Resolving Continuous Symmetries in Holography

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Based on arXiv:2510.19812 and 2602.22377 with I. Bah, F. Bonetti, and M. Chitoto

Developments on Generalized Symmetries of Quantum Systems
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MAX PLANCK INSTITUTE
FOR MATHEMATICS IN THE SCIENCES

Outline

Generalized symmetries

- Global symmetries are useful for characterizing QFTs
- The notion has been generalized to
  - Higher-form symmetries
  - Higher-group symmetries
  - Non-invertible/categorical symmetries
- Modern formulation in terms of topological operators/defects

  [Gaiotto-Kapustin-Seiberg-Willett '14]

Symmetries in holography

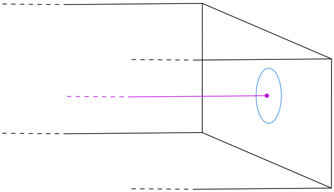
- Symmetry structure of a QFT is encoded in its topological sector
  - Symmetry Topological Field Theory (SymTFT) [Ji-Wen '19, Gaiotto-Kulp '20, Apruzzi-Bonetti-García Etxebarria-Hosseini-Schäfer-Nameki '21, Freed-Moore-Teleman '22]
  - A d-dim QFT can be resolved into a $(d+1)$-dim bulk TQFT sandwiched by a gapped boundary and a gapless boundary
  - Isolate symmetry data from complicated dynamics
- AdS/CFT correspondence is a playground to examine these ideas
  - QFT on the conformal boundary dual to some string/M-theory background
  - Boundary global symmetry dual to bulk gauge symmetry

Symmetries in holography (cont'd)

- Tremendous success in modeling the holographic duals of symmetry generators and charged operators as D-branes

  [Apruzzi-Bah-Bonetti-Schäfer-Nameki '22, García Etxebarria '22, Heckman-Hübner-Torres-Zhang '22, ...]

  - Symmetry generators lie on the boundary
  - Charged operators end on the boundary



- Works well for finite symmetries, both invertible and non-invertible
- Captures field theory results using brane dynamics [Bah-**EL**-Waddleton '23]

From finite to continuous

- The categorical language inherent in the SymTFT construction fails when the symmetry is not finite
- Several proposals to extend to continuous symmetries [Brennan-Sun '24, Antinucci-Benini '24, Bonetti-Del Zotto-Minasian '24 '25, Antinucci-Benini-Rizi '24, Arbalestrier-Argurio-Tizzano '25, Jia-Luo-Tian-Wang-Zhang '25, Apruzzi-Dondi-García Etxebarria-Lam-Schäfer-Nameki '25]
- A precise mathematical description is still lacking
- More subtle regularization issues, especially if gravity is turned on [Bah-Jefferson-Roumpedakis-Waddleton '24, Calvo-Mignosa-Rodriguez-Gomez '25]
- String theory and holography may give us hints on how to tackle these problems!

Outline

Low-energy gauged supergravity

- Study 0-form continuous G symmetries in a d-dim QFT
  - G is a compact connected, and possibly non-Abelian, Lie group
- Some $\mathrm{AdS}_{d+1} \times X$ solution in Type IIB string theory or M-theory

$$S_{\text{eff}} \supset \int_{\mathrm{AdS}_{d+1}} \left[-\frac{1}{2} \tau_{IJ} F^I \wedge * F^J + \mathrm{CS}_{d+1}(A) \right]$$

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- $A^I = (A^i, A^\alpha)$ is a collective notation
  - A^i from isometries of X, with $i = 1, \ldots, \dim G_{\text{iso}}$
  - A^α from homological cycles of X, with $\alpha = 1, \ldots, n_{\text{Betti}}$
    - E.g. reduce C_4 or C_3 in IIB/M-theory over 3-cycles/2-cycles
- For now, focus on low-energy theory; top-down perspective later

Wilson lines

- Key players: charged operators and symmetry generators
- The former are bulk Wilson lines ending on the boundary
- Formal expression ($\mathbf{R}$ is an irrep of $G = G_{\text{iso}} \times \mathrm{U}(1)^{n_{\text{Betti}}}$):

$$W_{\mathbf{R}}(M^1) = \mathrm{Tr}_{\mathbf{R}} \mathrm{Pexp}\left(i \int_{M^1} A \right)$$

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- Use the following quantum mechanical model (see, e.g. [Tong-Wong '14])

$$S_{\text{WL}} = \int_{M^1} \langle q, \chi^{-1} d_A \chi \rangle$$

  - q constant parameter valued in $\mathfrak{g}^* \overset{\text{metric}}{\cong} \mathfrak{g}$
  - χ dynamical scalar field valued in G
  - $\chi^{-1} d_A \chi = \chi^{-1}(d + A)\chi$ valued in $\mathfrak{g}$
  - $\langle -, - \rangle$ pairing between $\mathfrak{g}^*$ and $\mathfrak{g}$

Wilson lines (cont'd)

$$S_{\mathrm{WL}} = \int_{M^1} \langle q, \chi^{-1} d_A \chi \rangle$$

- Quantization condition: q is a (dominant) analytically integral weight

  (cf. [Bah-Bonetti-Chitoto-**EL** '26])

- Theorem of the highest weight tells us that the parameter q is in 1-to-1 correspondence with the irrep **R** labeling the Wilson line
  - Determining **R** amounts to measuring q

- Advantage: q arises geometrically from the internal space X

Example: $G = \mathrm{SO}(n+1)$ for $n \geq 2$

$$S_{\mathrm{WL}} = \frac{1}{2}\int_{M^1} q_{ab}(\chi^{-1} d_A \chi)^{ab}\,, \quad q_{ab} = \begin{pmatrix} 0 & q_1 & & & & & \\ -q_1 & 0 & & & & & \\ & & 0 & q_2 & & & \\ & & -q_2 & 0 & & & \\ & & & & 0 & q_3 & \\ & & & & -q_3 & 0 & \\ & & & & & & \ldots \end{pmatrix}$$

- $a, b = 1, \ldots, n+1$ and $q_1, \ldots, q_k \in \mathbb{Z}$ with $k = \lfloor \frac{n+1}{2} \rfloor$
- From isometry of $S^n \subset X$, parametrized by y^a with $y^a y_a = 1$

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- $a, b = 1, \ldots, n+1$ and $q_1, \ldots, q_k \in \mathbb{Z}$ with $k = \lfloor \frac{n+1}{2} \rfloor$
- From isometry of $S^n \subset X$, parametrized by y^a with $y^a y_a = 1$
- Wrap a brane on $M^1 \times S^{n-2}$, where $S^{n-2} \subset S^n$ by setting, say, $y^{c_1} = y^{c_2} = 0$ for some c_1 and c_2

$$S_{\mathrm{brane}} \supset 2\pi \int_{N^2 \times S^{n-2}} \mathcal{P}[G_n]\,, \quad G_n \supset \# \epsilon_{ab_1 \ldots b_{n-2} c_1 c_2} y^a Dy^{b_1} \ldots Dy^{b_{n-2}} F^{c_1 c_2}$$

- Pullback $\mathcal{P}[G_n]$ induces dependence on real worldvolume scalars ${\chi^a}_b$
  - Describe rotation of brane while preserving a maximal $S^{n-2} \subset S^n$

Example: $G = \mathrm{SO}(n+1)$ for $n \geq 2$ (cont'd)

- With some algebra, one finds (with $M^1 = \partial N^2$)

$$S_{\text{brane}} = \frac{1}{2} Nw \int_{M^1} n_{ab} (\chi^{-1} d_A \chi)^{ab}$$

- Identify $q_{ab} = Nwn_{ab}$ to obtain $\mathrm{SO}(n+1)$ Wilson lines
  - N some integer flux number
  - w winding number around S^{n-2}
  - n_{ab} encodes choice of $y^a = y^b = 0$ defining maximal $S^{n-2} \subset S^n$
- In Type IIB on $\mathrm{AdS}_5 \times S^5$, these correspond to giant gravitons [Myers '99, McGreevy-Susskind-Toumbas '00]

Other examples

- $G = \mathrm{U}(1)$
  - From isometry of $S^1 \subset X$, or some cycle in $H_*(X;\mathbb{R})$
  - worldvolume scalar χ decouples
  - $S_{\mathrm{WL}} = q \int_{M^1} A$ with $q \in \mathbb{Z}$
  - $q \propto$ wrapping number of brane around the cycle

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- $G = \mathrm{Spin}(n+1)$
  - Double cover of $\mathrm{SO}(n+1)$
  - Follows previous discussion except that $q_1, \ldots, q_k \in \mathbb{Z}$ or $\mathbb{Z} + \frac{1}{2}$
  - Latter corresponds to spin representations

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- $G = \mathrm{SU}(n+1)$
  - From isometry of $\mathbb{CP}^n \subset X$
  - Brane wraps some $\mathbb{CP}^{n-1} \subset \mathbb{CP}^n$
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- Compatible with exceptional isomorphisms
  - E.g. $\mathrm{SO}(3) \cong \mathrm{Spin}(3)/\mathbb{Z}_2 \cong \mathrm{SU}(2)/\mathbb{Z}_2$
  - $\mathrm{pt}_+ \sqcup \mathrm{pt}_- \cong S^0 \subset S^2$ vs. $\mathrm{pt} \cong \mathbb{CP}^0 \subset \mathbb{CP}^1$

Outline

Gauss' law constraints

- Need symmetry generators to measure the charge q of Wilson lines
- Recall $S_{\text{eff}} \supset \int_{\text{AdS}_{d+1}} \left[-\frac{1}{2}\tau_{IJ} F^I \wedge *F^J + \text{CS}_{d+1}(A) \right]$
- Gauss' law constraint:

$$\mathbf{G}_I = \tau_{IJ} D * F^J + \mathbf{G}_I^{\text{CS}} \stackrel{!}{=} 0$$

  - $D * F^I = d * F^I - f_{JK}{}^I A^J \wedge *F^K$
  - Generates bulk gauge transformations

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  - Generates bulk gauge transformations

- Dirichlet b.c. $A^I = 0$ realizes global symmetry in the dual CFT

$$\mathbf{G}_I|_{\text{boundary}} = \tau_{IJ} d * F^J = 0$$

- Naïve symmetry generator: $U_\theta(M^{d-1}) = \exp\left(i\theta^I \int_{M^{d-1}} \tau_{IJ} * F^J\right)$
  - θ^I valued in $\mathfrak{g}$, and $\tau_{IJ} * F^J$ valued in $\mathfrak{g}^*$

Need for regularization

- Ward identity in QFT takes the form

$$\langle \partial_\mu j^\mu(x)\,\phi(x_1)\ldots\rangle = \sum_i q_i \delta(x-x_i)\langle\phi(x_1)\ldots\rangle$$

- Inserting the defect $U = \exp\left(i\theta \int_{M^{d-1}} * j\right)$ in correlator yields powers of delta functions which are not well-behaved

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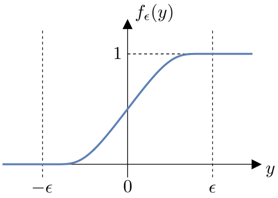
- Regularize by thickening [Bah-Jefferson-Roumpedakis-Waddleton '24]

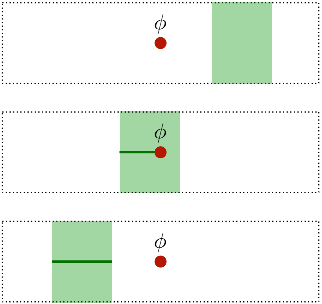
$$U_{\text{reg}} = \exp\left[\int d^{d-1}x\,dy\left(i\theta f_\epsilon'(y)\,j_y + \text{h.o.t.}\right)\right], \quad \lim_{\epsilon\to 0} f_\epsilon(y) = \Theta(y)$$



Charge measurement

- Observe $a_1 = \theta f'_\epsilon(y)dy$ can be viewed as a flat connection
- More generally, $U_{\text{reg}} = \exp\left(i \int_{[-\epsilon,\epsilon]\times M^{d-1}} a_1 \wedge *j\right)$ with $da_1 = 0$
- Dragging defect U_{reg} past local operator ϕ produces a phase

$$\phi'(x,y) = \exp\left(i \int_{-\epsilon}^{y} a_1\right)\phi(x,y)$$



Hanging branes in holography

- What objects in the supergravity bulk can mimic this behavior?
- Consider hanging brane with effective action

$$S_{\text{hanging}} = \int_{\gamma^1 \times M^{d-1}} a_1 \wedge n^I \tau_{IJ} * F^J$$

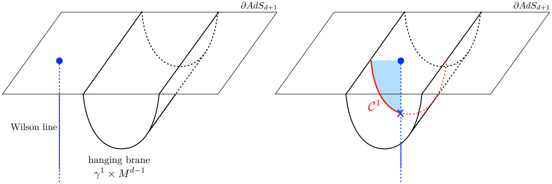
  - γ^1 an arc with endpoints on conformal boundary
  - a_1 flat worldvolume U(1) gauge field with Dirichlet b.c.
  - n^I encodes wrapping number of internal submanifold in X

- Identify $\theta^I = \alpha n^I$
  - $\alpha = \int_{\gamma^1} a_1$ encodes "magnitude," n^I encodes "direction" of $\theta \in \mathfrak{g}$

- Operator labeled by $g = \exp(\theta) \in G$ where $\exp : \mathfrak{g} \to G$

Hanany-Witten transition

- Hanging geometry as brane and anti-brane recombining in bulk
  [Calvo-Mignosa-Rodriguez-Gomez '25]
- Dynamically stable (cf. [Bah-Bonetti-Chitoto-**EL** '26])
- (Partial) charge measurement $\leftrightarrow$ Hanany-Witten transition



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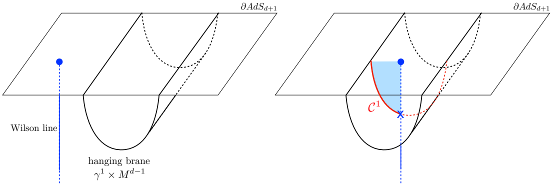


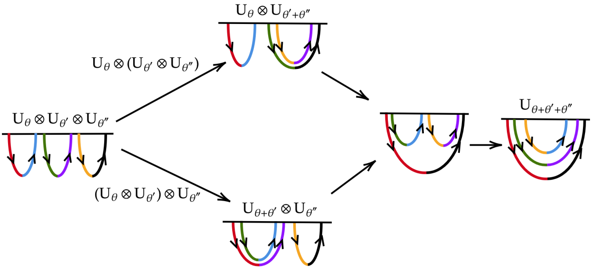
- Zero-width limit?
  - Tachyon condensation $\Rightarrow$ non-BPS brane localized on boundary [Bergman-Garcia-Valdecasas-Mignosa-Rodriguez-Gomez '24, Calvo-Mignosa-Rodriguez-Gomez '25]
  - Incompatible with gravity turned on; must have finite width and tension [Bah-Jefferson-Roumpedakis-Waddleton '24]

Hanging brane fusion

- BCH formula gives non-commutative fusion

$$U_\theta \otimes U_{\theta'} = \exp\left(i\left(\theta + \theta' + \tfrac{1}{2}[\theta,\theta'] + \dots\right)^I \int_{M^{d-1}} \tau_{IJ} * F^J \right)$$

- If $[\theta, \theta'] = 0$, brane recombination:



- If $[\theta, \theta'] \neq 0$, non-trivial rearrangement of internal geometries

Outline

$\mathrm{AdS}_5 \times S^5$ in Type IIB

- SO(6) isometry $\leftrightarrow$ R-symmetry in 4d $\mathcal{N} = 4$ SU(N) YM theory

$$S_{\mathrm{eff}} = \int_{\mathrm{AdS}_5} \left[-\frac{1}{4} \tau F_{ab} \wedge * F^{ab} - k \mathrm{CS}_5(A) \right]$$

  - $\tau = \tau_{\mathrm{flux}} + \tau_{\mathrm{EH}} = \frac{N^2}{8\pi^2 L}$, $k = N^2 - 1$

- Wilson line: D3-brane wrapping some maximal $S^3 \subset S^5$
  - Useful to treat as Hopf fibration $S^1 \hookrightarrow S^3 \to S^2 \subset \mathbb{CP}^2 \cong S^5 / S^1$

$\text{AdS}_5 \times S^5$ in Type IIB (cont'd)

- Symmetry generator: bound state of D5-brane and KK monopole
- Responsible respectively for τ_{flux} and τ_{EH}
- Wrap internal 2-cycle Σ^2 that intersects non-trivially with $S^2 \subset \mathbb{CP}^2$

$$S_{\text{D5}} \supset 2\pi \int_{\gamma^1 \times M^3 \times \Sigma^2} a_1 \wedge \mathcal{P}[G_5]$$

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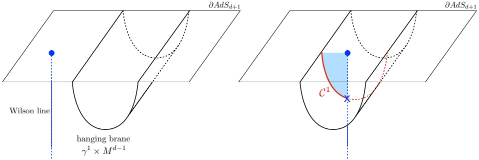
$$S_{\text{D5}} \supset 2\pi \int_{\gamma^1 \times M^3 \times \Sigma^2} a_1 \wedge \mathcal{P}[G_5] = \frac{\alpha_{\text{D5}}\, m^{ab}}{2 g_{\text{flux}}^2} \int_{M^3} * F_{ab}$$

- Worldvolume gauge field a_1 identified with
  - Chan-Paton gauge field on D5-brane
  - Composite gauge field constructed out of two compact scalars $\varphi, \widetilde{\varphi}$ on KK monopole, i.e. $a_1 = \frac{1}{2}(\widetilde{\varphi} d\varphi - \varphi d\widetilde{\varphi})$

$AdS_5 \times S^5$ in Type IIB (cont'd)



- Passing D3 through D5 leaves behind F1-strings with phase

$$\exp\left(2\pi i n_{\mathrm{F1}} \int_{\gamma^1} a_1\right) = \exp\left(i \tfrac{1}{2} q_{ab} \theta^{ab}\right)$$

  - n_{F1} intersection number of D3 and D5 within $\mathbb{CP}^2$
- Irrep **R** of Wilson line can be determined by measuring q_{ab} of D3 with multiple different D5s
- Can get spin rep. in SU(4) by setting q_{ab} half-integral

$\mathrm{AdS}_5 \times T^{1,1}$ in Type IIB

- Dual to 4d $\mathcal{N} = 1$ $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory [Klebanov-Witten '98]
- Hopf fibration: $S^1 \hookrightarrow T^{1,1} \to S_1^2 \times S_2^2$
  - Topologically equivalent to $S^2 \times S^3$
- $(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2)/\mathbb{Z}_2 \times \mathrm{U}(1)_R$ isometry + $\mathrm{U}(1)_B$ baryonic symmetry
  - $\mathrm{SU}(2)_1$ from $\mathbb{CP}_1^1 \cong S_1^2$, and $\mathrm{SU}(2)_2$ from $\mathbb{CP}_2^1 \cong S_2^2$
  - $\mathrm{U}(1)_R$ from Hopf fiber S^1
  - $\mathrm{U}(1)_B$ from reducing RR 4-form C_4 over homological S^3

$\mathrm{AdS}_5 \times T^{1,1}$ in Type IIB (cont'd)

- Construct 3-spheres

$$S^1 \hookrightarrow S^3_1 \to \mathbb{CP}^1_1, \qquad S^1 \hookrightarrow S^3_2 \to \mathbb{CP}^1_2$$

- $\mathrm{SU}(2)_1$ Wilson line: D3-brane wrapping $\mathbb{CP}^0_1 \times S^3_2$
- $\mathrm{SU}(2)_2$ Wilson line: D3-brane wrapping $\mathbb{CP}^0_2 \times S^3_1$

$\mathrm{AdS}_5 \times T^{1,1}$ in Type IIB (cont'd)

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- $\mathrm{U}(1)_R$ Wilson line: D3-brane wrapping $S^1 \hookrightarrow \Sigma^3 \to \mathrm{PD}(J)$
  - J is the Kähler form of the base $\mathbb{CP}^1_1 \times \mathbb{CP}^1_2$
- $\mathrm{U}(1)_B$ Wilson line: D3-brane wrapping homological S^3

$\mathrm{AdS}_5 \times T^{1,1}$ in Type IIB (cont'd)

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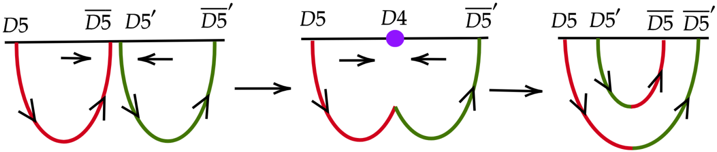
    - J is the Kähler form of the base $\mathbb{CP}^1_1 \times \mathbb{CP}^1_2$
- $\mathrm{U}(1)_B$ Wilson line: D3-brane wrapping homological S^3
- Symmetry generators: D5-KK bound states wrapping suitable internal submanifolds in $\mathbb{CP}^1_1 \times \mathbb{CP}^1_2$

Fusion dynamics



- Each hanging D5 can be viewed as a D5-$\overline{\text{D5}}$ pair
- Tachyon condensation on inner $\overline{\text{D5}}$-D5' gives non-BPS D4
- D4 reopens into D5'-$\overline{\text{D5}}$ with correct orientation
- Viewed as a stack of two hanging D5s
- (Non-Abelian) relative modes decouple [Bah-**EL**-Waddleton '23]

$\mathrm{AdS}_4 \times S^7$ in M-theory

- SO(8) isometry $\leftrightarrow$ R-symmetry in 3d $\mathcal{N} = 8$ SCFT
- Wilson line: M5-brane wrapping maximal $S^5 \subset S^7$
  - Useful to treat as Hopf fibration $S^1 \hookrightarrow S^5 \to \mathbb{CP}^2 \subset \mathbb{CP}^3 \cong S^7/S^1$
- Symmetry generator: bound state of M5-brane and KK monopole

$\mathrm{AdS}_4 \times S^7$ in M-theory (cont'd)

- M5 has a chiral 2-form b_2 but not a 1-form like in D5, how do we get an effective worldvolume 1-form a_1 on the defect?
- Take $b_2 \approx a_1 \wedge \mathrm{vol}(S^1)$

$$S_{\mathrm{M5}} \supset -2\pi \int_{\gamma^1 \times M^2 \times S^3} b_2 \wedge \mathcal{P}[G_4] = -2\pi \int_{\gamma^1 \times M^2 \times \mathbb{CP}^1} a_1 \wedge \mathcal{P}[G_4]$$

  - Internal S^3 constructed as $S^1 \hookrightarrow S^3 \to \mathbb{CP}^1 \subset \mathbb{CP}^3$
- IIA description: D4-brane wrapping $\mathbb{CP}^1$; Chan-Paton field a_1

$\mathrm{AdS}_4 \times S^7$ in M-theory (cont'd)

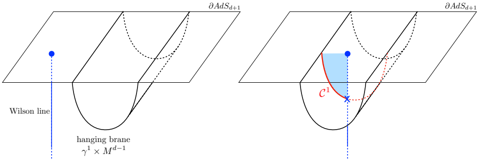
- M5 has a chiral 2-form b_2 but not a 1-form like in D5, how do we get an effective worldvolume 1-form a_1 on the defect?
- Take $b_2 \approx a_1 \wedge \mathrm{vol}(S^1)$

$$S_{\mathrm{M5}} \supset -2\pi \int_{\gamma^1 \times M^2 \times S^3} b_2 \wedge \mathcal{P}[G_4] = -2\pi \int_{\gamma^1 \times M^2 \times \mathbb{CP}^1} a_1 \wedge \mathcal{P}[G_4]$$

  - Internal S^3 constructed as $S^1 \hookrightarrow S^3 \to \mathbb{CP}^1 \subset \mathbb{CP}^3$
- IIA description: D4-brane wrapping $\mathbb{CP}^1$; Chan-Paton field a_1
- KK monopole in M-theory has a 7d worldvolume, but it also has a 1-form worldvolume field
  - Compensate by wrapping an internal 4-manifold $\mathbb{CP}^2$ such that

$$\mathbb{CP}^1_{\mathrm{M5}} \subset \mathbb{CP}^2_{\mathrm{KK}} \subset \mathbb{CP}^3$$

$\mathrm{AdS}_4 \times S^7$ in M-theory (cont'd)



- Passing $\mathrm{M5_{WL}}$ through $\mathrm{M5_{sym}}$ leaves behind M2-branes with phase

$$\exp\left(2\pi i n_{\mathrm{M2}} \int_{\gamma^1 \times S^1} b_2\right) = \exp\left(2\pi i n_{\mathrm{F1}} \int_{\gamma^1} a_1\right)$$

  - n_{M2} intersection number of $\mathrm{M5_{WL}}$ and $\mathrm{M5_{sym}}$ within $\mathbb{CP}^3$

- IIA description: D4-D4 Hanany-Witten transition producing F1s

Generalizations

- $\mathrm{AdS}_5 \times \mathrm{SE}_5$ in Type IIB
  - S^5 and $T^{1,1}$ are regular Sasaki-Einstein 5-manifolds
  - Generically, $S^1 \hookrightarrow \mathrm{SE}_5 \to B^4$ with Kähler-Einstein base B^4
    - E.g. $B^4 = dP_k$ del Pezzo surfaces, $k = 3, \ldots, 8$
  - D3 Wilson lines & D5-KK symmetry generators

- $\mathrm{AdS}_4 \times \mathrm{SE}_7$ in M-theory
  - S^7 is a regular Sasaki-Einstein 7-manifold
  - Generically, $S^1 \hookrightarrow \mathrm{SE}_7 \to B^6$ with Kähler-Einstein base B^6
    - E.g. $B^6 = \mathbb{CP}^2 \times \mathbb{CP}^1$ or $\mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1$
  - M5 Wilson lines & M5-KK symmetry generators

Summary & open questions

- Defining continuous symmetry operators in QFTs is surprisingly subtle
- Regularization should be taken into account
- String theory and holography provide natural candidates
- Clear geometric interpretation of charge measurement in terms of Hanany-Witten transitions in the bulk
- Non-Sasaki-Einstein examples?
- Ingredients to construct SymTFT for continuous symmetries?
- Fate of global symmetries in quantum gravity?