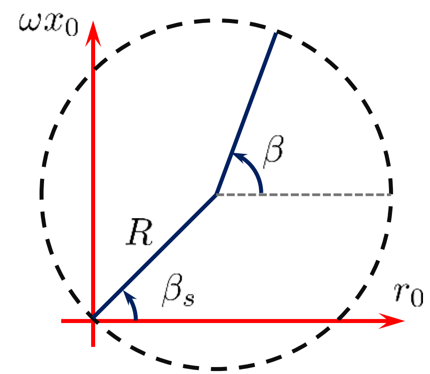
Coefficients k, n, ϵ, b_0, b_1, b_2, b_3, b_4, $\tilde{a}_3$ and $\tilde{a}_4$ are not independent and related to m, a, l, α, c

Parameters r_0 and x_0 are not independent

$$r_0 = R\cos\beta_s + R\cos\beta$$
$$\omega x_0 = R\sin\beta_s + R\sin\beta$$



Electromagnetic field

Electromagnetic 1-form is given by

$$\mathbf{A} = \frac{1}{4\alpha\,\bar{c}}\,\frac{\omega}{a}\left[\Omega_{,r}\,\frac{a dt - [(r+r_0)^2 + (a+l+\omega x_0)^2]d\varphi}{(r+r_0) + i(a\cos\theta + l + \omega x_0)} + \frac{i\,\Omega_{,\theta}}{\sin\theta}\,\frac{dt - [a\sin^2\theta + 2(l+\omega\,x_0)(1-\cos\theta)]\,d\varphi}{(r+r_0) + i\,(a\cos\theta + l + \omega\,x_0)} + \Omega\,d\varphi\right].$$

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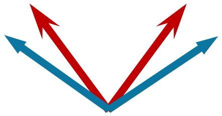

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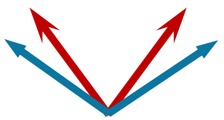

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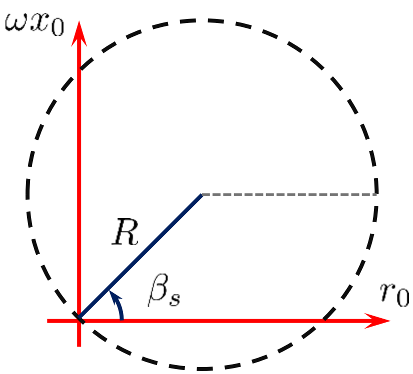

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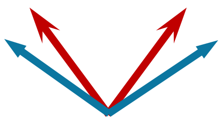

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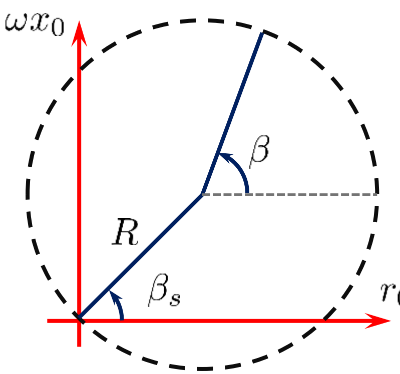

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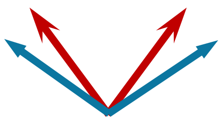

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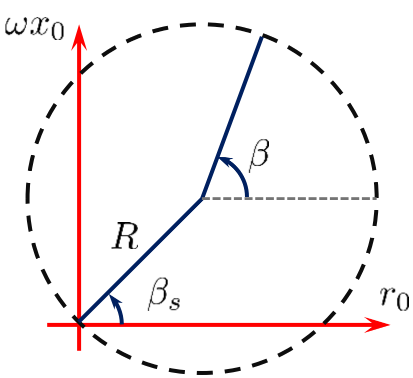

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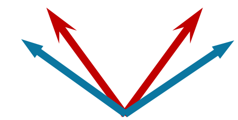

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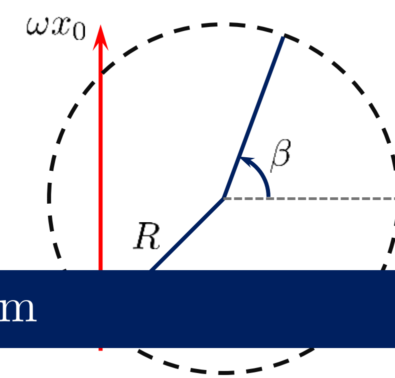

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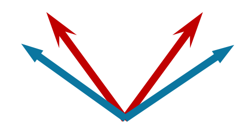

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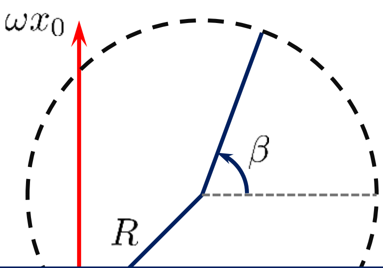

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To interpret this non-aligned field physically, let us consider some special subcases

Special limit

Kerr Black Hole in a Uniform Bertotti-Robinson Magnetic Field: An Exact Solution

Phys. Rev. Lett. **135** (2025) 18, 181401

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Special limit we would like to investigate is the one in which

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$$Q = \big(1 + B^2 r^2\big)\,\Delta\,,$$

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Subcases of this solution

- Taking $B \to 0$ brings the metric to the **Kerr** spacetime

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Brings the metric to the **Bertotti-Robinson** spacetime.

$$ds^2 = -\left(1+\frac{R^2}{e^2}\right)d\tau^2 + \left(1+\frac{R^2}{e^2}\right)^{-1}dR^2 + e^2(d\Theta^2 + \sin^2\Theta\, d\phi^2)$$

It is conformally flat

$$\Psi_0 = \Psi_1 = \Psi_2 = \Psi_3 = \Psi_4 = 0$$

Represents uniform electromagnetic field

$$F^*_{\mu\nu}\,F^{*\,\mu\nu} = 4e^{i\,2\gamma}\,B^2(1-B^2a^2) = (2e^{i\,\gamma}/e)^2 = \text{const}$$

γ – duality rotation parameter

Kerr-Bertotti-Robinson black hole

New solution

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Represents spacetime with the uniform electromagnetic field

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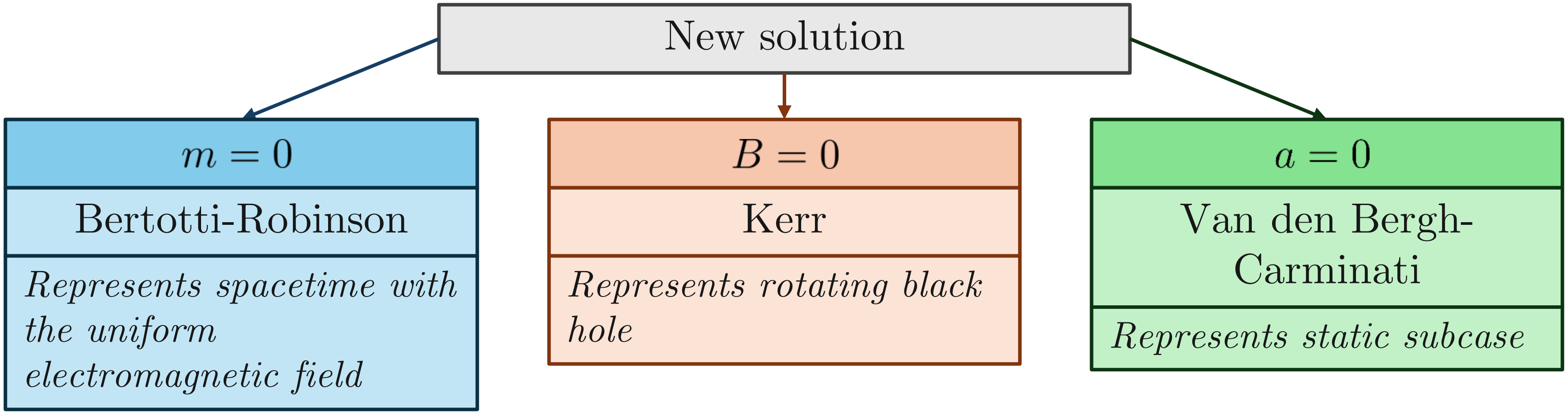
Kerr

Represents rotating black hole

Kerr-Bertotti-Robinson black hole



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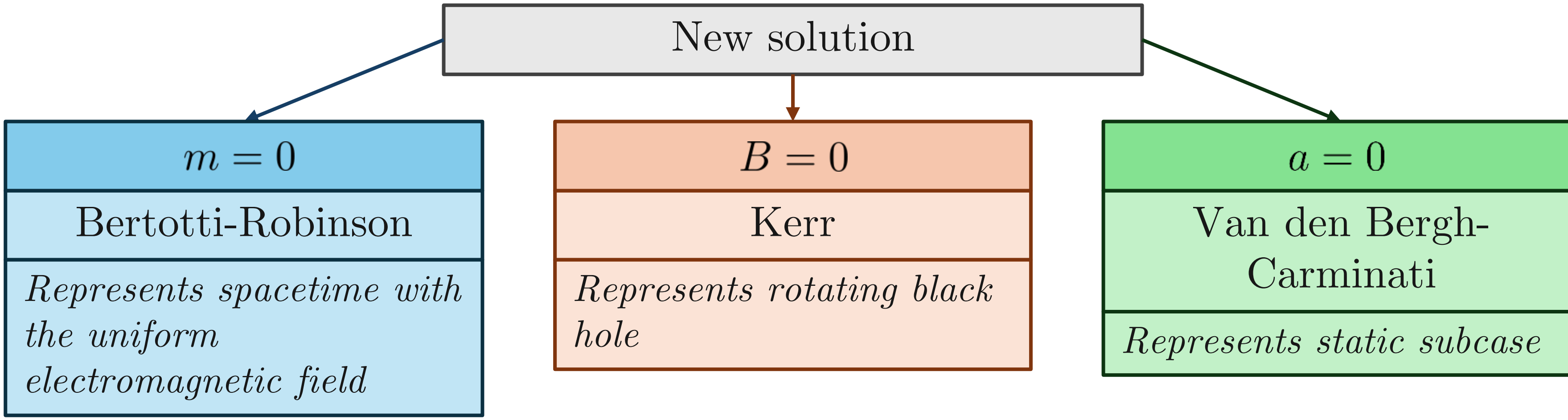

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Kerr-Bertotti-Robinson black hole

New solution

- $m = 0$
  - Bertotti-Robinson
  - *Represents spacetime with the uniform electromagnetic field*
- $B = 0$
  - Kerr
  - *Represents rotating black hole*
- $a = 0$
  - Van den Bergh-Carminati
  - *Represents static subcase*

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Does our solution possess some more realistic properties?

Singularities of the Kerr-BR

- Curvature singularity

The only non-zero scalar, characterising this spacetime, is Ψ_2

$$\Psi_2 = -\frac{m}{(r + i\,a\cos\theta)^3}\left[\Big(1 - i\,B^2 a\,r\cos\theta\Big)I_1 I_2 + B^2\,\frac{m\,r^2\cos^2\theta}{r - i\,a\cos\theta}\right]\frac{\Omega^2}{I_1^2}$$

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with the unique choice

$$C = \left[1 + B^2\left(m^2\,\frac{I_2}{I_1^2} - a^2\right)\right]^{-1}$$

Horizons

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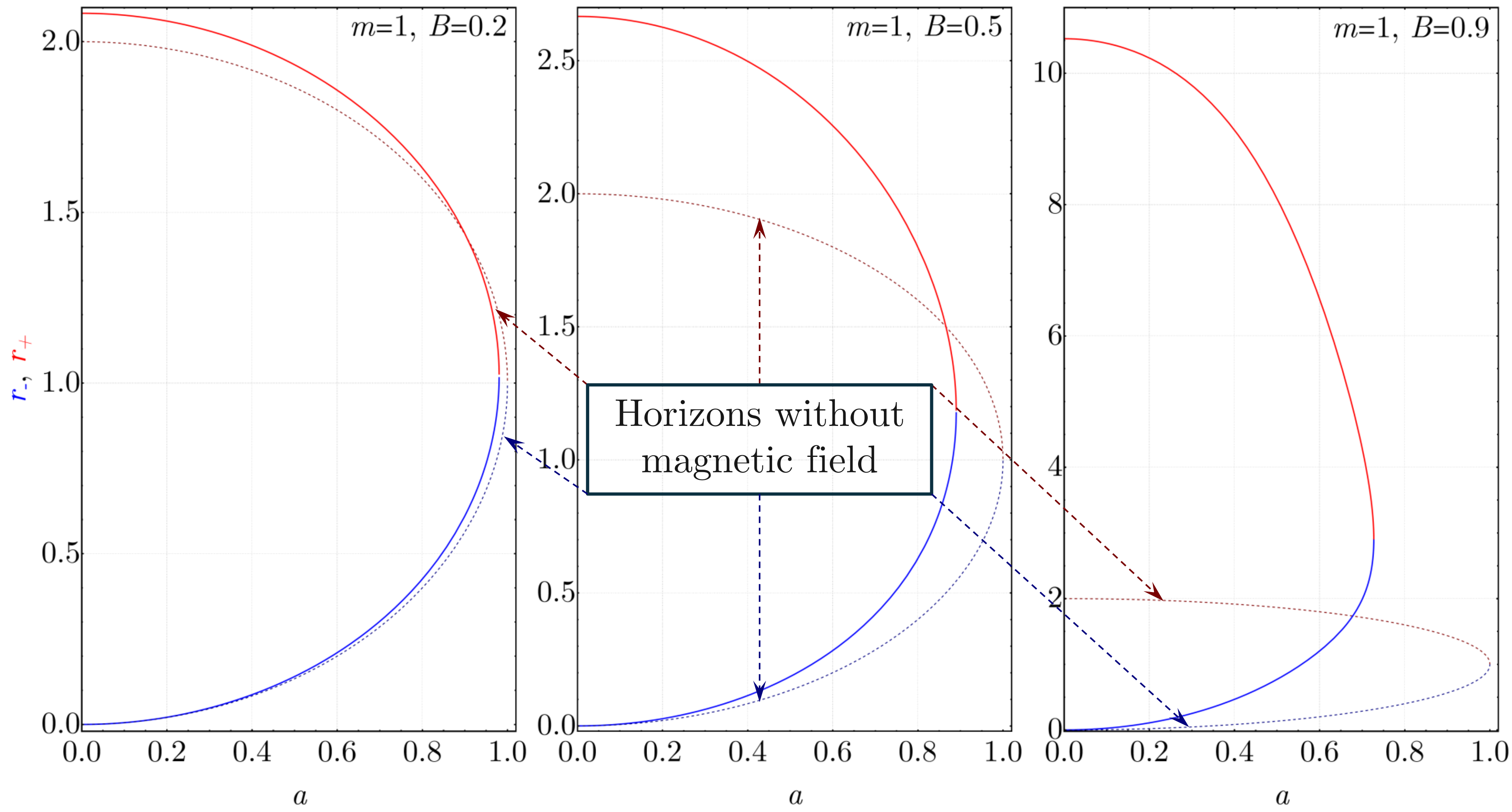
Horizon becomes extreme for $B^2 = B^2_{\rm extr} = \dfrac{2}{a^4}\big(m - \sqrt{m^2 - a^2}\,\big)\sqrt{m^2 - a^2}$

$$r_{\rm extr} = \frac{m}{I_1} = \frac{a^2}{m - \sqrt{m^2 - a^2}}$$

Horizons



Electromagnetic field

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To visualize the electromagnetic field, we conduct the 3+1 splitting of this spacetime, adapted to a ZAMO observer with 4-velocity

$$\mathbf{u} = N^{-1}\big(\partial_t + \omega\,\partial_\varphi\big)\,, \quad \text{where} \quad N^2 = \frac{PQ\rho^2}{R\,\Omega^2}\,,$$

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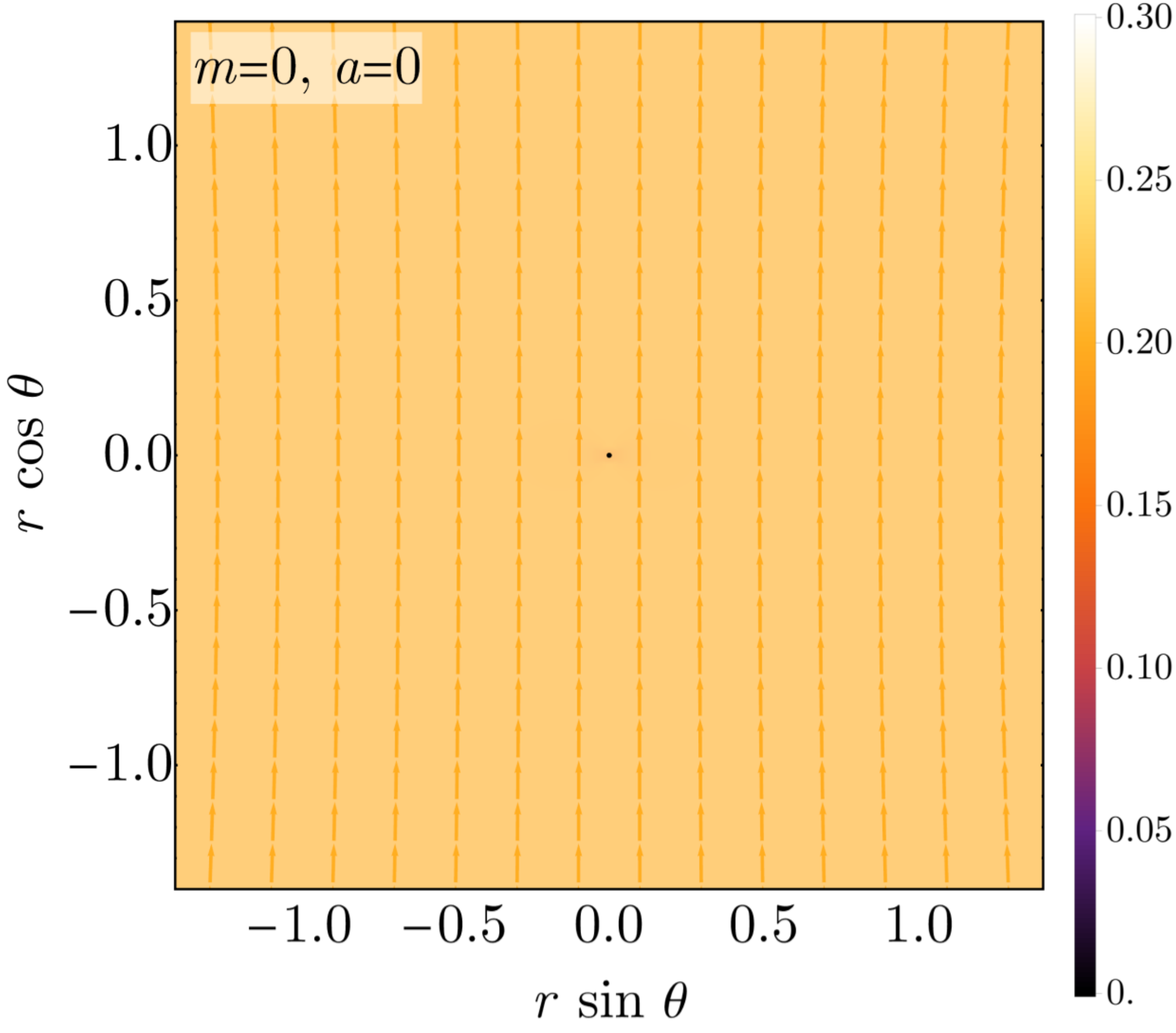
From the axial symmetry directly follows that

$$E^{((\varphi)} = B^{(\varphi)} = 0$$

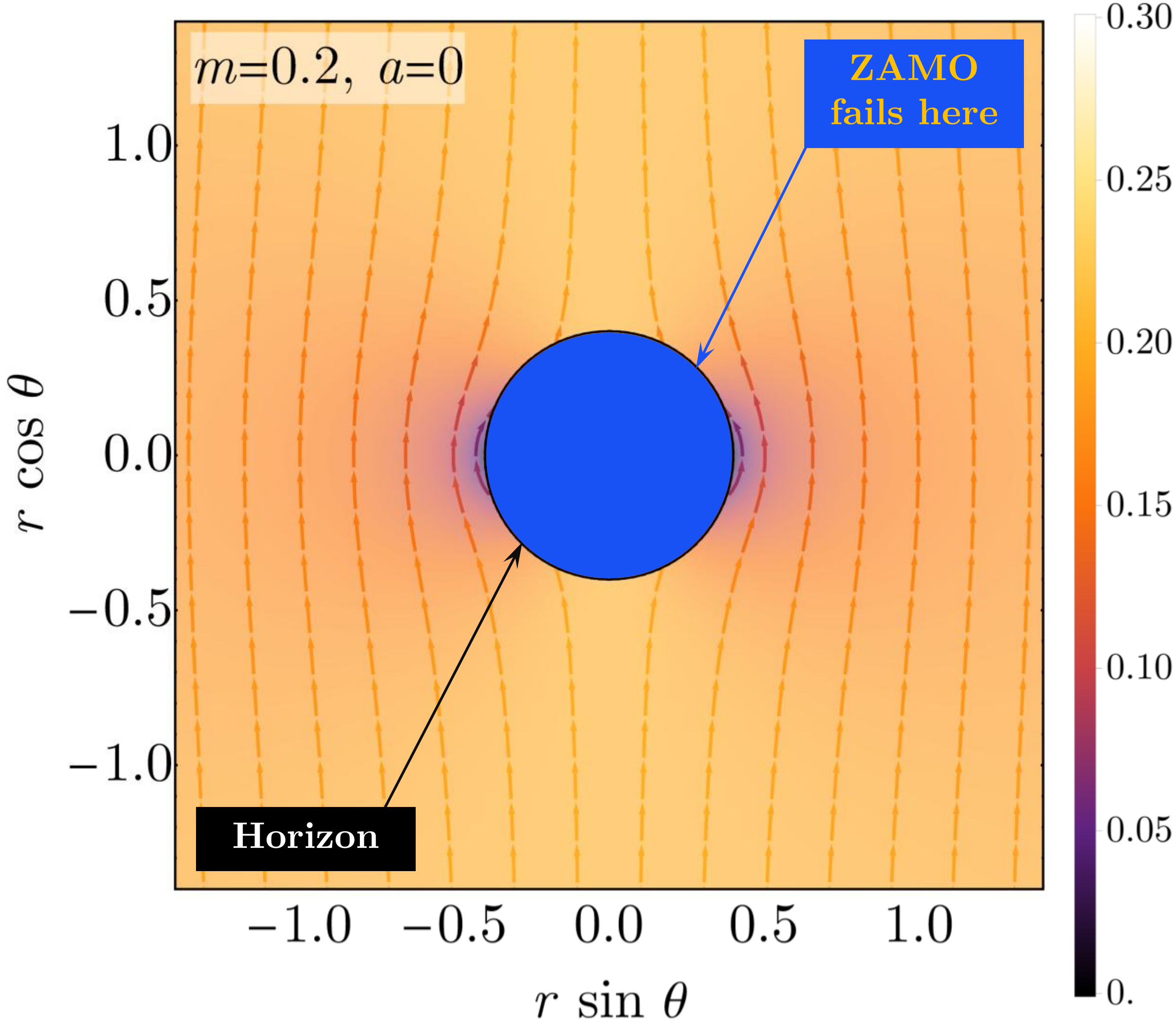
Electromagnetic field (B=0.2)



No rotation
No mass

Uniform
electromagnetic field

Electromagnetic field (B=0.2)

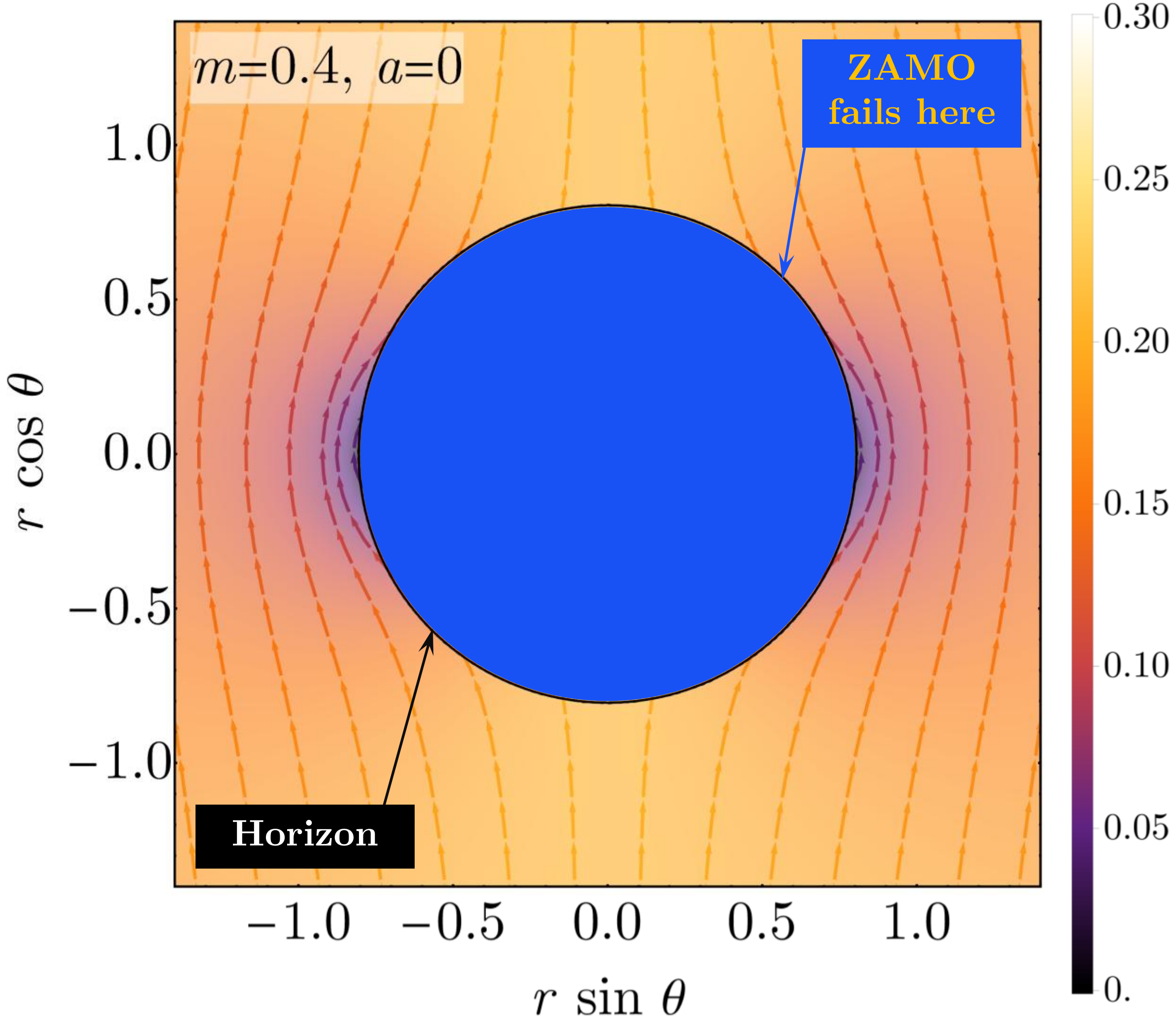


When mass is added, electromagnetic field is being expelled

This is known as the black hole Meissner effect

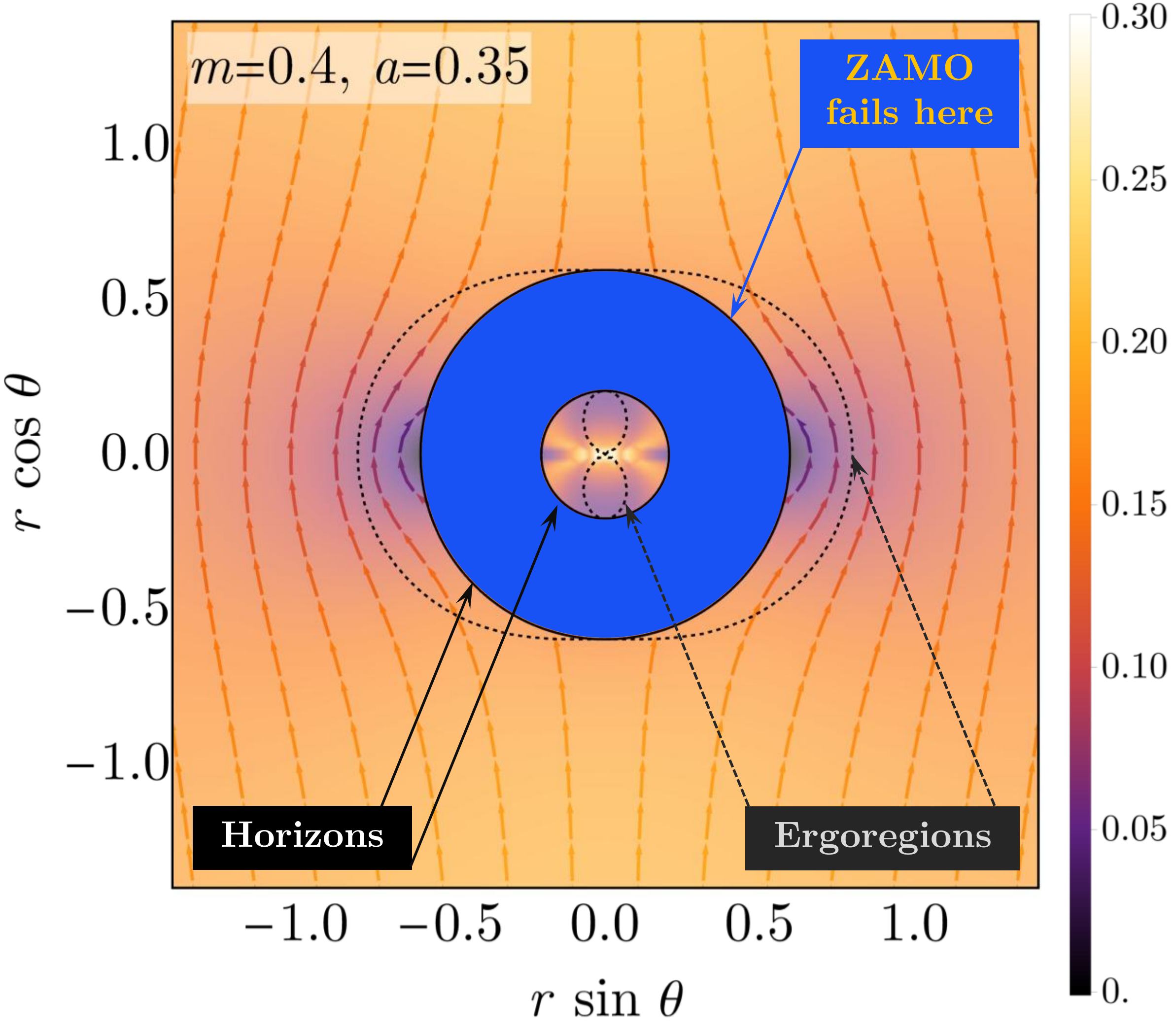
Expulsion is not full as a BH is not charged

Electromagnetic field (B=0.2)



As we increase mass, phenomenon becomes stronger

Electromagnetic field (B=0.2)



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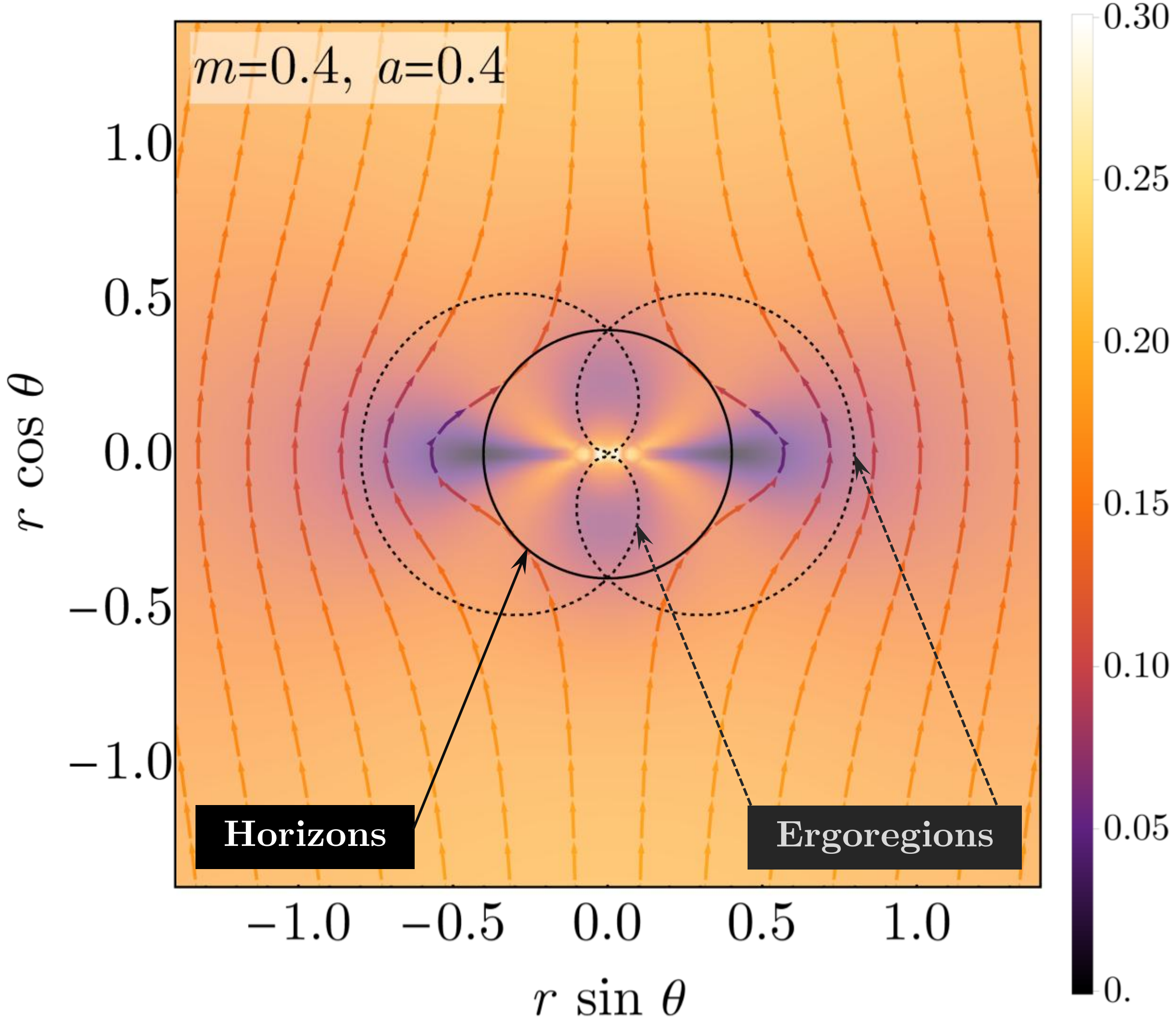

Geodesic motion and thermodynamics

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Thorne (1965), Melvin and Wallingford (1966)

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But for equatorial motion (and for zero Kerr parameter a) equation of radial motion is given by

$$u_r^2 = \Omega^4 \Big[E^2/m_0^2 - V(r) \Big]$$

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But for equatorial motion (a=*0*) equation of radial motion is given by

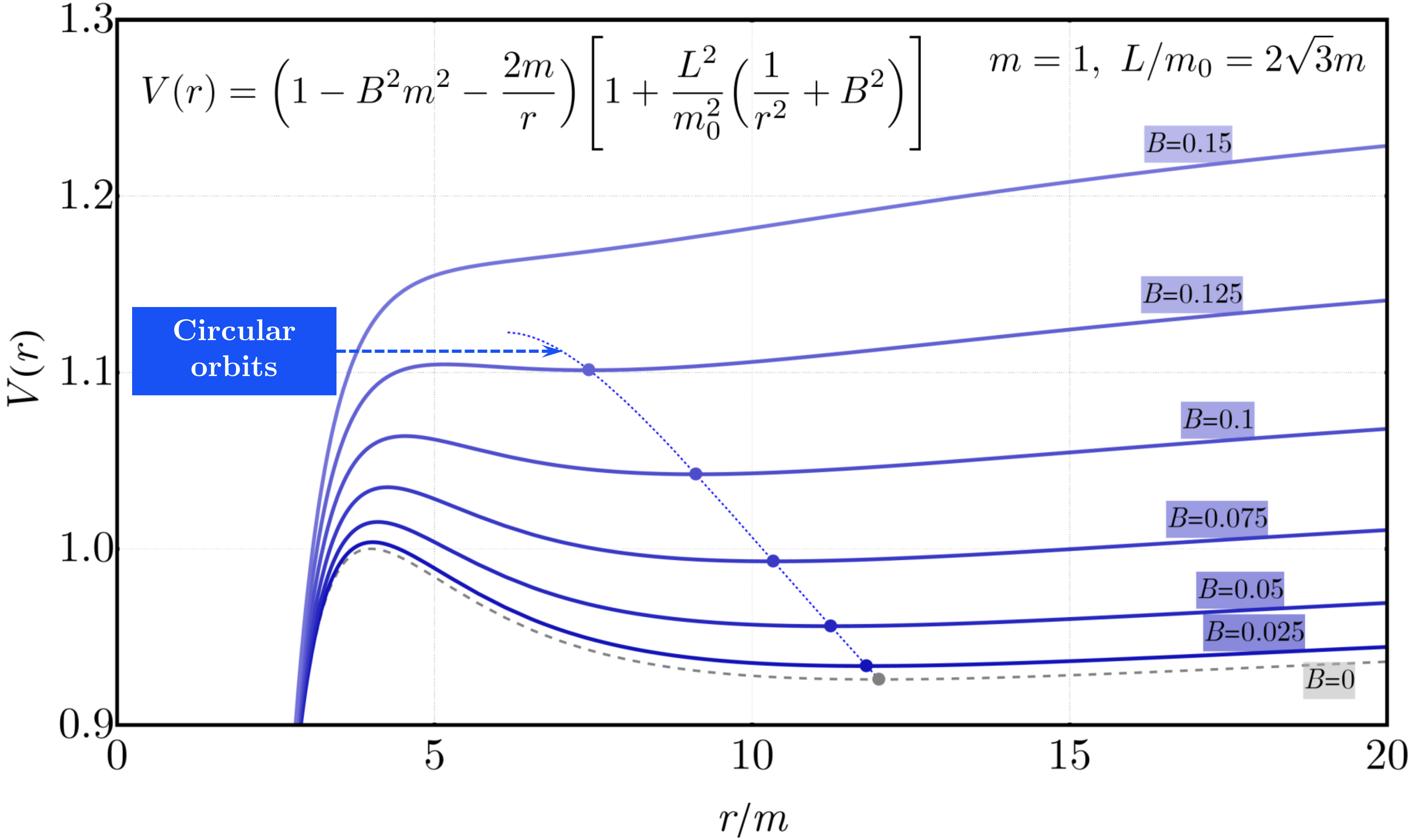
$$u_r^2 = \Omega^4 \Big[E^2/m_0^2 - V(r) \Big]$$

where

$$\Omega^2 = 1 + B^2 r^2$$

$$V(r) = \left(1 - B^2 m^2 - \frac{2m}{r}\right)\left[1 + \frac{L^2}{m_0^2}\left(\frac{1}{r^2} + B^2\right)\right]$$

Effective potential



Geodesics and ISCO

For large radii one gets

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Direct computation shows $r_{\text{ISCO}} = 3r_h$ the same relation as for Schwarzschild!

However, in this case the horizon is further then for the Schwarzschild case

$$r_h = \frac{2m}{1 - B^2m^2} > 2m$$

Thermodynamics

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Variation of the external parameter (magnetic field) does not have to enter the 1-st law for a BH

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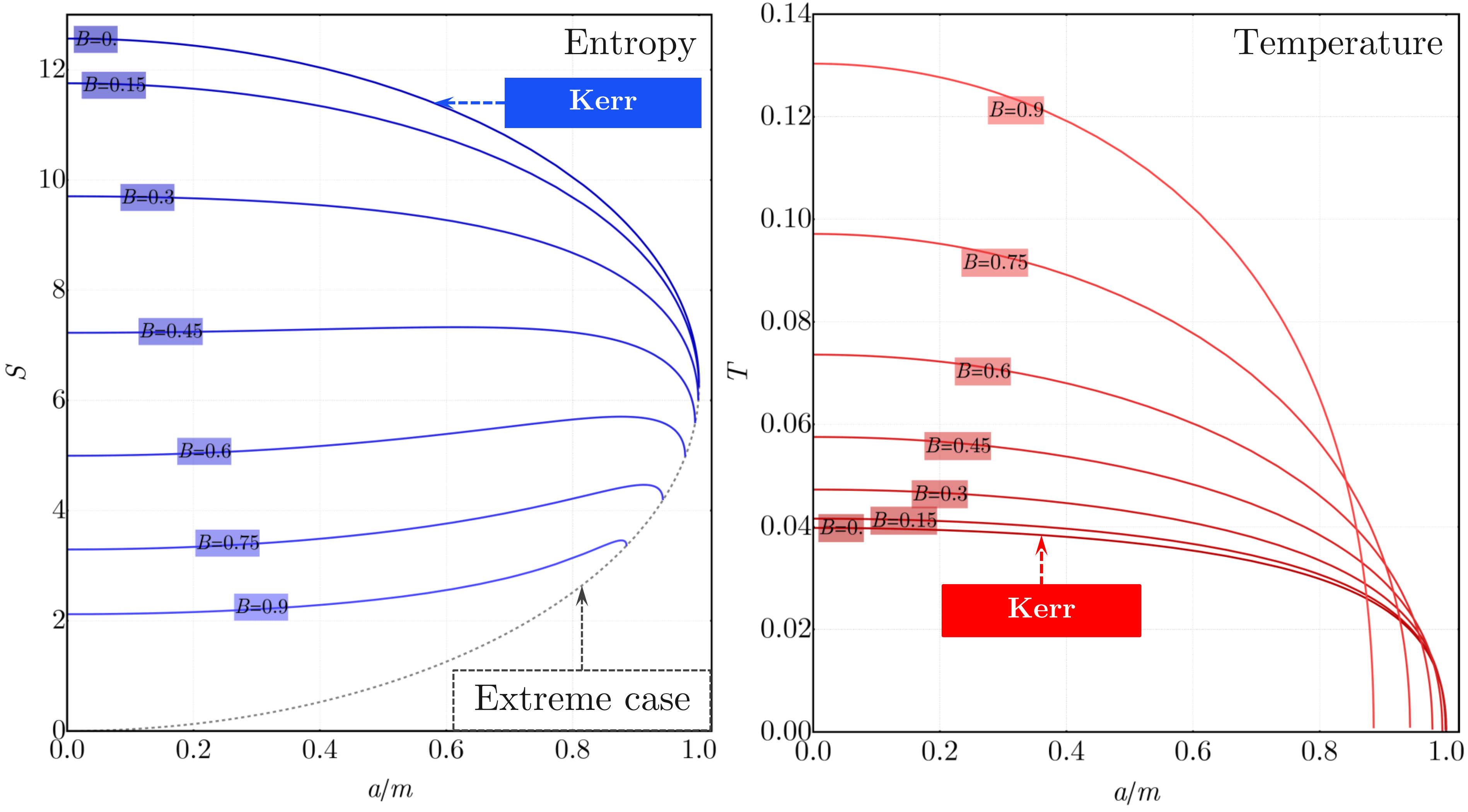

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- Black holes within this class exhibit expulsion of the magnetic field the so-called ***Meissner effect***

Thank you for your attention!

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