Global aspects of 3-form fields in axion Yang-Mills systems

Mohamed Anber
Center for Particle Theory
Durham University

Durham University

Developments on Generalized Symmetries of Quantum Systems, Murcia, May 2026

Based on arXiv:2407.03416 with Samson Chan

Why axion-YM systems? The old and new

Axions are important!

✓**Axion solves the strong CP problem** (Peccei-Quinn 1977):

- QCD θ-vacuum $\dfrac{\theta}{8\pi^2}\displaystyle\int \mathrm{tr} F \wedge F$
- $\theta \neq 0$ ➡ CP-violating effects (neutron electric dipole moment)
- Experimentally $\theta < 10^{-10}$, **why θ is small**?

Why axion-YM systems? The old and new

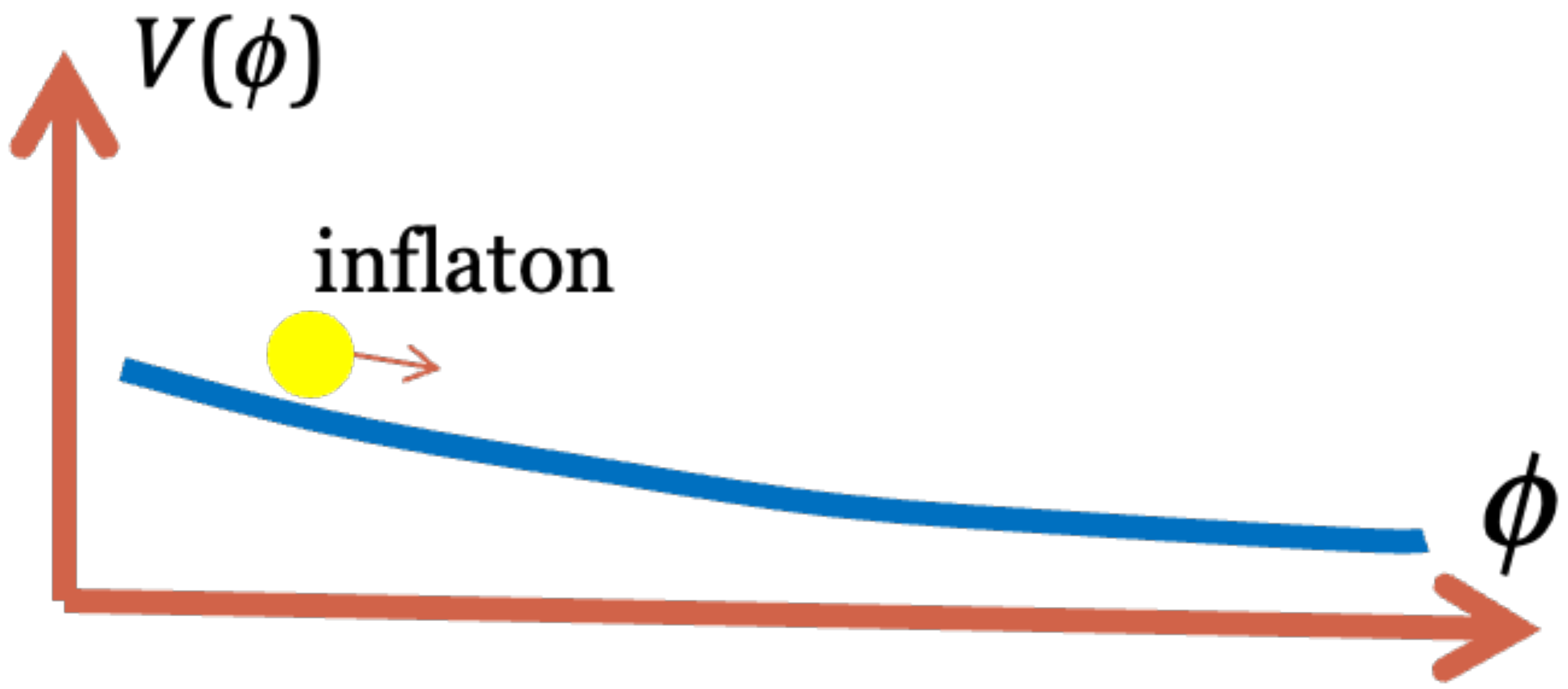
✓**Axion is a dark matter candidate**

- Dark matter accounts for 25 % of the Universe (not SM)
- Important for structure formation, etc. Many current experiments.

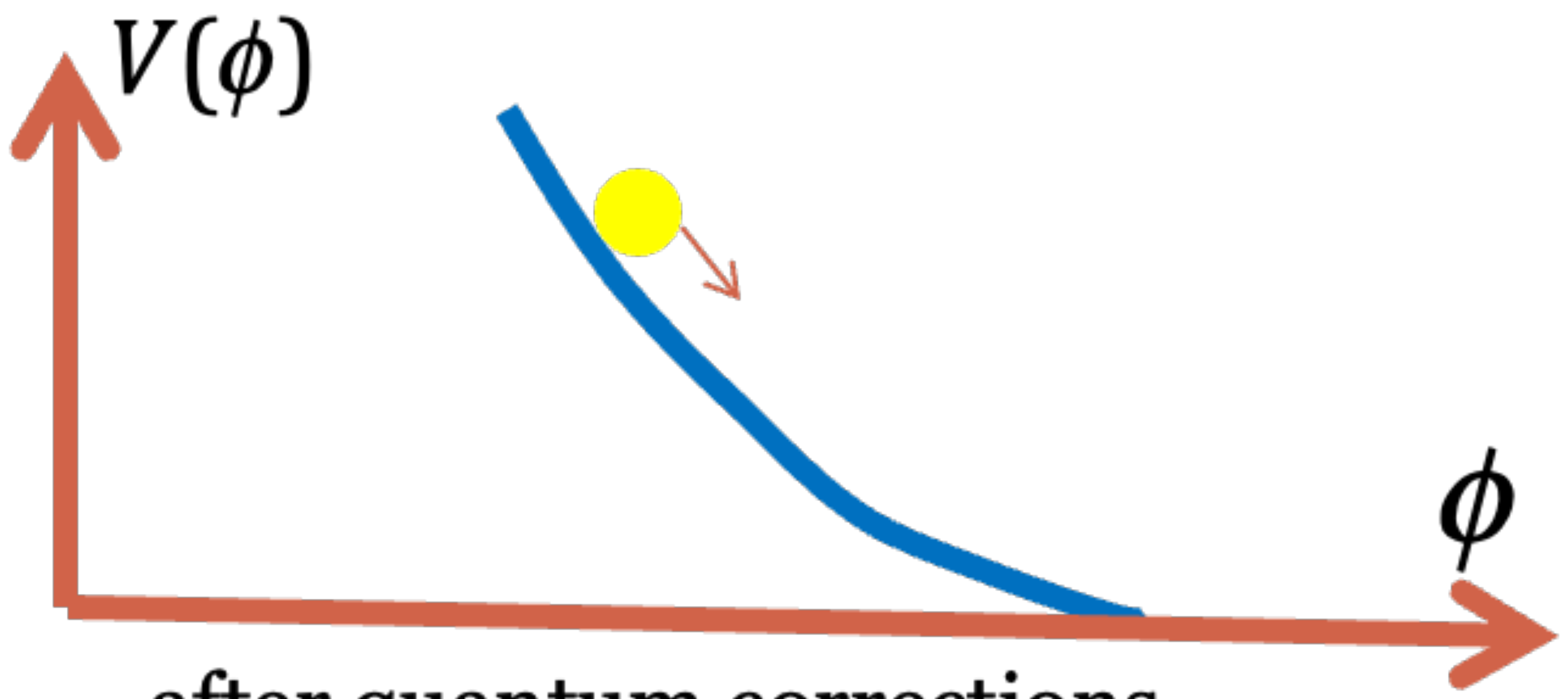
Why axion-YM systems? The old and new

✓Axion is a candidate for inflation

- Inflation is a period of accelerating expansion before the hot big bang. It solves flatness, horizon, etc.
- To sustain inflation, we need flat potential of a scalar field

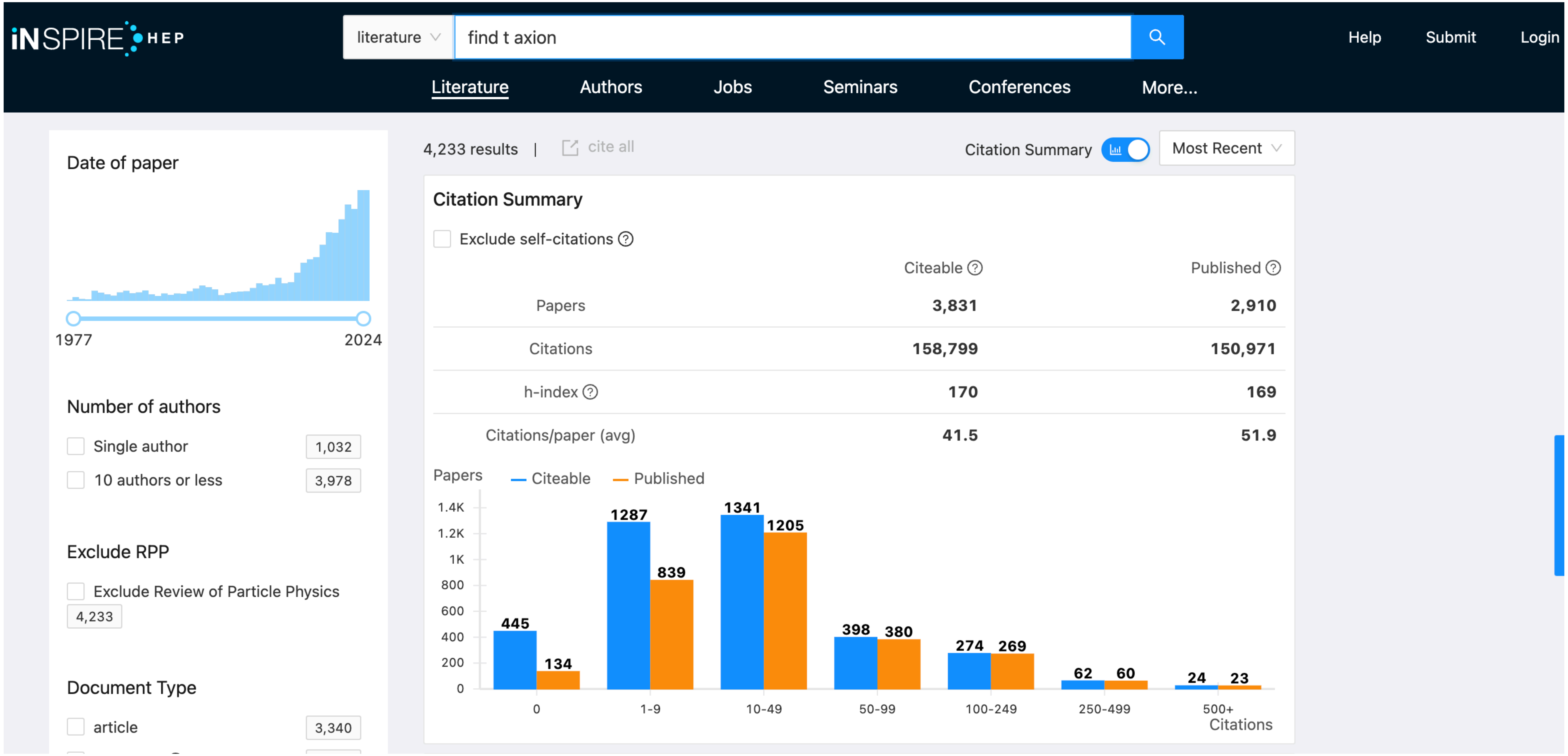


before quantum corrections



after quantum corrections

Why axion-YM systems? The old and new

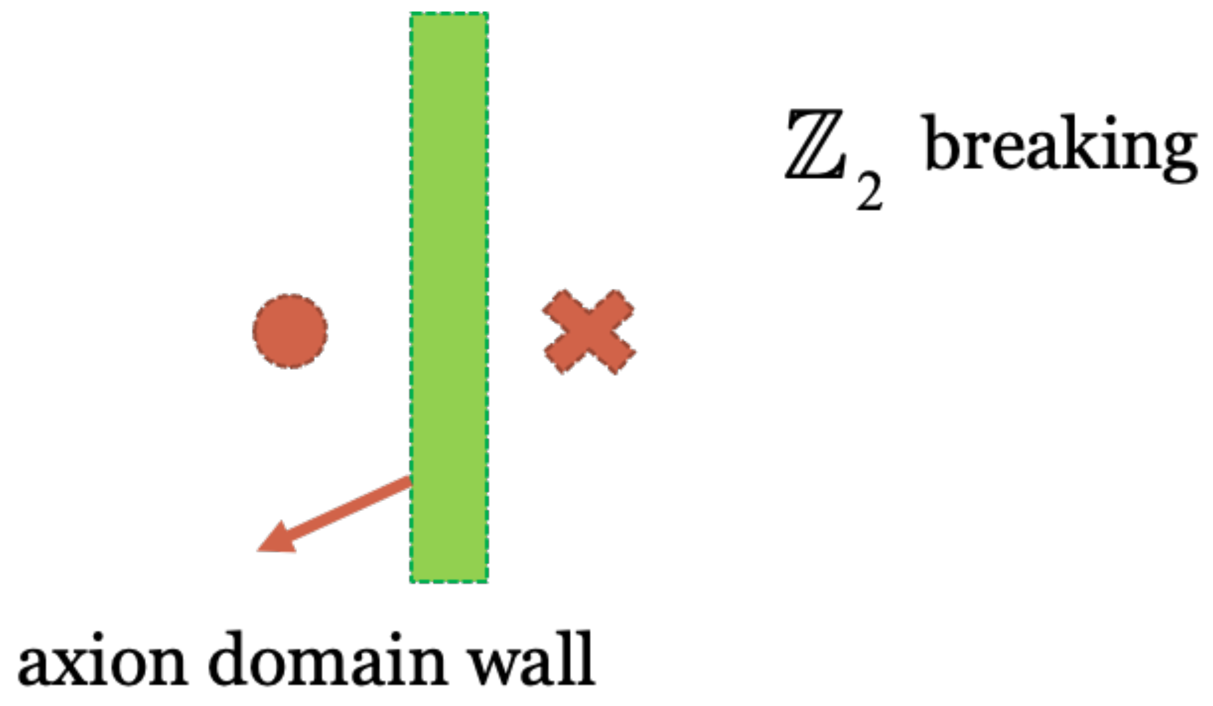


Why axion-YM systems? The old and new

- Fixing the setup: KSVZ model, UV theory is
  YM + fermions+$\underbrace{\text{complex scalar } \Phi = \rho e^{ia}}_{\text{axion=phase}}$+Yukawa term

Kim–Shifman–Vainshtein– Zakharov, 1979-80

- $U(1)$ shift symmetry $a \to a$+const. broken by instantons to $\mathbb{Z}_q$
- IR: $m_a \cong \dfrac{\Lambda^2}{v} \ll \Lambda\,, \mathscr{L} = \dfrac{v^2}{2}(\partial_\mu a)^2 + \Lambda^4 \cos(qa)\,, \quad q \geq 1$
- Can give rise to DWs for $q > 1$
- It hides many details (anomalies, light/heavy d.o.f. intertwining, etc.)



Why axion-YM systems? The old and new

- YM IR **can be** described by 3-form field $c_3 \sim c_{[\alpha\beta\gamma]}$: Luscher, Veneziano (1978-79)
- c_3+axion: Dvali (2005)
- They did not consider global aspects: Dirac quantization, anomalies, etc.
- **Generalized global symmetries (GGS)/anomalies** imply (M.A. , Chan 2024):

✓ c_3 **provides a consistent description of axions in IR:** anomalies, light/heavy d.o.f. intertwining

✓**IR physics is sensitive to the global structure of the YM gauge group (FQH states, etc.)**

Take-home messages

★GGS provide a guiding principle to build consistent EFTs

★They can lead to new/new perspectives on nonperturbative phenomena

Outline

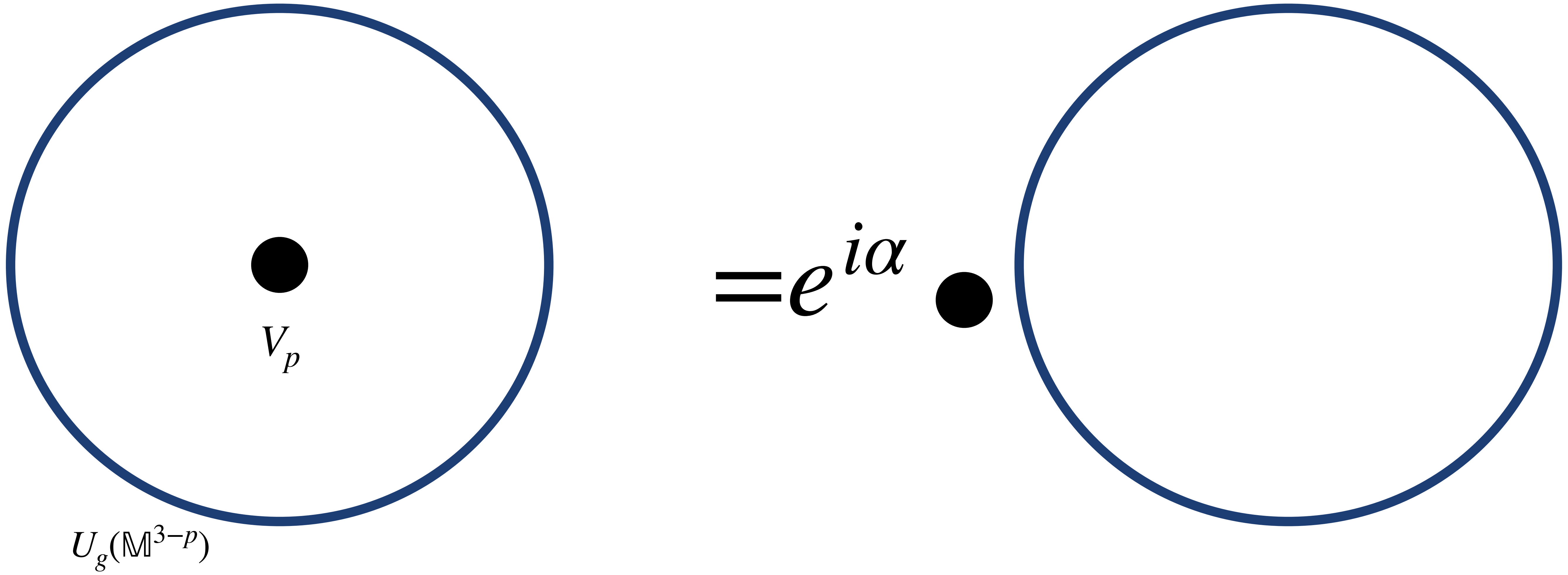
✦Simple Lagrangians and lots of symmetries/anomalies

✦Axion-YM: anomalies, hadronic structure, FQH states on walls, etc.

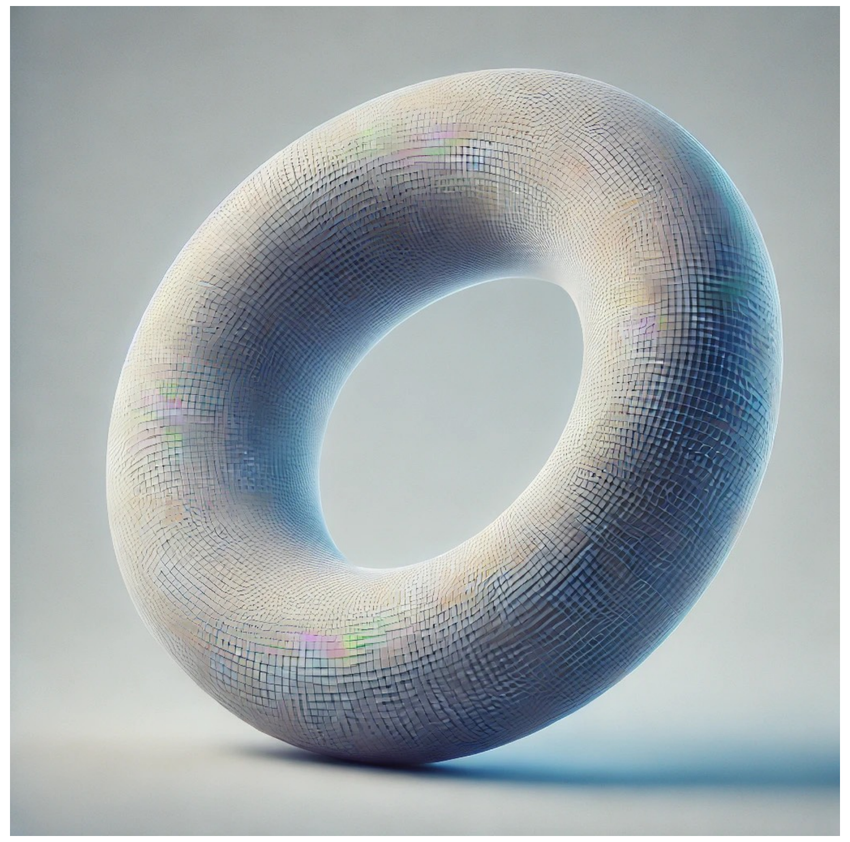
Simple Lagrangians and lots of symmetries/ anomalies

Symmetries: modern perspective

- Generalization of old symmetries: p-form symmetry ➡ $d \star j_{p+1} = 0$
- Symmetry defect $U_g(\mathbb{M}^{3-p}) = e^{i\alpha \oint_{\mathbb{M}^{3-p}} \star j_{p+1}}$
- The symmetry acts on p-dimensional object (Wilson's line): $V_p = e^{i \oint_{\mathbb{M}^p} C_p}$
- $U_g V_p = e^{i\alpha} V_p U_g$

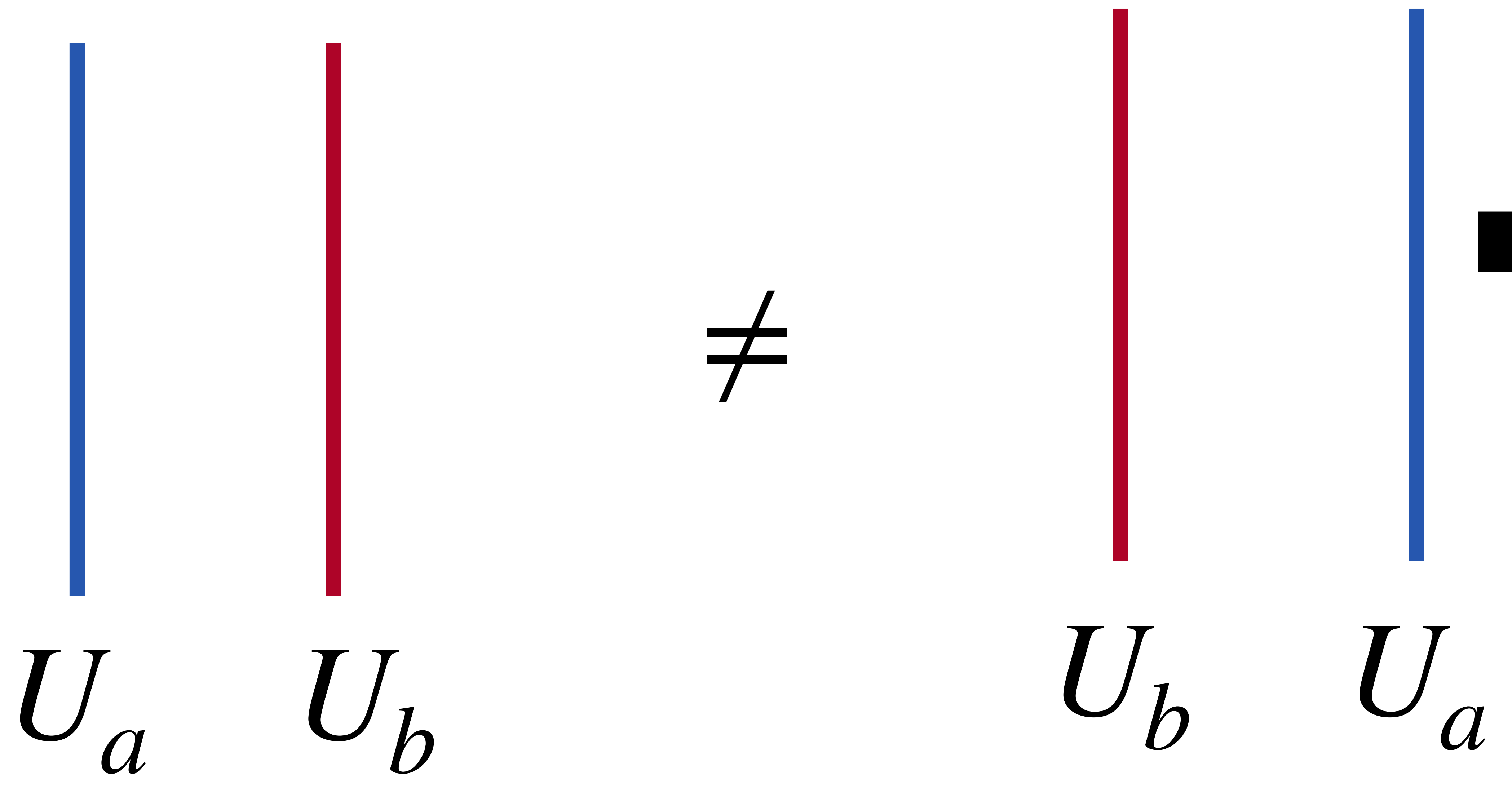


Gaiotto, Kapustin, Seiberg, Willett, 2014



Symmetries: modern perspective

- **’t Hooft anomaly**
- Given abelian-symmetry defects $\{U_a, U_b, U_c, \ldots\}$, the **lack of commutativity** signals an anomaly:



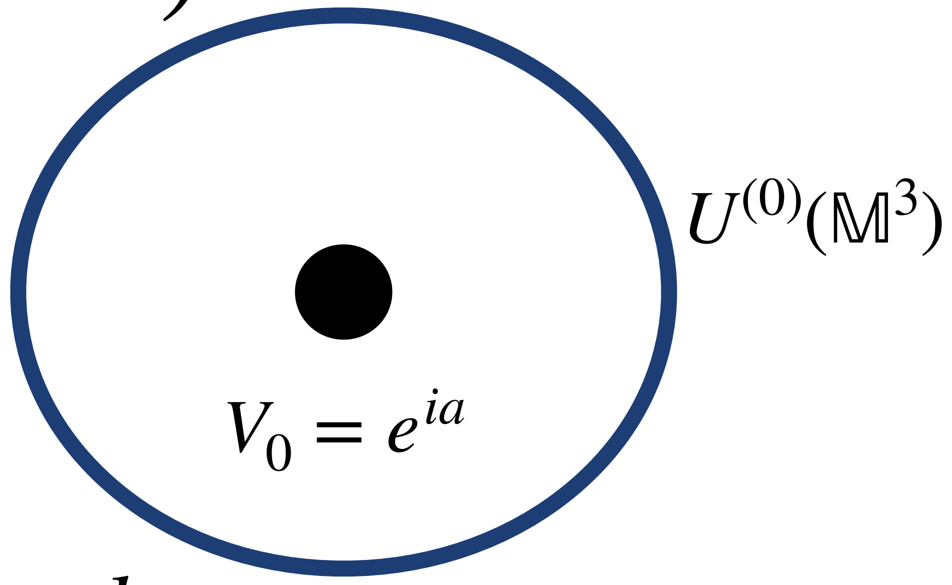
$$\Rightarrow \quad Z[A_{(a)}] \xrightarrow{b} = e^{i\alpha} Z[A_{(a)}]$$

anomalies are RG invariants:
vacuum is not trivially gapped

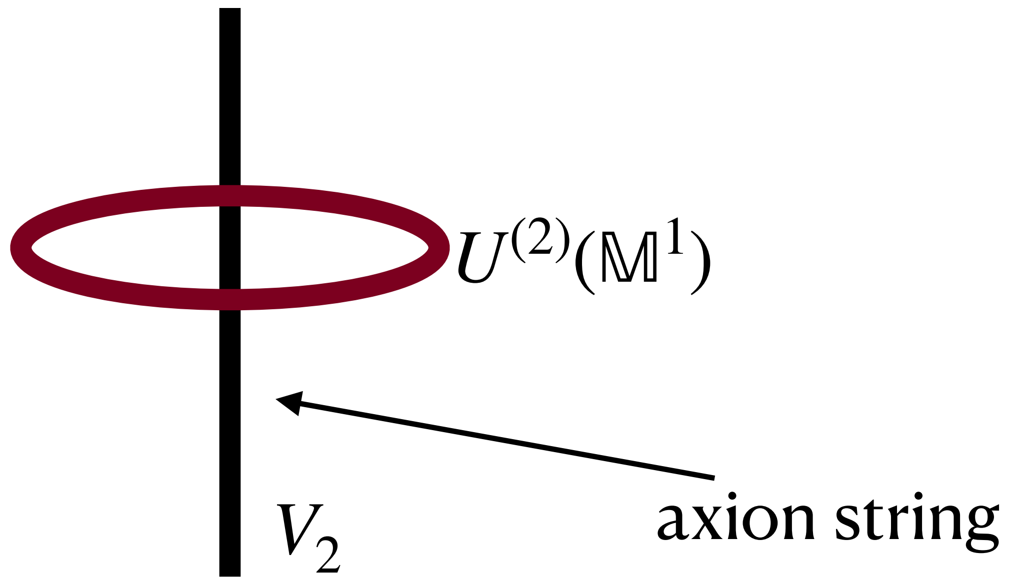
Simple Lagrangian, lots of symmetries

- $\mathscr{L} = \frac{v^2}{2}(\partial_\mu a)^2 \equiv \frac{v^2}{2} da \wedge \star da = \frac{v^2}{2} |da|^2 , \quad a \sim a + 2\pi , \quad v =$ PQ scale
- **Two conserved currents**: $d \star j_1 = v^2 d \star da = 0 , \quad d \star j_3 = dda = 0$

❖ $U(1)$ 0-form shift symmetry $a \to a$+const.: $U^{(0)}(\mathbb{M}^3) = e^{i\alpha v \oint_{\mathbb{M}^3} \star da}$



❖ $U(1)$ 2-form winding symmetry: $U^{(2)}(\mathbb{M}^1) = e^{i\beta \oint_{\mathbb{M}^1} da}$



Simple Lagrangian, lots of symmetries

- There is a mixed ’t Hooft anomaly between the 0-form and 2-form symmetries:
- $$U^{(0)}(\mathbb{M}^3)U^{(2)}(\mathbb{M}^1) = \underbrace{e^{-i\alpha\beta}}_{\text{anomaly}} \; U^{(2)}(\mathbb{M}^1)U^{(0)}(\mathbb{M}^3)$$
- The anomaly implies that:

✓ **We cannot gauge a symmetry without breaking the other (ABJ type)**

Simple Lagrangian, lots of symmetries

- Gauge $U(1)^{(2)}$: $d \star j_3 = 0 \leftrightarrow \oint_{\mathbb{M}^4} \star j_3 \wedge C_3 = \oint_{\mathbb{M}^4} da \wedge C_3 \,, \quad C_3 \to C_3 + d\Lambda_2$
- $Z = \int [Dc_3][Da]e^{-S}, \quad S = \oint_{\mathbb{M}^4} \frac{v^2}{2} |da|^2 - \frac{q}{2\pi} da \wedge c_3 + \frac{|dc_3|^2}{2\Lambda^4}$
- Λ is a **new scale** and $q \in \mathbb{N}$ is a **free parameter**.
- DQC: $\oint_{\mathbb{M}^4} (dc_3 \equiv f_4) \in 2\pi\mathbb{Z}$
- The term $\frac{q}{2\pi} da \wedge c_3$ implies that only $\mathbb{Z}_q$ survives as a 0-form shift symmetry (result from ABJ anomaly).

Simple Lagrangian, lots of symmetries

- Consider $\mathscr{L} = \frac{v^2}{2}|da|^2 - \frac{q}{2\pi} da \wedge c_3 + \frac{|dc_3|^2}{2\Lambda^4}$
- c_3 is **not propagating**, $f_4 = dc_3$ is a **CC** and $\oint_{\mathbb{M}^4} f_4 = 2\pi\mathbb{Z}$
- 2 conserved currents (EOM) ➡ 2 global symmetries:

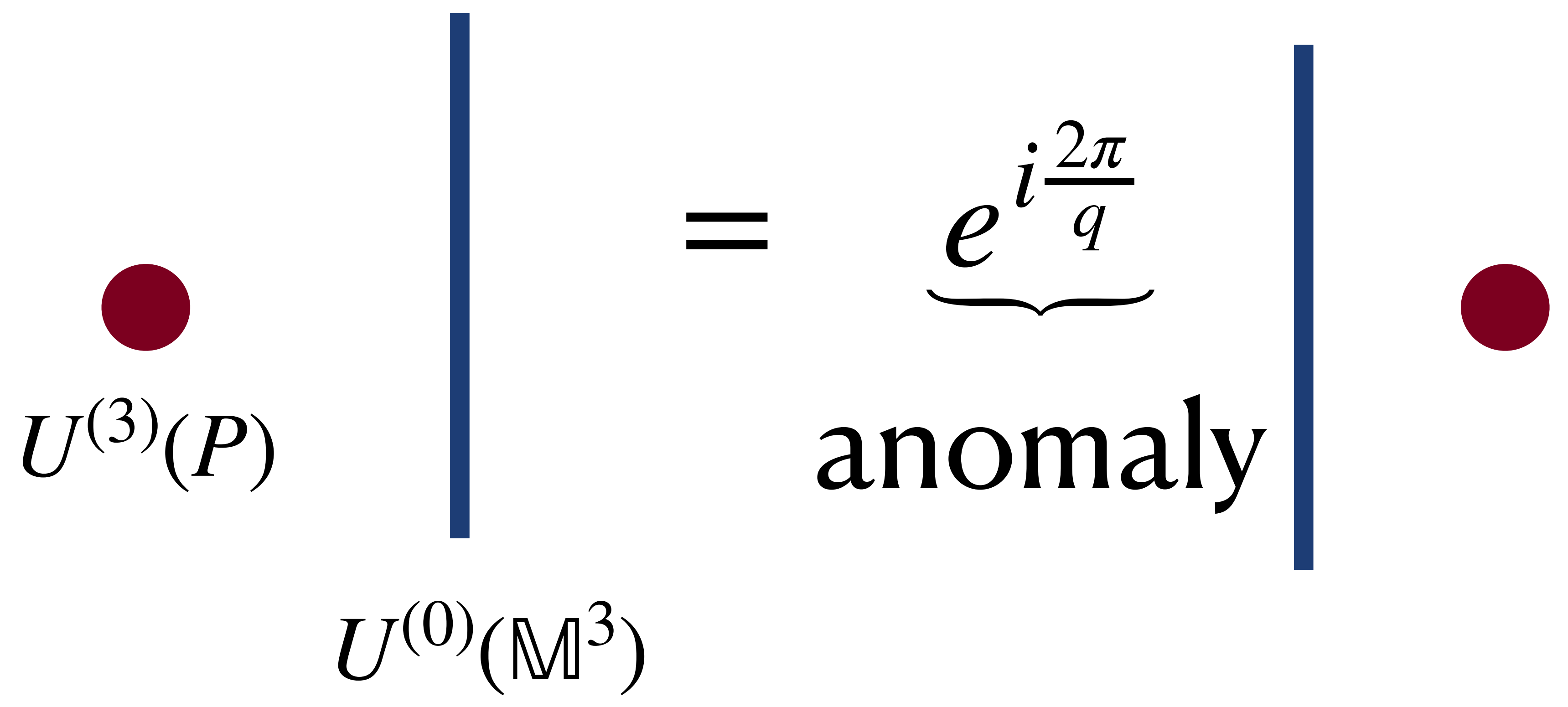
✓ $\mathbb{Z}_q^{(0)}$ shift symmetry: $d\star j_1 = v^2 d\star da - \frac{q}{2\pi}dc_3 = 0$ ➡ $U^{(0)}(\mathbb{M}^3) = e^{\underbrace{i\frac{2\pi}{q}\oint_{\mathbb{M}^3} v^2\star da - \frac{q}{2\pi}c_3}}$

✓ $\mathbb{Z}_q^{(3)}$ symmetry: $d\star j_4 = \frac{1}{\Lambda^4} d\star dc_3 + \frac{q}{2\pi}da = 0$ ➡ $\underbrace{U^{(3)}(P = \mathbb{M}^0)}_{\text{act on } 3\text{–surfaces}} = \underbrace{e^{ia + \frac{2\pi}{q}\frac{\star f_4}{\Lambda^4}}}$

$U(1)^{(2)}$–invariants

Simple Lagrangian, lots of symmetries

- $U^{(0)}$ and $U^{(3)}$ do not commute:



- The anomaly ➡ vacuum is not unique: $\mathbb{Z}_q^{(0)}$ or $\mathbb{Z}_q^{(3)}$ or both are **broken**.

Simple Lagrangian, lots of symmetries

- Kalb-Ramond frame: $$\mathscr{L} = \frac{v^2}{2} da \wedge \star da - \frac{q}{2\pi} da \wedge c_3 + \frac{|dc_3|^2}{2\Lambda^4} - \underbrace{\frac{1}{2\pi} b_2 \wedge d^2 a}_{d^2a=0}$$

- Integrating out a: $$\mathscr{L} = \frac{1}{2(2\pi)^2} |db_2 + \underbrace{q}\, c_3|^2 + \frac{|dc_3|^2}{2\Lambda^4}$$

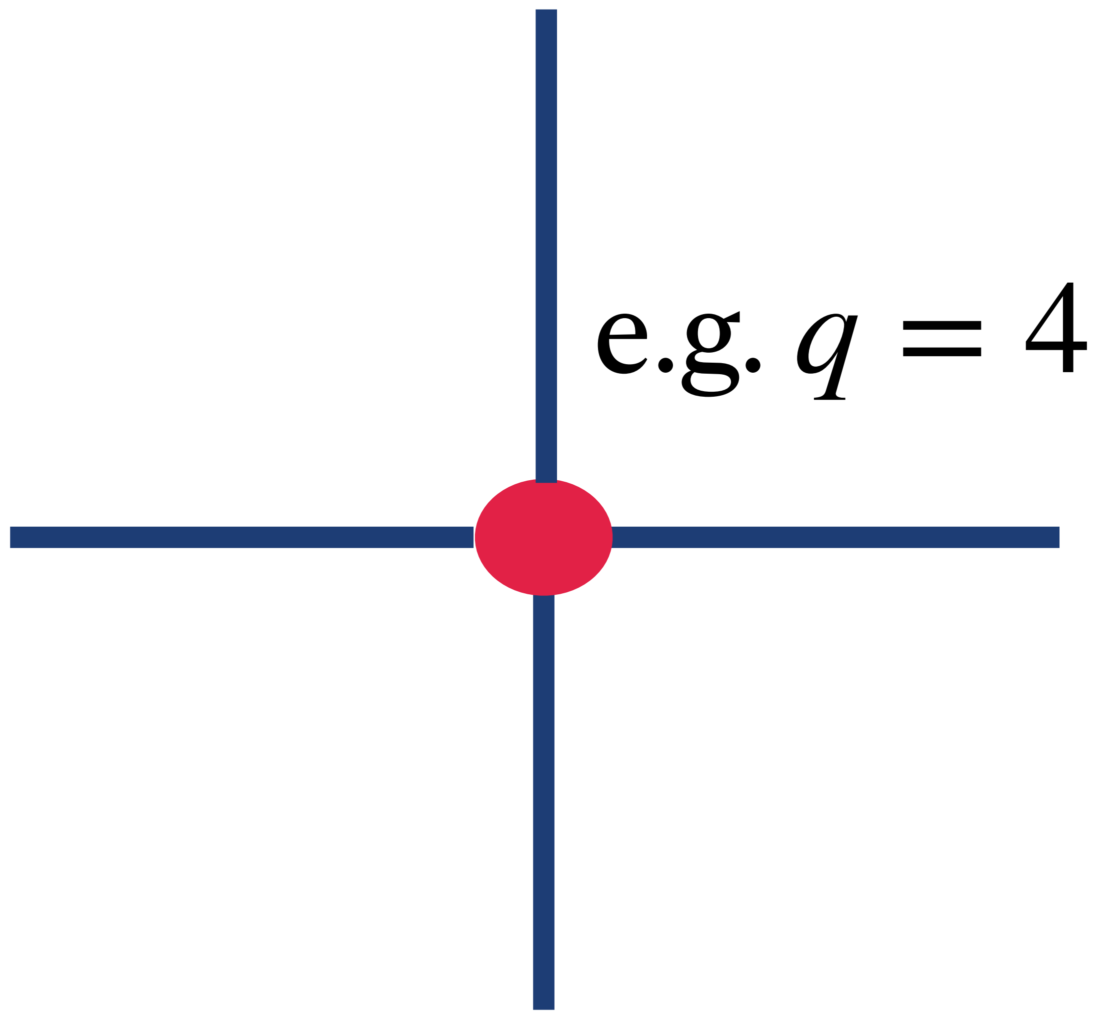
Kalb-Ramond field:
Stuckelberg field

Charge of b_2

- c_3 eats b_2 ➡ c_3 becomes massive

Simple Lagrangian, lots of symmetries

- Gauge-invariant operator: $\langle e^{i\int_{M_3} db_2 + qc_3} \rangle = \langle e^{i\int_{M_2=\partial M_3} b_2} e^{iq\int_{M_3} c_3} \rangle$
- e.g.



- This matches the mixed $Z_q^{(0)} - \mathbb{Z}_q^{(3)}$ anomaly.

✦ Axion-YM: anomalies, hadronic structure, etc.

3-form fields in axion-YM theory

- UV completion:

$$\mathscr{L} = \underbrace{\text{YM}}_{SU(N=6)} + \underbrace{\text{Dirac fermion}}_{\text{in rep.}\, R= \begin{smallmatrix}\square\\ \square\end{smallmatrix}} + |d\Phi|^2 + \lambda(|\Phi|^2 - v^2)^2 + (y\Phi\tilde{\psi}\psi + h\,.\,c)$$

- $\Phi = \rho e^{ia}$, $a \sim a + 2\pi$, and take $v \gg \Lambda$(strong scale)
- UV symmetries:

✓$Z_{T_R}^{(0)\chi} = \mathbb{Z}_4^{(0)\chi}$

✓$\mathbb{Z}_2^{(1)}$ 1-form center symmetry (acts on Wilson's lines)

3-form fields in axion-YM theory

- Mixed anomaly: $\mathbb{Z}_2^{(1)}/\mathbb{Z}_4^{(0)\chi}$
- UV description $E \gg v$: $\mathscr{L} \supset \dfrac{1}{8\pi^2}\mathrm{tr}\left(\hat{f}_2^c - B_2^c\right)\wedge\left(\hat{f}_2^c - B_2^c\right)$

$U(N)$ field

Background of $\mathbb{Z}_2^{(1)}$: $\oint_{\mathbb{M}^2} B_2^c = \dfrac{2\pi\mathbb{Z}}{2}$

Kapustin, Seiberg, 2014
Gaiotto, Kapustin, Komargodski, Seiberg, 2017

- $U(1) \subset U(N)$ unphysical ➡ postulate $U(1)^{(1)}$ gauge symmetry:

$$B_2^c \to B_2^c - d\Lambda_1\,, \quad \hat{f}_2^c \to \hat{f}_2^c - d\Lambda_1$$

- Anomaly: $Z[B_2^c] \xrightarrow{\mathbb{Z}_4^{(0)\chi}} e^{i\pi}Z[B_2^c]$

3-form fields in axion-YM theory

- At $\Lambda \ll E \ll v$:
$$\mathscr{L} = \text{YM} + \frac{v^2}{2}|da|^2 + \frac{(T_R = 4)}{8\pi^2}\, a\, \mathrm{tr}\left(\hat{f}_2^c - B_2^c\right) \wedge \left(\hat{f}_2^c - B_2^c\right) + \text{heavy } \cancel{\text{fermions}}$$
- Axion $\mathbb{Z}_4^{(0)}$ shift symmetry $a \to a + \dfrac{2\pi}{4}$ (you can see the anomaly)
- Global symmetry: $\mathbb{Z}_{T_R=4}^{(0)} \times \left(\mathbb{Z}_{N=6}^{(1)} \tilde{\times}\, U(1)^{(2)}\right)$
- $U(1)^{(2)}$ is emergent, $\mathbb{Z}_N^{(1)}$ is an enhanced center with a higher group $\tilde{\times}$
- $E_{\mathbb{Z}_N^{(1)}} \lesssim E_{U(1)^{(2)}}$ and $E_{\mathbb{Z}_N^{(1)}} \cong \sqrt{\lambda} v, \quad E_{U(1)^{(2)}} \cong yv, \quad \lambda \ll y^2 \ll 1$

Cordova, Dumitrescu, Intriligator, 2018
Brennan, Cordova 2020

3-form fields in axion-YM theory

- The trouble is at $E \ll \Lambda$: the color is gapped

$$\mathscr{L} = \cancel{\text{YM}} + \frac{v^2}{2}|da|^2 + \frac{(T_R = 4)}{8\pi^2} a\, \mathrm{tr}\left(\cancel{\hat{f}_2^c} - B_2^c\right) \wedge \left(\cancel{\hat{f}_2^c} - B_2^c\right) + \text{heavy }\cancel{\text{fermions}}$$

- This violates $U(1)^{(1)}$ gauge symmetry!
- The way out: $U(1)^{(2)}$ winding symmetry at $E \ll v$

✓ We gauge $U(1)^{(2)}$ at $E \ll \Lambda$

3-form fields in axion-YM theory

✓ Replace $\mathrm{tr}(\hat{f}_2^c \wedge \hat{f}_2^c) \longrightarrow dc_3$

- $$\mathcal{L}_{E\ll\Lambda} = \frac{v^2}{2}|da|^2 + \frac{(T_R = 4)}{2\pi} a \wedge \underbrace{\left(dc_3 - \frac{N}{4\pi} B_2^c \wedge B_2^c\right)}_{U(1)^{(1)}\ \text{invariant}} + \frac{|dc_3|^2}{2\Lambda^4}$$

- dc_3 is the long tail incarnation of $\mathrm{tr}(f_2 \wedge f_2)$ below Λ

$$\mathcal{L} = \frac{v^2}{2}(\partial_\mu a)^2 + \Lambda^4 \cos(4a)$$

Luscher, 1978
Veneziano, 1979

3-form fields in axion-YM theory

- DQC $\oint_{\mathbb{M}^4} (f_4 = dc_3) \in 2\pi m\,, m \in \mathbb{Z}$
- Performing the sum over $m \in \mathbb{Z}$:

$$Z[a] \sim \sum_{k\in\mathbb{Z}} \underbrace{\exp\left[-i\frac{Nk}{4\pi}\int_{\mathbb{M}^4} B_2^c \wedge B_2^c\right]}_{\text{mixed anomaly}:a\to a+2\pi/(T_R=4)} \exp\left[-\oint_{\mathbb{M}^4} \frac{v^2}{2}\,|\,da\,|^2 + \frac{\Lambda^4}{8\pi^2}\left(T_R\,a + 2\pi k\right)^2\right]$$

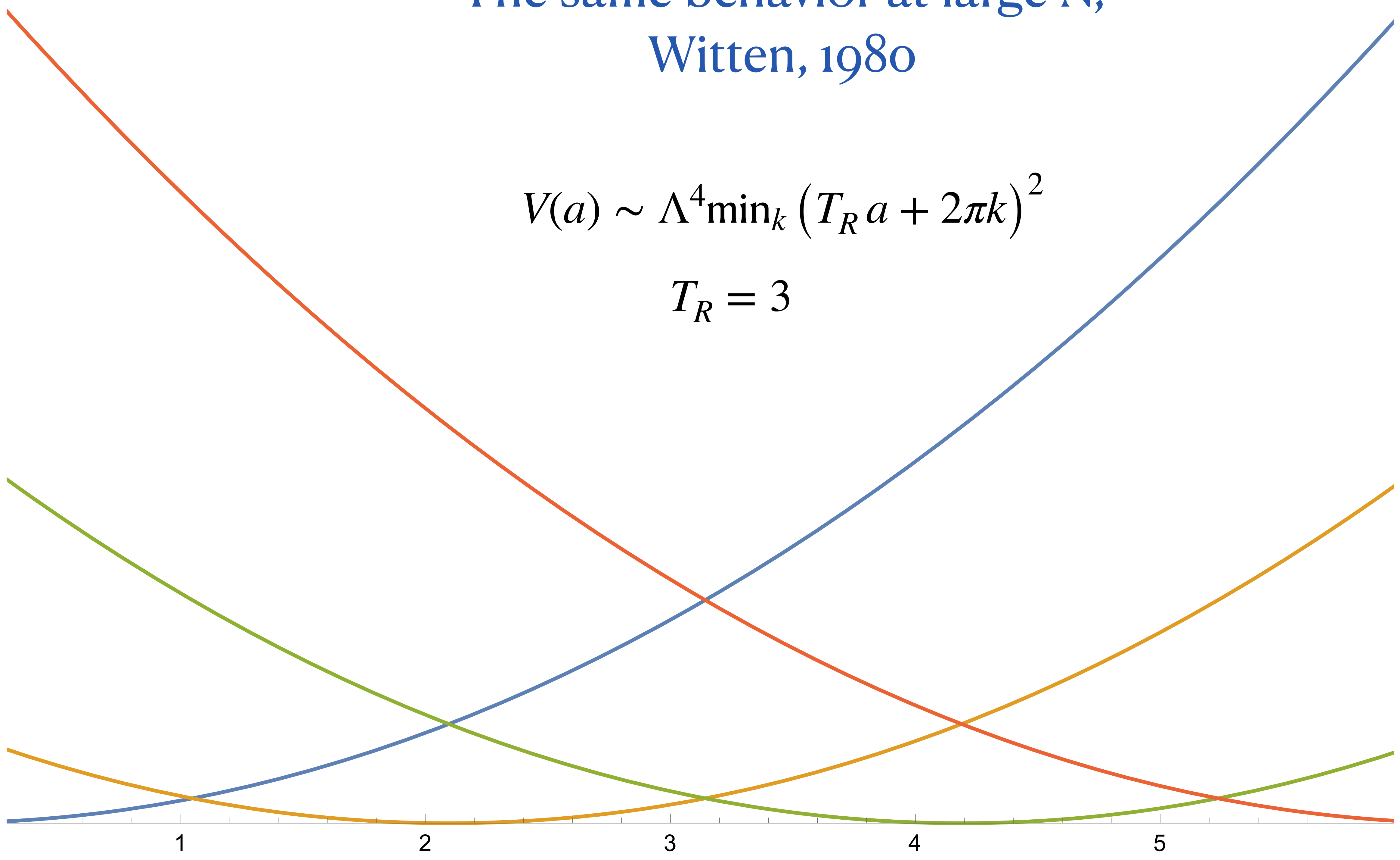
M.A. , Chan 2024

- $V(a) \sim \Lambda^4 \text{min}_k \left(T_R\,a + 2\pi k\right)^2$
- vacua $= \dfrac{2\pi\ell}{T_R = 4}$, cusps $= \dfrac{\pi(2\ell+1)}{T_R = 4}$

3-form fields in axion-YM theory

- Axion+hadronic walls: $\delta_{DW} \sim \dfrac{v}{\Lambda^2} \gg \Lambda^{-1}\,, \quad \delta_H \sim \Lambda^{-1}$

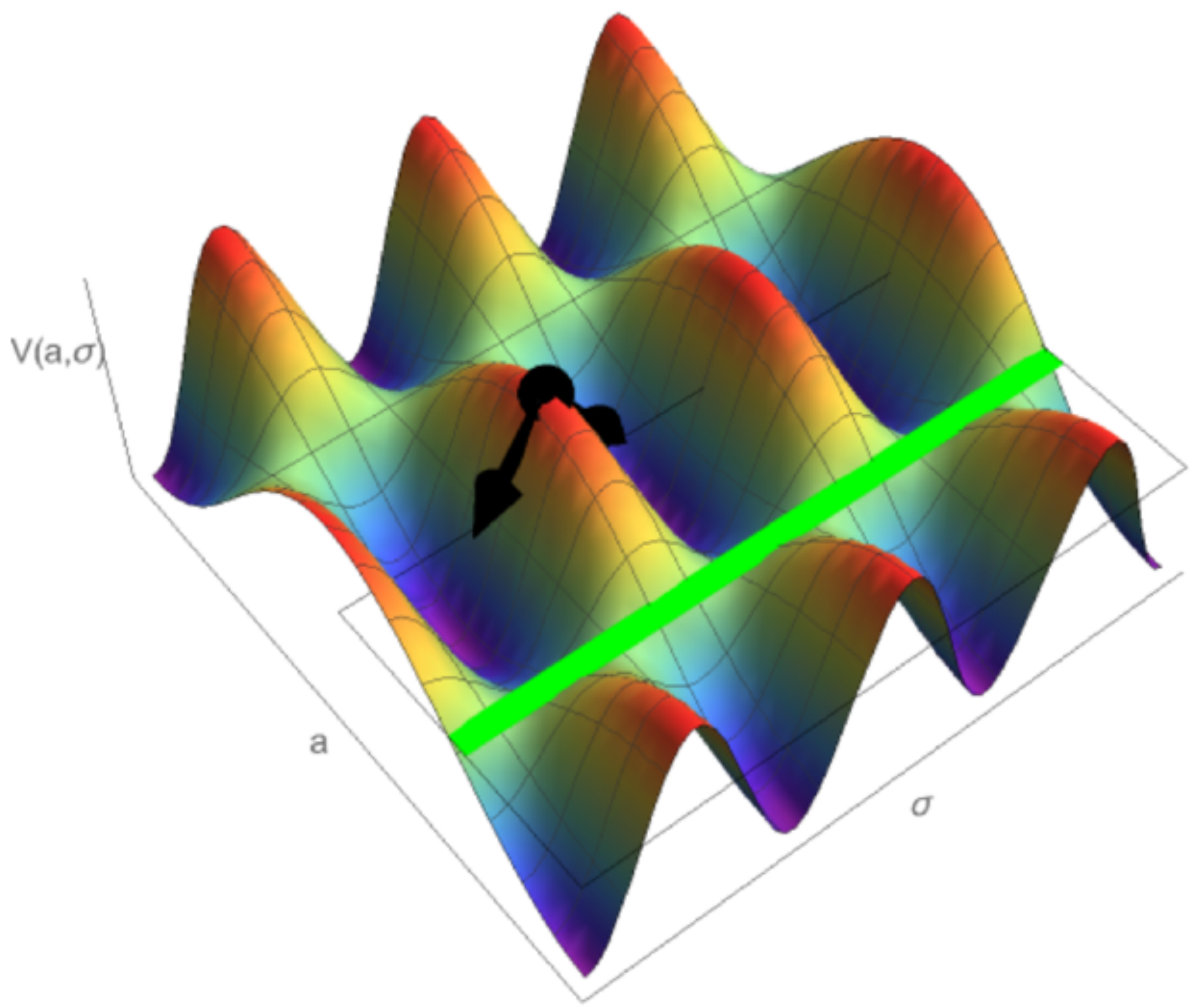
The same behavior at large N,
Witten, 1980



3-form fields in axion-YM theory

- Take-home message: the simple IR $\mathscr{L} = \frac{v^2}{2}(\partial_\mu a)^2 + \Lambda^4 \cos(M\, a)$ breaks down under large-field excursion $a \to a + 2\pi$
- Application: natural models of inflation:

$$\Lambda^4 \min_k \left(T_R\, a + 2\pi k\right)^2 \to V(a, \sigma) = \Lambda^4 \left[2 - \cos\left(\frac{a}{f} + \frac{\sigma}{\Lambda}\right) - \cos\left(\frac{a}{f} - \frac{\sigma}{\Lambda}\right)\right]$$



MA, Baker 2020

3-form fields in axion-YM theory

- Deep in the IR: $G = \mathbb{Z}^{(0)}_{T_R=4} \times \left(\mathbb{Z}^{(1)}_{N=6} \tilde{\times} \mathbb{Z}^{(3)}_{T_R=4} \right)$
- Notice the higher-group structure: we cannot gauge the daughter without the parent

✓ For $\mathbb{Z}^{(1)}_2$, the higher group trivializes

3-form fields in axion-YM theory

- Consider $SU(6)/\mathbb{Z}_2$ gauge group by gauging $\mathbb{Z}_2^{(1)}$ center symmetry:

$$Z = \sum_{b_2^c} \int [Dc_3][Da] e^{-S}, \quad S = \oint_{\mathbb{M}^4} \frac{v^2}{2} |da|^2 + \frac{(T_R = 4)}{2\pi} a \wedge \left(dc_3 - \frac{N}{4\pi} b_2^c \wedge b_2^c \right) + \frac{|dc_3|^2}{2\Lambda^4} \Bigg|_{c_3 \longrightarrow c_3 - \frac{N}{4\pi(2)^2} b_1^c \wedge db_1^c}$$

- Recall $U^{(0)}(\mathbb{M}^3) = e^{i\frac{2\pi}{T_R}\oint_{\mathbb{M}^3} v^2 \star da - \frac{T_R}{2\pi} c_3}$ of $\mathbb{Z}_{T_R}^{(0)\chi}$
- Replace $c_3 \longrightarrow c_3 - \dfrac{N}{4\pi(2)^2} b_1^c \wedge db_1^c, \quad 2b_2^c = db_1^c$
- $\tilde{U}^{(0)}(\mathbb{M}^3) = \displaystyle\sum_{(b_1^c, b_2^c)} e^{i\frac{2\pi}{T_R}\oint_{\mathbb{M}^3} v^2 \star da - \frac{T_R}{2\pi}\left(c_3 - \frac{N}{4\pi(2)^2} b_1^c \wedge db_1^c\right)}, \quad \tilde{U}^{(0)}(\mathbb{M}^3)\tilde{U}^{(0)}(\bar{\mathbb{M}}^3) \neq 1$
- This is non-invertible symmetry (NIS) with FQH states on DW.

Cordova, Ohmori 2023
M.A., Chan 2024
Cordova, Hong, Wang 2024

Summary/Future

★Generalized symmetries are a powerful tool to build EFTs

★Ex: Axion+YM system with a fully consistent picture in the IR.

★New phenomena on axion DW (FQH states)

★Decay of axion DW in external B (work in progress)

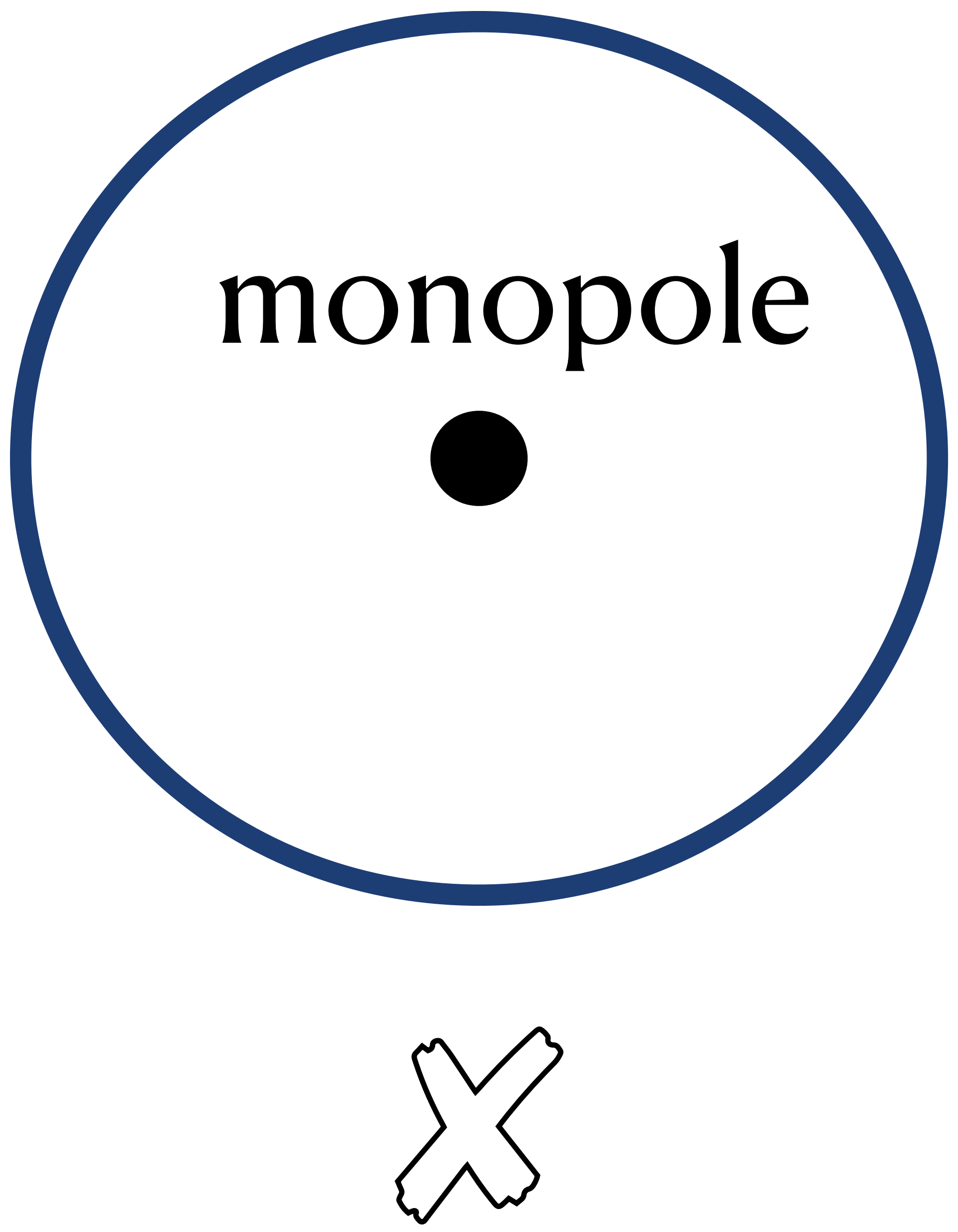
★Two 3-forms fields and CC (a la 't Hooft)? Recall Brown-Teitelboim mechanism

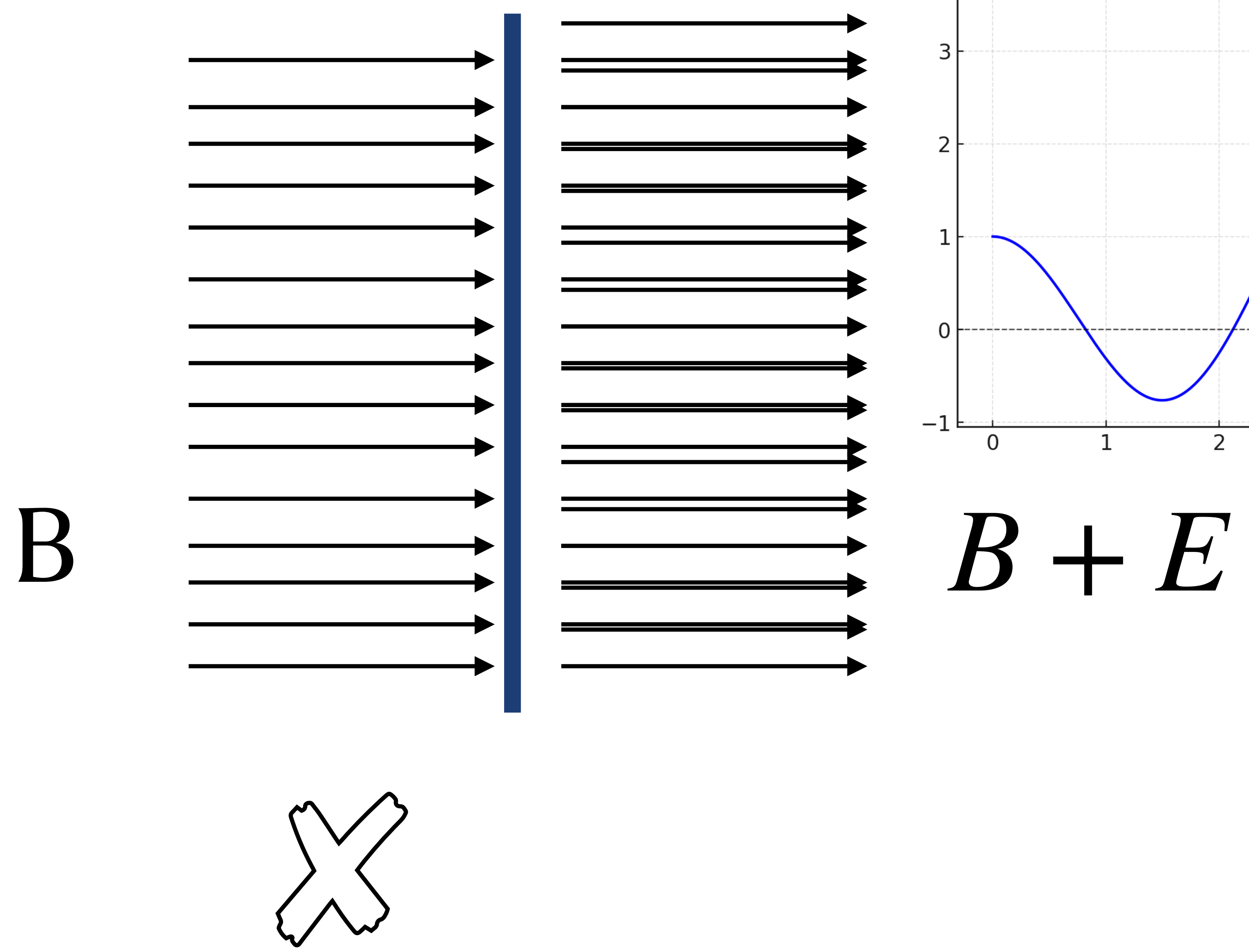
★What happens when we turn on gravity in the presence of 3-forms?

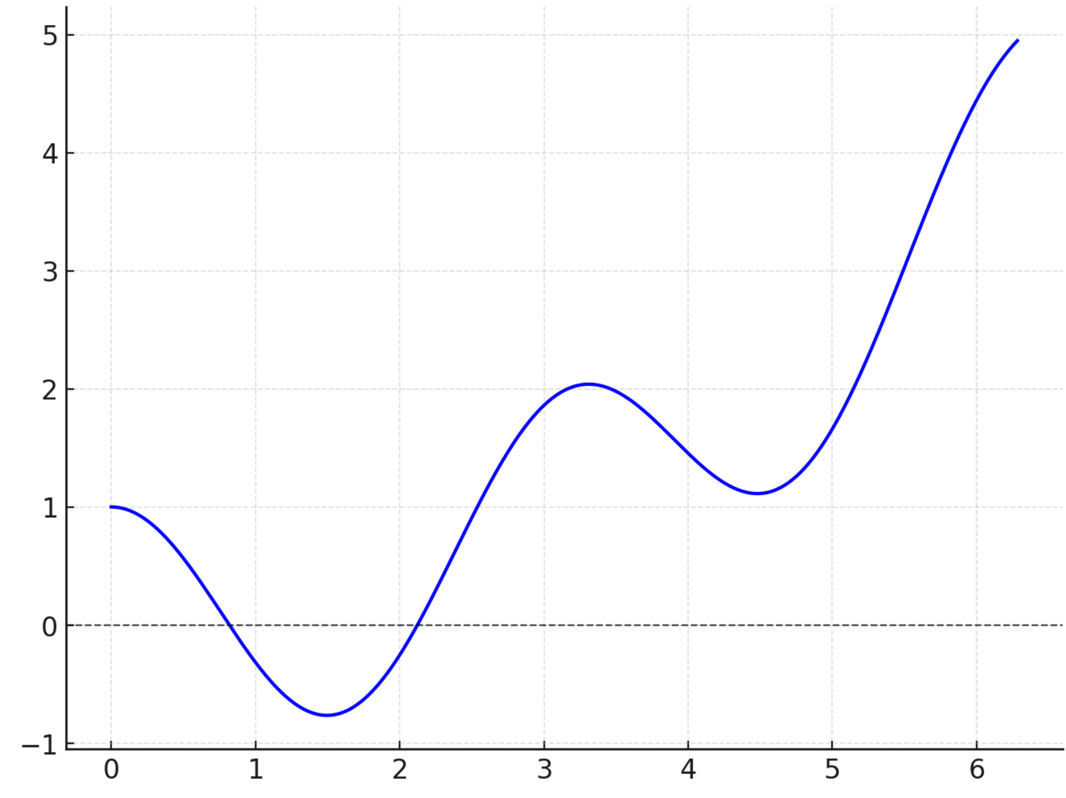
3-form fields in axion-YM theory

- Consequence of **NIS** (can be proved using Hilbert-space construction M.A., Poppitz, 2023; M.A, Chan 2023): **instability in external magnetic field** (work in progress)

Classically stable, Kogan 1993
QM unstable (in progress)







Classically and QM unstable
(in progress)