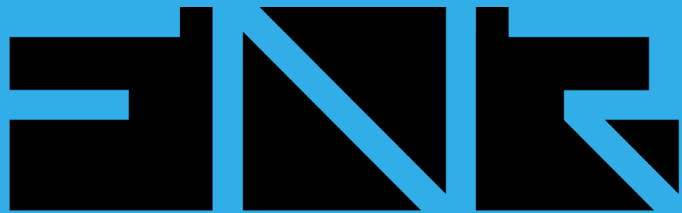


Spontaneous quantum turbulence via U(1) symmetry-breaking phase transition in a dilute Bose gas

Seong-Ho Shinn (Univ. Luxembourg)

Matteo Massaro (Univ. Luxembourg),
Mithun Thudiyangal (CQTCS, Christ Univ.),
Adolfo del Campo (Univ. Luxembourg, DIPC)



Luxembourg
National
Research Fund





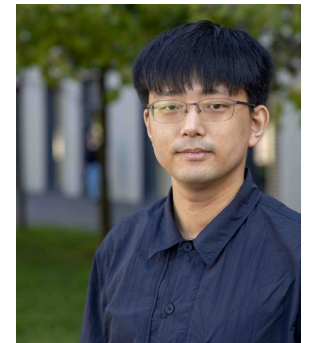
Prof. Dr. Adolfo del Campo



Dr. Budhaditya Bhattacharjee



Dr. András Grabarits



Dr. Seong-Ho Shinn



Preethi Gopalakrishnan



Matteo Massaro



Nilanjan Sasmal



Ankit Wenju Shrestha



Kasturi Ranjan Swain

- (News & Views)
 - A. del Campo and **S.-H. Shinn**, “Universal symmetry breaking passes the superfluid test”, Nat. Phys. **20**, 1523 (2024).
- (Articles)
 - **S.-H. Shinn** and A. del Campo, “Electrodynamics of Vortices in Quasi-2D Scalar Bose-Einstein Condensates”, Phys. Rev. Research **7**, 013217 (2025).
 - **S.-H. Shinn**, M. Massaro, M. Thudiyangal, and A. del Campo, “Spontaneous Quantum Turbulence in a Newborn Bose-Einstein Condensate via the Kibble-Zurek Mechanism, arXiv:2506.21670 [**cond-mat.quant-gas**].
 - A. del Campo, A. Grabarits, D. Makarov, and **S.-H. Shinn**, “Quantum Transition Rates in Arbitrary Physical Processes”, arXiv:2506.21672 [**quant-ph**]. Accepted in Phys. Rev. Lett. (2026).
 - M. Massaro, **S.-H. Shinn**, M. Thudiyangal, and A. del Campo, “Circulation Statistics and Migdal Area Rule Beyond the Kibble-Zurek Mechanism in a Newborn Bose-Einstein Condensate”, arXiv:2508.11047 [**cond-mat.quant-gas**].
 - S. Patra, A. Roy, **S.-H. Shinn**, A. del Campo, and M. Thudiyangal, “Kibble-Zurek Scaling and Spatial Statistics in Quenched Binary Bose Superfluids”, arXiv:2510.12770 [**cond-mat.quant-gas**]. Accepted in Commun Phys (2026).

1. $U(1)$ symmetry-breaking, quantum turbulence, and Kolmogorov scaling
2. Kibble-Zurek mechanism and power-law
3. Stochastic projected Gross-Pitaevskii equation
4. Numerical results – spontaneous quantum turbulence (SQT)
 - Incompressible kinetic energy spectra
 - Universality of the incompressible kinetic energy spectra
5. Numerical results – Migdal's area rule in SQT
6. Summary



1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



■ Hamiltonian of the dilute Bose gas

Number density of air (27°C) $\sim O(10^{25} \text{ m}^{-3})$

Number density of BEC $\sim O(10^{20} \text{ m}^{-3})$

Temperature of BEC $\lesssim O(100 \text{ nK})$

$$[\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}'), \quad \hat{N} := \int d^D r \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}).$$

$$\hat{H} = \int d^D r \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}) + \frac{1}{2} \int d^D r \int d^D r' V_{\text{int}}(\mathbf{r} - \mathbf{r}') \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}') \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r}).$$

$$V_{\text{int}}(\mathbf{r} - \mathbf{r}') \rightarrow g\delta(\mathbf{r} - \mathbf{r}'), \quad \hat{\psi}(\mathbf{r}) \rightarrow \psi(\mathbf{r}) \text{ in the limit } T \rightarrow 0.$$

$$\hat{H} - \mu(t) \hat{N} \rightarrow \int d^D r \left\{ \frac{\hbar^2}{2m} |\nabla \psi(\mathbf{r})|^2 - [\mu(t) - V_{\text{trap}}(\mathbf{r})] |\psi(\mathbf{r})|^2 + \frac{g}{2} |\psi(\mathbf{r})|^4 \right\}.$$

U(1) symmetry-breaking!

$$\pi_1(U(1)) \cong \mathbb{Z}$$

Vortices can exist!

Y. Castin and R. Dum, Phys. Rev. A **57**, 3008 (1998).

Y. Kawaguchi and M. Ueda, Phys. Rep. **520**, 253 (2012).

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

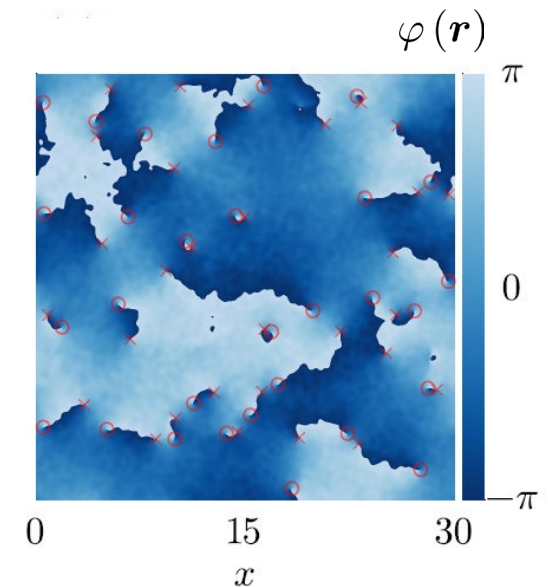
- Schrödinger equation:
$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) \right] \psi(\mathbf{r}, t).$$

- Madelung transformation:
$$\psi(\mathbf{r}, t) = \sqrt{\rho(\mathbf{r}, t)} e^{i\varphi(\mathbf{r}, t)}.$$

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot [\rho(\mathbf{r}, t) \mathbf{v}_s(\mathbf{r}, t)] = 0,$$

$$\rho(\mathbf{r}, t) \mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m} \text{Im} [\psi^*(\mathbf{r}, t) \nabla \psi(\mathbf{r}, t)] = \rho(\mathbf{r}, t) \frac{\hbar}{m} \nabla \varphi(\mathbf{r}, t).$$

$$\mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \varphi(\mathbf{r}, t).$$



$$\oint d\mathbf{l} \cdot \mathbf{v}_s(\mathbf{r}, t) = \frac{2\pi\hbar}{m} Q(t).$$

E. Madelung, Z. Phys. **40**, 322 (1927), D. Bohm, Phys. Rev. **85**, 166 (1952), T. Takabayasi, Prog. Theor. Phys. **8**, 143 (1952).

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

■ Gross-Pitaevskii equation:
$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(\mathbf{r}, t) + g |\psi(\mathbf{r}, t)|^2 \right] \psi(\mathbf{r}, t).$$

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot [\rho(\mathbf{r}, t) \mathbf{v}_s(\mathbf{r}, t)] = 0, \quad \mathbf{v}_s(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \varphi(\mathbf{r}, t). \quad \oint d\mathbf{l} \cdot \mathbf{v}_s(\mathbf{r}, t) = \frac{2\pi\hbar}{m} Q(t).$$

$$m\rho(\mathbf{r}, t) \left\{ \frac{\partial \mathbf{v}_s(\mathbf{r}, t)}{\partial t} + [\mathbf{v}_s(\mathbf{r}, t) \cdot \nabla] \mathbf{v}_s(\mathbf{r}, t) \right\} = -\nabla p_{\text{sf}}(\mathbf{r}, t) + \nabla \cdot \boldsymbol{\sigma}_{\text{sf}}(\mathbf{r}, t) + \mathbf{f}_{\text{sf}}(\mathbf{r}, t),$$

$$p_{\text{sf}}(\mathbf{r}, t) = \frac{g}{2} \rho^2(\mathbf{r}, t),$$

$$\boldsymbol{\sigma}_{\text{sf}}(\mathbf{r}, t) = \frac{\hbar^2}{m} \left\{ \frac{1}{4} [\nabla^2 \rho(\mathbf{r}, t)] \mathbb{I} - \left[\nabla \sqrt{\rho(\mathbf{r}, t)} \right] \otimes \left[\nabla \sqrt{\rho(\mathbf{r}, t)} \right] \right\},$$

$$\mathbf{f}_{\text{sf}}(\mathbf{r}, t) = \rho(\mathbf{r}, t) [-\nabla V_{\text{trap}}(\mathbf{r}, t) + m\boldsymbol{\omega}_s(\mathbf{r}, t) \times \mathbf{v}_s(\mathbf{r}, t)], \quad \boldsymbol{\omega}_s(\mathbf{r}, t) = \nabla \times \mathbf{v}_s(\mathbf{r}, t).$$

R. P. Feynman, Prog. Low Temp. Phys.
1, 17 (1955)

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

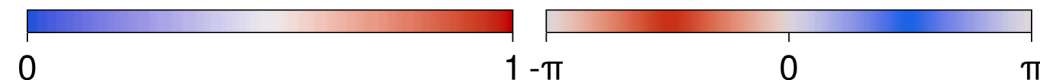
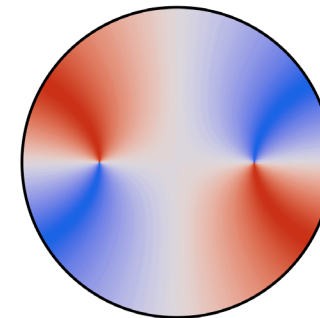
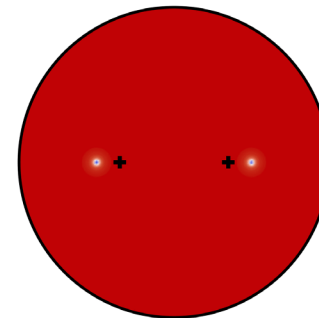
FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

least, if no forces act directly on the vortex line. In general the force per unit length on a vortex line equals the density, $\rho_0 m$, times the vector cross product of the circulation, $2\pi\hbar m^{-1}$, and the velocity of fluid where the vortex is.

$$\varepsilon = 0.01, \quad t = 0.0$$

squared modulus

phase



Gif from Gaspard Kemlin's homepage
<https://gkemlin.pages.math.cnrs.fr/assets/images/case1.gif>

10. Turbulence

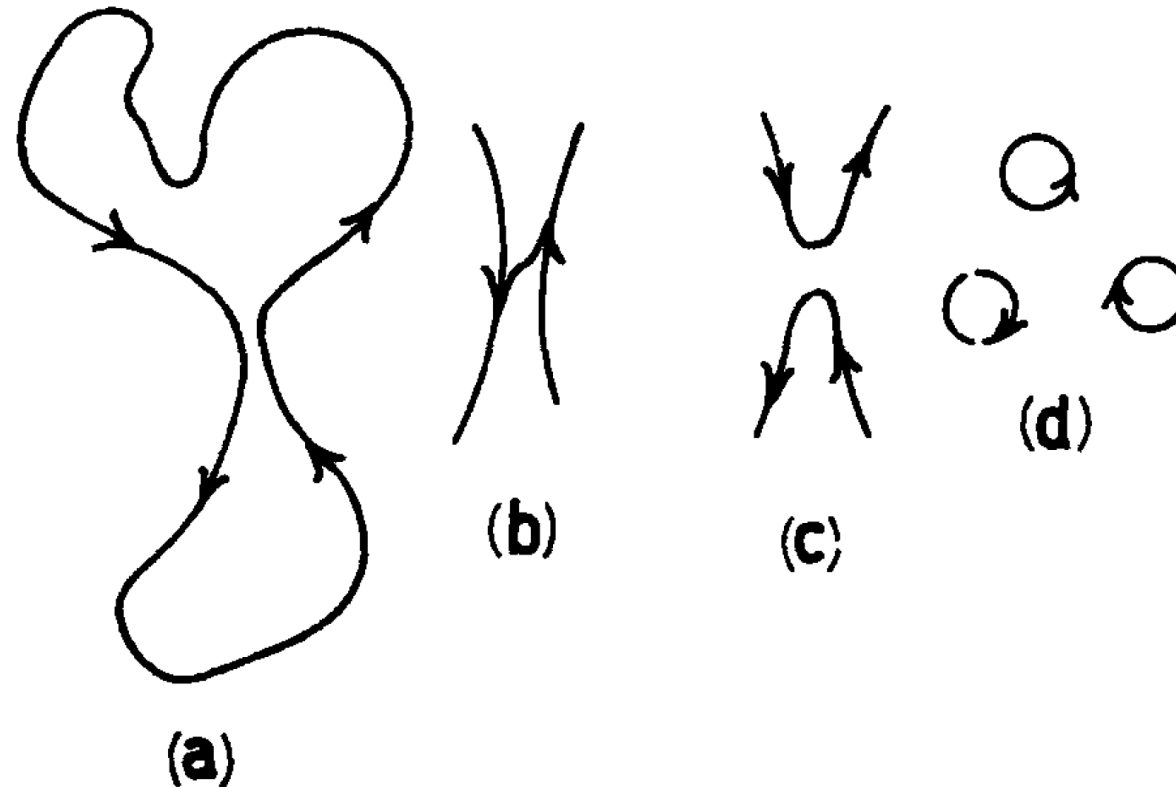


Fig. 10. A vortex ring (a) can break up into smaller rings if the transition between states (b) and (c) is allowed when the separation of vortex lines becomes of atomic dimensions. The eventual small rings (d) may be identical to rotons.

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

the aforementioned *ad hoc* assumptions. To do so requires a *microscopic* quantum mechanical model of superfluid helium, since it is on the core length scale that the relevant dynamics takes place. The simplest such model is the nonlinear Schrödinger equation for the boson wave function [6,7]

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + (v_0 |\Psi|^2 + w) \Psi, \quad (1)$$

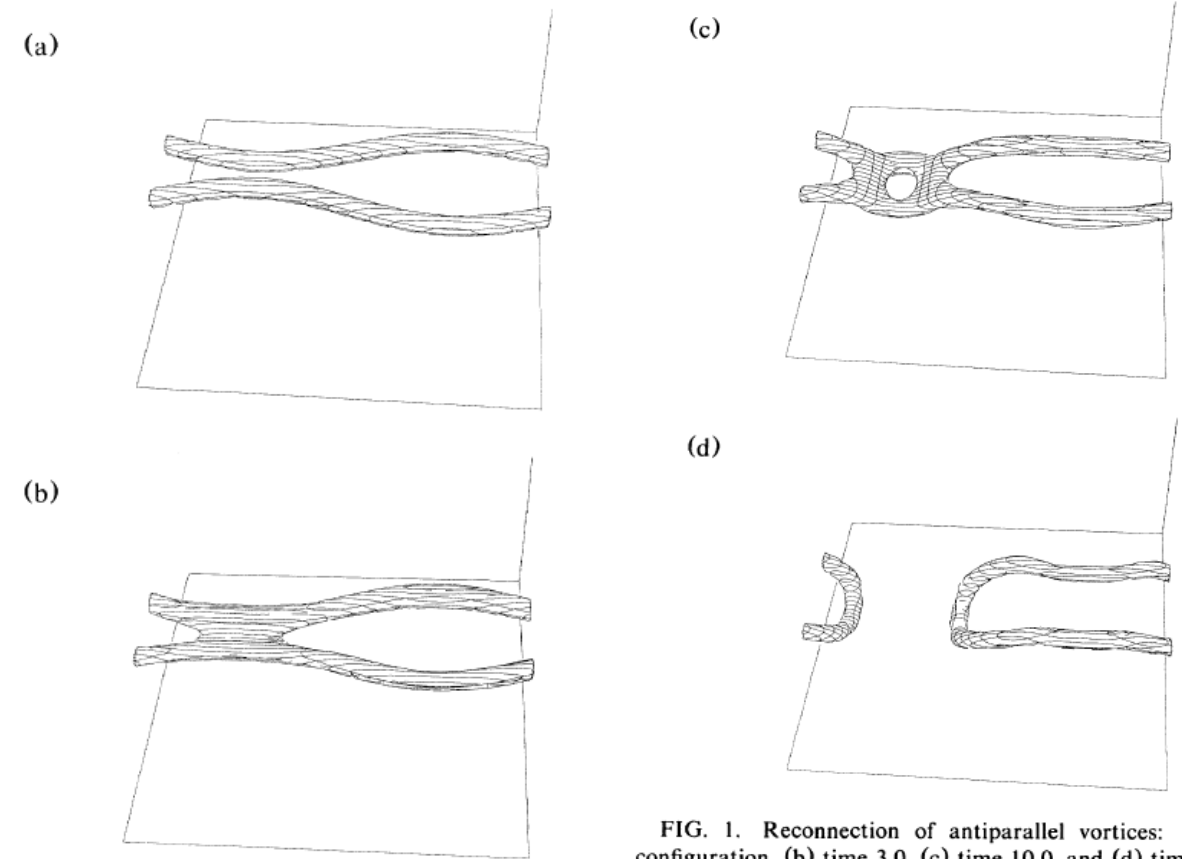


FIG. 1. Reconnection of antiparallel vortices: (a) initial configuration, (b) time 3.0, (c) time 10.0, and (d) time 20.

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

📖 FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



- Inertial range (IR)

$$\mathbf{v}(\mathbf{r}) = \mathbf{v}_c(\mathbf{r}) + \mathbf{v}_i(\mathbf{r}),$$

$$\nabla \times \mathbf{v}(\mathbf{r}) = \nabla \times \mathbf{v}_i(\mathbf{r}).$$

$$\int_0^\infty dk E_i(k) := \frac{\rho_m}{2} \int d^D r |\mathbf{v}_i(\mathbf{r})|^2.$$

- Details of energy injection and dissipation can both be neglected
- Eddies are assumed to follow universal statistical properties

- Kolmogorov scaling from the dimensional analysis

- (Assumption 1) Kinetic energy dissipation rate ϵ per unit mass is independent of the wavevector k
- (Assumption 2) (Incompressible) Kinetic energy is independent of the viscosity

$$\Rightarrow E_i(k) = C \epsilon^{2/3} k^{-5/3} \quad \text{in the IR}$$



C. F. Barenghi, L. Skrbek, and K. R. Sreenivasan, Quantum Turbulence, 1st ed.
(Cambridge University Press, Cambridge, 2024).

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

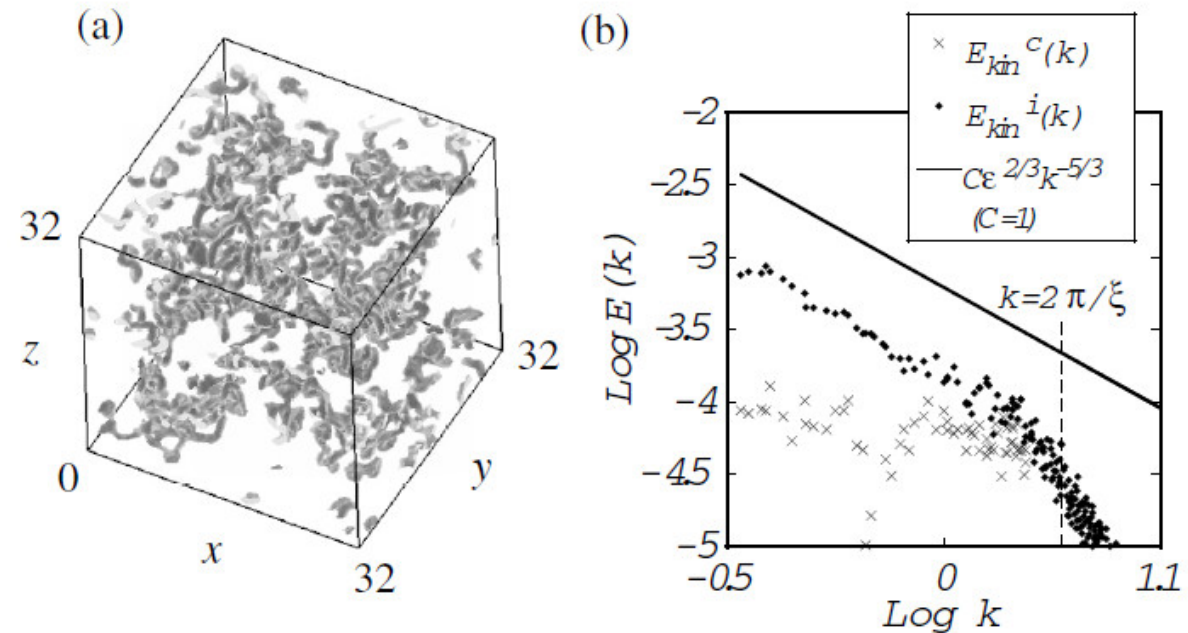
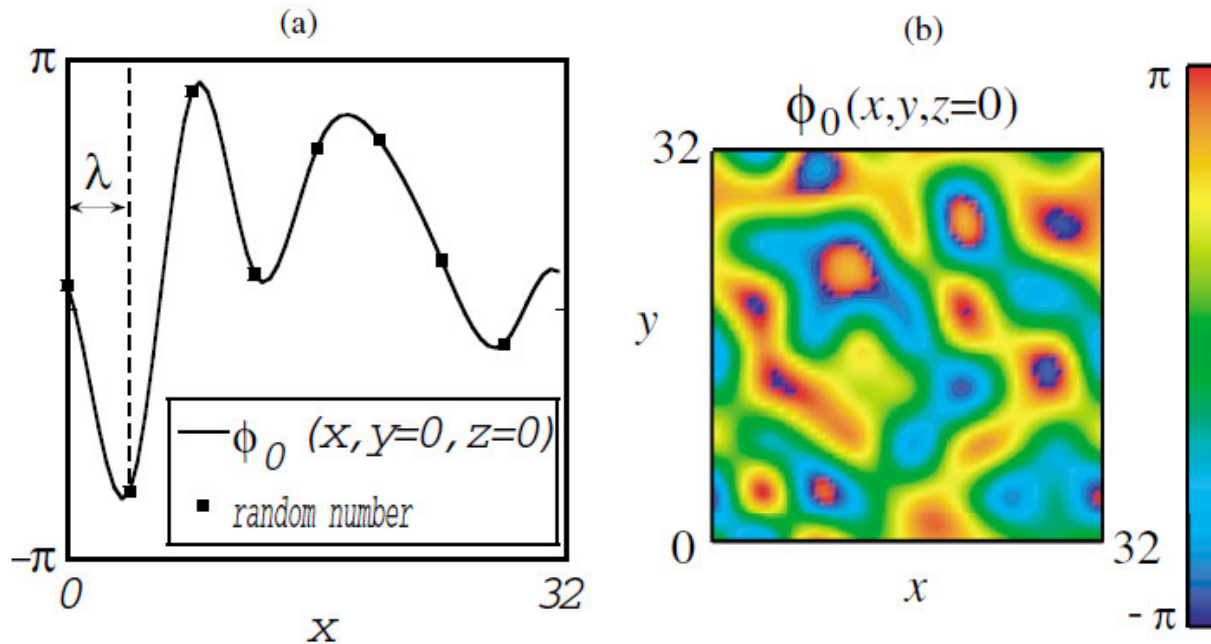
FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

- Kolmogorov scaling is also observed in quantum turbulence

$$i \frac{\partial}{\partial t} \Phi(\mathbf{x}, t) = [-\nabla^2 - \mu + g|\Phi(\mathbf{x}, t)|^2] \Phi(\mathbf{x}, t), \quad (2)$$

$$\mathbf{v}_s(\mathbf{x}, t) = \nabla \phi_0(\mathbf{x}, t),$$

$$\begin{aligned} \mathbf{u}(\mathbf{x}, t) &= |\Phi(\mathbf{x}, t)| \mathbf{v}_s(\mathbf{x}, t) \\ &= \mathbf{u}_c(\mathbf{x}, t) + \mathbf{u}_i(\mathbf{x}, t). \end{aligned}$$



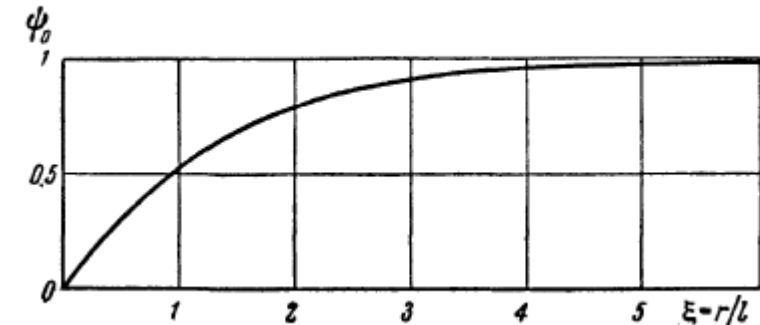
1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

The velocity v_s has only a component $v_{s\varphi}$,
and (see also Ref. 10)*

$$\oint \mathbf{v}_s ds = v_{s\varphi} \cdot 2\pi r = \frac{2\pi n \hbar}{m} . \quad (18)$$

The result for $n = 1$ is given in the figure.



V. L. Ginzburg and L. P. Pitaevskii, ZhETF **34**, 1240 (1958) [Sov. Phys. JETP **7**, 858 (1958)].

1. U(1) symmetry-breaking, quantum turbulence, and Kolmogorov scaling

FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

PHYSICAL REVIEW LETTERS **134**, 083402 (2025)

Editors' Suggestion

Featured in Physics

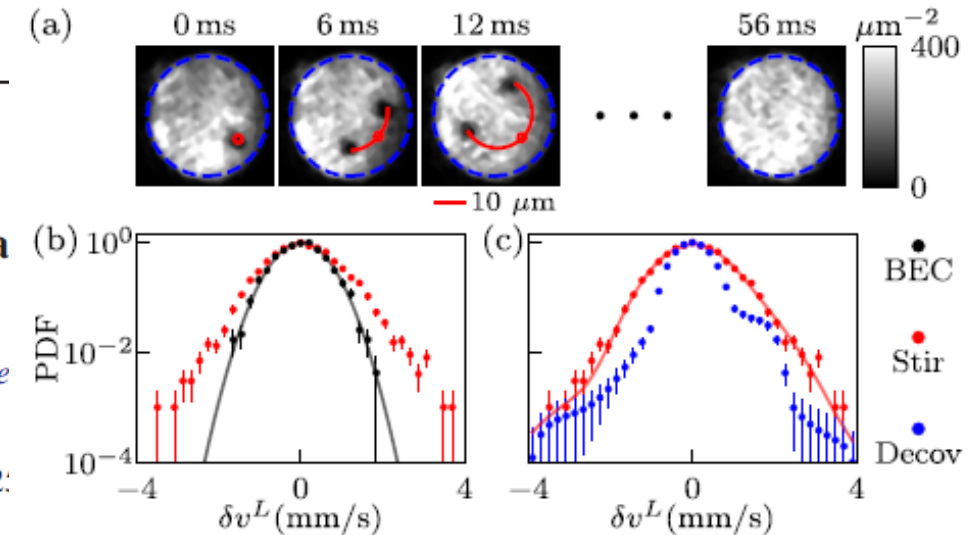
Kolmogorov Scaling in Turbulent 2D Bose-Einstein Condensate

M. Zhao¹, J. Tao, and I. B. Spielman^{1*}

Joint Quantum Institute, University of Maryland and National Institute of Standards and Technology, College Park, Maryland 20742, USA



(Received 8 August 2024; revised 17 October 2024; accepted 18 December 2024; published 2:)

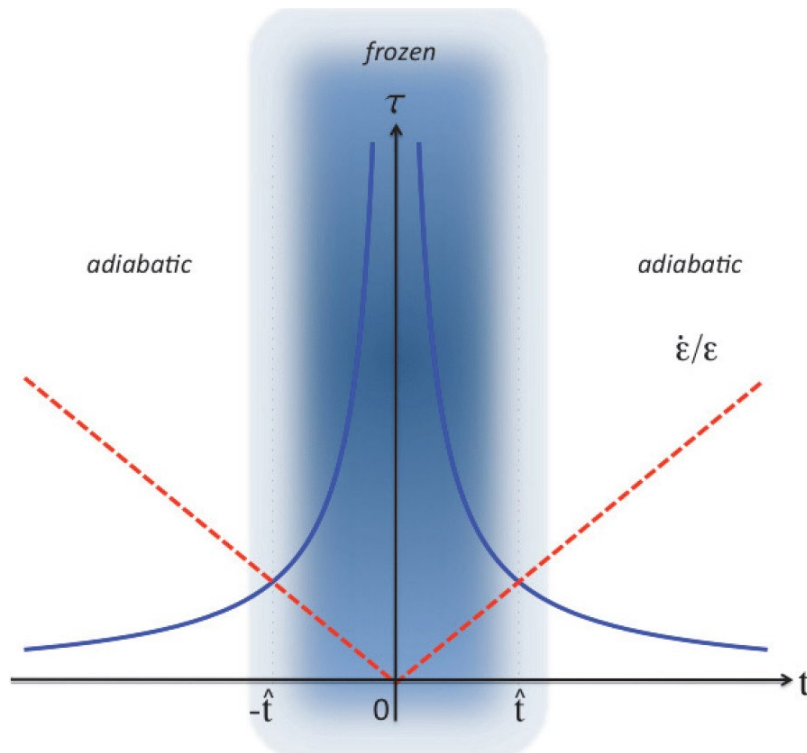


No other way (other than manual dissipation or stirring the system) to study quantum turbulence?

2. Kibble-Zurek mechanism and power-law

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

$$\tau(t) \propto |\epsilon(t)|^{-\nu z}, \quad \xi(t) \propto |\epsilon(t)|^{-\nu}.$$



$$\hat{H} - \mu(t) \hat{N} \rightarrow \int d^D r \left\{ \frac{\hbar^2}{2m} |\nabla \psi(\mathbf{r})|^2 - [\mu(t) - V_{\text{trap}}(\mathbf{r})] |\psi(\mathbf{r})|^2 + \frac{g}{2} |\psi(\mathbf{r})|^4 \right\}.$$

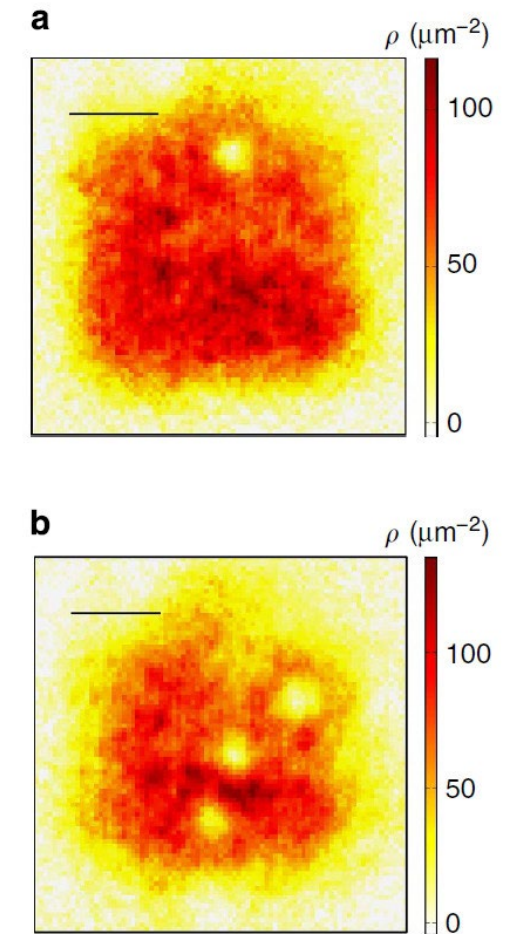
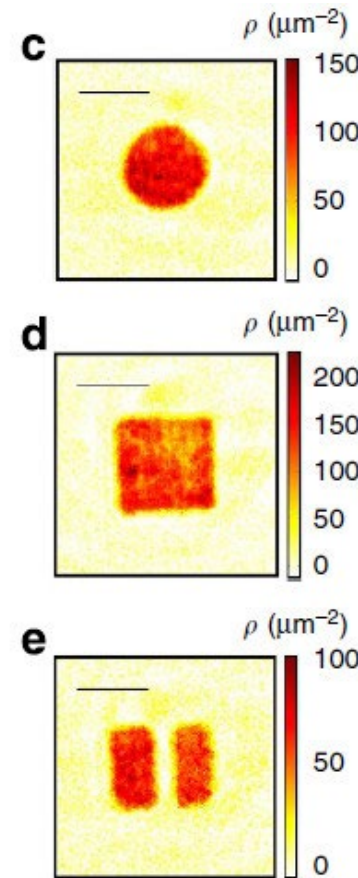
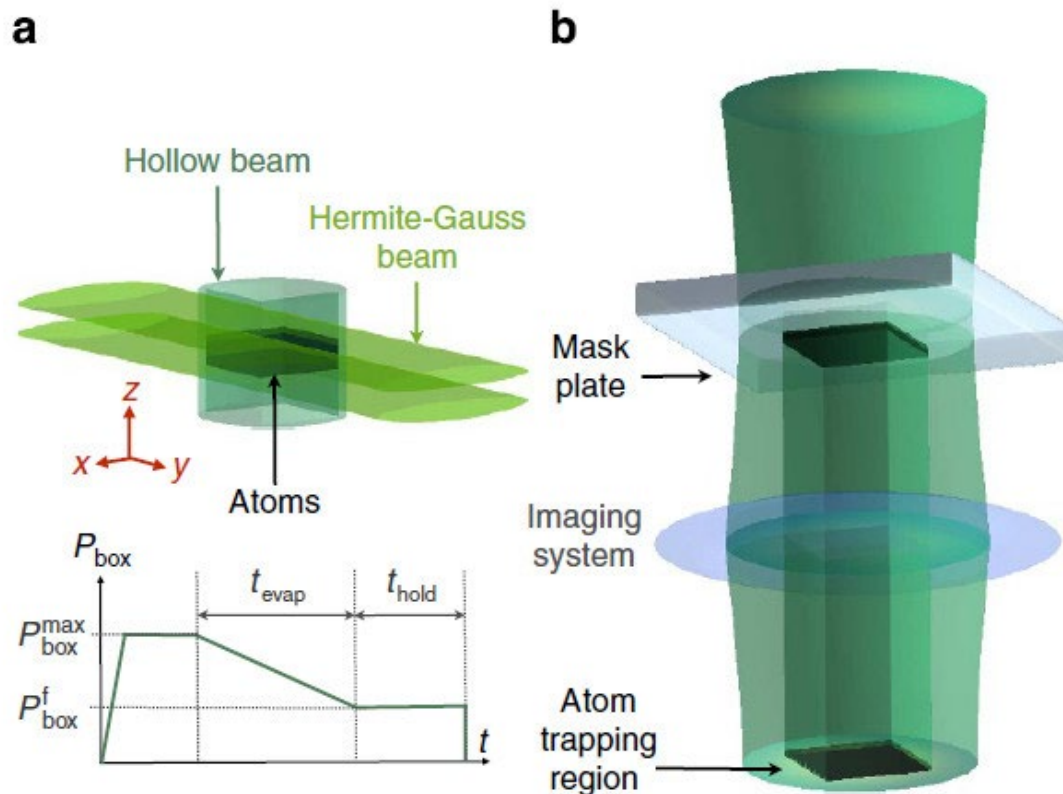
$$\epsilon(t) = \mu(t) = t/\tau_Q$$

$$\Rightarrow \hat{t} \propto \tau_Q^{\nu z/(1+\nu z)}, \quad \xi(\hat{t}) \propto \tau_Q^{\nu/(1+\nu z)}.$$

2. Kibble-Zurek mechanism and power-law

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

■ Experiments on quasi-2D dilute Rb Bose gas



2. Kibble-Zurek mechanism and power-law

📖 FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

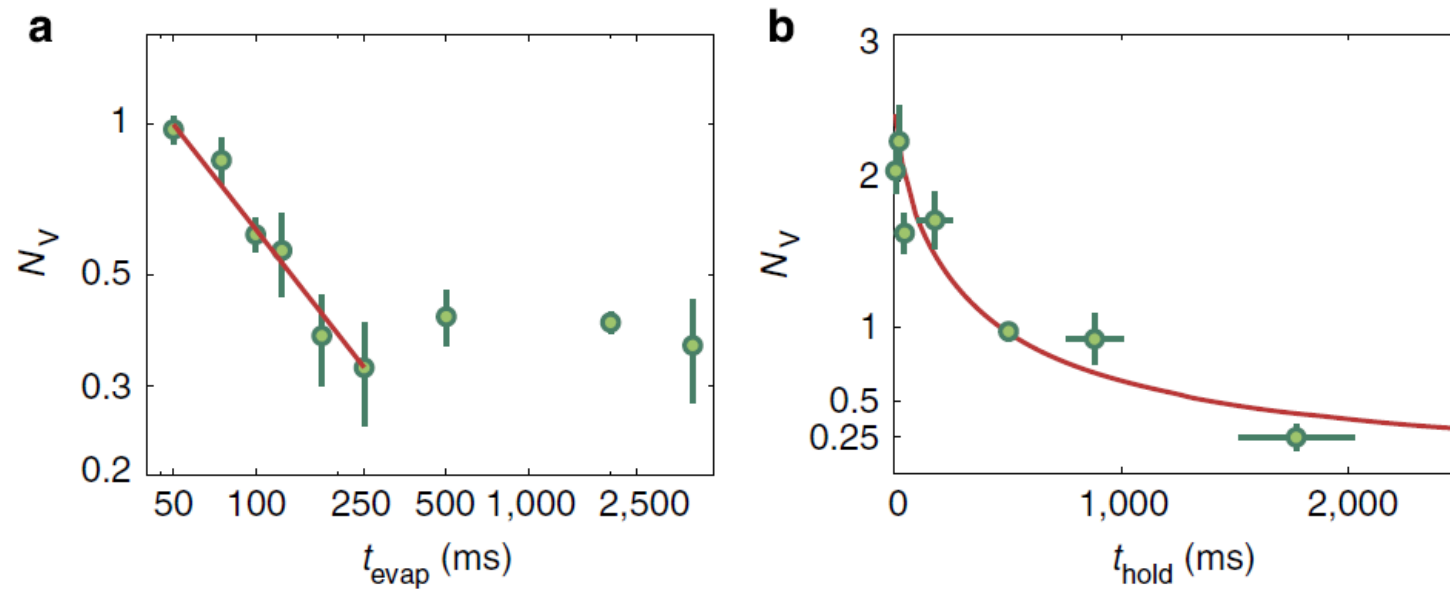


Figure 6 | Dynamics of vortex nucleation. (a) Circle symbols: evolution of the mean vortex number N_v with the quench time t_{evap} (fixed $t_{\text{hold}} = 500$ ms) for a gas initially confined in a square of size $L = 30 \mu\text{m}$ ($v_z = 365$ Hz) and observed after a 3D ToF of $\tau = 4.5$ ms. The number of images per point ranges from 10 to 100.

2. Kibble-Zurek mechanism and power-law

📖 FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

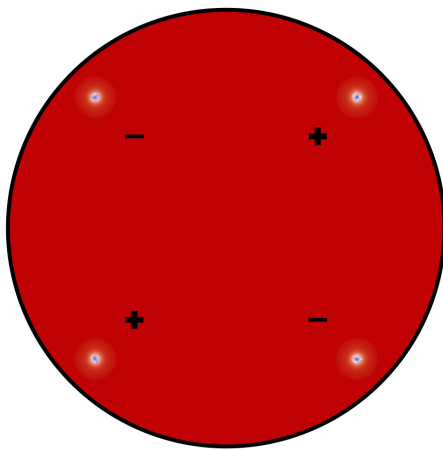
Dynamical origin of the topological defects. In principle, the vortices observed in the gas could be due to steady-state thermal fluctuations. BKT theory indeed predicts that vortices should be present in an interacting 2D Bose gas around the superfluid transition point¹¹. Such ‘thermal’ vortices have been observed in non-homogeneous atomic gases, either interferometrically¹⁴ or as density holes in the trap region corresponding to the critical region²⁰. However, for the large and uniform phase-space densities that we obtain at the end of the cooling process ($\rho\lambda_T^2 \geq 100$), ref. 44 predicts a vanishingly small probability of occurrence for such thermal excitations. This supports a dynamical origin for the observed defects.

3. Stochastic projected Gross-Pitaevskii equation

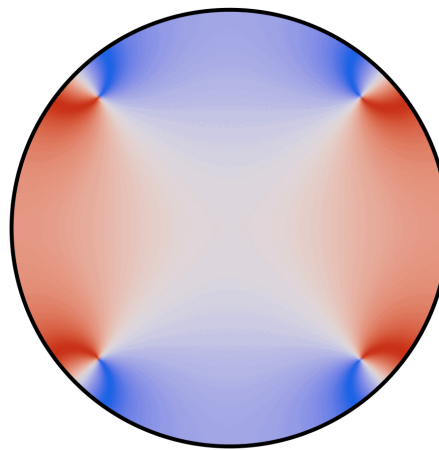
□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

$$\varepsilon = 0.01, \quad t = 0.0$$

squared modulus



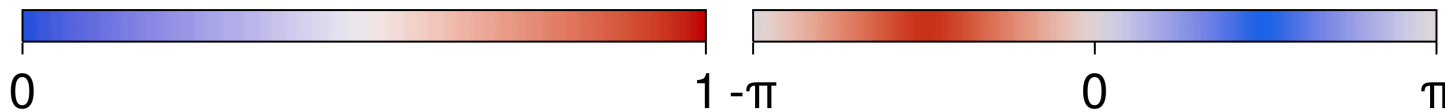
phase



$$i \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\nabla^2 \psi(\mathbf{r}, t) + \frac{1}{\varepsilon^2} (1 - |\psi(\mathbf{r}, t)|)^2.$$

Gif from Gaspard Kemlin's homepage

<https://gkemlin.pages.math.cnrs.fr/assets/images/case4.gif>

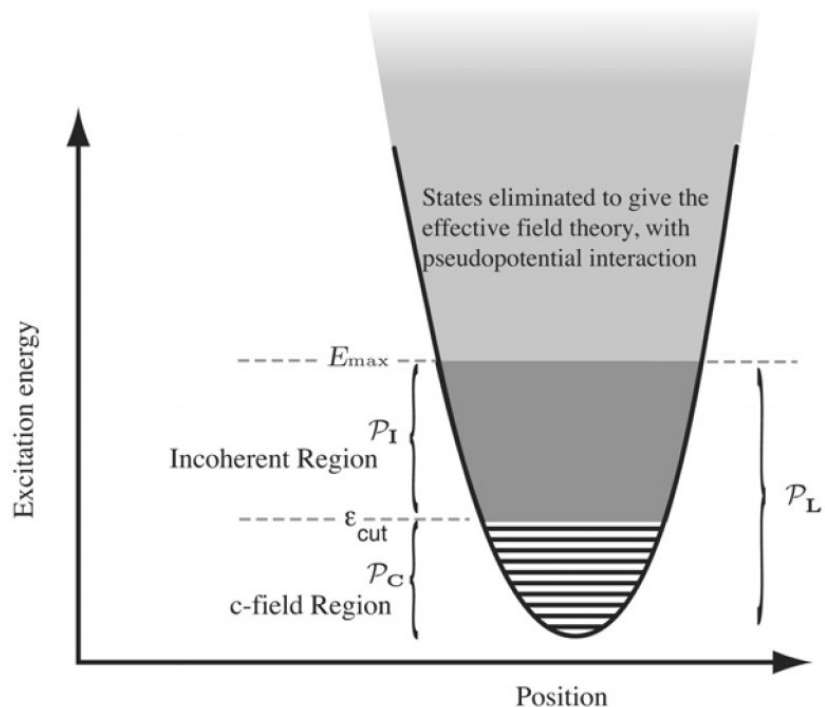


3. Stochastic projected Gross-Pitaevskii equation

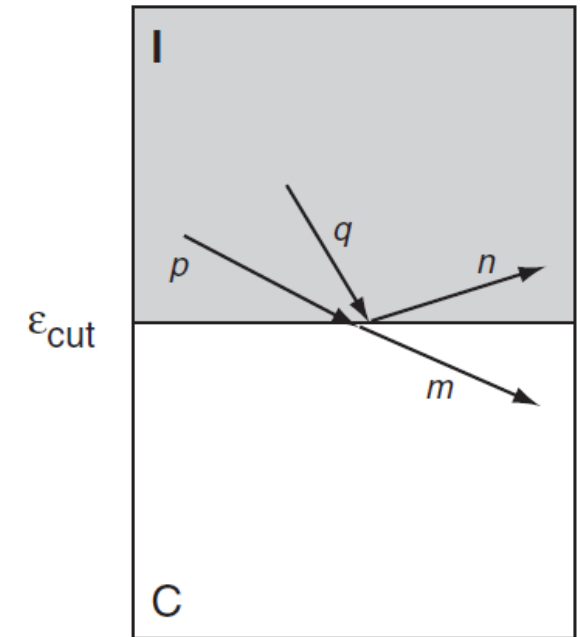
FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

- Equations we use: stochastic projected Gross-Pitaevskii equation (SPGPE)

$$d\psi_C(\mathbf{r}, t) = \hat{P}_C \left[-\frac{i + \gamma}{\hbar} \left\{ -\frac{\hbar^2}{2m} \nabla^2 + g |\psi_C(\mathbf{r}, t)|^2 - \mu(t) \right\} \psi_C(\mathbf{r}, t) dt + d\eta(\mathbf{r}, t) \right],$$



$$\begin{aligned} \langle d\eta^*(\mathbf{r}_1, t) d\eta(\mathbf{r}_2, t) \rangle \\ = 2\gamma \frac{k_B T}{\hbar} \delta_C(\mathbf{r}_1 - \mathbf{r}_2) dt. \end{aligned}$$



3. Stochastic projected Gross-Pitaevskii equation

📖 FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



$$\rho(\mathbf{r}, t)$$

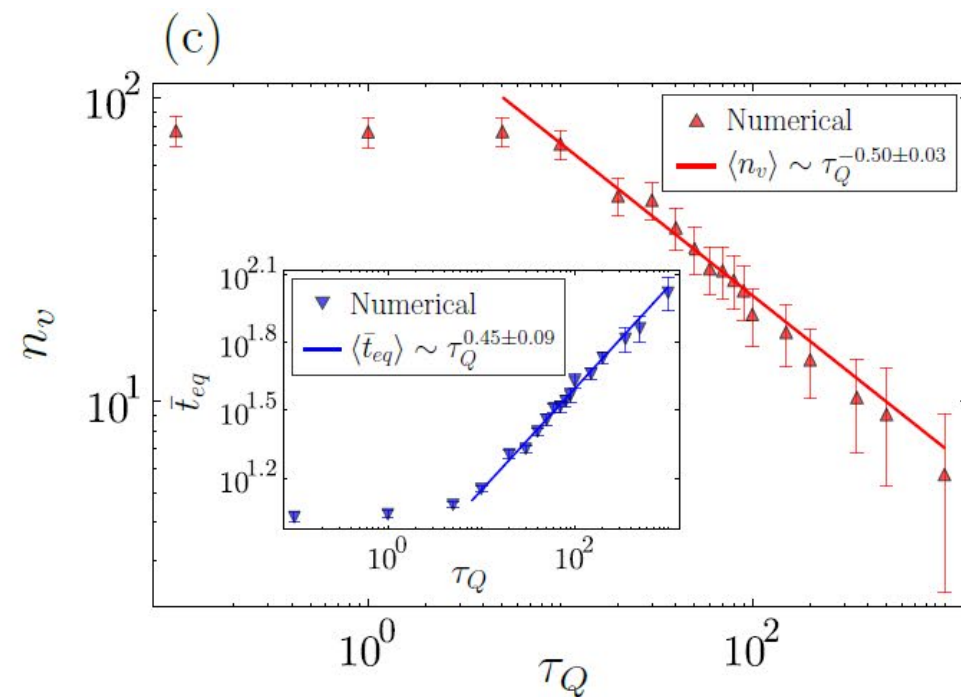
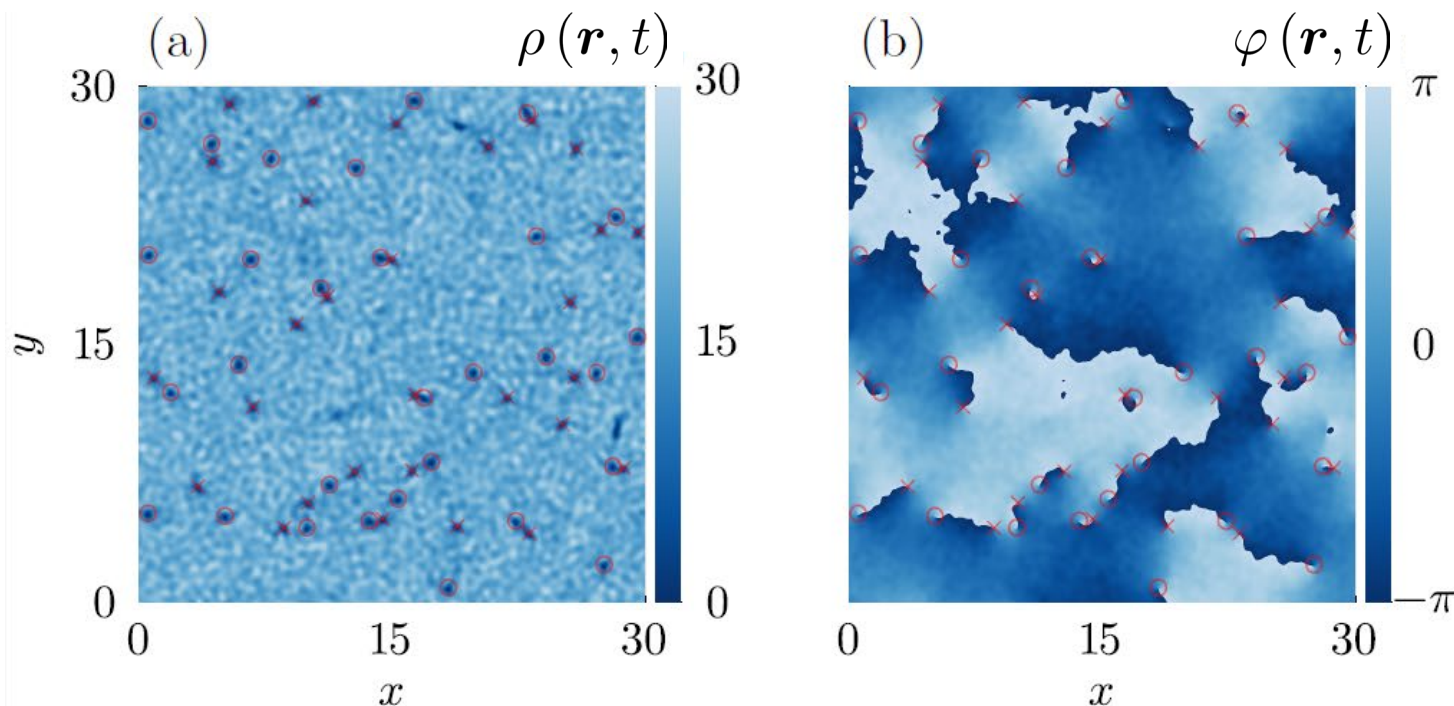


$$\rho(\mathbf{r}, t)$$



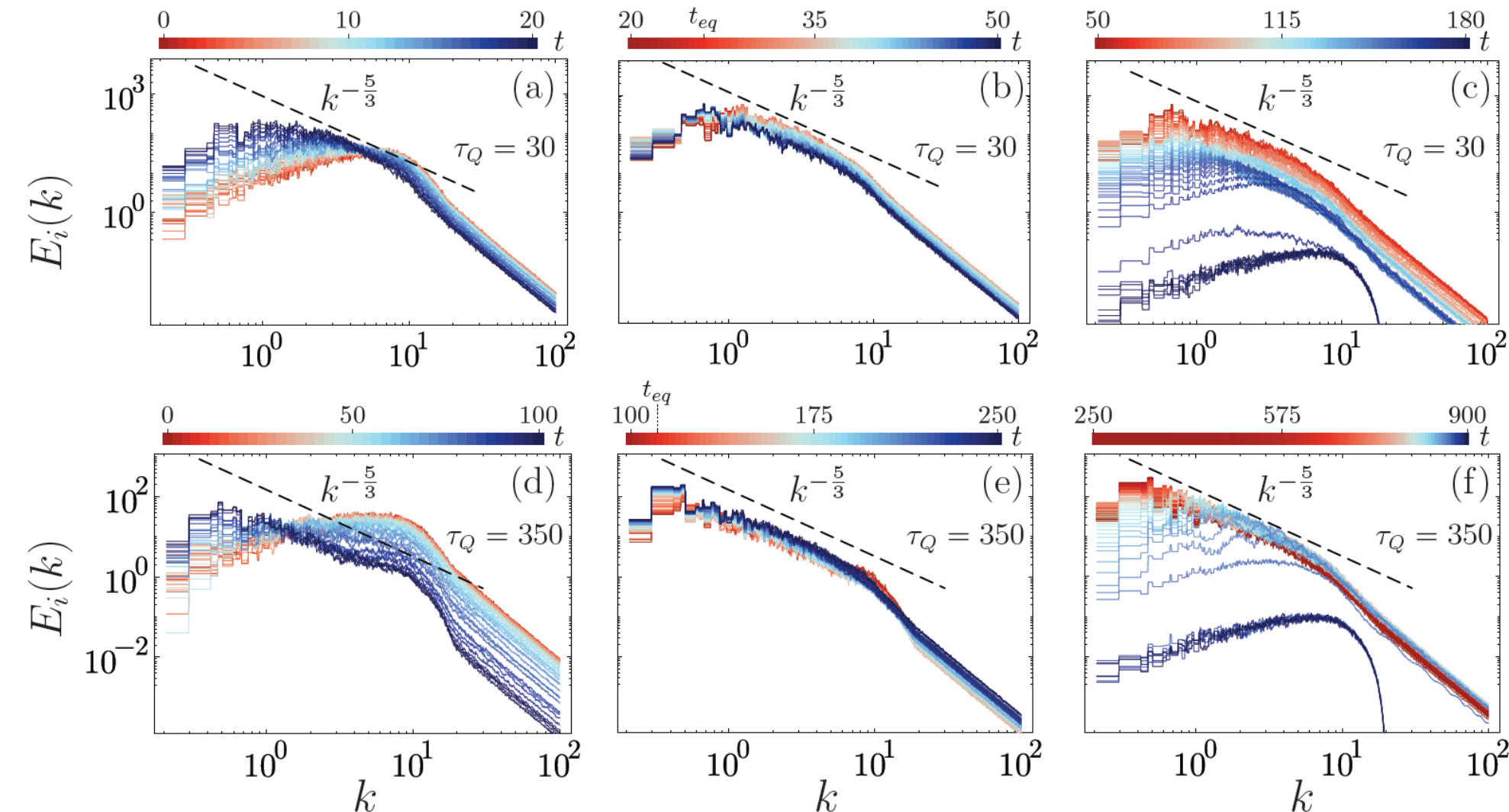
4. Numerical results – spontaneous quantum turbulence (SQT)

FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



4. Numerical results – spontaneous quantum turbulence (SQT)

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

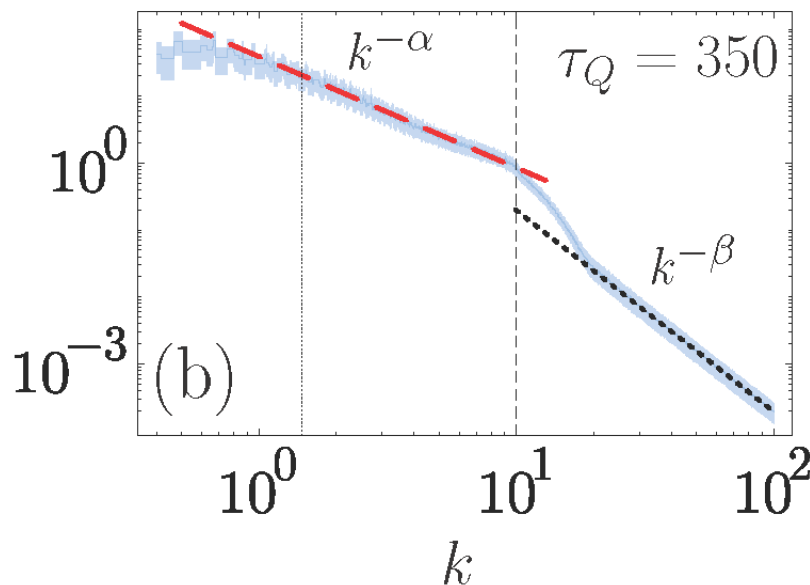
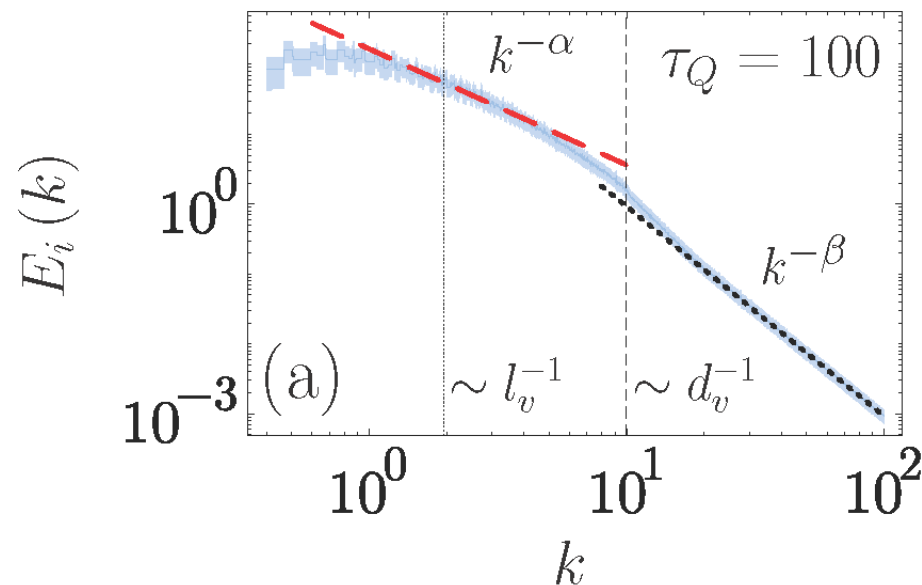


$$\begin{aligned} \mathbf{u}(\mathbf{r}) &= \sqrt{\rho(\mathbf{r})} \mathbf{v}_s(\mathbf{r}) \\ &= \mathbf{u}_c(\mathbf{r}) + \mathbf{u}_i(\mathbf{r}), \end{aligned}$$

$$\int_0^\infty dk E_i(k) = \frac{m}{2} \int d^2r |\mathbf{u}_i(\mathbf{r})|^2$$

4. Numerical results – spontaneous quantum turbulence (SQT)

FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



$$l_v \propto \hat{\xi}(t_{eq})$$
$$\propto \tau_Q^{\nu/(1+z\nu)}.$$

5/3 ≈ 1.67

	(a)	(b)
$\alpha (\eta)$	1.662 ± 0.022	1.661 ± 0.019
β	2.999 ± 0.018	3.007 ± 0.016

Due to the 2D system

G. Krstulovic and M. Brachet,
PRL **105**, 129401 (2010)

4. Numerical results – spontaneous quantum turbulence (SQT)

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

$$E_i(t_{eq}) = \frac{1}{2} \int d^2r |\mathbf{u}_i(\mathbf{r}, t_{eq})|^2 = \int_0^\infty dk E_i(k, t_{eq}).$$

$$E_i(t_{eq}) \simeq \frac{1}{2} n_v(t_{eq}) \int_{\mathcal{D}_{\hat{\xi}(t_{eq})}} d^2r \left| \frac{\sqrt{\rho(r, t_{eq})}}{r} \mathbf{e}_\varphi \right|^2$$

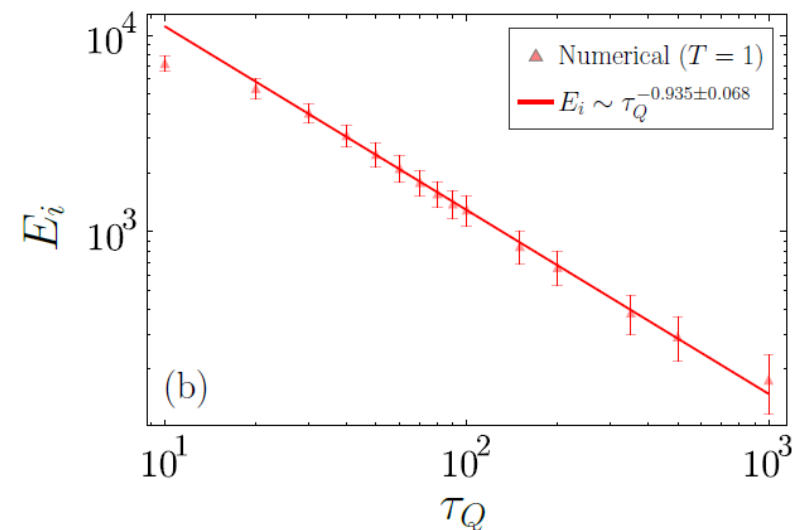
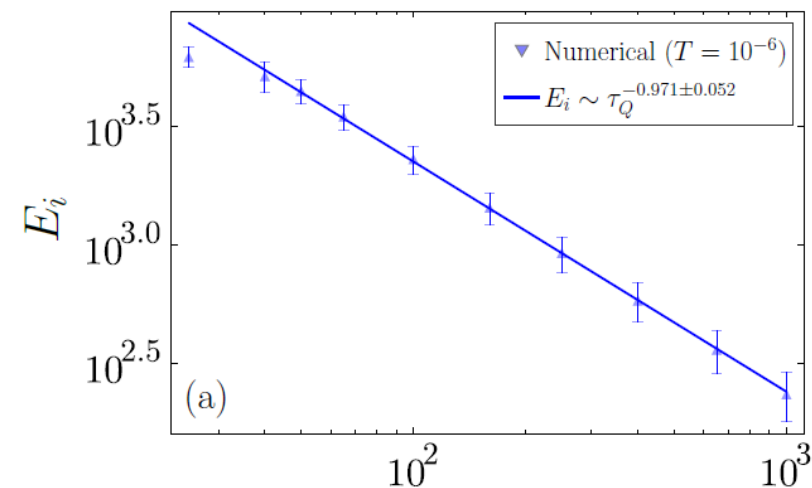
$$\simeq \frac{1}{2} n_v(t_{eq}) 2\pi \rho(r, t_{eq}) \int_{r_v}^{\hat{\xi}(t_{eq})} dr \frac{1}{r}$$

$$\propto n_v(t_{eq}) \rho(r, t_{eq})$$

$$\propto n_v(t_{eq}) \mu(t_{eq})$$

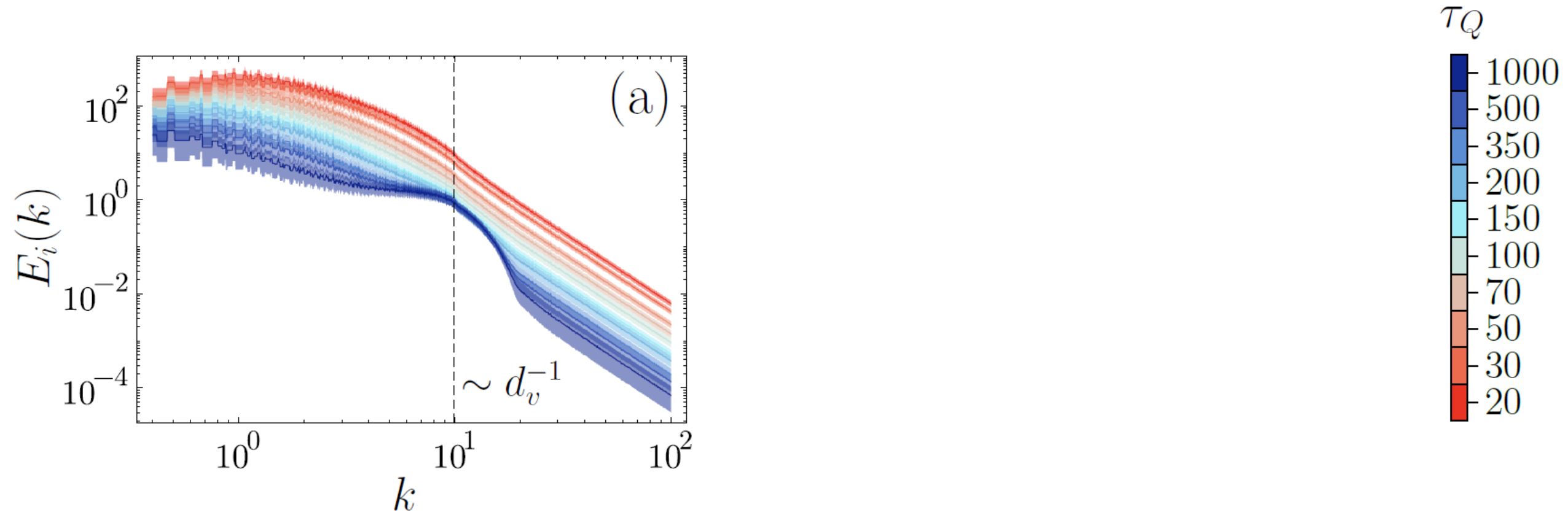
$$\propto \tau_Q^{-2\nu/(1+z\nu)} t_{eq}/\tau_Q$$

$$\propto \tau_Q^{-2\nu/(1+z\nu)} \tau_Q^{z\nu/(1+z\nu)} / \tau_Q = \tau_Q^{-(1+2\nu)/(1+z\nu)}$$



4. Numerical results – spontaneous quantum turbulence (SQT)

FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

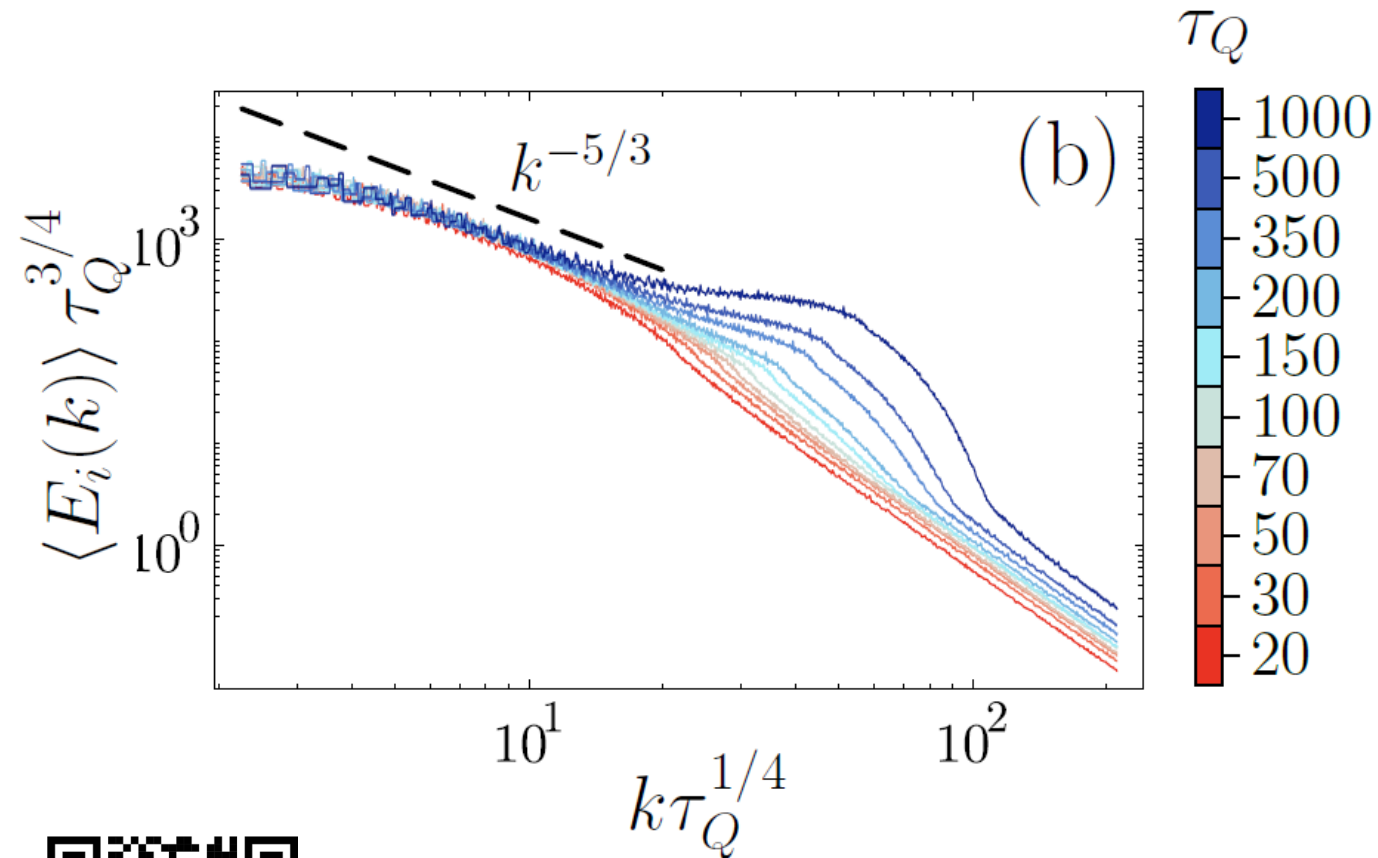
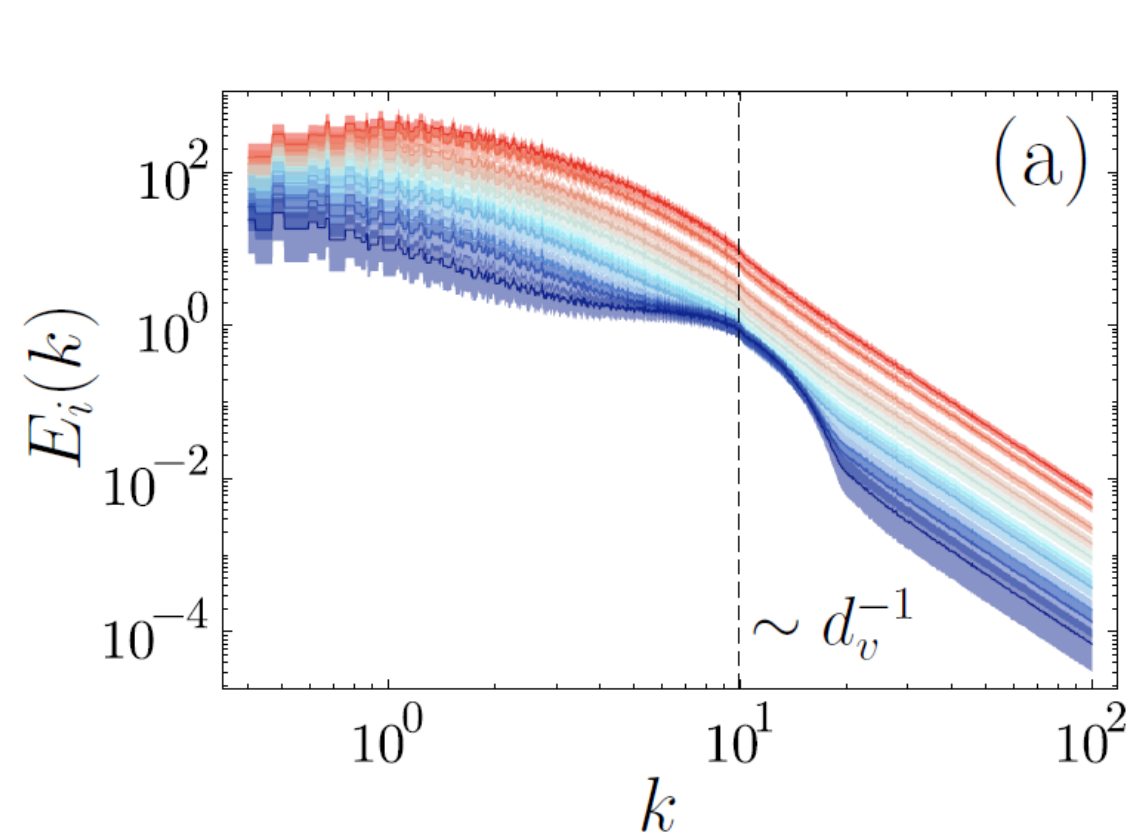


$$E_i(k, t_{eq}) \propto \tau_Q^{-(1+\nu)/(1+z\nu)} F\left(k\tau_Q^{\nu/(1+z\nu)}\right).$$
$$\propto \tau_Q^{-3/4} F\left(k\tau_Q^{1/4}\right) \text{ for } \nu = 1/2, z = 2.$$



4. Numerical results – spontaneous quantum turbulence (SQT)

FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



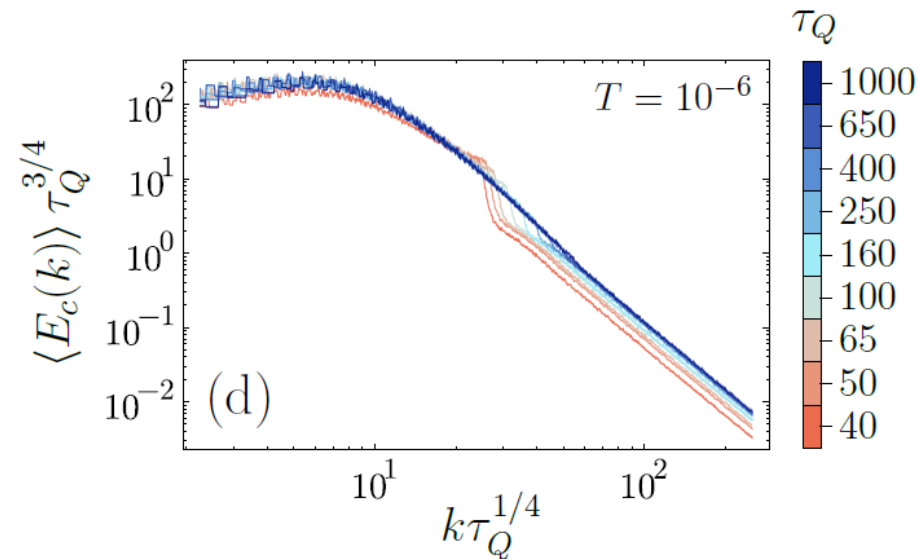
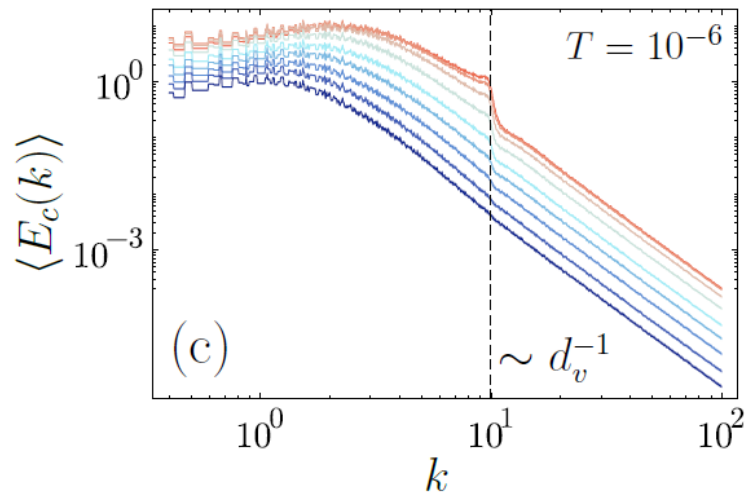
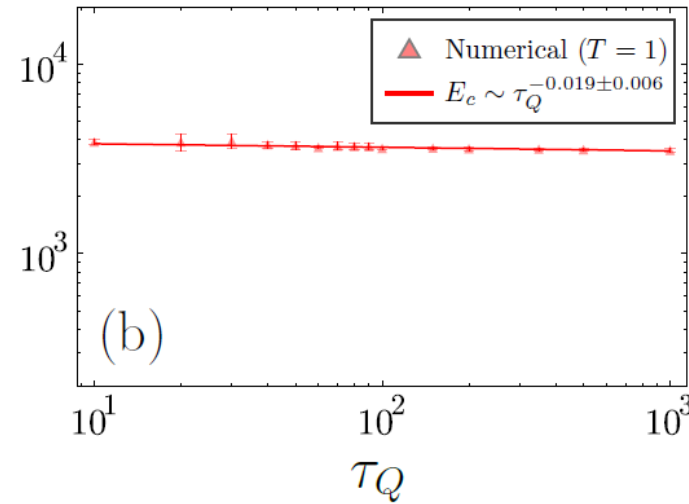
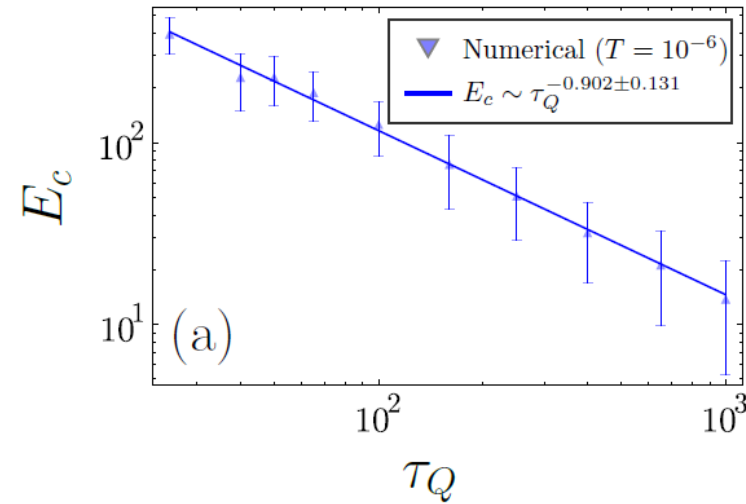
$$E_i(k, t_{eq}) \propto \tau_Q^{-(1+\nu)/(1+z\nu)} F\left(k\tau_Q^{\nu/(1+z\nu)}\right).$$

$$\propto \tau_Q^{-3/4} F\left(k\tau_Q^{1/4}\right) \text{ for } \nu = 1/2, z = 2.$$



4. Numerical results – spontaneous quantum turbulence (SQT)

FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



- More details can be seen
in arXiv:2506.21670



- Or in Q&A session

5. Numerical results – Migdal's area rule in SQT

- Studies on statistics of topological defects during continuous phase transition
 - F. J. Gómez-Ruiz, J. J. Mayo, and A. del Campo, *Full Counting Statistics of Topological Defects After Crossing a Phase Transition*, Phys. Rev. Lett. **124**, 240602 (2020).
 - J. J. Mayo, Z. Fan, G.-W. Chern, and A. del Campo, *Distribution of kinks in an Ising ferromagnet after annealing and the generalized Kibble-Zurek mechanism*, Phys. Rev. Research **3**, 033150 (2021).
 - A. del Campo, F. J. Gómez-Ruiz, Z.-H. Li, C.-Y. Xia, H.-B. Zeng, and H.-Q. Zhang, *Universal statistics of vortices in a newborn holographic superconductor: beyond the Kibble-Zurek mechanism*, JHEP **2021**, 61 (2021).
 - F. Balducci, M. Beau, J. Yang, A. Gambassi, and A. del Campo, *Large Deviations beyond the Kibble-Zurek Mechanism*, Phys. Rev. Lett. **131**, 230401 (2023).
 - M. Thudiyangal and A. del Campo, *Universal vortex statistics and stochastic geometry of Bose-Einstein condensation*, Phys. Rev. Research **6**, 033152 (2024).
 - C.-Y. Xia, H.-B. Zeng, A. Grabarits, and A. del Campo, *Kibble-Zurek mechanism and beyond in a holographic superfluid disk*, Nat Commun **17**, 3668 (2026).
 - S. Patra, A. Roy, **S.-H. Shinn**, A. del Campo, and M. Thudiyangal, *Kibble-Zurek scaling and spatial statistics in quenched binary Bose superfluids*, Commun Phys (Published on May 9, 2026)

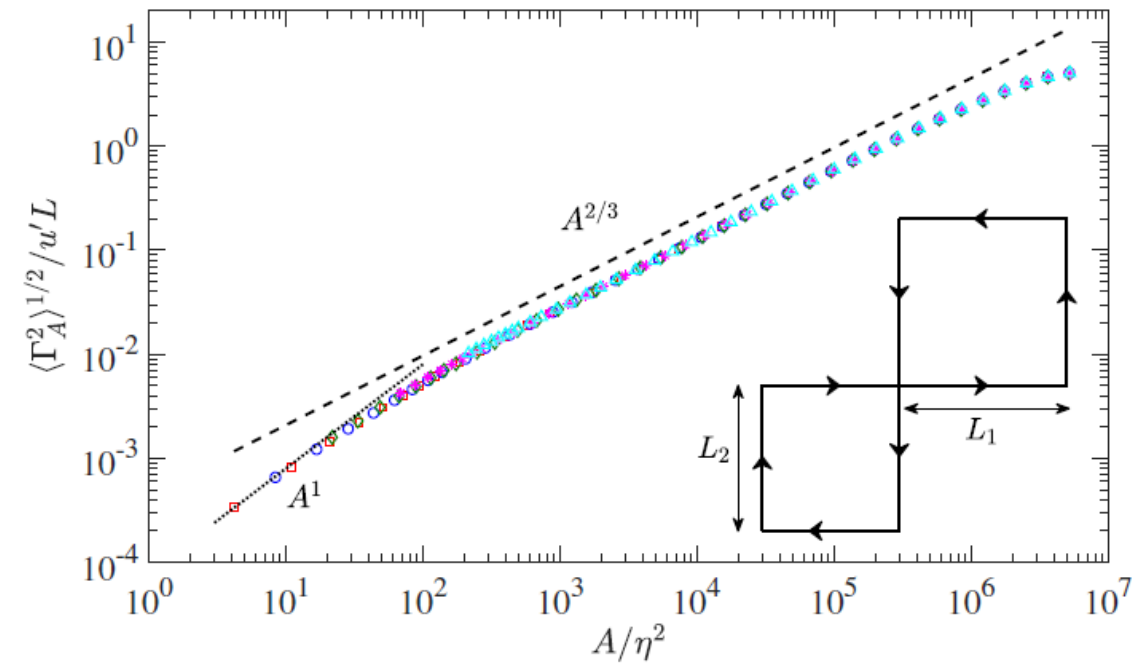
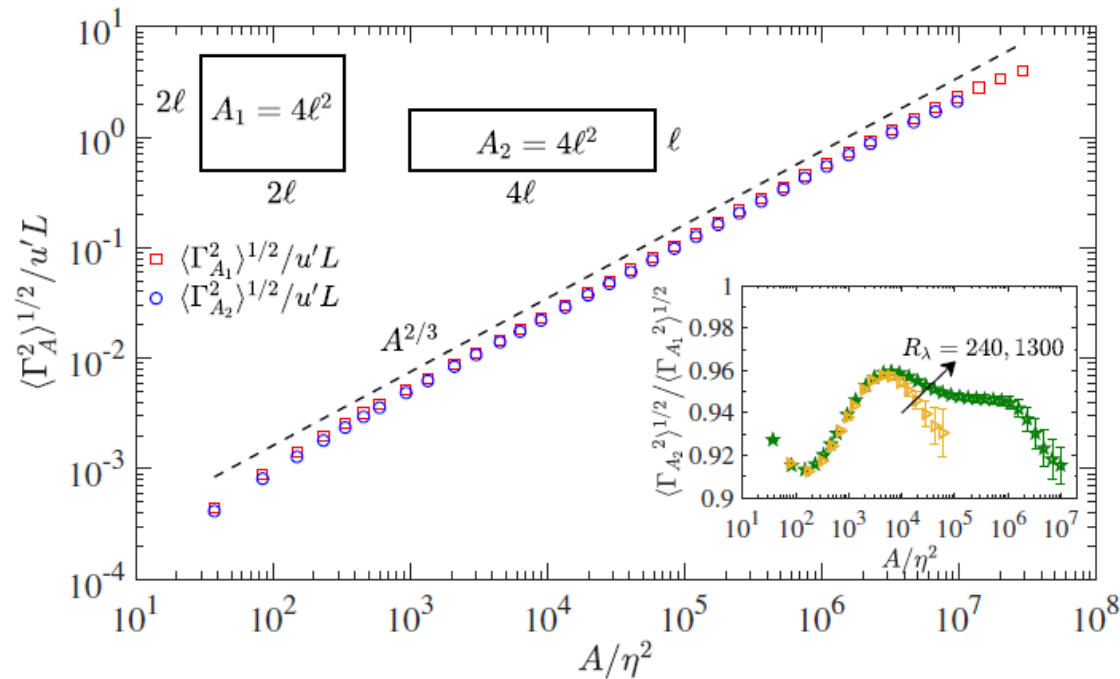
5. Numerical results – Migdal's area rule in SQT

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE

■ Migdal's area rule in classical turbulence

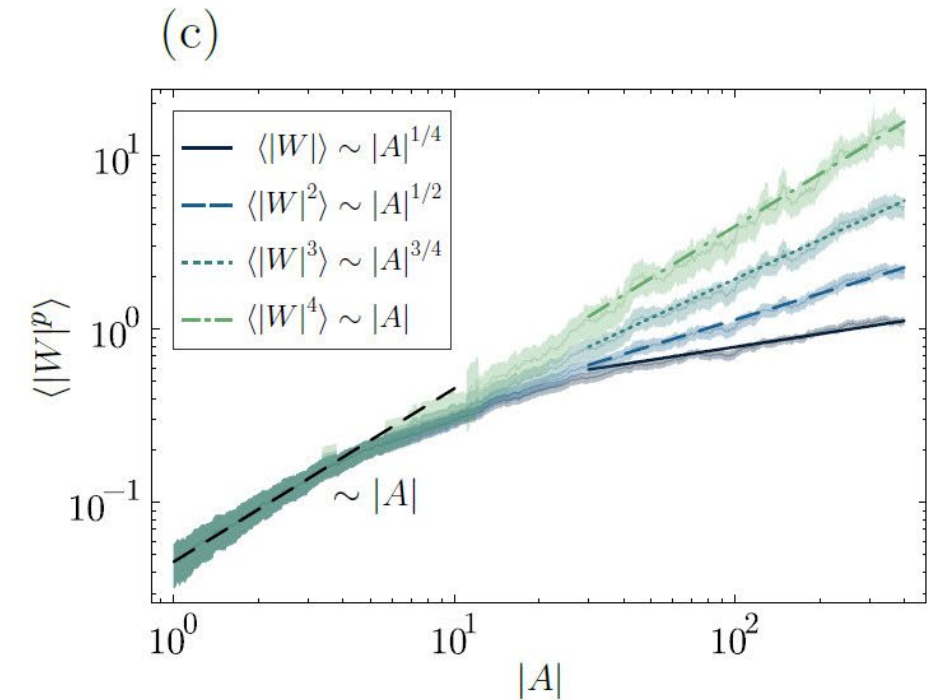
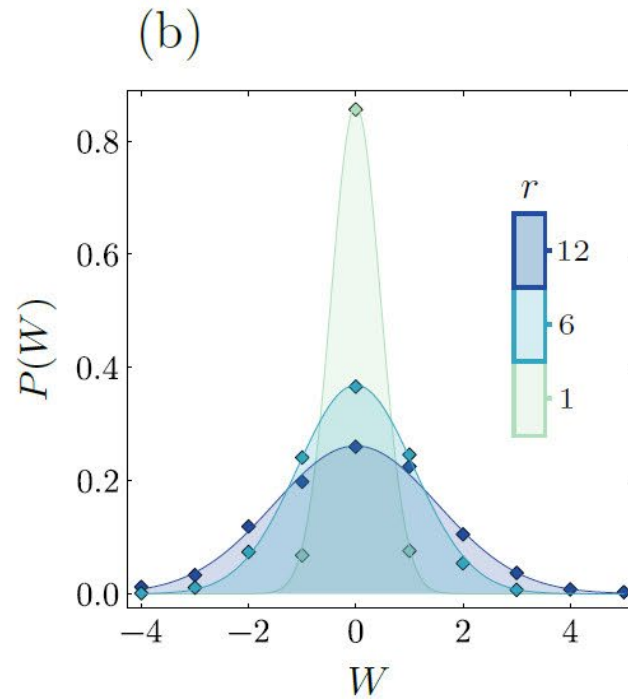
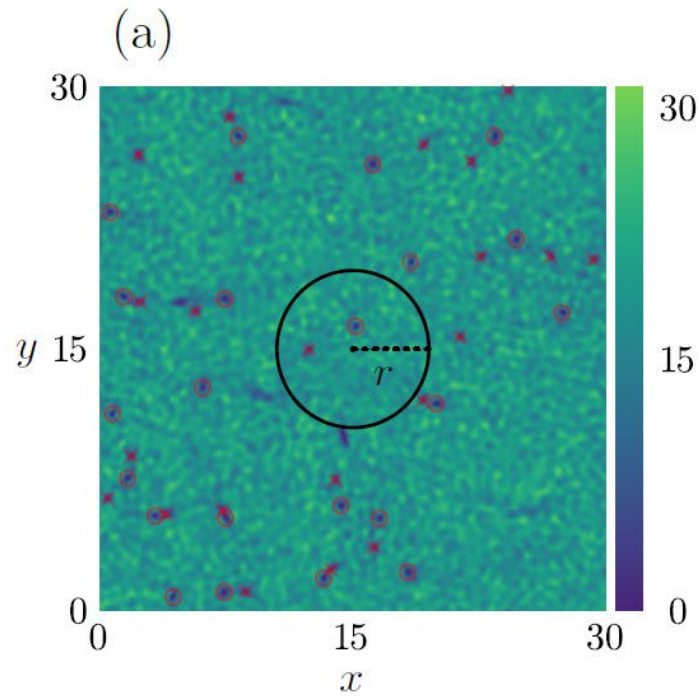
IYER, SREENIVASAN, and YEUNG

PHYS. REV. X **9**, 041006 (2019)



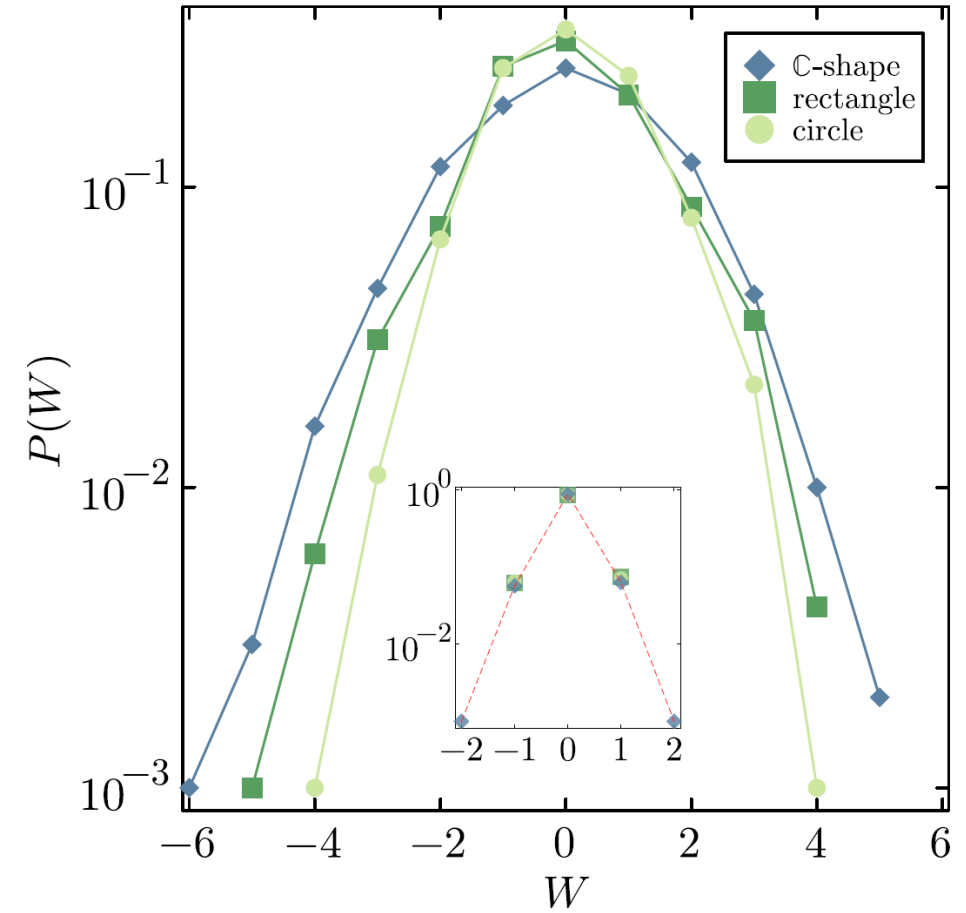
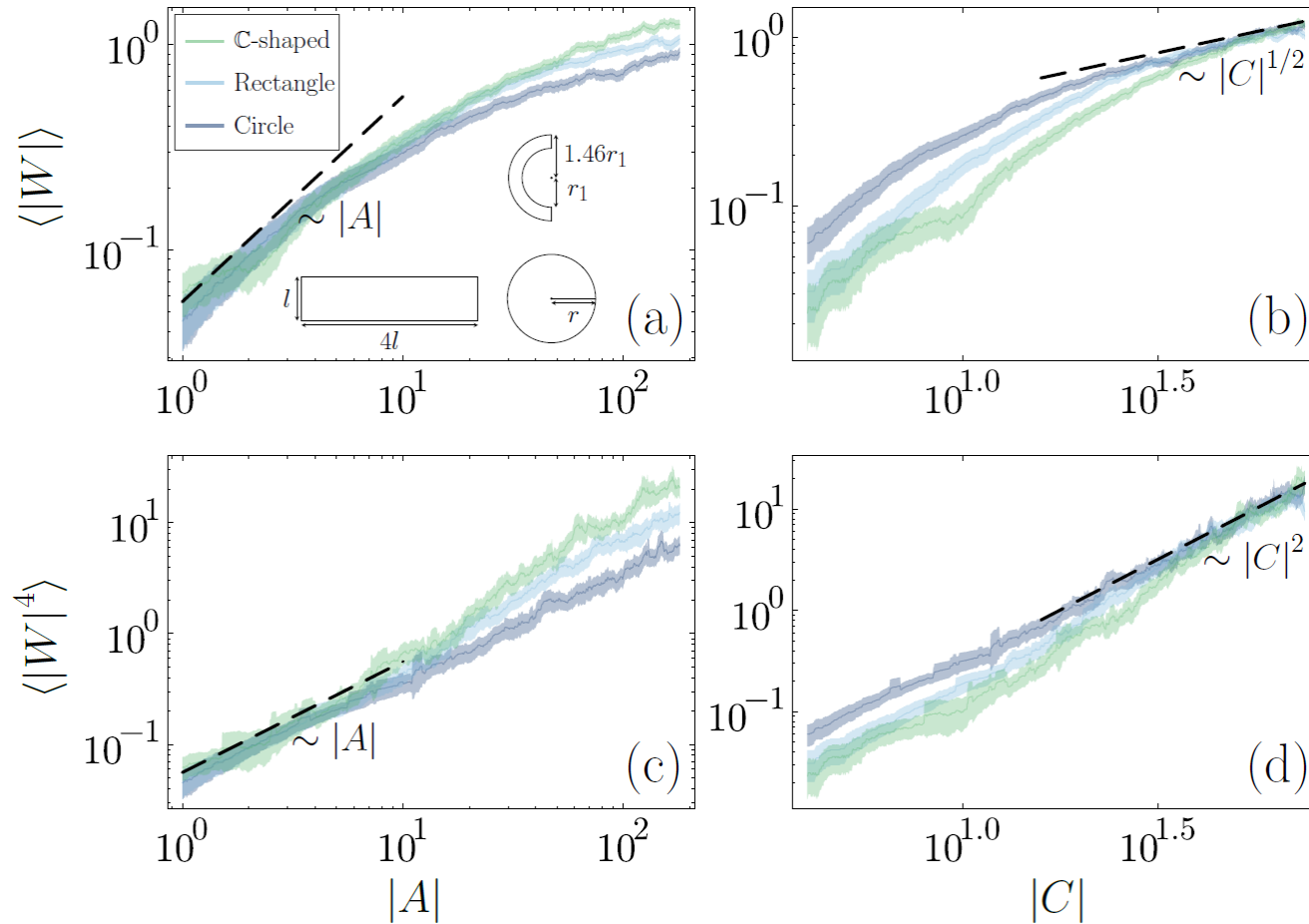
5. Numerical results – Migdal's area rule in SQT

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



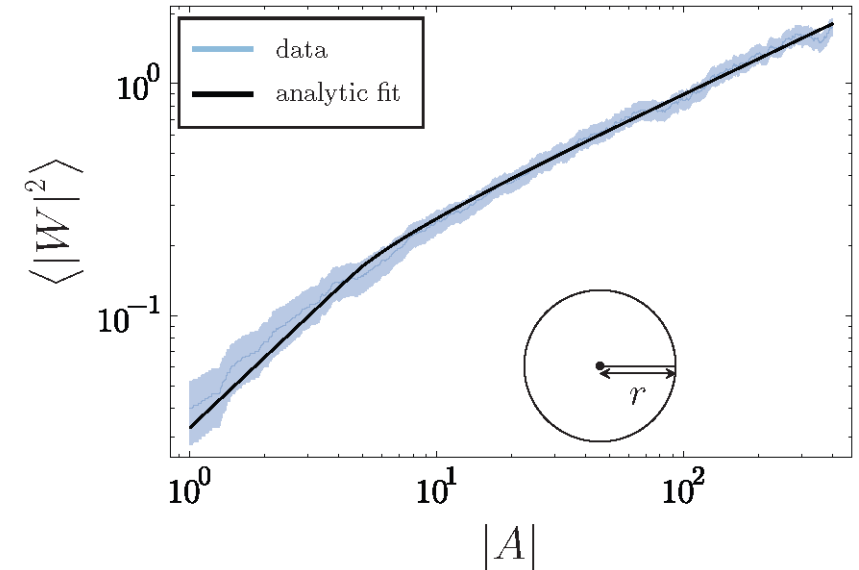
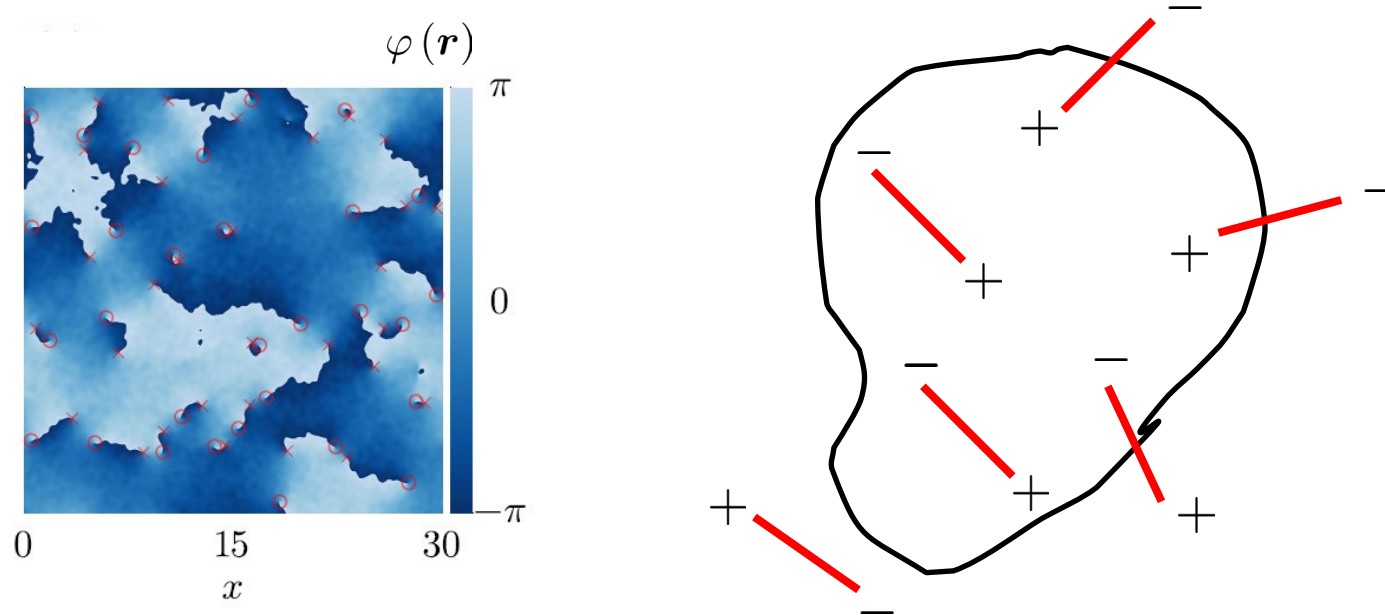
5. Numerical results – Migdal's area rule in SQT

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



5. Numerical results – Migdal's area rule in SQT

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



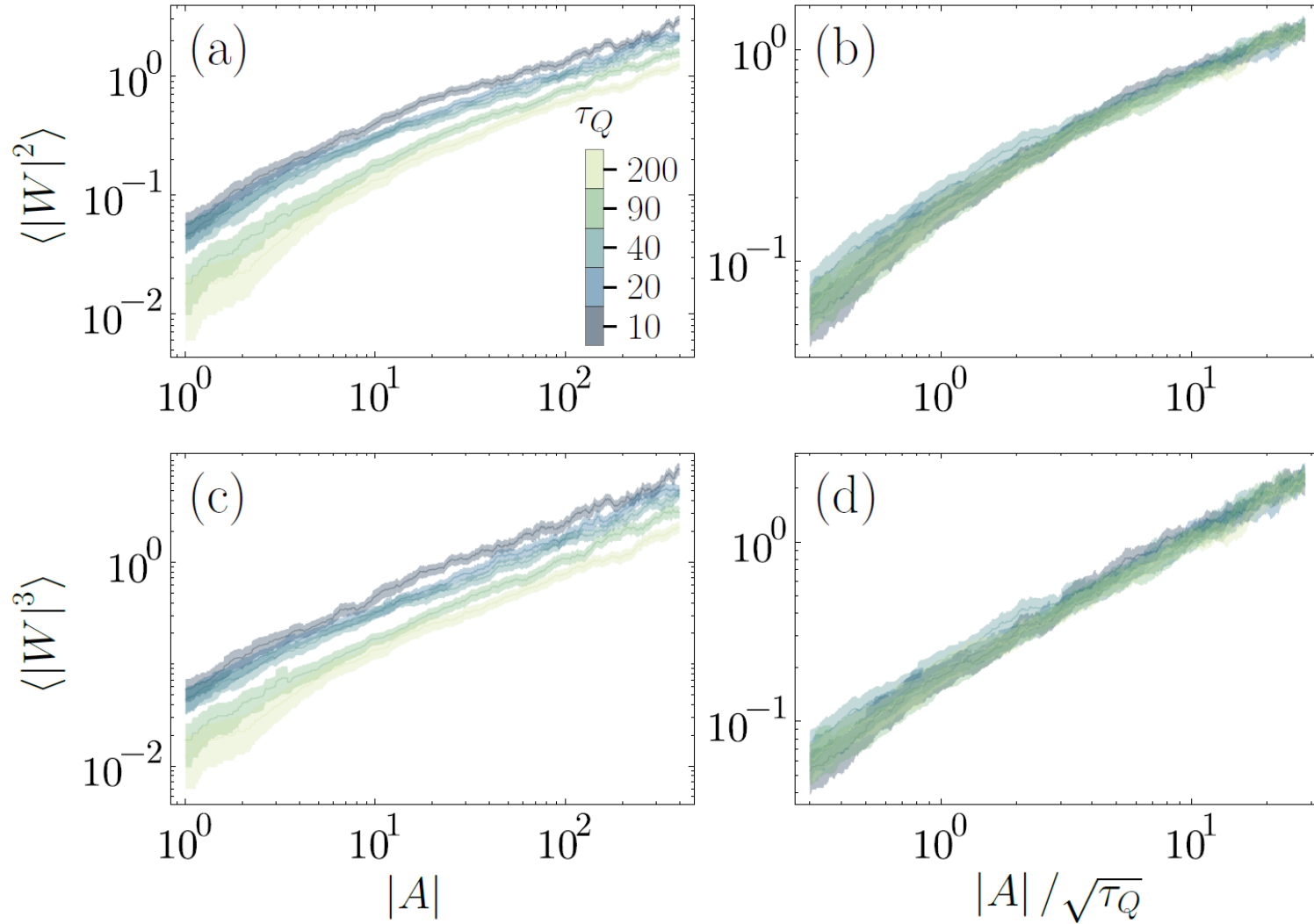
Vortex pair model

- W. H. Zurek, J. Phys: Condens. Matter **25**, 404209 (2013).
- S.-Z. Lin, X. Wang, Y. Kamiya, G.-W. Chern, F. Fan, D. Fan, B. Casas, Y. Liu, V. Kiryukhin, W. H. Zurek, C. D. Batista, and S.-W. Cheong, Nature Phys **10**, 970 (2014).



5. Numerical results – Migdal's area rule in SQT

□ FACULTY OF SCIENCE, TECHNOLOGY AND MEDICINE



$$\begin{aligned}\hat{\xi}(t_{eq}) &\propto \tau_Q^{\nu/(1+z\nu)} \\ &\propto \tau_Q^{1/4}.\end{aligned}$$

6. Summary

- Continuous phase transition triggered by a finite-time quench may induce spontaneous quantum turbulence (SQT), characterized by the existence of the Kolmogorov scaling
- Kolmogorov scaling admits a universal collapse (dictated by the KZM) for different values of the quench time
- Migdal's area rule (shape invariance) also holds in SQT, and the circulation within the arbitrary area A also admits a universal collapse



Thank you!

arXiv:2506.21670

arXiv:2508.11047

Email: seongho.shin@uni.lu





$$E_c(t_{eq}) = \frac{1}{2} \int d^2r |\mathbf{u}_c(\mathbf{r}, t_{eq})|^2 = \int_0^\infty dk E_c(k, t_{eq}).$$

Assumption 1: At low temperature, phonons are due to vortex-antivortex annihilation.

$$\Rightarrow N_p(t_{eq}) \propto n_v(t_{eq}) \propto \tau_Q^{-2\nu/(1+\nu z)}.$$

$$\Rightarrow \int_0^\infty dk n_p(k, t_{eq}) := N_p(t_{eq}).$$

Assumption 2: $n_p(k, t_{eq}) = \tau_Q^{-\alpha} F_p(k\hat{\xi})$.

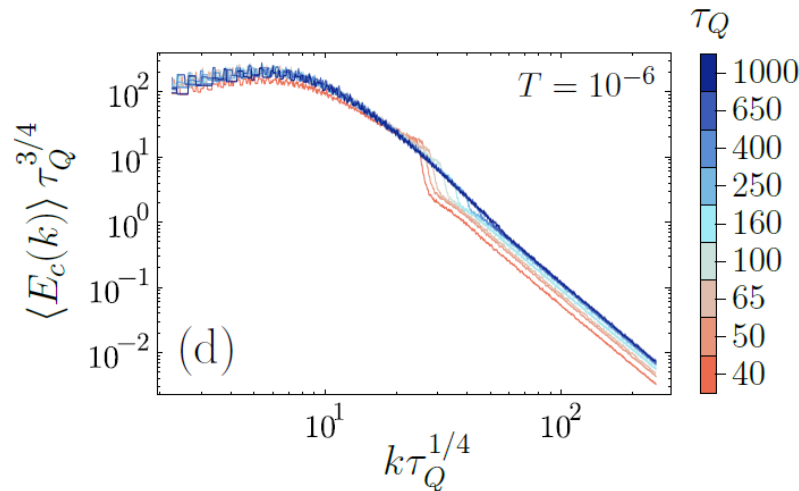
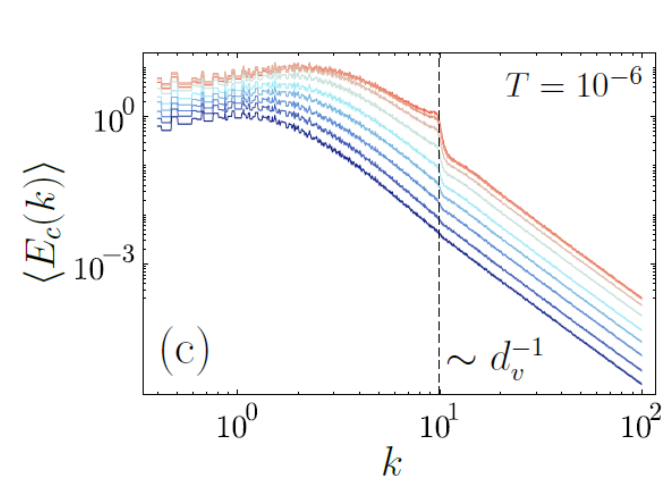
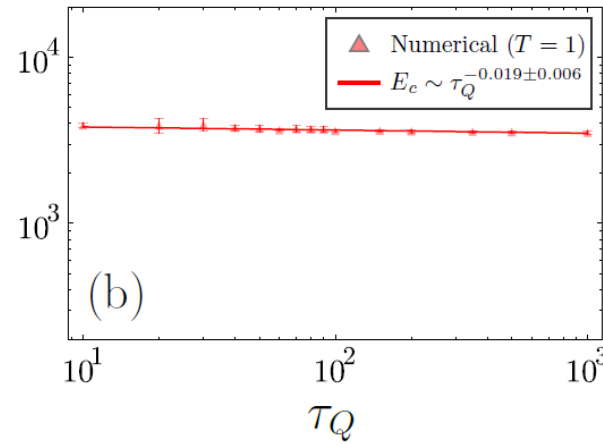
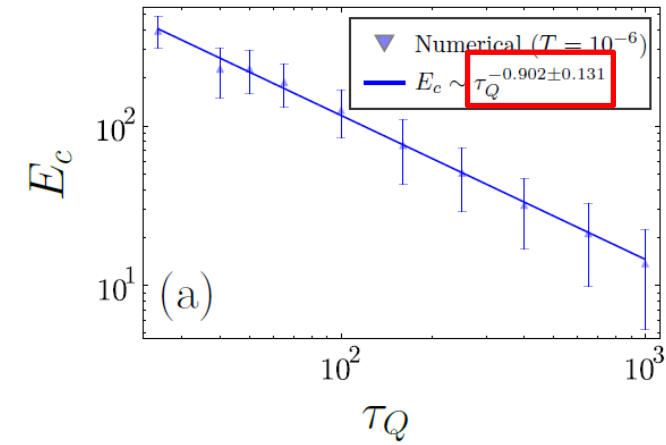
$$\Rightarrow \int_0^\infty dk n_p(k, t_{eq}) \propto \tau_Q^{-\alpha} \hat{\xi}^{-1} \propto \tau_Q^{-\alpha} \tau_Q^{-\nu/(1+z\nu)}. \quad \Rightarrow \alpha = \nu/(1+z\nu).$$

Phonon energy: $\hbar k c_s(t_{eq}) = \hbar k \sqrt{\mu(t_{eq})/m} \propto k \sqrt{\mu(t_{eq})} \propto k \tau_Q^{-1/2(1+z\nu)}.$

$$\Rightarrow E_p(t_{eq}) = \int_0^\infty dk n_p(k, t_{eq}) \hbar k c_s(t_{eq}) \propto \tau_Q^{-\alpha} \hat{\xi}^{-2} \tau_Q^{-1/2(1+z\nu)} \propto \tau_Q^{-(3\nu+1/2)/(1+z\nu)}.$$



$$E_c(t_{eq}) = \frac{1}{2} \int d^2r |\mathbf{u}_c(\mathbf{r}, t_{eq})|^2 = \int_0^\infty dk E_c(k, t_{eq}).$$



$$E_p(t_{eq}) \propto \tau_Q^{-(3\nu+1/2)/(1+z\nu)}.$$

$$\propto \tau_Q^{-1} \text{ for } \nu = 1/2, z = 2.$$

$$E_p(k, t_{eq}) \propto \tau_Q^{-(2\nu+1/2)/(1+z\nu)} G\left(k \tau_Q^{\nu/(1+z\nu)}\right).$$

$$\propto \tau_Q^{-3/4} G\left(k \tau_Q^{1/4}\right).$$



$$\langle n_v(t_{eq}) \rangle = \mathcal{N}_{\text{event}} p_v,$$

$$\text{Var}(n_v(t_{eq})) = \mathcal{N}_{\text{event}} p_v (1 - p_v).$$

$$P(W = n) = \sum_{\substack{m \geq |n| \\ m+n \text{ even}}}^{\mathcal{N}} \binom{\mathcal{N}}{m} \binom{m}{\frac{m+n}{2}} \left(\frac{p_v p_C}{2}\right)^{(m+n)/2} \left(\frac{p_v p_C}{2}\right)^{(m-n)/2} (1 - p_v p_C)^{\mathcal{N}-m},$$

C.-Y. Xia and H.-B. Zeng, *Winding up a finite size holographic superconducting ring beyond Kibble-Zurek mechanism*, Phys. Rev. D **102**, 126005 (2020).

$$p_C = \begin{cases} 2 \frac{\pi r^2}{A_{\text{tot}}}, & r < \frac{\hat{\xi}}{2} \\ \frac{2}{A_{\text{tot}}} \left[\frac{1}{2} \hat{\xi} \sqrt{4r^2 - \hat{\xi}^2} - r^2 \left(2 \cos^{-1} \left(\frac{\hat{\xi}}{2r} \right) - \pi \right) \right], & r > \frac{\hat{\xi}}{2}. \end{cases}$$

F. J. Gómez-Ruiz, J. J. Mayo, and A. del Campo, *Full Counting Statistics of Topological Defects After Crossing a Phase Transition*, Phys. Rev. Lett. **124**, 240602 (2020).