Symmetric Mass Generation as a multicritical point with enhanced symmetry

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Collaborators and Papers

Sandip Maiti

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- Symmetric mass generation as a multicritical point with enhanced symmetry, arXiv:2512.24836.
- Phase diagram of a lattice fermion model with symmetric mass generation, arXiv:2602.18360.

Outline

Mass generation paradigms in QFT

Microscopic model and Symmetries

Fermion Bags

Results

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Masses in QFTs

Mass terms in QFTs are often an emergent property:

- whether a fermion bilinear condenses,
- whether a symmetry is broken – either explicitly or spontaneously,
- whether topologies or the anomaly structure plays a role,
- whether the IR theory is trivial or interacting,
- trivial case: explicit masses

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + m \bar{\psi} \psi$$

- dynamical (continuous) symmetry breaking:

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + g (\bar{\psi} \psi)^2$$

At strong coupling $\langle \bar{\psi} \psi \rangle \neq 0$, giving rise to a mass term.

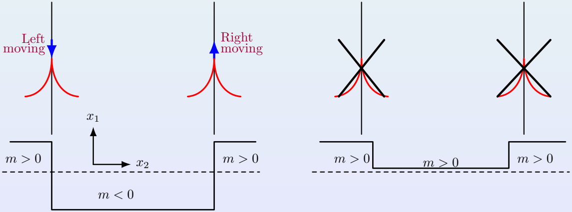
Masses in QFTs

- spontaneous symmetry breaking (SSB) with scalars:

  $$\mathcal{L} = |D_\mu \phi|^2 - V(\phi) + y\phi\bar{\psi}_L\psi_R$$

  After SSB: $\langle\phi\rangle \neq 0$: massive fermions $m_\psi = y\langle\phi\rangle$ and massless Goldstones.

- Topological origins: $\mathcal{L} = \bar{\psi} i \not{\partial} \psi - m\bar{\psi}\psi$ (e.g. Callan-Harvey mechanism, 1984)



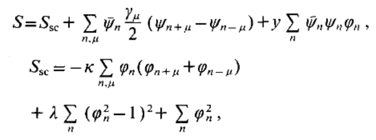
Symmetric Mass Generation (SMG)

Generate masses without topology or symmetry breaking?
symmetric mass generation.

- Require the presence of strong interactions:
  typically multi-fermion interactions
- All fermion bilinears are forbidden by symmetry
- The IR is a featureless symmetric gapped phase
- No 't Hooft anomalies: anomaly cancellation important
- LFT community recognised this as a possibility from 1980's, but were widely regarded as lattice artefacts.
- Research on pure fermionic theories, fermions + gauge theories, and anomaly cancelling mechanism now consider this phase seriously.
- Microscopic models where this phase is present?
  New fixed point in the RG flow of fermionic theories?

Hints of critical points

Previous studies of Hasenfratz and Neuhaus PLB 220,3 (1989) with a Yukawa model in $d = 4$.



$$S = S_{\mathrm{sc}} + \sum_{n,\mu} \bar{\psi}_n \frac{\gamma_\mu}{2} (\psi_{n+\mu} - \psi_{n-\mu}) + y \sum_n \bar{\psi}_n \psi_n \varphi_n ,$$

$$S_{\mathrm{sc}} = -\kappa \sum_{n,\mu} \varphi_n (\varphi_{n+\mu} + \varphi_{n-\mu})$$

$$+ \lambda \sum_n (\varphi_n^2 - 1)^2 + \sum_n \varphi_n^2 ,$$

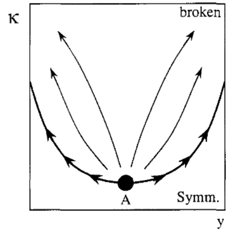
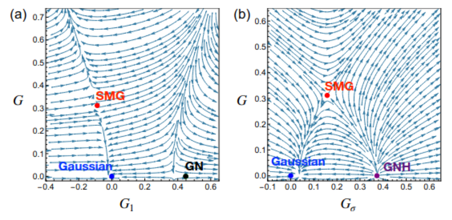


Fig. 2. The sketch of the flow lines in the κ–y plane. The solid line represents the symmetry breaking phase transition.

- Tuning of two bare-couplings to access the tricritical point.
- Fermion masses in the symmetric phase were at the cut-off scale.

Hints of critical points

- Ayyar and Chandrasekharan, PRD 91, 065035 (2015).
- Caterall, JHEP 01, 121 (2016).
- Both observed a direct transition between a MF ↔ SMG phase.
- (Many) studies in condensed matter theory reported similar results.
- Study in the ϵ-expansion seems to produce the presence of a fixed point governing the SMG physics. Martin, Grover, arXiv:2507.23032



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Lattice model

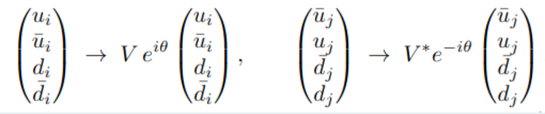
- Model in $(2+1)$-d Euclidean space–time system, $n_f = 2$ massless staggered fermions, denoted u and d.
- Four Grassmann-valued staggered fermion fields at each site i, namely $\bar{u}_i$, u_i, $\bar{d}_i$, and d_i.

$$\begin{aligned} S = & \sum_{\langle ij \rangle} \eta_{ij} \left(\bar{u}_i u_j - \bar{u}_j u_i + \bar{d}_i d_j - \bar{d}_j d_i \right) - U_I \sum_i \left(\bar{u}_i u_i \, \bar{d}_i d_i \right) \\ & - U_B \sum_{\langle ij \rangle} \left(\bar{u}_i u_i \, \bar{u}_j u_j + \bar{d}_i d_i \, \bar{d}_j d_j \right). \end{aligned}$$

- Nearest neighbour couplings on a cubical lattice $t, x, y = 0, \cdots, L-1$.
- Phase factors η_{ij} implement a π-flix through each plaquette: $\eta_{\hat{t}} = 1$, $\eta_{\hat{x}} = (-1)^t$, $\eta_{\hat{y}} = (-1)^{t+x}$.

Symmetries 1

- $S_F = \sum_{\langle ij \rangle} \eta_{ij} \left(\bar{u}_i u_j - \bar{u}_j u_i + \bar{d}_i d_j - \bar{d}_j d_i \right) = \sum_{\langle ij \rangle} (\bar{u}_i M_{ij} u_j + \bar{d}_i M_{ij} d_j)$
- Free fermion hopping; invariant under internal $SU(4) \times U(1)$

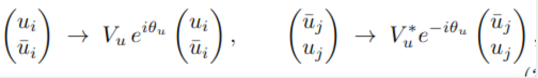


$$\begin{pmatrix} u_i \\ \bar{u}_i \\ d_i \\ \bar{d}_i \end{pmatrix} \to V\, e^{i\theta} \begin{pmatrix} u_i \\ \bar{u}_i \\ d_i \\ \bar{d}_i \end{pmatrix}, \qquad \begin{pmatrix} \bar{u}_j \\ u_j \\ \bar{d}_j \\ d_j \end{pmatrix} \to V^* e^{-i\theta} \begin{pmatrix} \bar{u}_j \\ u_j \\ \bar{d}_j \\ d_j \end{pmatrix}$$

- Site i is even and j odd, V is an element of $SU(4)$,
- $e^{i\theta}$ represents a $U(1)$ phase → axial symmetry of staggered fermions.
- $S_I = -U_I \sum_i (\bar{u}_i u_i \, \bar{d}_i d_i)$
- Invariant under $SU(4)$ but explicitly breaks the $U(1)$ symmetry.
- Behaves like a 't Hooft vertex, introducing instanton like effects.

Symmetries 2

- $S_B = -U_B \sum_{\langle ij \rangle} \left(\bar{u}_i u_i \, \bar{u}_j u_j + \bar{d}_i d_i \, \bar{d}_j d_j \right)$ → current-current interaction.
- Invariant under $SU(2) \times U(1)_{\mathrm{u-quark}}$ and $SU(2) \times U(1)_{\mathrm{d-quark}}$.
- Thus, the transformation on the u-quarks is, with $V_u \in SU(2)$:



$$\begin{pmatrix} u_i \\ \bar{u}_i \end{pmatrix} \to V_u \, e^{i\theta_u} \begin{pmatrix} u_i \\ \bar{u}_i \end{pmatrix}, \qquad \begin{pmatrix} \bar{u}_j \\ u_j \end{pmatrix} \to V_u^* e^{-i\theta_u} \begin{pmatrix} \bar{u}_j \\ u_j \end{pmatrix}$$

- Action on d-quarks is similar, with $V_u \to V_d$ and $e^{i\theta_u} \to e^{i\theta_d}$.
- For $\theta_u = \theta_d = \theta$, one obtains the $U(1)$ symmetry of S_I.
- Another independent $U_\chi(1)$ symmetry of S_B is obtained for $\chi = \theta_u = -\theta_d$.
- The $U_\chi(1)$ is a sub-group of the $SU(4)$ symmetry of S_I.
- For $U_B = 0$, $U_I \neq 0$, the full $SU(4)$ is intact, but for $U_B \neq 0$, $U_I \neq 0$ one has $SU(2) \times SU(2) \times U_\chi(1)$.

Condensates and Scalings

- Bilinear condensates $\langle \bar{u} u \rangle$ and $\langle \bar{d} d \rangle$ spontaneously break $U_\chi(1)$.
- Detect by measuring the following susceptibilities:

$$\chi_{ud} = \frac{1}{2L^3} \sum_{i,j} \langle \bar{u}_i u_i \; \bar{d}_j d_j \rangle \, ,$$

$$\chi_{uu} = \frac{1}{2L^3} \sum_{i,j} \langle \bar{u}_i u_i \; \bar{u}_j u_j \rangle \, ,$$

$$\chi_{dd} = \frac{1}{2L^3} \sum_{i,j} \langle \bar{d}_i d_i \; \bar{d}_j d_j \rangle \, .$$

- Note: $\chi_{uu} = \chi_{dd}$ (symmetry of action), but $\chi_{ud} \approx \chi_{uu}$ (empirically) on large lattices, except when U_I is small.
- In a phase with SSB, $\chi \sim L^3$, while in both the MF phase and the SMG phase we expect $\chi \sim$ const.
- At a critical point, in contrast, we expect the finite-size scaling behavior $\chi \sim L^{2-\eta}$.

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Traditional methods

- E.g.: $S = \sum_{\langle ij \rangle} \bar{\psi}_i M_{ij} \psi_j - U \sum_{\langle ij \rangle} \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j$.
- Use auxiliary bosons (continuous or discrete) and Hubbard-Stratanovich transformations:

$$e^{U \sum_{\langle ij \rangle} \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j} = \prod_{\langle ij \rangle} \left[\frac{1}{2} \sum_{\sigma_{ij} = \pm 1} e^{\sigma_{ij} \sqrt{U} (\bar{\psi}_i \psi_j - \bar{\psi}_j \psi_i)} \right]$$

$$= \frac{1}{2^{N_b}} \sum_{\{\sigma_{ij}\}} \exp \left[\sqrt{U} \sum_{\langle ij \rangle} \sigma_{ij} (\bar{\psi}_i \psi_j - \bar{\psi}_j \psi_i) \right].$$

- Rewrite the partition function:

$$Z = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \quad e^{- \sum_{\langle ij \rangle} \bar{\psi}_i M_{ij} \psi_j + U \sum_{\langle ij \rangle} \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j},$$

$$= \frac{1}{2^{N_b}} \sum_{\{\sigma_{ij}\}} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \, e^{- \sum_{\langle ij \rangle} \bar{\psi}_i M'_{ij} [\sigma_{ij}] \psi_j} = \frac{1}{2^{N_b}} \sum_{\{\sigma_{ij}\}} \det M'_{ij} [\{\sigma_{ij}\}]$$

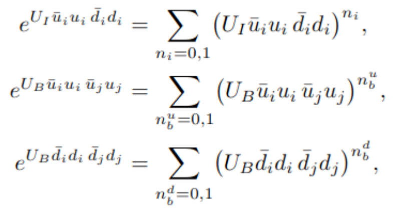
$$M'_{ij} = M_{ij} - \sigma_{ij} \sqrt{U}; \quad M'_{ji} = M_{ji} + \sigma_{ij} \sqrt{U}$$

- Can be simulated if there is no sign problem.

Dual variables

$$Z = \int [\mathcal{D}\bar{d}\mathcal{D}d\mathcal{D}\bar{u}\mathcal{D}u]\ e^{-\left(\bar{u}_i M_{ij} u_j + \bar{d}_i M_{ij} d_j\right)} \prod_i e^{U_I \bar{u}_i u_i\ \bar{d}_i d_i} \prod_{b\equiv\langle ij\rangle} e^{U_B\left(\bar{u}_i u_i \bar{u}_j u_j + \bar{d}_i d_i \bar{d}_j d_j\right)}$$

Expand the interactions:



$$e^{U_I \bar{u}_i u_i\ \bar{d}_i d_i} = \sum_{n_i=0,1} \left(U_I \bar{u}_i u_i\ \bar{d}_i d_i\right)^{n_i},$$

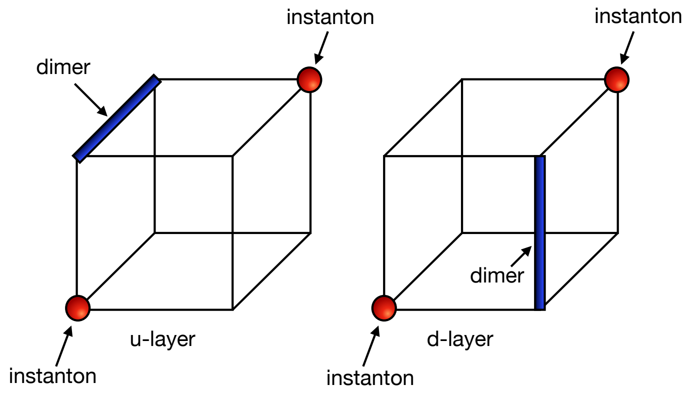
$$e^{U_B \bar{u}_i u_i\ \bar{u}_j u_j} = \sum_{n_b^u=0,1} \left(U_B \bar{u}_i u_i\ \bar{u}_j u_j\right)^{n_b^u},$$

$$e^{U_B \bar{d}_i d_i\ \bar{d}_j d_j} = \sum_{n_b^d=0,1} \left(U_B \bar{d}_i d_i\ \bar{d}_j d_j\right)^{n_b^d},$$

$n_i = 0, 1$ instanton at site i; $n_b^u, n_b^d = 0, 1$ u, d-type dimer on bond b.

Dimers and instantons

Physically, the set of all n_i, n_b^u and n_b^d define a instanton-dimer configuration.



Rewriting Z

$$
\begin{aligned}
Z &= \sum_{[n_i, n_b^u, n_b^d]} \int [\mathcal{D}\bar{d}\,\mathcal{D}d\,\mathcal{D}\bar{u}\,\mathcal{D}u]\; e^{-\left(\bar{u}_i M_{ij} u_j + \bar{d}_i M_{ij} d_j\right)} \prod_i (\bar{u}_i u_i\, \bar{d}_i d_i)^{n_i} \prod_b (\bar{u}_i u_i\, \bar{u}_j u_j)^{n_b^u} (\bar{d}_i d_i\, \bar{d}_j d_j)^{n_b^d} \\
&= \sum_{[n_i, n_b^u, n_b^d]} U_I^{n_I} U_B^{n_B} \quad \left\{ \int [\mathcal{D}\bar{u}\,\mathcal{D}u]\; e^{-\bar{u}_i M_{ij} u_j} \prod_i (\bar{u}_i u_i)^{n_i} \prod_b (\bar{u}_i u_i\, \bar{u}_j u_j)^{n_b^u} \right\} \\
&\qquad\qquad \times \quad \left\{ \int [\mathcal{D}\bar{d}\,\mathcal{D}d]\; e^{-\bar{d}_i M_{ij} d_j} \prod_i (\bar{d}_i d_i)^{n_i} \prod_b (\bar{d}_i d_i\, \bar{d}_j d_j)^{n_b^d} \right\},
\end{aligned}
$$

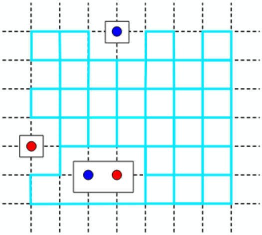
Grassmann integrals have been separated into u and d fields, and we defined $n_I = \sum_i n_i, \quad n_B = \sum_b (n_b^u + n_b^d)$.

Fermion partition function is a sum over all paths that does not contain the interaction sites.

$$
\int' [d\overline{\psi} d\psi]\; \mathrm{e}^{-\sum_{x,\alpha} \frac{\eta_{x,\mu}}{2} \left(\overline{\psi}_x\ \psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\ \psi_x\right)}
$$

$$
= \ \mathrm{Det}(W_k[n])
$$

which is positive!

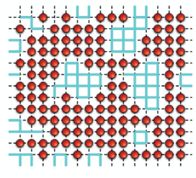


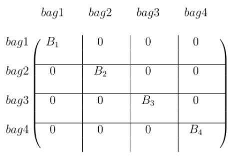
Strong Coupling Bags

- Integrating over Grassmann variables on sites occupied by bonds or instantons:

$$Z = \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u+N_d} \det(W_u) \det(W_d)$$

- W_u and W_d are free staggered fermion matrices restricted to the unoccupied lattice sites.
- Connected clusters of these sites define strong-coupling fermion bags.
- Bags are regions where fermions can freely propagate.





$$\begin{array}{c|c|c|c|c} & bag1 & bag2 & bag3 & bag4 \\ bag1 & B_1 & 0 & 0 & 0 \\ \hline bag2 & 0 & B_2 & 0 & 0 \\ \hline bag3 & 0 & 0 & B_3 & 0 \\ \hline bag4 & 0 & 0 & 0 & B_4 \end{array}$$

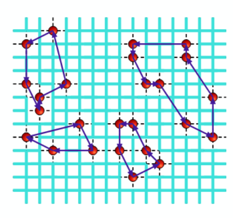
Weak Coupling Bags

- Treat interactions as operator insertions in an expansion about the free staggered fermion theory.

$$Z = \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u+N_d} \langle \bar{u}_{a_1} u_{a_1} \cdots \bar{u}_{a_l} u_{a_l} \rangle \langle \bar{d}_{b_1} d_{b_1} \cdots \bar{d}_{b_k} d_{b_k} \rangle$$

- $\{a_1, \ldots, a_l\}$ and $\{b_1, \ldots, b_k\}$ denote the lattice sites at which interaction insertions occur in the u and d fermion sectors.
- Wick's theorem → determinants of matrices built from staggered fermion propagators G_u, G_d constructed from M^{-1}:

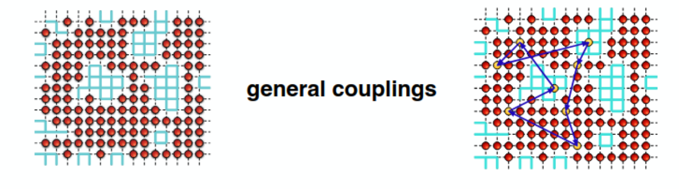
$$Z = (\det M)^2 \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u+N_d} \det(G_u) \det(G_d)$$



Fermion bags identified with connected clusters of interaction sites. Diagrammatic Determinantal Monte Carlo.

Monte Carlo updates

- Use the weak coupling fermion bag approach while starting from small values of U_I and U_B.
- Local update algorithms to add/remove bonds and two instantons at a time.
- Worm algorithm for non-local updates to fermionic configurations.
- Improved estimators for susceptibilities.

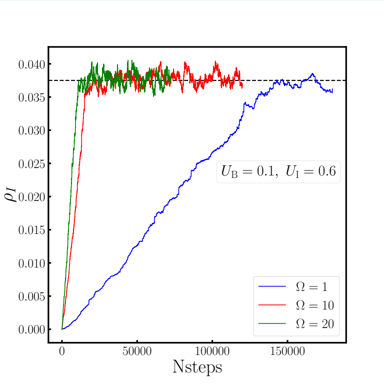


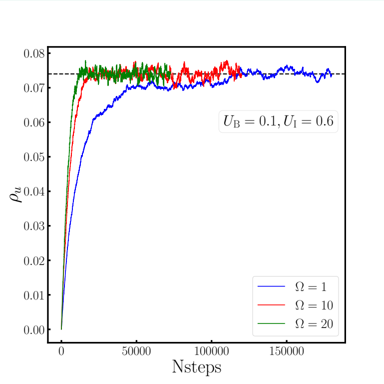
Fluctuation Matrices

- Each MC step involves the computation of determinant ratios.
- Computation of full determinant ratios both numerically unstable and computationally inefficient. Use fluctuation matrices.
- Given a base configuration (propagator: G_B) the update proposes a new configuration G (by deleting k_d rows and adding k_a rows).
- Then, $\frac{\det G}{\det G_B} = \det F$, where F is a square matrix of $k = k_d + k_a$ from appropriate elements of G_B^{-1}.
- Allows a fast computation of local updates using the formula $\frac{\det G_1}{\det G_2} = \frac{\det F_1}{\det F_2}$.
- Algorithm controls the size of fluctuation matrices, if the exceed a certain size, then base config is refreshed to the current configuration.
- The reset is the most time consuming step of the algorithm, but this happens infrequently.

Thermalisation and Autocorrelations

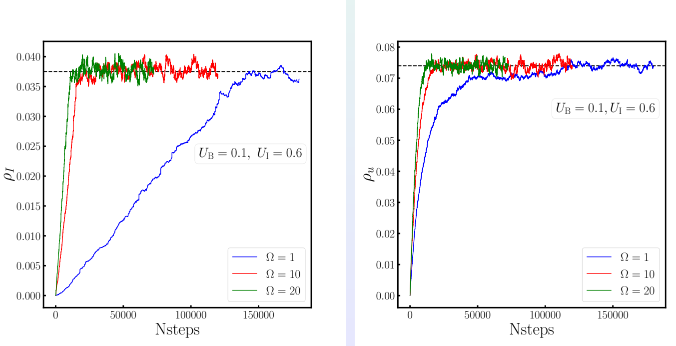
- Reweighting factor Ω is used to control autocorrelations and thermalisation.
- $\Omega = 1 : 1.8 \times 10^5$ steps: 8 h.
- $\Omega = 10 : 0.72 \times 10^5$ steps: 43h (=38h [f-matrix] + 5h [reset]).
- $\Omega = 20 : 0.72 \times 10^5$ steps: 88h (=78h [f-matrix] + 10h [reset]).





Thermalisation and Autocorrelations

- Mostly used $\Omega = 20$.
- Statistics of $2.5 - 8 \times 10^6$ for GN transition, $L = 28 - 56$.
- Autocorrelations $\tau \approx 4000$ close to the phase transition.
- Statistics of $2 - 4 \times 10^6$ for XY transition, $L = 12 - 32$.
- Autocorrelations $\tau \approx 1500$ close to the phase transition



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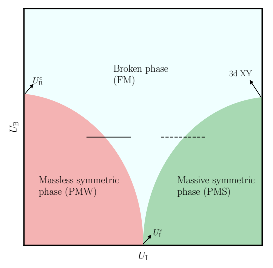
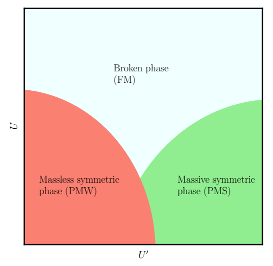
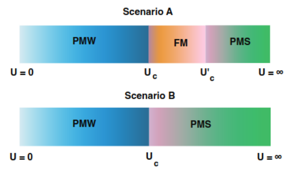
What is the phase diagram?

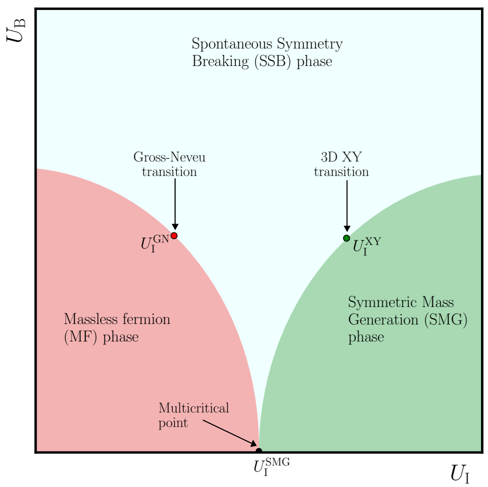




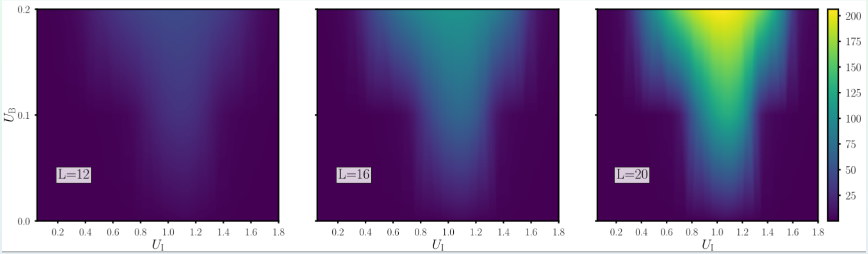
There is a direct transition from MF to SMG (Ayyar and Chandrasekharan, PRD 91, 065035 (2015))



This is the phase diagram

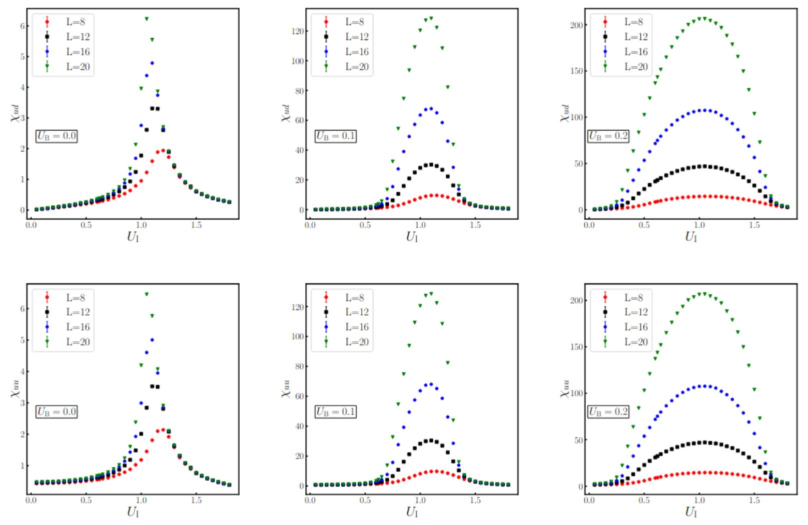


Evidence from susceptibility

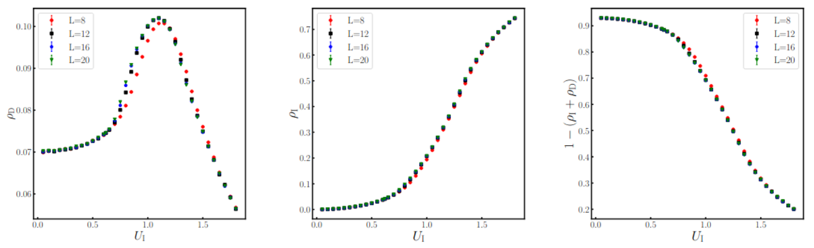


- χ_{ud} heat maps for $L = 12, 16, 20$, illustrating the structure of the phase diagram.
- Enhanced susceptibilities indicative of a SSB phase are visible for U_B values as small as $U_B = 0.01$ at $U_I = 1.0$.

Evidence from susceptibility

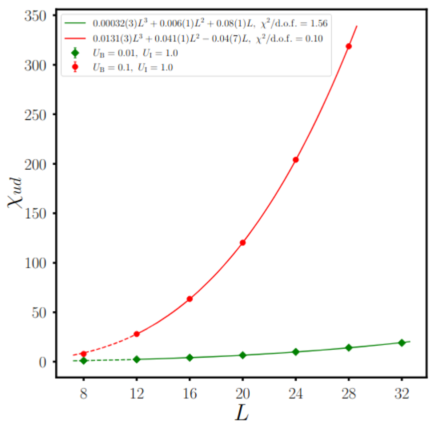


Bonds, Instantons, and Free Bags



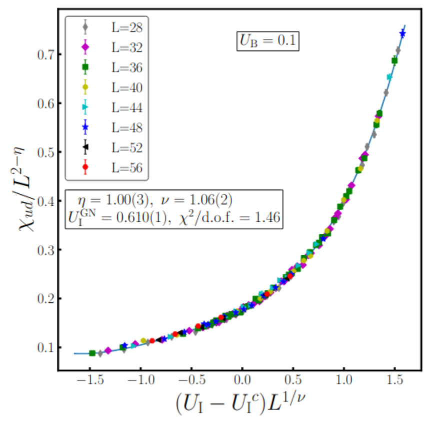
- Phase transition is triggered by about 20% of sites having monomer insertions.
- For a 50^3 lattice, this means that fermion bag algorithm is already dealing with matrices of size $\mathcal{N} = 2.5 \times 10^4$.
- Ideally need to switch to the strong coupling bag.

$U_I(c)$ at $U_B = 0$: multicritical point?



- Even $U_B = 0.01$ causes SSB at $U_I(c)$, detect by χ-PT inspired fits.
- SSB breaks the $U_\chi(1)$ subgroup in $SU(4) = SU(2) \times SU(2) \times U_\chi(1)$.

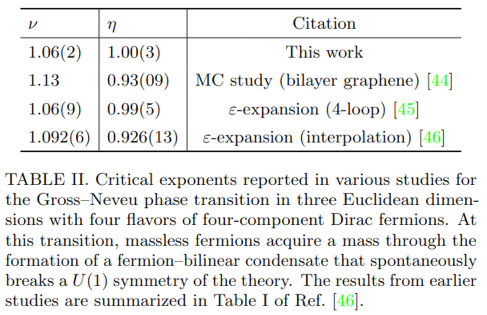
MF to SSB phase transition



Exponents extracted by fitting χ_{uu} and χ_{dd} using the finite-size scaling relation $\chi/L^{2-\eta} = f((U_I - U_I^C)L^{1/\nu})$ at the critical point.

MF to SSB: Gross-Neveu universality

- Staggered $n_f = 1$ in d=3 corresponds to $N_f = 2$ of four-component Dirac fermions at long distances.
- We should expect $N_f = 4$ four-component Dirac fermions and mean-field critical exponents.
- Exponents checked using various stability fits.

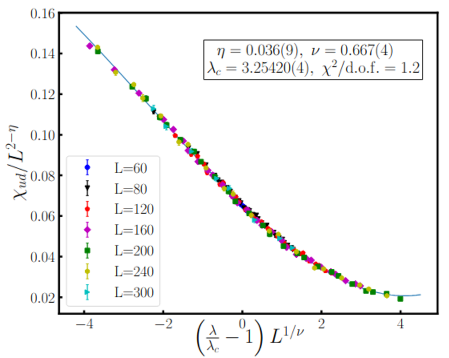


ν	η	Citation
1.06(2)	1.00(3)	This work
1.13	0.93(09)	MC study (bilayer graphene) [44]
1.06(9)	0.99(5)	ε-expansion (4-loop) [45]
1.092(6)	0.926(13)	ε-expansion (interpolation) [46]

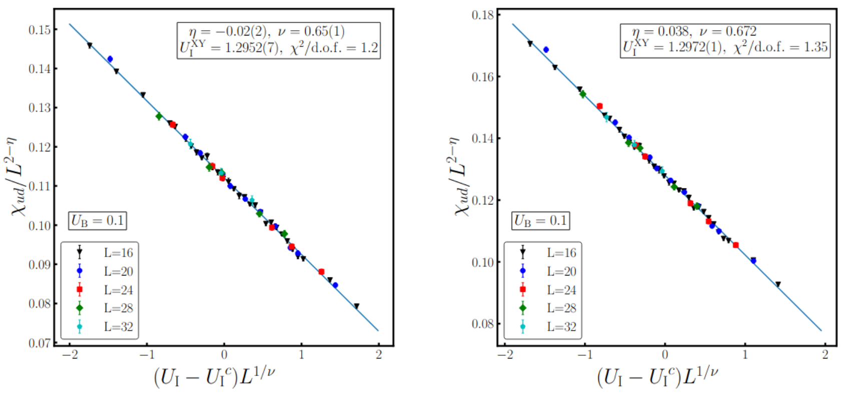
TABLE II. Critical exponents reported in various studies for the Gross–Neveu phase transition in three Euclidean dimensions with four flavors of four-component Dirac fermions. At this transition, massless fermions acquire a mass through the formation of a fermion–bilinear condensate that spontaneously breaks a $U(1)$ symmetry of the theory. The results from earlier studies are summarized in Table I of Ref. [46].

SSB to SMG phase transition

- Low-energy modes in the SSB phase are Goldstone bosons associated with $U_\chi(1)$, while fermionic excitations are massive.
- Long-distance critical behaviour = 3d XY universality.
- Not a-priori obvious if all fermionic effects are irrelevant at this PT.
- bosonic limit: (easy) $U_B \sim U_I \gg t$, and define $\lambda = U_I/U_B$.



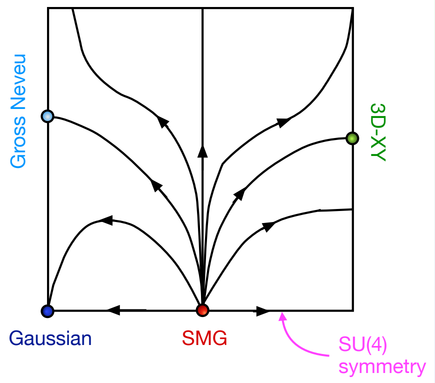
SSB to SMG: 3d XY universality



Different analysis: (left) fits critical exponents, while (right) fixes the critical exponents to the 3d XY values.

RG flow diagram

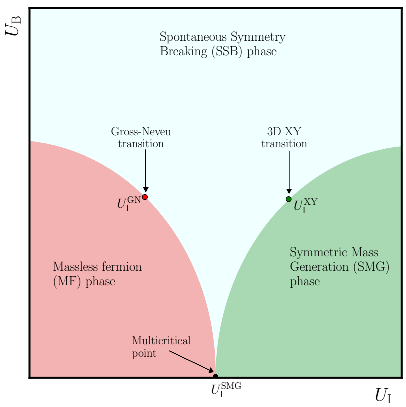
- Conjectured RG flows in $d = 3$ for fermionic Gross-Neveu-Yukawa models near the SMG fixed point.
- Gaussian FP: massless fermions,
- Gross-Neveu FP: interacting massless fermions and bosons,
- XY FP: interacting massless bosons,
- SMG FP: massless fermions, massive and massless bosons.



Further, for $N_f = 4$ four-component Dirac fermions there is no parity anomaly. It should be the case that anomalies due to the other global symmetries cancel as well.

Overview

- Confirmed the role of U_I as a multicritical point.
- Larger lattices
- Strong coupling bags at small U_B to look for fermionic effects.
- Spectroscopy in the SMG phase.
- Similar scenario in $d = 4$?



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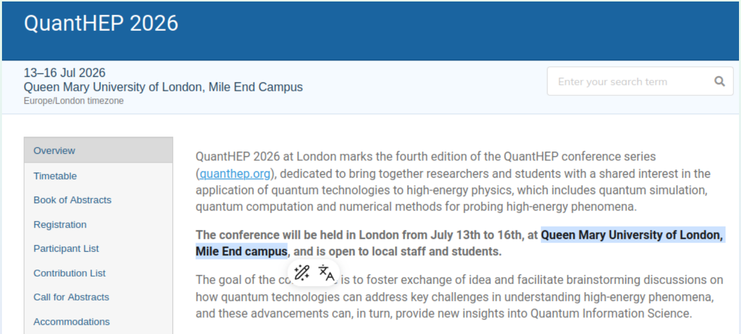

QuantHEP 2026

13–16 Jul 2026
Queen Mary University of London, Mile End Campus
Europe/London timezone

QuantHEP 2026 at London marks the fourth edition of the QuantHEP conference series (quanthep.org), dedicated to bring together researchers and students with a shared interest in the application of quantum technologies to high-energy physics, which includes quantum simulation, quantum computation and numerical methods for probing high-energy phenomena.

The conference will be held in London from July 13th to 16th, at Queen Mary University of London, Mile End campus, and is open to local staff and students.

The goal of the co[illegible] is to foster exchange of idea and facilitate brainstorming discussions on how quantum technologies can address key challenges in understanding high-energy phenomena, and these advancements can, in turn, provide new insights into Quantum Information Science.