Information-Theoretic Signatures of Categorical Symmetry Breaking

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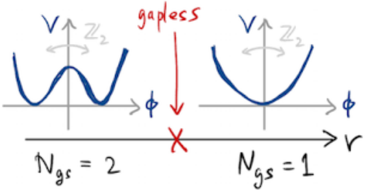
Outline

Motivation

Landau Paradigm

Phases of matter are classified by how they represent their symmetries.

- A continuous 2nd order phase transition is a symmetry breaking transition
- G is symmetry group spontaneously broken to a subgroup H resulting in $|G|/|H|$ vacua acted by the broken symmetry.
- Order parameters to distinguish phases are given by charged operators under the symmetry.



Basis of most condensed matter understanding!

Categorical Landau Paradigm [L. Bhardwaj,et al, PRL (2024)]

- Consider symmetries that go beyond groups, so-called categorical symmetries.
- Which features characterize the SSB of non-invertible symmetries?

Using

1. The SymTFT construction
2. Algebraic methods
3. Information theoretic tools

Results

We provide a class of entropic order parameters characterising phases in $(1+1)$-d with broken generalized symmetries.

Setting

We consider

- $(1+1)$-d QFT's with a global symmetry, which can be non-invertible and described by a fusion category $\mathcal{C}$.
- The latter acts on $\mathcal{F}$, the algebra generated by the full operator content of the theory.
- The theory has operator algebra $\mathcal{O}$, which is the subalgebra of $\mathcal{F}$ generated by the operators invariant (i.e. 'uncharged') under $\mathcal{C}$
- Each gapped phase for such a QFT corresponds to a way of gauging the global symmetry.

Outline

Algebras $\mathcal{F}$ and $\mathcal{O}$

Consider operators $V_W \in \mathcal{F}$ on a region W such that $|\psi\rangle = V_W^\dagger|0\rangle$

Operators $b \in \mathcal{O} = \mathcal{F}/G$ invariant under the group G: neutral operators

[H. Casini,et al, JHEP (2020) (2021)]

One can produce a representation ($\neq$ vacuum) ρ_W of $\mathcal{O}$ by

$$\langle\psi\,|\,b\,|\,\psi\rangle = \langle 0\,|\,V_W\,b\,V_W^\dagger|\,0\rangle = \langle 0\,|\,\rho_W(b)|\,0\rangle$$

$$\rho_W(b) = V_W\,b\,V_W^\dagger$$

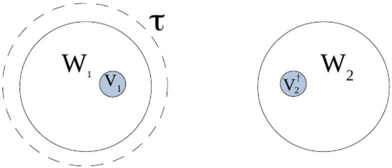
ρ_W are endomorphisms of the algebra $\mathcal{O}$, that is

$$\text{Reps} \equiv \text{End}(\mathcal{O})$$

We would like to understand ρ_W, in terms of certain limit of operators in $\mathcal{O}$.

Non-Local operators = Charge-Anticharge operators $\in \mathcal{O}$

$$\mathcal{I}_{12} = V_2\, V_1^\dagger$$



$$\mathcal{I}_{12}\, \rho_1(b) = \rho_2(b)\, \mathcal{I}_{12}\,, \quad b \in \mathcal{O}\,.$$

$\mathcal{I}_{12}$ translates the charge from W_1 to W_2, leaving intact all operators localized outside W_1 and W_2. Can produce ρ_W creating charge at infinity and transporting it as

$$\rho_1(b) = V_1\, b\, V_1^\dagger = \lim_{x \to \infty} \mathcal{I}_{1\,(2+x)}\, b\, \mathcal{I}_{1\,(2+x)}$$

Algebraic Theory of Superselection Sectors

In a theory with a global symmetry G, superselection sectors $\equiv$ irreps r

DHR endomorphisms ρ_r of the algebra $\mathcal{O}$: from the vacuum representation of $\mathcal{O}$, the other representations can be constructed using ρ_r [Haag, 1996], [Bischoff et al, 2014]

$$V_r^i \in \mathcal{F} \quad \text{w/} \quad (V_r^i)^\dagger V_r^j = \delta_{ij} \quad \text{s.t} \quad |\psi_i\rangle = \sqrt{d_r}\,(V_r^i)^\dagger\,|0\rangle$$

$$\langle\psi_i|b|\psi_i\rangle = \langle 0|\rho_r(b)|0\rangle \quad \text{w/} \quad \rho_r(b) = \sum_j V_r^j\, b\,(V_r^j)^\dagger$$

τ_g are twist operators implementing group actions

$$\tau_g\, V_r^i\, \tau_g^\dagger = D_{ij}^r(g)\, V_r^j \quad \tau_g\, \tau_h = \tau_{gh}$$

Some Quantum Information Tools

$$b \in \mathcal{O} \longrightarrow b = \sum_r C_r \, \mathcal{I}^r_{12}\,, \quad \mathcal{I}^r_{12} = \sum_j V_1^{r,j}\,(V_2^{r,j})^\dagger$$

$$a \in \mathcal{F} \longrightarrow a = \lim_{x\to\infty} \sum_r C_r \, \mathcal{I}^r_{1\,(2+x)} = \sum_{r,i} C_{r,i} \, V^i_r$$

Here we make a notational abuse of a charged operator as the limit of an intertwiner $\mathcal{I}^r_{1\,(2+\infty)} = \mathcal{I}^r_{1\,\bar{2}} \equiv$ Charged intertwiner

Conditional Expectation $E : \mathcal{F} \longrightarrow \mathcal{O}$

Positive linear map such that, given any $a \in \mathcal{F}$ and $b_1,\; b_2 \in \mathcal{O}$

$$E(1) = 1 \quad E(b_1\, a\, b_2) = b_1\, E(a)\, b_2\,,$$

Some Quantum Information Tools

E is the symmetry-averaging map

$$E(a) = \frac{1}{|G|} \sum_{g \in G} \tau_g \, a \, \tau_g^{\dagger} \, ;$$

that annihilates everything charged, that is

$$E(V_r^i) = E(\mathcal{I}^r_{1\bar{2}}) = \delta_{r,0} \, \mathbf{1}_{\mathcal{O}}$$

Therefore, E projects $\mathcal{F}$ onto $\mathcal{O}$.

$$a_{\mathrm{inv}} = E(a) = E\left(\sum_r C_r \, \mathcal{I}^r_{1\bar{2}} \right) = C_0 \, \mathbf{1} \in \mathcal{O}$$

Extension to categorical symmetries $\mathcal{C}$ in [Longo and Rerhen, 1994]

For categorical symmetries $\mathcal{C}$, the role of the Haar average is played by the counit/trace of a condensable algebra (Q-system) given in terms of the canonical endomorphism ($\sim$ regular representation in groups)

$$\theta = \bigoplus_{r \in \mathrm{Irreps}(\mathcal{C})} N_r\, \rho^r \;\in \mathrm{End}(\mathcal{O})$$

and the conditional expectation is given by

$$E(a) = \frac{1}{\dim(\iota)}\, w^*\, \bar{\iota}(a)\, w$$

where $\iota : \mathcal{O} \to \mathcal{F}$ and $\bar{\iota} : \mathcal{F} \to \mathcal{O}$ such that $\theta := \bar{\iota}\iota$ and

$$w \in \mathrm{Hom}(\mathbf{1}_{\mathcal{O}}, \theta) \quad \text{and} \quad \dim(\iota) = \sqrt{\dim(\theta)}$$

Outline

EOP from Relative Entropy

Given ρ and σ two states on a finite matrix algebra $\mathcal{M}$, relative entropy quantifies the distinguishability of ρ and σ,

$$S(\rho|\sigma) := \langle \log \Delta_{\rho|\sigma} \rangle_\rho = \mathrm{Tr}\left[\rho \log \Delta_{\rho|\sigma}\right] , \quad \mathrm{w/} \quad \Delta_{\rho|\sigma} = \rho \cdot \sigma^{-1}$$

with $\Delta_{\rho|\sigma}$ being the Tomita-Takesaki relative modular operator.

Entropic Order Parameter

$$S(\omega \,|\, \omega \circ E)$$

Relative entropy between the vacuum ω and its invariant part $\omega \circ E$. Serves as EOP signalling SSB by measuring the statistics (expectation values) of non local symmetry operators.

[H. Casini et al, 2021], [Ares et al, 2023], [JMV et al, 2025], [Benini et al, 2023]

Vacua of Gapped Phases in $(1+1)$d

In a $(1+1)$d theory with a symmetry $\mathcal{C}$ the *vacua*

$$\{v_1, v_2, \dots, v_N\}$$

form a basis of the Hilbert space.

State-Operator map $v_k \to \hat{v}_k$ s.t the OPE

$$v_k v_{k'} = \delta_{kk'} v_k, \quad k, k' = 1, \dots, N.$$

The vacua are orthogonal idempotents.

Relative Euler Terms

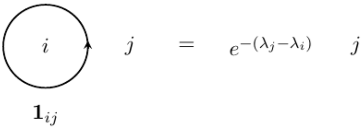
Euler number: The partition function of the vacuum v_i on the sphere S^2 is given by

$$Z_i = e^{-2\lambda_i}\,.$$

Properties of interfaces between different vacua $\rightarrow$ *relative Euler terms*

[L. Bhardwaj, et al, 2025]

$$\mathbf{1}_{ij}\,:\; v_i \longrightarrow e^{-(\lambda_j-\lambda_i)}\, v_j$$



We are interested in ρ_k, the state operator associated to the vacua $v_k \in \mathcal{F}$, which can be expressed in terms intertwiners $\mathcal{I}^x$ as

$$v_k = \sum_x C_{k,x}\, \mathcal{I}^x \,.$$

$\{v_k\}$ is an orthonormal basis $\implies$ ρ_{v_k} is diagonal.

$$v_k \mapsto v_k \circ E = C_{k,0}\, \mathcal{I}^{(0)}, \qquad \text{or} \qquad \rho_{v_k \circ E} = C_{k,0}\, \mathbb{I}.$$

EOP

$$S_{\mathcal{I}}(v_k | v_k \circ E) = -\log\, C_{k,0}$$

Algebraic characterization of Relative Euler Terms

GNS representation of $\mathcal{F} \implies S_{k'k}$ satisfying

$$S_{k'k}|k\rangle = \mathrm{e}^{-(\lambda_{k'}-\lambda_k)}|k'\rangle, \qquad S_{k'k} = J_{k'k}\Delta_{k'k}^{1/2},$$

where $J_{k'k}$ is the modular conjugation between $|k\rangle$ and $|k'\rangle$ and

$$\Delta_{k'k} = \rho_{v_{k'}\circ E}\,(\rho_{v_k\circ E})^{-1} = \frac{C_{k',0}}{C_{k,0}}\,\mathbb{I}$$

is the Tomita-Takesaki relative modular operator.

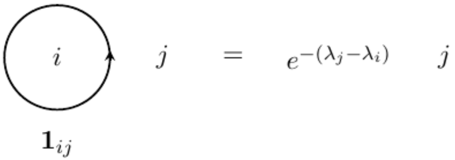
Our result

$$\lambda_{k',k} = \frac{1}{2}\log\left(\frac{C_{k,0}}{C_{k',0}}\right) \quad \Rightarrow \quad S_{k'k}|k\rangle = \mathrm{e}^{-\lambda_{k',k}}|k'\rangle = \sqrt{\frac{C_{k',0}}{C_{k,0}}}\,|k'\rangle\,;$$

Entropic Relative Euler Terms

$$\lambda_{j,i} = \frac{1}{2}\left[S_{\mathcal{I}}(v_i|v_i \circ E) - S_{\mathcal{I}}(v_j|v_j \circ E)\right] \,,$$

$$\lambda_{j,i} = -\log \Delta_{j|i}^{1/2} \,.$$



Outline

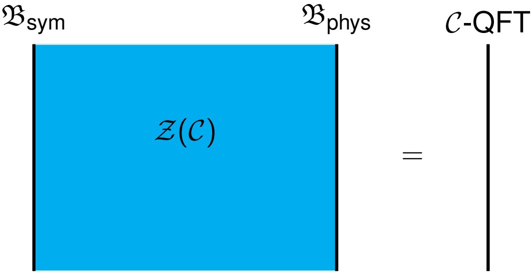
Topological Holography/SymTFT

SymTFT in our work

Systematic building of vacua of SSB gapped phases in $(1+1)$-d in terms of intertwiners

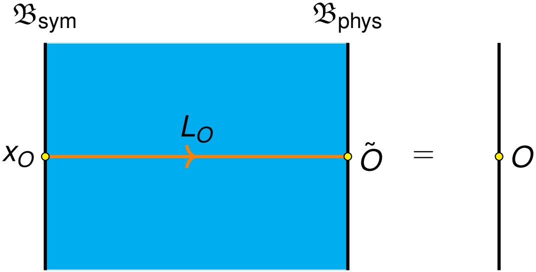
$$v_k = \sum_x C_{k,x}\,\mathcal{I}^x\,.$$

SymTFT



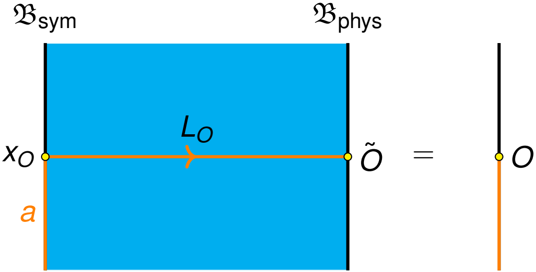
$\mathcal{C}$-QFT represented as a 'sandwich' consisting of the $(2+1)$d SymTFT in with topological lines in $\mathcal{Z}(\mathcal{C})$, $\mathfrak{B}_{\mathsf{sym}}$ where the SymTFT has Dirichlet boundary condition, and $\mathfrak{B}_{\mathsf{phys}}$ which is not necessarily topological.

Local Operators



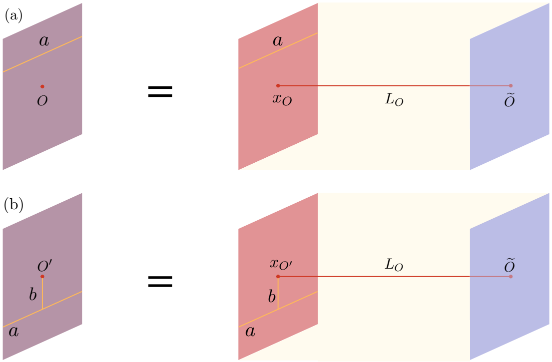
A local operator $O \in \mathcal{F}$ is represented as a triple $(x_O, L_O, \tilde{O})$. The operator O is a genuinely local operator, which refers to one that is not attached to a non-trivial topological defect line.

Non-Genuine Operators



An operator residing at the end of a semi-infinite non-trivial line is said to be in a twisted sector. The algebra $\mathcal{F}$ does not include these twisted-sector operators.

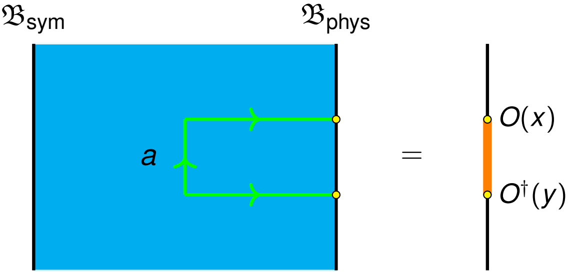
Symmetry Actions



Action of $a \in \mathcal{C}$ on $O \in \mathcal{F}$ by pushing it on the $\mathfrak{B}_{\text{sym}}$ through x_O. This leaves behind another defect line b attaching the junction to the line a.

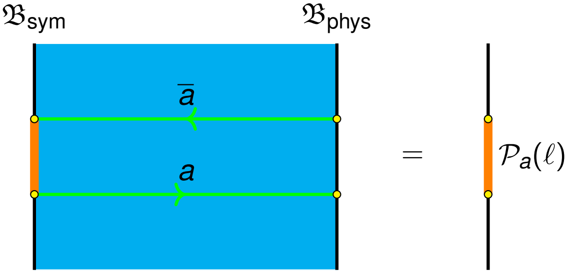
If L_O is the trivial line, the corresponding operator O is invariant under the symmetry and belongs to $\mathcal{O}$

Bi-Local Patch operator



[K. Inamura and X.-G. Wen, 2023], [S. Schafer-Nameki et al, 2025] [Q.Jia and J. Tian, 2025]

Bi-Local Patch operator

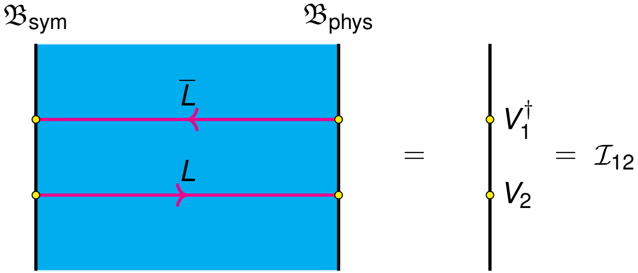


The bulk topological line a is merged into the symmetry boundary ; $\bar{a}$ is the orientation reversal of a.

Proposed as order/disorder OP $\langle\psi|\mathcal{P}_a(\ell)|\psi\rangle \neq 0$ for generalized symmetries in

[K. Inamura and X.-G. Wen, 2023], [S. Schafer-Nameki et al, 2025] [Q.Jia and J. Tian, 2025]

Intertwiners



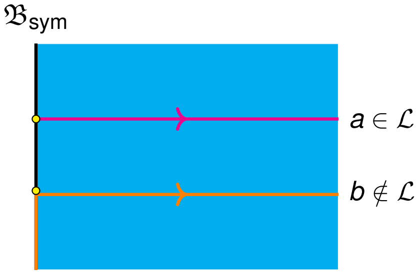
$\mathcal{I}_{12} \in \mathcal{O}$ invariant under $\mathcal{C}$.

When $\bar{L} \to \infty$; the operator resulting is denoted as $\mathcal{I}_{1\bar{2}} \in \mathcal{F}$

Gapped Phases from SymTFT

Gapped boundary specified by maximal set of anyons that can end on it $\leftrightarrow$ Lagrangian Algebras in $\mathcal{Z}(\mathcal{C})$ [L. Bhardwaj et al 2025]

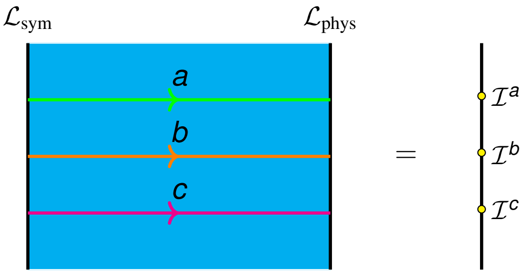
$$\mathcal{L} = \bigoplus_{a\in\mathcal{Z}(\mathcal{C})} n_a\, a \qquad \text{s.t} \qquad \dim(\mathcal{L}) = \bigoplus_{a\in\mathcal{Z}(\mathcal{C})} n_a \dim(a) = \dim(\mathcal{C})$$



- Anyons $b \notin \mathcal{L}$ survive on $\mathfrak{B}_{\text{sym}}$ and generate $\mathcal{C}$ (Neumann b.c.)
- Anyons $a \in \mathcal{L}$ are annihilated on $\mathfrak{B}_{\text{sym}}$ (Dirichlet b.c.)

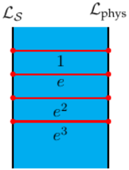
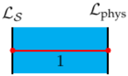
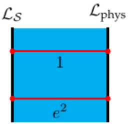
$\mathcal{C}$-symmetric gapped phases in $(1+1)$d

- From $\mathcal{C}$ build $\mathcal{Z}(\mathcal{C})$. All gapped b.c's $\leftrightarrow$ $\mathcal{L}$ in $\mathcal{Z}(\mathcal{C})$
- $\mathcal{C}$- symmetric phases $\equiv$ fix $\mathfrak{B}_{\text{sym}} = \mathcal{L}_{\mathcal{C}}$. Varying $\mathcal{L}_{\text{phys}}$ we span all -symmetric gapped phases.
- Number of vacua $\leftrightarrow$ anyons completely ending on both boundaries



$$v_k v_k' = \delta_{k,k'}\, v_k \Longrightarrow v_k = \sum_x C_{k,x}\, \mathcal{I}^x\,, \qquad k = 1,2,3$$

Gapped Phases with $G = \mathbb{Z}_4$ via SymTFT

$\mathcal{L}_{\text{phys}} = \mathcal{L}_{\mathcal{S}} = \mathcal{L}_{\text{Dir}}$	$\mathcal{L}_{\text{phys}} = \mathcal{L}_{\text{Neu}}$	$\mathcal{L}_{\text{phys}} = \mathcal{L}_{\text{Neu}(\mathbb{Z}_2)}$
		
$\mathbb{Z}_4$ SSB: 4 identical vacua, permuted by $\mathbb{Z}_4$	$\mathbb{Z}_4$ Trivial Phase: single vacuum with $\mathbb{Z}_4$ acting trivially	$\mathbb{Z}_2$ SSB: 2 identical vacua, permuted by $\mathbb{Z}_2$

Outline

Group Symmetries

$$v_k = \frac{|H|}{|G|} \sum_r d_r \sum_{i,j} D_r^{ij}(k) \mathcal{I}_{ji}^r,$$

where $k = 1, 2, \ldots, N = \sum_r d_r^2 = |G|/|H|$.

$$E_{\mathcal{I}} : v_k \mapsto C_{k,0}\, \mathcal{I}^{(0)} = \frac{|H|}{|G|} \mathcal{I}^{(0)}, \quad \text{and} \quad S_{\mathcal{I}}(v_k | v_k \circ E_{\mathcal{I}}) = \log\left(|G|/|H|\right)$$

is the same for all the symmetry-breaking vacua.

$$\lambda_{k',k} = \frac{1}{2} \log \left(\frac{C_{k,0}}{C_{k',0}} \right) = 0 \quad \forall\, k, k',$$

Non-Invertible Symmetries

$$v_a = \mathbb{S}^*_{\mathbf{1}a} \sum_b \mathbb{S}_{ba} \mathcal{I}^b,$$

$$E_{\mathcal{I}} : v_a \mapsto |\mathbb{S}_{0a}|^2 \cdot \mathcal{I}^0 = \frac{d_a^2}{\dim(\mathcal{C})} \cdot \mathcal{I}^0, \qquad S_{\mathcal{I}}(v_a | v_a \circ E_{\mathcal{I}}) = \log \frac{\dim(\mathcal{C})}{d_a^2},$$

$$\lambda_{a,b} = \frac{1}{2} \log \left(\frac{C_{b,0}}{C_{a,0}} \right) = \log \frac{d_b}{d_a} \,,$$

which are not all necessarily vanishing as in the case of group symmetries.

Rep(G)

Gauging G sends $\mathcal{L}_{\mathrm{Dir}} \Rightarrow \mathcal{L}_{\mathrm{Neu}}$ and transforms the symmetry $G \Rightarrow \mathrm{Rep}(G)$. Under this operation, intertwiners $\mathcal{I}^r$ are exchanged with twist operators $\tau_{[h]}$.

The vacua admit a character expansion over conjugacy classes of H,

$$v_r = \frac{d_r}{|H|} \sum_{[h]} \chi_r([h])\, \tau_{[h]},$$

analogous to a discrete Fourier transform, with the EOP

$$S_\tau(v_r \,|\, v_r \circ E_\tau) = \log \frac{|H|}{d_r^2}.$$

G**-theory**	**Rep**(G)**-theory**
$\mathcal{L}_{\text{Dir}}$	$\mathcal{L}_{\text{Neu}}$
Intertwiners $\mathcal{I}^r$	Twist operators $\tau_{[h]}$
$E_{\mathcal{I}}$ averages over charge	E_τ averages over twist
$S_{\mathcal{I}}(v_k \mid v_k \circ E_{\mathcal{I}}) = \log \lvert G/H \rvert$	$S_\tau(v_r \mid v_r \circ E_\tau) = \log \dfrac{\lvert H \rvert}{d_r^2}$

Examples: $G = \mathbb{Z}_2$

$$G = \mathbb{Z}_2 = \{\mathbf{1}, \eta\}$$
$$\mathcal{Z}(G) = \{\mathbf{1} = ([\mathbf{1}], +)\,, \mathbf{e} = ([\mathbf{1}], -)\,, \mathbf{m} = ([\eta], +)\,, \mathbf{e} \otimes \mathbf{m} = ([\eta], -)\}.$$

$G = \mathbb{Z}_2$ SSB phase $\longleftrightarrow \mathcal{L}_{\mathrm{Dir}} = \mathbf{1} \oplus \mathbf{e}$

$$v_{\mathbf{1}} = \frac{1}{2}\left(\mathcal{I}^0 + \mathcal{I}^-\right), \quad v_\eta = \frac{1}{2}\left(\mathcal{I}^0 - \mathcal{I}^-\right).$$

$$S_{\mathcal{I}}(v_{\mathbf{1}} | v_{\mathbf{1}} \circ E_{\mathcal{I}}) = S_{\mathcal{I}}(v_\eta | v_\eta \circ E_{\mathcal{I}}) = \log 2$$

$$\lambda_{\mathbf{1},\mathbf{2}} = 0$$

Examples: $\mathrm{Rep}(S_3)$ SSB phase $\sim H = G = S_3$

Bulk lines that can end topologically on both boundaries are then identified with the twist operators $\tau_{[g]}$ for $g = e, a, b$.

$$v_{\mathbf{1}_+} = \frac{1}{6}\left(\tau_{[e]} + \tau_{[a]} + \tau_{[b]}\right), \qquad v_{\mathbf{1}_-} = \frac{1}{6}\left(\tau_{[e]} + \tau_{[a]} - \tau_{[b]}\right),$$
$$v_{\mathbf{2}} = \frac{1}{3}\left(2\tau_{[e]} - \tau_{[a]}\right).$$

The conditional expectation E_τ kills all the non-trivial twist operators, and

$$S_\tau(v_{\mathbf{1}_+}|v_{\mathbf{1}_+} \circ E_\tau) = S_\tau(v_{\mathbf{1}_-}|v_{\mathbf{1}_-} \circ E_\tau) = \log 6,$$
$$S_\tau(v_{\mathbf{2}}|v_{\mathbf{2}} \circ E_\tau) = \log \frac{3}{2}.$$

$$\lambda_{\mathbf{1}_+,\mathbf{1}_-} = 0, \quad \lambda_{\mathbf{1}_\pm,\mathbf{2}} = \log 2.$$

Example: Ising Category

$\mathrm{Ising} = \{\mathbf{1}, \eta, \mathcal{N}\}$, with $\mathcal{Z}(\mathrm{Ising}) = \mathrm{Ising} \boxtimes \overline{\mathrm{Ising}}$

$$\mathcal{L}_{\mathrm{Dir}} = (\mathbf{1}, \mathbf{1}) \oplus (\eta, \eta) \oplus (\mathcal{N}, \mathcal{N}).$$

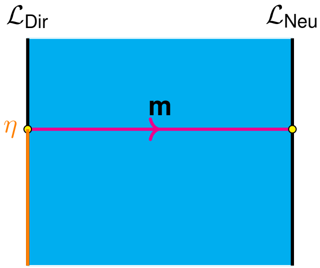
$$\begin{aligned}
v_{\mathbf{1}} &= \frac{1}{4}\left(\mathcal{I}^{(\mathbf{1},\mathbf{1})} + \mathcal{I}^{(\eta,\eta)} + \sqrt{2}\mathcal{I}^{(\mathcal{N},\mathcal{N})}\right), \\
v_{\eta} &= \frac{1}{4}\left(\mathcal{I}^{(\mathbf{1},\mathbf{1})} + \mathcal{I}^{(\eta,\eta)} - \sqrt{2}\mathcal{I}^{(\mathcal{N},\mathcal{N})}\right), \\
v_{\mathcal{N}} &= \frac{1}{2}\left(\mathcal{I}^{(\mathbf{1},\mathbf{1})} - \mathcal{I}^{(\eta,\eta)}\right).
\end{aligned}$$

$$S_{\mathcal{I}}(v_{\mathbf{1}} | v_{\mathbf{1}} \circ E_{\mathcal{I}}) = S_{\mathcal{I}}(v_{\eta} | v_{\eta} \circ E_{\mathcal{I}}) = \log 4, \quad S_{\mathcal{I}}(v_{\mathcal{N}} | v_{\mathcal{N}} \circ E_{\mathcal{I}}) = \log 2.$$

$$\lambda_{\mathbf{1},\eta} = 0, \quad \lambda_{\mathbf{1},\mathcal{N}} = \lambda_{\eta,\mathcal{N}} = \frac{1}{2}\log 2.$$

SPT Phases from SymTFT

Non-trivial bulk lines do not end both on $\mathcal{L}_{\rm Dir}$ and $\mathcal{L}_{\rm Neu}$ $\Rightarrow$ **single vacuum**, no SSB.



- No untwisted intertwiners. $\mathcal{I}_{1\bar{2}}$ lives in the twisted sector:
- string order parameter of the SPT

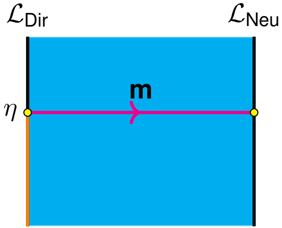
EOP in the SPT phase $S_{\mathcal{I}}(v \,|\, v \circ E_{\mathcal{I}}) = 0$

The string order parameter is the twisted-sector dual of the intertwiner $\mathcal{I}_{1\bar{2}}$,that is invisible to $E_{\mathcal{I}}$ but detectable via the dual (twist) E_{τ}.

SPT $\longleftrightarrow$ SSB: Kramers–Wannier Duality

Gauging $\mathbb{Z}_2$ on $\mathfrak{B}_{\text{sym}} \implies \mathcal{L}_{\text{Dir}} \longrightarrow \mathcal{L}_{\text{Neu}}$ **KW**-duality

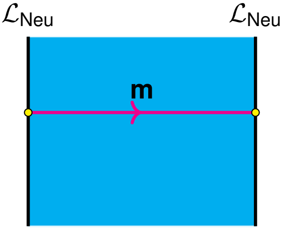
(a) SPT phase: $\mathcal{L}_{\text{Dir}} \,/\, \mathcal{L}_{\text{Neu}}$



String OP $=$ twisted-sector $\mathcal{I}_R$

$S_{\mathcal{I}} = 0$

(b) SSB phase: $\mathcal{L}_{\text{Neu}} \,/\, \mathcal{L}_{\text{Neu}}$



Ordinary (untwisted) $\mathcal{I}_R$

$S_{\mathcal{I}} = \log |G|$

Conclusions

1. SymTFT and EOP provide a natural and unified framework for characterizing SSB of generalized symmetries in $(1+1)$ dimensions.
2. This framework places SSB of invertible and non-invertible symmetries on equal footing.
3. Clear information-theoretic understanding of the distinguishability of symmetry-broken vacua.
4. SSB of non-invertible symmetries yields vacua that are intrinsically distinguishable. Our approach shows that this happens already at the level of the observable algebra.
5. Entropic interpretation of Relative Euler terms

Thanks for your attention!