Automorphism Is All You Need: Transversal Non-Clifford Gates in 2+1D & Higher Symmetry

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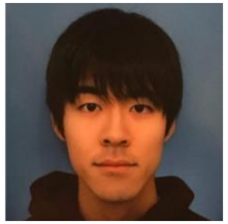
Related works

- 26xx.xxxxx: + Alison Warman and Sakura Schafer-Nameki (Oxford)
- 2511.02900, 2511.15783: automorphism-induced transversal non-Clifford logical gates and higher symmetries
- 2411.15848 Classifying logical gates in homological codes, new $C^m R_k$ transversal logical gates for quantum Fourier transform
- 2405.11719 non-Abelian self-correcting quantum memories with transversal CCZ logical gate

Collaborators:



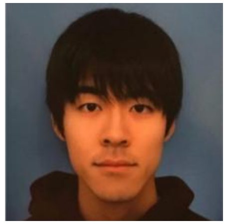
Ryohei Kobayashi (U Tokyo)

Guanyu Zhu (IBM)

Related works

- 2211.11764, 2311.05674 (Gauged SPT defects and Higher Group Symmetries provide new logical CZ, CS, S, CCZ, T gates)

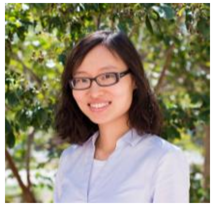
Collaborators:

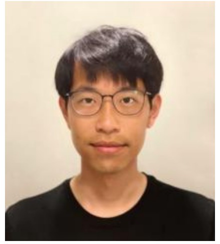


Maissam Barkeshli (UMD/Meta) Ryohei Kobayashi (U Tokyo)

- 2112.02137 new logical CZ gate in 4+1d self-correcting loop toric code

Collaborators:





Xie Chen (Caltech) Arpit Dua (QuEra) Chaoming Jian (Cornell) Cenke Xu (UCSB)

Quantum Computation: Why We Care

- Simulate quantum dynamics exponentially faster than classical computers, especially in 2+1d and higher dimensions

Hilbert space dimension grows exponentially with size: too resource-demanding for classical computers above 1+1d

- 2+1d Fermion Hubbard model (high Tc superconductors $\mathrm{La_3Ni_2O}$) [Quantinuum's Helios QPU]
- Quantum chemistry: new materials, new drugs
- Tackle computational demanding tasks

Quantum cryptography, [10000 physical qubits: 2603.28627]

Quantum optimization for complex data such as finance portfolio [IBM]

Brief History of Quantum Computing

- Early 1980: Manin and Feynman suggested quantum computers can simulate quantum dynamics much faster than classical computers
- 1994: Shor discovered the first error-correcting code (also 1996: Steane)
- 1997: Kitaev discovered surface code, use anyon and topological gauge theories in quantum computing.
- 1998: Threshold theorem by Kitaev, Knill,Laflamme,Zurek (also: 1999 Aharonov, Ben-Or)
- 2000: Freedman, Larsen, Wang showed non-Abelian anyon braiding can perform universal quantum computing (improved error correction by Lyons&Brown 2026)
- 2022: Panteleev,Kalachev discovered asymptotically good qLDPC code
- 2024: Google's Willow processor reached below critical error threshold-- logical error reduces as quantum computer scales up
- 2025: Microsoft's road map for topological quantum computing, Google's Echoes algorithm

Basic Components of Quantum Computing

- Replace classical bits by quantum states qubits
$$0 \to |0\rangle, \qquad 1 \to |1\rangle$$
- Encode logical qubits as ground states wavefunctions of many-body gapped lattice models of physical qubits (gap: protection)
- Quantum memory stores logical qubits and maintains coherence
- Computation: logical gates, unitary operator on logical qubits
- Performance: $[[n, k, d]]$ encodes k logical qubits with n physical qubits. Distance d: how many errors can be corrected
- Quantum simulation $H = \sum H_j$: $e^{-iHt} = \left(\prod_j e^{-\frac{iH_j t}{N}}\right)^N = \prod(\text{logical gates})$

(Emergent) Lattice Symmetry = Logical Gates

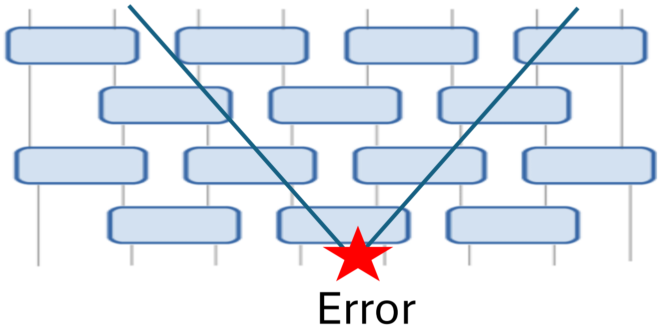
- Emergent symmetry of lattice model preserves ground state subspace: logical gates acting on logical qubits
- Example: SWAP on 2 code block copies by exchanging everything
- Fault-tolerant: locality-preserving of symmetry generator (error does not spread too much after applying the logical gate).

E.g. finite depth local unitary circuit (FDLC) or QCA

We will call FDLC as (nearly) transversal gates

Support of error bounded by code distance

Unitary with finite # layers



Challenge of Universal Computation with Low Spacetime Overhead

- Needs logical non-Clifford gates: universal quantum computation and outperform classical computers
- Reducing space overhead: interaction with small number of physical qubits, e.g. low space dimension
- Fault-tolerance: desirable to have transversal logical gates
- Transversal gates also important for reducing depth of quantum algorithms (max depth is limited by hardware)
- But transversal non-Clifford gates are hard to find in low dimension (no-go theorems for Pauli stabilizer: Eastin-Knill, Bravy-König...)
- How to find transversal non-Clifford logical gates in 2+1d?
- Reducing preparation overhead: wavefunction as simple as possible

New Routes for non-Clifford gates: Non-Abelian Topological Order & Symmetries

- Logical gates in non-Abelian topological orders are less explored

[Laubscher,Loss,Wootton], [PH,Kobayashi,Zhu], [Chen,Huang],[Davydova,Bauer,de la Fuente,Webster,Williamson, Brown],[Huang,Warman,Schafer-Nameki,Chen],[Warman,Schafer-Nameki] [Manjunath,Mattei,Tiwari,Ellison]

- Non-Abelian topological orders have been realized on device [Iqbal et al]
- Pauli stabilizer models $H=\sum H_i, H_i=\prod\text{Pauli}, H_iH_j=H_jH_i$, cannot describe non-Abelian topological order, but Clifford stabilizer models can give non-Abelian topological order: next simplest candidates
- Today: (generalized) symmetries from automorphisms gives new construction for transversal non-Clifford logical gates
- Other approaches: magic state distillation, dimension jump…

Outline

- Twisted Gauge Theory Lattice Model: Clifford stabilizer model for non-Abelian topological code

- Automorphism in twisted gauge theory as generalized symmetry: higher group & non-invertible symmetries

- Transversal non-Clifford logical gates from automorphisms:

T,CS gates in 2+1D, CCZ gate in 5+1D self-correcting quantum memory. Byond Bravy-König bound.

Twisted Gauge Theory Lattice Model For Non-Abelian Topological Code

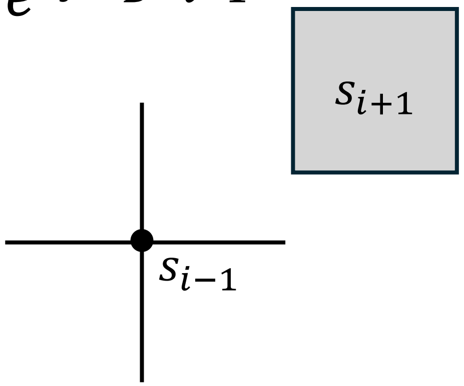
Review: Physics of Homological Codes

- Homological codes: construct CSS code from triangulated manifold
- Physically: higher-form gauge theories on space manifold M
- i-form Z_N gauge field a in D spacetime dimension (focus on $N = 2$)

$$\frac{N}{2\pi}\int a_i d b_{D-i-1}$$

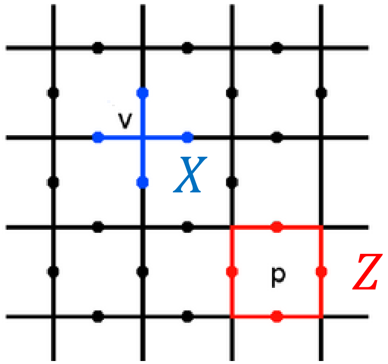
- Equivalently: $(D-i-1)$-form magnetic gauge field b
- Ground state degeneracy: $\dim H^i(M, \mathrm{Z}_N) \coloneqq N^k$ #logical qubit k
- Symmetries: Electric Wilson operator $e^{i\int a_i}$, magnetic operator $e^{i\int b_{D-i-1}}$
- CSS Stabilizer Hamiltonian: physical qubit on i-simplices

$$H = -\sum S_i = -\sum_{s_{i-1}} \prod_{s_{i-1}\in\partial s_i} X_{s_i} - \sum_{s_{i+1}} \prod_{s_i\in\partial s_{i+1}} Z_{s_i}$$



Review: Z_2 1-form Gauge Theory / Toric Code

[Kitaev]



- Each edge has a qubit, acted by Pauli $XZ = -ZX$

Z: Z_2 gauge field, X: electric field

- Hamiltonian $H = -\sum S_i = -\sum A_v - \sum B_p$

Gauss law: $\sum A_v \to (-1)^{\nabla\cdot E}$ Flux term: $\sum B_p \to (-1)^{\oint a}$

- 2 logical qubits on torus space (holonomies on 2 independent 1-cycles)
- Onsite 1-form symmetry: Wilson lines $\overline{Z_\gamma} = \prod_e Z_e$ for edges along 1-cycle γ
- Onsite 1-form symmetry: magnetic lines $\overline{X_{\gamma'}} = \prod_e X_e$ for edges intersect 1-cycle γ' on dual lattice

$\overline{X_{\gamma'}}$ is transversal logical NOT gate for logical qubit on cycle γ st $\#(\gamma,\gamma') = \text{odd}$

Clifford Stabilizer Model for Non-Abelian Topological Order: Twisted $(\mathrm{Z}_2)^3$ Gauge Theory

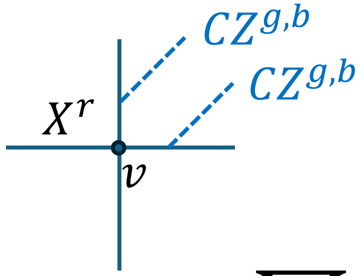
- Twisted $G = (\mathrm{Z}_2)^3$ gauge theory in 2+1D

$$\int \omega = \pi \int a_r \cup a_g \cup a_b$$

Include dual fields: $\pi \int a_r \cup a_g \cup a_b + \pi \int (a_r \cup d\widetilde{a_r} + a_g \cup d\widetilde{a_g} + a_b \cup d\widetilde{a_b})$

- Lattice model: (1) lattice model for group cocycle ω symmetry protected topological (SPT) phase [Chen,Gu,Liu,Wen]

and (2) gauge G symmetry



- Clifford stabilizer model: $\{S_X^r, S_X^g, S_X^b, S_Z^r, S_Z^g, S_Z^b\}$

$$(S_X^r)_v = \left(\prod_{\partial e \supset v} X_e^r\right) \prod_{e', e'' \in E^{gb}} CZ_{e', e''}^{g,b}, \qquad (S_Z^r)_f = \prod_{e \in \partial f} Z_e^r$$

E^{gb}: $\int \tilde{v} \cup \widetilde{e'} \cup \widetilde{e''} = 1,\ \tilde{v}(v') = \delta_{v,v'},\ \tilde{e}(e') = \delta_{e,e'}$

Clifford Stabilizer Model for Non-Abelian Topological Order: Twisted $(\mathrm{Z}_2)^3$ Gauge Theory

- Twisted $(\mathrm{Z}_2)^3$ Gauge Theory is equivalent to untwisted D8 gauge theory [Coste]...

$a_g \cup a_b = d\tilde{a}_r \Rightarrow$ extend $\mathrm{Z}_2^g \times \mathrm{Z}_2^b$ by dual of Z_2^r: D_8 group

- Electric operator E: logical Pauli Z loops, obey Z_2 fusion
- Magnetic operator M: Pauli X loops need to decorate with projector to be logical operators:

$$M^r = (X^r \text{ loop}) \cdot (1 + Z^g \text{ loop})(1 + Z^b \text{ loop})$$

- Fusion rule: $M^r \times M^r = (1 + E^g)\left(1 + E^b\right) = 1 + E^g + E^b + E^g E^b$

Same fusion rule as 2d-irrep of D8

Automorphism Symmetry in Twisted Gauge Theories

Automorphism In Twisted Gauge Theories

- Gauge Group G, Topological action: group cocycle $\omega \in H^D(BG, U(1))$
- Automorphism $\rho: G \to G$.

Faithful symmetry is outer automorphism (= $\mathrm{Aut}(G)/\mathrm{Inn}(G)$), but we will focus on automorphism $\mathrm{Aut}(G)$ here

- Example: $G = \mathrm{Z}_2^{(1)} \times \mathrm{Z}_2^{(2)}$, ρ permutes $(g_1, g_2) \leftrightarrow (g_2, g_1)$

Gauge fields are also permuted $a_1 \leftrightarrow a_2$

Invertible Automorphism Symmetry

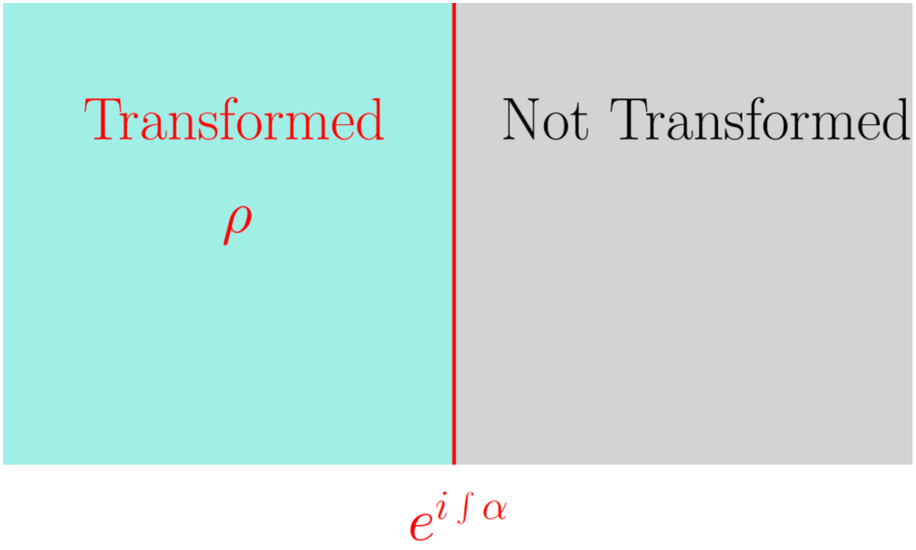
- Case 1: automorphism preserves the topological action

$$[\rho^*\omega] = [\omega], \qquad \rho^*\omega = \omega + d\alpha(\rho)$$

- Automorphism symmetry:

$$U_\rho = e^{i\int \alpha(\rho)} V_\rho,$$

V_ρ: automorphism action



- Example: $\mathrm{Z}_N^{(1)} \times \mathrm{Z}_N^{(2)}$ gauge theory

Swap automorphism ρ: $a_1 \leftrightarrow a_2$

$$\omega = \frac{k}{2\pi} a_1 da_2 \Rightarrow \rho^*\omega = \frac{k}{2\pi} a_2 da_1 = \omega + d\left(\frac{k}{2\pi} a_1 a_2\right) \Rightarrow U_\rho = e^{\frac{ik}{2\pi}\int a_1 a_2} V_\rho$$

Fusion Rule of Automorphism Symmetry Can be Extension of $\mathrm{Aut}(G)$

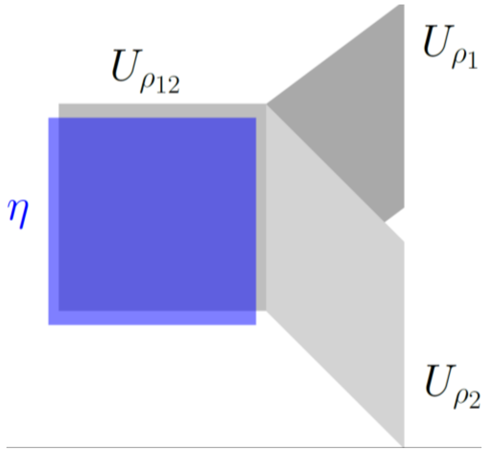
- Automorphism symmetry may not obey the same fusion rules as automorphism group $\mathrm{Aut}(G)$ multiplication law
- Automorphisms ρ_1, ρ_2 and $\rho_2\rho_1 := \rho_{12}$, symmetries:

$$U_{\rho_1} = e^{i\int \alpha(\rho_1)} V_{\rho_1}, \qquad U_{\rho_2} = e^{i\int \alpha(\rho_2)} V_{\rho_2}, \qquad U_{\rho_{12}} = e^{i\int \alpha(\rho_{12})} V_{\rho_{12}}$$

- While $V_{\rho_2} V_{\rho_1} = V_{\rho_{12}}$, symmetry U obeys

$$U_{\rho_2} \cdot U_{\rho_1} = e^{i\int \alpha(\rho_2) + \rho_2(\alpha(\rho_1))} V_{\rho_{12}} = \eta\, U_{\rho_{12}}$$

Fusion rule of automorphism symmetry is extended by gauged SPT: $\eta = e^{i\int \alpha(\rho_2) + \rho_2(\alpha(\rho_1)) - \alpha(\rho_{12})}$



Example: Another Twisted $(\mathrm{Z}_2)^4$ Gauge Theory in 2+1D

- Another twisted $G=(\mathrm{Z}_2)^4$ Gauge Theory in 2+1D with action

$$\omega=\frac{\pi}{2}(a_1\cup da_4-da_1\cup a_3)$$

- Automorphism ρ: $(g_1,g_2,g_3,g_4)\to(g_1,g_2,g_3+g_2,g_4+g_2)$ mod 2,

Z_2 multiplication law $\rho^2=1$ in $\mathrm{Aut}(G)$

Transforms gauge fields $a_3\to a_3+a_2,\ \ a_4\to a_4+a_2$

- Action transforms as $\omega\to\omega+\frac{\pi}{2}(a_1\cup da_2+da_1\cup a_2)=\omega+d\alpha$,

$\alpha=\frac{\pi}{2}a_1\cup a_2\Rightarrow$ Automorphism symmetry $U_\rho=i^{\int a_1\cup a_2}V_\rho$

$$U_\rho(M)^2=(-1)^{\int_M a_1\cup a_2}=(\text{Cluster state SPT on } M)$$

Automorphism is extended from Z_2 to be Z_4 symmetry

Example: Previous Twisted $(\mathrm{Z}_2)^3$ Gauge Theory in 2+1D

- Twisted $(\mathrm{Z}_2)^3$ Gauge Theory in 2+1D

$$\int \omega = \pi \int a_r \cup a_g \cup a_b$$

- Z_2 Automorphism ρ: $\left(g_r, g_g, g_b\right) \rightarrow (g_r, g_g + g_r + g_b \bmod 2, g_b)$

Transform gauge fields $a_g \rightarrow a_g + a_r + a_b$

- $\rho^* \omega = \omega + \pi\left(a_r^2 \cup a_b + a_r \cup a_b^2\right) = \pi\left(\frac{da_r}{2} \cup a_b + a_r \cup \frac{da_b}{2}\right)$

$= \omega + d\alpha, \alpha = \frac{\pi}{2} a_r \cup a_b \Rightarrow U_\rho = i^{\int a_r \cup a_b} V_\rho$

$U_\rho(M) \cdot U_\rho(M) = (-1)^{\int_M a_r \cup a_b} = 1$ still Z_2 symmetry, does not extend

- Automorphism symmetry generated by finite depth circuit that commute with Clifford stabilizers

Higher Group Automorphism Symmetry

Related: [Chen,Dua,PH,Jian,Shirley,Xu]

- $\mathbb{Z}_2 \times \mathbb{Z}_2$ 2-form gauge theory in 4+1D

$$\frac{1}{2\pi} b_1 d b_2 + \left(\frac{2}{2\pi} b_1 d\tilde{b}_1 + \frac{2}{2\pi} b_2 d\tilde{b}_2 \right)$$

Integrate out Lagrangian multipliers $\tilde{b}_1, \tilde{b}_2$: $\omega = \frac{1}{2\pi} b_1 d b_2$

- $\mathbb{Z}_2$ Automorphism $b_1 \to b_1 + b_2$:

$$\omega = \frac{1}{2\pi} b_1 d b_2 \to \omega + \frac{1}{2\pi} b_2 d b_2 = \omega + d\alpha, \qquad \alpha = \frac{1}{4\pi} b_2 b_2$$

$\mathbb{Z}_4$ Automorphism symmetry $U = e^{\frac{i}{4\pi}\int (b_2)^2} V$, [Walker,Wang],[Gaiotto,Kapustin,Seiberg,Willet], [PH,Lam,Seiberg],[Tsui,Wen]...

$$U(M) \cdot U(M) = e^{\frac{2}{4\pi} i \int_M (b_2)^2} = (\text{Gauged 1 form SPT on } M)$$

$U(T^4)^4 = 1$, $U(\mathrm{CP}^2)^4 = e^{i \int_\kappa b}$ fusing 4D domain walls produces 2D membrane

$\Rightarrow$ 3-group symmetry

Non-Invertible Automorphism Symmetry

- Case 2: automorphism does not preserve topological action

$$[\rho^*\omega] \neq [\omega]$$

In such case, automorphism becomes non-invertible symmetry

- Sandwich construction of non-invertible symmetry domain wall:

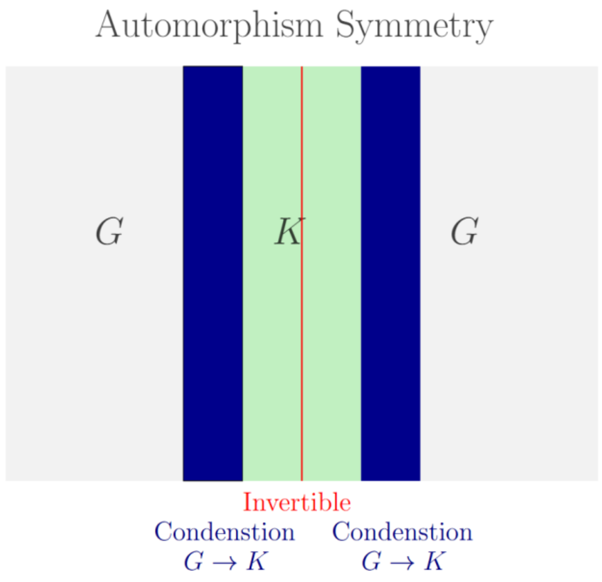
Subgroup $\iota: K \to G$ such that $[\rho^*\iota^*\omega] = [\iota^*\omega]$

- Symmetry operator:

$$U_\rho = \prod \left(e^{i\int \alpha(\rho)} V_\rho\right) \prod$$

$\prod$: projection to K gauge theory

[PH,Kobayahi,Zhang],[Costa,Cordova,PH]



Example: Mixed Theta Term in 3+1D

- $U(1)\times U(1)$ gauge theory with topological action

$$\int \omega = \frac{\theta}{(2\pi)^2}\int da_1 da_2$$

- Automorphism $a_1 \to a_1 + a_2 : \omega \to \omega + d\alpha,\ \ \alpha = \frac{\theta}{(2\pi)^2} a_2 da_2, U = e^{i\int \alpha} V$

For $\theta = 0, \pi$: well-defined by itself (IQH); For $\theta \neq 0, \pi$: needs other dof (FQH)

- For $\theta = 0$: invertible automorphism symmetry (group $\mathbb{Z}$)
- For $\theta = \pi$: invertible automorphism symmetry (group $\mathbb{Z}$), 2-group

$$\rho^* \omega - \omega = \pi \frac{da_2}{2\pi} \cup w_2 = \pi \frac{da_2}{2\pi} \cup \Delta B_2, B_2: \text{magnetic 1 form symmetry}$$

- For $\theta \neq 0, \pi,\ \ \theta = 2\pi \frac{p}{q}$: non-invertible automorphism symmetry

Fractional quantum Hall state on domain wall $\Rightarrow$ non-invertible

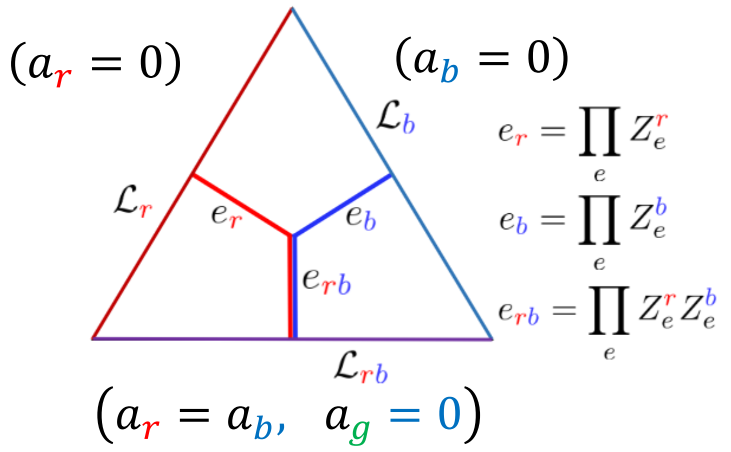
Related: [Choi,Lam,Shao],[Cordova,Ohmori],...

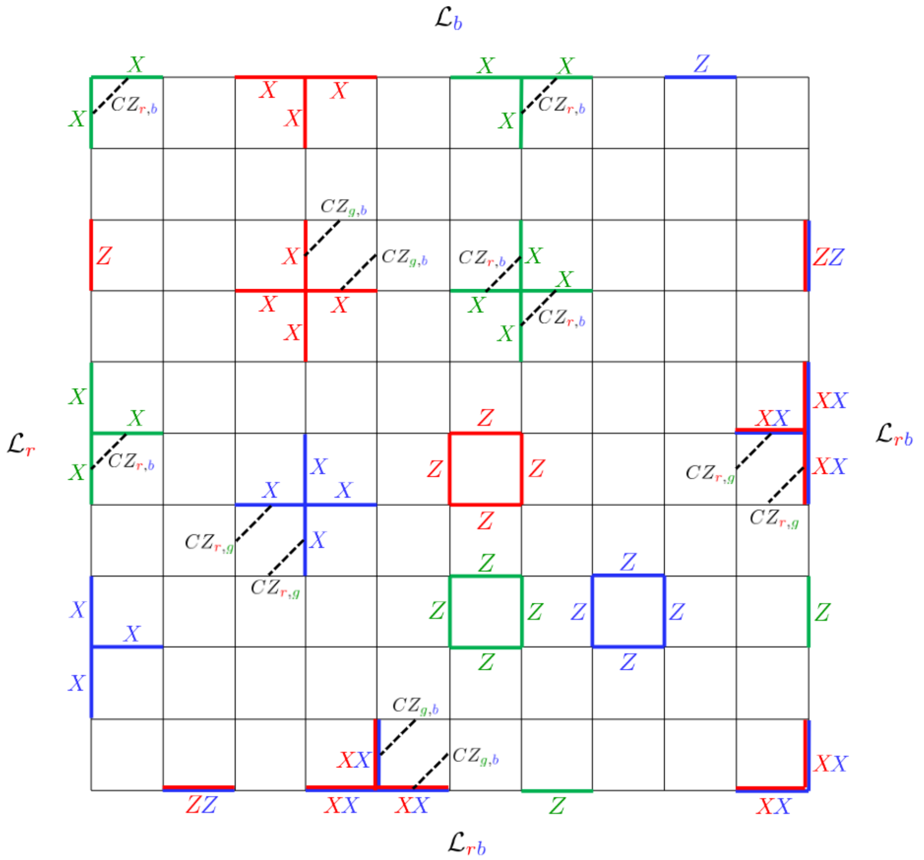
Transversal Non-Clifford Logical Gates From Automorphism Symmetry

Logical T Gate in 2+1D

[PH,Kobayashi,Zhu]

- Triangular region with gapped boundaries: single logical qubit





Logical qubit can be labelled by

Holonomy of $a_r = a_b$ on bottom

Logical T Gate in 2+1D

[PH,Kobayashi,Zhu]

- Automorphism symmetry $a_g \to a_g + a_r + a_b$ without boundaries: finite depth circuits that commute with Clifford stabilizers

$$U = i^{\int a_r \cup a_b} V$$

- Automorphism symmetry in presence of gapped boundary: extra boundary gauged SPT

[PH,Kobayashi,Zhu]

$$a_r \cup a_b \Big|_{a_r = a_b := a} = \frac{da}{2} \Rightarrow U = \left(i^{\int a_r \cup a_b} e^{-\frac{\pi i}{4} \int_{\mathrm{bdy}\, L_{rb}} a} \right) V$$

Finite depth circuit $U = \left(\prod \mathrm{CS}^{r,b} \prod \left(\mathrm{CS}^{r,b}\right)^{\dagger} \prod_{\mathrm{bdy}} \mathrm{T}^{\dagger} \right) \prod \mathrm{CNOT}^{r,g} \mathrm{CNOT}^{b,g}$

- Transversal logical T gate in 2+1D Clifford stabilizer: $U|m\rangle = \bar{T}^{\dagger}|m\rangle$

$m = 0{,}1$: eigenvalue of holonomy of $a_r = a_b$ on boundary L_{rb}

Logical T Gate in 2+1D

[PH,Kobayashi,Zhu]

Protocol for T magic state on triangle region:

- $\mathbb{Z}_2^r \times \mathbb{Z}_2^b$ toric code state $\overline{|+\rangle}$, initialize $\mathbb{Z}_2^g$ qubit to be $\otimes\, |0\rangle$
- Switch to twisted $\mathbb{Z}_2^r \times \mathbb{Z}_2^g \times \mathbb{Z}_2^b$ Clifford stabilizer by measuring S_X^g stabilizers over entire space (needs just-in-time decoder to correct errors as in [Davydova,Bauer,de la Fuente,Webster,Williamson, Brown])
- Apply transversal logical $\overline{\mathrm{T}}^\dagger$ gate U
- Measure Pauli Z^g to switch back to $\mathbb{Z}_2^r \times \mathbb{Z}_2^b$ toric code

Final state: $\overline{\mathrm{T}}^\dagger\overline{|+\rangle}$

- Universal quantum computation: Cliffords + T magic state

Logical T Gate in 2+1D

[PH,Kobayashi,Zhu]

- Comparison with Magarita et al (2503.15751) from code switching
- Relation between domain walls:

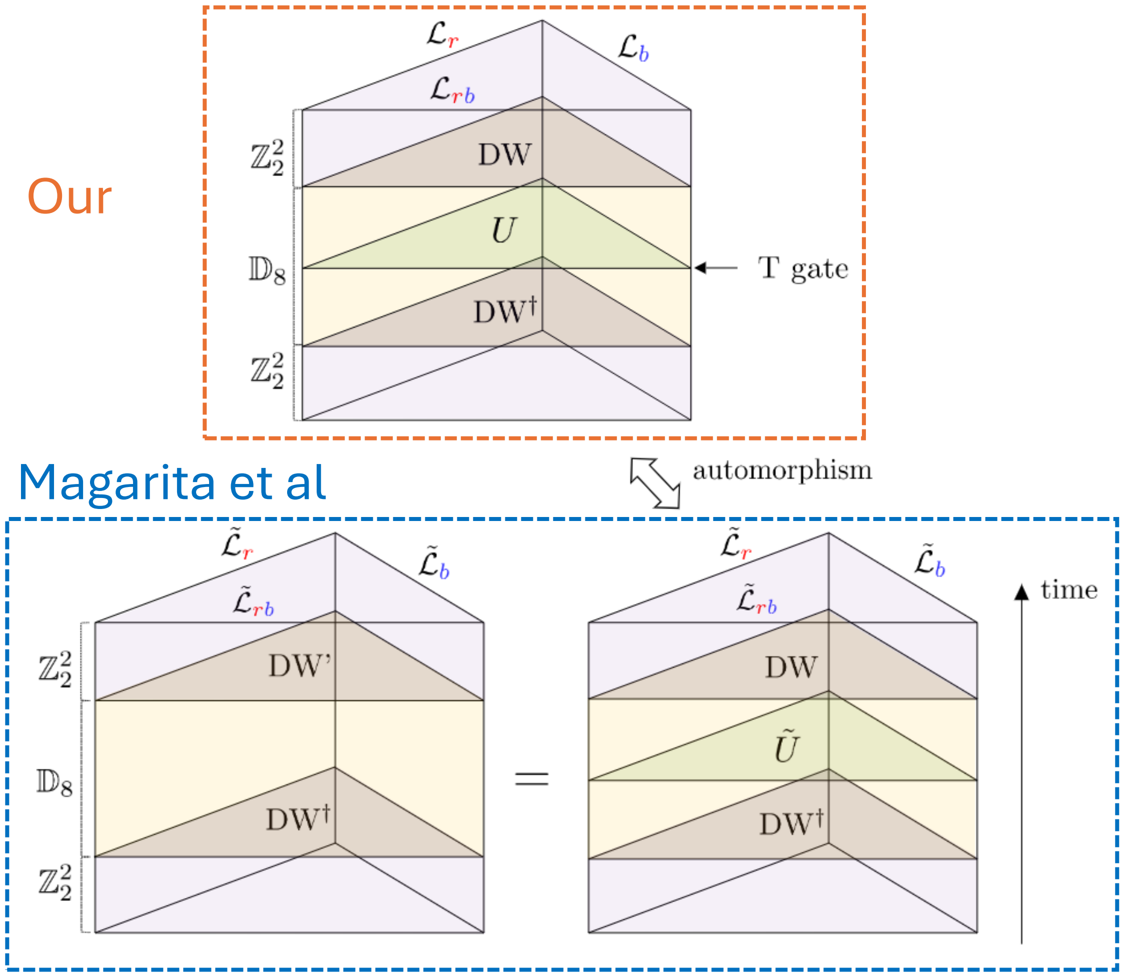
$\mathrm{DW}' = \mathrm{DW} \cdot \widetilde{U}$

$\widetilde{U}$: $m_g \leftrightarrow m_b, \quad e_g \leftrightarrow e_b$

- Automorphism relating our setup & Magarita et al:

$m_r \leftrightarrow m_{rb}, \ m_g \leftrightarrow m_{gb},$

$e_b \leftrightarrow e_{rgb}$



Logical CS Gate in 2+1D

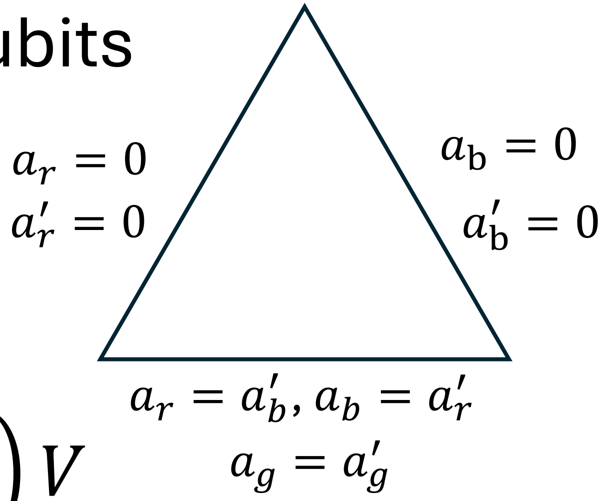
[PH,Kobayashi,Zhu]

- 6 layers of Clifford stabilizer models, diagonal automorphism

$$U \otimes U' \otimes \prod \mathrm{SWAP} = \left(i^{\int a_r \cup a_b} i^{\int a_r' \cup a_b'} (-1)^{\text{higher cup prod}} \right) V$$

- Triangular region with gapped boundaries: 2 logical qubits

Labelled by holonomy of a_r, a_b on bottom boundary



- Logical CS gate from automorphism with bdy

$$\left(i^{\int a_r \cup a_b} i^{\int a_r' \cup a_b'} (-1)^{\text{higher cup prod}} i^{\int_{\text{bdy 3}} a_r \cup_1 a_b} \right) V$$

$$U \otimes U' \otimes \prod \mathrm{SWAP} |m_r, m_b\rangle = \overline{\mathrm{CS}} |m_r, m_b\rangle$$

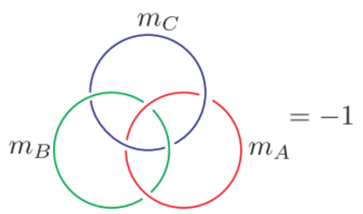
Transversal CCZ Gate in 5+1D Self-Correcting Memory

[PH,Kobayashi,Zhu]

- Clifford stabilizer model for $(\mathbb{Z}_2)^3$ 2-form gauge fields

$$\int \omega = \pi \int a_A \cup a_B \cup a_C$$

- Self-correcting non-Abelian quantum memory: 2-form gauge theory without particle excitations, has non-Abelian m excitations



- Distance $d = O(n^{\frac{2}{5}})$ overcome Pauli stabilizer barrier $O(n^{\frac{1}{3}})$ from Bravy-König theorem

Transversal CCZ Gate in 5+1D Self-Correcting Memory

[PH,Kobayashi,Zhu]

- Automorphism of $(\mathrm{Z}_2)^3$: $a_A \leftrightarrow a_B$

$$\omega = \pi a_A \cup a_B \cup a_C \to \pi a_B \cup a_A \cup a_C = \omega + d\alpha,$$
$$\alpha = \pi(a_A \cup_1 a_B) \cup a_C$$
$$U = (-1)^{\int (a_A \cup_1 a_B) \cup a_C} V$$

- Transversal Logical CCZ gate: Wu 5-manifold space $SU(3)/SO(3)$

Single 2-cycle with even self intersection: 3 logical qubits for A,B,C

$$a_A = s_A w_2, \qquad a_B = s_B w_2, \qquad a_C = s_C w_2, \qquad s_{I=A,B,C} \in \{0,1\}$$

$$U|s_A, s_B, s_C\rangle = (-1)^{s_A s_B s_C \int w_3 \cup w_2} = (-1)^{s_A s_B s_C} = \overline{\mathrm{CCZ}}|s_A, s_B, s_C\rangle$$

- Universal quantum computation: code switching to 5D toric code similar to Magarita et al (2503.15751). Advantage here: single shot, more efficient

Beyond Bravy-König Bound

- Bravy-König bound: transversal Nth level Clifford hierarchy in Pauli stabilizer codes needs N space dimensions (e.g. color code)

Nth level Clifford hierarchy P_N: $P_1 = \mathrm{Pauli}$, $P_2 = \mathrm{Cliffords}$

$$P_{N>1} = \{\text{unitary } p_N : p_N p_1 p_N^{-1} p_1^{-1} \in P_{N-1}\}$$

- Pauli stabilizers in 2D can only do transversal Clifford gate
- Generalized Bravy-König bound [PH,Kobayashi,Zhu]: non-Abelian codes in 2D can do transversal non-Clifford gate
- In fact, general 2D non-Abelian codes can do arbitrary hierarchy transversal gates $\overline{R_m}$ [Warman, Schafer-Nameki]

Conclusion

- Automorphism symmetry in twisted gauge theories carry additional gauged SPT

$\Rightarrow$ Automorphism can become extension and/or higher group and/or non-invertible symmetries

- Automorphism symmetry in twisted gauge theories can give rise to new transversal non-Clifford logical gates in non-Abelian codes, such as T,CS gates in 2+1D and CCZ gate in 5+1D self correcting memory, described by Clifford stabilizer models
- Generalized Bravy-König bound