

Reconstructing the Full Spin Density Matrix for a Pair of Z bosons in Higgs decays at ATLAS

Article

Observation of quantum entanglement with top quarks at the ATLAS detector

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The ATLAS Collaboration*

Entanglement is a key feature of quantum mechanics^{1–3}, with applications in fields such as metrology, cryptography, quantum information and quantum computation^{4–8}. It has been observed in a wide variety of systems and length scales, ranging from the microscopic^{9–13} to the macroscopic^{14–16}. However, entanglement remains largely unexplored at the highest accessible energy scales. Here we report the highest-energy observation of entanglement, in top–antitop quark events produced at the Large Hadron Collider, using a proton–proton collision dataset with a centre-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 inverse femtobarns (fb^{-1}) recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D , inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top–antitop quark production threshold, at which the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from the limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.537 \pm 0.002$ (stat.) ± 0.019 (syst.) for $340 \text{ GeV} < m_{t\bar{t}} < 380 \text{ GeV}$. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement so far.



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Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration*

Abstract

Entanglement is an intrinsic property of quantum mechanics and is predicted to be exhibited in the particles produced at the Large Hadron Collider. A measurement of the extent of entanglement in top quark-antiquark ($t\bar{t}$) events produced in proton-proton collisions at a center-of-mass energy of 13 TeV is performed with the data recorded by the CMS experiment at the CERN LHC in 2016, and corresponding to an integrated luminosity of 36.3 fb^{-1} . The events are selected based on the presence of two leptons with opposite charges and high transverse momentum. An entanglement-sensitive observable D is derived from the top quark spin-dependent parts of the $t\bar{t}$ production density matrix and measured in the region of the $t\bar{t}$ production threshold. Values of $D < -1/3$ are evidence of entanglement and D is observed (expected) to be $-0.480^{+0.026}_{-0.029}$ ($-0.467^{+0.026}_{-0.029}$) at the parton level. With an observed significance of 5.1 standard deviations with respect to the non-entangled hypothesis, this provides observation of quantum mechanical entanglement within $t\bar{t}$ pairs in this phase space. This measurement provides a new probe of quantum mechanics at the highest energies ever produced.

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)



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Measurements of Z-boson pair entanglement in decays of Higgs bosons at the ATLAS experiment

The ATLAS Collaboration

Entanglement is a key property of quantum systems. In this Letter the first measurements of quantum entanglement between spins in pairs of Z bosons are reported, using proton–proton collision data from the Large Hadron Collider (LHC) at center-of-mass energies of 13 TeV and 13.6 TeV, recorded with the ATLAS detector. Measurements of angular observables sensitive to ZZ^* spin-density-matrix elements in the $H \rightarrow ZZ^* \rightarrow \ell^+\ell^-\ell^+\ell^-$ process yield coefficients $C_{2,1,2,-1} = -0.71 \pm 0.45$ and $C_{2,2,2,-2} = 0.08 \pm 0.44$, consistent with their Standard Model predictions. A complementary hypothesis test using the full angular distribution, and relying on several Standard Model assumptions in the decays, provides substantially higher sensitivity to quantum correlations and disfavors the separable-state hypothesis at a significance of 4.7 standard deviations (expected 4.9σ) relative to the entangled Standard Model hypothesis. These results provide strong evidence of quantum entanglement between massive bosons (spin qutrits) at the electroweak scale.

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6} (\sqrt{2} A_{2,0}^1 + 2) & 0 & \frac{1}{3} C_{2,1,2,-1} & 0 & \frac{1}{3} C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} C_{2,1,2,-1} & 0 & \frac{1}{3} (1 - \sqrt{2} A_{2,0}^1) & 0 & \frac{1}{3} C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} C_{2,2,2,-2} & 0 & \frac{1}{3} C_{2,1,2,-1} & 0 & \frac{1}{6} (\sqrt{2} A_{2,0}^1 + 2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\frac{1}{9} \left(-\frac{3C[2,2,1,1]}{\sqrt{2}} - \frac{3C[2,2,2,1]}{\sqrt{2}} \right)$	$\frac{1}{9} \left(-\frac{3C[2,2,1,1]}{\sqrt{2}} + \frac{3C[2,2,2,1]}{\sqrt{2}} \right)$
$\frac{1}{9} \left(\sqrt{3} A1[2, 2] - \sqrt{6} C[2, 2, 2, 0] \right)$	$\frac{1}{9} \left(-\frac{3C[2,2,1,1]}{\sqrt{2}} + \frac{3C[2,2,2,1]}{\sqrt{2}} \right)$
$\frac{1}{9} \left(\frac{3C[2,2,1,-1]}{\sqrt{2}} - \frac{3C[2,2,2,-1]}{\sqrt{2}} \right)$	$\frac{1}{9} \left(\sqrt{3} A1[2, 2] - \frac{3C[2,2,1,0]}{\sqrt{2}} + \sqrt{\frac{3}{2}} C[2, 2, 2, 0] \right)$
$\frac{1}{9} \left(\frac{3}{2} C[1, 1, 1, 1] + \frac{3}{2} C[1, 1, 2, 1] - \frac{3}{2} C[2, 1, 1, 1] - \frac{3}{2} C[2, 1, 2, 1] \right)$	$\frac{1}{9} \left(-\frac{3C[1,1,2,2]}{\sqrt{2}} + \frac{3C[2,1,2,2]}{\sqrt{2}} \right)$
$\frac{1}{9} \left(-\sqrt{\frac{3}{2}} A1[1, 1] + \sqrt{\frac{3}{2}} A1[2, 1] + \sqrt{3} C[1, 1, 2, 0] - \sqrt{3} C[2, 1, 2, 0] \right)$	$\frac{1}{9} \left(\frac{3}{2} C[1, 1, 1, 1] - \frac{3}{2} C[1, 1, 2, 1] - \frac{3}{2} C[2, 1, 1, 1] + \frac{3}{2} C[2, 1, 2, 1] \right)$
$\frac{1}{9} \left(-\frac{3}{2} C[1, 1, 1, -1] + \frac{3}{2} C[1, 1, 2, -1] + \frac{3}{2} C[2, 1, 1, -1] - \frac{3}{2} C[2, 1, 2, -1] \right)$	$\frac{1}{9} \left(-\sqrt{\frac{3}{2}} A1[1, 1] + \sqrt{\frac{3}{2}} A1[2, 1] + \frac{3}{2} C[1, 1, 1, 0] - \frac{1}{2} \sqrt{3} C[1, 1, 2, 0] - \frac{3}{2} C[2, 1, 1, 0] + \frac{1}{2} \sqrt{3} C[2, 1, 2, 0] \right)$
$\frac{1}{9} \left(-\sqrt{\frac{3}{2}} A2[1, 1] - \sqrt{\frac{3}{2}} A2[2, 1] + \frac{3}{2} C[1, 0, 1, 1] + \frac{3}{2} C[1, 0, 2, 1] - \frac{1}{2} \sqrt{3} C[2, 0, 1, 1] - \frac{1}{2} \sqrt{3} C[2, 0, 2, 1] \right)$	$\frac{1}{9} \left(\sqrt{3} A2[2, 2] - \frac{3C[1,0,2,2]}{\sqrt{2}} + \sqrt{\frac{3}{2}} C[2, 0, 2, 2] \right)$
$\frac{1}{9} \left(1 - \sqrt{\frac{3}{2}} A1[1, 0] + \frac{A1[2,0]}{\sqrt{2}} - \sqrt{2} A2[2, 0] + \sqrt{3} C[1, 0, 2, 0] - C[2, 0, 2, 0] \right)$	$\frac{1}{9} \left(-\sqrt{\frac{3}{2}} A2[1, 1] + \sqrt{\frac{3}{2}} A2[2, 1] + \frac{3}{2} C[1, 0, 1, 1] - \frac{3}{2} C[1, 0, 2, 1] - \frac{1}{2} \sqrt{3} C[2, 0, 1, 1] + \frac{1}{2} \sqrt{3} C[2, 0, 2, 1] \right)$
$\frac{1}{9} \left(\sqrt{\frac{3}{2}} A2[1, -1] - \sqrt{\frac{3}{2}} A2[2, -1] - \frac{3}{2} C[1, 0, 1, -1] + \frac{3}{2} C[1, 0, 2, -1] + \frac{1}{2} \sqrt{3} C[2, 0, 1, -1] - \frac{1}{2} \sqrt{3} C[2, 0, 2, -1] \right)$	$\frac{1}{9} \left(1 - \sqrt{\frac{3}{2}} A1[1, 0] + \frac{A1[2,0]}{\sqrt{2}} - \sqrt{\frac{3}{2}} A2[1, 0] + \frac{A2[2,0]}{\sqrt{2}} + \frac{3}{2} C[1, 0, 1, 0] - \frac{1}{2} \sqrt{3} C[1, 0, 2, 0] - \frac{1}{2} \sqrt{3} C[2, 0, 1, 0] + \frac{1}{2} C[2, 0, 2, 0] \right)$

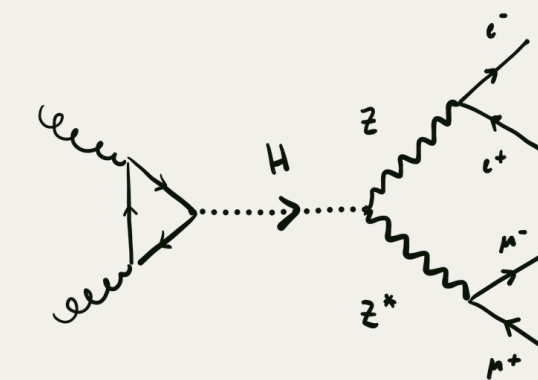
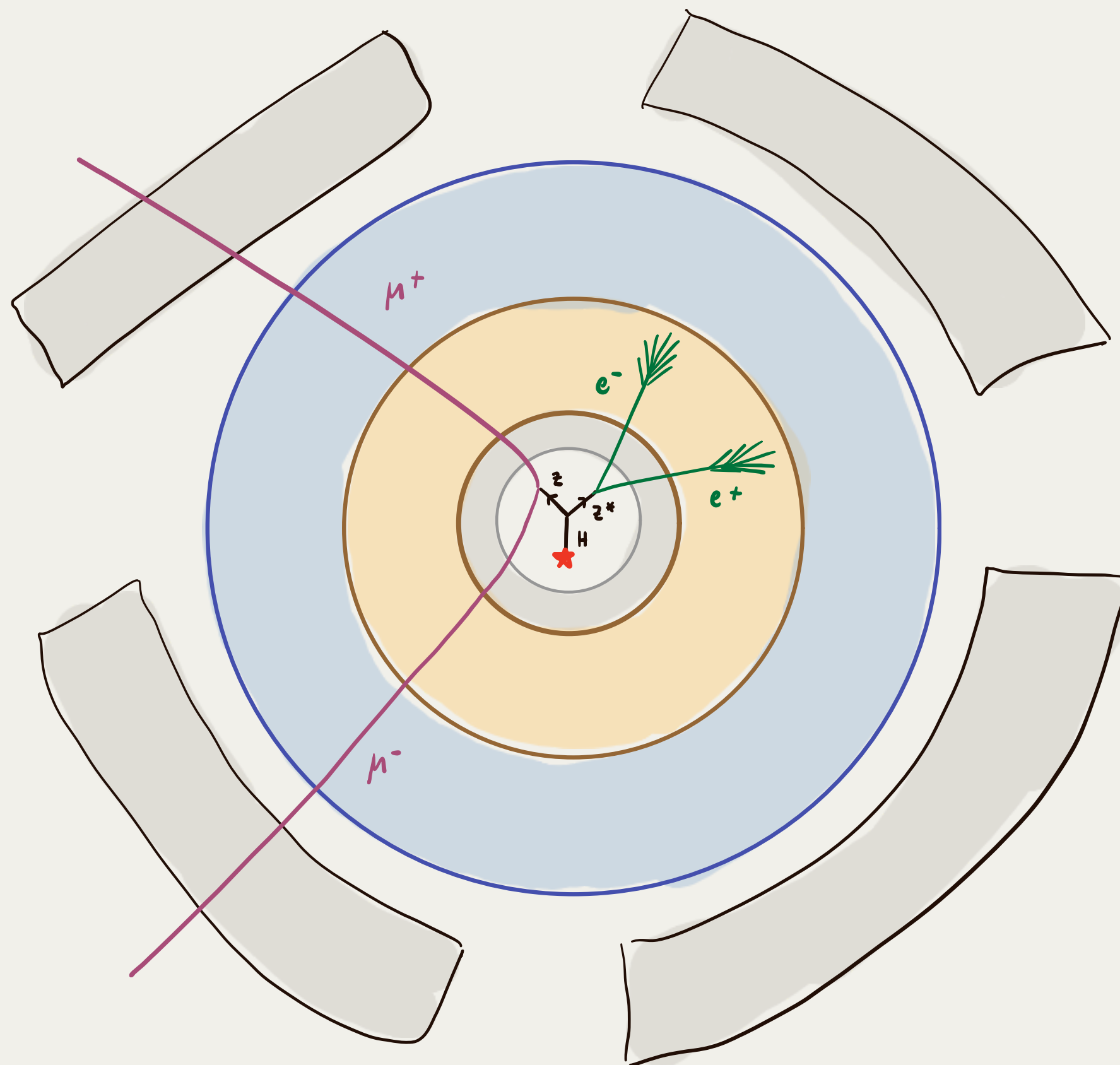
- What is quantum entanglement?
- How do you test quantum entanglement at high energies?
 - Reconstructing the full spin density matrix for $H \rightarrow ZZ \rightarrow 4l$
- What's wrong with this method?

A state is entangled iff it cannot be written as a separable state

State vector vs density matrix formulation of quantum mechanics

	State vector	Density Matrix
A state	$ \psi\rangle = \frac{1}{\sqrt{2}} (\uparrow\rangle + \downarrow\rangle)$	$\rho = \psi\rangle\langle\psi $
Separable state	$ \psi\rangle = \psi_A\rangle \otimes \psi_B\rangle$	$\rho = \sum_i p_i \rho_i^{(A)} \otimes \rho_i^{(B)}$

How to test entanglement at high energies?



$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_2} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \{ \rho (\Gamma_1 \otimes \Gamma_2)^T \} \quad \text{eq. 1}$$

$$\rho = \frac{1}{9} [1_3 \otimes 1_3 + A_{LM}^1 T_L^M \otimes 1_3 + A_{LM}^2 1_3 \otimes T_L^M + C_{L_1 M_1 L_2 M_2} T_{L_1}^{M_1} \otimes T_{L_2}^{M_2}] \quad \text{eq. 2}$$

(where $L = 1, 2$ and $-L \leq M \leq L$ and $B_1 = -\sqrt{2\pi\eta_\ell}$, $B_2 = \sqrt{\frac{2\pi}{5}}$)

$$\int \frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_2} Y_L^M(\Omega_j) d\Omega_j = \frac{B_L}{4\pi} A_{LM}^j, \quad j = 1, 2,$$

$$\int \frac{1}{\sigma} \frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2) d\Omega_1 d\Omega_2 = \frac{B_{L_1} B_{L_2}}{(4\pi)^2} C_{L_1 M_1 L_2 M_2}. \quad \text{eq. 3}$$

Run 2 (2015-2018) Monte Carlo Samples

Channels:

- $H \rightarrow ZZ^* \rightarrow 4e$
- $H \rightarrow ZZ^* \rightarrow 4\mu$
- $H \rightarrow ZZ^* \rightarrow 2e2\mu$

} 220 signal events

Cuts:

$$m_{4\ell} \in (115, 130) \text{ GeV}$$

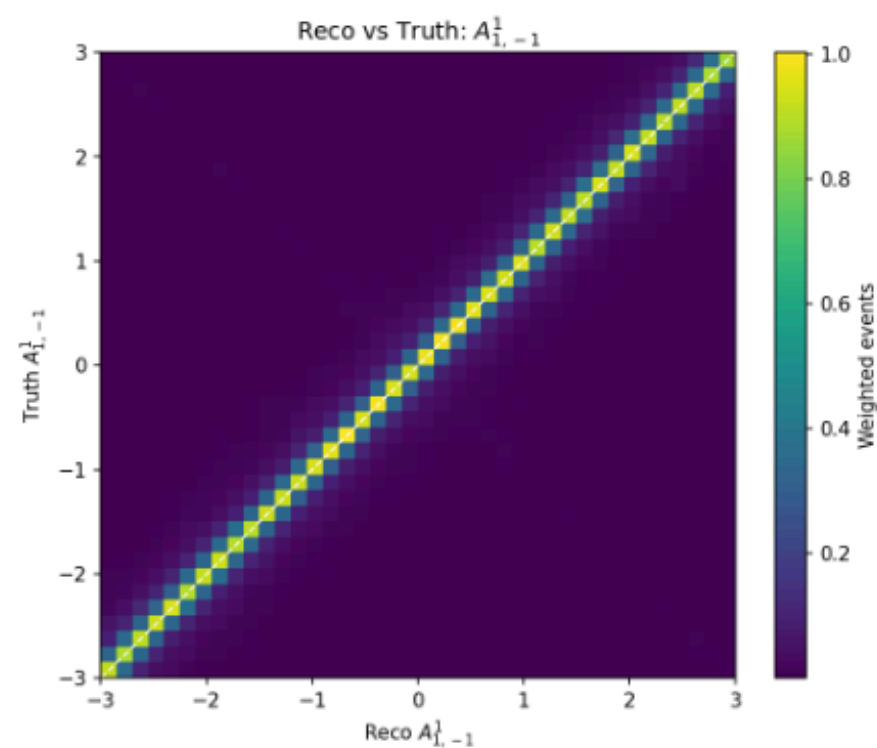
p_T thresholds for three leading leptons in the quadruplet: 20, 15 and 10 GeV

$$m_{Z_1} \in (50, 106) \text{ GeV}$$

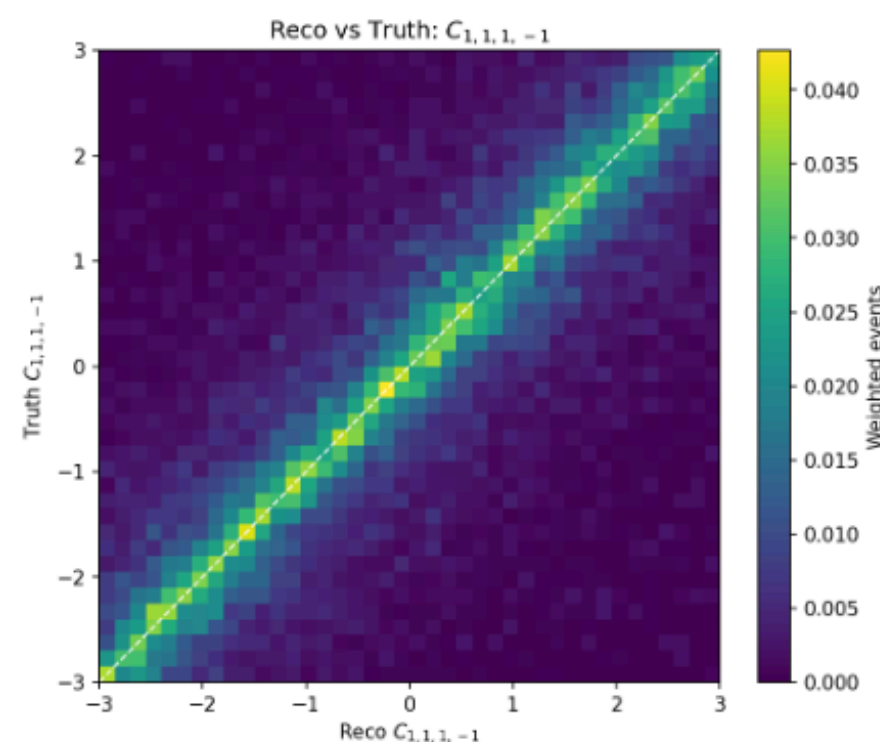
SFOC pair must have an invariant mass greater than 5 GeV

$$m_{Z_2} \in (12, 115) \text{ GeV}$$

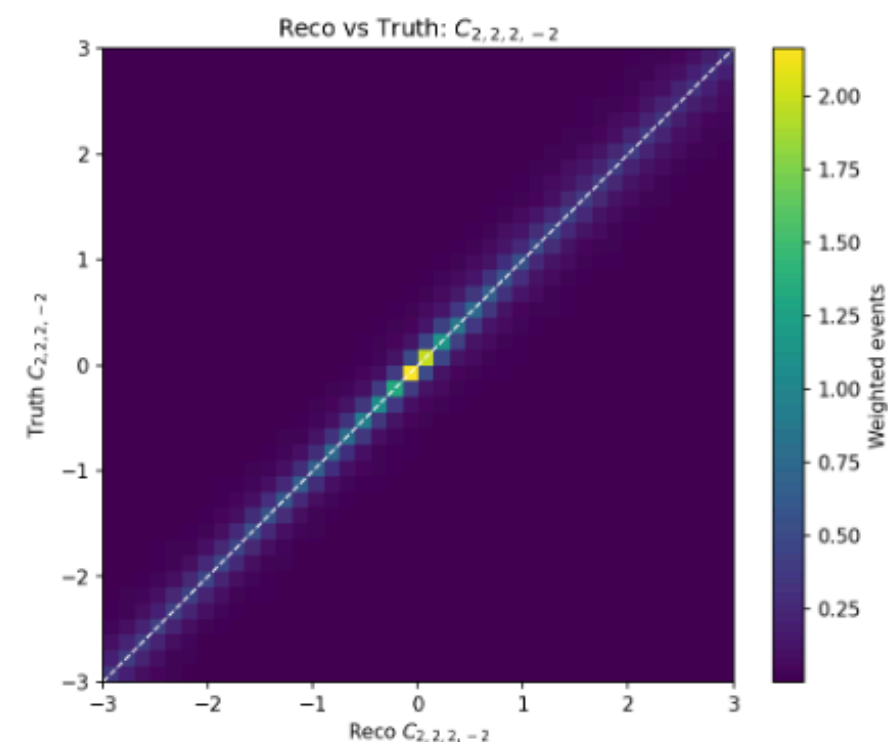
Extension to full phase space - Migration functions



(a) $A_{1,-1}^1$



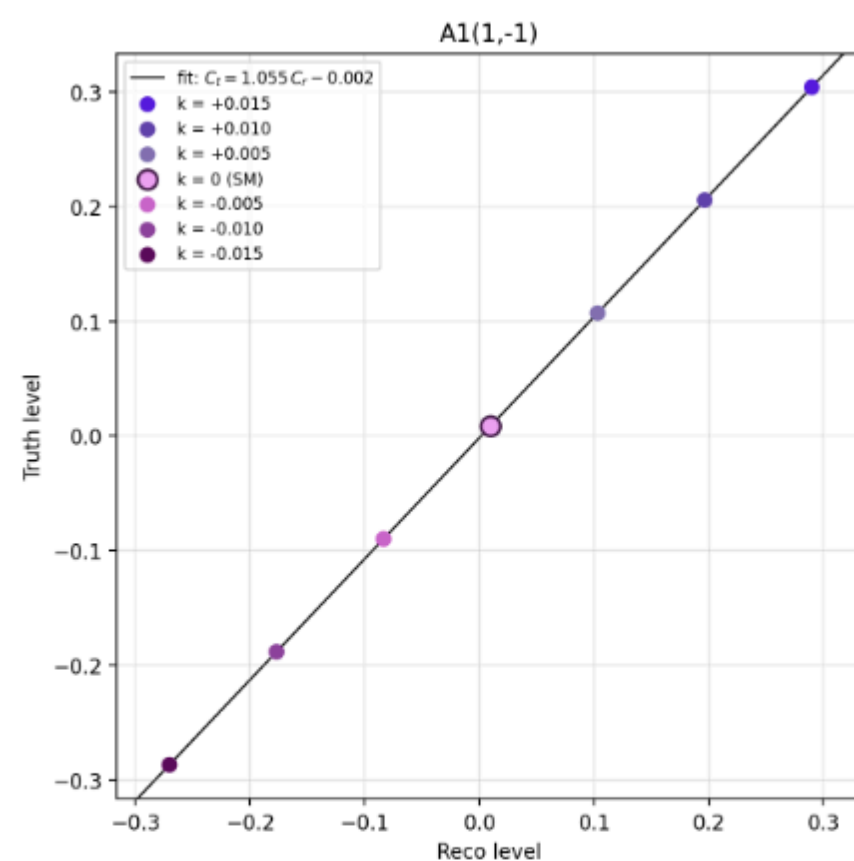
(b) $C_{1,1,1,-1}$



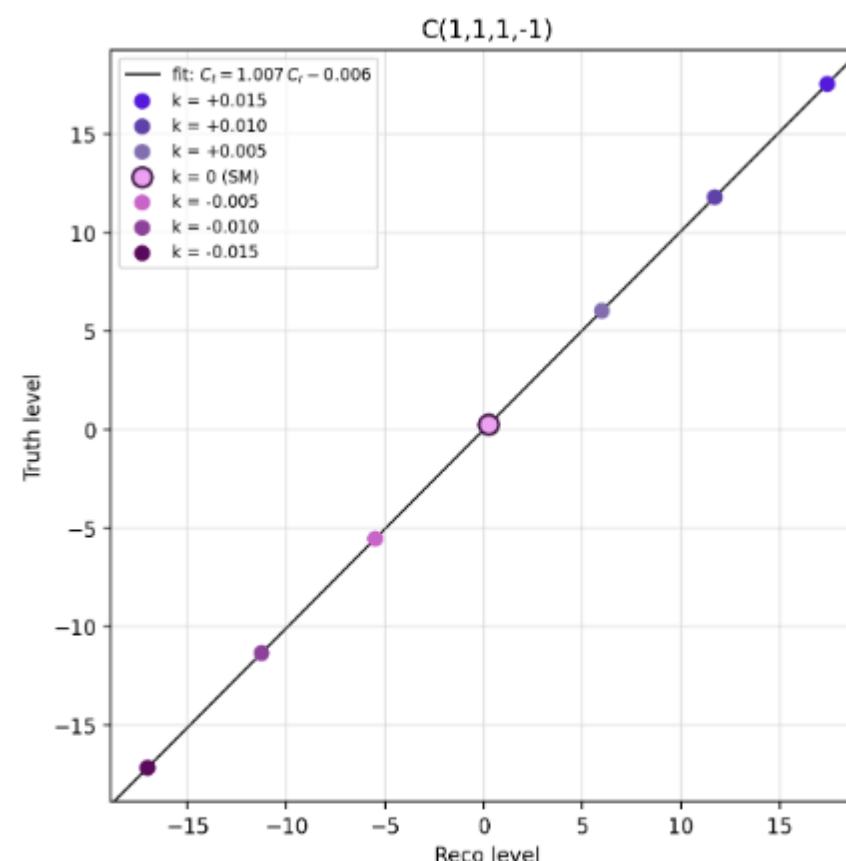
(c) $C_{2,2,2,-2}$

Extension to full phase space - migration functions

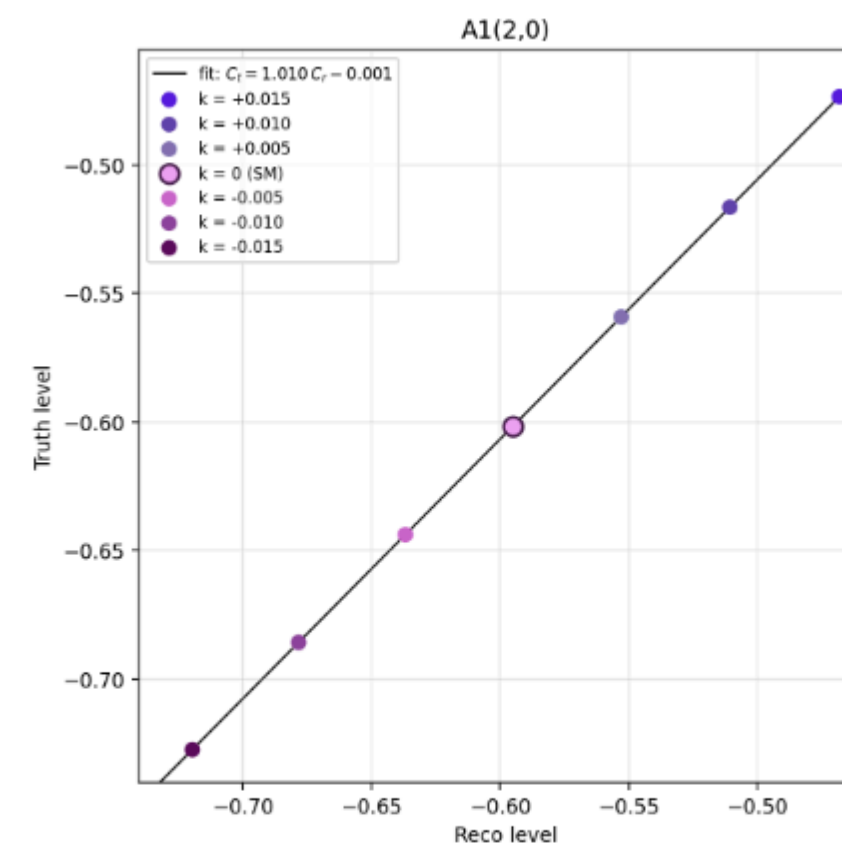
$$w_i \rightarrow w_i^{\text{reweighted}} = (k \cdot c_i + 1) w_i \quad k \in \{-0.015, -0.01, -0.005, 0, 0.005, 0.01, 0.015\}, \quad c_i: \text{event observable}$$



(a) $A_{1,-1}^1$ for $2e2\mu$



(b) $C_{1,1,1,-1}$ for $2e2\mu$



(c) $A_{2,0}^1$ for 4μ

Results!

$$\rho = \begin{pmatrix} -0.197 & 0.008 & -0.057 & 0.007 & 0.023 & -0.023 & -0.012 & -0.002 & 0.054 \\ 0.008 & 0.052 & -0.035 & -0.150 & 0.064 & 0.076 & 0.007 & -0.172 & 0.001 \\ -0.057 & -0.035 & \mathbf{0.351} & -0.014 & \mathbf{-0.439} & 0.029 & \mathbf{0.252} & -0.010 & -0.017 \\ 0.007 & -0.150 & -0.014 & -0.038 & 0.015 & -0.171 & -0.035 & 0.081 & 0.024 \\ 0.023 & 0.064 & \mathbf{-0.439} & 0.015 & \mathbf{0.678} & -0.021 & \mathbf{-0.446} & -0.060 & 0.023 \\ -0.023 & 0.076 & 0.029 & -0.171 & -0.021 & -0.043 & 0.015 & -0.128 & -0.002 \\ -0.012 & 0.007 & \mathbf{0.252} & -0.035 & \mathbf{-0.446} & 0.015 & \mathbf{0.351} & 0.040 & -0.045 \\ -0.002 & -0.172 & -0.010 & 0.081 & -0.060 & -0.128 & 0.040 & 0.036 & -0.003 \\ 0.054 & 0.001 & -0.017 & 0.024 & 0.023 & -0.002 & -0.045 & -0.003 & -0.190 \end{pmatrix}$$

Entangled! (Peres-Horodecki condition)

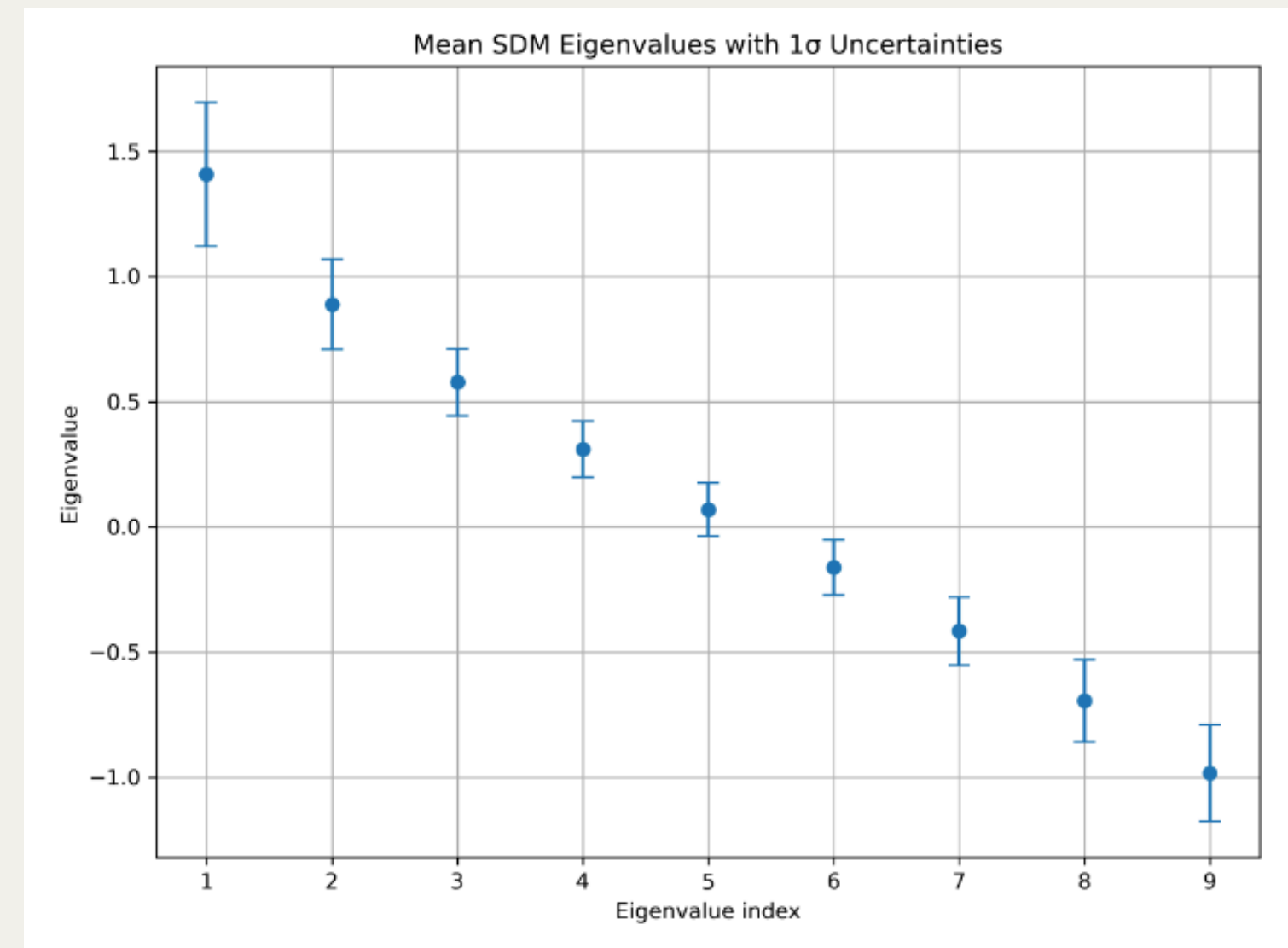
Negative eigenvalues: -0.264 -0.233 -0.151 -0.109, -0.051, 0.014, 0.116, 0.399, 1.278

Formal definition of a density matrix

A density operator ρ on a Hilbert space $\mathcal{H}$ satisfies:

1. Hermitian: $\rho^\dagger = \rho$
2. Trace one: $\text{tr}(\rho) = 1$
3. Non-negative: $\langle v | \rho | v \rangle \geq 0$

Uncertainties of eigenvalues (run 2 + 3 estimate)



Thank you!

JATLAS Collaboration. “Observation of quantum entanglement with top quarks at the ATLAS detector”. In: Nature 633 (2024), pp. 542–547. doi: 10.1038/s41586-024-07824-z

CMS Collaboration. “Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV”. In: Reports on Progress in Physics 87.11 (2024), p. 117801.
doi:10.1088/1361-6633/ad7e4d.

ATLAS Collaboration. “Measurements of Z-boson pair entanglement in decays of Higgs bosons at the ATLAS experiment”. In: arXiv preprint arXiv:2603.26463 (2026). doi:
10.48550/arXiv.2603.26463. arXiv: 2603.26463 [hep-ex]

A. Peres. “Separability Criterion for Density Matrices”. In: Physical Review Letters 77 (1996), pp. 1413–1415. doi: 10.1103/PhysRevLett.77.1413.

J. A. Aguilar-Saavedra et al. “Testing entanglement and Bell inequalities in $H \rightarrow ZZ$ ”. In: Phys. Rev.D 107 (2023), p. 016012. doi: 10.1103/PhysRevD.107.016012. arXiv: 2209.13441 [hep-ph]

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Higgs parity conservation

$$\{|++\rangle, | + 0\rangle, | + -\rangle, |0+\rangle, |00\rangle, |0-\rangle, | - +\rangle, | - 0\rangle, | --\rangle\}$$

$$|\psi_{ZZ}\rangle = \frac{1}{\sqrt{2 + \beta^2}} (| + -\rangle - \beta|00\rangle + | - +\rangle)$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{6}(\sqrt{2}A_{2,0}^1 + 2) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{3}(1 - \sqrt{2}A_{2,0}^1) & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}C_{2,2,2,-2} & 0 & \frac{1}{3}C_{2,1,2,-1} & 0 & \frac{1}{6}(\sqrt{2}A_{2,0}^1 + 2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Irreducible tensor operators

$$T_1^1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_0^1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T_{-1}^1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T_2^2 = \sqrt{3} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_{-2}^2 = \sqrt{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T_1^2 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_{-1}^2 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \quad T_0^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

J. A. Aguilar-Saavedra et al. “Testing entanglement and Bell inequalities in $H \rightarrow ZZ$ ”. In: Phys. Rev.

D 107 (2023), p. 016012. doi: 10.1103/PhysRevD.107.016012. arXiv: 2209.13441 [hep-ph]

Decay density matrix of a Z boson into leptons

$$\Gamma = \frac{1}{4} \begin{pmatrix} 1 + \cos^2 \theta - 2\eta_\ell \cos \theta & \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin \theta)e^{i\varphi} & (1 - \cos^2 \theta)e^{i2\varphi} \\ \frac{1}{\sqrt{2}}(\sin 2\theta - 2\eta_\ell \sin \theta)e^{-i\varphi} & 2 \sin^2 \theta & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin \theta)e^{i\varphi} \\ (1 - \cos^2 \theta)e^{-i2\varphi} & -\frac{1}{\sqrt{2}}(\sin 2\theta + 2\eta_\ell \sin \theta)e^{-i\varphi} & 1 + \cos^2 \theta - 2\eta_\ell \cos \theta \end{pmatrix}$$

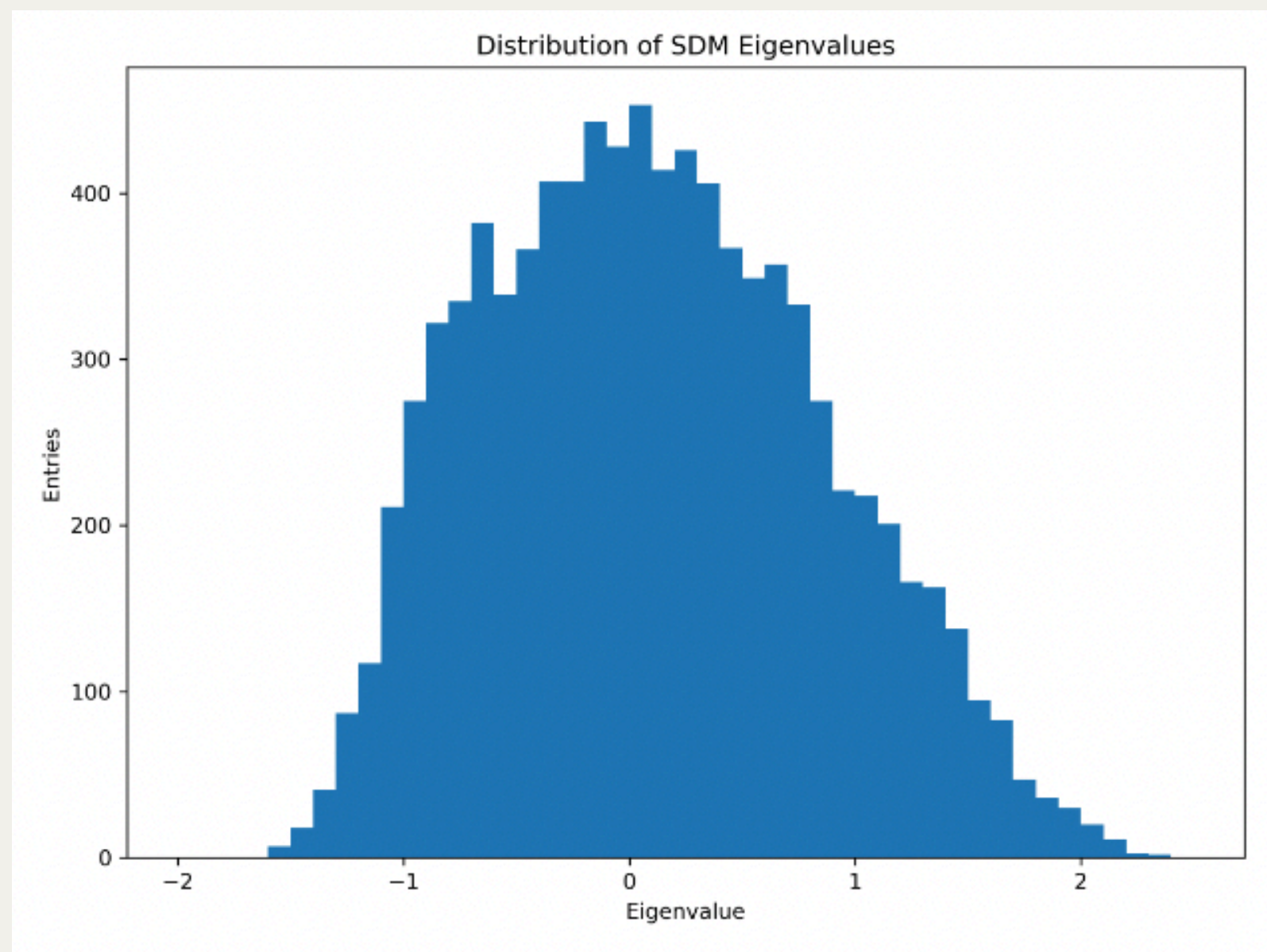
$$\eta_\ell = \frac{1 - 4s_W^2}{1 - 4s_W^2 + 8s_W^4}$$

J. A. Aguilar-Saavedra et al. “Testing entanglement and Bell inequalities in $H \rightarrow ZZ$ ”. In: Phys. Rev.

D 107 (2023), p. 016012. doi: 10.1103/PhysRevD.107.016012. arXiv: 2209.13441 [hep-ph]

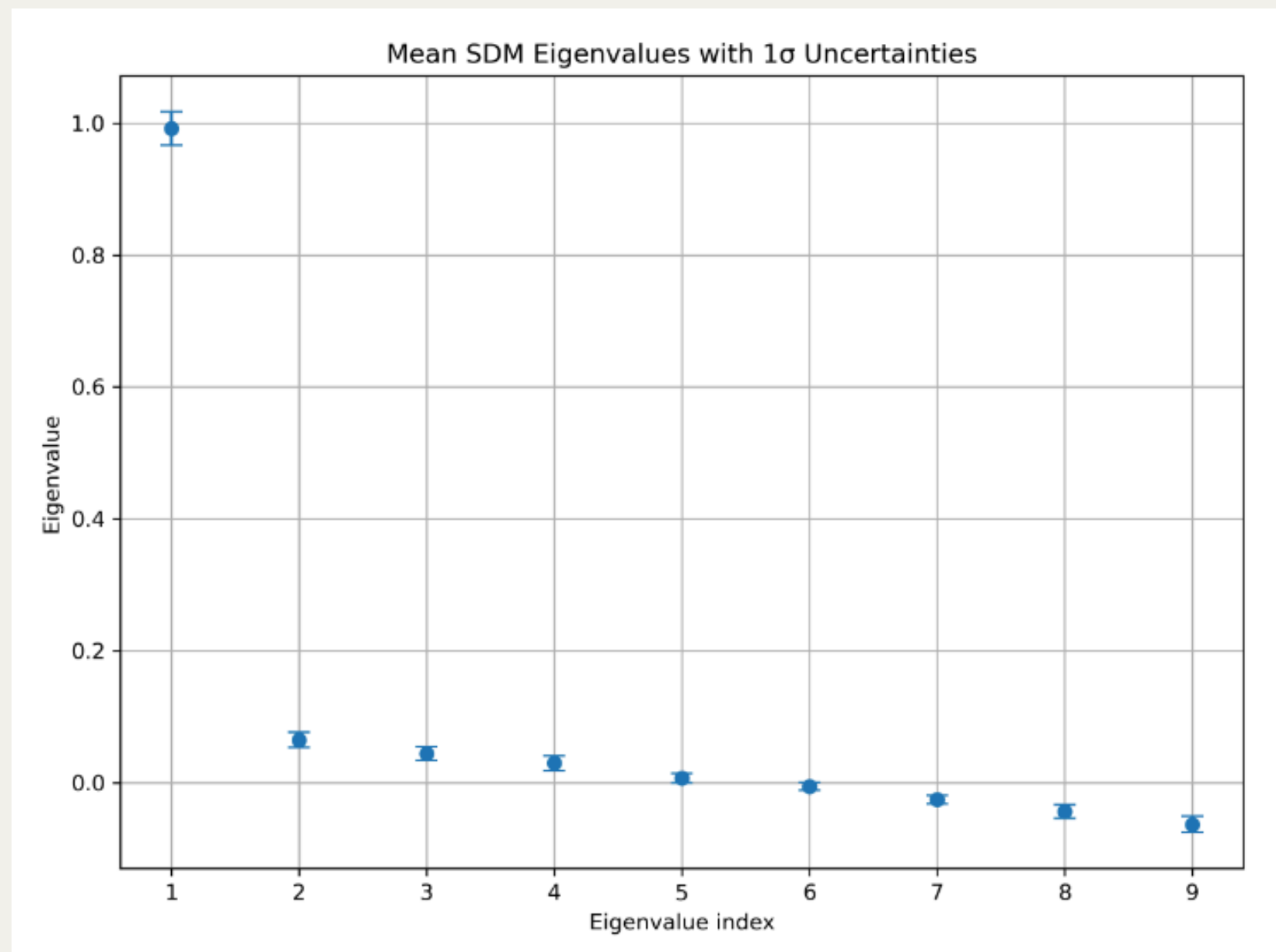
Distribution of eigenvalues $H \rightarrow ZZ$

(1000 toy experiments with 470 events each - run 2 + 3)



High statistics distribution of eigenvalues $H \rightarrow ZZ$

(100k events per toy experiment)



Truth level SDM high statistics - 1 million events (LO)

$$\rho = \begin{pmatrix} 0.004 & -0.007 & 0.001 & 0.009 & -0.002 & 0.006 & -0.002 & -0.007 & 0.003 \\ -0.007 & -0.005 & -0.012 & 0.011 & -0.001 & 0.004 & 0.005 & -0.002 & -0.002 \\ 0.001 & -0.012 & 0.197 & 0 & -0.311 & -0.006 & 0.196 & 0.005 & 0.004 \\ 0.009 & 0.011 & 0 & 0.002 & -0.001 & 0.001 & 0.005 & 0 & 0.003 \\ -0.002 & -0.001 & -0.311 & -0.001 & 0.611 & 0.003 & -0.311 & -0.001 & -0.002 \\ 0.006 & 0.004 & -0.006 & 0.001 & 0.003 & -0.002 & -0.001 & 0.001 & -0.001 \\ -0.002 & 0.005 & 0.196 & 0.005 & -0.311 & -0.001 & 0.191 & 0.009 & -0.002 \\ -0.007 & -0.002 & 0.005 & 0 & -0.001 & 0.001 & 0.009 & 0.006 & 0.009 \\ 0.003 & -0.002 & 0.004 & 0.003 & -0.002 & -0.001 & -0.002 & 0.009 & -0.003 \end{pmatrix}$$

Some example migration functions...

Observable	Migration Function
A1(1,-1)	$C_t = C_r \cdot 1.029 - 0.001$
A1(1,0)	$C_t = C_r \cdot 1.005 + 0.003$
A1(1,1)	$C_t = C_r \cdot 1.029 + 0.001$
A1(2,-2)	$C_t = C_r \cdot 1.014 + 0.004$
A1(2,-1)	$C_t = C_r \cdot 1.033 - 0.009$
A1(2,0)	$C_t = C_r \cdot 1.010 - 0.001$
A1(2,1)	$C_t = C_r \cdot 1.033 + 0.009$
A1(2,2)	$C_t = C_r \cdot 1.014 + 0.004$

(a) A1 migration functions 4mu (NTuples 2.a)

Observable	Migration Function
A2(1,-1)	$C_t = C_r \cdot 1.028 + 0.007$
A2(1,0)	$C_t = C_r \cdot 1.006 + 0.001$
A2(1,1)	$C_t = C_r \cdot 1.028 - 0.007$
A2(2,-2)	$C_t = C_r \cdot 1.014 + 0.005$
A2(2,-1)	$C_t = C_r \cdot 1.033 - 0.006$
A2(2,0)	$C_t = C_r \cdot 1.011 + 0.004$
A2(2,1)	$C_t = C_r \cdot 1.033 + 0.006$
A2(2,2)	$C_t = C_r \cdot 1.014 + 0.005$

(b) A2 migration functions 4mu (NTuples 2.a)