Entanglement asymmetry and higher form symmetries.

The limits of symmetry breaking

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Based on 2512.15898 with Francesco Benini and Stathis Vitouladitis.

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Introduction

Motivation

1. **Symmetries** are a powerful tool in physics. In quantum field theory:
   - They are an organizing principle.
   - They are a handle to hold on in strongly coupled regimes.
2. **Quantum information** measures have proven fruitful to study in quantum field theory and holography:
   - Entanglement entropy has given the Ryu-Takayanagi formula.
   - Relative entropies have been used to prove monotonicity theorems (such as the c-theorem) and the averaged null energy condition (ANEC).

Entropic order parameters are entropic measures that characterize the breaking (or not) of a symmetry in a quantum state.

In this talk we explore such a measure in systems with spontaneously broken continuous p-form symmetries.

Spontaneously broken symmetries

- Once you know the symmetry of a system, you would like to know how is it realized in the vacuum. **Is it spontaneously broken?**
- Typically the answer to such question is answered using **order parameters:**
  - For 0-form symmetries: local order parameters $\mathcal{O}(x)$.

$$\langle \mathcal{O}(x) \rangle \neq 0 \quad \Rightarrow \quad \text{SSB}$$

  - For p-form symmetries: extended (p-dimensional) order parameters $W(\gamma_p)$.

$$\langle W(\gamma_p) \rangle \sim e^{-\text{Perimeter}(\gamma_p)} \quad \Rightarrow \quad \text{SSB}$$

- This is a very successful story,
  - Universal physics in the IR.
  - Landau classification and generalizations such as topological order.
  - Characterization of confinement.

The star of this show: Entanglement Asymmetry

Consider a system $\mathcal{M}$ with a symmetry G and in a quantum state ρ. Define a symmetrized state ρ_{S},

$$\rho_{\mathsf{S}} = \int_G da U_a \rho U_a^\dagger, \qquad a \in G \, .$$

The **entanglement asymmetry** is (Ares *et al.*, 2023),

$$\boxed{\Delta S(\rho) = S(\rho || \rho_{\mathsf{S}}) = \operatorname{tr}\Big(\rho\,(\log\rho - \log\rho_{\mathsf{S}})\Big)}$$

It has the following properties,

$$\begin{aligned} &\Delta S(\rho) \geqslant 0 \, , \\ &\Delta S(\rho) = 0 \quad \Leftrightarrow \quad \rho = \rho_{\mathsf{S}} \, . \\ &\Delta S(\rho) \geqslant \Delta S(\xi(\rho)) \end{aligned}$$

Sometimes easier to compute the **Rényi asymmetries**,

$$\Delta\mathcal{S}_N(\rho) \coloneqq \frac{1}{1-N}\log\frac{\operatorname{tr}\left(\rho_{\mathsf{S}}^N\right)}{\operatorname{tr}\left(\rho^N\right)} \, , \qquad \lim_{N\to 1}\Delta\mathcal{S}_N(\rho) = \Delta S(\rho)$$

The star of this show: Entanglement Asymmetry

Consider a **subsystem** $\mathcal{A} \subset \mathcal{M}$ in a reduced quantum state $\rho_{\mathcal{A}}$. Assume the Hilbert space factorizes $\mathcal{H}_{\mathcal{M}} \simeq \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{A}'}$ and that the symmetry operators split,

$$U_a(\mathcal{M}) \to U_a(\mathcal{A})\, U_a(\mathcal{A}')\ ,$$

The symmetrized reduced density matrix is, $\rho_{\mathcal{A},\mathrm{S}}$,

$$\rho_{\mathcal{A},\mathrm{S}} = \int_G da U_a(\mathcal{A}) \rho_{\mathcal{A}} U_a^\dagger(\mathcal{A}), \qquad a \in G\ .$$

The entanglement asymmetry is (Ares *et al.*, 2023),

$$\boxed{\Delta S(\rho_{\mathcal{A}}) = S(\rho_{\mathcal{A}} || \rho_{\mathcal{A},\mathrm{S}}) = \mathrm{tr}\Big(\rho_{\mathcal{A}} \left(\log \rho_{\mathcal{A}} - \log \rho_{\mathcal{A},\mathrm{S}}\right)\Big)}$$

Again, it will sometimes be easier to compute the Rényi asymmetries.

Entanglement Asymmetry in Quantum Field Theory

In Quantum Field Theory, $\mathcal{A}$ is typically a spatial subsystem. Some comments are in order,

- One must *regulate* the entangling region $\partial\mathcal{A}$ and specify *boundary conditions*. The regulator drops from $\Delta S(\rho_{\mathcal{A}})$, which is **UV finite if the symmetry is spontaneously broken (or softly).**
  $\rightarrow$ Our results will not depend on the choice of boundary conditions or the regulator.
- Consider two spherical subregions $\mathcal{A}, \mathcal{B}$ of radii $R_{\mathcal{A}} > R_{\mathcal{B}}$. Partial tracing $\rho_{\mathcal{B}} = \mathrm{Tr}_{\mathcal{H}_{\mathcal{A}\setminus\mathcal{B}}}(\rho_{\mathcal{A}})$ is a quantum channel. It follows that, on top of the previous properties,

$$\frac{d}{d\mu}\Delta S \leqslant 0 \quad (\mu \sim R^{-1}) \quad \rightarrow \quad \Delta S \text{ is an } \textbf{RG monotone} \, .$$

The definition above can be generalized in several ways,

- Non-invertible symmetries (Molina-Vilaplana *et al.*, 2024; Benini *et al.*, 2025; Ali Ahmad *et al.*, 2025).
- Here we consider the following generalization for **higher form symmetries**,

$$\rho_{\mathcal{A},\mathsf{S}} = \int_{H_{d-p-1}(\mathcal{A},\partial\mathcal{A};G)} \mathrm{d}\mu(a)\; U_a(\mathcal{A})\; \rho_{\mathcal{A}}\; U_a^{\dagger}(\mathcal{A}) \, .$$

Entanglement asymmetry in systems with spontaneously broken symmetries

$U(1)^{(p)}$ symmetries

We consider **systems** with a spontaneously broken $U(1)$ p-form symmetry. For concreteness take a spatial slice $\Sigma_{d-1} = S^p_r \times S^{d-p-1}_R$.

- By Goldstone theorem, the low energy theory contains a massless p-form gauge field. To lowest order in derivatives,

$$S = \frac{\lambda\,\mu^\Delta}{2}\int_{X_d} da_p \wedge \star\, da_p\ , \qquad \Delta = d - 2(p+2)\ .$$

- We will compute ΔS in the ground state to probe spontaneous symmetry breaking. The spontaneously broken symmetry is non-linearly realized,

$$a_p \to a_p + \Lambda_p, \qquad \oint \Lambda_p \in [0, 2\pi)$$

It acts on Wilson lines $W = e^{i\oint_{S^p} a_p}$. The conserved charge is the electric flux,

$$Q = \lambda\mu^\Delta \oint_{S^{d-p-1}} \star f_{p+1} \equiv \Phi_e\ ,$$

The symmetry operators are,

$$U_\alpha = e^{i\alpha\Phi_e}$$

$U(1)^{(p)}$ symmetries

Our aim is to find the ground state and compute $\Delta S(\rho)$. We will proceed by **canonical quantization** on the spatial slice $\Sigma_{d-1} = S^p_r \times S^{d-p-1}_R$.

- The theory is free. States are a superposition of harmonic oscillators and classical field configurations. Consider a basis of oscillator-free charge eigenstates $|n\rangle$,

$$\Phi_e|n\rangle = n|n\rangle, \qquad a^i_{\boldsymbol{k}}|n\rangle = 0\ .$$

  They have energy,

$$H|n\rangle = \frac{1}{R}\frac{n^2}{2\lambda}(\mu r)^p(\mu R)^{p+2-d}$$

- The states $|n\rangle$ are not clustering. Consider the **clustering** state,

$$|\theta\rangle = e^{-\epsilon R H}\sum_{n\in\mathbb{Z}} e^{in\theta}|n\rangle\ , \qquad U_\alpha|\theta\rangle = |\theta+\alpha\rangle\ ,$$

  where ϵ is a *regulator* that we can tune to project to lowest energy states. These are symmetry breaking states.

The entropic Mermin-Wagner-Coleman theorem

Take the pure global state $\rho = |\theta\rangle\langle\theta|$. After normalizing it, compute the N-th Rényi asymmetry. We find,

$$\Delta S_N(\rho) = \frac{1}{1-N} \log \frac{\mathrm{tr}\left(\rho_{\mathsf{S}}^N\right)}{\mathrm{tr}\left(\rho^N\right)} = \frac{1}{1-N} \log \left(\frac{\sum_{n\in\mathbb{Z}} e^{\frac{-N\epsilon n^2}{\lambda}(\mu r)^p (\mu R)^{p+2-d}}}{\left(\sum_{n\in\mathbb{Z}} e^{\frac{-\epsilon n^2}{\lambda}(\mu r)^p (\mu R)^{p+2-d}}\right)^N} \right) .$$

Now, to address the question of spontaneous symmetry breaking we must,

1. Take the Thermodynamic limit: $\mu R \gg 1$.
2. Ensure that $|\theta\rangle$ is a ground state.

The result depends on d, p,

- $d > p+2$. The asymmetry diverges with the size of the system. There is **SSB**.

$$\Delta\mathcal{S} \sim \frac{d-p-2}{2} \log(\mu R) + \text{const} ,$$

- $d \leqslant p+2$. The asymmetry vanishes. We find a contradiction and conclude that there is **no SSB**.

$$\Delta\mathcal{S} \to 0 .$$

The entropic Mermin-Wagner-Coleman theorem

- **Summary**. The proof is by contradiction. We assumed an spontaneously broken $U(1)^{(p)}$ symmetry and that Goldstone theorem holds. We found a vanishing $\Delta\mathcal{S}$ in $d \leqslant p+2$. We conclude that a $U(1)^{(p)}$ symmetry can't be spontaneously broken in $d \leqslant p+2$.

Non-abelian 0-form symmetries

- **Non-abelian 0-form symmetries.** Consider a G symmetry spontaneously broken to nothing. The low energy effective theory is a non-linear σ-model with target space G.
- At weak coupling the dynamics of the lowest lying, spatially constant mode, can be decoupled and the system reduces to the quantum mechanics of a particle on the group manifold G. Depending on d, we find,
  - $d > 2$. The σ-model is weakly coupled in the IR. We show that, in the $\mu R \to \infty$ limit,

$$\Delta \mathcal{S}_G \sim \frac{(d-2)\dim G}{2} \log(\mu R) + \text{const}\,.$$

  - $d = 2$. The σ-model is strongly coupled in the IR. Our perturbative methods fail, but the σ-model is known to be gapped at infinite volume and $\Delta\mathcal{S}$ should therefore vanish.
  - $d = 1$. The σ-model is already quantum mechanics. One concludes that $\Delta\mathcal{S} = 0$.

Entanglement asymmetry in subsystems with spontaneously broken symmetries

$U(1)^{(\mathrm{p})}$ symmetries in a subsystem

- Consider now the case of a subsystem in a system with a spontaneously broken $U(1)$ p-form symmetry. We take the spatial slice to be $\Sigma_{d-1} = S^p_r \times \mathbb{R}^{d-p-1}$ and the spatial subregion $\mathcal{A} = S^p_r \times \mathbb{B}^{d-p-1}_R$.
- We will compute the Renyi asymmetries,

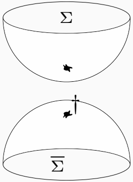
$$\Delta\mathcal{S}_N(\rho) := \frac{1}{1-N} \log \frac{\mathrm{tr}\left(\rho^N_{\mathcal{A},\mathsf{s}}\right)}{\mathrm{tr}\left(\rho^N_{\mathcal{A}}\right)} \,, \qquad \lim_{N\to 1} \Delta\mathcal{S}_N(\rho) = \Delta S(\rho) \,.$$

- We need to evaluate two path integrals: $\mathrm{tr}\left(\rho^N_{\mathcal{A}}\right)$ and $\mathrm{tr}\left(\rho^N_{\mathcal{A},\mathsf{s}}\right)$. We will use path integral methods, following (Agon *et al.*, 2014).

Crash course on path integral methods

- A Euclidean path integral on an open manifold X_d with boundary $\partial X_d = \Sigma_{d-1}$ prepares a state $|\Psi\rangle$ on Σ_{d-1}. Likewise, a path integral with two boundaries defines an (unnormalized) density matrix ρ.

$\rho =$



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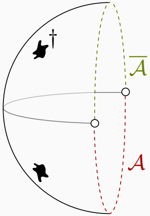
- A reduced density matrix $\rho_{\mathcal{A}}$ for a subregion $\mathcal{A} \subset \Sigma$ is computed by taking a partial trace of ρ over the complement $\mathcal{A}'$ of $\mathcal{A}$. In the path-integral language, this is implemented by a partial gluing along $\mathcal{A}' \subset \Sigma$ and $\overline{\mathcal{A}'} \subset \overline{\Sigma}$.

$\rho_{\mathcal{A}} =$



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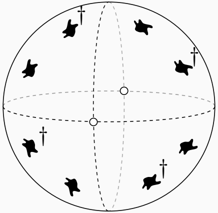
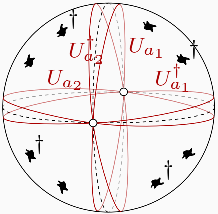
Crash course on path integral methods

- Traces $\mathrm{tr}\left(\rho_{\mathcal{A}}^{N}\right)$ are constructed by gluing cyclically N copies of $\rho_{\mathcal{A}}$. The resulting manifold is called the **replica manifold** X_N,

$\mathrm{tr}\left(\rho_{\mathcal{A}}^{N}\right) =$  (here for $N = 4$) .

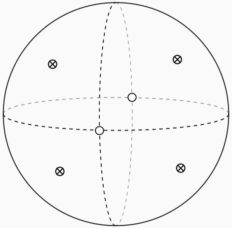
- Symmetrized traces are found by decorating the previous expression with symmetry operators inserted along the appropriate cycles of the replica manifold and integrating over the symmetry group.

$\mathrm{tr}\left(\rho_{\mathcal{A},\mathrm{S}}^{N}\right) = \int da_1 da_2 da_3 da_4$ 

(here for $N = 4$) .

The unsymmetrized path integral

For the case at hand, the Euclidean spacetime is $M_d = S^p_r \times \mathbb{R}^{d-p}$ and the spatial subregion $\mathcal{A} = S^p_r \times \mathbb{B}^{d-p-1}_R$. The replica manifold $X_N = S^p_r \times Y_{d-p}$ is found by taking N copies of M_d and gluing them along $\mathcal{A}$. Schematically, the path integral to be evaluated is,

$$\mathrm{tr}\left(\rho_{\mathcal{A}}^N\right) = \mathcal{Z}[X_N] =$$  $$\times\, S^p_r\ .$$

It has N asymptotic infinities, where we choose a Dirichlet boundary condition,

$$\int_{S^p_r} a_p \to 0$$

The unsymmetrized path integral

Decompose the p-form field strength into a globally defined p-form a_p and a magnetic flux contribution f_{p+1}^{flux},

$$f_{p+1} = f_{p+1}^{\text{flux}} + \mathrm{d}a_p \ .$$

The theory is free, therefore $\mathcal{Z}[X_N]$ splits,

$$\mathcal{Z}[X_N] = \mathcal{Z}^{\text{flux}}[X_N]\, \mathcal{Z}^{\text{osc}}[X_N] \ ,$$

The p-form symmetry does not act on $\mathcal{Z}^{\text{osc}}[X_N]$. This contribution will drop from $\Delta\mathcal{S}_N(\rho)$ and we ignore it. Let $\{\, \tau_i \,\}$ be a basis of 1-forms on Y_{d-p} such that $\tau_i\big|_{\partial X_d} = 0$ and with integer periods,

$$f_{p+1}^{\text{flux}} = 2\pi w^i \tau_i \wedge \Omega_p \ , \qquad w^i \in \mathbb{Z} \ ,$$

$\mathcal{Z}^{\text{flux}}[X_N]$ is a sum over flux contributions with action ,

$$S^{\text{flux}} = \frac{\lambda\, \mu^{d-p-2}}{2} \int_{X_N} f_{p+1}^{\text{flux}} \wedge \star\, f_{p+1}^{\text{flux}} = 2\pi^2 \lambda\, \frac{(\mu R)^{d-p-2}}{(\mu r)^p}\, w^i\, \mathbb{M}_{ij}\, w^j \ , \qquad \mathbb{M}_{ij} = \int_{Y_{d-p}} \tau_i \wedge \star\, \tau_j \ .$$

Therefore,

$$\mathcal{Z}^{\text{flux}}[X_N] = \sum_{\substack{w^i \in \mathbb{Z} \\ i=1,\ldots,b_1}} \exp\left(-2\pi^2 \lambda\, \frac{(\mu R)^{d-p-2}}{(\mu r)^p}\, w^i\, \mathbb{M}_{ij}\, w^j \right)$$

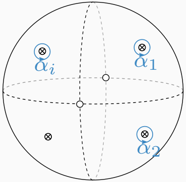
The symmetrized path integral

The path integral is evaluated on the same replica manifold X_N, but with symmetry operators inserted. One may deform the symmetry operators until they surround the asymptotic infinities, where they act on the boundary conditions as,

$$\int_{S_r^p} a_p \to \alpha_i$$

An integral is performed over the symmetry operators parameters. In pictures,

$$\operatorname{tr}\left(\rho_{\mathsf{S}}^N\right)\Big|_{\text{flux}} = \int \frac{d\alpha_1 \ldots d\alpha_N}{(2\pi)^N}$$



$$\times S_r^p = \left[\det\left(\frac{2\pi\lambda(\mu R)^{d-p-2}}{(\mu r)^p}\mathbb{M}_{N,d-p}\right)\right]^{-\frac{1}{2}} .$$

All in all the Rényi asymmetries become,

$$\Delta\mathcal{S}_N = \frac{1}{1-N}\log\frac{\operatorname{tr}\left(\rho_{\mathsf{S}}^N\right)}{\operatorname{tr}\left(\rho^N\right)} = \frac{1}{N-1}\log\left[\sum_{\omega^i\in\mathbb{Z}} e^{\frac{-(\mu r)^p}{2\lambda(\mu R)^{d-p-2}}\omega^i\mathbb{M}_{ij}^{-1}\omega^j}\right] .$$

The flux matrix for $p = 0$

- The matrix $\mathbb{M}_{N,d}$ evaluates the action of the Laplace equation on the replica manifold X_N with Dirichlet boundary conditions. It takes the form,

$$[\mathbb{M}_{N,d}]_{j\ell} = \frac{1}{N} \sum_{k=1}^{N-1} \mathrm{e}^{2\pi i \frac{(j-\ell)k}{N}} C_d\left(\frac{k}{N}\right) .$$

- The function $C_d\big(k/N\big)$ can be computed numerically and, in some cases an analytic expression has been proposed. For $d = 3$, its form was proposed in (Agon *et al.*, 2014),

$$C_{d=3}\left(\frac{k}{N}\right) = 4\pi\left(1 - \frac{2k}{N}\right)\tan\left(\frac{\pi k}{N}\right) .$$

  We have proposed the form for $d = 4$.

$$C_{d=4}\left(\frac{k}{N}\right) = 8\pi^2\,\frac{k}{N}\left(1 - \frac{k}{N}\right) .$$

The entropic Coleman-Mermin-Wagner theorem

In $d \leqslant p+2$ the matrix $\mathbb{M}$ vanishes identically. It follows that,

$$\Delta\mathcal{S}_N = 0, \qquad d \leqslant p+2$$

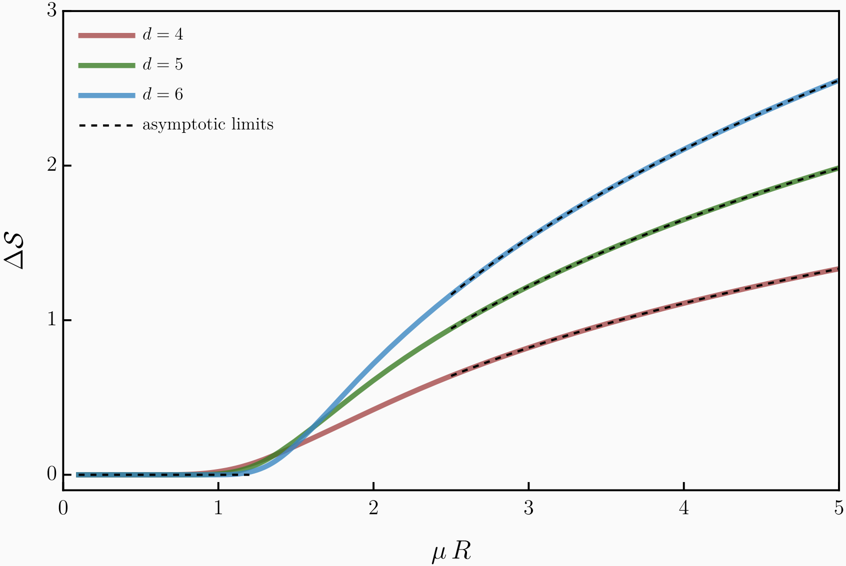
In $d > p+2$ the expression is involved but simplifies in the large and small subregion limits. All Rényi asymmetries coincide in those limits and the entanglement asymmetry becomes,

$$\Delta\mathcal{S} = \frac{d-p-2}{2}\log(\mu R) + \text{const.}\,, \qquad \mu R \gg 1$$
$$\Delta\mathcal{S} = 0\,, \qquad \mu R \ll 1$$

We thus learn that,

- In $d \leqslant p+2$, a p-form symmetry can't spontaneously break.
- In $d > p+2$, the entanglement asymmetry vanishes towards the UV.
- In $d > p+2$, the entanglement asymmetry diverges towards the IR, signalling that spontaneous symmetry breaking indeed occurs.

The 0-form case, in more detail



Summary and Outlook

We have presented an entropic version of the Mermin-Wagner-Coleman theorem. It states that,

1. A continuous p-form symmetry can't spontaneously break in $d \leqslant p + 2$.
2. In $d > p + 2$ the entanglement asymmetry of the Goldstone theory vanishes in the UV.
3. In $d > p + 2$ the entanglement asymmetry of the Goldstone theory diverges in the IR, signalling SSB.
4. If we interpret the Goldstone theory as an effective theory, we loose control around $\mu R \sim 1$, the scale at which the symmetry is restored.

Entropic order parameters, and entanglement asymmetry in particular, have hardly been studied in quantum field theory. There are many exciting avenues that we plan to study in the future.

Thanks for listening!

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