SymTFT for continuous spacetime symmetries

2509.07965 with N. Dondi, I. García-Etxebarria, H-T. Lam, S. Schäfer-Nameki

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SymTFT for Continous Spacetime Symmetries

Continuous spacetime symmetries are a bit different from all the other symmetries:

- They do not commute with the hamiltonian (or energy momentum tensor), schematically

$$\frac{d}{dt}\mathcal{U}^X = [\mathcal{U}^X, H] + \frac{\partial \mathcal{U}^X}{\partial t} = 0$$

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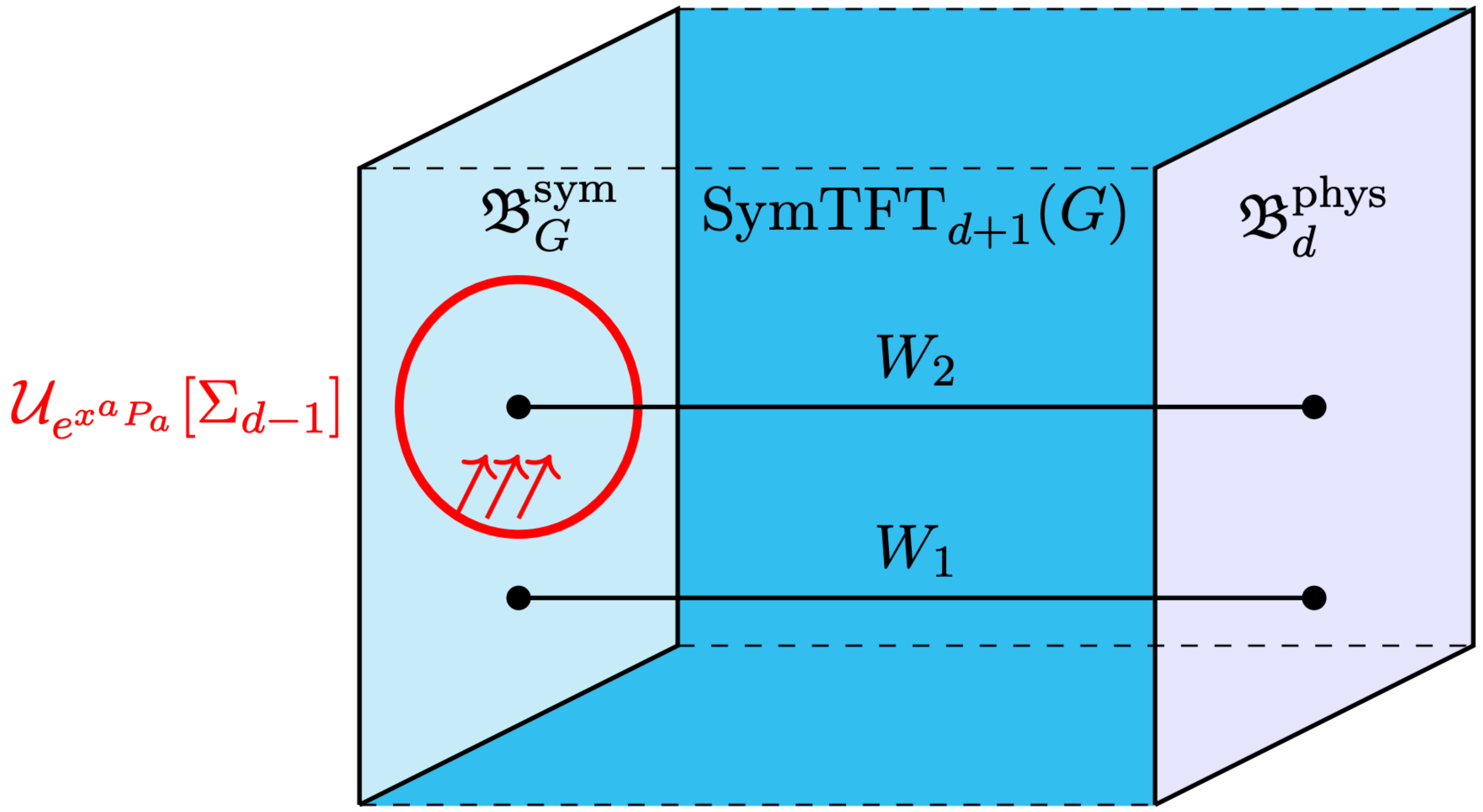
- The symmetry defects “do not act topologically”

$$\mathcal{U}^X(\Sigma)\mathcal{O}(x) = e^{X^\mu \partial_\mu}\mathcal{O}(x) = \sum_{n=0}^{\infty} \frac{1}{n!}(X^\mu \partial_\mu)^n \mathcal{O}(x)$$

Goal

Extract all the topological feature of spacetime symmetries by implementing the SymTFT approach.

Implement the (non-topological) action on spacetime points, lines, regions on the boundary while keeping the action topological in the bulk



SymTFT proposal

Non-Abelian BF-theory

[**FA**, Dondi, Garcia-Etxebarria, Lam, Schafer-Nameki
Cattaneo, Rossi; Brennan, Sun; Antinucci,
Benini; Bonetti, Del Zotto, Minasian;
Jia, Luo, Tian, Wang, Zhang]

Our starting point is the following BF-theory

$$S_{\mathrm{BF}} = \frac{i}{2\pi} \int_{M_{d+1}} \langle B_{d-1}, F_A \rangle$$

where $\langle \, , \rangle$ is the canonical inner product between $\mathfrak{g}$ and $\mathfrak{g}^*$, and

$$F_A = dA + A \wedge A, \quad B_{d-1} \in \Omega^{d-1}(M_{d+1}, \mathfrak{g}^*)$$

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$$A \to A^{(g)} = g^{-1} A g + g^{-1} dg, \quad B_{d-1} \to B_{d-1}^{(g,\sigma)} = g^{-1}(B_{d-1}) g + d_{A^{(g)}} \sigma_{d-2}, \quad \sigma_{d-2} \in \Omega^{d-2}(M_{d+1}, \mathfrak{g}^*)$$

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$$F_A = 0, \quad d_A B_{d-1} = 0 \quad \text{with} \quad d_A \omega_p = d\omega_p + A \wedge \omega_p - (-1)^p \omega_p \wedge A$$

Non-Abelian BF-theory, boundary and defects

We get topological line defects

$$W_{\mathscr{R}}[\gamma] = \mathrm{Tr}_{\mathscr{R}}\, P\left[e^{i\oint_\gamma A}\right]$$

In analogy with abelian BF traced holonomies of *B* suffer of path(ological) ordering issue

$$\mathrm{Tr}P\left[e^{i\oint_{\Sigma_{d-1}} B}\right](?)$$

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$$\mathscr{U}^{[X]}(\Sigma_{d-1}) = \int \mathscr{D}U\mathscr{D}\beta_{d-1}\, \mathrm{exp}\left\{ i\oint_\Sigma \langle UXU^{-1}, d_A\beta_{d-2} + B\rangle \right\}, \quad U \in \Omega^0(\Sigma, G)$$

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Defect labelled by the conjugacy class $e^{[X]} \subset G$,

Topological Defects

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$$\langle\mathscr{U}^{[X]}[\Sigma]\,\mathscr{W}^{\mathscr{R}}[\gamma]\rangle=\mathrm{Tr}_{\mathscr{R}}\left[e^{-2\pi X\mathrm{Link}(\Sigma,\gamma)}\right]\langle\mathscr{W}^{\mathscr{R}}[\gamma']\mathscr{U}^{[X]}[\Sigma]\rangle$$

Boundary conditions

In the presence of a boundary $\delta S_{\mathrm{BF}} = \dfrac{i}{2\pi}\displaystyle\int_{\partial M_{d+1}} \langle \delta A \wedge B \rangle$, gauge transformations imply

$$S_{\mathrm{BF}}[A^{(g)}, B^{(g,\sigma)}] = S_{\mathrm{BF}}[A, B] + \int_{\partial M_{d+1}} \langle \sigma_{d-2} \wedge g^{-1} F_A g \rangle$$

⇒Boundary conditions must be compatible with bulk EOM to guarantee a proper variational problem, e.g. Dirichlet implies $F_A \,|_{\partial M} = 0$ **(Flatness condition)**

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labelled by element in $g = e^X$

Neumann boundary conditions

Let us now discuss (partial) Neumann boundary conditions. Consider $H < G$, such that

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}, \quad [\mathfrak{h}, \mathfrak{h}] \subseteq \mathfrak{h}, \quad \langle \mathfrak{h}, \mathfrak{m}^* \rangle = \langle \mathfrak{m}, \mathfrak{h}^* \rangle = 0$$

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H-gauge transformations: $A \mapsto h^{-1} A h + h^{-1} dh\,, \quad B \mapsto h^{-1} B h\,, \quad \mathscr{A}_{\mathfrak{h}} \mapsto h^{-1} \mathscr{A}_{\mathfrak{h}} h + h^{-1} dh$

BF-theory with $G=SO(d+1,1)$

Component decomposition of BF

We now decompose the BF-theory into the Lorentz, translation, special conformal, dilatation generators

$$(L_{ab}, P_a, K_a, D)$$

The A and B gauge field decompose

$$A = \frac{1}{2}\omega^{ab}L_{ab} + e^a P_a + f^a K_a + bD, \quad B = \frac{1}{2}j^{ab}L_{ab} + s^a P_a + t^a K_a + \phi D$$

By using the previously introduced inner product for $\mathfrak{so}(d+1,1)$ the BF Lagrangian becomes,

$$\mathscr{L}_{BF} = i\langle B \wedge F\rangle_{\mathrm{BF}} = \frac{i}{2}j_{ab}(d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb} - 2e^{[a} \wedge f^{b]}) - i\phi(db - 2e_a \wedge f^a) + \cdots$$

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$$g_{\mu\nu} = \Omega(x)^2\eta_{\mu\nu}$$

Gapped boundary conditions

From the equations of motion for B we get,

$$dj^{ab} - 2e^{[a} \wedge t^{b]} = 0, \quad dt^a = 0, \quad ds^a + e^b \wedge j_b{}^a - e^a \wedge \phi = 0, \quad d\phi - 2e^a \wedge t_a$$

By choosing the flat spacetime background

$$\mathscr{A} = dx^a \otimes P_a \quad \Longleftrightarrow \quad e^a_\mu = \delta^a_\mu \;, b_\mu = w^{ab}_\mu = f^a_\mu = 0$$

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$$\mathscr{P}^a = \oint t^a, \quad \mathscr{J}^{ab} = \frac{1}{2}\oint \left(j^{ab} - 2x^{[a}t^{b]}\right), \quad \mathscr{K}^a = \oint (s^a + x^b j_b{}^a - x^a\phi + 2x^a x^b t_b - x^b x_b t^a), \quad \mathscr{D} = \frac{1}{2}\oint \left(2x^a t_a - \phi\right)$$

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We now pick a gauge $j^{ab} = s^a = \phi = 0$, where we can identify

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We now pick a gauge $j^{ab} = s^a = \phi = 0$, where we can identify

$$t^a = 2\pi \star T^a, \quad T^a \equiv e^{a\mu} T_{\mu\nu} dx^\nu$$

Gapped boundary conditions

From the equations of motion for B we get,

$$dj^{ab} - 2e^{[a} \wedge t^{b]} = 0, \quad dt^a = 0, \quad ds^a + e^b \wedge j_b{}^a - e^a \wedge \phi = 0, \quad d\phi - 2e^a \wedge t_a$$

By choosing the flat spacetime background

$$\mathscr{A} = dx^a \otimes P_a \quad \Longleftrightarrow \quad e^a_\mu = \delta^a_\mu \ , b_\mu = w^{ab}_\mu = f^a_\mu = 0$$

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Where $\star$ is the hodge operator in flat space

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The flatness condition implies that the **stress tensors is symmetric conserved and traceless**

$$\mathcal{F} = 0 \Rightarrow \begin{cases} \partial^{\nu} T_{\mu\nu} = 0 \\ T^{\mu}{}_{\mu} = 0 \end{cases}$$

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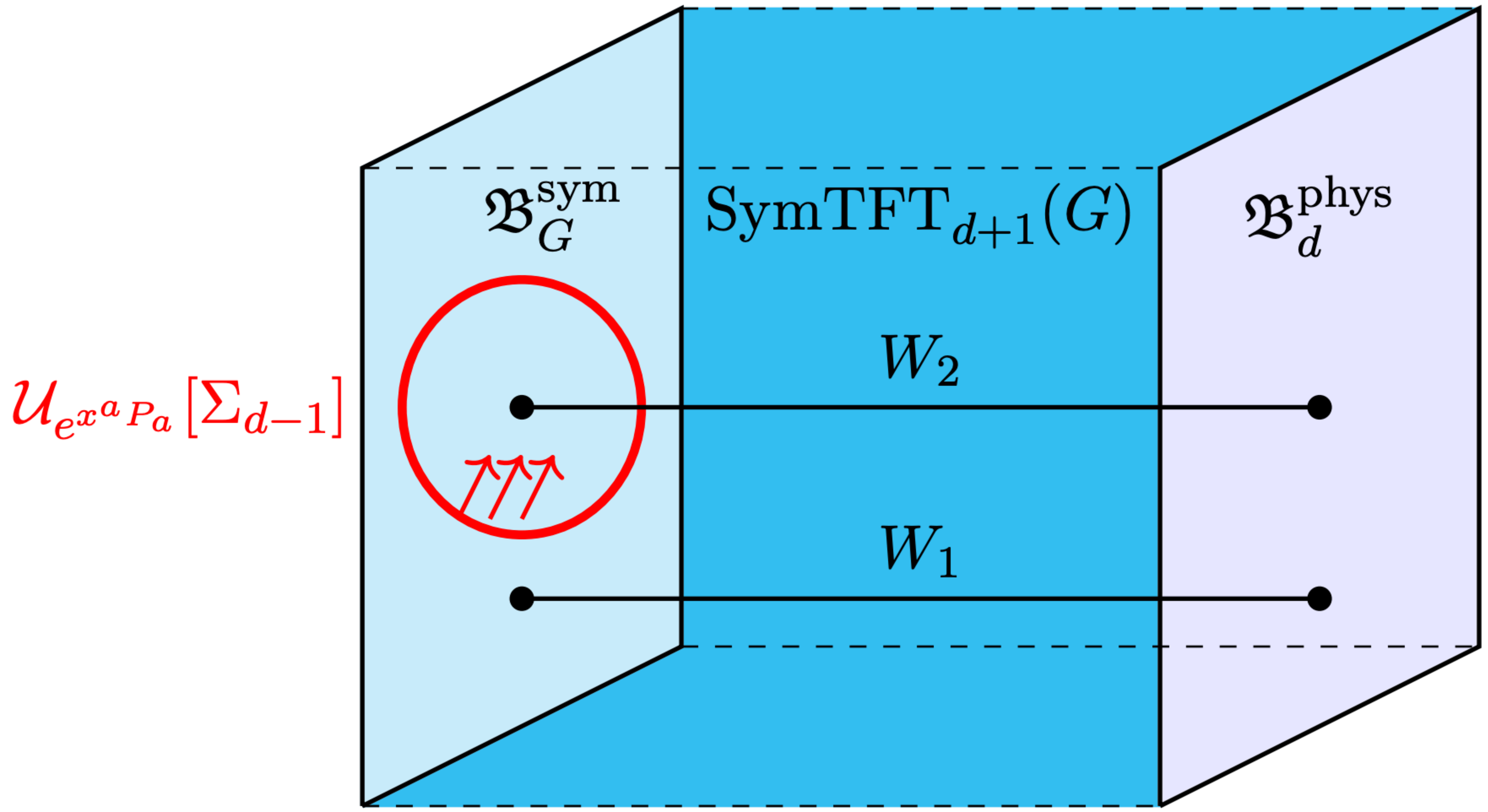
We defined the conformal generator on the Dirichlet boundary in terms of *B*

How to move points and lines

How to move a point



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We start from our Dirichlet boundary (flat space) given by

$$\mathscr{A} = \delta^a_\mu dx^\mu \otimes P_a = e^{-\delta^a_\mu x^\mu P_a} d\left(e^{\delta^a_\mu x^\mu P_a}\right)$$

We can insert a topological symmetry operator labelled by a group element $g = e^X$, by focusing on **translation**, $\mathscr{A}$ modifies as follows

$$A\,|_{\partial M_{d+1}} = h^{-1}dh + (h^{-1}Xh)\,\delta^{(1)}(\Sigma_{d-1}), \quad X = X^a P_a, \quad h = e^{\delta^a_\mu x^\mu P_a}$$

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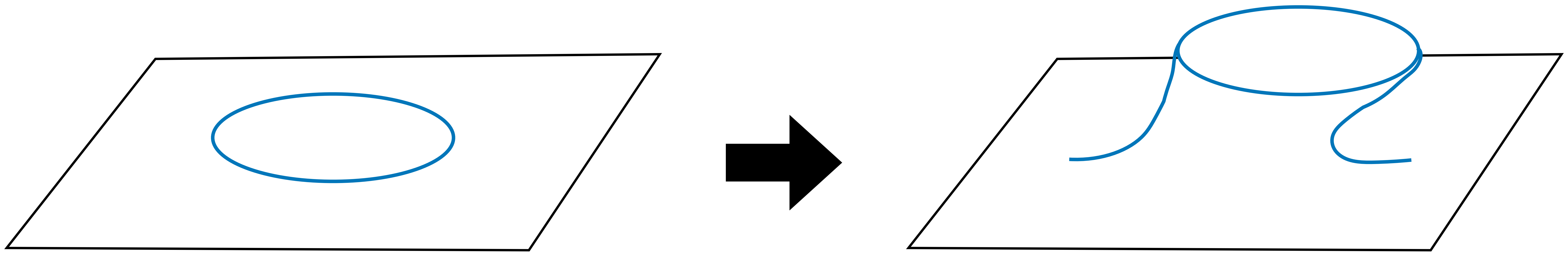
We can regularize the delta function by cutting out a tubular neighbourhood $D_\epsilon(\Sigma_{d-1}) \cong \Sigma_{d-1} \times [-\epsilon, \epsilon]$, and smear $\delta^{(1)}(\Sigma_{d-1}) \mapsto \rho(r)dr$



How to move a point (alternative point of view)

Even for the singular background we can apply a gauge transformation back to flat space

$$A\,\big|_{\partial M_{d+1}} = A^{(\alpha)} = e^{-\alpha} A_0 e^{\alpha} + e^{-\alpha} d e^{\alpha}, \quad \alpha = X^a P_a \vartheta(r - r_0)$$

Equivalently by applying the diffeomorphism by generated by the vector $\xi^\mu = X^\mu \vartheta(r - r_0)$.

Let us now consider the following correlation with a symmetry operator insertion

$$\langle \mathcal{O}(x_1) \mathcal{U}^X(\Sigma_{d-1}) \mathcal{O}(x_2) \rangle_{\mathscr{A}} = \langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle_{\mathscr{A}^{(\alpha)}}, \qquad Link(\Sigma_{d-1}, x_1) = 1$$

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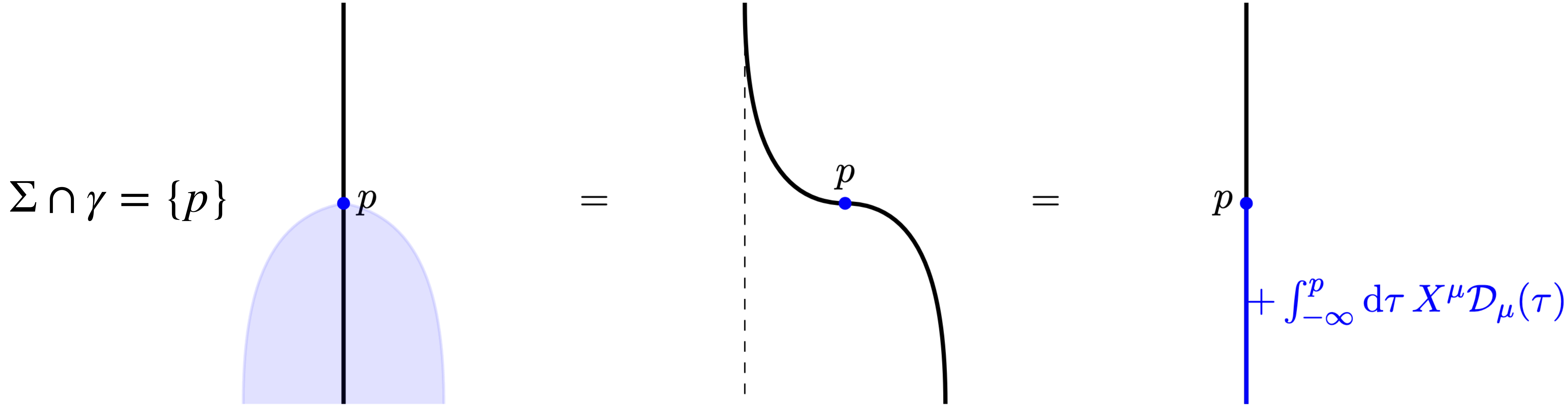
For this specific case the net effect inside the correlator above is to leave invariant the operator insertion at x_2 and move the one at x_1 to $x_1 - X$

How to move a line



The story is very similar. Let us focus on the correlation of the line with the symmetry defect

$$\langle U_X(\Sigma) L(\gamma) \rangle_{\mathscr{A}} = \langle L(\gamma) \rangle_{\mathscr{A}^{(\alpha)}} = \langle L(f_X \circ \gamma) \rangle_{\mathscr{A}}$$

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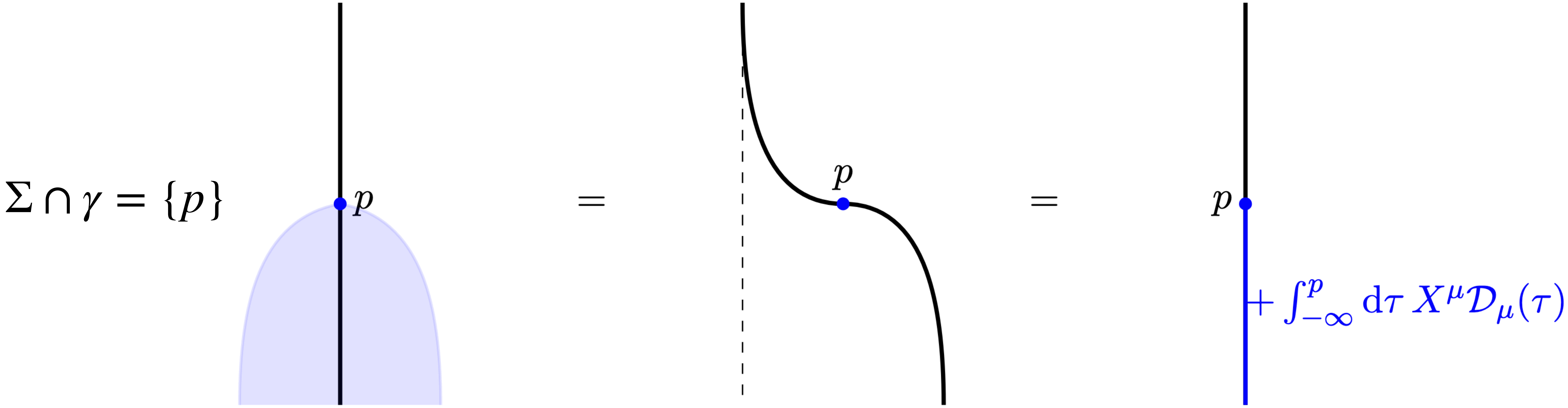

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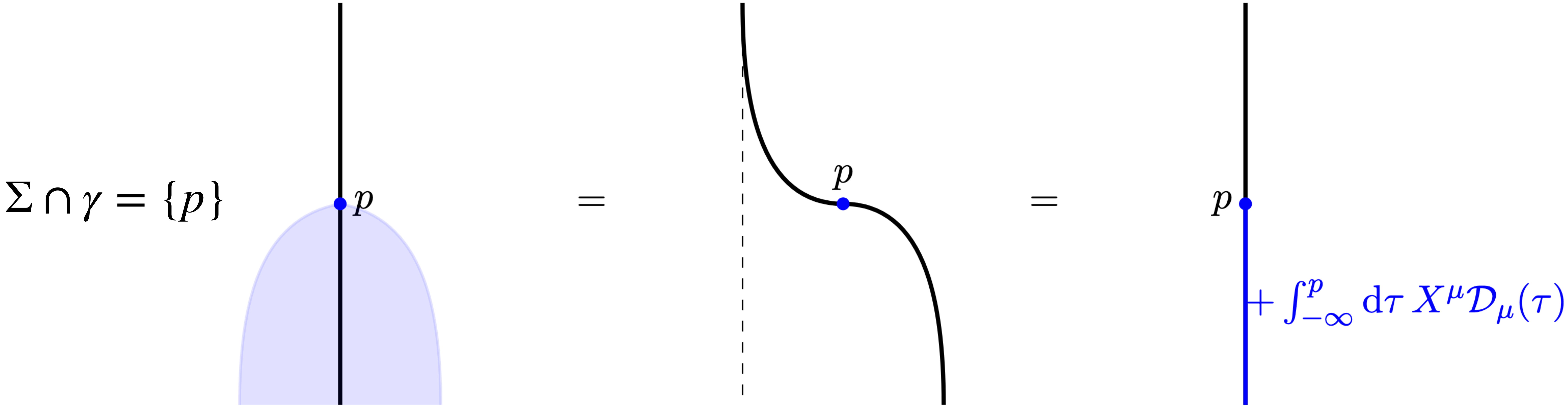

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$$\langle L(f_X \circ \gamma)\rangle_{\mathscr{A}} = \left\langle L(\gamma)\, e^{\int_{-\infty}^{p} d\tau X^{\mu} \mathscr{D}_{\mu}} \right\rangle, \quad \partial_{\mu} T^{\mu\nu} = \delta(\gamma) D^{\nu}$$

Symmetry Breaking Boundary

Gapped SSB boundary conditions

We can restore the gauge invariance on the Dirichlet boundary by

$$\mathrm{Dir}(G): \qquad S_{\mathrm{Dir}(G)} = -\frac{i}{2\pi}\int_{\partial M_{d+1}} \left\langle A - \mathscr{A}^{(U^{-1})}, B \right\rangle$$

Where $\mathscr{A}^{(U^{-1})} = U\mathscr{A}U^{-1} - dUU^{-1}, \quad \mathscr{F} = 0$, with $U \in \Omega^0(\partial M_{d+1}, G)$,

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This BC is invariant under both the bulk G gauge symmetry and the boundary H

Singleton boundary condition

Gappless boundary condition, leading order non-topological term that can be written in terms of the available bulk fields

[Maldacena, Moore, Seiberg; Belov, Moore; Hofman, Iqbal; Kravek, McGrevy; Antinucci, Benini, Rizi; Antinucci, Copetti Di Pietro; **FA**, Bedogna, Dondi; Garcia-Etxebarria, Hosseini Argurio, Collinucci, Galati, Hulk, Paznokas]

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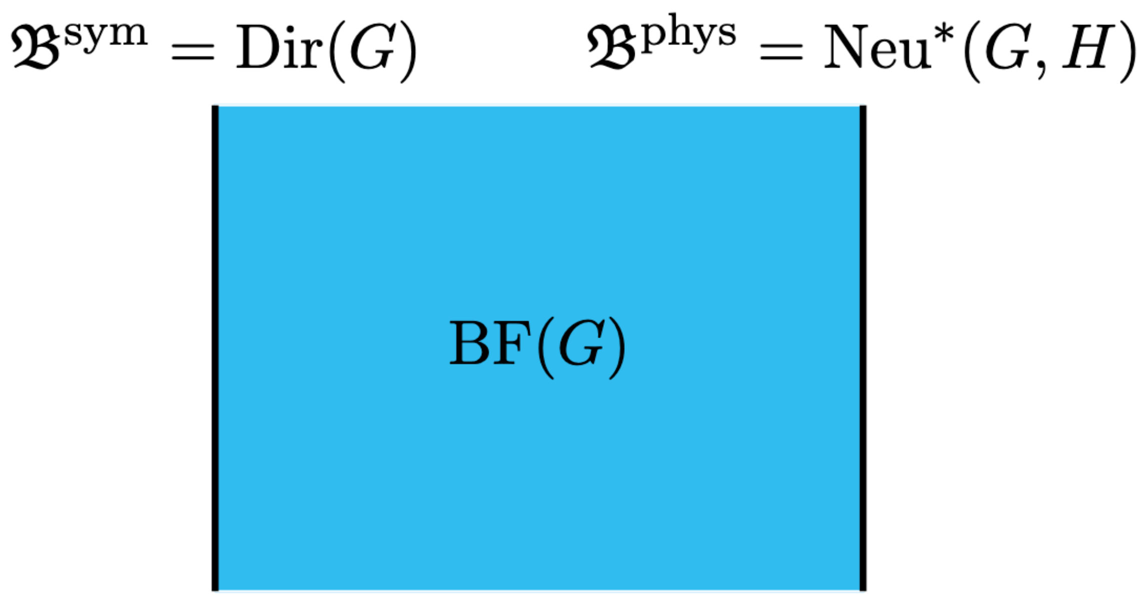
Where $\langle\ ,\ \rangle_\kappa$ denotes $\kappa_{ab} = \text{Tr}(T_a T_b)$ with $T_a, T_b \in \mathfrak{g}$

SymTFT sandwich

We now consider the following sandwich

Similar construction for internal symmetries in [Argurio, Collinucci, Galati, Hulk, Paznokas]



By integrating out the bulk for internal symmetries G we get

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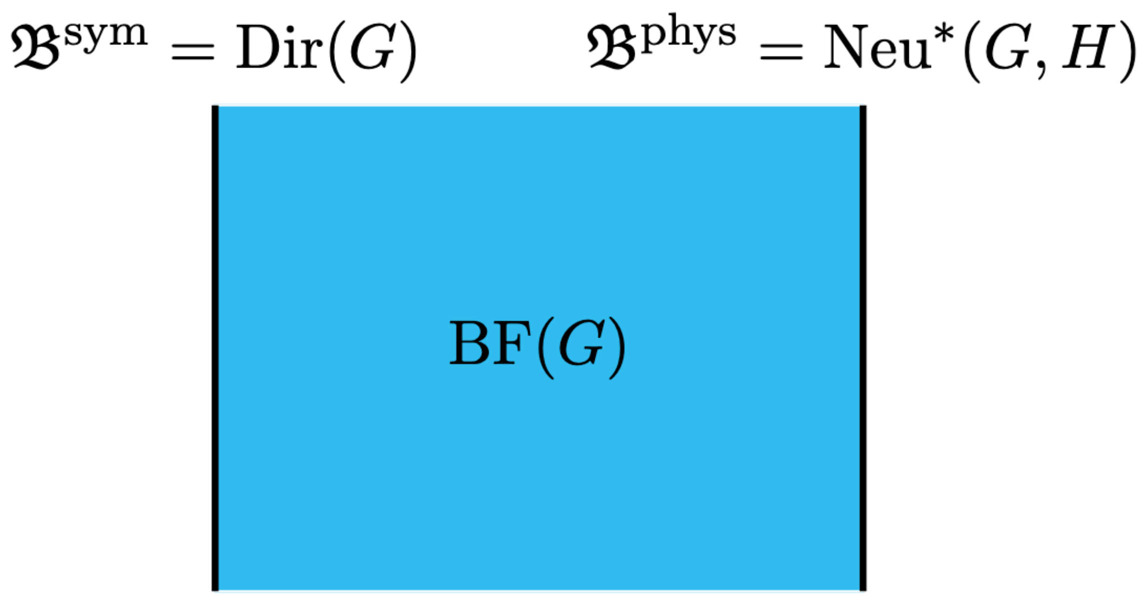

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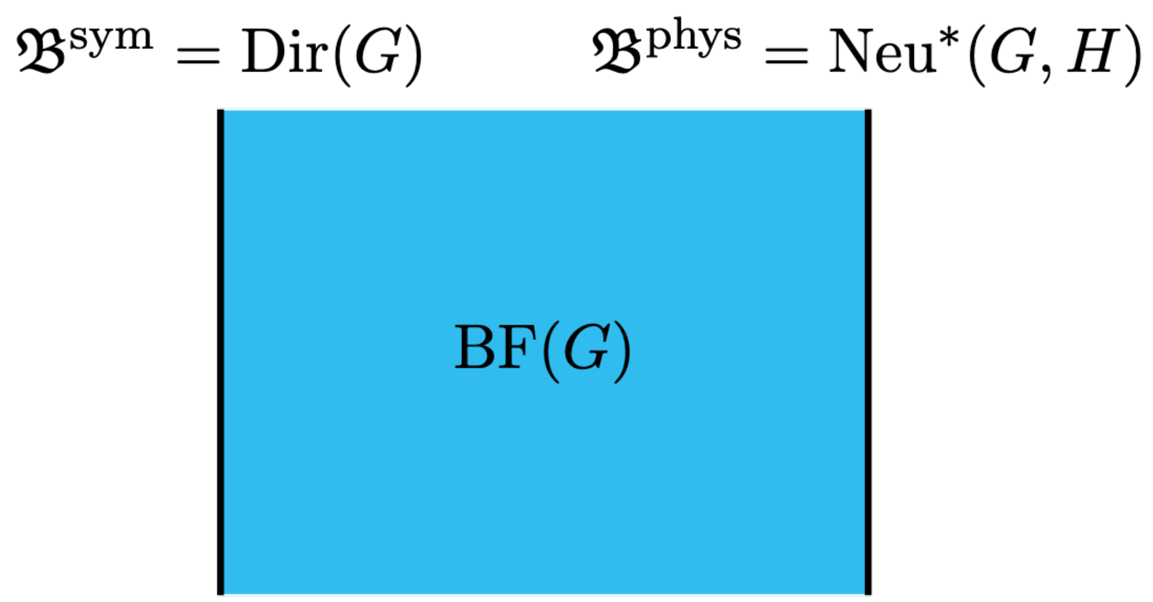

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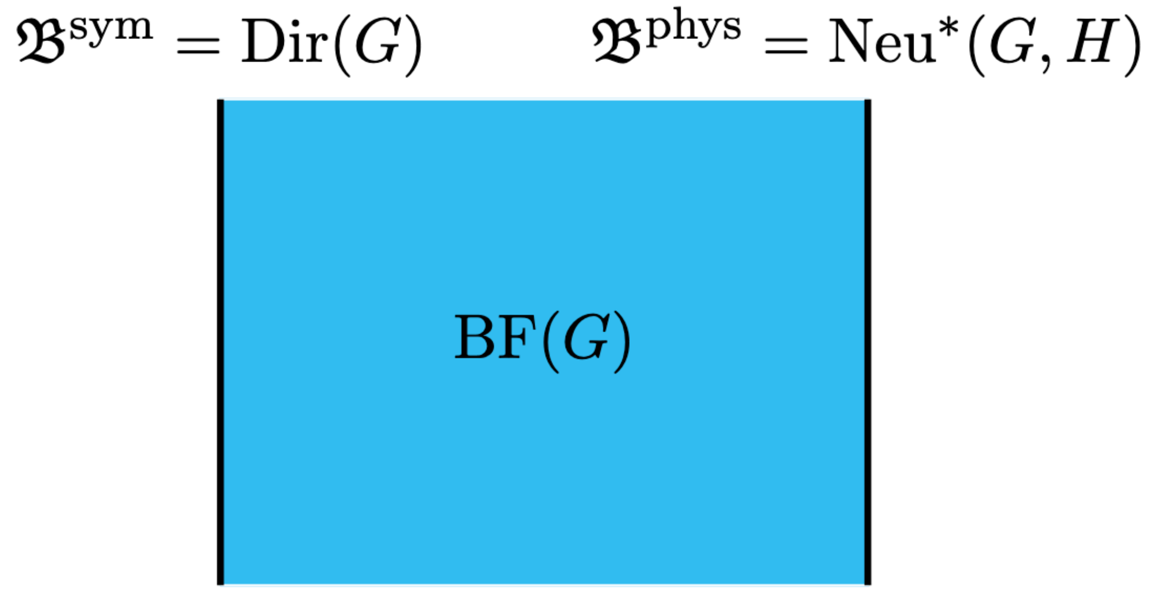

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Gauge transformation $U \mapsto \tilde{g}^{-1} U h,\ \tilde{g} \in G$

SO(d+1,1)/ISO(d) Breaking

Any flat ISO(d) connection can be written as $\mathscr{A}_{\mathfrak{h}} = W^{-1}dW$ for $W \in \mathfrak{h}$

By parametrizing the coset as $U = e^{-\pi^a K_a} e^{-\sigma D}$ with σ the GB for dilaton and π^a for special conformal, we get from by collapsing the SymTFT sandwich

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[Brauner, Watanabe;
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$$S_{\text{total}} = \frac{f^2(2d-3)}{2\pi}\int_{\partial M_{d+1}} e^{-(d-2)\sigma}d\sigma\wedge\star d\sigma$$

[Brauner, Watanabe; Delacretaz,Endlich, Monin, Penco, Riva; Monin, Prtskhalava,Rattazzi, Sebold]

Conformal anomaly (A-Type)

Chern Simons Gravity functional

Again we start from a proposal, which consists of adding the Chern-Simons term (d even)

$$\frac{ik}{(2\pi)^n} CS_{2n+1}(A,F) = \frac{ik}{(2\pi)^n}(n+1)\int_0^1 dt\langle a[tF + (t^2-t)A^2]^n\rangle_{\mathrm{CS}}$$

Where $\langle , \rangle_{CS}$ is the inner product defined for instance in Banados, Schwimmer, Theisen 0404245

$$\langle X_1, \cdots, X_{n+1}\rangle_{\mathrm{CS}} = \frac{1}{(n+1)!}\sum_{\sigma\in S_{n+1}} \mathrm{Tr}[X_{\sigma(1)}\cdots X_{\sigma(n+1)}]$$

This proposal is motivated by

- In Holography by choosing Fefferman-Graham gauge the CS action produces A-type anomaly

  [Brown-Hanneaux; Banados, Schwimmer, Theisen;….]
- Various attempts at a descendant mechanism and inflow with and without supersymmetry

  [Imbimbo, Rovere, Warman; Hebner, Heid, Lopes-Cardoso]
- Depending on n we can have also other CS type, like for n=1 $\langle , \rangle_\kappa$ (Gravitational anomaly)
- Does the Chern-Simons term reproduce the anomaly matching term in the dilation action?

  [Komargodski, Schwimmer;Luty, Polchinski, Rattazzi; ..]

3d SymTFT

Consider the d=2 case $S_{\text{SymTFT}} = S_{\text{BF}} + S_{\text{CS}'}$ (physical boundary CFT with $c = c_L + c_R$)

$$S_{\text{CS}'} = \frac{c}{24\pi} \int_{M_3} \left[\langle A, F \rangle_{\text{CS}} - \frac{1}{3} \left\langle A, A^2 \right\rangle_{\text{CS}} \right]$$

The boundary condition modify due to boundary variation

$$\delta S_{\text{SymTFT}} |_{\partial M_3} = \frac{i}{2\pi} \int_{\partial M_3} \left\{ \langle \delta A, B \rangle_{\text{BF}} + \frac{k}{2} \langle \delta A, A \rangle_{\text{CS}} \right\}$$

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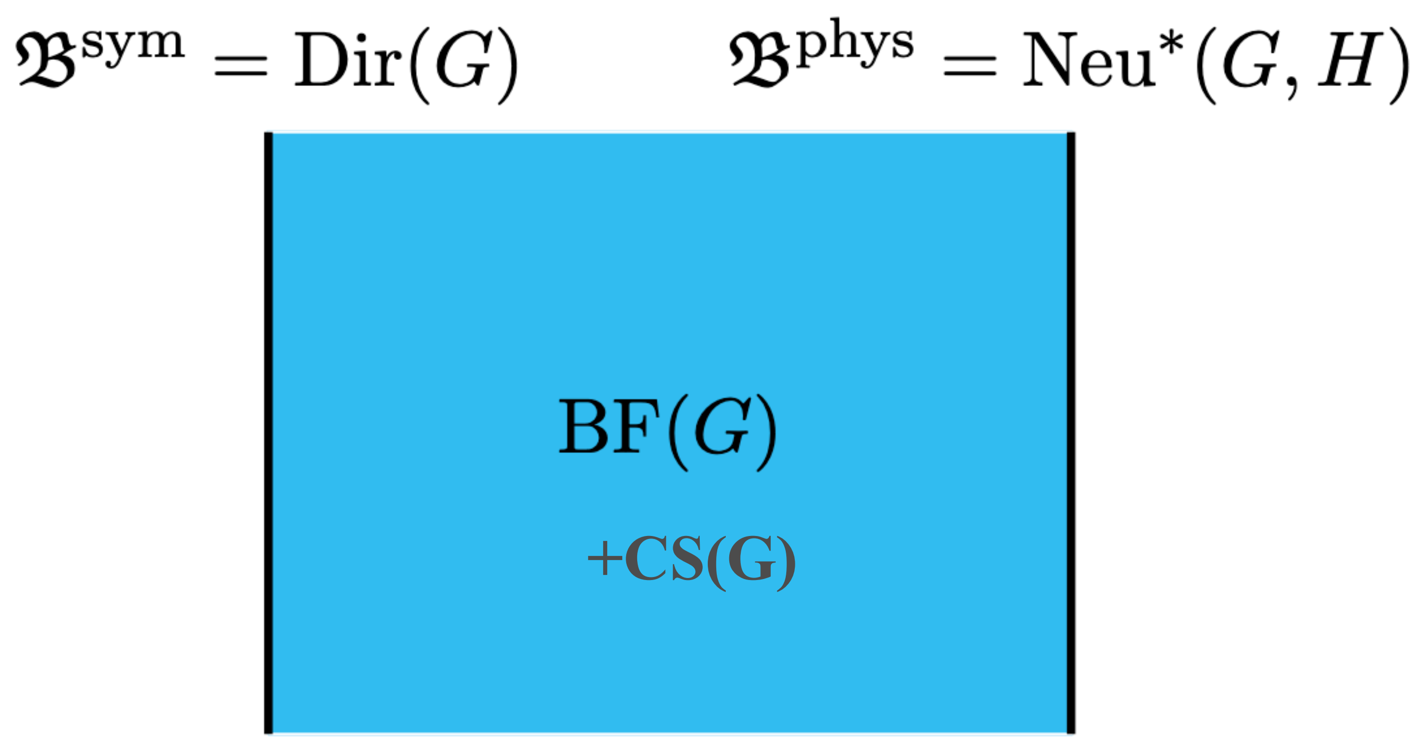
Where $\Gamma_3(U, A) = -\frac{1}{3}\left\langle (UdU^{-1})^3\right\rangle_{\text{CS}} - d\left\langle UdU^{-1}, A\right\rangle_{\text{CS}}$

Dilaton action

Let us now consider the sandwich construction (where on the RHS we add the singleton)



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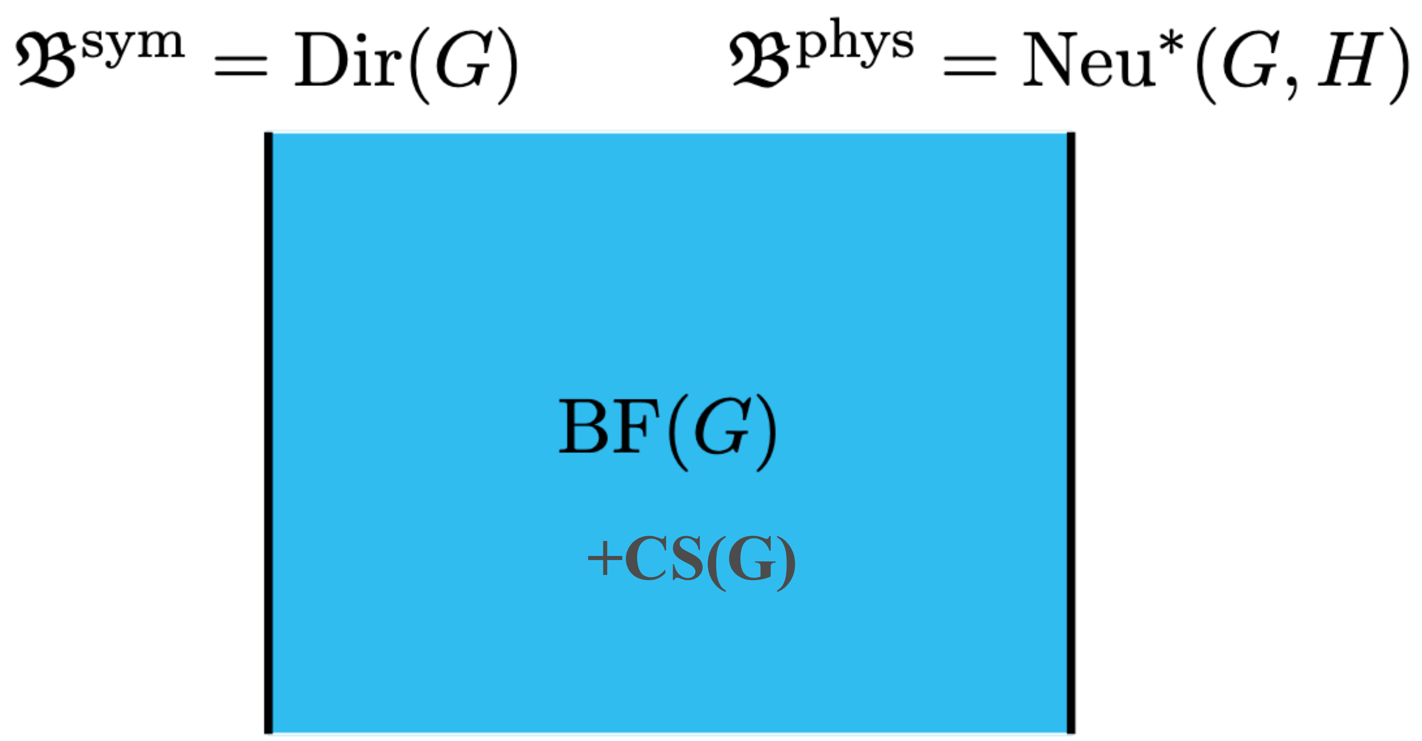

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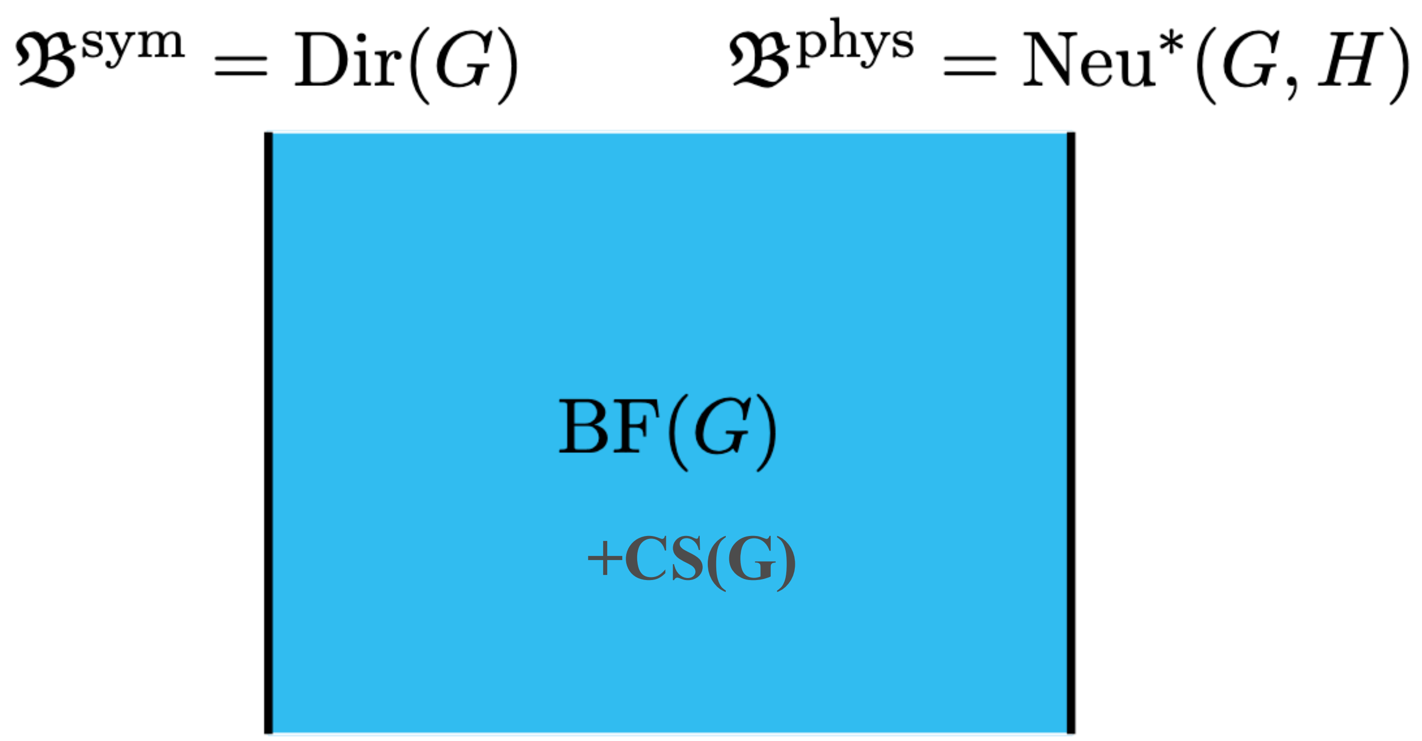

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Gravity origin

How is gravity related to our proposal?

In 2d the SymTFT looks like JT gravity. However this equivalence holds at only at a classical level.

- No sum over topologies
- Second order formalism implies invertible vielbein (restriction on the path integral over flat SL(2,R))
- Modding out by (large) diffeomorphisms.

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So if we forgot about the divergence piece we could truncate to BF.

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This picture is further supported by the MacDowell-Mansouri (MM) formulation

$$S_{BF} = \int_{X^4} B^{IJ} \wedge F_{IJ} - \frac{1}{2} B_{IJ} \wedge B_{LM} \epsilon^{IJKLM} v_M, \quad v = (0,0,0,0,G'_N \Lambda/6)$$

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In addition the action necessarily vanishes in the AdS vacuum.

Summary

- Non-Abelian BF theory captures spacetime and conformal symmetry properties
- The Chern-Simons terms in SymTFT captures A-type anomalies
- SymTFT and SSB boundary conditions lead to dilation action (in 3d)
- SymTFT differs from gravity in many ways, in certain cases is the topological truncation
- Adding SUSY, Mixing internal symmetries, defect conformal anomalies, other spacetime symmetries breaking pattern, systematic analysis of gravitational SPT phases, beyond topology, B-type anomalies…

Thank you!

Topological defect linking

We compute how topological defects act on each other by evaluating

$$S = \int_M \langle B \wedge F_A \rangle + i \int_{\Sigma_{d-1}} \langle [X], B \rangle \Rightarrow F_A + i[X]\delta^{(2)}(\Sigma_{d-1}) = 0$$

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$$\mathscr{M}_{(\Delta,\ell)} = \mathrm{span}\left\{P_{\mu_1}\dots P_{\mu_n}|\Delta,\ell\rangle \,|\, n \geq 0\right\}, \quad D|\Delta,\ell\rangle = i\Delta|\Delta,\ell\rangle, \quad J^2|\Delta,\ell\rangle = \ell(\ell+d-2)|\Delta,\ell\rangle$$

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Where we take a representation built out of a primary vector $|\,\Delta,\ell\rangle$

$$\mathscr{M}_{(\Delta,\ell)} = \mathrm{span}\left\{P_{\mu_1}\ldots P_{\mu_n}|\,\Delta,\ell\rangle \,|\, n \geq 0\right\}, \quad D\,|\,\Delta,\ell\rangle = i\Delta\,|\,\Delta,\ell\rangle, \quad J^2\,|\,\Delta,\ell\rangle = \ell(\ell+d-2)\,|\,\Delta,\ell\rangle$$

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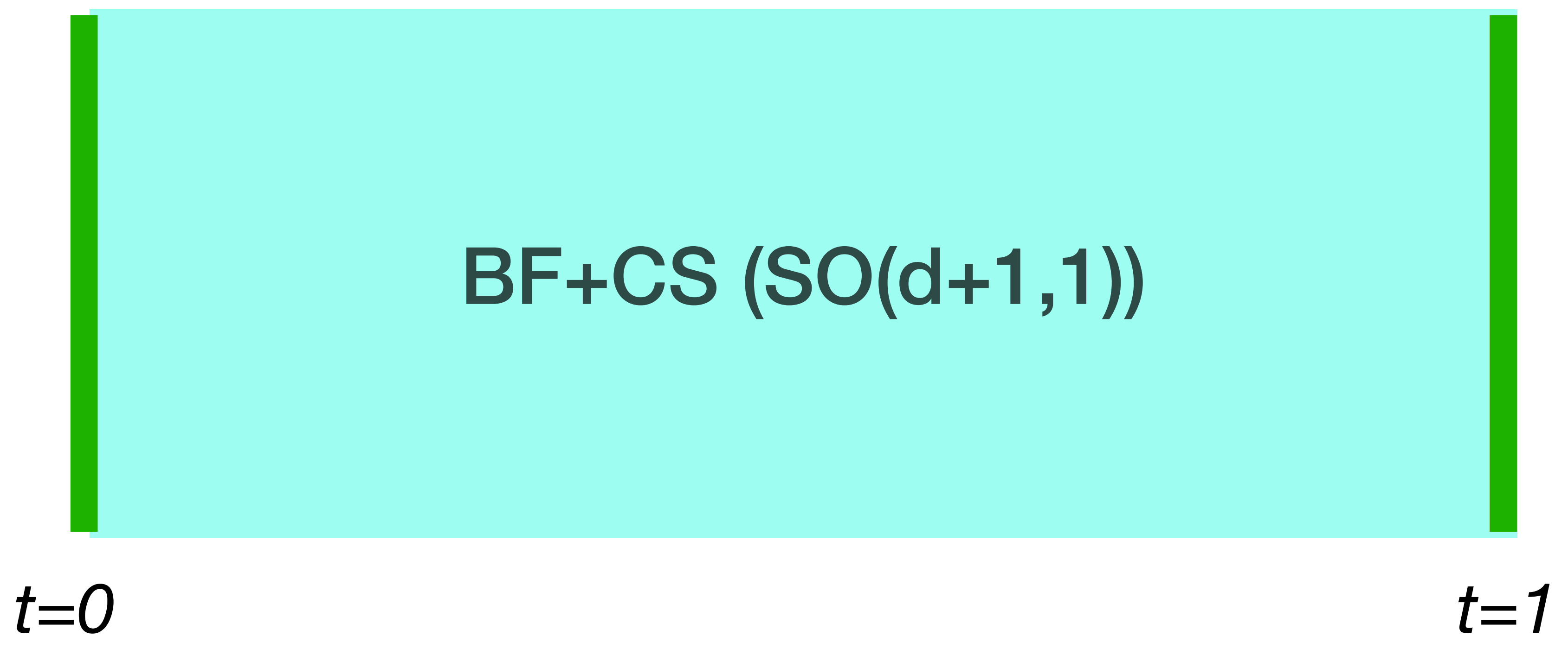
Where $\sum_n$ is the sum over the descendant in the multiplets and the binomial factor is due to $O(d)$ degeneracy

Gravitational CS SPT for Weyl Anomaly

Let us consider the following sandwich



We impose Dirichlet boundary conditions at the two boundaries

$$A\,|_{t=0} = A_0^{(\tau)} = \alpha e^{-\tau}\bar{e}^a P_a + \frac{1}{2}\omega^{ab}(e^{-\tau}\bar{e})L_{ab} + \alpha e^{\tau}\bar{e}^a K_a,$$

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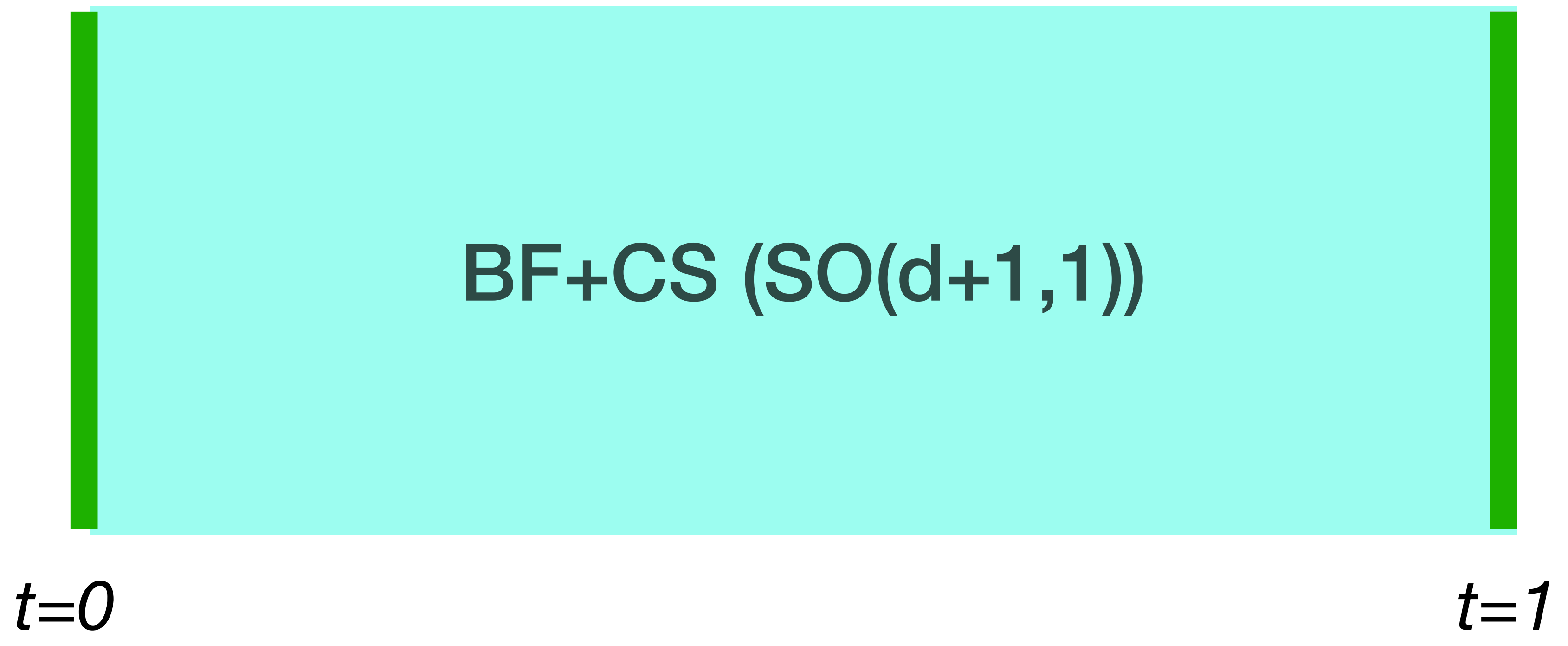

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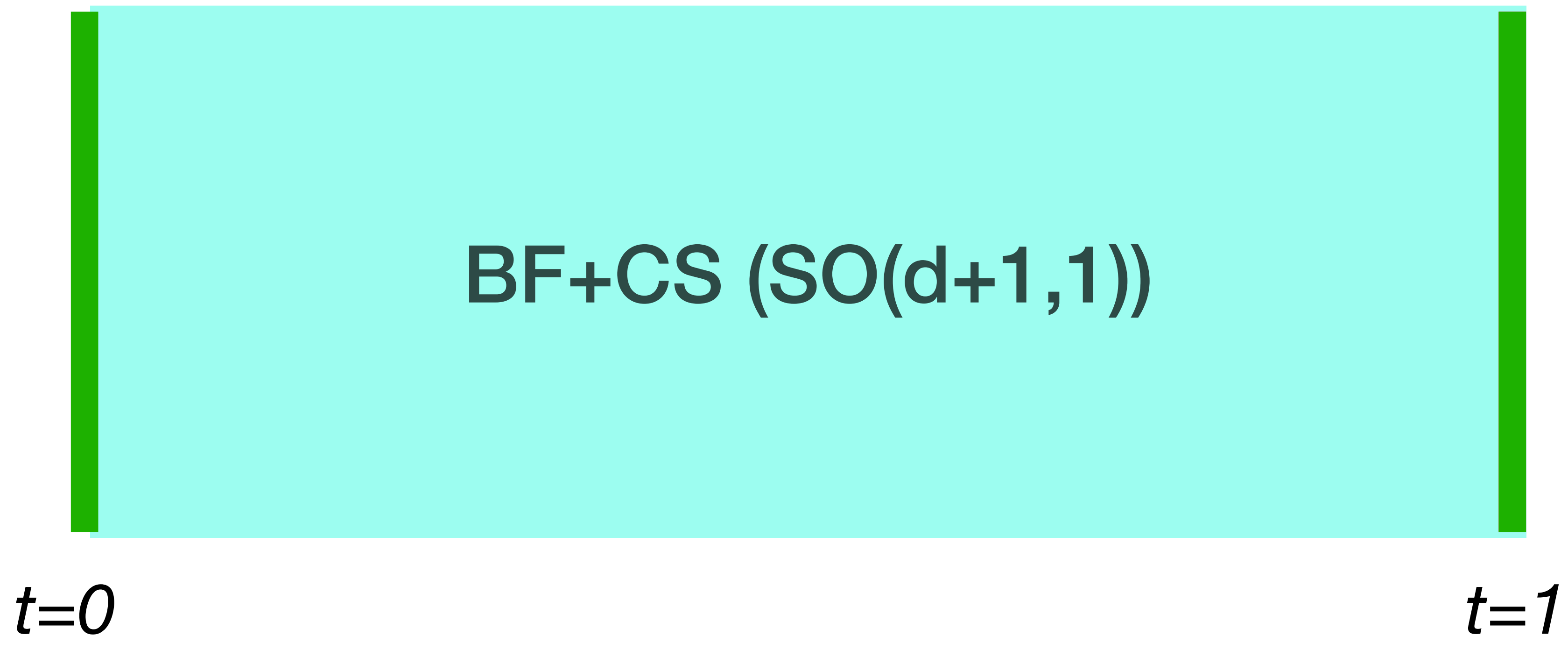

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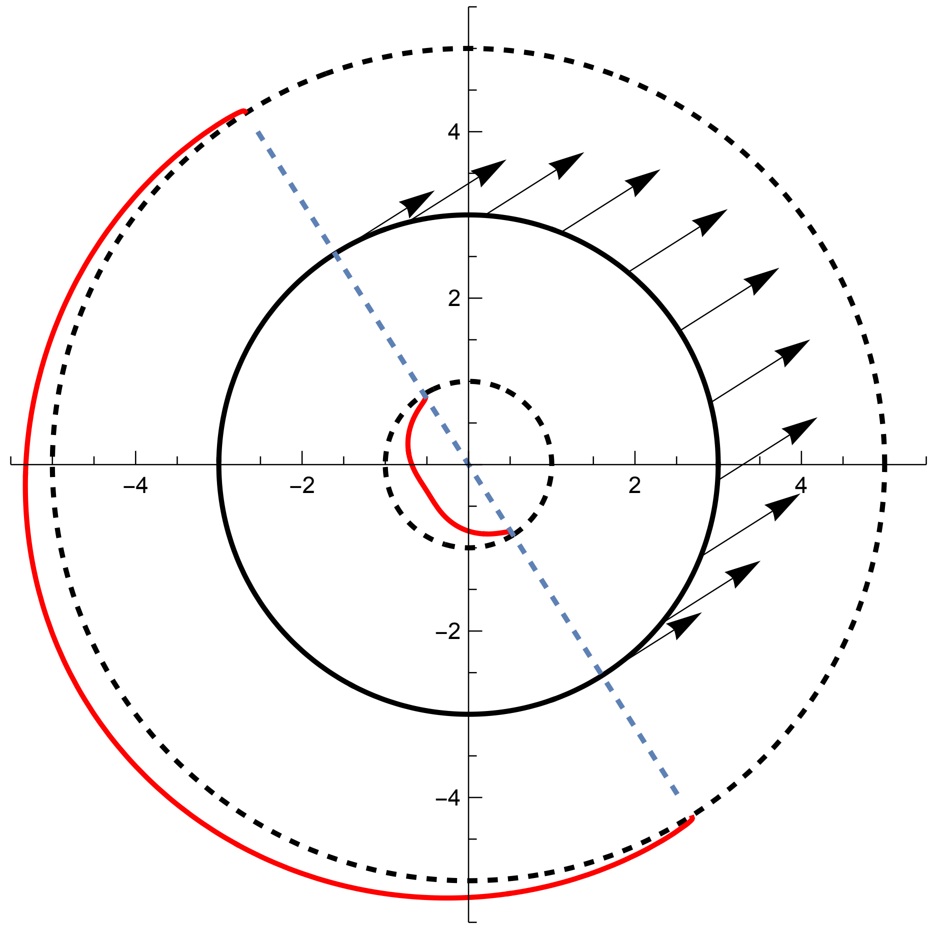
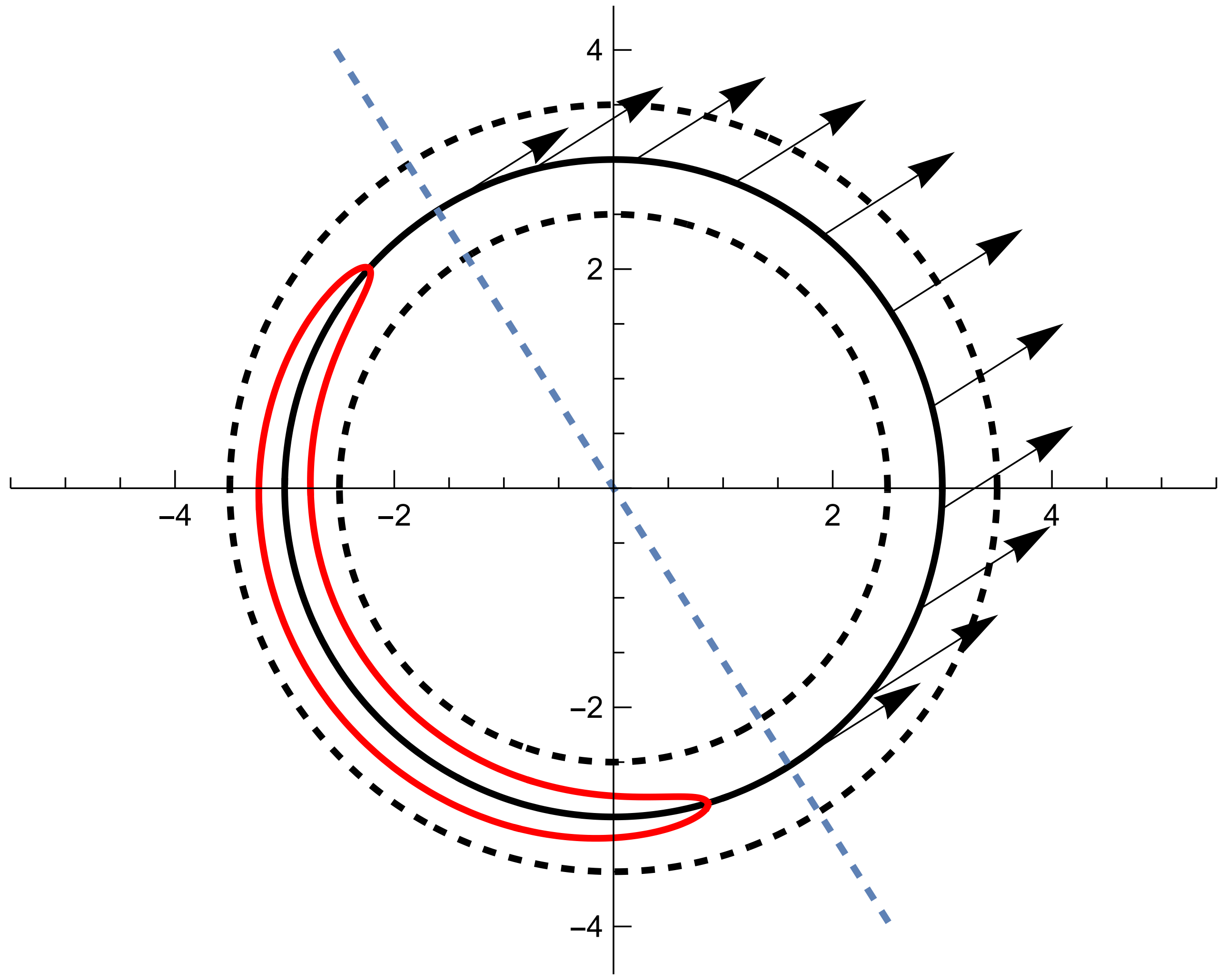
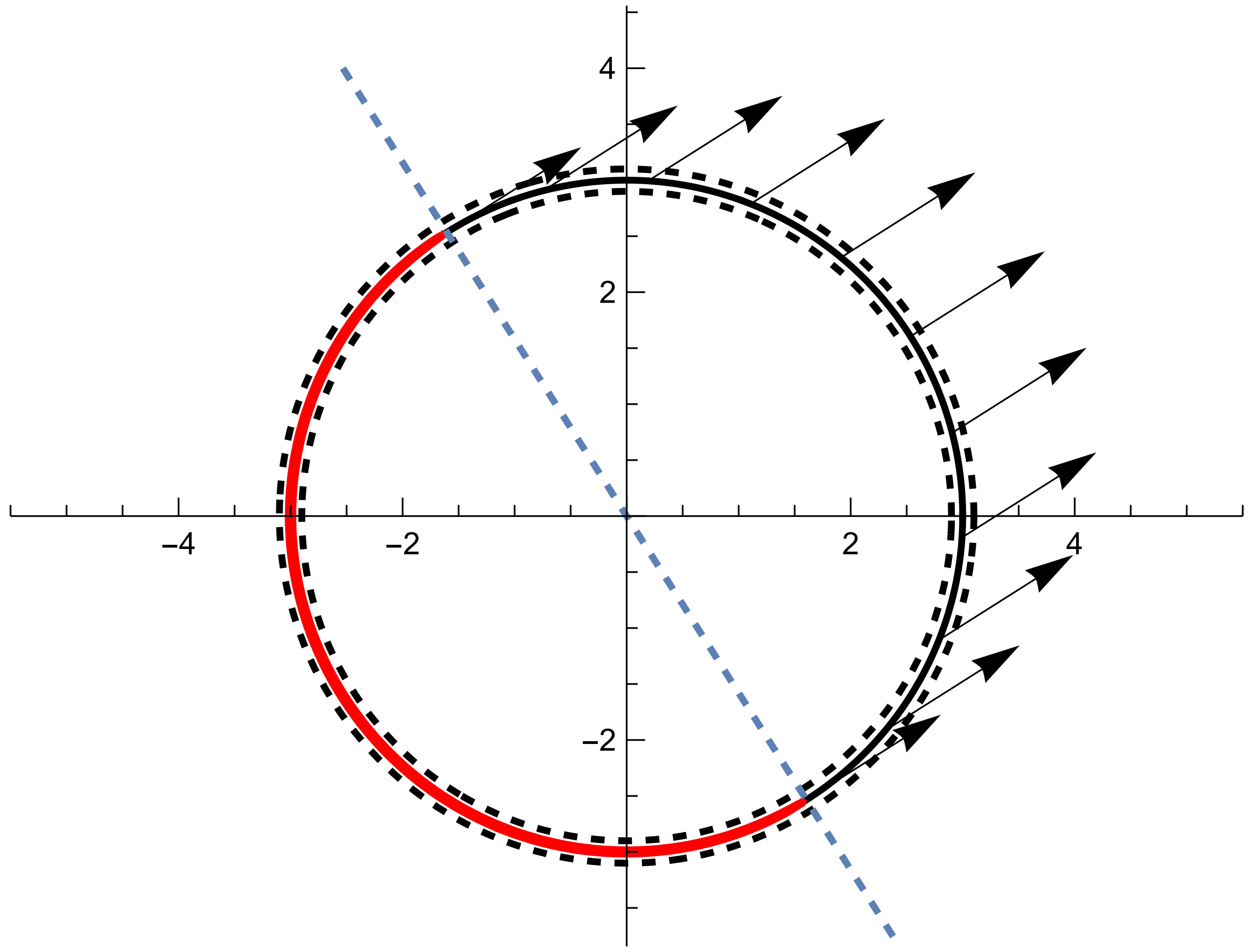
The bulk e.o.m. sets $A = \tilde{U}^{-1}(t)A_0^{(\tau)}\tilde{U}(t) + \tilde{U}(t)^{-1}d\tilde{U}(t), \quad U(0) = 1, \quad \tilde{U}(1) \equiv U = e^{\sigma D}$

How to move a point

It is always possible to find a tubular neighbourhood large enough such that the metric does not degenerate, for $\epsilon \to 0_+$







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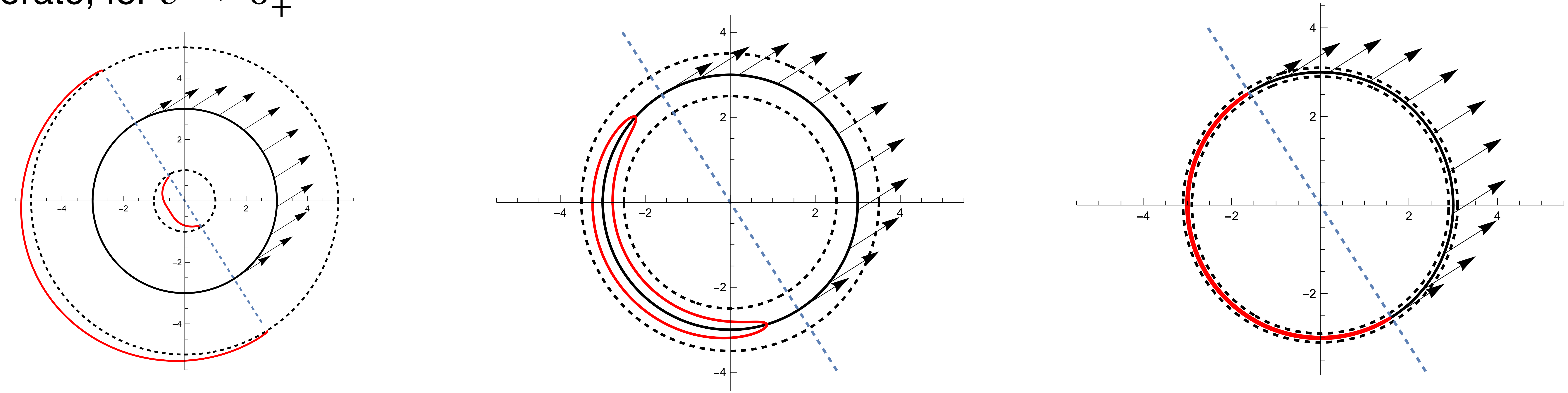

These are strange geometries, on the other hand geometry is diffeomorphic to ordinary flat space

$$A\,\big|_{\partial M_{d+1}} = A^{(\alpha)} = e^{-\alpha} A_0 e^{\alpha} + e^{-\alpha} d e^{\alpha}, \quad \alpha = X^a P_a \int_{r_0-\epsilon}^{r} dr' \rho(r')$$

Gapped SSB boundary conditions

[Maldacena, Moore, Seiberg;
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Kravek, McGrevy;
Antinucci, Benini, Rizi;
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FA, Bedogna, Dondi;
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We can restore the gauge invariance on the Dirichlet boundary by

$$\mathrm{Dir}(G): \qquad S_{\mathrm{Dir}(G)} = -\frac{i}{2\pi}\int_{\partial M_{d+1}} \left\langle A - \mathscr{A}^{(U^{-1})}, B - d_A\lambda \right\rangle - \frac{i}{2\pi}\int_{\partial M_{d+1}} \langle \lambda, F \rangle$$

Where $\mathscr{A}^{(U^{-1})} = U\mathscr{A}U^{-1} - dUU^{-1}, \quad \mathscr{F} = 0$, with $U \in \Omega^0(\partial M_{d+1}, G), \quad \lambda \in \Omega^{d-2}(\partial M_{d+1}, \mathfrak{g}^*)$,

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This BC is invariant under both the bulk G gauge symmetry and the boundary H