Non-BPS branes as holographic symmetry operators

Diego Rodriguez-Gomez
University of Oviedo

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(w/ Oren Bergman, Hugo Calvo, Eduardo Garcia Valdecasas and Francesco Mignosa)

- Over the last decade there has been a revolution in our understanding of global symmetries in field theory and their fate upon the inclusion of dynamical gravity.

- It was key to realize that symmetries are associated to the existence of defect operators supported on submanifolds of spacetime on which they depend topologically

Gaiotto, Kapustin, Seiberg & Willett'14

- extension to higher-form symmetries (continuous or discrete)

- possible "exotic OPEs" for defects (non-invertible symmetries)

- It is natural to ask about the holographic avatar of these developments
    - Make more precise the holographic dictionary
    - Potential bridge to quantum gravity insights
    - Further understanding of String Theory
    - ...
- Along a different, yet related, line, it has proven useful to strip-off the notion of symmetry from particular realizations. This gives rise to the so-called symTFT paradigm. It is also natural to ask about its holographic avatar

Today: we'll try to (partially!) address these questions

- what is the holographic realization of (continuous) symmetry operators?
- can holography accommodate the symTFT?

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- Continuous symmetries in holography and non-BPS D-branes
- R-symmetry and anomalies
- non-BPS D-branes and brane/antibrane pairs
- SymTFT’s and holography?
- Conclusions

Continuous symmetries in holography and non-BPS D-branes

- It is very interesting to study holographic CFT's. A natural question is what is the bulk avatar of global symmetries

- Let us look to continuous symmetries

$$\text{Global symmetry in } QFT_d \leftrightarrow \text{Gauge field in } AdS_{d+1}$$

- Yet it is natural to look for the holographic avatar of symmetry in terms of topological defects

- First of all, let us consider 10d string theory: it has RR potentials. Associated to those there would higher form U(1) global symmetries…but the existence of dynamical branes explicitly breaks these symmetries.

- Consider a Dp brane: it sources a p+2-form RR field strength satisfying

$$\frac{1}{2\pi}\int_{M_{8-p}} \star F_{p+2} = 1\,.$$

- Alternatively, the Dp brane links with a 7-p dimensional object: it would be a "wrong dimension" brane. String theory contains something like this: non-BPS branes!!!

$$S_{\widetilde{D}p} = -\int_{\Sigma_{p+1}} d^{p+1}x\, e^{-\Phi}\, V(T)\sqrt{-\det(G_{\mu\nu} + \partial_\mu T \partial_\nu T)} + \int_{\Sigma_{p+1}} W(T)\, dT \wedge C_p\,.$$

Sen '04

Warning: not a low-energy effective theory, but rather a theory whose eom. reproduce the BCFT describing the unstable D-brane

- The relevant facts here are

  - $V(T), W(T) > 0 \qquad \& \qquad \lim_{T\to\infty} V(T) \sim \lim_{T\to\infty} W(T) \sim e^{-\frac{T}{\sqrt{2}}}\,.$

  - $V(0) = \tilde{T}_p = \sqrt{2}\, T_p\,.$

  - The Dp-1 brane is a tachyon kink in the non-BPS Dp, which requires

$$\int_{-\infty}^{\infty} dT\, V(T) = \int_{-\infty}^{\infty} dT\, W(T) = T_{p-1}\,.$$

- In the presence of RR-field strengths we should use

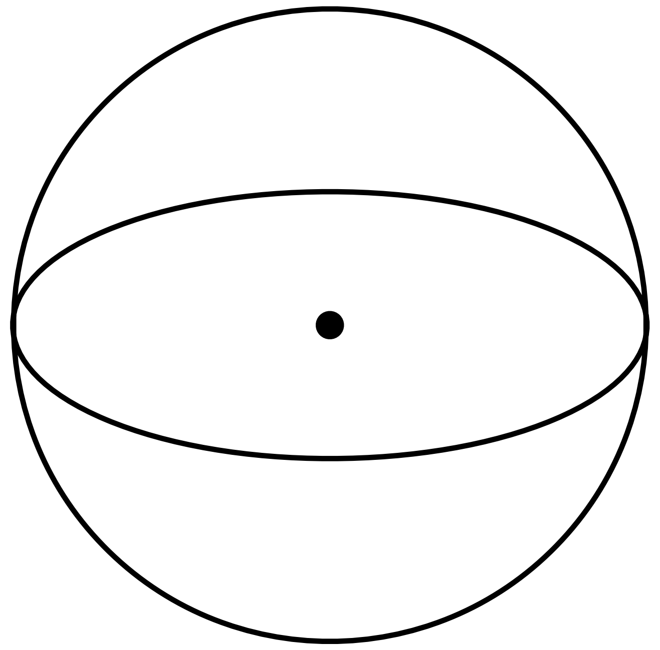
$$S_{WZ} = \int_{\Sigma_{p+1}} Z(T)\, F_{p+1}\,.$$

$dZ = W\, dT$: crucially, note that the above requirements leave an ambiguity (for a D-brane it would be a gauge transformation, but not here)

Similar to Cornalba, Costa & Penedones '04

- As a consequence, after the non-BPS brane decays to its vacuum it leaves behind a phase $e^{i\frac{\alpha}{2\pi}\int F_{p+1}}$.

- So if we link q Dp's with a non-BPS D7-p we find



$$= e^{iq\alpha} \quad .$$

- Thus the non-BPS brane measures the linked charge.

- However it is non-topological: D-branes are dynamical and the would-be symmetry is explicitly broken.

- Let us now go to AdS

$$ds^2 = \frac{dz^2 + d\vec{x}^2}{z^2} + ds_X^2 \,.$$

- Because of the z-factor, branes pushed to the boundary are infinitely heavy and become non-dynamical.

- Thus, a non-BPS D-brane at the boundary of AdS is topological!

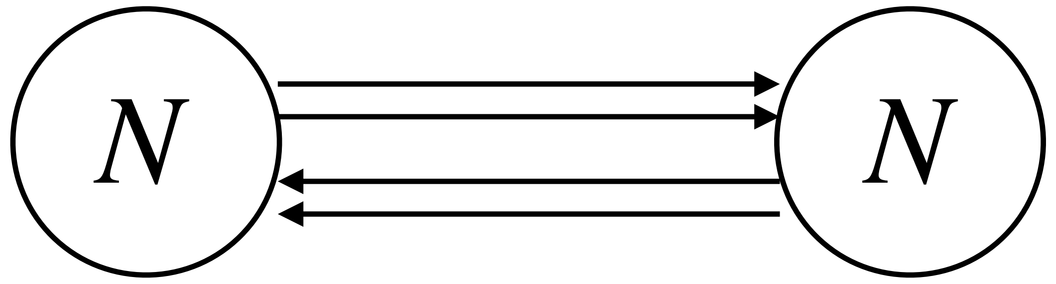
$\downarrow$

Natural candidates for the holographic realization of U(1) symmetry defects (note the emergence of the parameter labelling the operator!)

(see also Cvetic, Heckman, Hubner & Torres '23, in terms of fuxbranes)

(Waddleton'24)

- Let us see an explicit example: consider the Klebanov-Witten (conformal) field theory (with gauge group $SU(N) \times SU(N)$)



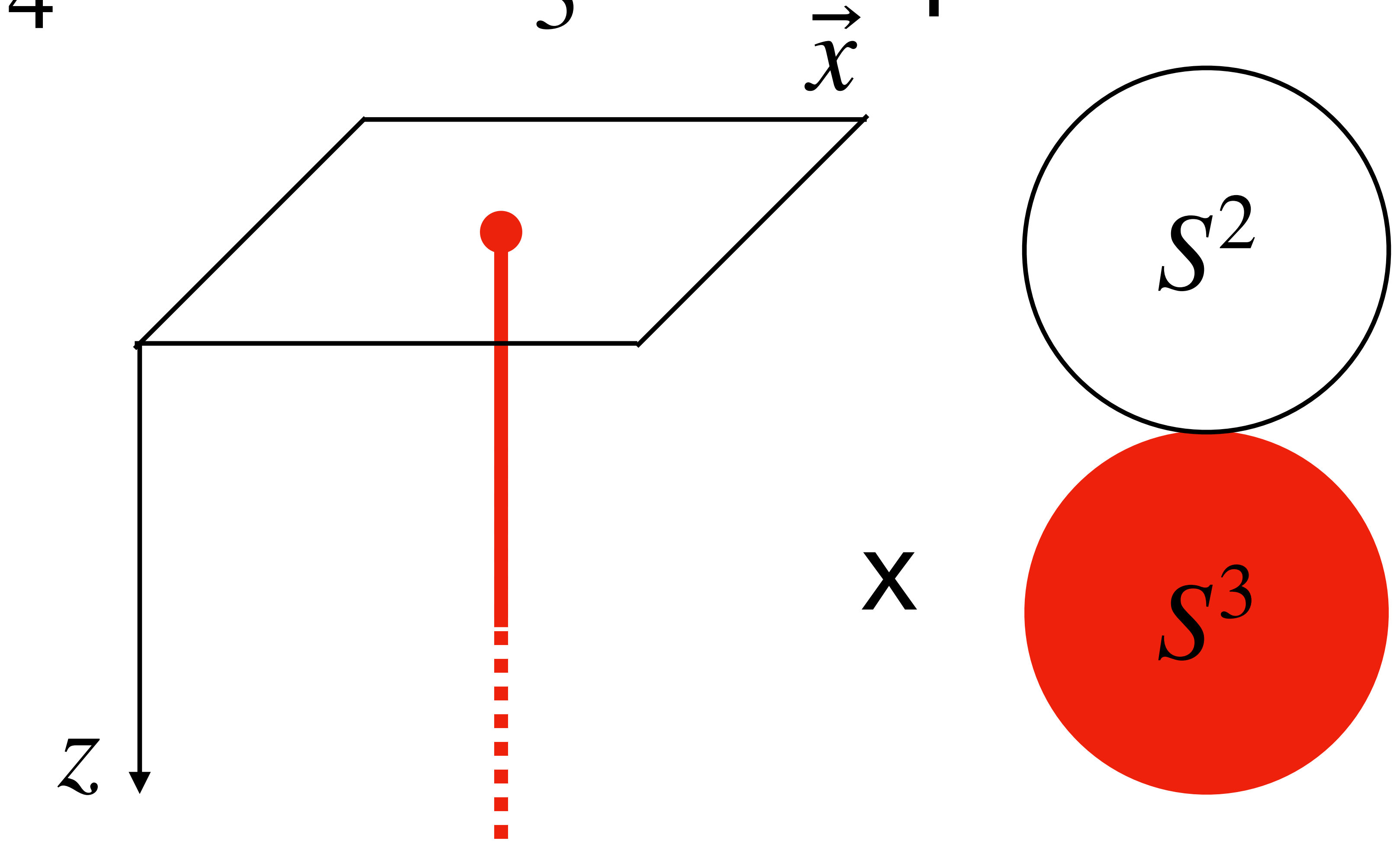
$$W = \epsilon^{ij}\epsilon^{mn} A_i B_m A_j B_n \,.$$

- There is a global U(1) baryonic symmetry $(A_i, B_m) \to (e^{i\theta}A_i, e^{-i\theta}B_m)$ under which determinant operators are charged (baryons).

- The holographic dual is $AdS_5 \times T^{1,1}$. For our purposes $T^{1,1} \sim S^2 \times S^3$.

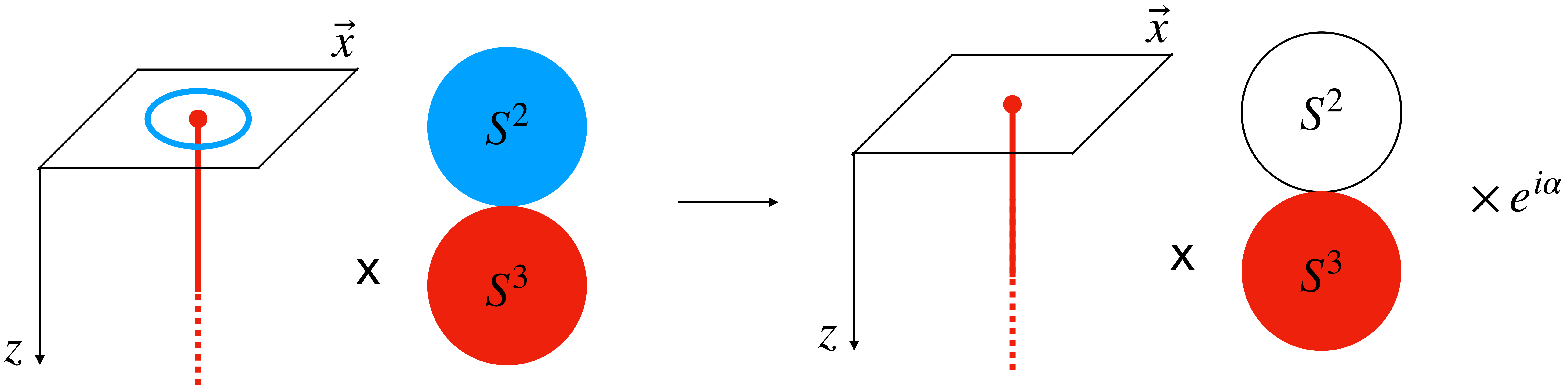
- The baryonic gauge field in AdS_5 comes from $C_4 = A \wedge \omega_3$. It couples to D3-branes wrapping the S^3: the baryons

Wilson line for baryon gauge field in AdS ←



Gubser & Klebanov '98

- Such D3 produces a 5-form field strength on the transverse space: consider a non-BPS D4 linking with the D5…



- …so we recover the expected action of the symmetry operator.

R-symmetries and anomalies

- Symmetries dual to isometries look more complicated: the gauge field is in the metric

$$ds_X^2 = g_{\phi\phi}(d\phi + \xi)^2 + ds_B^2 \to g_{\phi\phi}(d\phi + \xi + A)^2 + ds_B^2\,, \qquad (A, \phi) \to (A + d\lambda, \phi + \lambda)\,.$$

- Natural charged objects are KK-fluctuations, a.k.a. mesons in field theory. The corresponding particle is a gravitational wave.

$$S_{GW} \supset -\int_{AdS} A\,.$$

Lozano & Janssen '02

- What about the symmetry operators? It must feel the purely geometric flux…

- Natural candidate: non-BPS KK monopole, predicted by dualities

$$
\left|\begin{array}{c} \overline{\mathrm{D6}} \\ \\ \Downarrow \\ \\ \mathrm{D5} \end{array}\right| \xrightarrow{S} \left|\begin{array}{c} \overline{\mathrm{NS6}}_B \\ \\ \Downarrow \\ \\ \mathrm{NS5}_B \end{array}\right| \xrightarrow{T_\perp} \left|\begin{array}{c} \boxed{\overline{\mathrm{KK}}_A} \\ \\ \Downarrow \\ \\ \mathrm{KK}_A \end{array}\right| \xrightarrow{T_\parallel} \left|\begin{array}{c} \boxed{\overline{\mathrm{KK}}_B} \\ \\ \Downarrow \\ \\ \mathrm{KK}_B \end{array}\right|
$$

- Its wv. action can be constructed by consistency with the KK monopole wv action constructed by Eyras, Janssen & Lozano in 1998

$$
S_{\overline{KK}_B} \supset -\,T_{\overline{KK}_B}\int_{M_7} W\,dT \wedge \iota_k \mathcal{N} \sim \alpha\, T_{\overline{KK}_B}\int_{M_7} d(\iota_k \mathcal{N}) \sim \alpha \int_{AdS} \star_{AdS}\, dA
$$

$$
\iota_k \mathcal{N} \sim \tilde{B}_6 \quad \rightsquigarrow \quad d(\iota_k \mathcal{N}) \sim e^{-\tilde{\Phi}}\, \tilde{\star}\, \tilde{H}_3
$$

- From Cassani & Faedo'2010 we can read-off a consistent truncation including the baryon and the R-symmetry gauge fields. Restricting to the gauge sector

essentially the symTh

Apruzzi, Bedogna & Dondi'24

$$S_{\text{SymTh}} = \frac{1}{2\pi}\int_{M_5}\left(-\frac{1}{2g_R^2}|dA_R|^2 - \frac{1}{2g_B^2}|dA_B|^2 + \frac{\kappa_R}{12\pi}A_R\wedge dA_R\wedge dA_R + \frac{\kappa_B}{4\pi}A_R\wedge dA_B\wedge dA_B\right)$$

$$\frac{1}{g_R^2} = \frac{9N^2}{16\pi}\left(\frac{2}{k}\right)^{\frac{2}{3}}, \qquad \frac{1}{g_B^2} = \frac{3N^2}{2\pi}\left(\frac{2}{k}\right)^{\frac{2}{3}}. \qquad \kappa_R = \mathcal{A}_{RRR} = \frac{3}{2}N^2, \qquad \kappa_B = \mathcal{A}_{RBB} = -2N^2$$

correct anomaly coefficients!

- The operators are (we list only electric for simplicity)

$$W_B^{(1)}(M_1) = e^{in_B\oint_{M_1}A_B}, \qquad W_R^{(1)}(M_1') = e^{in_R\oint_{M_1'}A_R}$$

$$U_B^{(3)}(M_3) = e^{i\alpha_B\oint_{M_3}(g_B^{-2}\star dA_B - \frac{\kappa_B}{2\pi}A_R\wedge dA_B)},$$
$$U_R^{(3)}(M_3') = e^{i\alpha_R\oint_{M_3'}(g_R^{-2}\star dA_R - \frac{\kappa_R}{4\pi}A_R\wedge dA_R - \frac{\kappa_B}{4\pi}A_B\wedge dA_B)},$$

$$\alpha_B\in\frac{\mathbb{Z}}{\kappa_B}, \qquad \alpha_R\in\frac{\mathbb{Z}}{\gcd(\kappa_R,\kappa_B)}$$

enlarge with a TQFT

Garcia-Extebarria & Iqbal'22

$$U_B^{(3)}(M_3) = e^{i\alpha_B\oint_{M_3}g_B^{-2}\star dA_B}\int\mathcal{D}\rho_R\, e^{-i\alpha_B\frac{\kappa_B}{4\pi}\oint_{M_3}(A_R+d\rho_R)\wedge dA_B},$$
$$U_R^{(3)}(M_3') = e^{i\alpha_R\oint_{M_3'}g_R^{-2}\star dA_R}\int\mathcal{D}\rho_R'\mathcal{D}\rho_B e^{-i\alpha_R\oint_{M_3'}\left(\frac{\kappa_B}{4\pi}(A_B+d\rho_B)\wedge dA_B + \frac{\kappa_R}{4\pi}(A_R+d\rho_R')\wedge dA_R\right)},$$

non-invertible

- The RRR anomaly forbids N bc for R-symmetry

$$(A_B, A_R) = \begin{cases}(D,D) : KW \\ (N,D) : \text{gauging } \mathrm{U(1)_B} : \text{non} - \text{invertible symmetries}\end{cases}$$

- What about branes? Take for instance the non-BPS D4. Looking to its worldvolume including all the fluctuations of the background AND wv scalar fluctuations

$$S_{\mathrm{D4}}^{\mathrm{WZ}} = \alpha \int_{M_3} \left(g_B^{-2} \star dA_B - \frac{\kappa_B}{2\pi} (A_R + d\rho_R) \wedge dA_B \right)$$

- What about charged states? For mesons (GW) we saw how they are charged under the R-symmetry. Take a baryonic D3. Including again all gauge fluctuations

$$S_{\mathrm{D3}}^{\mathrm{WZ}} = N \int_{M_1} A_B - \frac{N}{2} \int_{M_1} A_R$$

baryons have both baryon and R-charge

- So far we have implicitly consider abelian symmetries. What about non-abelian? Isometry symmetries are a prototypical example…in particular the SO(6) R-symmetry of N=4 SYM. see also Bah, Bonetti, Chitoto & Leung'25, 26

- Charged states first: let's restrict to arguably the simplest class of operators… 1/2 BPS: [0,J,0] of SO(6) and dimension=J. Their SUGRA realization is

$J \ll \sqrt{N}$	Supergravity KK fluctuation
$J \sim \sqrt{N}$	Macroscopic string spinning
$J \sim N$	Giant/dual giant graviton
$J \sim N^2$	Black hole in AdS

$\longrightarrow$ the GW we discussed

$\longrightarrow$ giants=GW's puffed up due to Myers effect into a spinning D3

Janssen, Lozano & D.R-G'2005

but D3's link also with 4-branes…so both 4-branes and non-BPS KK monopoles link??

turns out that non-BPS KK can be regarded as a Myers expansion of 4-branes

GW $\leftrightarrow$4-branes

?

giants $\leftrightarrow$nonBPS KK

details in Mignosa & D.R-G'26

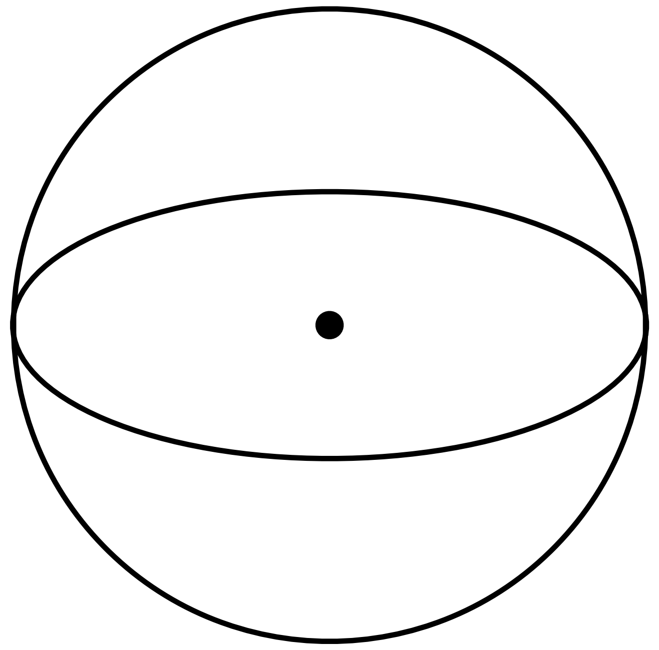
non-BPS D-branes, symmetry operators and brane/antibrane pairs

- Let us restrict to abelian 0-form global symmetries in QFT's in d-dimensions. For continuous symmetries, Noether's theorem ensures (barring anomalies) the existence of a conserved 1-form current

- The modern perspective is that associated to the symmetry there are symmetry operators defined on d-1 dimensional manifolds whose dependence on the manifold is only topological

- On a charge q operator, the symmetry operator acts as

$$\mathcal{O}_q(x)\, U_\alpha(M) = e^{iq\alpha}\, \mathcal{O}_q \qquad \leftrightarrow$$



$$= e^{iq\alpha} \quad .$$

- Let us try to see this by explicit computations. Consider a free complex scalar in d-dimensions:

$$S = \int d^d x\, |\,\partial\phi\,|^2 - V(\,|\,\phi\,|\,) \qquad \rightsquigarrow \qquad j_\mu = i\,(\phi^\star \partial_\mu \phi - \phi \partial_\mu \phi^\star) \quad , \quad \partial_\mu j^\mu = 0\,.$$

we will mostly consider the free case

- Consider a flat defect located at $x^0 = 0$ and compute, say, the correlator $\langle \phi(x)\, \phi^\star(y) U_\alpha \rangle$ where

$$U_\alpha = e^{i\alpha \int d^{d-1}\vec{x}\, j^0|_{x^0=0}} = e^{i\alpha \int d^d x\, j^0\, \delta(x^0)}\,.$$

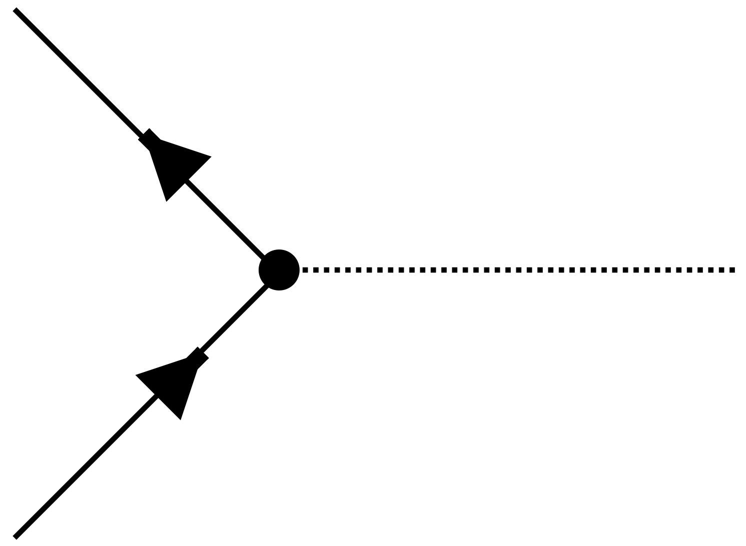
- Expanding the exponential, to order α^2

$$\langle \phi(x)\, \phi^\star(y) U_\alpha \rangle = \langle \phi(x)\, \phi^\star(y) \rangle +$$

$$+ i\alpha \int d^d z\, \delta(z^0)\, \langle \phi(x)\, \phi^\star(y)\, j^0(z) \rangle - \frac{\alpha}{2} \int d^d z_1 \int dz_2\, \delta z_1^0)\, \delta(z_2^0)\, \langle \phi(x)\, \phi^\star(y)\, j^0(z_1)\, j^0(z_2) \rangle + \cdots$$

- One can see that

$$\int d^d z\, \delta(z^0)\, \langle \phi(x)\, \phi^\star(y)\, j^0(z) \rangle = \langle \phi(x)\, \phi^\star(y) \rangle\, (\theta(x^0) - \theta(y^0)) \qquad \leftrightarrow$$

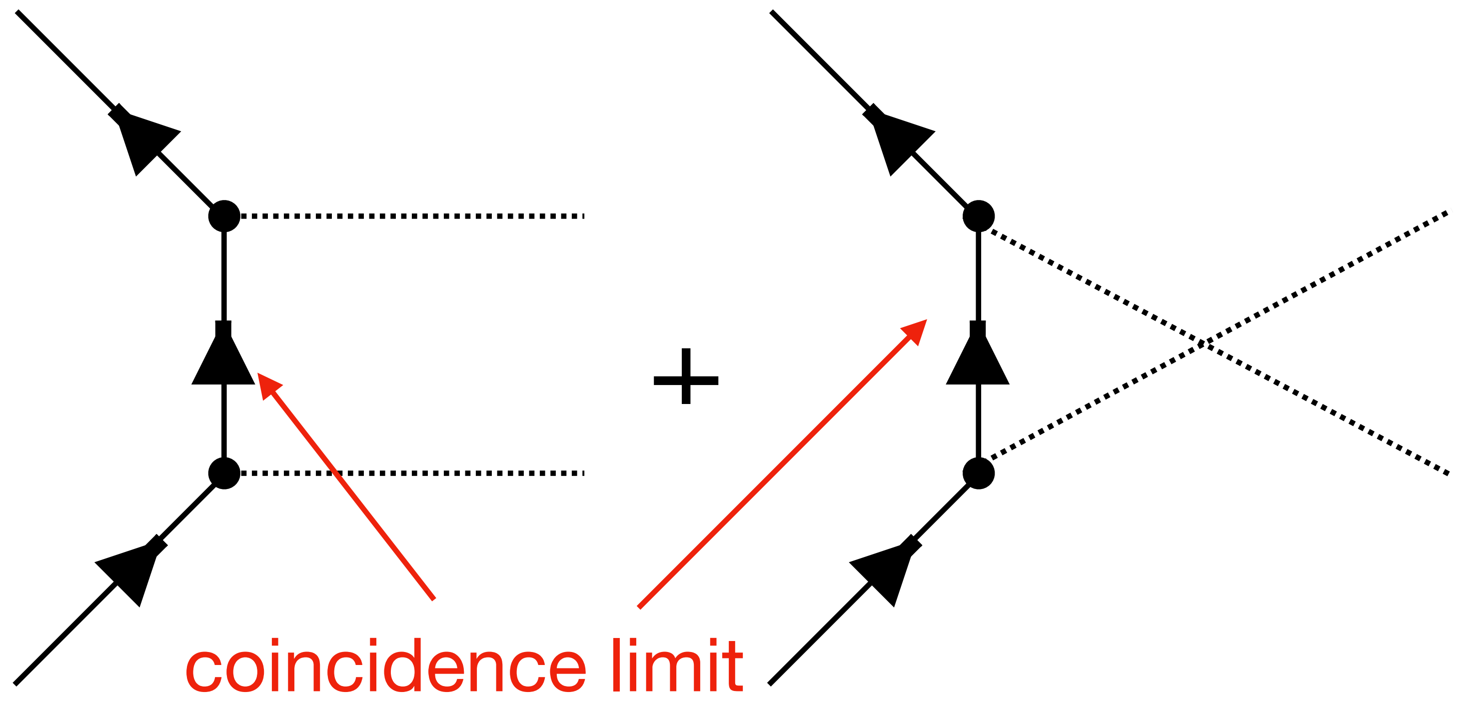


- Also

$$\int d^d z_1 \int dz_2\, \delta(z_1^0)\, \delta(z_2^0)\, \langle \phi(x)\, \phi^\star(y)\, j^0(z_1)\, j^0(z_2) \rangle = \langle \phi(x)\, \phi^\star(y) \rangle\, (\theta(x^0) - \theta(y^0))^2 + S\,,$$

- where S is

$$S = -\,2\,\delta(0) \int d^d z\, \delta(z^0)\, \langle \phi(x)\, \phi^\star(y)\, \phi\phi^\star(z) \rangle \qquad \leftrightarrow$$



- …so the defect doesn't quite act as expected: there is a contact term. But one can define a (very mildly) modified defect

$$\widehat{U}_\alpha = U_\alpha \, e^{-\alpha^2 \int d^d z \, \delta(z)^2 \, \phi\phi^\star(z)} \qquad \rightsquigarrow \qquad \langle \phi(x)\, \phi^\star(y)\ \widehat{U}_\alpha \rangle = e^{i\alpha\,(\theta(x^0) - \theta(y^0))} \langle \phi(x)\, \phi^\star(y) \rangle \,.$$

- In path integral language it all conspires so that

$$\langle \phi(x)\, \phi^\star(y)\ \widehat{U}_\alpha \rangle = \int \phi(x)\, \phi^\star(y)\, e^{i \int d^d x\, |D\phi|^2}\,, \qquad D_\mu = \partial + i\, J_\mu\,, \qquad J_\mu = \delta_{0,\mu}\, \alpha\, \delta(x^0)\,.$$

- So with the modified correlator, we are background-gauging the global symmetry. The background gauge field is actually a singular gauge transformation

$$J_\mu = \partial_\mu (\alpha\, \theta(x^0))\,,$$

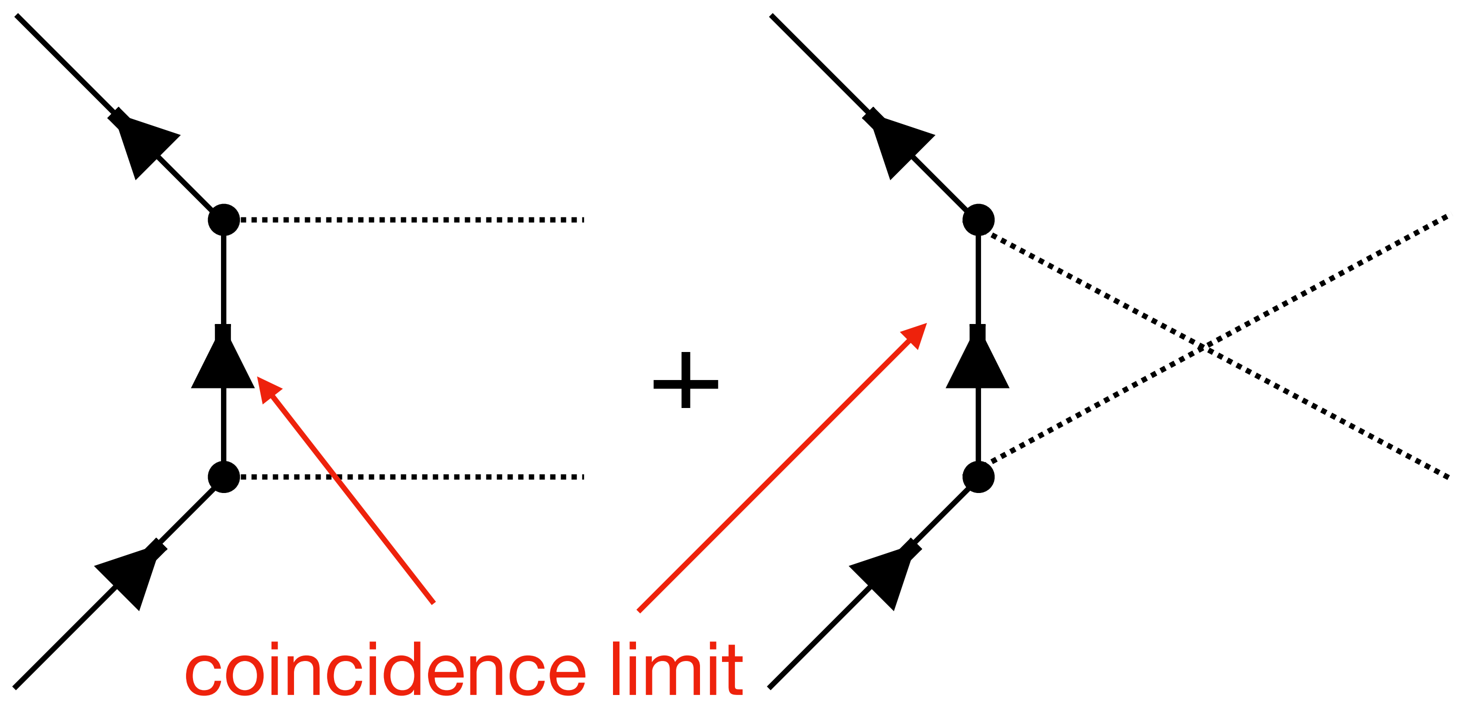
- but this is precisely the effect of the defect $\phi \to e^{i\alpha\,\theta(x^0)}\,\phi(x)$.

Bah, Jefferson, Roumpedakis & Waddleton '24

- In our field theory exercise we saw that to get the correct result we should consider a (very mild) modified defect which takes into account a contact term.

- The contact term came from a short distance singularity

$$S = -2\,\delta(0)\int d^d z\,\delta(z^0)\,\langle \phi(x)\,\phi^\star(y)\,\phi\phi^\star(z)\,|\,\rangle \quad \leftrightarrow$$



- The two delta's can be regularized in many ways. A particularly nice regularization is to regard the delta as a limit of a lorentzian

Like thickening the defect

Bah, Jefferson, Roumpedakis & Waddleton '24

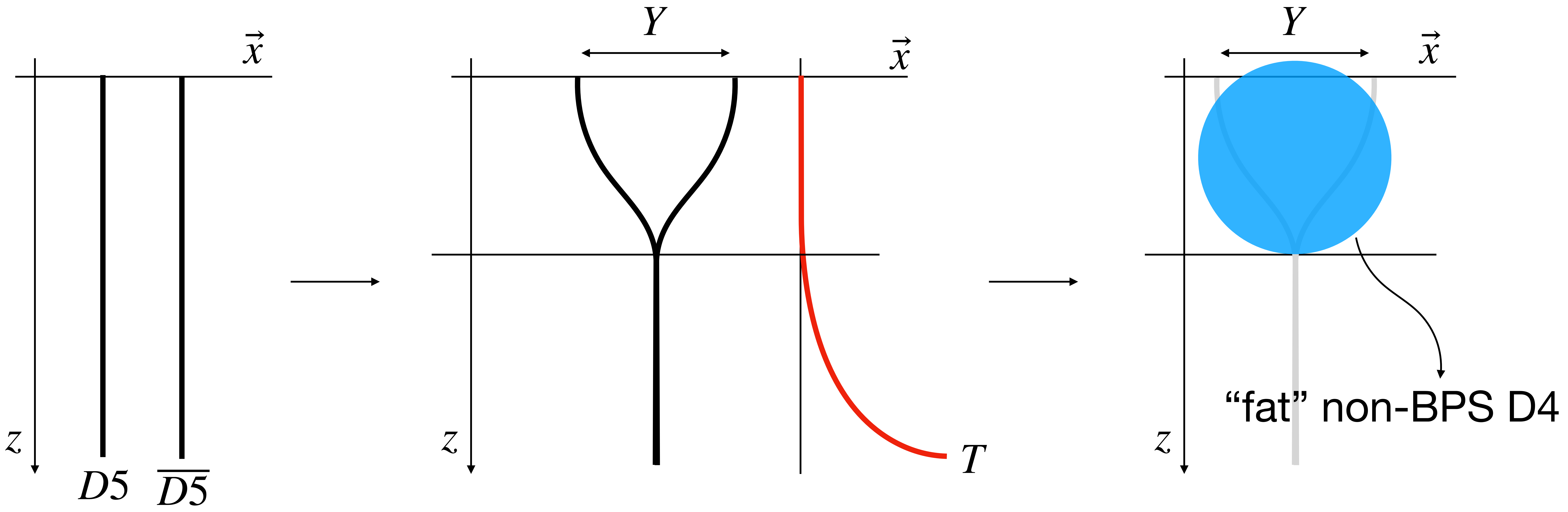
- Could one see this in the stringy version?
- Related to this, a non-BPS Dp brane can be regarded as an intermediate step in the decay of a Dp+1/anti Dp+1 system…can our symmetry operators be regarded in this way?

Sen '03

- The action is now

$$S = \int V(|T|, Y)\left(\sqrt{G+F+|DT|^2}\,|_{D5} + \sqrt{G+F+|DT|^2}\,|_{\overline{D5}}\right)$$

- The upshot is that the D5/anti D5 system looks like



- Alternatively: a U-shaped D5!

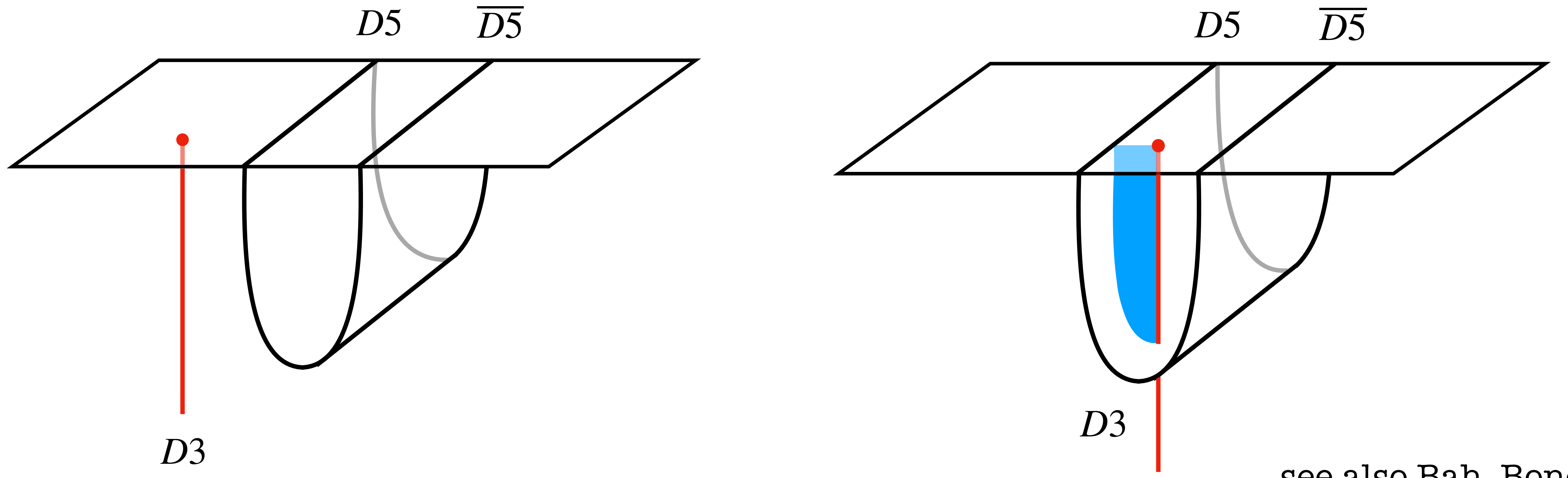
- As a comment: it is well-known that the D5 is a $SU(N) \times SU(N) \to SU(N) \times SU(N+1)$ DW

- Take a single D5: it contains a wv. coupling $\int_{10d} C_4 \wedge da \wedge \delta_{D5}$. As a consequence, the C_4 eom. is $d(F_5 - a \wedge \delta_{D5}) = 0$:

  Gauge-invariant under $(C_4, a) \to (C_4 + \lambda\, \delta_{D5}, a + d\lambda)$

  Green, Harvey & Moore '96

- As the D3 is dragged across the U, a string (=electric field) is created on the D5/antiD5



see also Bah, Bonetti, Chitoto & Leung'25, 26

- As the U becomes smaller, these the electric field on D5/antiD5 cancel each other… possibly up to a gauge transformation

- …but through the anomaly this generates a baryonic gauge field

symTFT and holography?

- So far we have been successful in finding the holographic avatar of symmetry operators, and they followed the rules dictated by the symTh read off from the appropriate SUGRA

- What about the symTFT? That looks more complicated

  - kosher holography does not allow arbitrary boundary conditions in AdS_{d+1}. For instance, gauge fields in $AdS_{>4}$ can only have Dirichlet boundary conditions: abelian gauge fields are IR free.

    Marolf & Ross'06

    incidentally: a loophole is a gauge field with no matter…like in U(N) N=4

  - even if we forgot that, changing boundary conditions in standard holography is tantamount to changing a global into a gauged symmetry, which in the symTFT is a dramatic change.

- Let's look to the first issue. Suppose first some CFT with a U(1) global symmetry. Through holography

$$Z = \int \mathscr{D}\phi\, e^{-S_{\text{gravity}}} = \int \mathscr{D}\phi_\star \left[\int \mathscr{D}\phi \Big|_{z<z_\star} e^{-S[\phi]\Big|_{z<z_\star}} \right] \left[\int \mathscr{D}\phi \Big|_{z>z_\star} e^{-S[\phi]\Big|_{z>z_\star}} \right] = \int \mathscr{D}\Phi_\star\, \Psi_{UV}[\phi_\star]\, \Psi_{IR}[\phi_\star]$$

Heemskerk & Polchinski'10

...

- Here $\Psi_{IR}[\phi_\star]$ is to be interpreted as the partition function for the CFT with cut-off $\Lambda \sim z_\star^{-1}$. Note that with the cut-off, any boundary condition is allowed

- …however by construction it imposes Dirichlet boundary conditions:

$$S = \int \frac{1}{2} dA_{p+1} \wedge \star dA_{p+1} \begin{cases} D : S = \int \frac{1}{2} dA_{p+1} \wedge \star dA_{p+1} \\ N : S = \int \frac{1}{2} d\tilde{A}_{d-p-2} \wedge \star d\tilde{A}_{d-p-2} \end{cases} \qquad \bigoplus \qquad D - \text{boundary conditions}$$

- While this is OK in the IR side, in the UV side it is generically not: arbitrary boundary conditions yield non-normalizable fields, so if we want to allow for this, the UV side must be "UV completed". This means that the gauge sector will be

$$S = \int_{\mathcal{I}} \frac{1}{2} dA_q \wedge \star_{\mathcal{I}} dA_q = \int_{\mathcal{I}} \frac{1}{2} f_{d-q} \wedge \star_{\mathcal{I}} f_{d-q} + \underbrace{i f_{d-q} \wedge dA_q}_{\text{topological!}}$$

- …so let's simply focus on the universal gauge part

$$\begin{cases} \mathrm{D}: & S_{\mathrm{D}}^{\mathrm{symTFT}} = \frac{i}{2\pi} \int_{\mathcal{I}_{d+1}} f_{d-p-1} \wedge dA_{p+1}\,, \\ \mathrm{N}: & S_{N}^{\mathrm{symTFT}} = \frac{i}{2\pi} \int_{\mathcal{I}_{d+1}} f_{p+2} \wedge d\tilde{A}_{d-p-2}\,. \end{cases} \longleftrightarrow \quad U(1)^{(p)}/U(1)^{(d-p-3)} \text{ symmetries}$$

Antinucci & Benini'24

- We can say a bit more…start with the D action and implement duality a la Rocek&Verlinde

$$S = \int \frac{1}{2}(dA_{p+1} - B_{p+2}) \wedge \star\, (dA_{p+1} - B_{p+2}) + i\, \tilde{A}_{d-p-2} \wedge dB_{p+2} \rightsquigarrow S = \int \frac{1}{2}(dA_{p+1} - B_{p+2}) \wedge \star\, (dA_{p+1} - B_{p+2}) + \frac{1}{2} dB_{p+2} \wedge \star dB_{p+2} + \frac{1}{2} d\tilde{A}_{d-p-2} \wedge \star d\tilde{A}_{d-p-2} + i\, \tilde{A}_{d-p-2} \wedge dB_{p+2}$$

$$S = i \int h_{d-p-1} \wedge dA_{p+1} - h_{d-p-1} \wedge B_{p+2} + g_{d-p-2} \wedge dB_{p+2} + f_{p+2} \wedge d\tilde{A}_{d-p-2} + \tilde{A}_{d-p-2} \wedge dB_{p+2}$$

gauged global symmetry!

Antinucci & Benini'24

Conclusions (and open directions)

- We proposed a holographic description of global (mostly continuous) symmetries in terms of non-BPS branes

- They agree with the predictions of the symTh which one obtains from the corresponding SUGRA description

- …including anomalies and non-invertible cases

- A natural question is the relation to the symTFT. It looks like generically one cannot “derive” it from holography

- …yet we proposed an embedding into holography (matching, quite non-trivially, the proposals in the literature!)

- Yet, it is fair to say that there are many open questions

    - true origin of the continuous parameter in the nonBPS brane action
    - full non-commutative structure (see Bah, Bonetti, Chitoto & Leung'25, 26)
    - the origin of the symTFT from holography
    - …

Many thanks