Study of the $\Lambda \to p\ell\bar{\nu}_\ell$ semi-leptonic decay in Lattice QCD

Speaker : Andreas Konstantinou

Supervisors: Constantia Alexandrou & Simone Bacchio

THE CYPRUS INSTITUTE

Republic of Cyprus

Co-funded by the European Union

RESEARCH & INNOVATION FOUNDATION

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SUPERMARIOLOGY 101



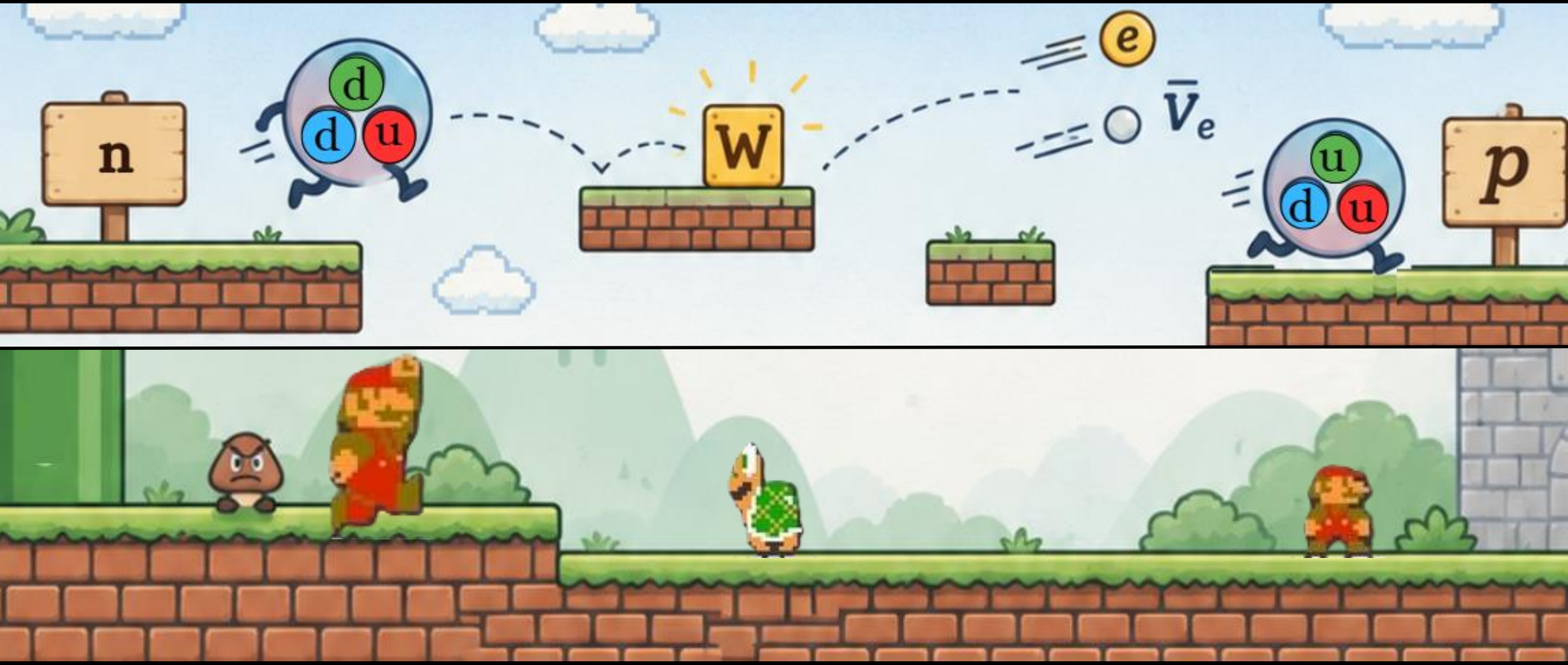
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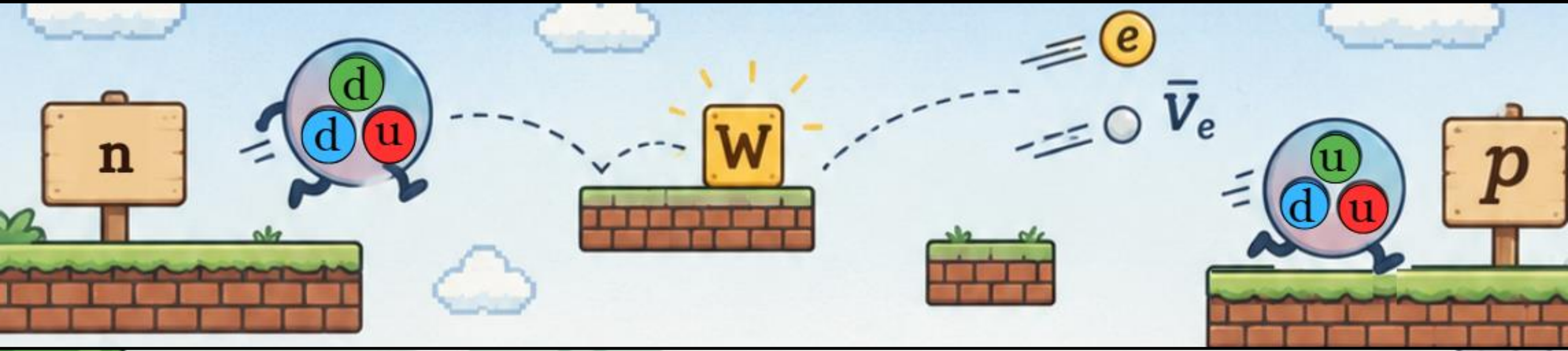
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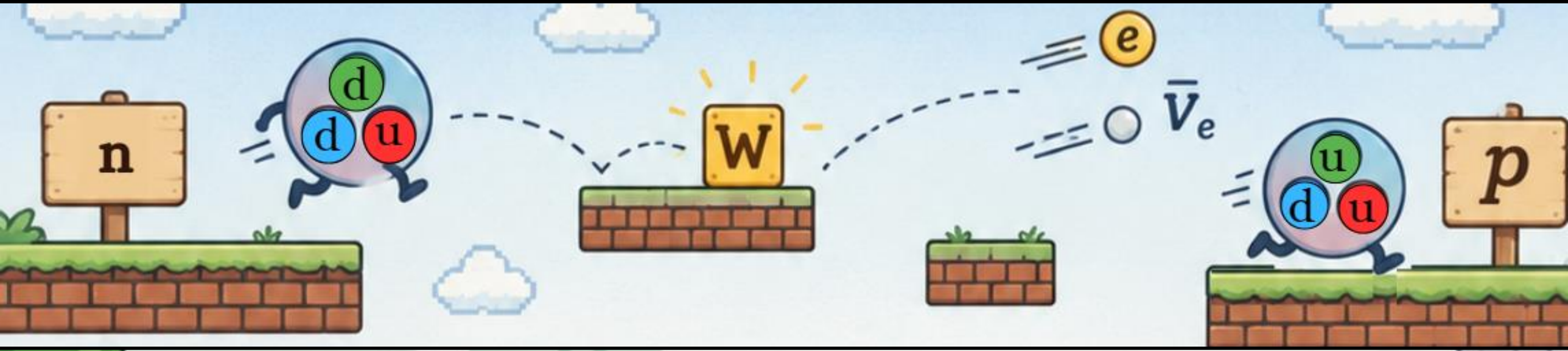


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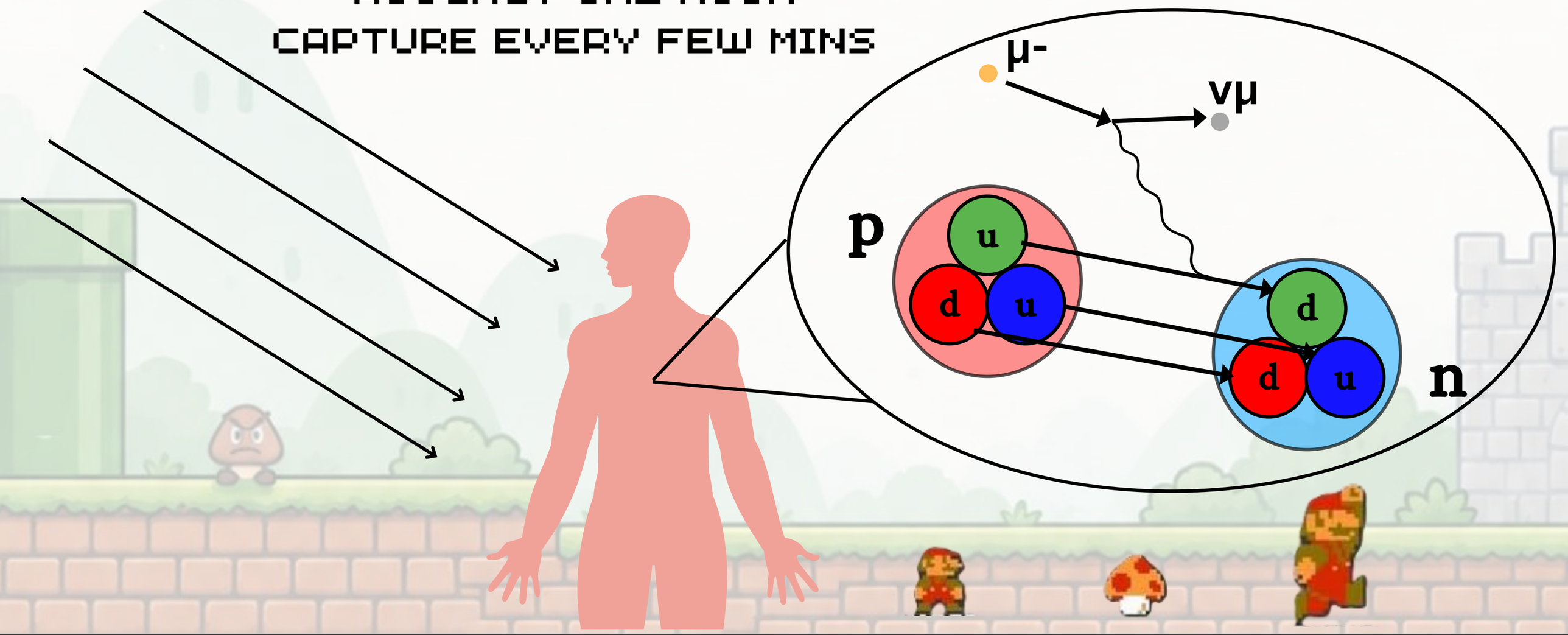
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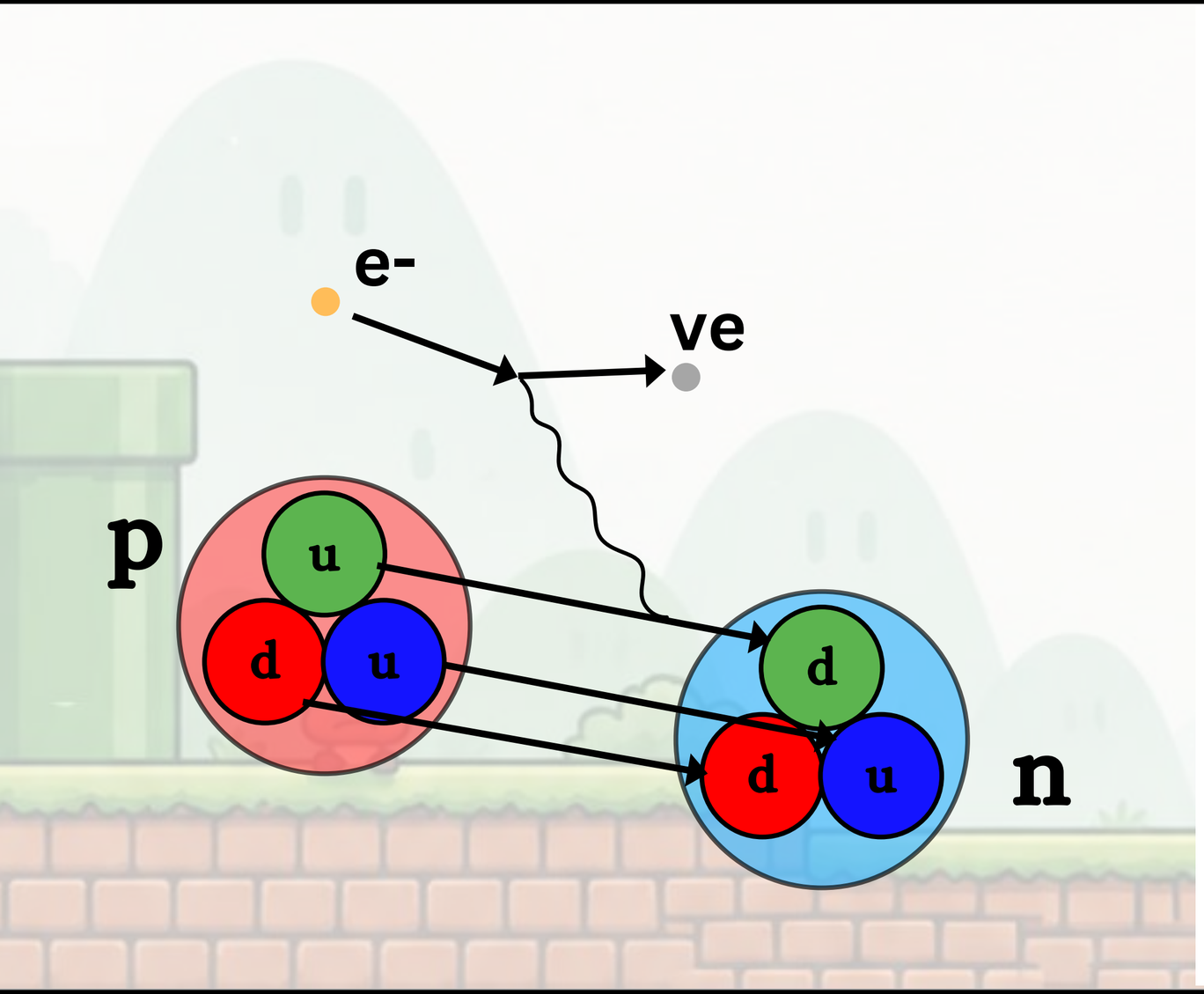


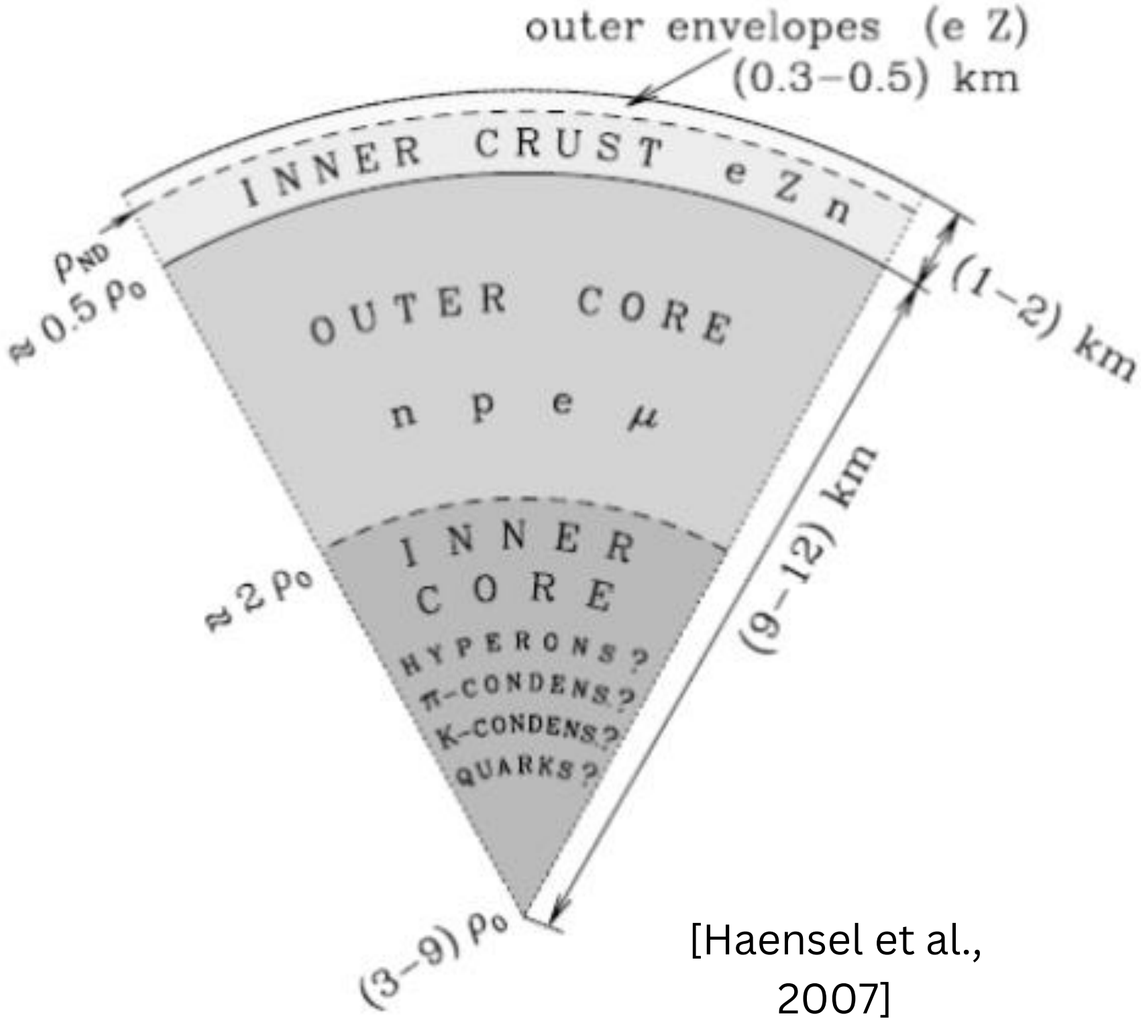
WHY CARING AT ALL?

ROUGHLY ONE MUON
CAPTURE EVERY FEW MINS



WHY CARING AT ALL?

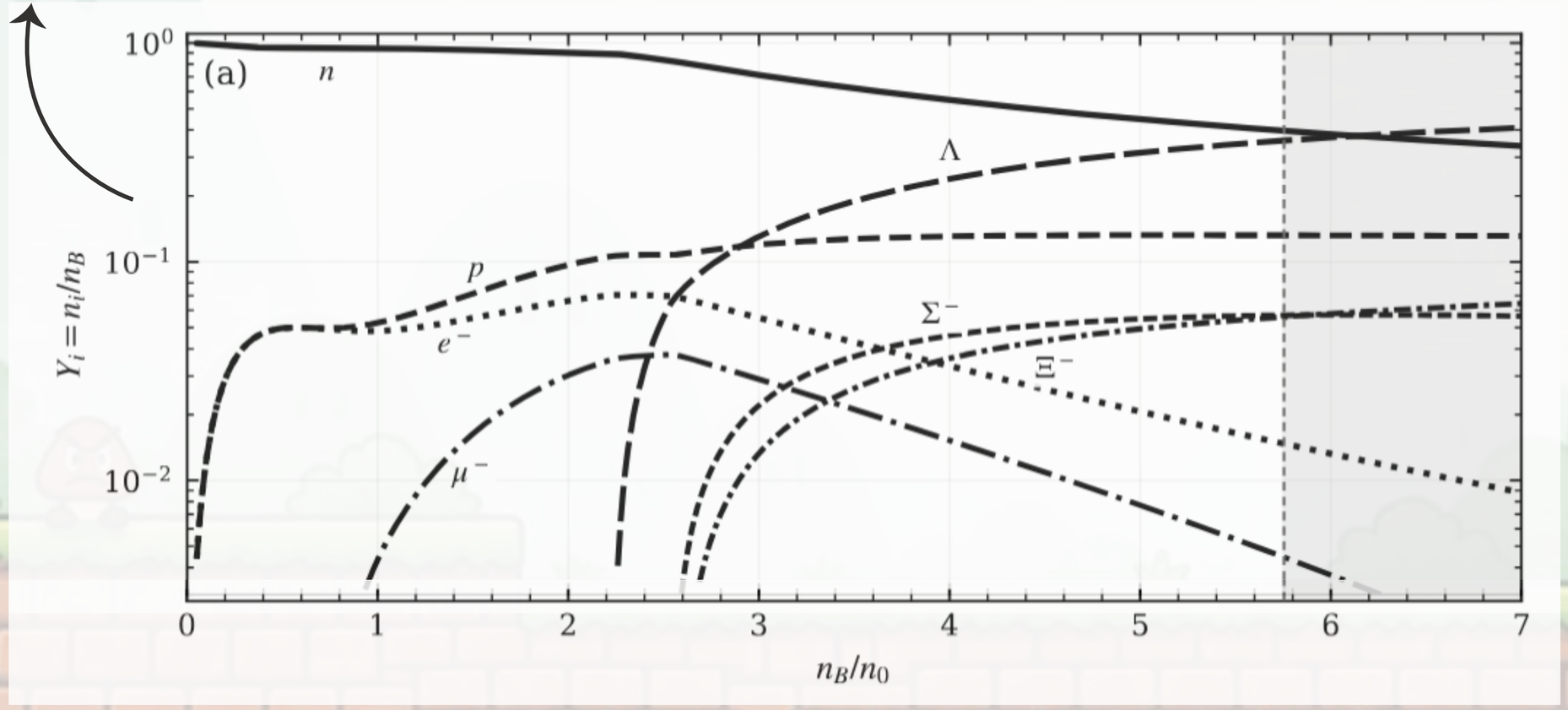




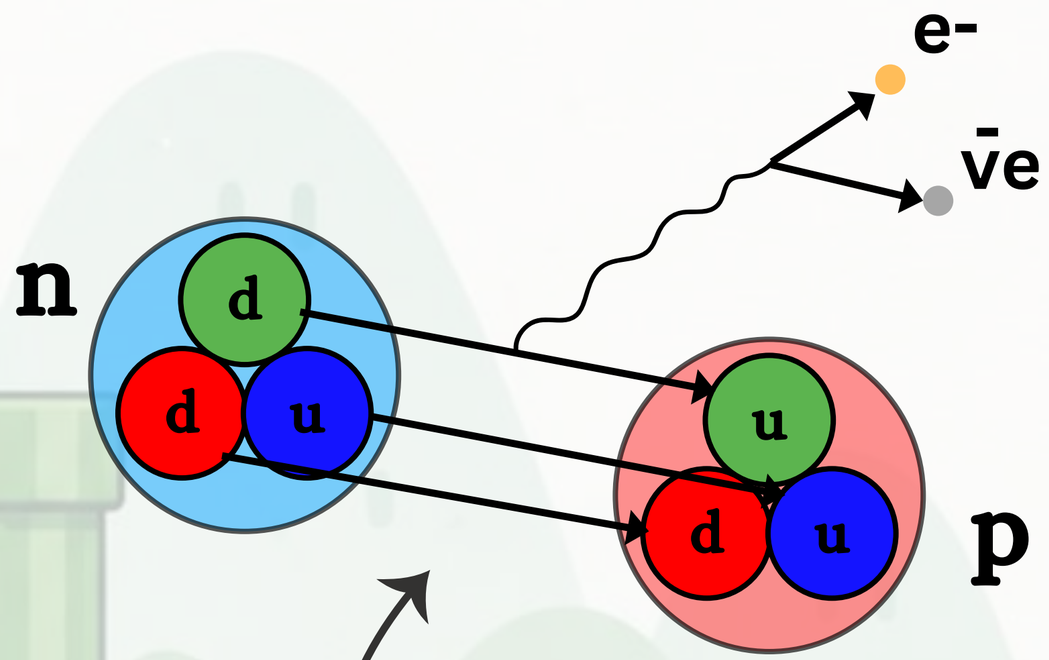
[Haensel et al., 2007]

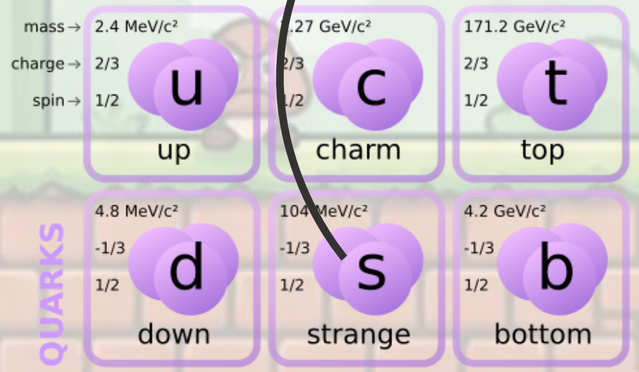
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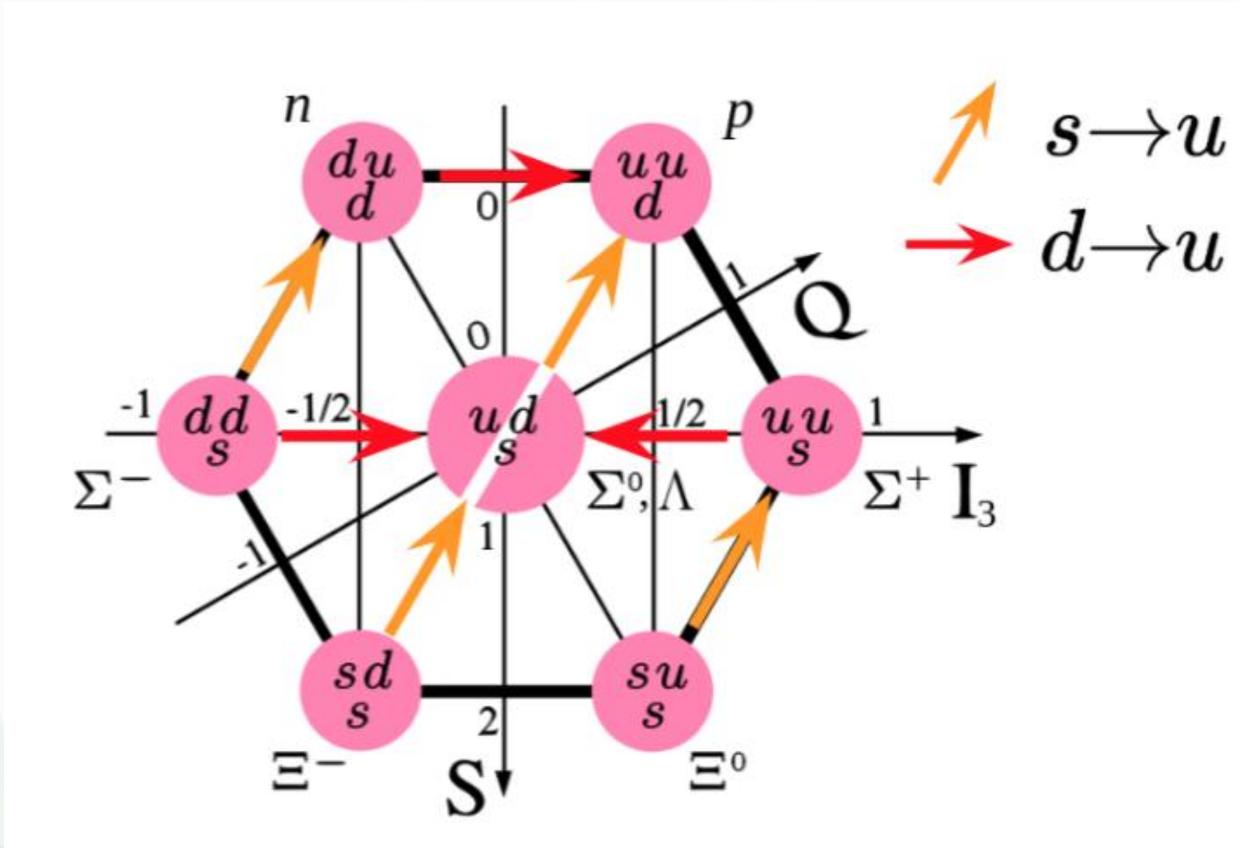
THIS IS WHY WE CALL NEUTRON STARS, NEUTRON STARS



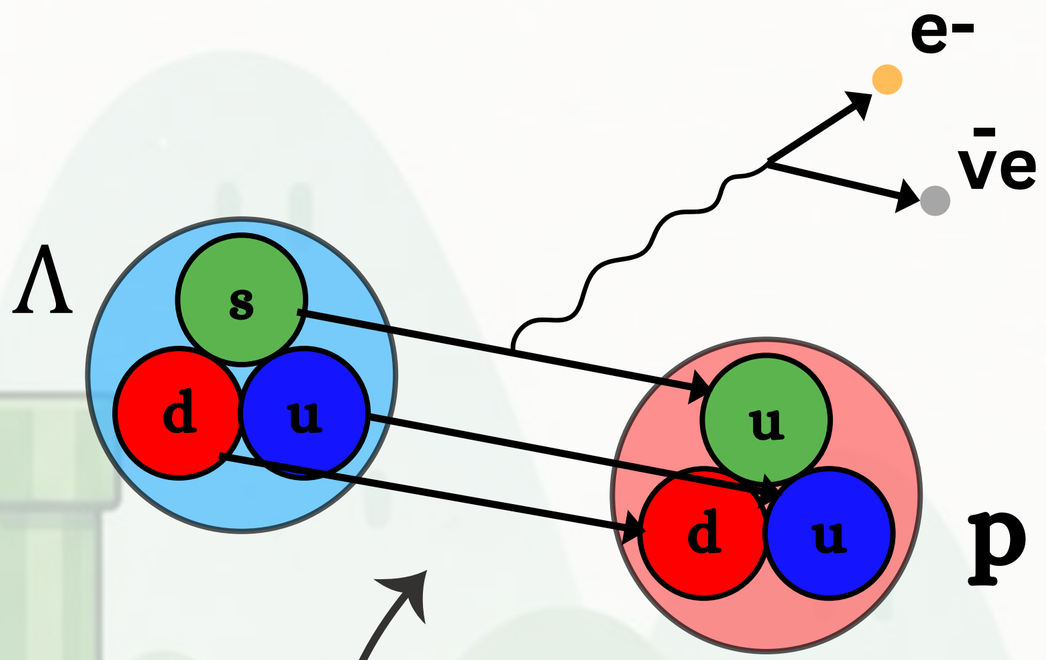
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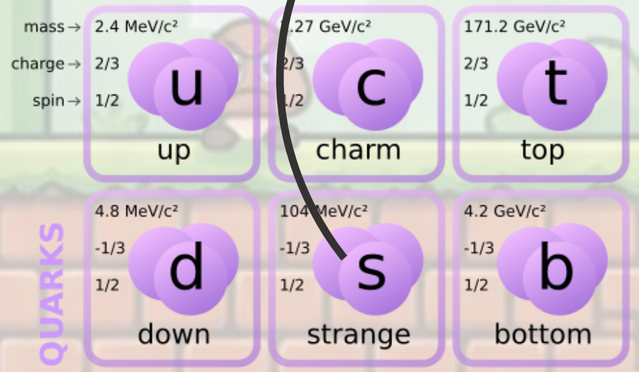


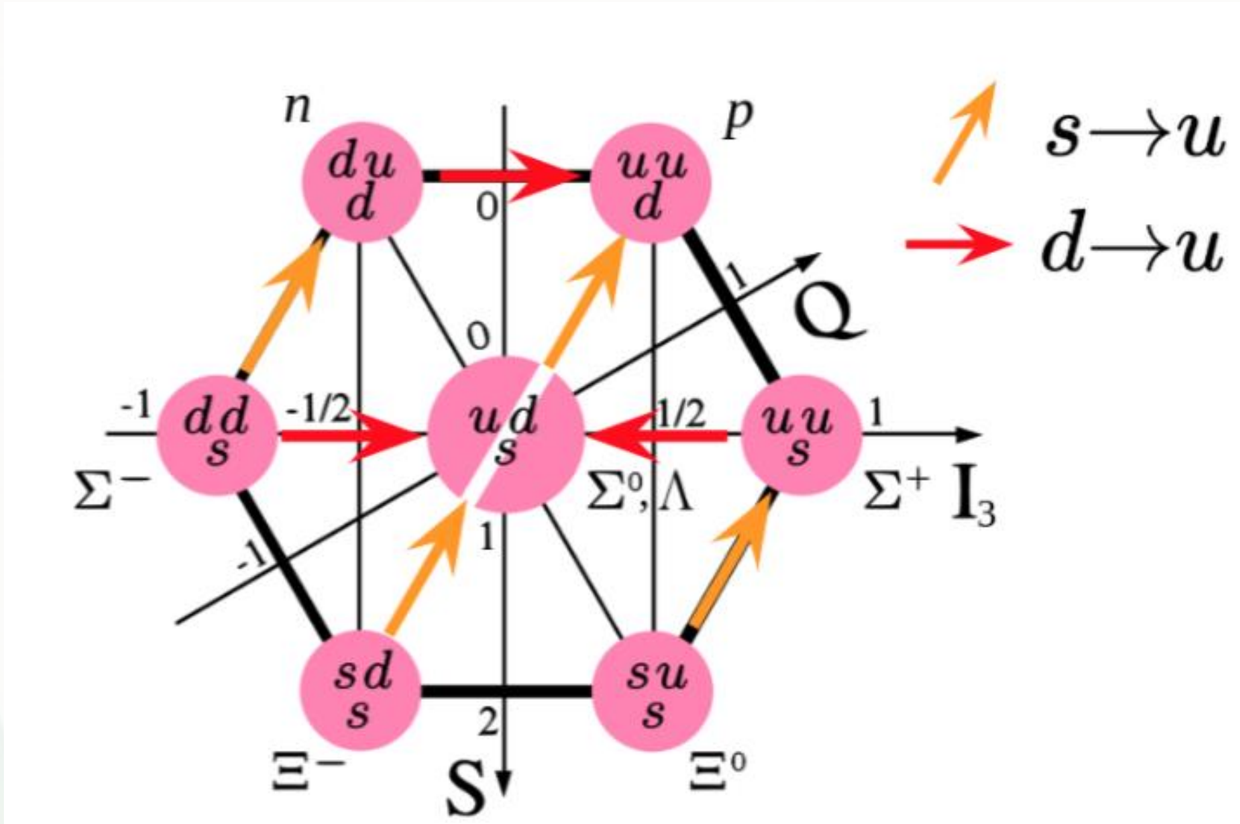




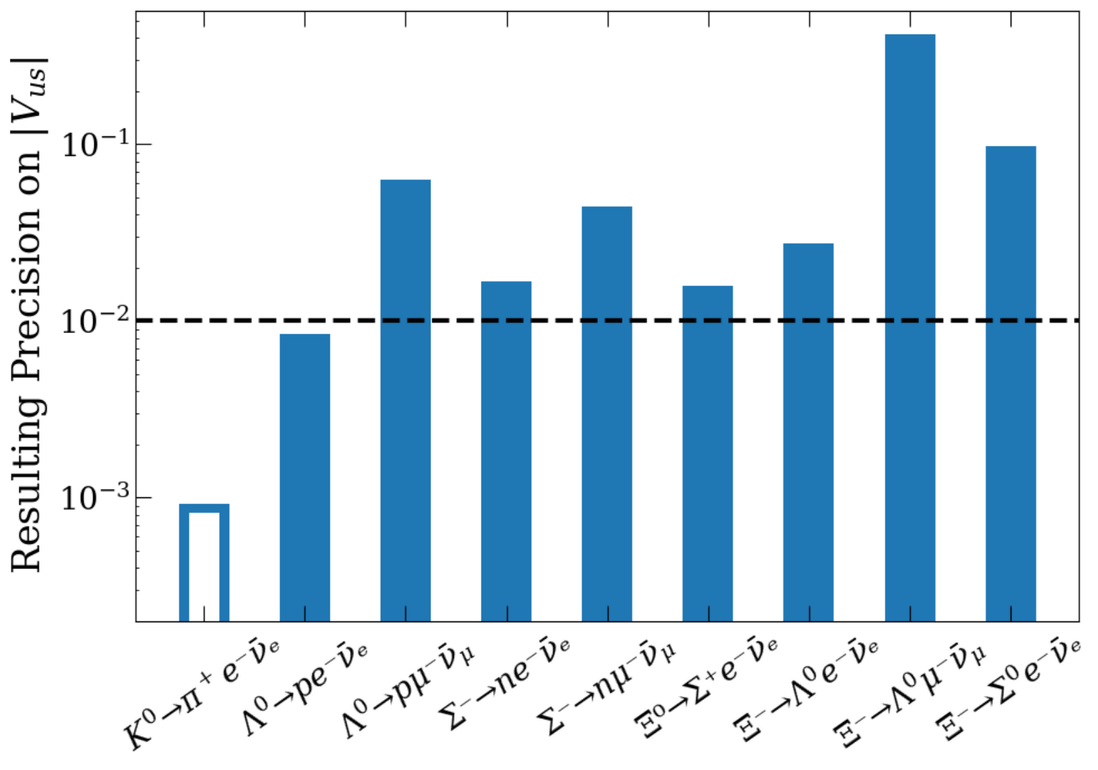
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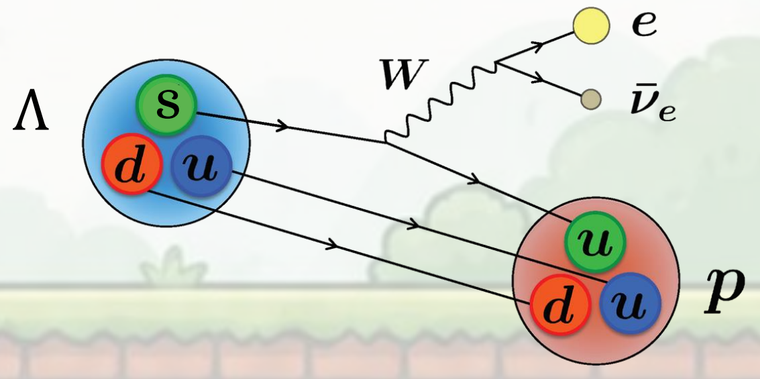




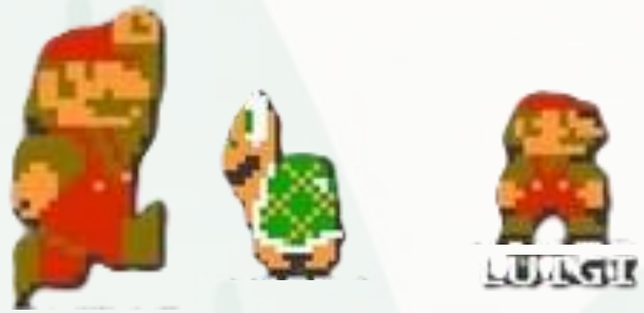


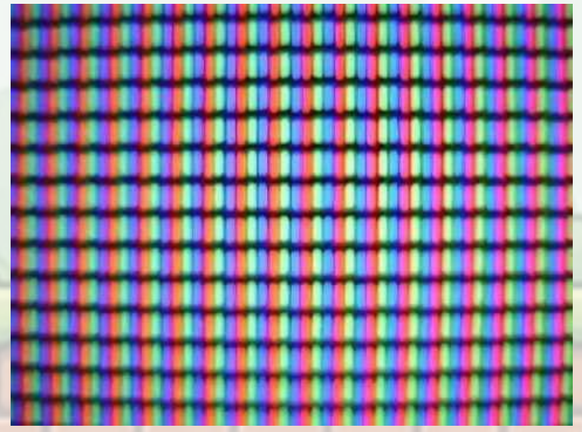
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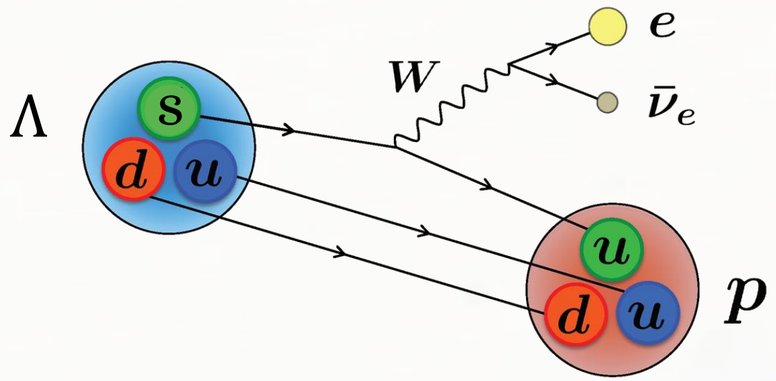


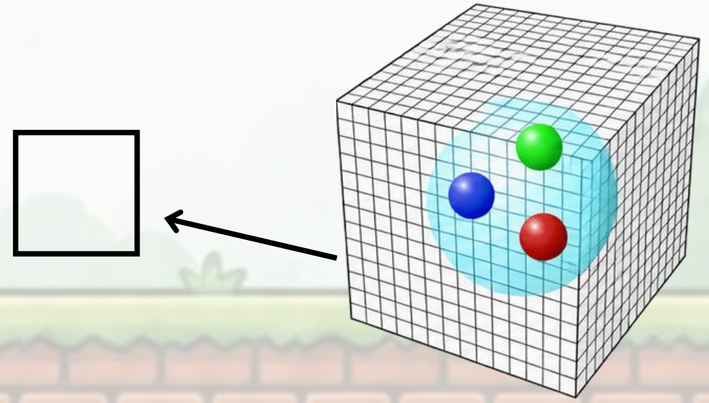
SuperMariology 101





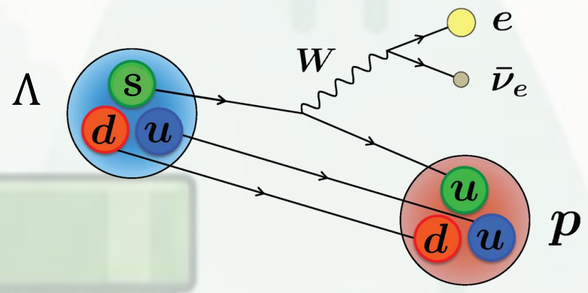
WORLD 1-1: WE DISCRETIZE SPACE AND TIME TO MAKE STANDARD MODEL PREDICTIONS ON A COMPUTER.

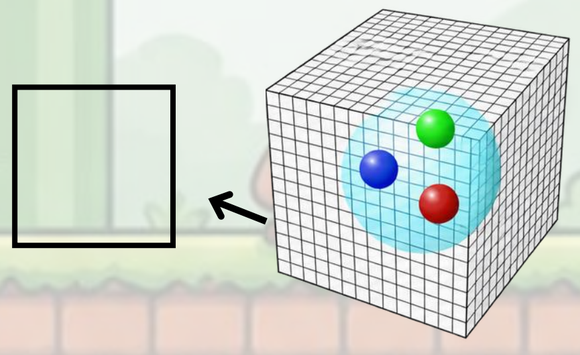


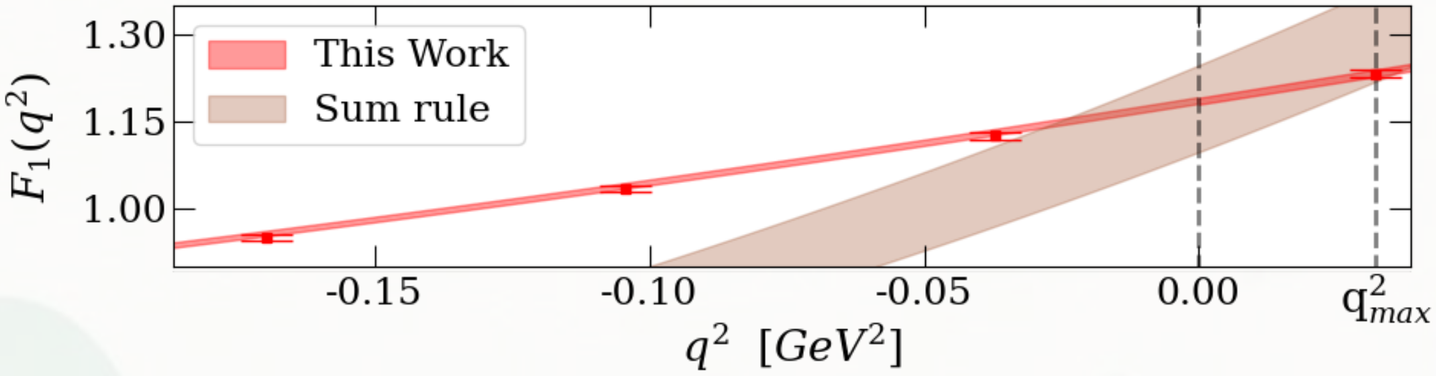


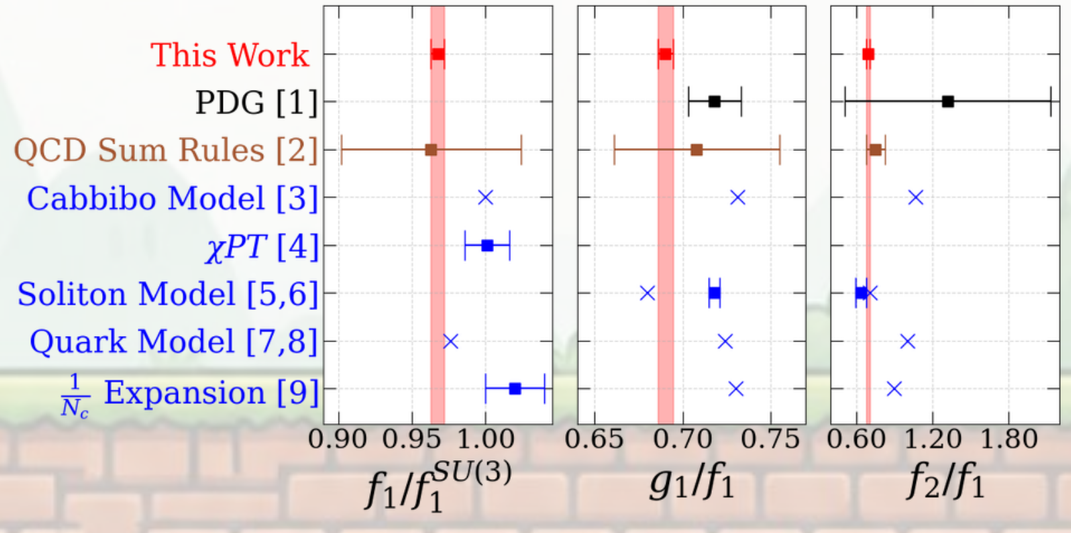
SuperMariology 101

WORLD 1-2 WE COMPUTE TWO- AND THREE-POINT CORRELATION FUNCTIONS TO EXTRACT THE FORM FACTORS, WHICH ENCODE THE EFFECTS OF THE STRONG INTERACTION.









SuperMariology 101

WORLD 1-3 BY COMBINING EXPERIMENTAL INPUTS [1], PERTURBATIVE LEPTONIC CONTRIBUTIONS, AND THE HADRONIC INPUTS FROM LQCD, WE CAN DETERMINE THE CKM MATRIX ELEMENTS

$$\mathcal{B}_{\Lambda\to pe\bar{\nu}_e} = \frac{\tau_\Lambda G_F^2 |V_{us}|^2}{192\pi^3 {m_\Lambda}^3} \int_{{m_\ell}^2}^{q^2_{\max}} dq^2 \sqrt{\lambda({m_\Lambda}^2, {m_p}^2, q^2)}\, L^{\mu\nu} H_{\mu\nu},$$



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

SuperMariology 101

WORLD 1-4: OUR DESIRED "PRINCESS" IS CKM MATRIX UNITARITY. ANY DEVIATION FROM UNITARITY COULD SIGNAL PHYSICS BEYOND THE STANDARD MODEL. OUR RESULTS AT ONE LATTICE SPACING ARE SHOWN BELOW.



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\begin{aligned} V_{ud}: \quad & n\,(udd) \to p\,(uud) \\ & \pi^+\,(u\bar{d}) \to \pi^0\,(d\bar{d}) \\ & \dots \\ V_{us}: \quad & \Lambda\,(usd) \to p\,(uud) \\ & \overline{K}^0\,(s\bar{d}) \to \pi^0\,(u\bar{d}) \\ & \dots \end{aligned}$$

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$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}^{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SuperMariology 101

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$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\sqrt{\left|V_{ud}^{\mathrm{PDG}}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}^{\mathrm{PDG}}\right|^2} = 1$$

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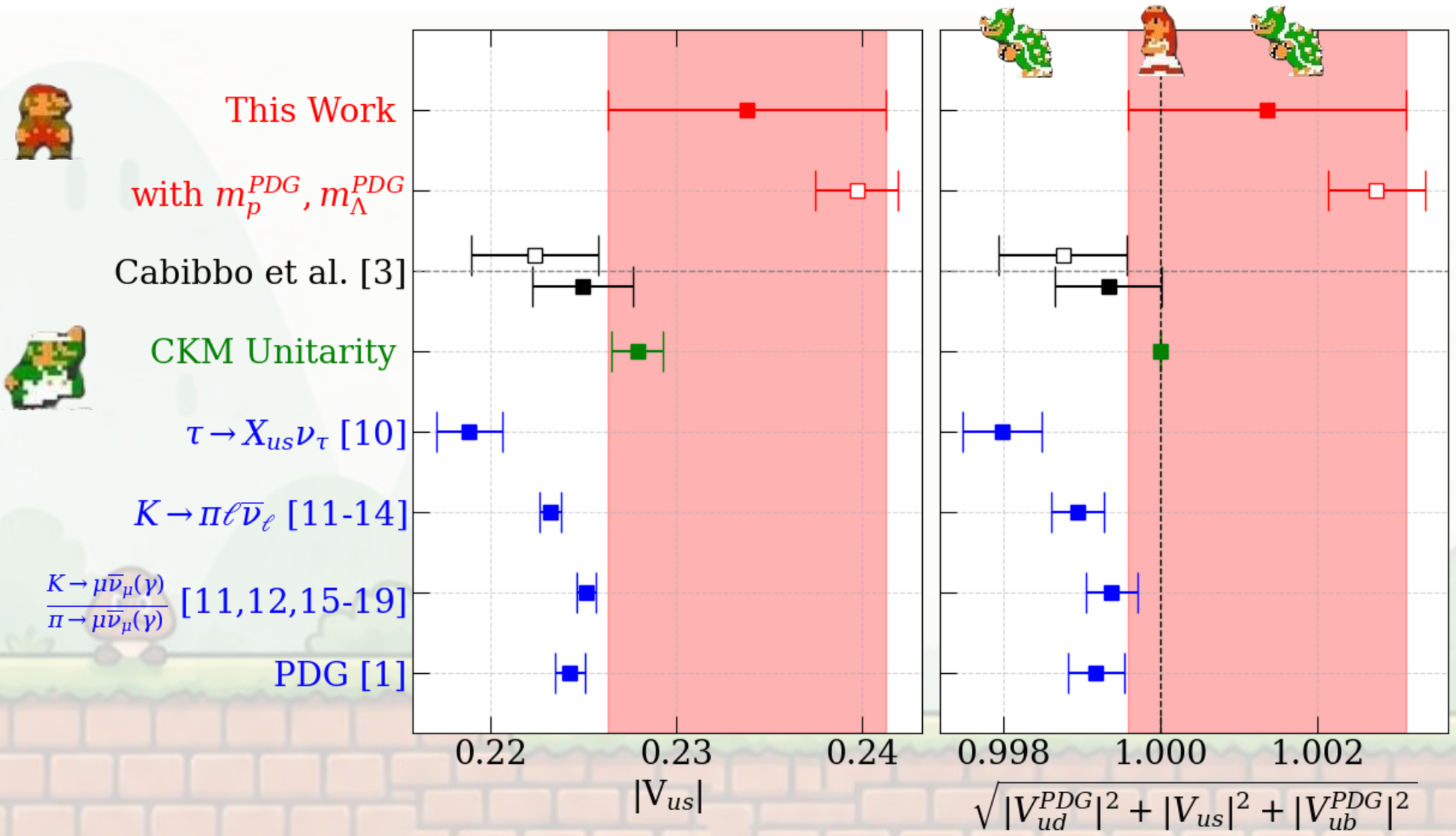
$$\sqrt{\left|V_{ud}^{\mathrm{PDG}}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}^{\mathrm{PDG}}\right|^2} = 1$$

$$\left|V_{us}\right| = 0.2338(75)\,,$$

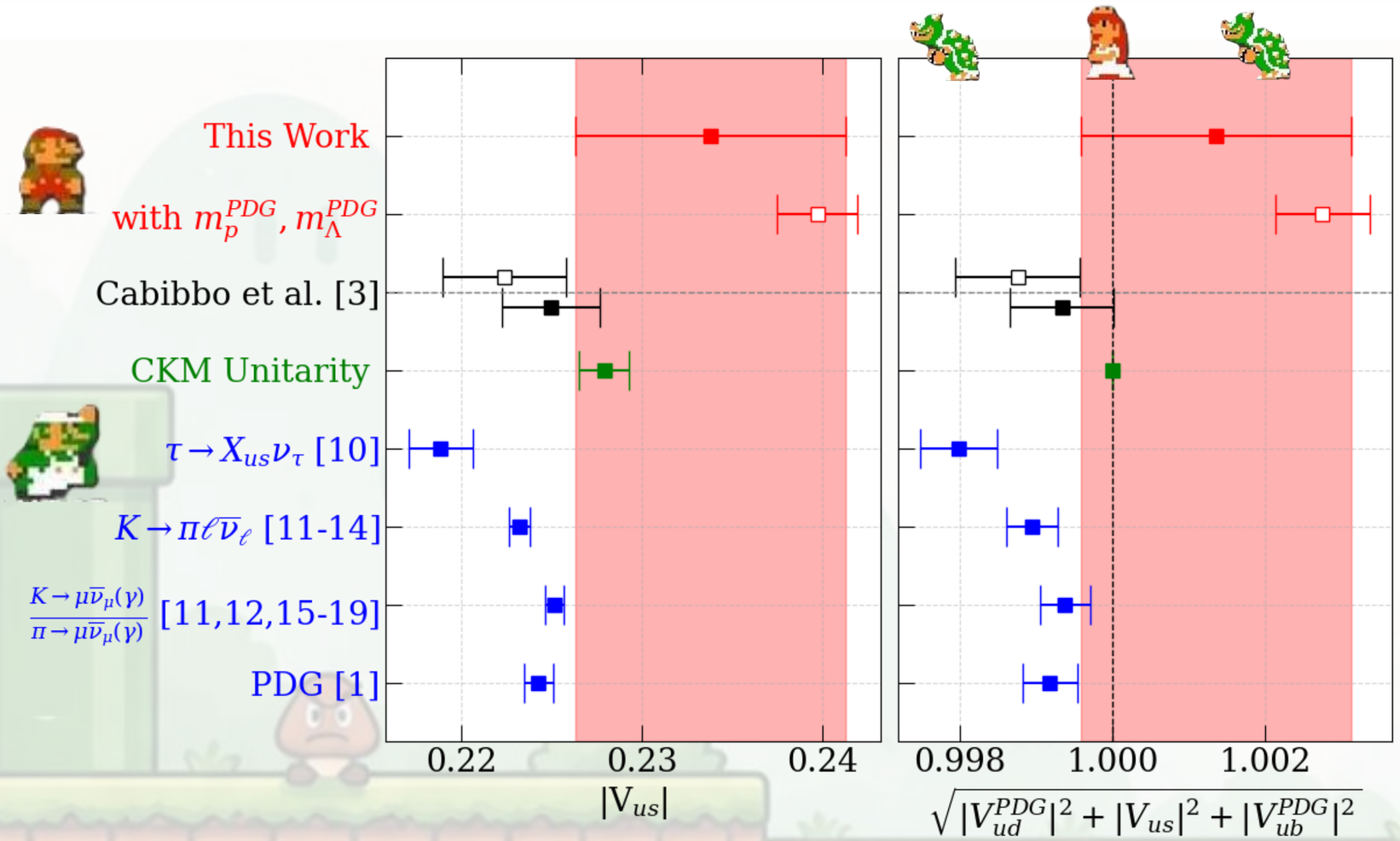


$$\sqrt{\left|V_{ud}^{\mathrm{PDG}}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}^{\mathrm{PDG}}\right|^2} = 1.0014(18)\,.$$

SCOREBOARD


This Work
with $m_p^{PDG}, m_\Lambda^{PDG}$
Cabibbo et al. [3]
CKM Unitarity
$\tau \to X_{us}\nu_\tau$ [10]
$K \to \pi\ell\overline{\nu}_\ell$ [11-14]
$\frac{K \to \mu\overline{\nu}_\mu(\gamma)}{\pi \to \mu\overline{\nu}_\mu(\gamma)}$ [11,12,15-19]
PDG [1]
0.22
0.23
0.24
$|V_{us}|$
0.998
1.000
1.002
$\sqrt{|V_{ud}^{PDG}|^2 + |V_{us}|^2 + |V_{ub}^{PDG}|^2}$

SCOREBOEARD



$$\mathcal{B}_{\Lambda \to p e \bar{\nu}_e} = \frac{\tau_\Lambda G_F^2 |V_{us}|^2}{192\pi^3 {m_\Lambda}^3} \times \int_{m_\ell{}^2}^{q^2_{\max}} dq^2 \sqrt{\lambda({m_\Lambda}^2, {m_p}^2, q^2)}\, L^{\mu\nu} H_{\mu\nu},$$

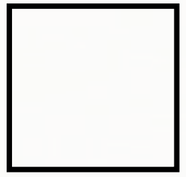
$$\mathcal{B}_{\Lambda \to p e \bar{\nu}_e} \propto m_\Lambda^5$$

$$(m_\Lambda^{LQCD})^5 = (1.1263(23))^5 = 1.812(19)$$

$$(m_\Lambda^{LQCD})^5 = (1.115683(6))^5 = 1.728638(46)$$

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CONTINUUM LIMIT.







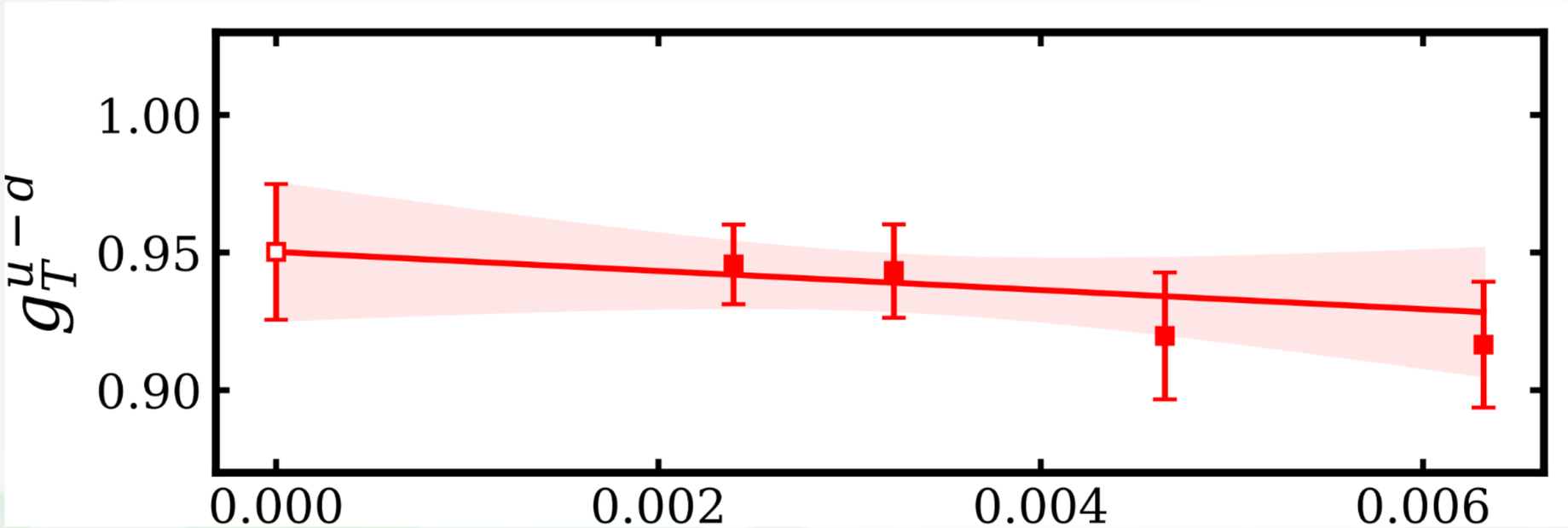




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Courtesy: Ch. Iona



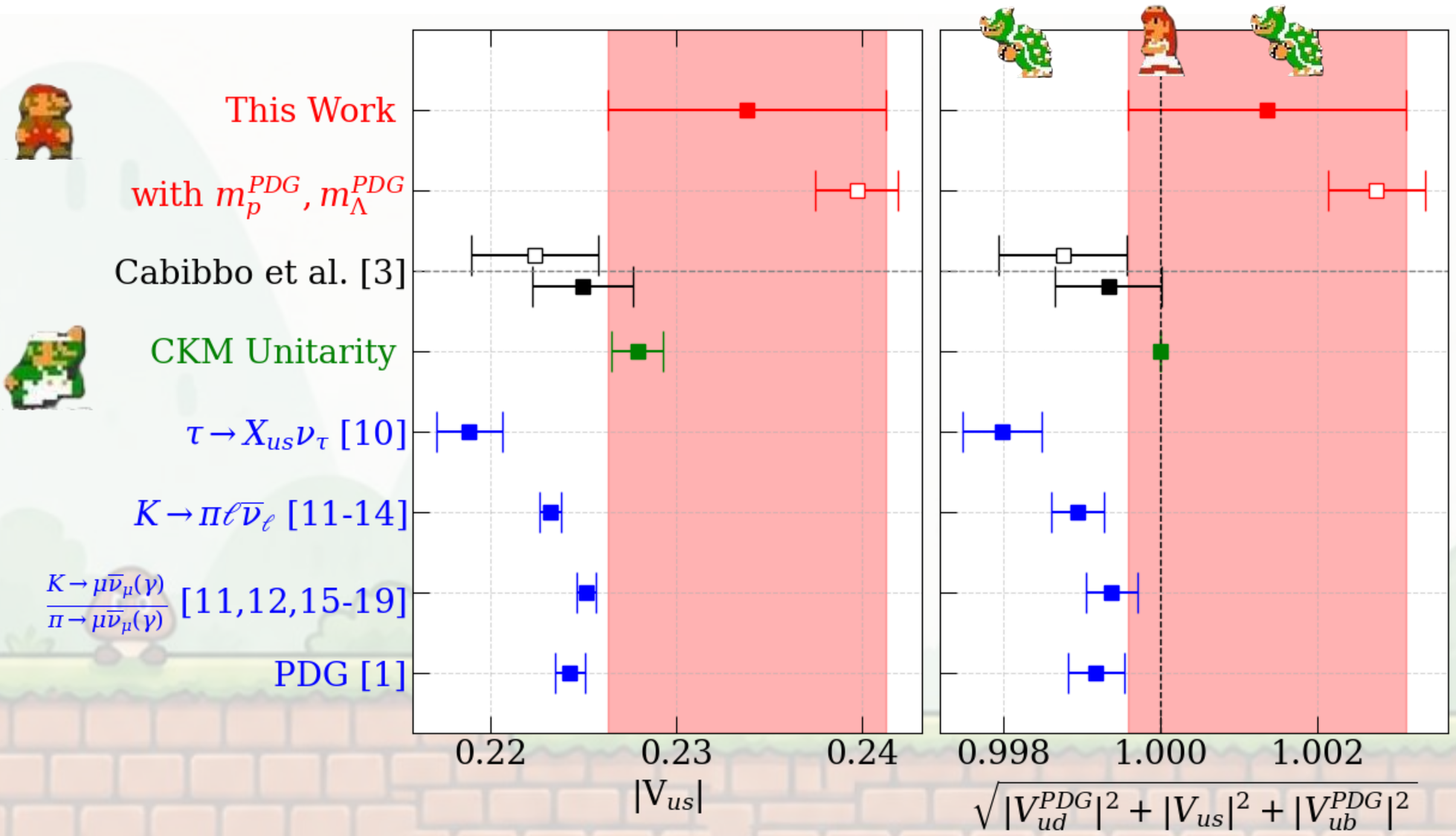






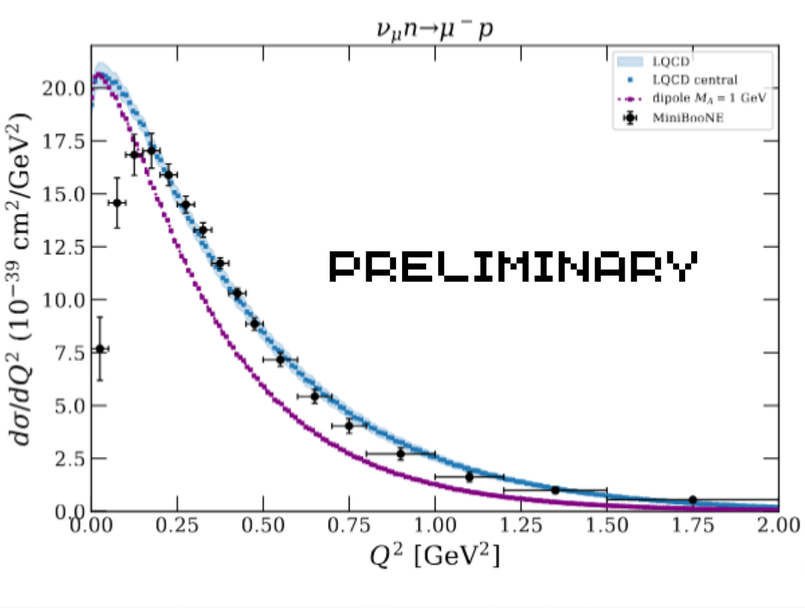


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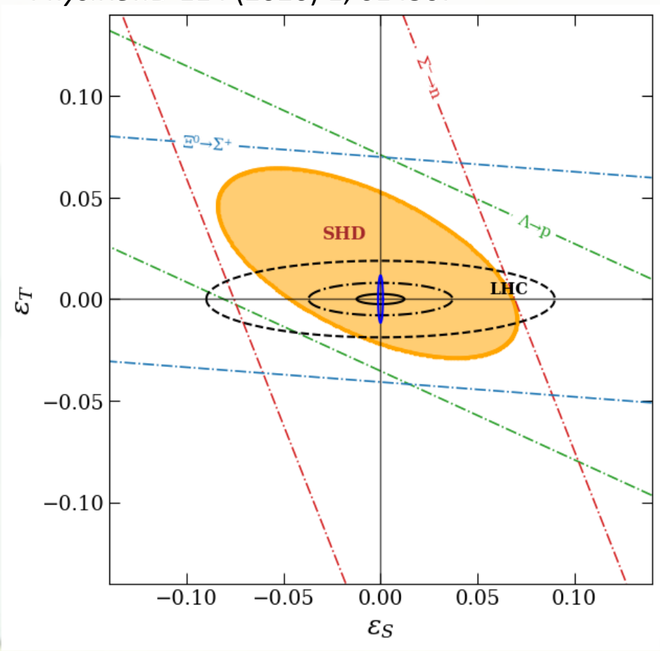
OTHER ASPECTS

NEUTRINO SCATTERIING



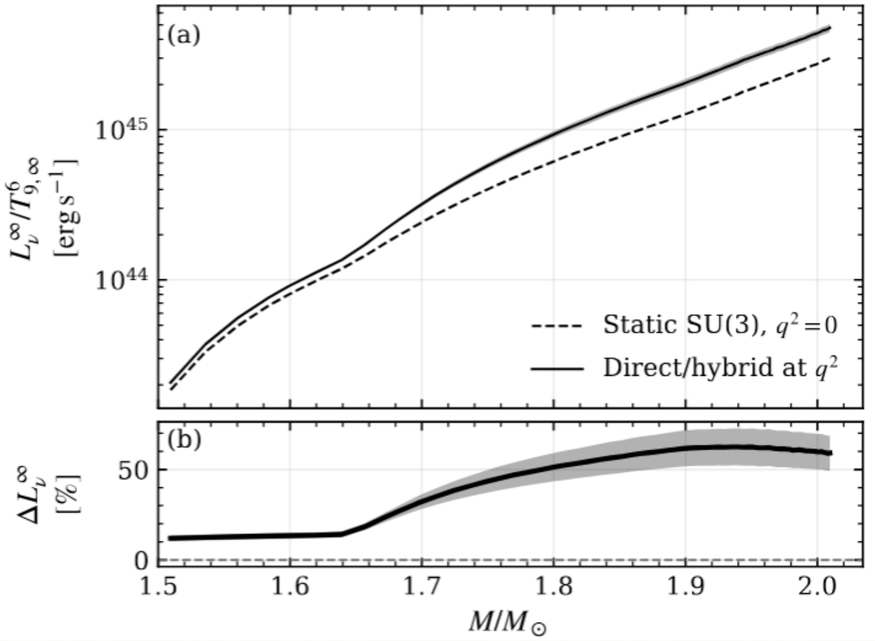
BSM INTERACTIONS

Phys.Rev.D 114 (2026) 1, 014507



NEUTRON STAR COOLING

Coming soon in ApJ



CONCLUSIONS

- WE INVESTIGATE THE LAMBDA TO PROTONSEMI-LEPTONIC DECAY IN LQCD FOR THE FIRST TIME.
- WE COMPUTE THE HELICITY AND WEINBERG FORM FACTORS' DEPENDENCE ON Q2.
- WE COMPUTE G1/F1 AND F2/F1 WITCH CAN BE DIRECTLY COMPARED TO EXPERIMENTAL RESULTS.
- BY COMBINING OUR RESULTS WITH THE EXPERIMENTAL DECAY RATE WE EXTRACT |VUS|.

CREDIT SCENE

THANK FOR YOUR ATTENTION!

THIS WORK WAS SUPPORTED BY THE PROJECTS HYPERON AND BARYONS CO-FUNDED BY THE EU WITHIN THE FRAMEWORK OF THE COHESION POLICY PROGRAMME "THALIA 2021-2027".

Co-funded by the European Union

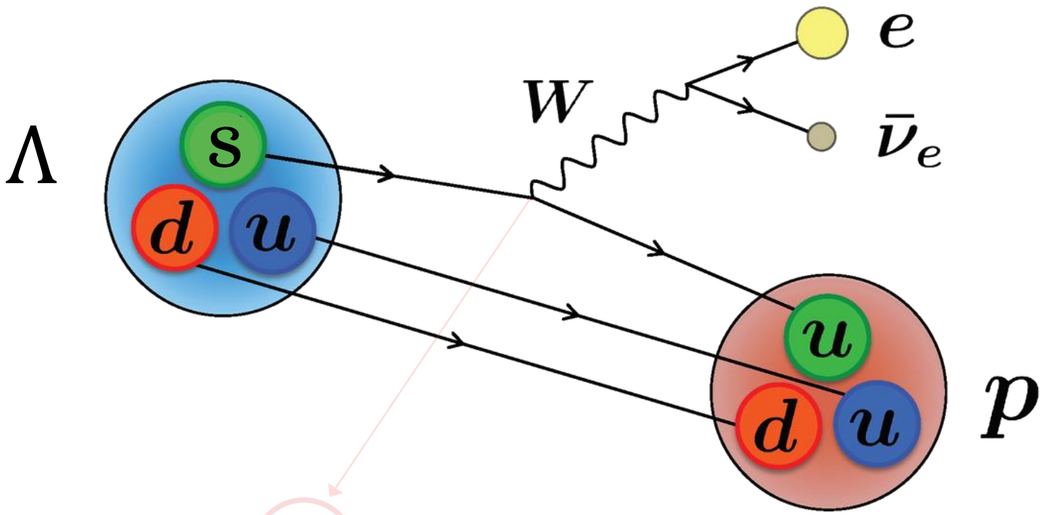
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Motivation

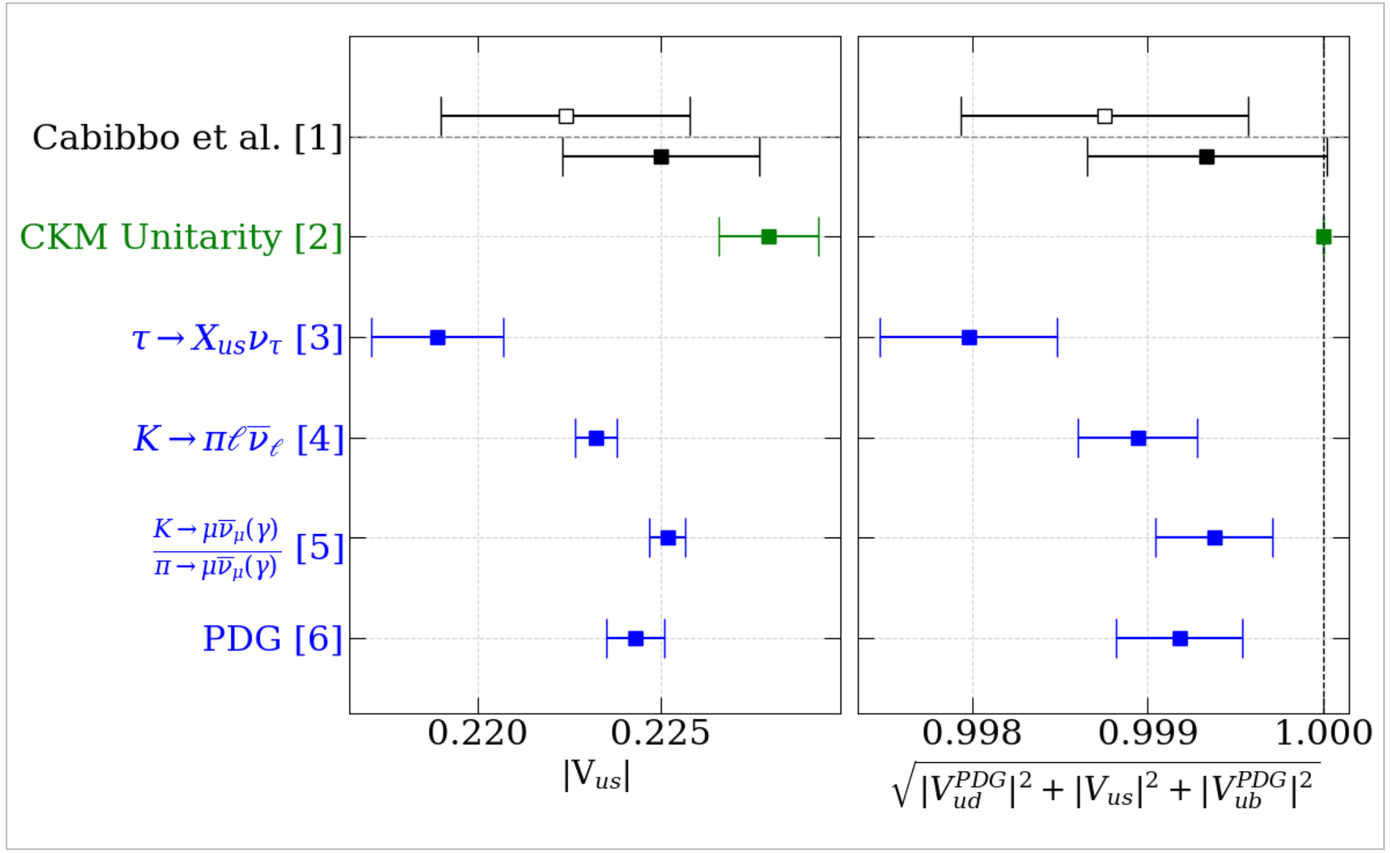


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM unitary matrix

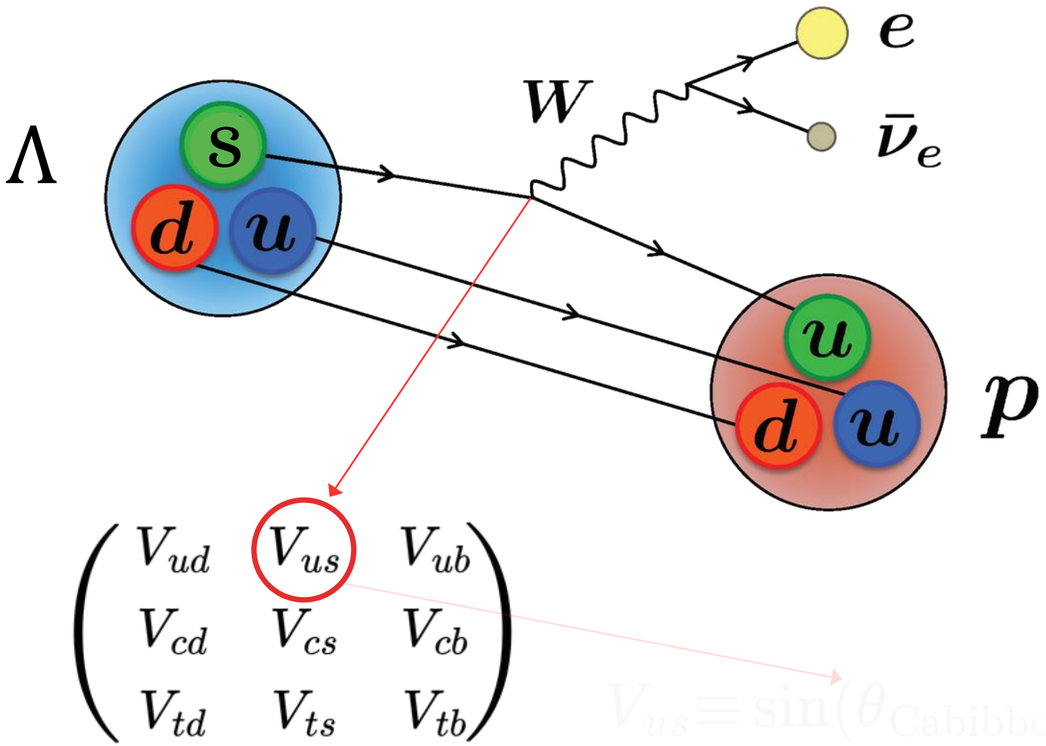
$$V_{us} = \sin(\theta_{\text{Cabibbo}})$$

Related to the Cabibbo angle



Determination of Vus (left) and the resulting CKM unitarity relation (right)

Motivation

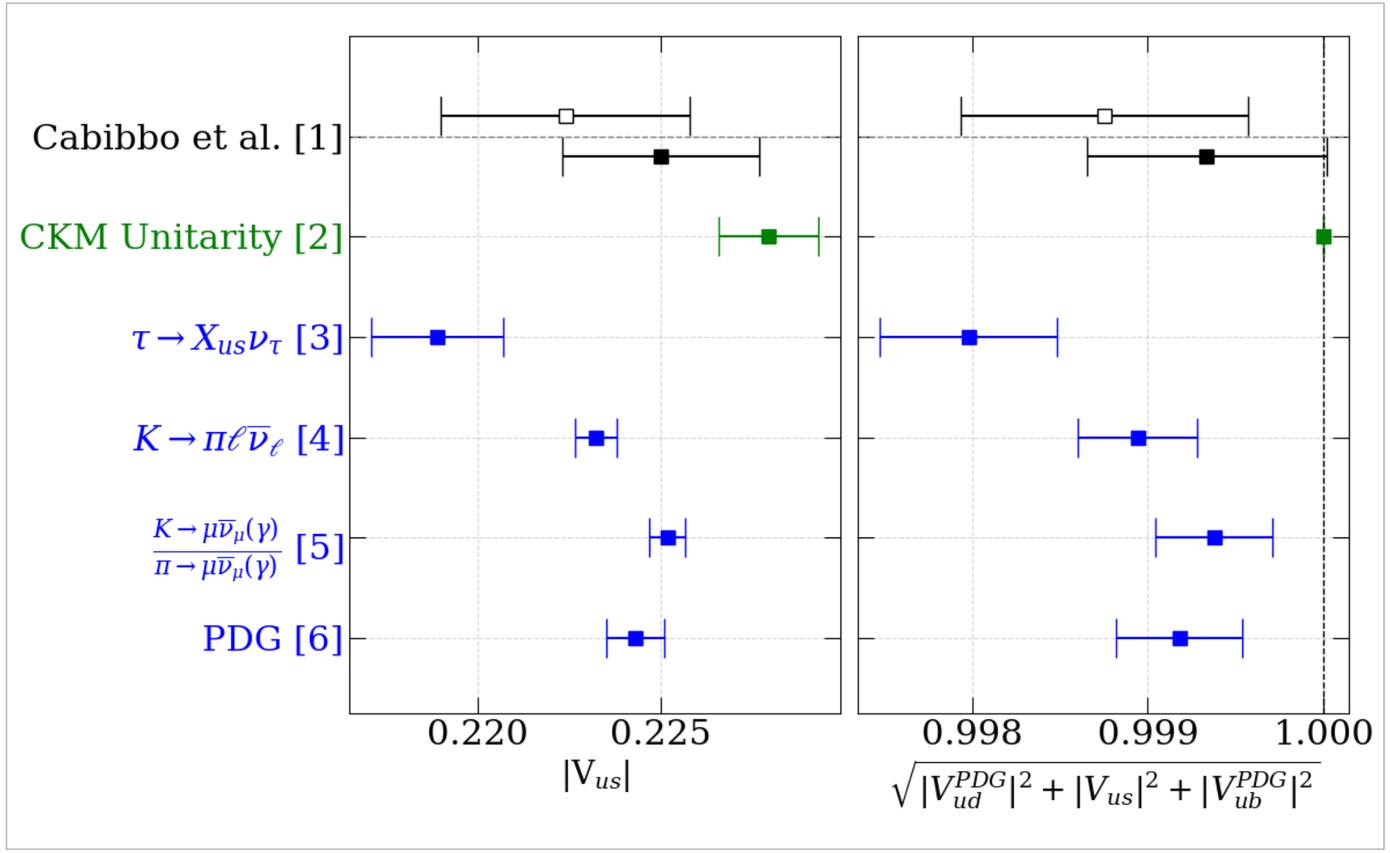


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CKM unitary matrix

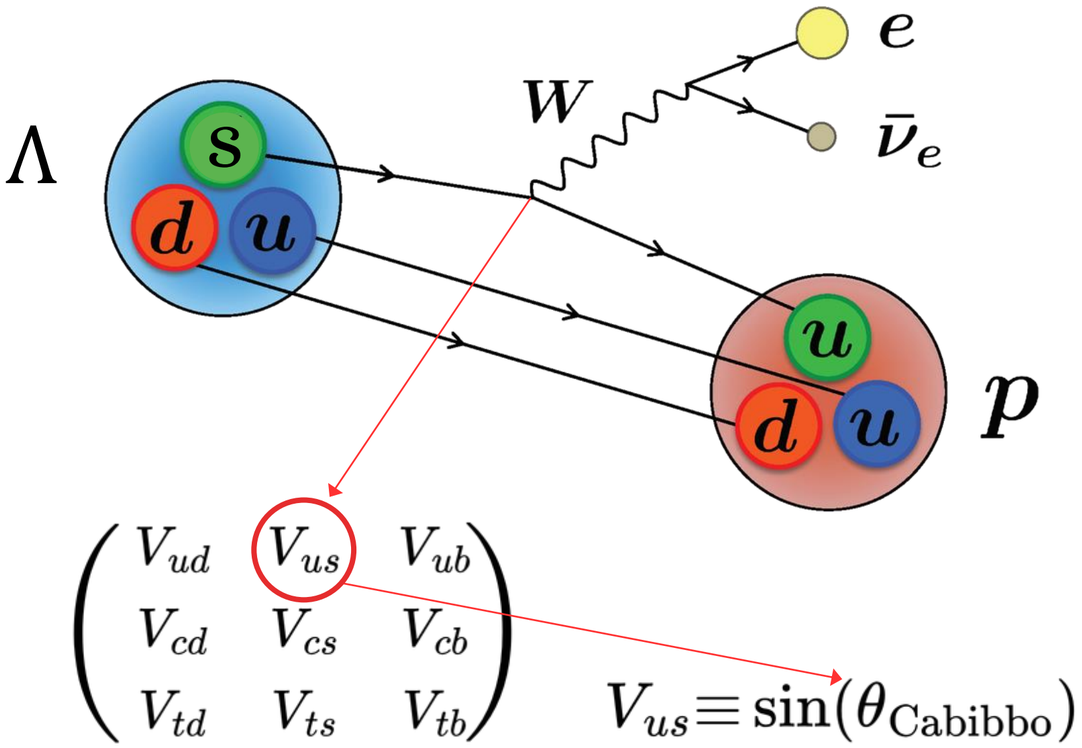
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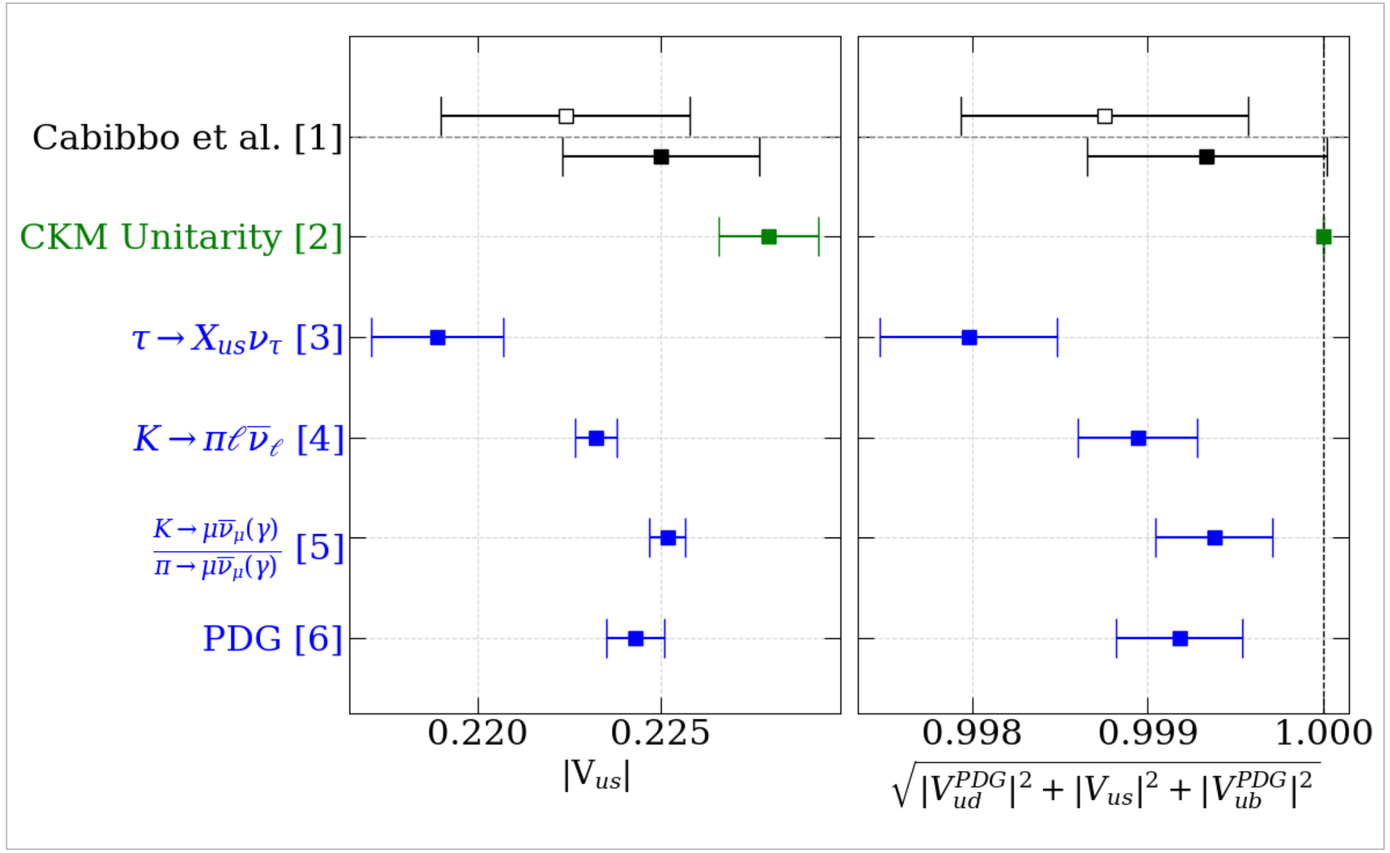


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM unitary matrix

$$V_{us} \equiv \sin(\theta_{\text{Cabibbo}})$$

Related to the Cabibbo angle



Determination of Vus (left) and the resulting CKM unitarity relation (right)

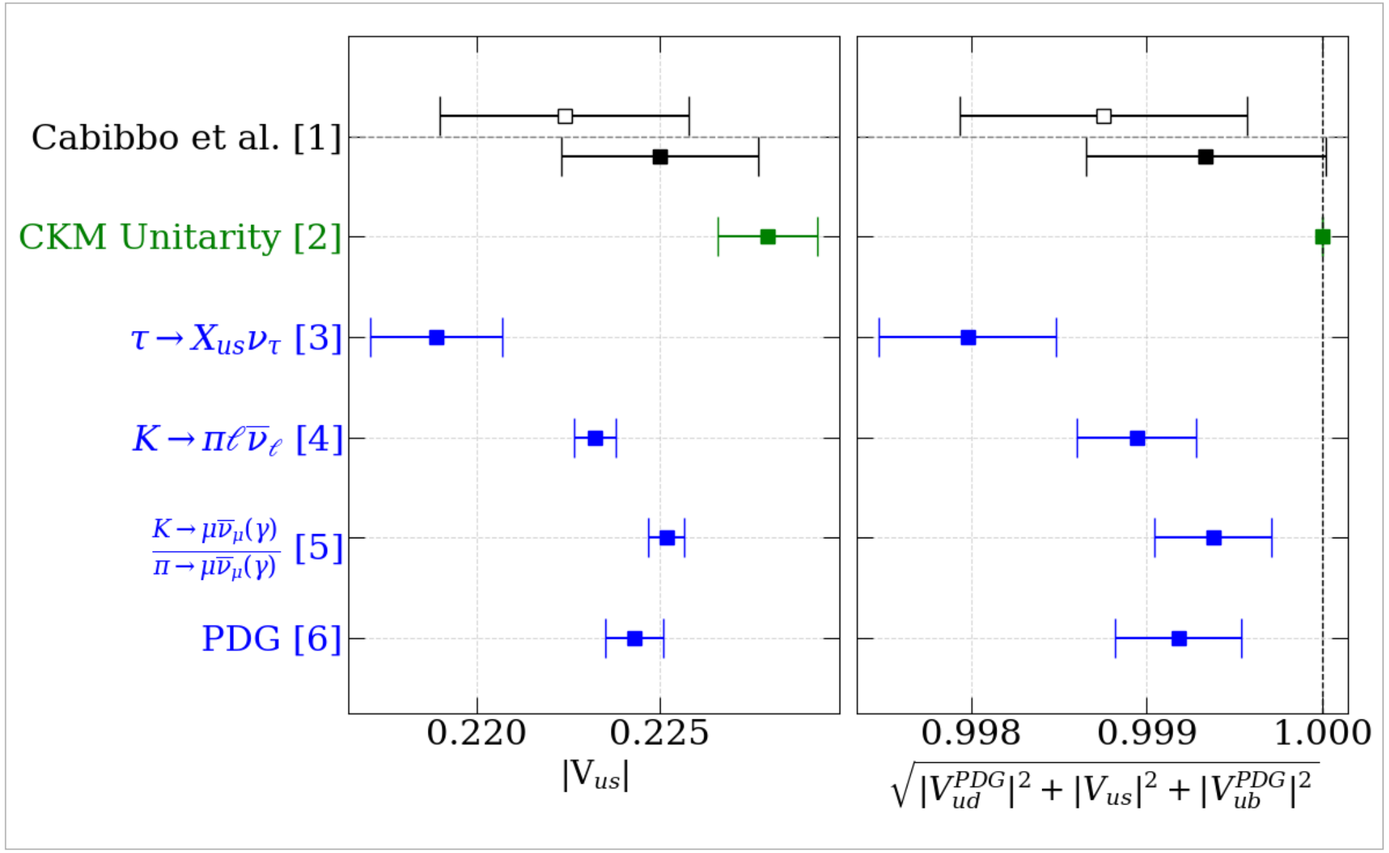
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Current computations of Vus

- **[1] Study the SU(3) limit. The empty point is the value derived from the Λ semi-leptonic decay, while the full point combines results from various hyperon decays (Cabibbo et al. 2003).** $|V_{us}| \propto \sqrt{\Gamma_{\Lambda \to p e^- \bar{\nu}_e}}$
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$$\text{with } |V_{ud}^{\text{PDG}}| = 0.97367(32), \quad |V_{ub}^{\text{PDG}}| = 3.82(20) \cdot 10^{-3}$$

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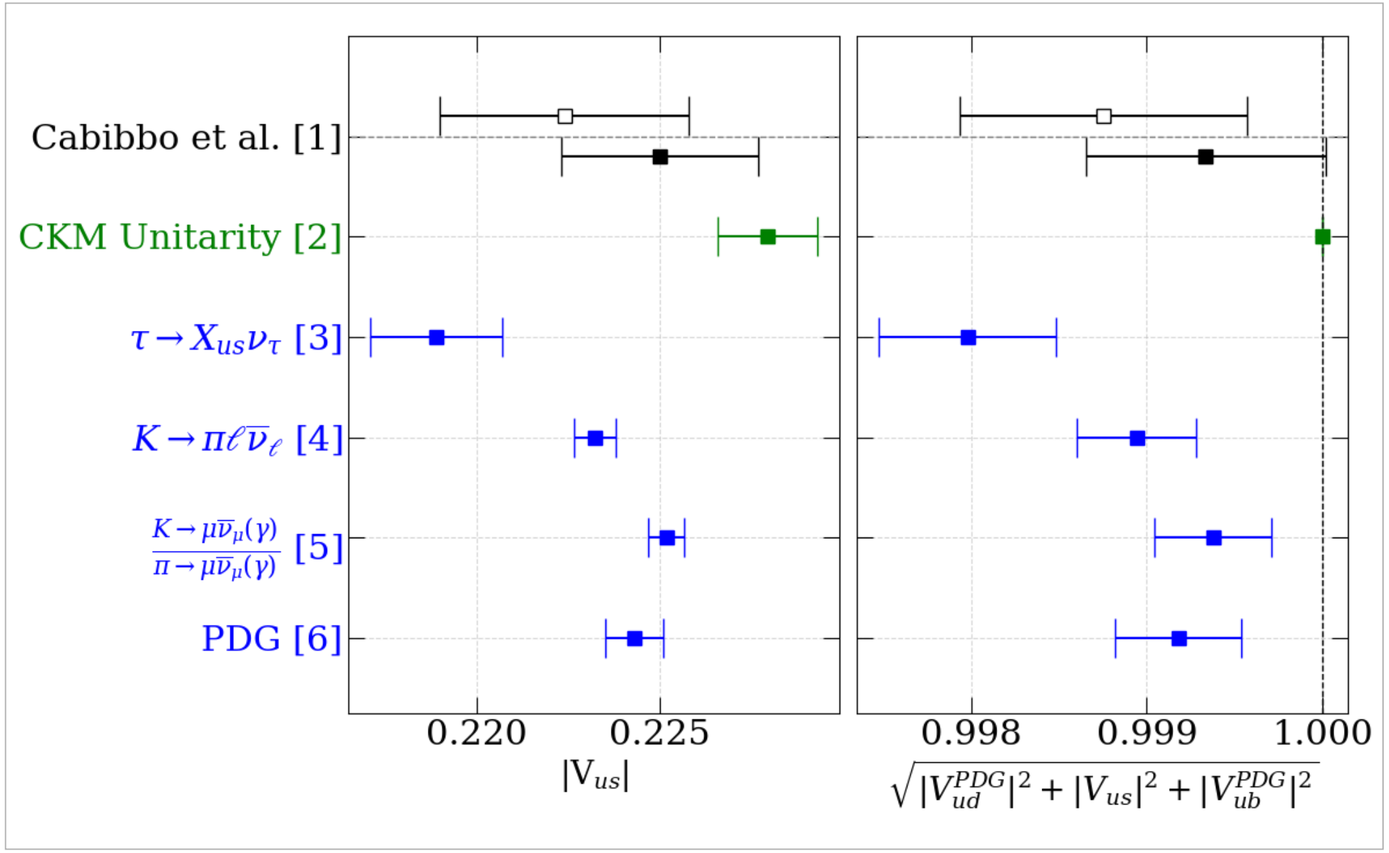
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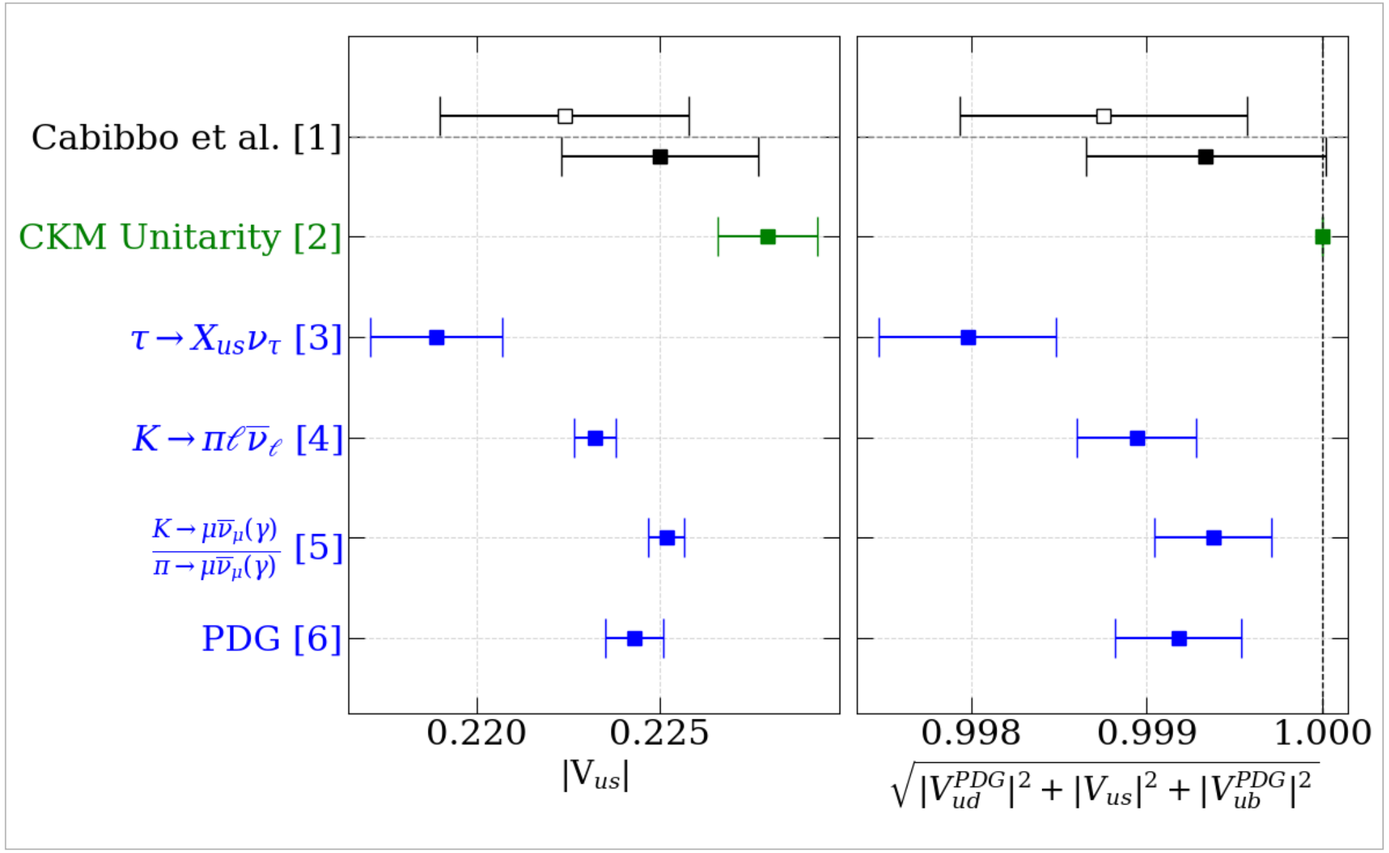
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- **LQCD + experimental data.**

[3] Inclusive τ decays
(C. Alexandrou et al. (ETMC) (2024))

$$R_{us}^{(\tau)} = \frac{\Gamma(\tau \to X_{us}\nu_\tau)}{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}$$

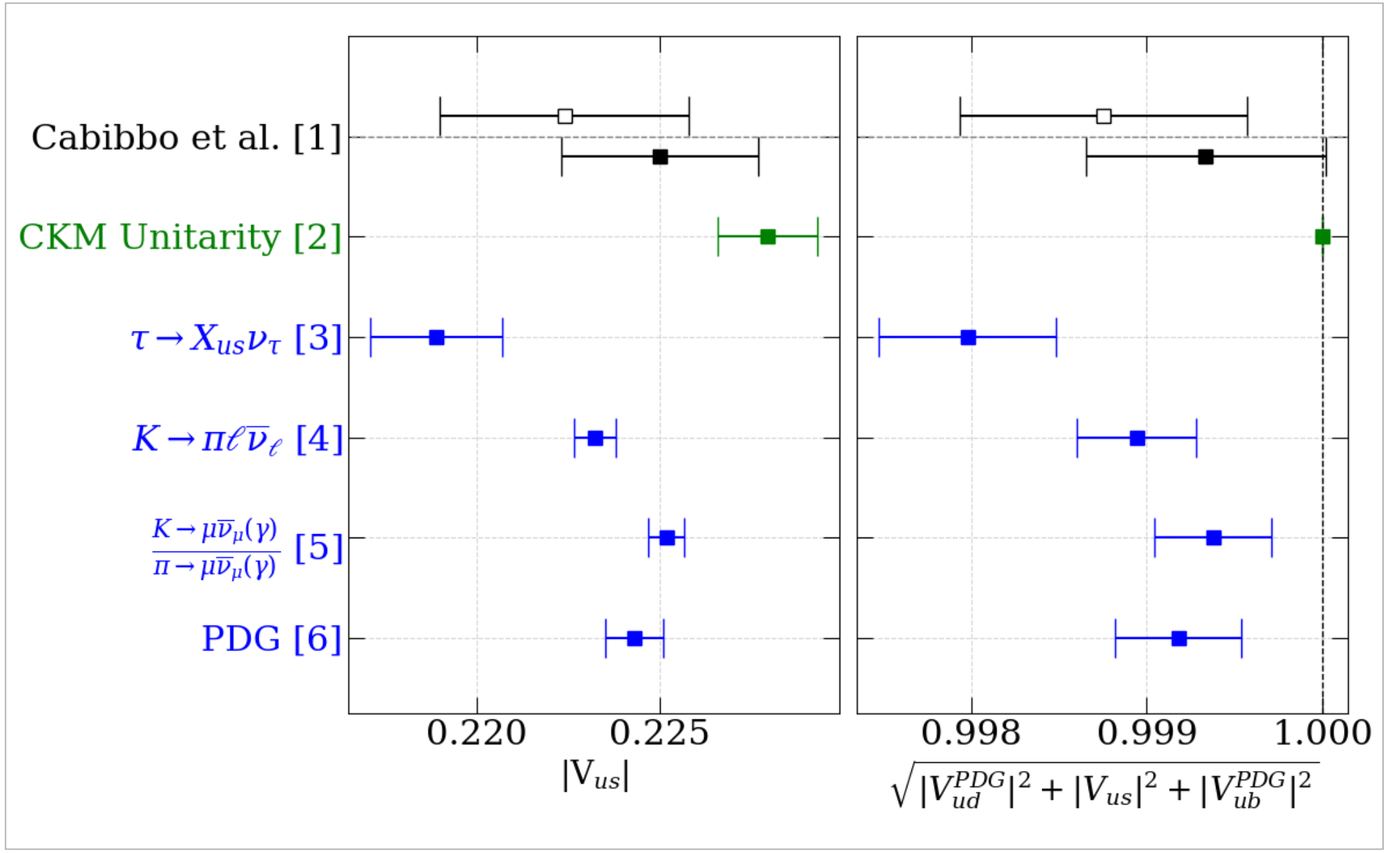
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[5] Ratio of K to π leptonic decays in the muonic channel
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$$\left|\frac{V_{us}}{V_{ud}}\right|\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4), \qquad \frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.1932(21)$$

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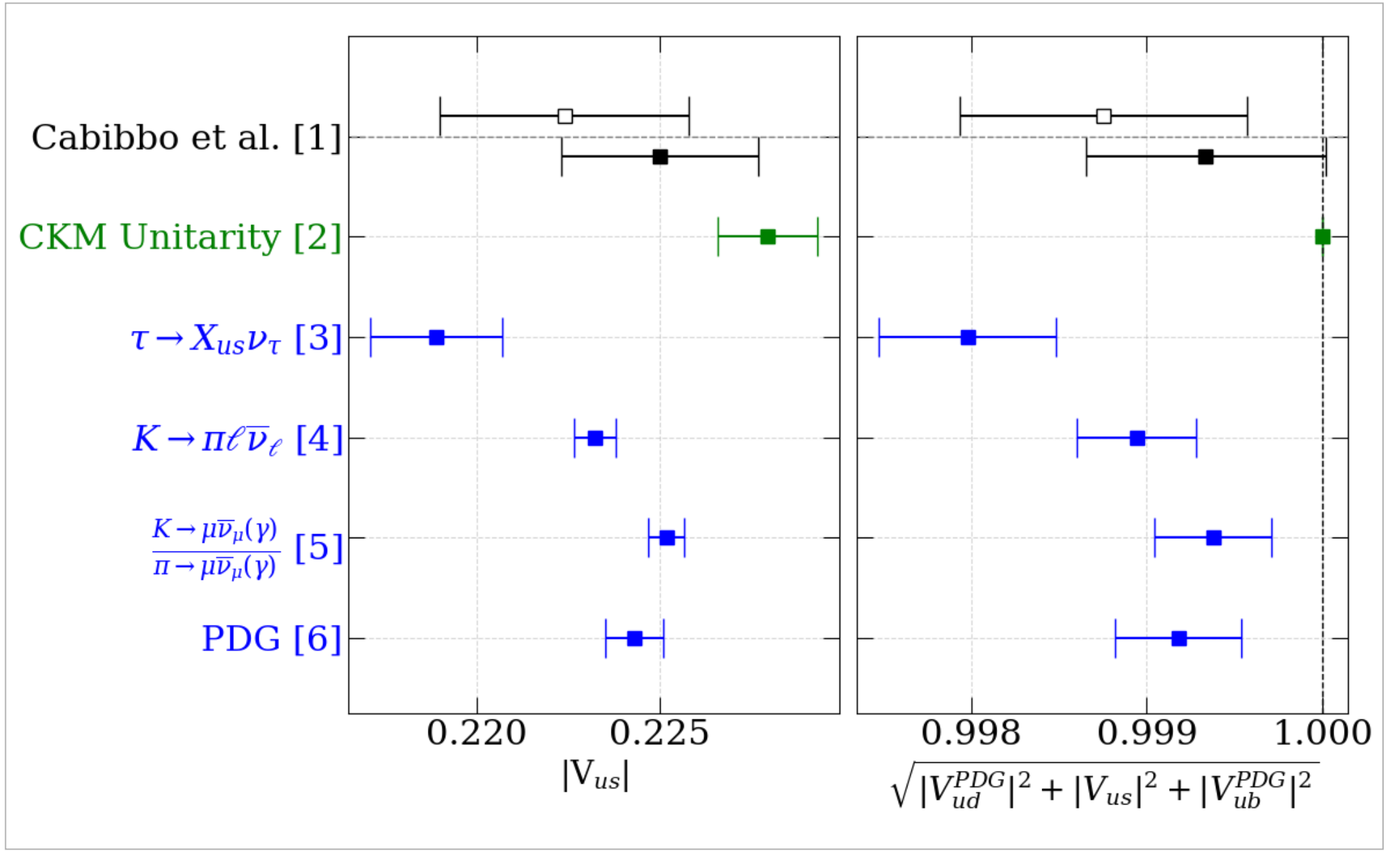
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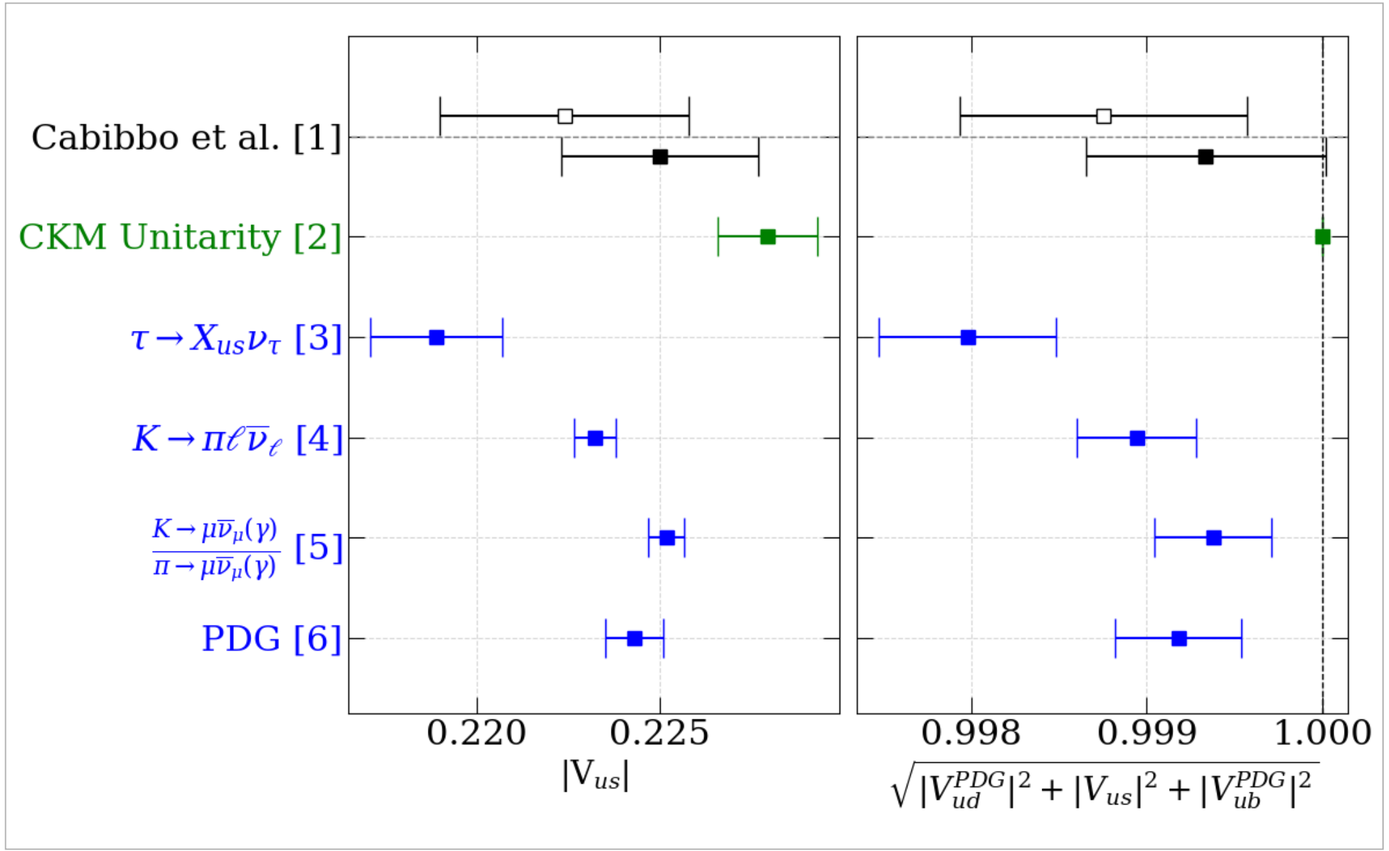
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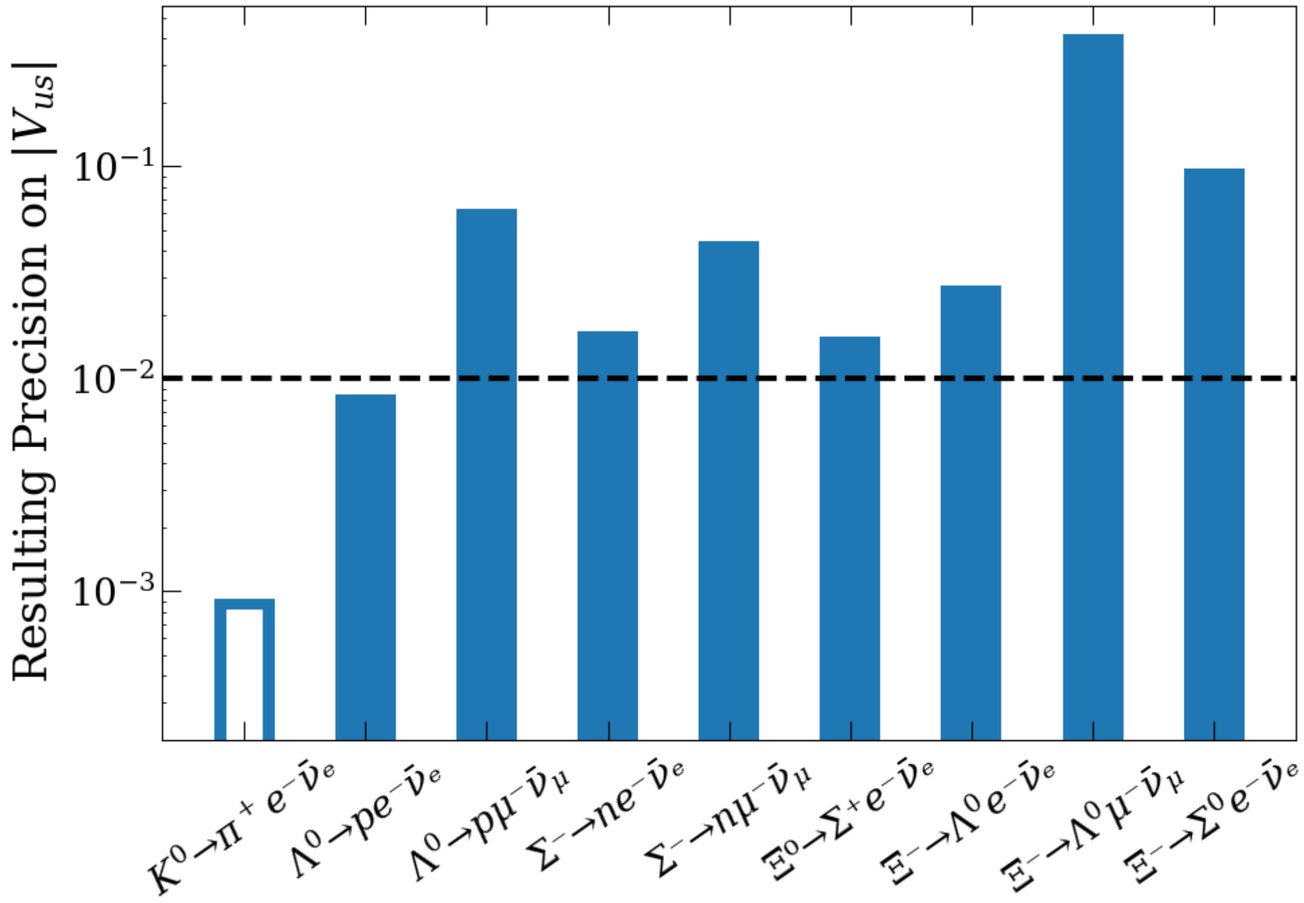
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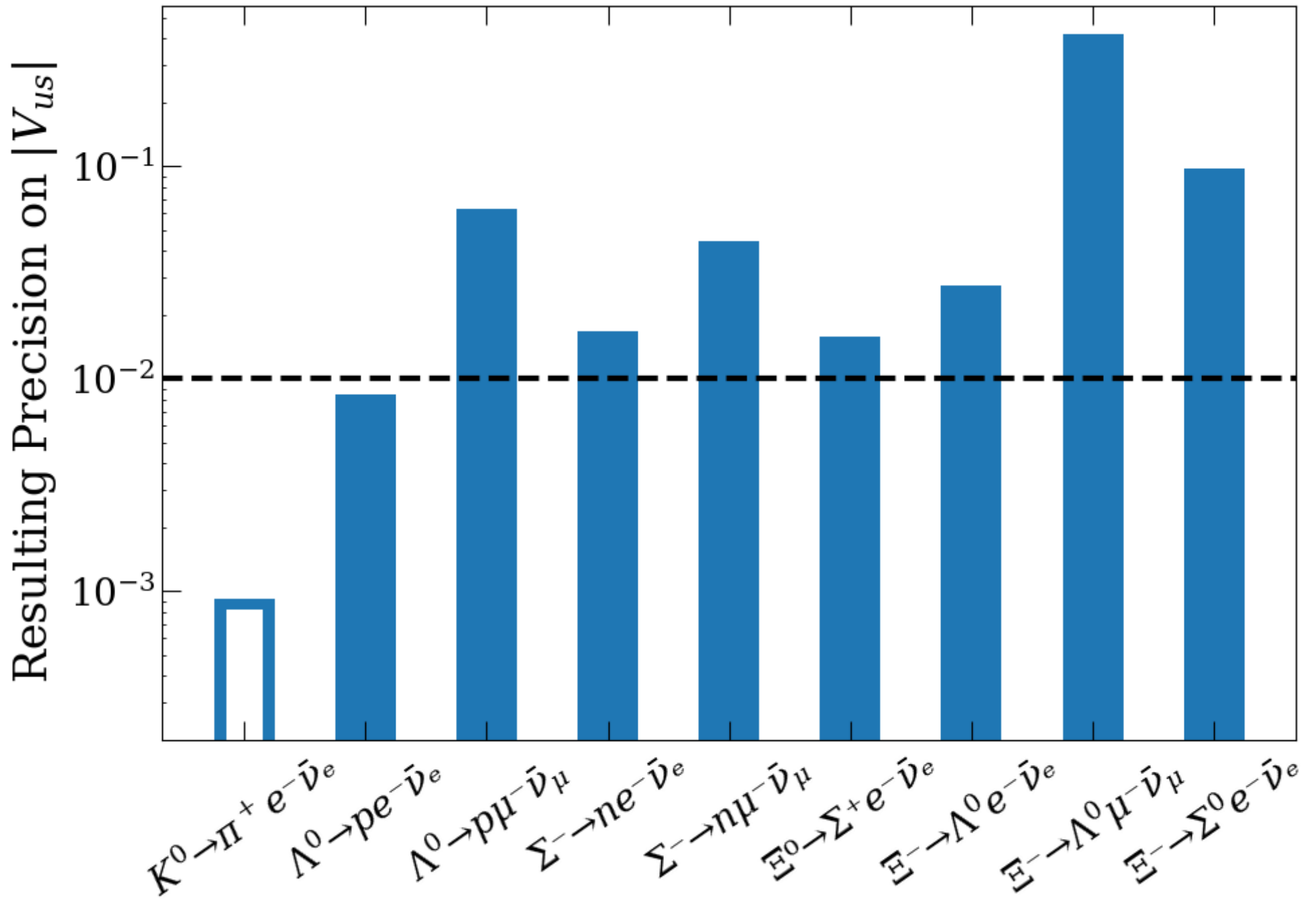


Resulting Precision on Vus from Branching fractions (Particle Data Group)

We would like to study a case where the uncertainty coming from the experimental results, contributes less.

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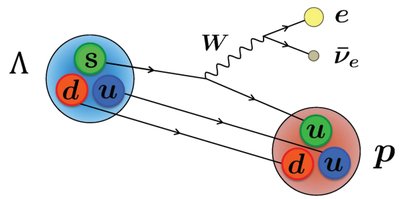
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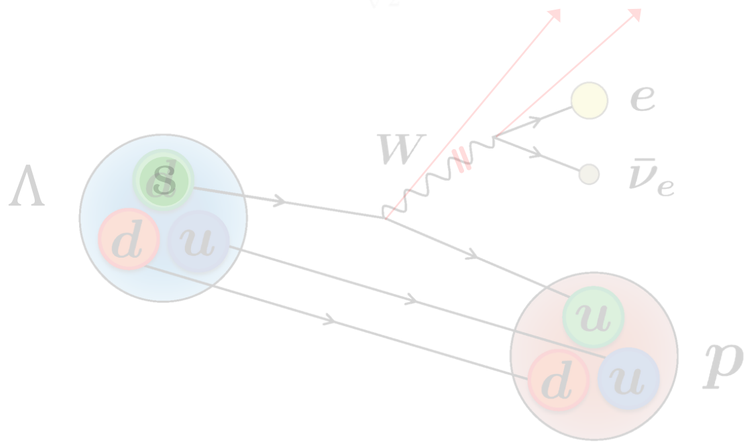
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We integrate with respect to q2 and obtain the decay rate

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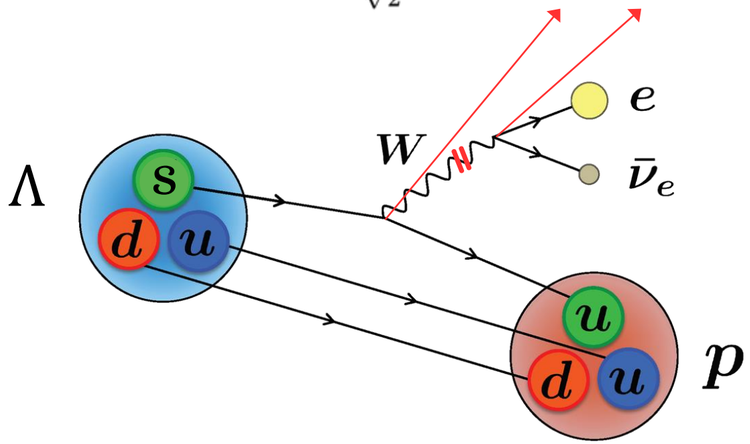
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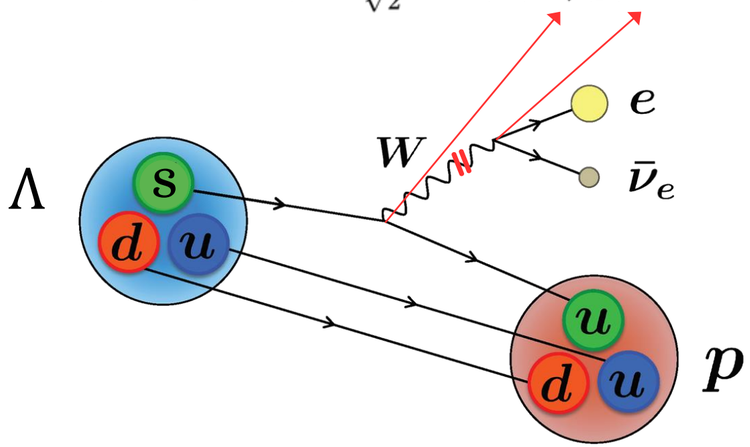
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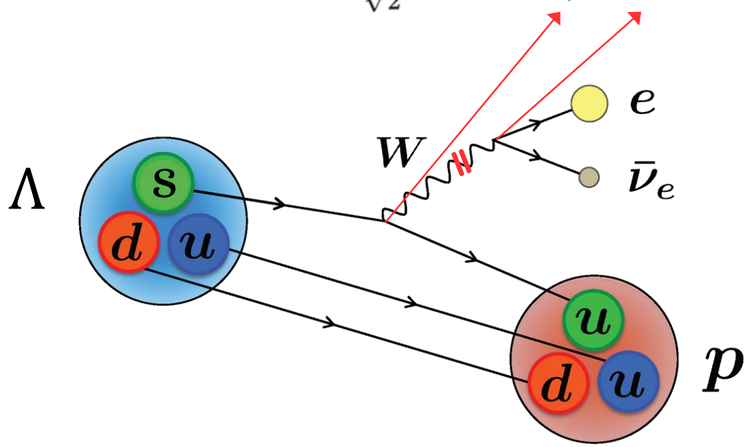
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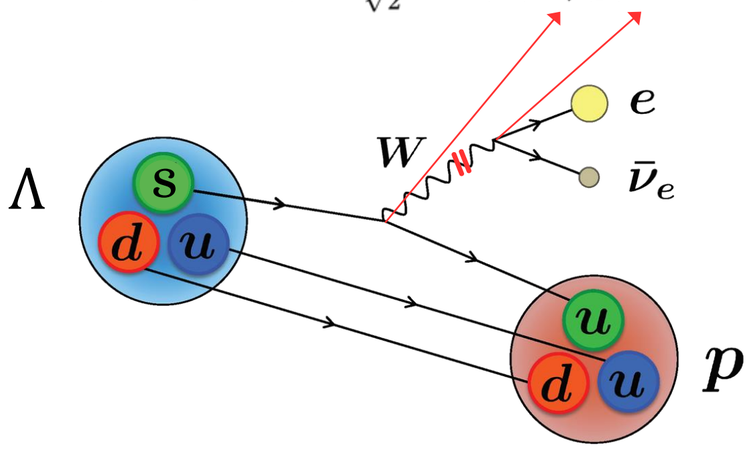
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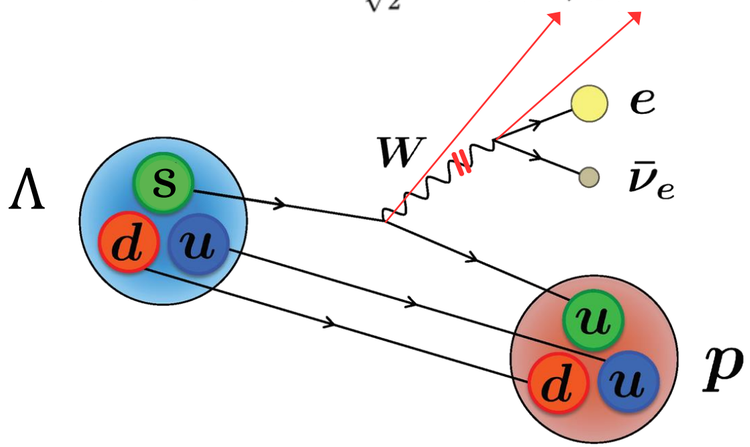
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We integrate with respect to q2 and obtain the decay rate

$$\Gamma = \frac{G_F{}^2|V_{us}|^2}{192\pi^3 m_\Lambda{}^3} \times \int_{m_\ell{}^2}^{q_{\max}{}^2} dq^2\sqrt{\lambda(m_\Lambda{}^2, m_p{}^2, q^2)}\,L^{\mu\nu}H_{\mu\nu}, \quad q^2 \equiv (p_\Lambda - p_p)^2$$

How?

The decay rate Γ is obtained by

$$\Gamma = \frac{1}{2m_\Lambda}\int \frac{d^3\vec{p}_p}{(2\pi)^3 2E_p}\frac{d^3\vec{p}_\ell}{(2\pi)^3 2E_\ell}\frac{d^3\vec{p}_\nu}{(2\pi)^3 2E_\nu} \times (2\pi)^4\delta^{(4)}(p_\Lambda - p_p - p_\ell - p_\nu)\,|\mathcal{M}|^2.$$

Hadronic matrix element

$$M_\mu^{\text{V-A}} \equiv \langle p(p_p, s_p)|\bar{u}\,\Gamma_\mu^{\text{V-A}} s|\Lambda(p_\Lambda, s_\Lambda)\rangle$$

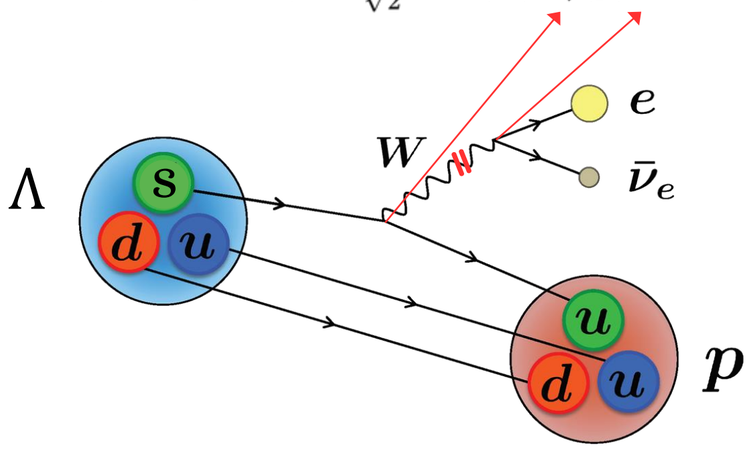
where V-A is the left-handed coupling in the Standard Model

Hadronic Tensor (Lattice QCD)

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The resulting decay amplitude is

$$\mathcal{M}(\Lambda \to p e^- \bar{\nu}_e) = \frac{G_F}{\sqrt{2}} V_{us}\,\langle \Lambda|W_\mu|p\rangle L^\mu$$



The Weinberg decomposition

$$M_\mu^{\text{V}} = \bar{u}_p\left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\nu}q^\nu\frac{F_2(q^2)}{m_\Lambda} + q_\mu\frac{F_3(q^2)}{m_\Lambda}\right]u_\Lambda$$

$$M_\mu^{\text{A}} = \bar{u}_p\left[\gamma_\mu G_1(q^2) - i\sigma_{\mu\nu}q^\nu\frac{G_2(q^2)}{m_\Lambda} + q_\mu\frac{G_3(q^2)}{m_\Lambda}\right]\gamma_5 u_\Lambda$$

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Computing |Vus|

Decay rate

$$\Gamma_{\Lambda\to p\ell\bar{\nu}_\ell} = |V_{us}|^2 \frac{G_F^2}{192\pi^3 m_\Lambda^3}\{\ldots\} = |V_{us}|^2 \tilde{\Gamma}_{\Lambda\to p\ell\bar{\nu}_\ell}$$

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$$\mathcal{B}(\Lambda \to p\ell\bar{\nu}_\ell) = \Gamma_{\Lambda\to p\ell\bar{\nu}_\ell}/\Gamma_{\text{tot}} = \tau_\Lambda \cdot \Gamma_{\Lambda\to p\ell\bar{\nu}_\ell}$$

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$$|V_{us}| = \left(\tau_\Lambda \frac{\tilde{\Gamma}_{\Lambda\to p\ell\bar{\nu}_\ell}}{\mathcal{B}(\Lambda \to p\ell\bar{\nu}_\ell)}\right)^{-1/2}$$

Use of experiential results

PDG (2024) :

$$\tau_\Lambda^{\text{PDG}} = 2.617(10)\cdot 10^{-10}\, s$$

BESIII (2025) :

$$\mathcal{B}^{\text{BESIII}}_{\Lambda\to pe\bar{\nu}_e} = 8.16(22)\cdot 10^{-4}$$

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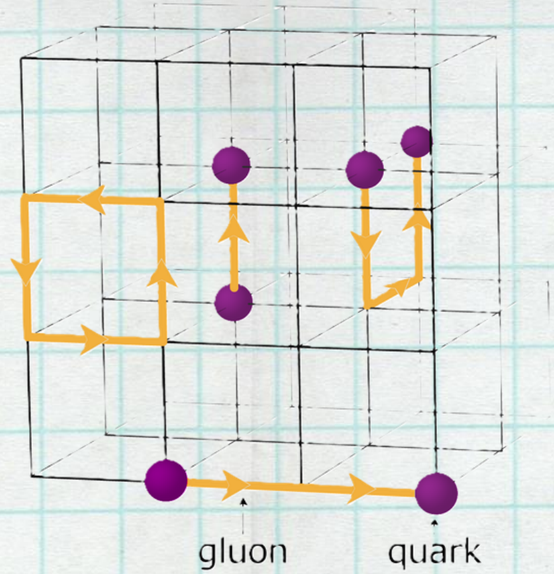
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Use of Lattice QCD



Two- and Three- point functions

The Nucleon and Λ-hyperons interpolating fields

$$\chi_N(x) = \epsilon^{abc} \left(u_a^T(x)\, C\gamma_5\, d_b(x)\right) u_c(x),$$

$$\chi_\Lambda(x) = \frac{1}{\sqrt{6}}\epsilon^{abc}\left[2\left(u_a^T C\gamma_5 d_b\right)s_c + \left(u_a^T C\gamma_5 s_b\right)d_c - \left(d_a^T C\gamma_5 s_b\right)u_c\right]$$

Two- and Three- point functions

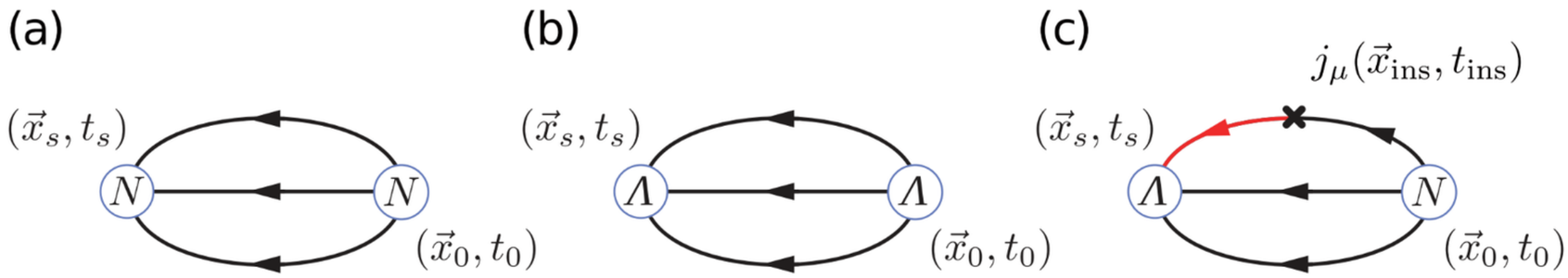
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Two- and Three- point functions



The Data

Parameters of the Nf = 2+1+1 Extended Twisted Mass Collaboration ensemble analyzed in this work.

(C. Alexandrou et al. (ETMC), (2023))

Ensemble	Abrv.	V/a^4	β	a [fm]	m_π [MeV]
cB211.072.64	B64	$64^3 \times 128$	1.778	0.07957(13)	140.2(2)

Smearing parameters for the light and strange quark interpolating fields.

Smearing parameters				
Flavor	a_G	n_G	a_{APE}	n_{APE}
Light	1.0	95	0.5	50
Strange	1.0	40	0.5	50

Statistics for the three- and two- point functions at various source-sink separations.

cB211.072.64 524 configurations		
t_s/a	t_s [fm]	n_{src}
8	0.64	1
10	0.80	3
12	0.95	9
14	1.12	27
16	1.28	27
18	1.44	27
Nucleon 2pt		349
Λ-hyperon 2pt		94

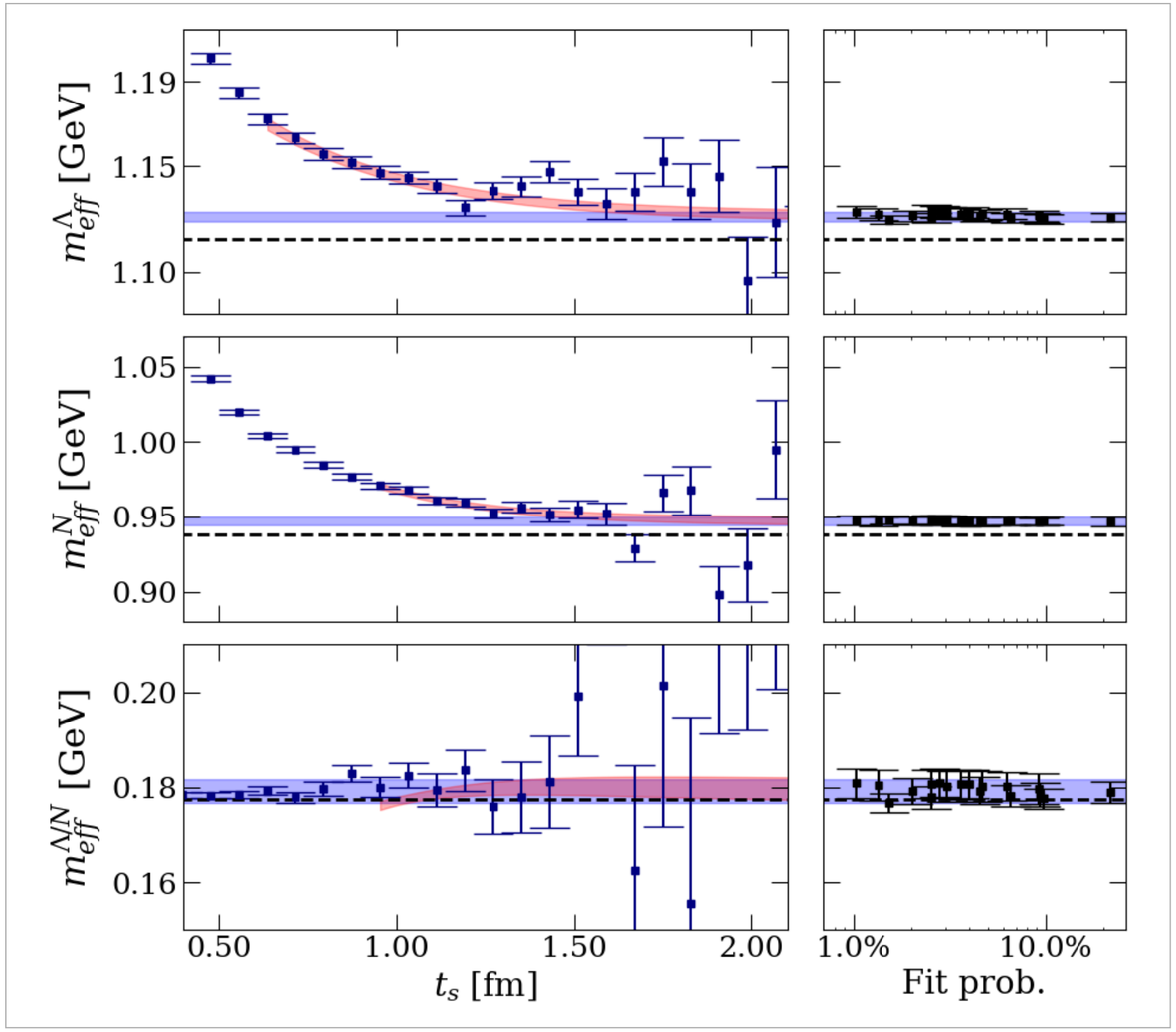
Data Analysis

Baryon masses : Two-point functions

$$C_B(\vec{p}, t_s) = e^{-t_s E_B^{0|\vec{p}|}} (c_B^{0|\vec{p}|} + c_B^{1|\vec{p}|} e^{-t_s \Delta E_B^{1|\vec{p}|}} + \dots)$$

$$M_B^{\text{eff}}(t_s) = \log \frac{C_B(t_s)}{C_B(t_s+1)} = m^B + \dots,$$

$$m_{\text{eff}}^{\Lambda/N}(t_s) = \log \frac{C^\Lambda(t_s) C^N(t_s+1)}{C^N(t_s) C^\Lambda(t_s+1)} = m_\Lambda - m_N + \dots,$$



Data Analysis

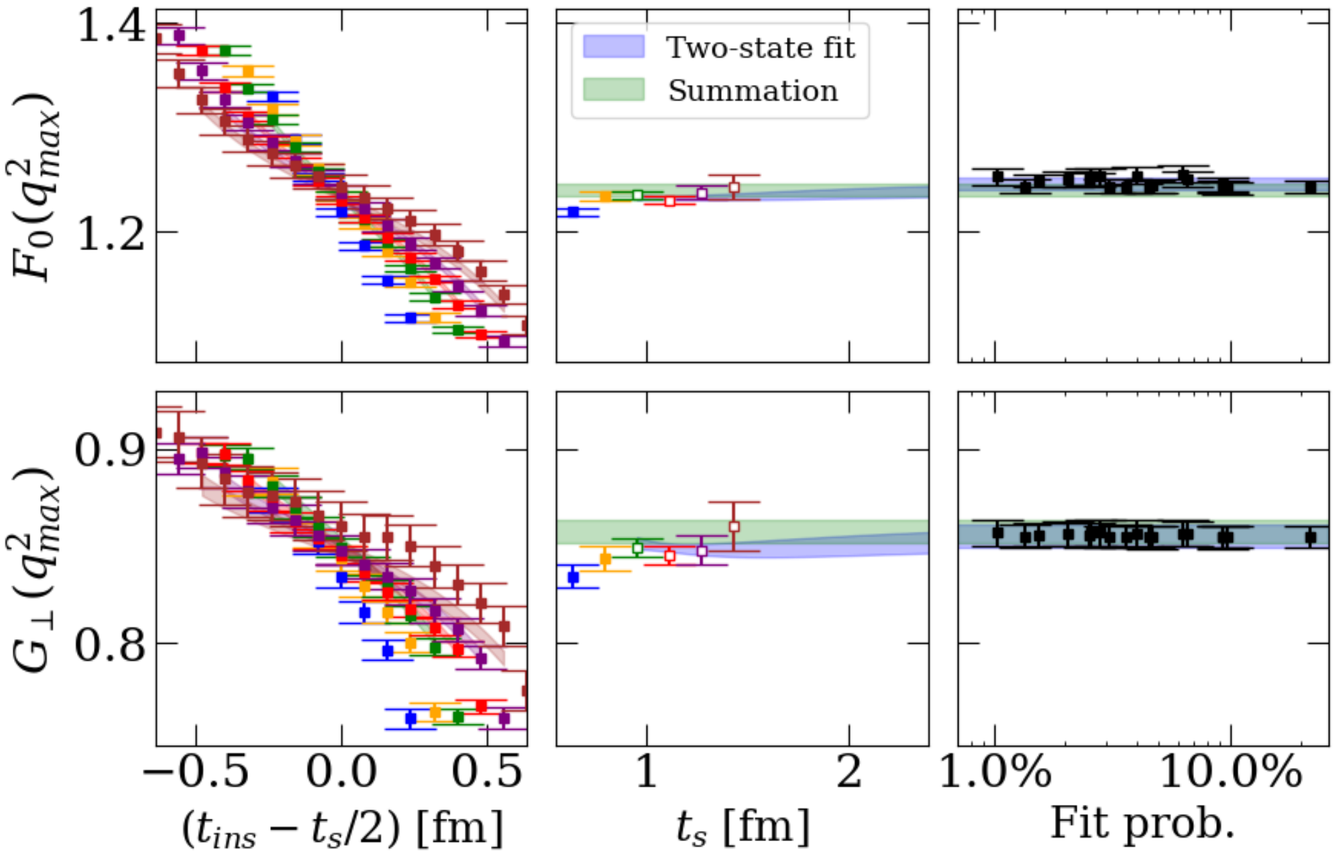
Form factors : Three-point functions

Two-state fit:

$$R^{\text{V-A}}_{\mu\nu}(\vec{p}_N,\vec{p}_\Lambda,t_\text{s},t_\text{ins}) \equiv \frac{C^{\text{V-A}}_{\mu\nu}(\vec{p}_N,\vec{p}_\Lambda,t_\text{s},t_\text{ins})}{\sqrt{C_\Lambda(\vec{p}_\Lambda,t_\text{s})C_N(\vec{p}_N,t_\text{s})}}$$

Summation method:

$$S^{\text{A}}_{\mu\nu}(\vec{p}_N,\vec{p}_\Lambda;t_\text{s}) \equiv \sum_{t_\text{ins}=n}^{t_\text{s}-n} R^{\text{A}}_{\mu\nu}(\vec{p}_N,\vec{p}_\Lambda;t_\text{s},t_\text{s}) \times \frac{\sqrt{C_\Lambda(\vec{p}_\Lambda;t_\text{ins})C_N(\vec{p}_N;t_\text{s}-t_\text{ins})}}{\sqrt{C_N(\vec{p}_N;t_\text{ins})C_\Lambda(\vec{p}_\Lambda,t_\text{s}-t_\text{ins})}}$$



Data Analysis

Form factors : Three-point functions

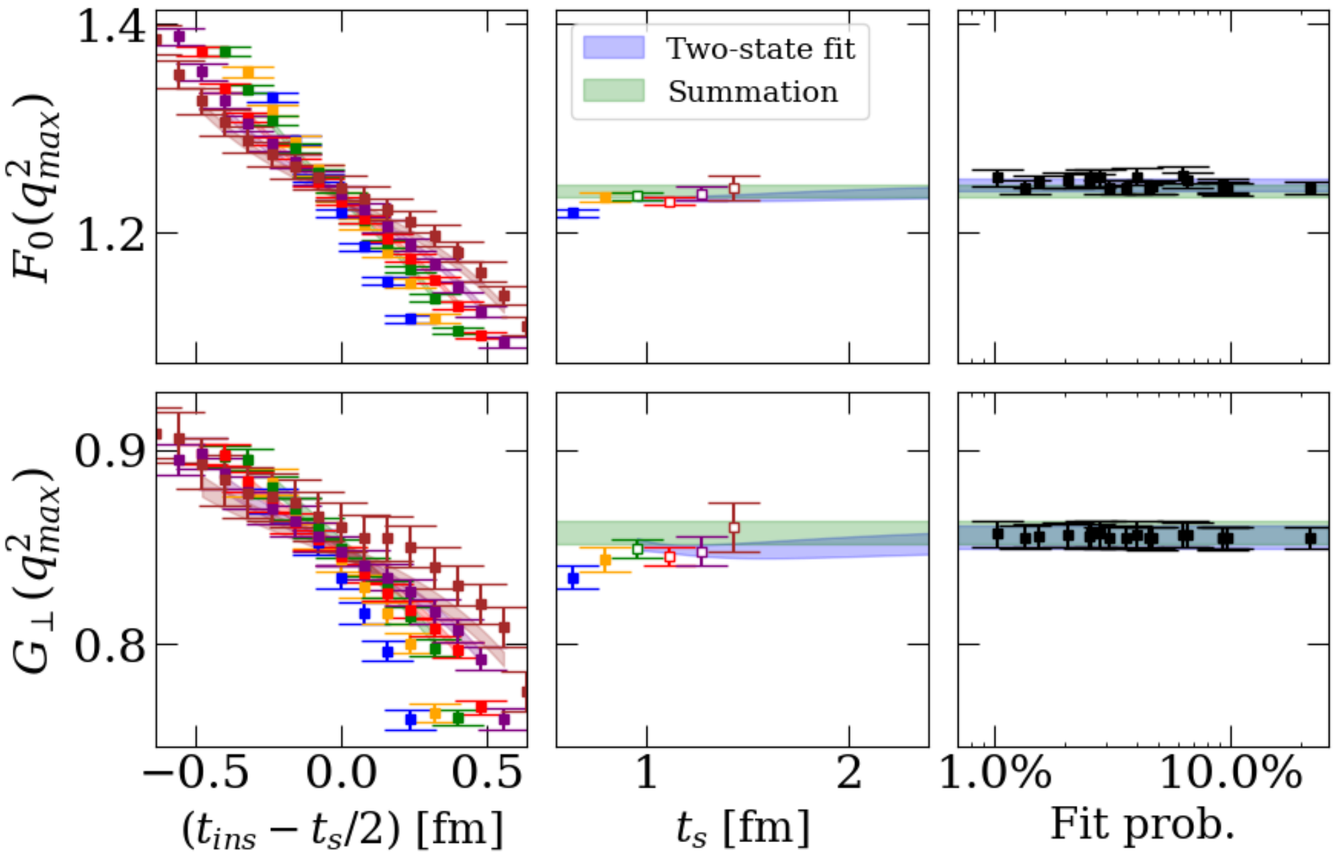
Two-state fit:

$$R^{\text{V-A}}_{\mu\nu}(\vec{p}_N, \vec{p}_\Lambda, t_s, t_s/2) \xrightarrow[t_s \gg a]{} \Pi^{\text{V-A}}_{\mu\nu}(\vec{p}_N, \vec{p}_\Lambda)$$

Summation method:

$$S^{\text{A}}_{\mu\nu}(\vec{p}_N, \vec{p}_\Lambda; t_s) = c + \Pi^{\text{A}}_{\mu\nu}(\vec{p}_N, \vec{p}_\Lambda)\, t_s + \dots$$

$$\Pi^{V}_{ij}(\vec{p}_N) = \frac{\epsilon_{ijk} p^k_N \mathcal{K}}{2m_N} F_\perp(q^2), \quad \mathcal{K} = \sqrt{\frac{2m_N^2}{E_N(E_N + m_N)}}.$$

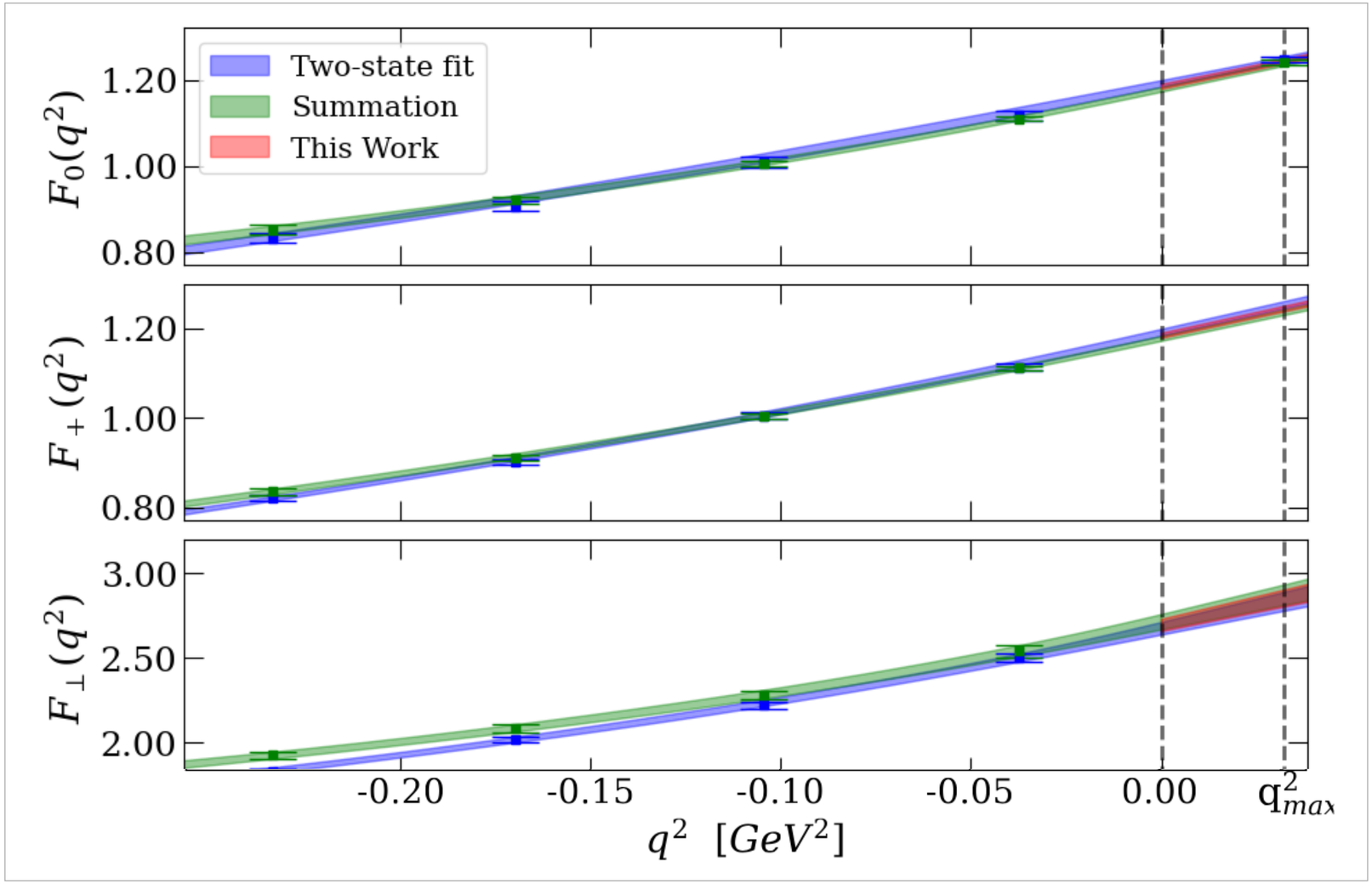


Z-Expansion

We employ the model-independent z-expansion

$$G(q^2) = \sum_{k=0}^{k_{\max}} \alpha_k z^k,$$

$$\text{with } z = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}}}}, \quad \text{and } t_{\text{cut}} = (m_\pi + m_K)^2$$



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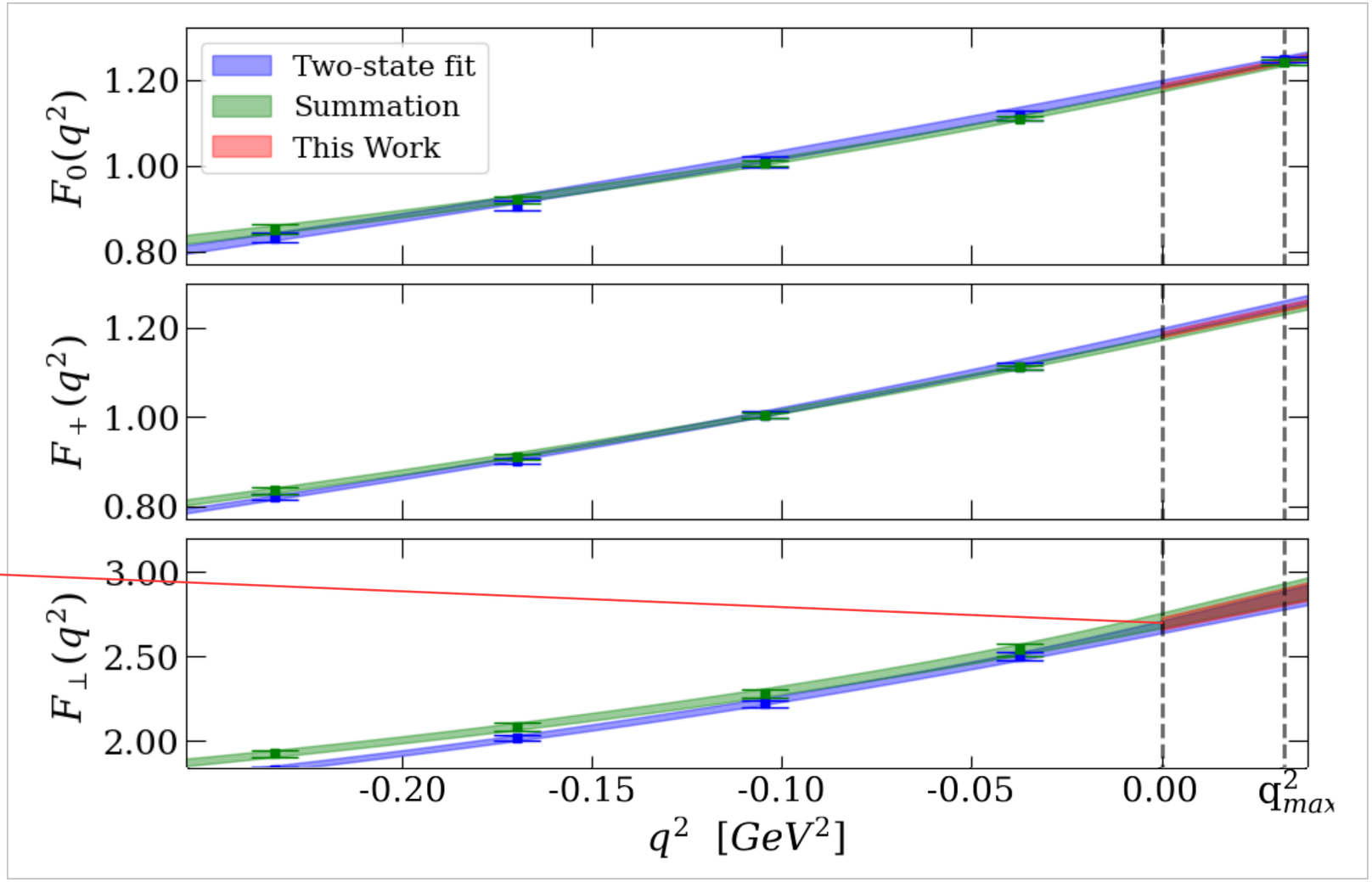
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$$\text{Charge :} \qquad g_i = G_i(0),$$

$$\text{Radii :} \qquad \langle r^2_{G_i} \rangle = \frac{6}{g_i} \frac{\partial G_i(q^2)}{\partial q^2}\bigg|_{q^2=0}.$$

$$G_i(q^2) = g_i \left(1 + \frac{\langle r^2_{G_i} \rangle}{6} q^2\right) + \mathcal{O}(q^4)$$



How?

The decay rate Γ is obtained by

$$\Gamma = \frac{1}{2m_\Lambda}\int \frac{d^3\vec{p}_p}{(2\pi)^3 2E_p}\frac{d^3\vec{p}_\ell}{(2\pi)^3 2E_\ell}\frac{d^3\vec{p}_\nu}{(2\pi)^3 2E_\nu} \times (2\pi)^4\delta^{(4)}(p_\Lambda - p_p - p_\ell - p_\nu)\,|\mathcal{M}|^2.$$

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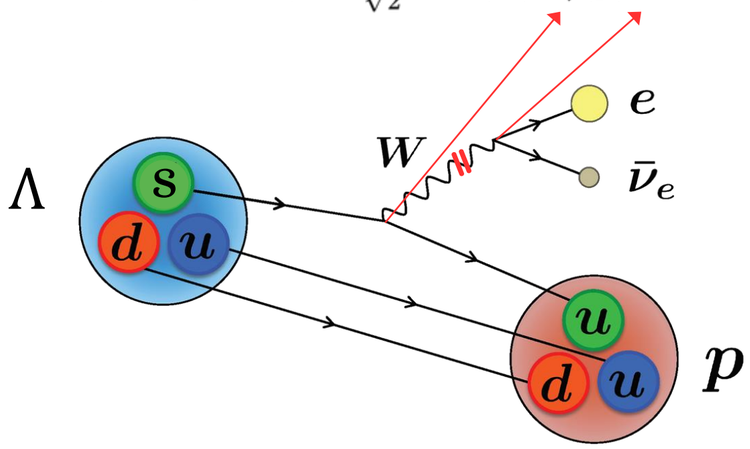
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$$\mathcal{M}(\Lambda \to p e^- \bar{\nu}_e) = \frac{G_F}{\sqrt{2}} V_{us} \langle \Lambda | W_\mu | p \rangle L^\mu$$



WE FOUND THE HADRONIC PART FROM LQCD

$$M_\mu^{\text{V}} = \bar{u}_p \left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\nu}q^\nu \frac{F_2(q^2)}{m_\Lambda} + q_\mu \frac{F_3(q^2)}{m_\Lambda}\right] u_\Lambda$$

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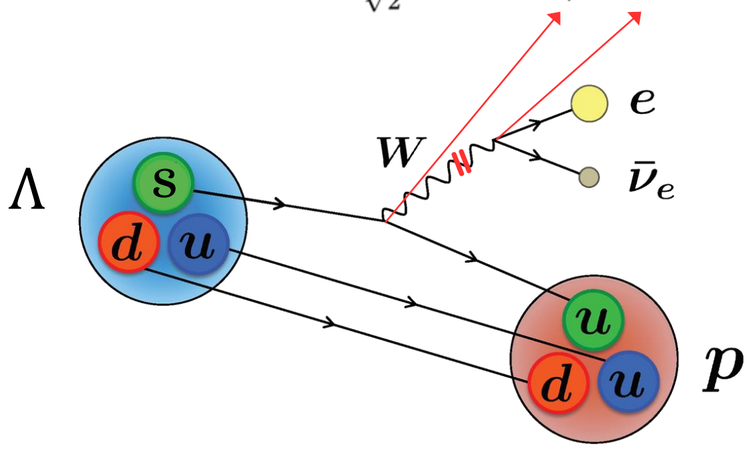
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WE CAN COMPUTE THE DECAY RATE NOW

We integrate with respect to q2 ... obtain the decay rate

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Decay Rate

For electron, adapting Weinberg FFs, and solve perturbatively (Cabibbo et al. 2003)

$$\Gamma_{\rm NLO} = \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^5}{60\pi^3}\left(1-\frac{3}{2}\delta\right)\left(f_1^2+3g_1^2\right), \qquad \delta \equiv q^2_{max}/m_\Lambda = (m_\Lambda - m_N)/m_\Lambda$$

$$\Gamma_{\rm NNLO} = \Gamma_{\rm NLO} + \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^7}{70\pi^3}\left\{ f_1 f_2 + \frac{2}{3} f_2^2 - \frac{14}{3}\frac{g_1 g_2}{\delta} + f_1^2\left(1+\frac{1}{9}\langle r^2_{F_1}\rangle m_\Lambda^2\right) + g_1^2\left(2+\frac{5}{9}\langle r^2_{G_1}\rangle m_\Lambda^2\right)\right\}$$

- f1 and g1 affect the leading order result
- f2 and g2 affect the NNLO result
- g3 and f3 contribute at higher orders
- g2 and f3 are second-class form factors (Weinberg (1958))

The deviations of the above from the integrated results are significant

$$\Delta_{\rm NLO} \simeq 4\% \quad \text{and} \quad \Delta_{\rm NNLO} \simeq -1\%,$$

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For electron, adapting Weinberg FFs, and solve perturbatively (Cabibbo et al. 2003)

$$\Gamma_{\rm NLO} = \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^5}{60\pi^3}\left(1-\frac{3}{2}\delta\right)\left(f_1^2+3g_1^2\right), \qquad \delta \equiv q^2_{max}/m_\Lambda = (m_\Lambda - m_N)/m_\Lambda$$

$$\Gamma_{\rm NNLO} = \Gamma_{\rm NLO} + \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^7}{70\pi^3}\left\{ f_1 f_2 + \frac{2}{3} f_2^2 - \frac{14}{3}\frac{g_1 g_2}{\delta} + f_1^2\left(1+\frac{1}{9}\langle r^2_{F_1}\rangle m_\Lambda^2\right) + g_1^2\left(2+\frac{5}{9}\langle r^2_{G_1}\rangle m_\Lambda^2\right)\right\}$$

- **f1 and g1 affect the leading order result**
- **f2 and g2 affect the NNLO result**

- **g3 and f3 contribute at higher orders**
- **g2 and f3 are second-class form factors**
  (Weinberg (1958))

The deviations of the above from the integrated results are significant

$$\Delta_{\rm NLO} \simeq 4\% \quad \text{and} \quad \Delta_{\rm NNLO} \simeq -1\%,$$

Decay Rate

For electron, adapting Weinberg FFs, and solve perturbatively (Cabibbo et al. 2003)

$$\Gamma_{\rm NLO} = \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^5}{60\pi^3}\left(1-\frac{3}{2}\delta\right)\left(f_1^2+3g_1^2\right), \qquad \delta \equiv q^2_{max}/m_\Lambda = (m_\Lambda - m_N)/m_\Lambda$$

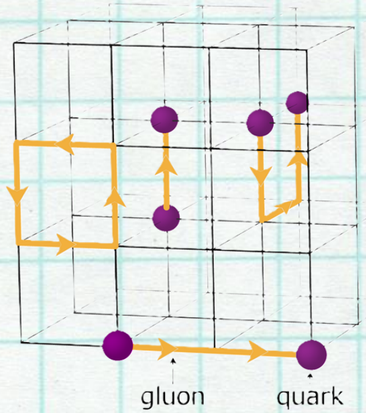
$$\Gamma_{\rm NNLO} = \Gamma_{\rm NLO} + \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^7}{70\pi^3}\left\{ f_1 f_2 + \frac{2}{3} f_2^2 - \frac{14}{3}\frac{g_1 g_2}{\delta} + f_1^2\left(1+\frac{1}{9}\langle r^2_{F_1}\rangle m_\Lambda^2\right) + g_1^2\left(2+\frac{5}{9}\langle r^2_{G_1}\rangle m_\Lambda^2\right)\right\}$$

- **f1 and g1 affect the leading order result**
- **f2 and g2 affect the NNLO result**
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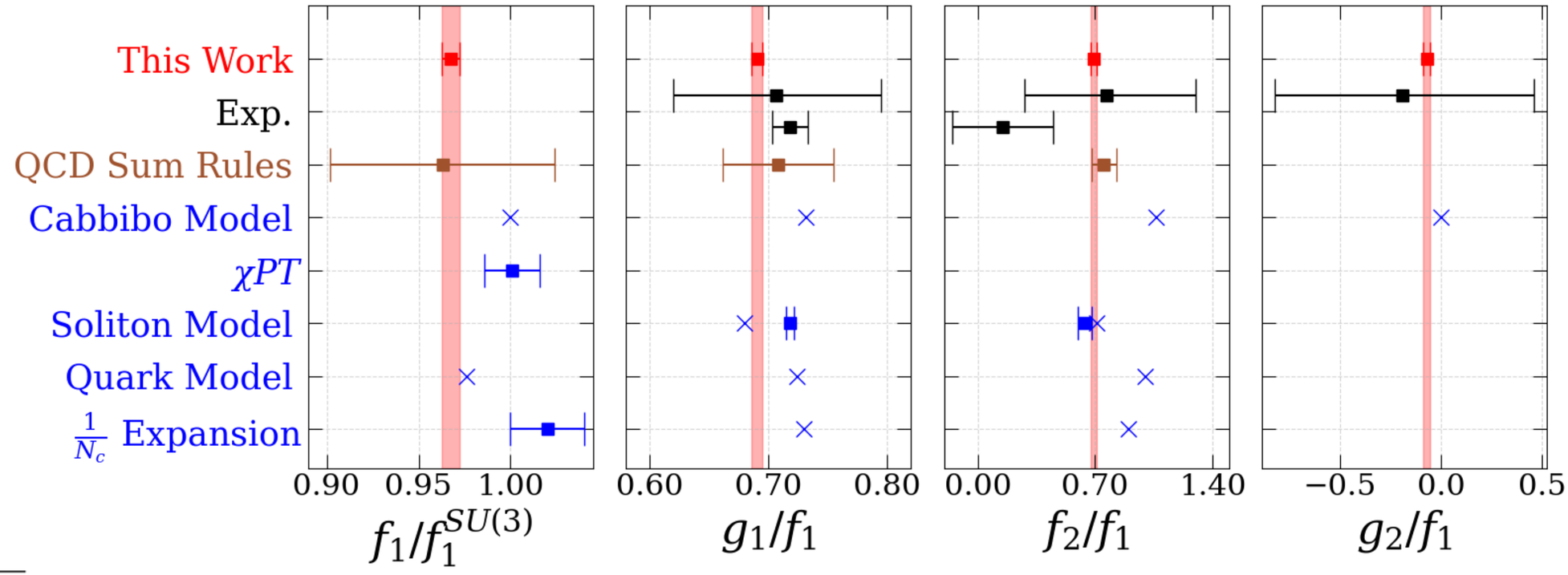
Comparison with Experimental Data & Main Results



→

PDG Live
particle data group

Comparison of charges



with $f_1^{SU(3)} = \sqrt{3/2}$

Exp. M. Ablikim et al. (BESIII), (2025),
S. Navas et al. (PDG), Phys. Rev. D 110, 030001 (2024)

QCD S.R. S.-Q, Zhang, X.-H. Zhang, and C.-F. Qiao, JHEP 06, 122 (2024),

Cabibbo N. Cabibbo, et al. Rev. Nucl. Part. Sci. 53, 39 (2003).

χPT L. S. Geng, et al, Phys. Rev. D 79, 094022 (2009).

Soliton G.-S. Yang and H.-C. Kim, Phys. Rev. C 92, 035206 (2015).

T. Ledwig, A. Silva, H.-C. Kim, and K. Goeke, JHEP 07, 132 (2008).

Quark Model F. Schlumpf, Phys. Rev. D 51, 2262 (1995).

A. Faessler, et al., Phys. Rev. D 78,094005 (2008).

1/Nc Exp. R. Flores-Mendieta, et al, Phys. Rev. D 58, 094028 (1998).

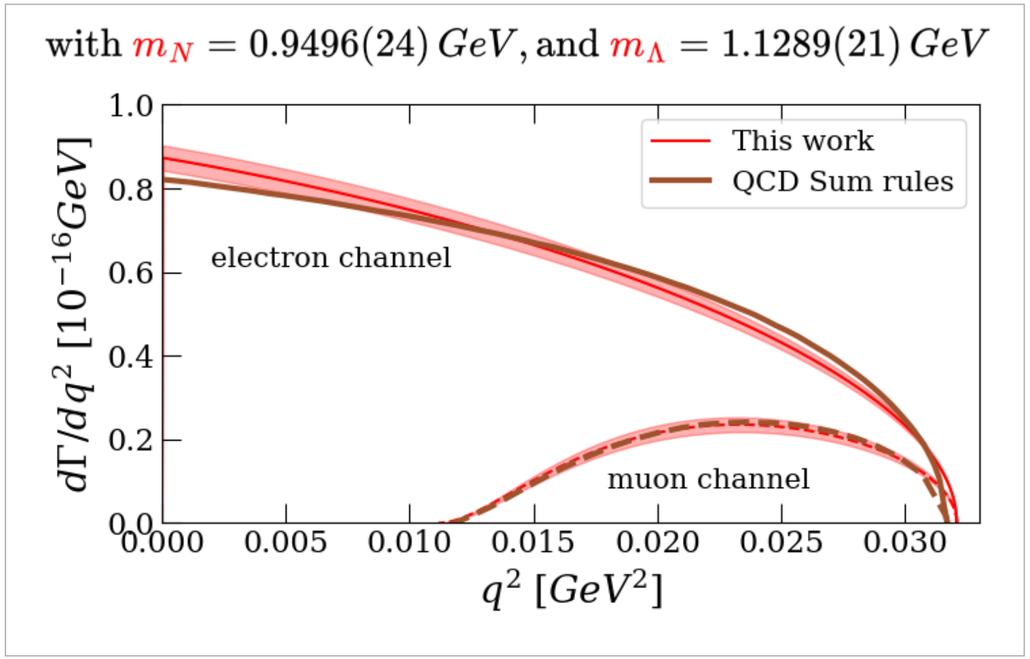
Masses Error Contribution

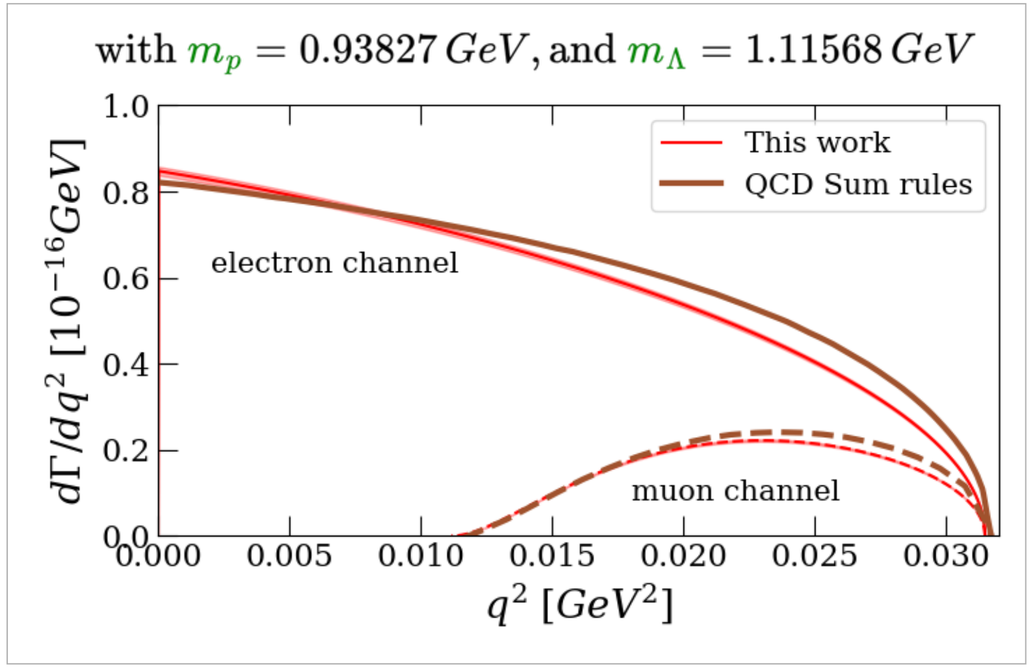
$$\Gamma = \int_{m_\ell{}^2}^{q^2_{\max}} dq^2 \frac{G_F^2 |V_{us}|^2}{192\pi^3 m_\Lambda^3} \sqrt{\lambda(m_\Lambda^2, m_p^2, q^2)}\, L^{\mu\nu} H_{\mu\nu}$$

Differential Decay Rate

Parameters

$$m_e = 0.511\, MeV, \qquad m_\mu = 0.10566\, GeV,$$
$$G_F = 1.16638 \cdot (1 + 0.0105) \cdot 10^{-5}\, GeV^{-2},$$





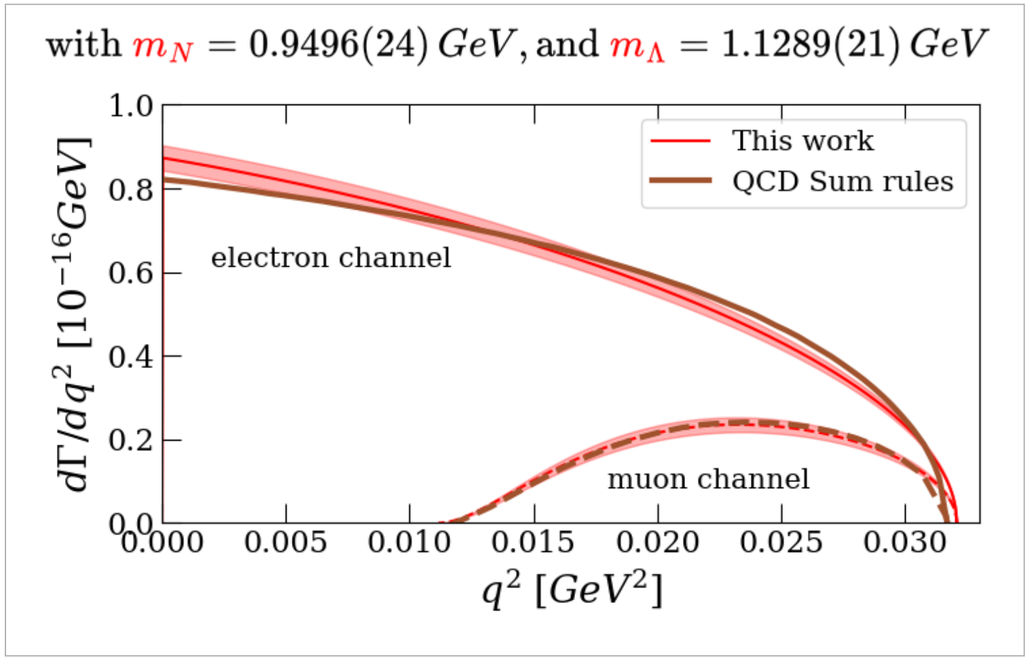
Masses Error Contribution

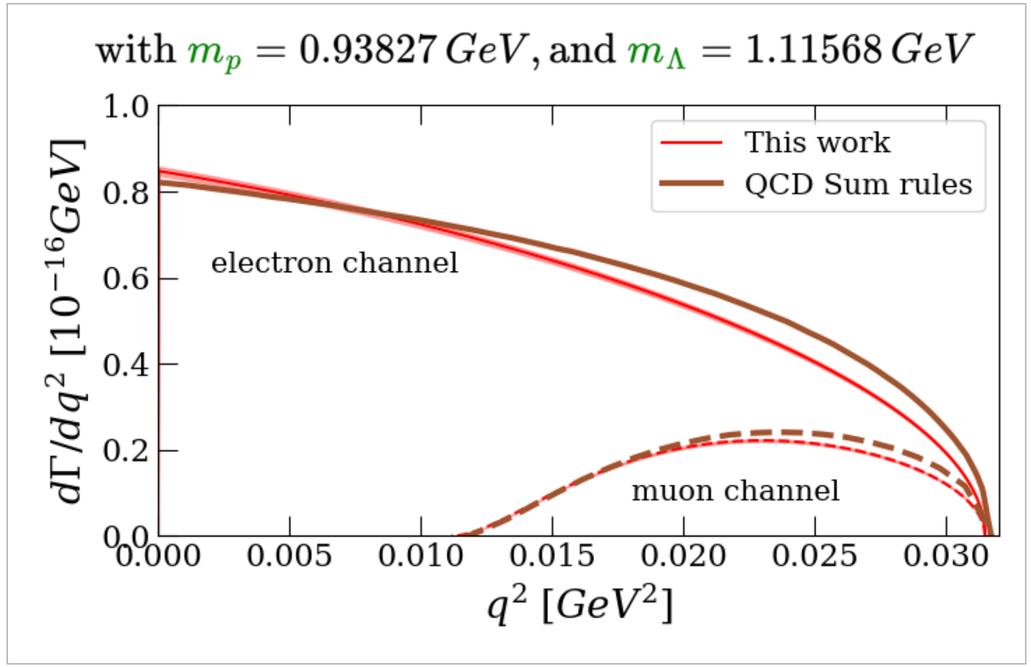
$$\Gamma_{\mathrm{NLO}} = \frac{G_F^2 |V_{us}|^2 m_\Lambda^5 \delta^5}{60\pi^3} \left(1 - \frac{3}{2}\delta\right)\left(f_1^2 + 3g_1^2\right),$$

Differential Decay Rate

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$$G_F = 1.16638 \cdot (1 + 0.0105) \cdot 10^{-5}\,GeV^{-2},$$





Beyond Standard Model

LQCD results

$$\tilde{\Gamma}_{pe\bar{\nu}_e} = 5.83(37) \cdot 10^7\,\mathrm{s}^{-1}\,,$$
$$\tilde{\Gamma}_{p\mu\bar{\nu}_\mu} = 1.01(12) \cdot 10^7\,\mathrm{s}^{-1}\,.$$

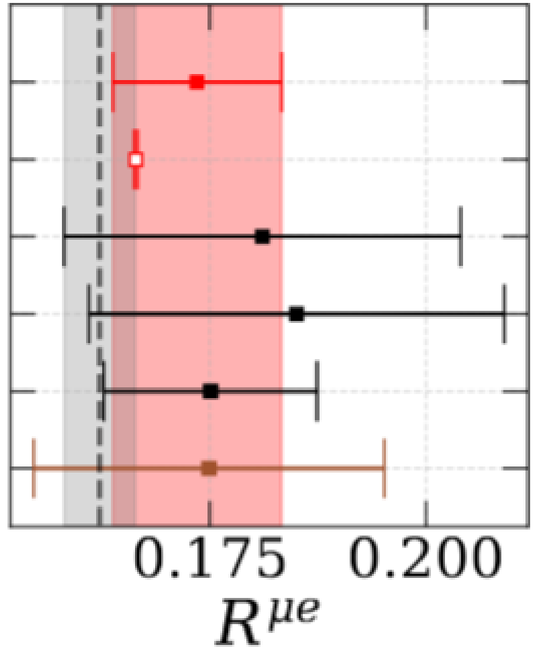
Decay rates ratio

$$R^{\mu e} \equiv \frac{\Gamma_{p\mu\bar{\nu}_\mu}}{\Gamma_{pe\bar{\nu}_e}} = 0.1735(98)$$

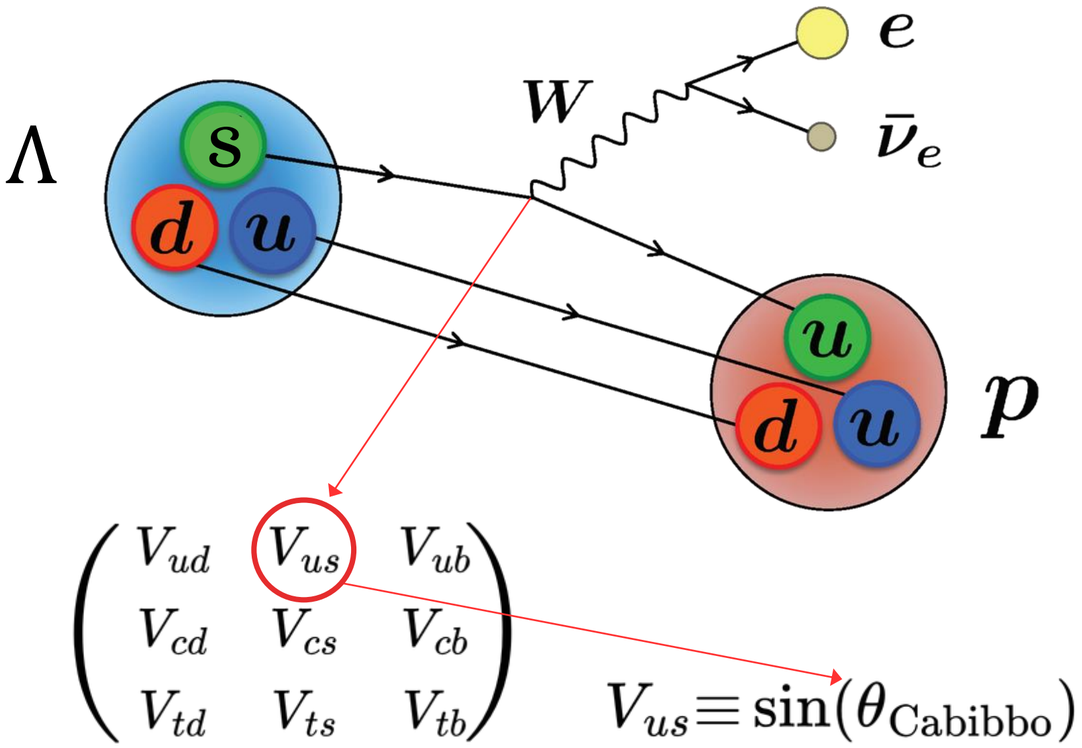
Up to NLO - FF Independent $(\Delta = m_\Lambda - m_p)$

$$R^{\mu e}_{\mathrm{SM}} = \sqrt{1-\frac{m_\mu^2}{\Delta^2}}\left(1-\frac{9}{2}\frac{m_\mu^2}{\Delta^2}-4\frac{m_\mu^4}{\Delta^4}\right)+\frac{15}{2}\frac{m_\mu^4}{\Delta^4}\operatorname{artanh}\left(\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)$$

This Work with $m_p^{PDG}, m_\Lambda^{PDG}$
PDG
BESIII
LHCb
QCD Sum Rules



Motivation

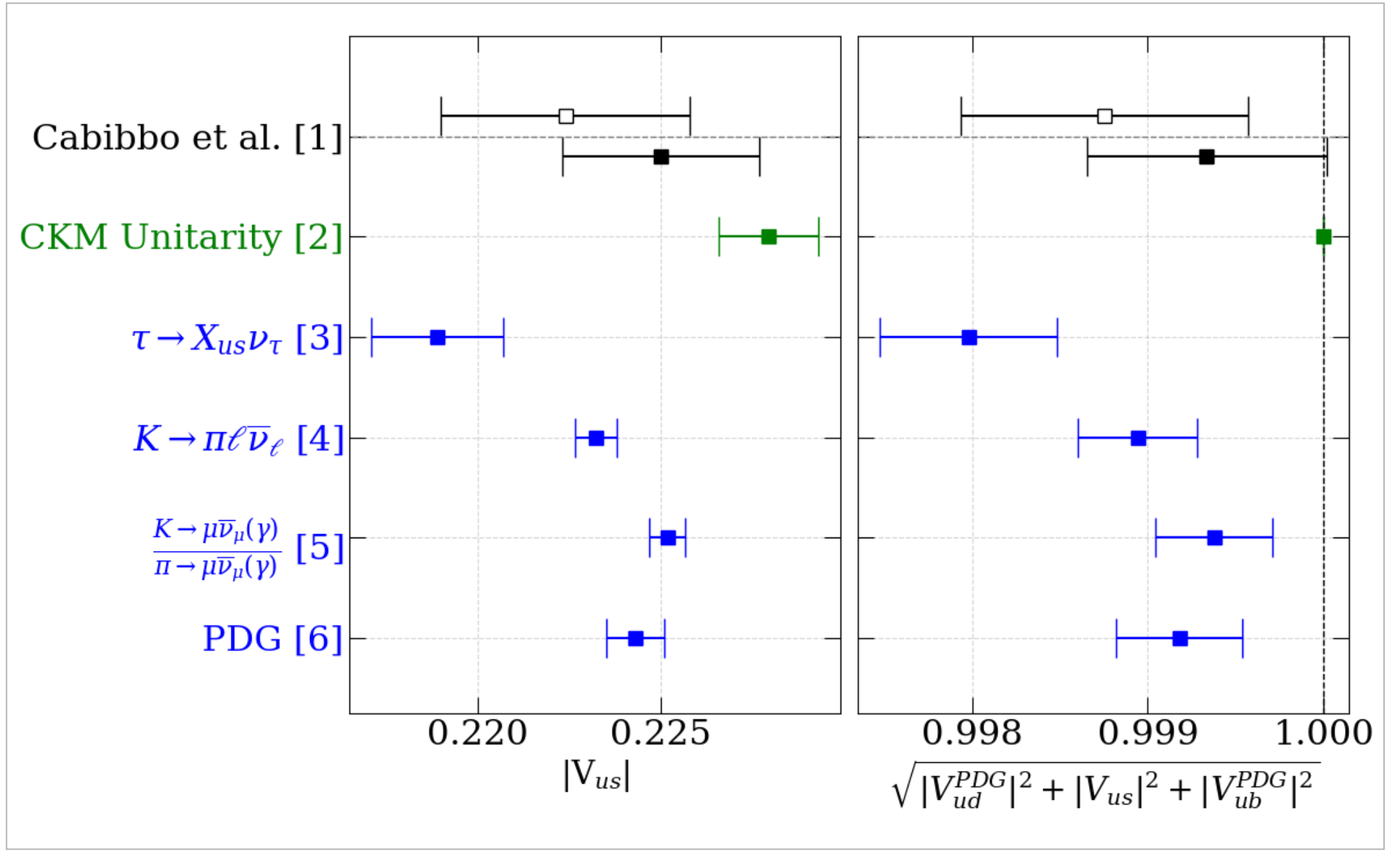


$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM unitary matrix

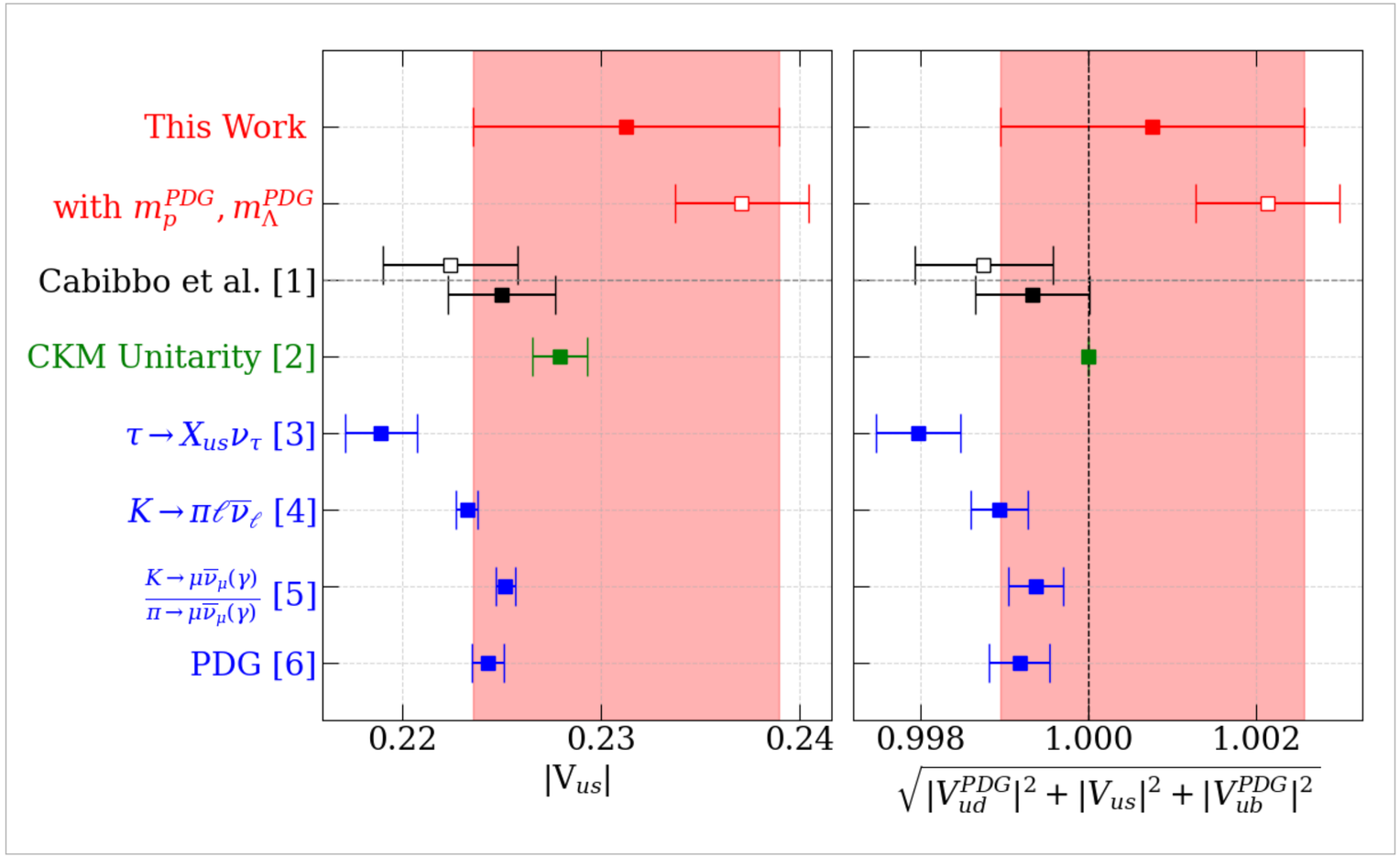
$$V_{us} \equiv \sin(\theta_{\text{Cabibbo}})$$

Related to the Cabibbo angle



Determination of Vus (left) and the resulting CKM unitarity relation (right)

|Vus| from LQCD



By combining LQCD and experimental results

$$\mathcal{B}^{\mathrm{BESIII}}_{\Lambda\to pe\bar{\nu}_e} = 8.16(22)\cdot 10^{-4}$$

$$\tau_\Lambda^{\mathrm{PDG}} = 2.617(10)\cdot 10^{-10} s$$

$$\tilde{\Gamma}^{LQCD}_{pe\bar{\nu}_e} = 5.83(37)\cdot 10^{7}\,\mathrm{s}^{-1}$$

$$|V_{us}| \equiv \left(\tau_\Lambda^{PDG}\frac{\tilde{\Gamma}^{LQCD}_{\Lambda\to pe\bar{\nu}_e}}{\mathcal{B}^{BESIII}(\Lambda\to pe\bar{\nu}_e)}\right)^{-1/2}$$

Beyond Standard Model

Scalar and Tensor Form Factors for Λ → pℓνℓ from Lattice QCD

Constantia Alexandrou, Simone Bacchio, **Andreas Konstantinou**, and Eleni Vakana

$$\mathcal{L}_{\text{eff}} = -\frac{G_F \tilde{V}_{us}}{\sqrt{2}} \sum_{\ell=e,\mu} \Big[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \; \bar{u}\gamma^\mu(1-\gamma_5)s$$

$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.},$$

Beyond Standard Model

Beyond Standard Model (BSM)

$$\mathcal{L}_{\text{eff}} = -\frac{G_F \tilde{V}_{us}}{\sqrt{2}} \sum_{\ell=e,\mu} \Big[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \; \bar{u}\gamma^\mu(1-\gamma_5)s$$

$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.}\,,$$

Form Factor independent up to NLO

$$R^{\mu e}_{\text{SM}} = \sqrt{1-\frac{m_\mu^2}{\Delta^2}}\left(1-\frac{9}{2}\frac{m_\mu^2}{\Delta^2}-4\frac{m_\mu^4}{\Delta^4}\right)+\frac{15}{2}\frac{m_\mu^4}{\Delta^4}\operatorname{artanh}\left(\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)+\mathcal{O}(\delta^2,\epsilon)$$

with $m=\frac{m_\mu}{\Delta}$, $\delta=\frac{\Delta}{m_\Lambda}$ and $\Delta = m_\Lambda - m_p$.

Differential decay rate

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_{LL}}{dq^2} + \epsilon_S \frac{d\Gamma_{LS}}{dq^2} + \epsilon_T \frac{d\Gamma_{LT}}{dq^2} + \mathcal{O}(\epsilon_S^2, \epsilon_T^2)\,,$$

Sensitivity coefficients

$$\frac{R^{\mu e}}{R^{\mu e}_{\text{SM}}} = \frac{\Gamma^e_{\text{SM}}}{\Gamma^\mu_{\text{SM}}}\frac{(\Gamma^\mu_{\text{SM}}+\Gamma^\mu_{\text{NP}})}{(\Gamma^e_{\text{SM}}+\Gamma^e_{\text{NP}})} \simeq 1 + r_S\,\epsilon_S + r_T\,\epsilon_T + \mathcal{O}(\epsilon^2)\,,$$

$$\text{with} \quad r_S = \frac{1}{R^{\mu e}_{\text{SM}}}\frac{\partial R^{\mu e}}{\partial \epsilon_S}\bigg|_{\epsilon=0} = \frac{\Gamma^\mu_{LS}}{\Gamma^\mu_{LL}} - \frac{\Gamma^e_{LS}}{\Gamma^e_{LL}},$$

$$\text{and} \quad r_T = \frac{1}{R^{\mu e}_{\text{SM}}}\frac{\partial R^{\mu e}}{\partial \epsilon_T}\bigg|_{\epsilon=0} = \frac{\Gamma^\mu_{LT}}{\Gamma^\mu_{LL}} - \frac{\Gamma^e_{LT}}{\Gamma^e_{LL}} \qquad (17)$$

Decay rate ratio |Vus| independent

$$R^{\mu e} = \frac{\Gamma(\Lambda \to p\,\mu\,\bar{\nu}_\mu)}{\Gamma(\Lambda \to p\,e\,\bar{\nu}_e)}\,.$$

Beyond Standard Model

Beyond Standard Model (BSM)

$$\mathcal{L}_{\text{eff}} = -\frac{G_F \tilde{V}_{us}}{\sqrt{2}} \sum_{\ell=e,\mu} \Big[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \; \bar{u}\gamma^\mu(1-\gamma_5)s$$

$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.}\,,$$

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$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_{LL}}{dq^2} + \epsilon_S \frac{d\Gamma_{LS}}{dq^2} + \epsilon_T \frac{d\Gamma_{LT}}{dq^2} + \mathcal{O}(\epsilon_S^2, \epsilon_T^2)\,,$$

Decay rate ratio |Vus| independent

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with $m=\frac{m_\mu}{\Delta}$, $\delta=\frac{\Delta}{m_\Lambda}$ and $\Delta = m_\Lambda - m_p$.

Sensitivity coefficients

$$\frac{R^{\mu e}}{R^{\mu e}_{\text{SM}}} = \frac{\Gamma^e_{\text{SM}}}{\Gamma^\mu_{\text{SM}}}\frac{(\Gamma^\mu_{\text{SM}}+\Gamma^\mu_{\text{NP}})}{(\Gamma^e_{\text{SM}}+\Gamma^e_{\text{NP}})} \simeq 1 + r_S\,\epsilon_S + r_T\,\epsilon_T + \mathcal{O}(\epsilon^2)\,,$$

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Beyond Standard Model

Beyond Standard Model (BSM)

$$\mathcal{L}_{\text{eff}} = -\frac{G_F \tilde{V}_{us}}{\sqrt{2}} \sum_{\ell=e,\mu} \Big[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \; \bar{u}\gamma^\mu(1-\gamma_5)s$$

$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.}\,,$$

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Decay rate ratio

|Vus| independent

$$R^{\mu e} \equiv \frac{\Gamma(\Lambda \to p\,\mu\,\bar{\nu}_\mu)}{\Gamma(\Lambda \to p\,e\,\bar{\nu}_e)}.$$

Form Factor independent up to NLO

$$R^{\mu e}_{\text{SM}} = \sqrt{1-\frac{m_\mu^2}{\Delta^2}}\left(1-\frac{9}{2}\frac{m_\mu^2}{\Delta^2}-4\frac{m_\mu^4}{\Delta^4}\right)+\frac{15}{2}\frac{m_\mu^4}{\Delta^4}\operatorname{artanh}\left(\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)+\mathcal{O}(\delta^2,\epsilon)$$

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$$\text{and} \quad r_T = \frac{1}{R^{\mu e}_{\text{SM}}}\frac{\partial R^{\mu e}}{\partial \epsilon_T}\bigg|_{\epsilon=0} = \frac{\Gamma^\mu_{LT}}{\Gamma^\mu_{LL}} - \frac{\Gamma^e_{LT}}{\Gamma^e_{LL}} \qquad (17)$$

Beyond Standard Model

Beyond Standard Model (BSM)

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$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.}\,,$$

Differential decay rate

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_{LL}}{dq^2} + \epsilon_S \frac{d\Gamma_{LS}}{dq^2} + \epsilon_T \frac{d\Gamma_{LT}}{dq^2} + \mathcal{O}(\epsilon_S^2, \epsilon_T^2)\,,$$

Decay rate ratio |Vus| independent

$$R^{\mu e} \equiv \frac{\Gamma(\Lambda \to p\,\mu\,\bar{\nu}_\mu)}{\Gamma(\Lambda \to p\,e\,\bar{\nu}_e)}.$$

Form Factor independent up to NLO

$$R^{\mu e}_{\text{SM}} = \sqrt{1-\frac{m_\mu^2}{\Delta^2}}\left(1-\frac{9}{2}\frac{m_\mu^2}{\Delta^2}-4\frac{m_\mu^4}{\Delta^4}\right)+\frac{15}{2}\frac{m_\mu^4}{\Delta^4}\operatorname{artanh}\left(\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)+\mathcal{O}(\delta^2,\epsilon)$$

with $\hat{m} \equiv \frac{m_\mu}{\Delta}$, $\delta \equiv \frac{\Delta}{m_\Lambda}$ and $\Delta = m_\Lambda - m_p$.

Sensitivity coefficients

$$\frac{R^{\mu e}}{R^{\mu e}_{\text{SM}}} = \frac{\Gamma^e_{\text{SM}}}{\Gamma^\mu_{\text{SM}}} \frac{(\Gamma^\mu_{\text{SM}} + \Gamma^\mu_{\text{NP}})}{(\Gamma^e_{\text{SM}} + \Gamma^e_{\text{NP}})} \simeq 1 + r_S\,\epsilon_S + r_T\,\epsilon_T + \mathcal{O}(\epsilon^2)\,,$$

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$$\text{and} \quad r_T = \frac{1}{R^{\mu e}_{\text{SM}}} \frac{\partial R^{\mu e}}{\partial \epsilon_T}\bigg|_{\epsilon=0} = \frac{\Gamma^\mu_{LT}}{\Gamma^\mu_{LL}} - \frac{\Gamma^e_{LT}}{\Gamma^e_{LL}} \qquad (17)$$

Beyond Standard Model

Beyond Standard Model (BSM)

$$\mathcal{L}_{\text{eff}} = -\frac{G_F \tilde{V}_{us}}{\sqrt{2}} \sum_{\ell=e,\mu} \Big[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \; \bar{u}\gamma^\mu(1-\gamma_5)s$$

$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.}\,,$$

Form Factor independent up to NLO

$$R^{\mu e}_{\text{SM}} = \sqrt{1-\frac{m_\mu^2}{\Delta^2}}\left(1-\frac{9}{2}\frac{m_\mu^2}{\Delta^2}-4\frac{m_\mu^4}{\Delta^4}\right)+\frac{15}{2}\frac{m_\mu^4}{\Delta^4}\text{artanh}\left(\sqrt{1-\frac{m_\mu^2}{\Delta^2}}\right)+\mathcal{O}(\delta^2,\epsilon)$$

with $\hat{m} \equiv \frac{m_\mu}{\Delta}\,, \quad \delta \equiv \frac{\Delta}{m_\Lambda}$ and $\Delta = m_\Lambda - m_p\,.$

Differential decay rate

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Sensitivity coefficients

$$\frac{R^{\mu e}}{R^{\mu e}_{\text{SM}}} = \frac{\Gamma^e_{\text{SM}}}{\Gamma^\mu_{\text{SM}}} \frac{(\Gamma^\mu_{\text{SM}} + \Gamma^\mu_{\text{NP}})}{(\Gamma^e_{\text{SM}} + \Gamma^e_{\text{NP}})} \simeq 1 + r_S\,\epsilon_S + r_T\,\epsilon_T + \mathcal{O}(\epsilon^2)\,,$$

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Decay rate ratio |Vus| independent

$$R^{\mu e} \equiv \frac{\Gamma(\Lambda \to p\,\mu\,\bar{\nu}_\mu)}{\Gamma(\Lambda \to p\,e\,\bar{\nu}_e)}.$$

Beyond Standard Model

Beyond Standard Model (BSM)

$$\mathcal{L}_{\text{eff}} = -\frac{G_F \tilde{V}_{us}}{\sqrt{2}} \sum_{\ell=e,\mu} \Big[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \; \bar{u}\gamma^\mu(1-\gamma_5)s$$

$$+\epsilon_S \, \bar{\ell}(1-\gamma_5)\nu_\ell \; \bar{u}s + \epsilon_T \, \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \; \bar{u}\sigma^{\mu\nu}s \Big] + \text{h. c.}\,,$$

Differential decay rate

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_{LL}}{dq^2} + \epsilon_S \frac{d\Gamma_{LS}}{dq^2} + \epsilon_T \frac{d\Gamma_{LT}}{dq^2} + \mathcal{O}(\epsilon_S^2, \epsilon_T^2)\,,$$

Decay rate ratio

|Vus| independent

$$R^{\mu e} \equiv \frac{\Gamma(\Lambda \to p\,\mu\,\bar{\nu}_\mu)}{\Gamma(\Lambda \to p\,e\,\bar{\nu}_e)}.$$

Perturbative result

$$r_S R^{\mu e}_{\text{SM}} = \frac{\Gamma^\mu_{LS}}{\Gamma^e_{LL}} + \mathcal{O}(m_e) = \frac{f_1 f_S}{f_1^2 + 3g_1^2}\,\Pi(\hat{m}_\mu) + \mathcal{O}\big(\delta^2\big)\,,$$

$$r_T R^{\mu e}_{\text{SM}} = \frac{\Gamma^\mu_{LT}}{\Gamma^e_{LL}} + \mathcal{O}(m_e) = \frac{12 g_1 f_T}{f_1^2 + 3g_1^2}\,\Pi(\hat{m}_\mu) + \mathcal{O}\big(\delta^2\big)\,,$$

where $g_i \equiv G_i(q^2 = 0)$ and $f_i \equiv F_i(q^2 = 0)$, and the phase-space function $\Pi(\hat{m}_\mu)$ is given by

$$\Pi(\hat{m}_\mu) = \frac{5}{2}\hat{m}_\mu \Bigg[\left(2 + 13\hat{m}_\mu^2\right)\sqrt{1-\hat{m}_\mu^2}$$

$$- 3\left(4\hat{m}_\mu^2 + \hat{m}_\mu^4\right) \text{artanh}\left(\sqrt{1-\hat{m}_\mu^2}\right) \Bigg].$$

Beyond Standard Model

Sensitivity coefficients

$$\text{with} \quad r_S = \frac{1}{R^{\mu e}_{\rm SM}} \frac{\partial R^{\mu e}}{\partial \epsilon_S}\bigg|_{\epsilon=0} = \frac{\Gamma^{\mu}_{LS}}{\Gamma^{\mu}_{LL}} - \frac{\Gamma^{e}_{LS}}{\Gamma^{e}_{LL}},$$

$$\text{and} \quad r_T = \frac{1}{R^{\mu e}_{\rm SM}} \frac{\partial R^{\mu e}}{\partial \epsilon_T}\bigg|_{\epsilon=0} = \frac{\Gamma^{\mu}_{LT}}{\Gamma^{\mu}_{LL}} - \frac{\Gamma^{e}_{LT}}{\Gamma^{e}_{LL}}$$

Perturbative result

$$r_S R^{\mu e}_{\rm SM} = \frac{\Gamma^{\mu}_{LS}}{\Gamma^{e}_{LL}} + \mathcal{O}(m_e) = \frac{f_1 f_S}{f_1^2 + 3g_1^2}\,\Pi(\hat{m}_\mu) + \mathcal{O}(\delta^2)\,,$$

$$r_T R^{\mu e}_{\rm SM} = \frac{\Gamma^{\mu}_{LT}}{\Gamma^{e}_{LL}} + \mathcal{O}(m_e) = \frac{12 g_1 f_T}{f_1^2 + 3g_1^2}\,\Pi(\hat{m}_\mu) + \mathcal{O}(\delta^2)\,,$$

Finding coefficients

$$\frac{R^{\mu e}}{R^{\mu e}_{\rm SM}} \simeq 1 + r_S\,\epsilon_S + r_T\,\epsilon_T + \mathcal{O}(\epsilon^2)$$

	This work	Using external inputs and expansions		
	$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^0 \to \Sigma^+$	$\Xi^- \to \Lambda$
g_1/f_1	0.6902(44)	−0.340(17)	1.220(5)	0.250(5)
f_S/f_1	1.925(36)	2.80(14)	1.36(7)	2.25(11)
f_T/f_1	0.5930(49)	−0.284(46)	1.28(11)	0.288(51)
r_S	1.272(27)	2.79(21)	0.469(48)	2.84(27)
r_T	2.940(24)	1.15(20)	6.45(69)	1.09(25)
$R^{\mu e}_{\rm SM}$	0.16638(20)	0.446(21)	0.00841(26)	0.2729(66)
$R^{\mu e}_{\rm exp}$	0.175(12)	0.442(42)	0.0092(14)	0.74(51)

Beyond Standard Model

90 % Confident Level contours from hyperon semi-leptonic decays

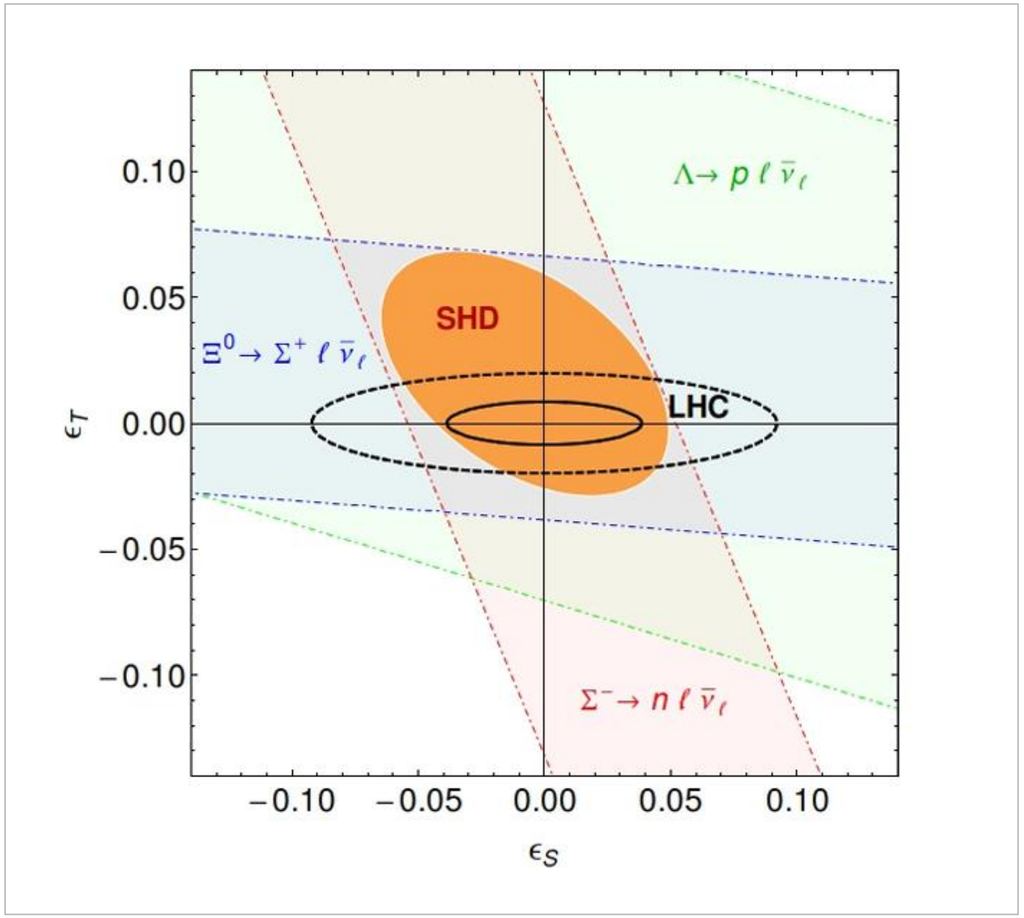
(H.-M. Chang, M. Gonzalez-Alonso, and J. Martin Camalich, Phys. Rev. Lett. 114, 161802 (2015))

90 % Confident Level contours from LHC Run 1 and Run 2

(V. Cirigliano, M. Gonzalez-Alonso, and M. L. Graesser, JHEP 02, 046 (2013))

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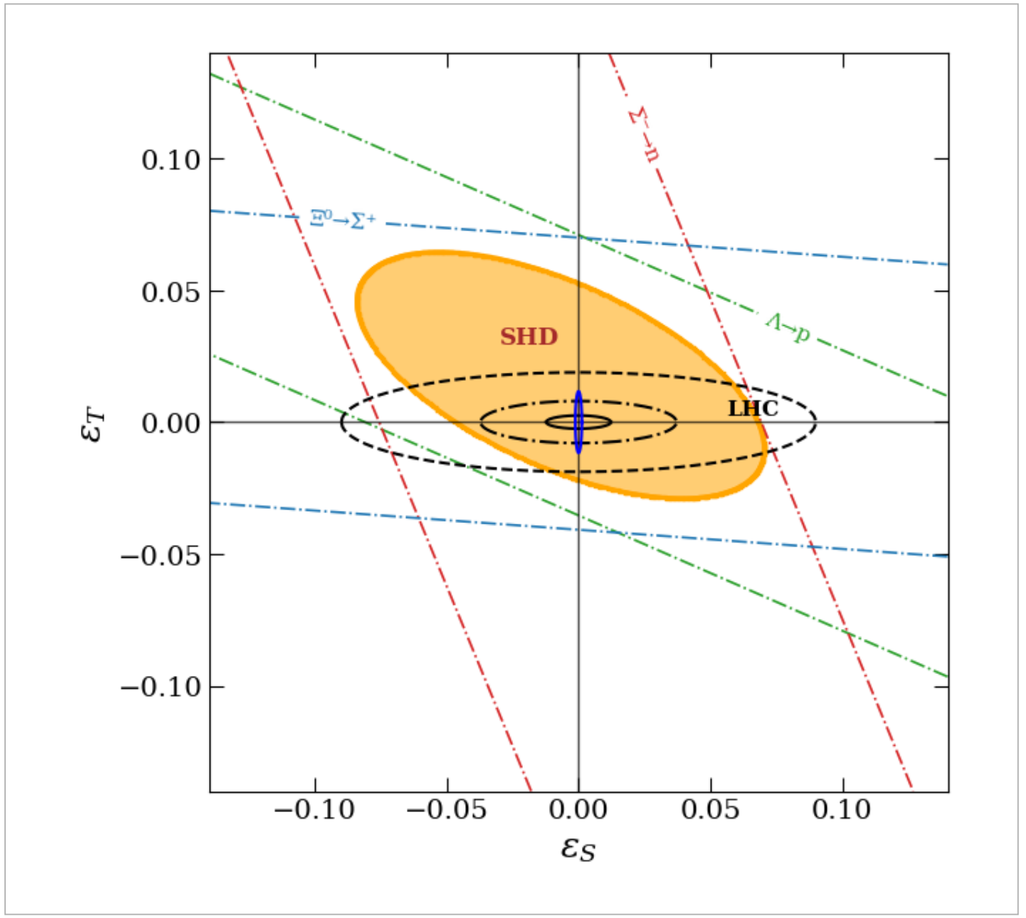
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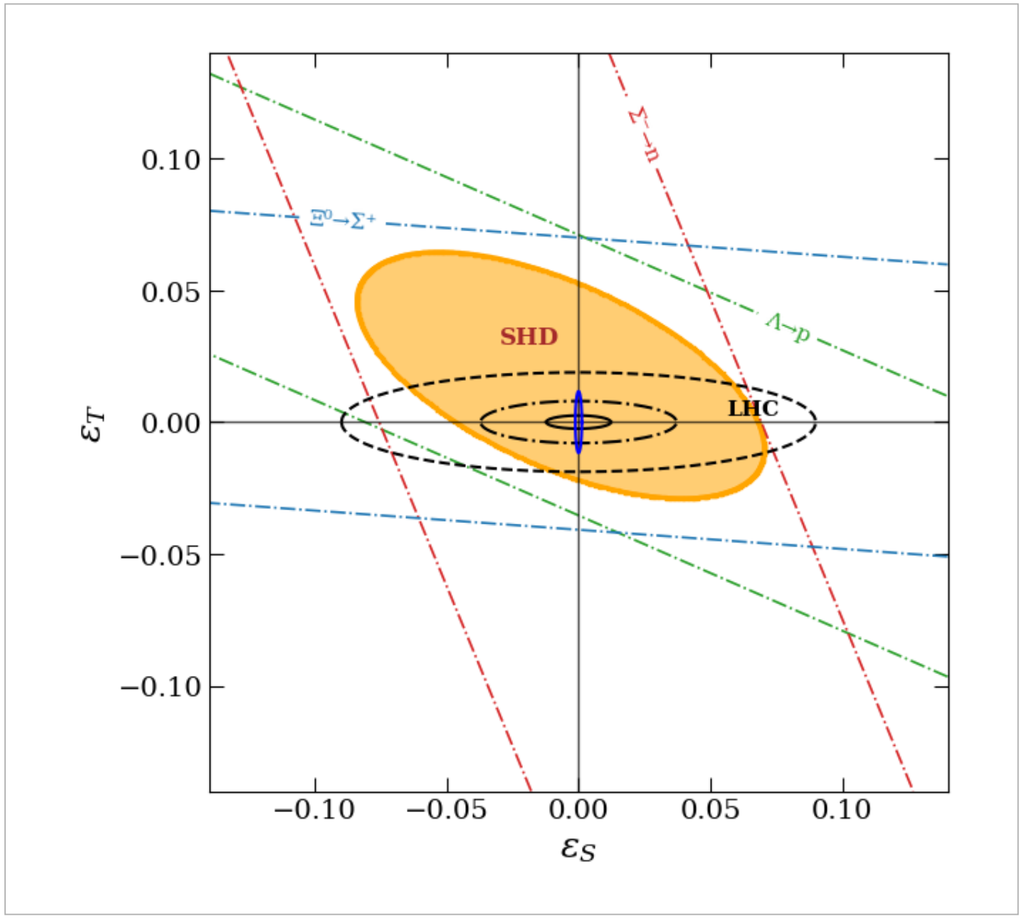
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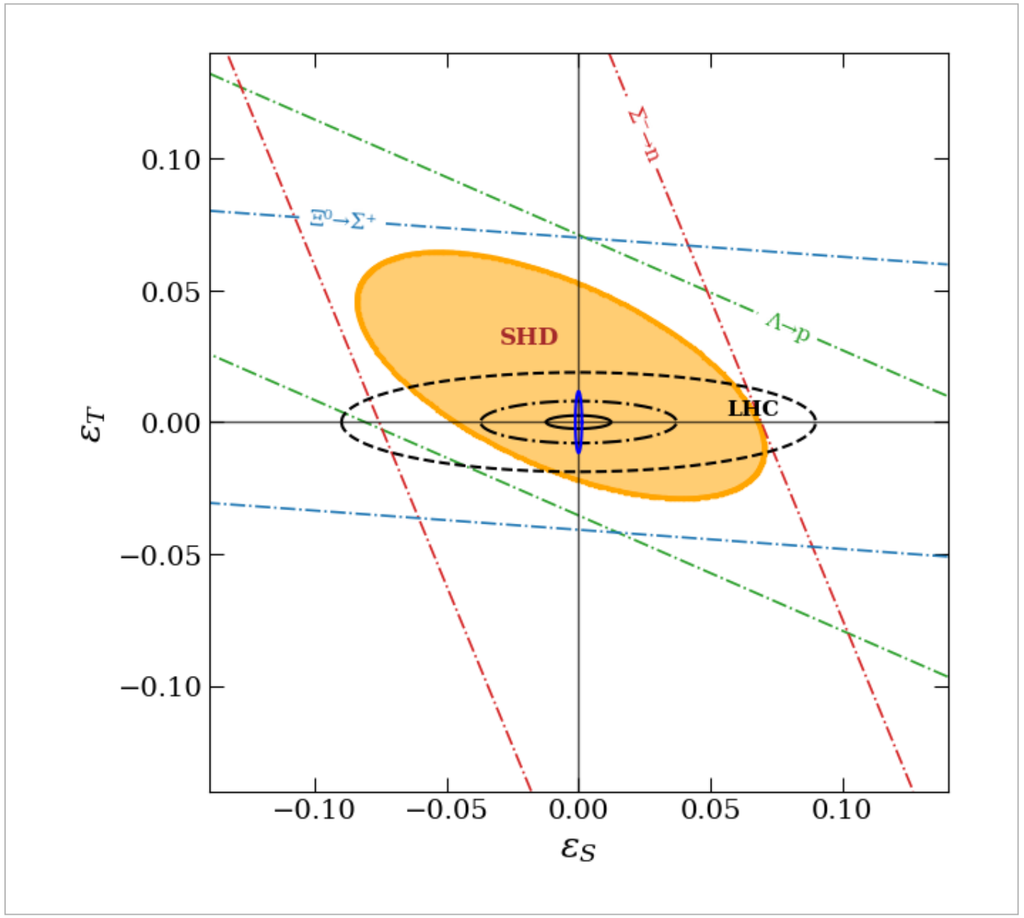
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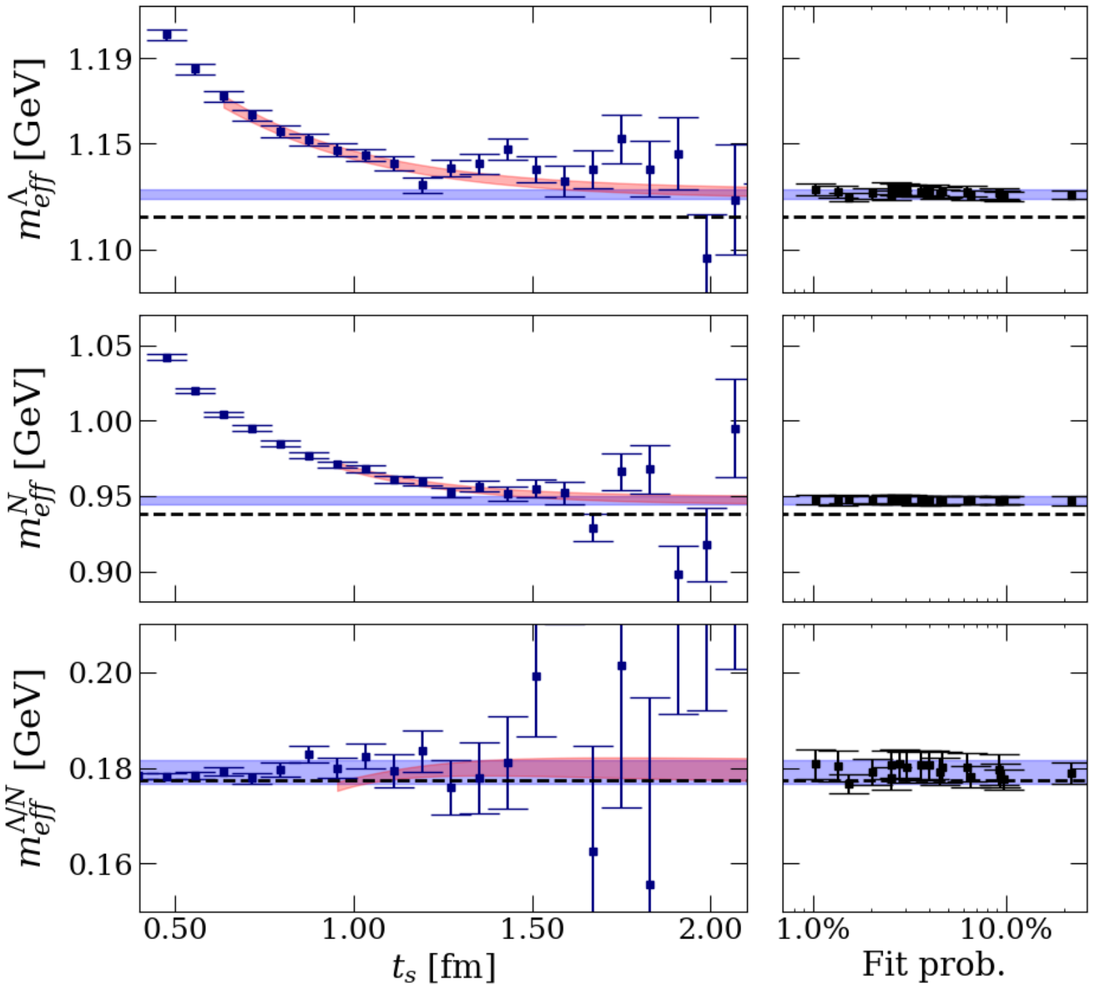
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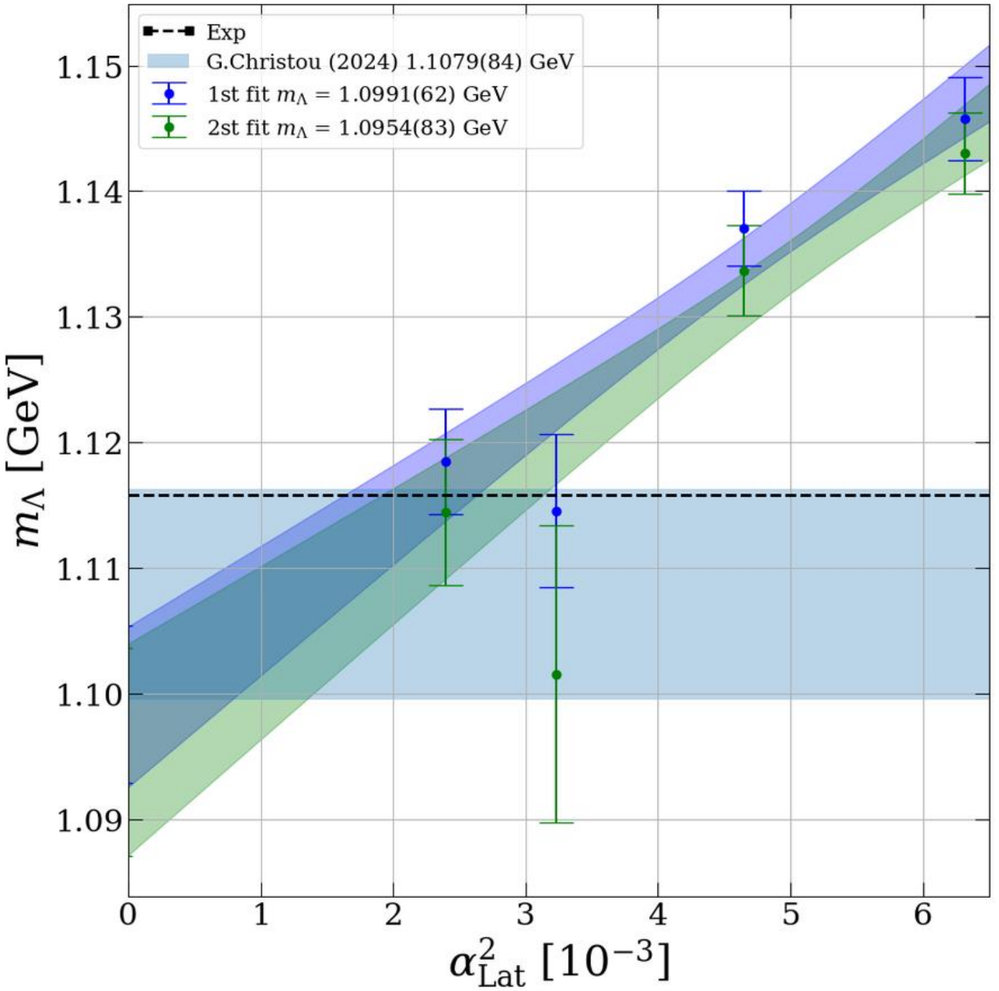
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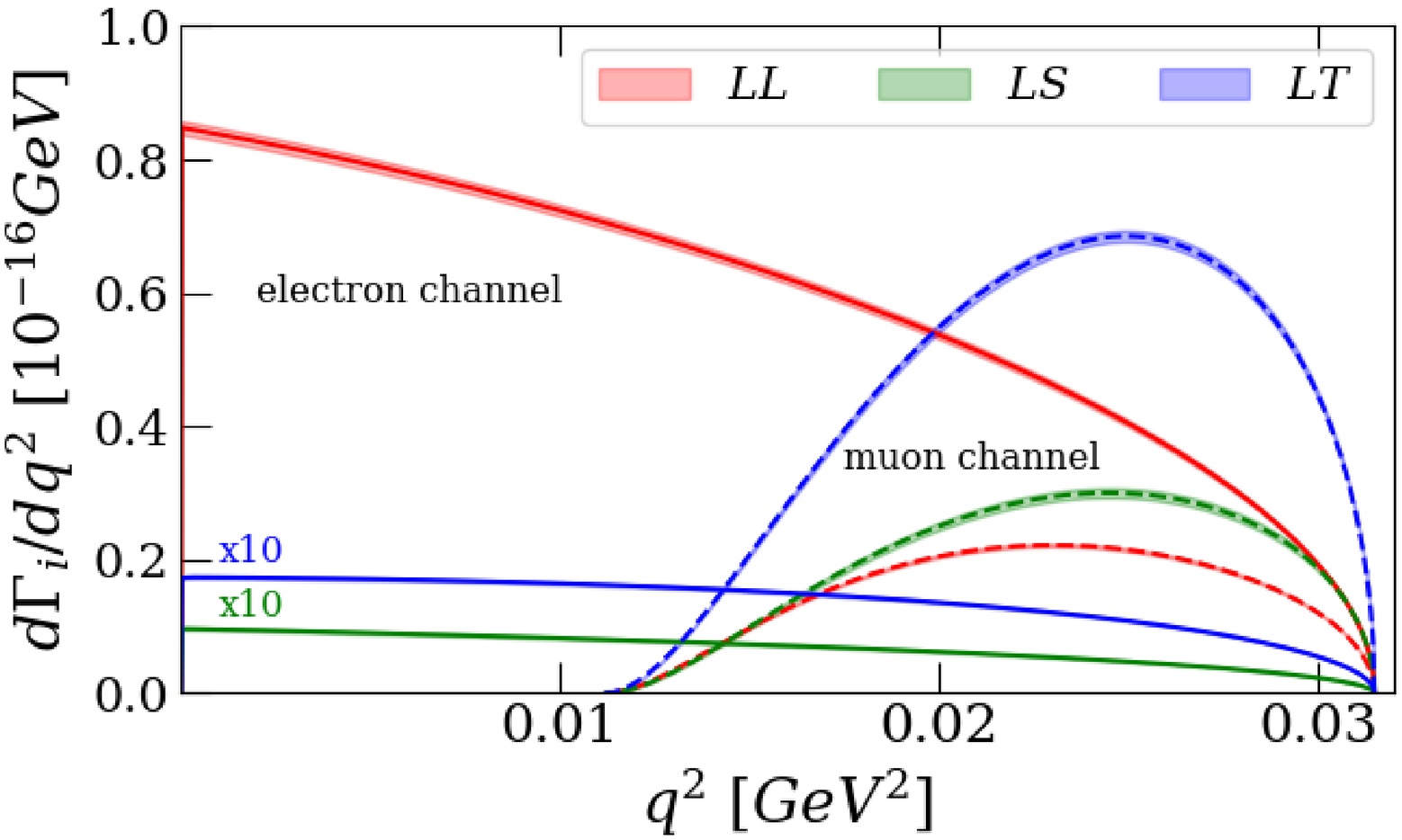


Baryon Masses

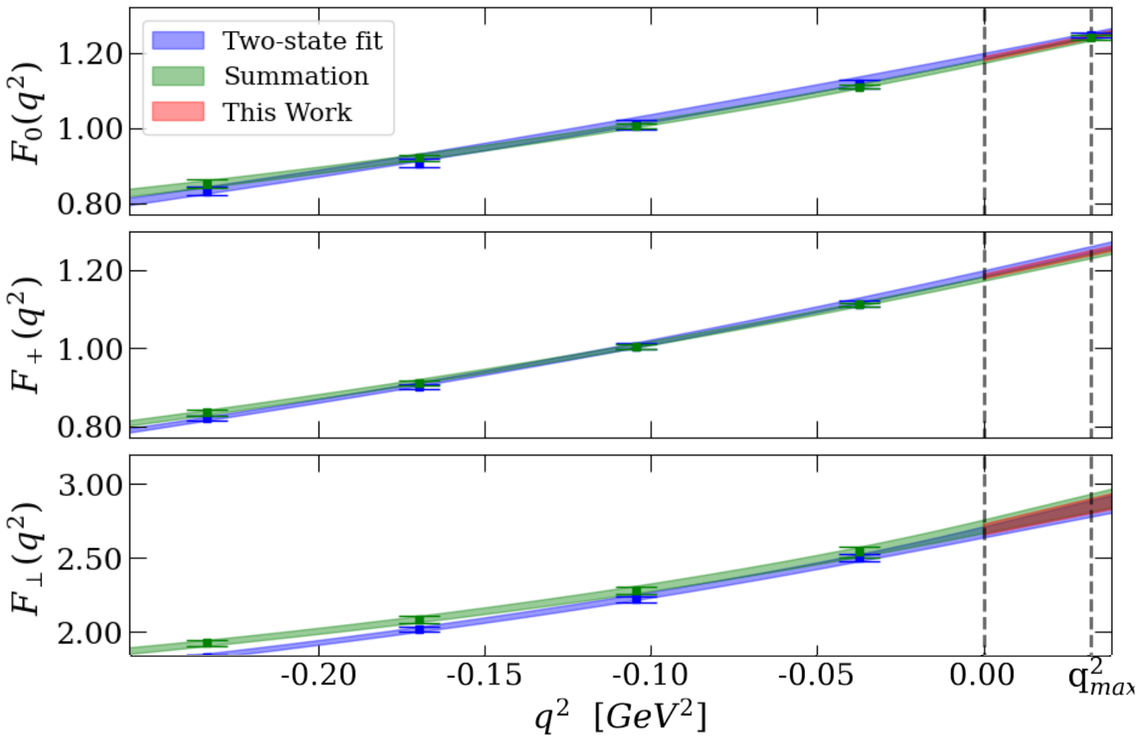


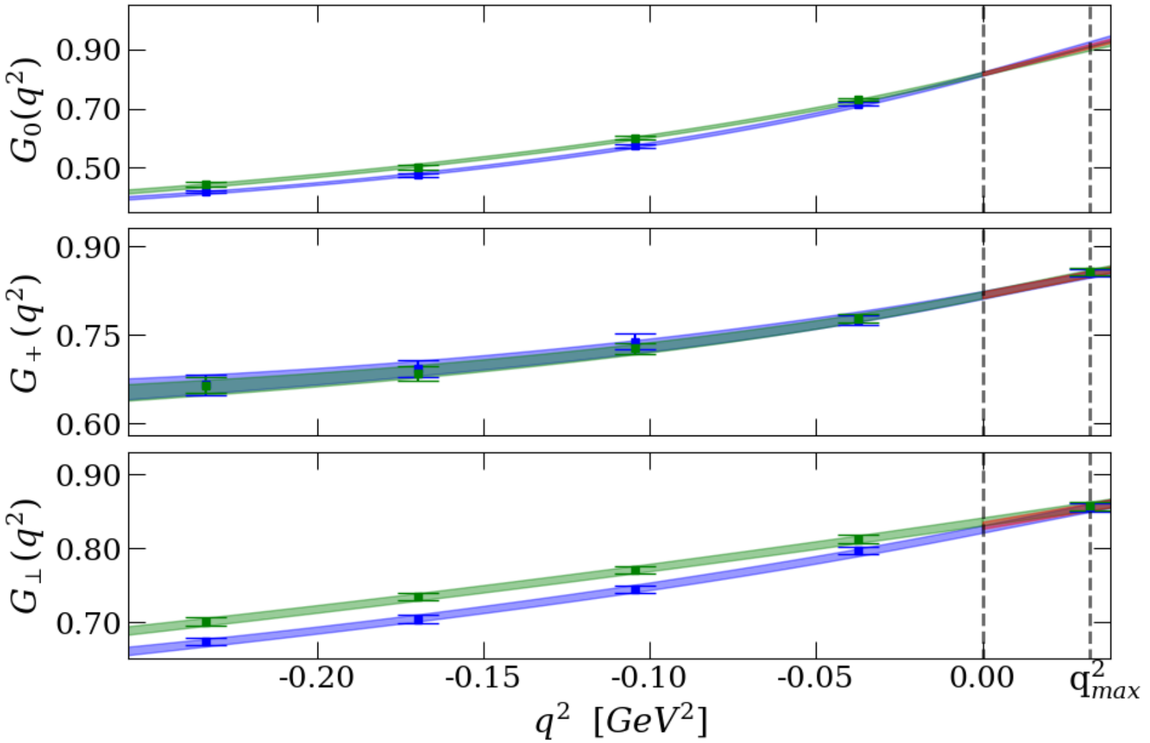


Baryon Masses

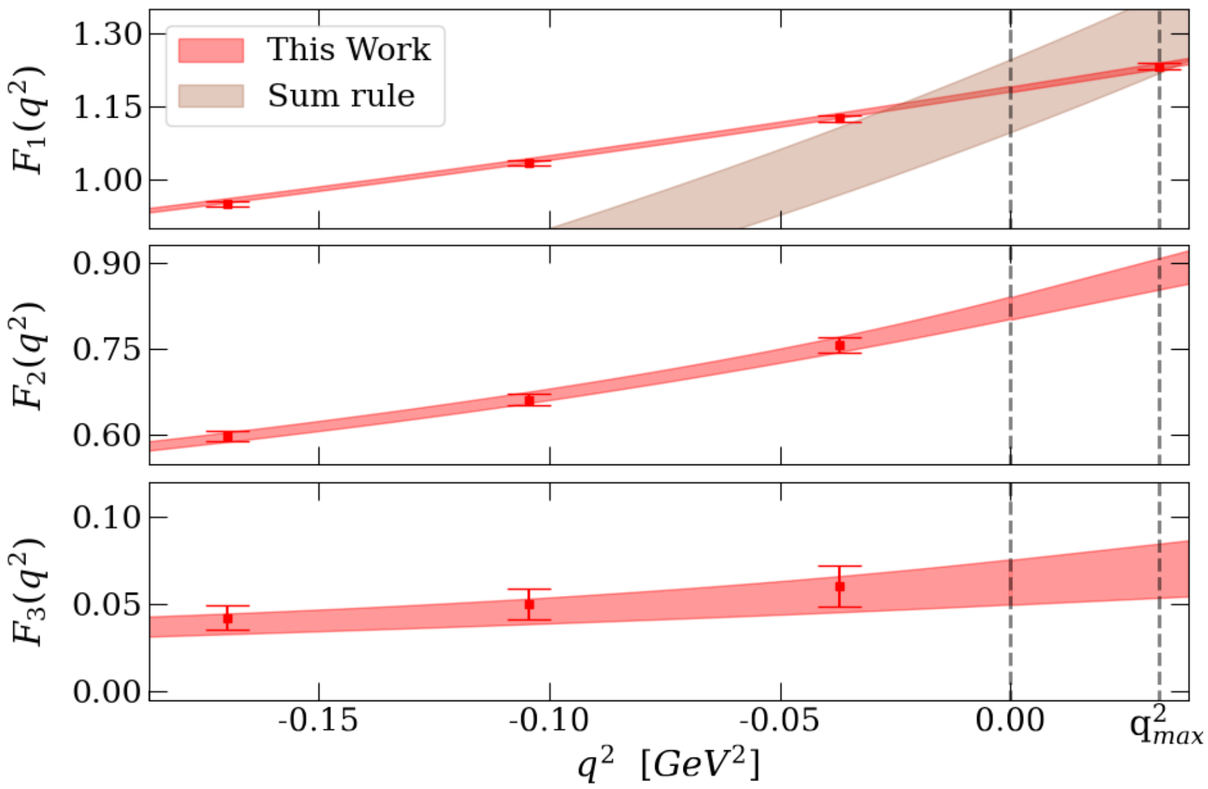


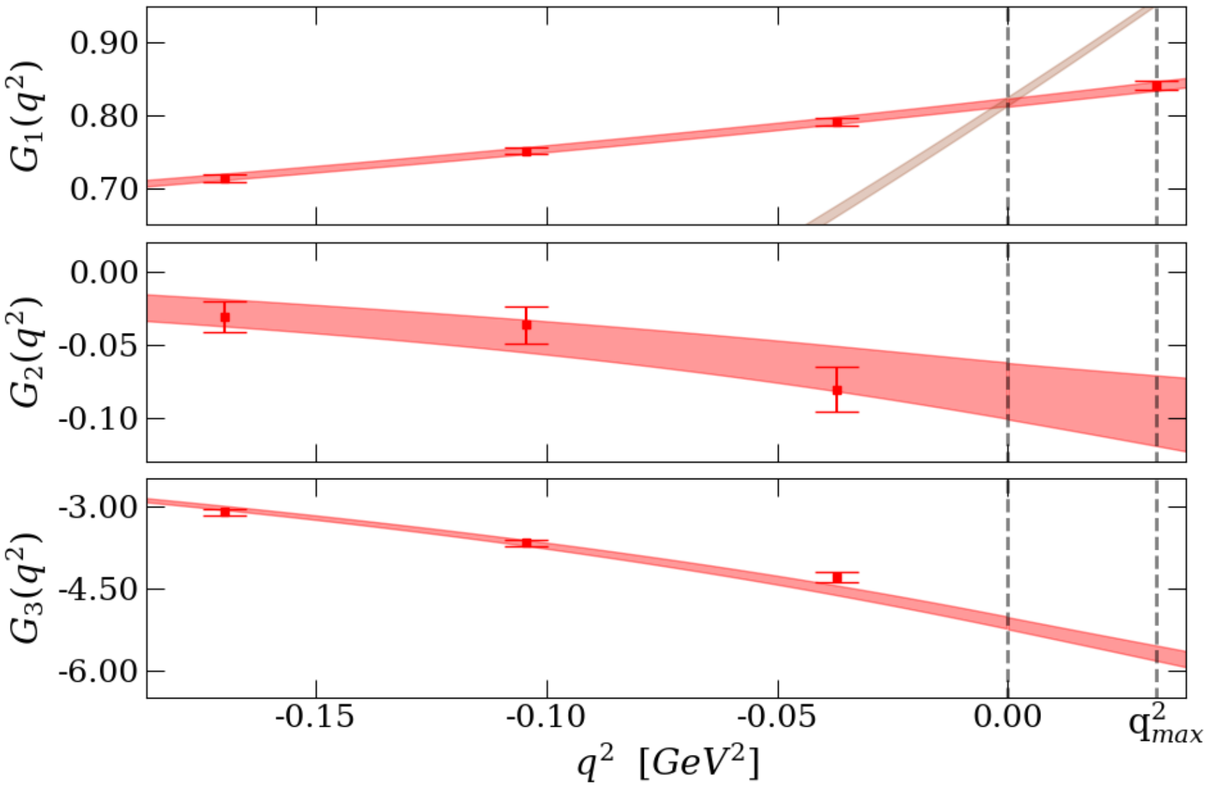
Helicity Form Factors



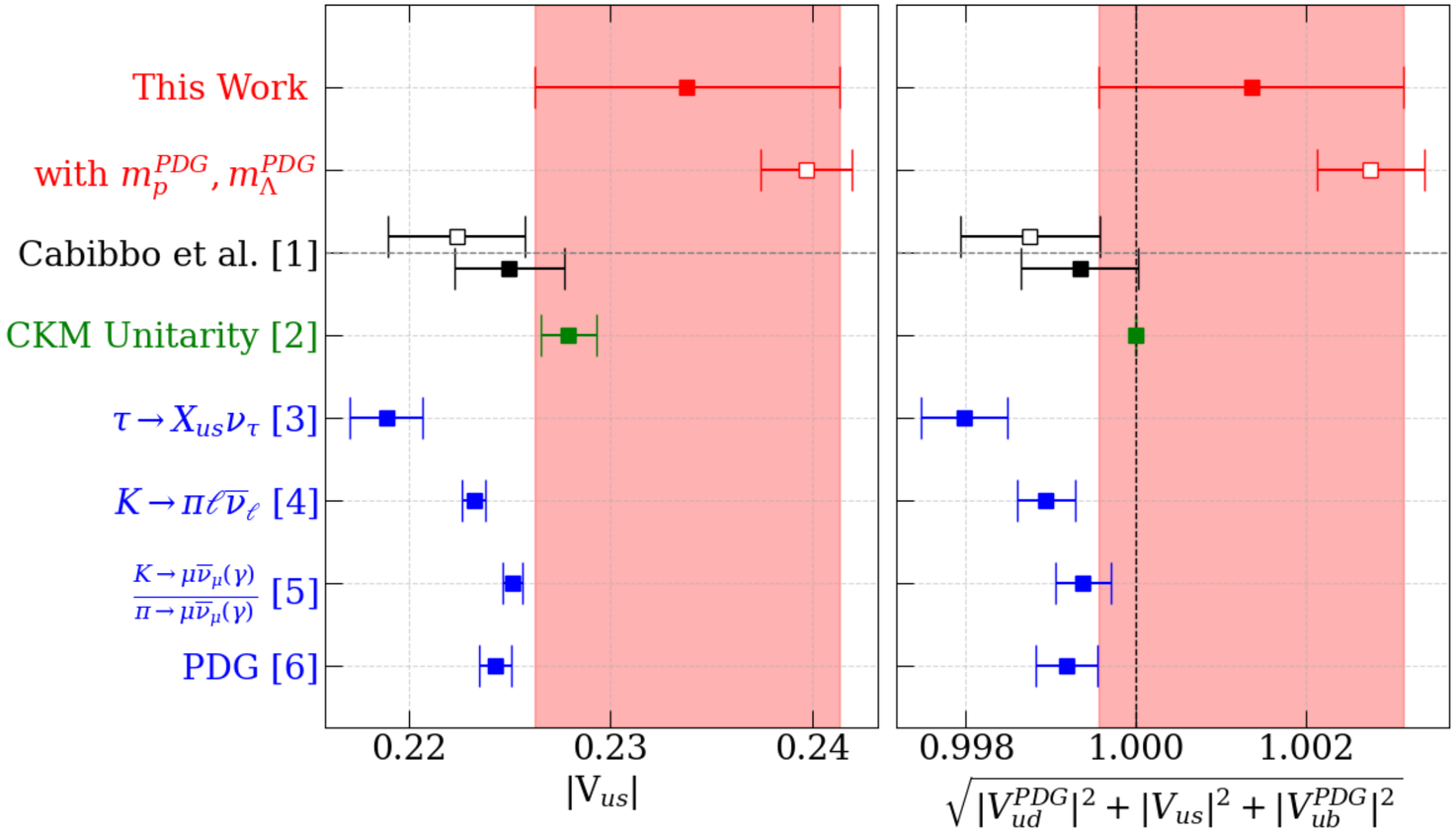


Weinberg Form Factors





|Vus| from LQCD

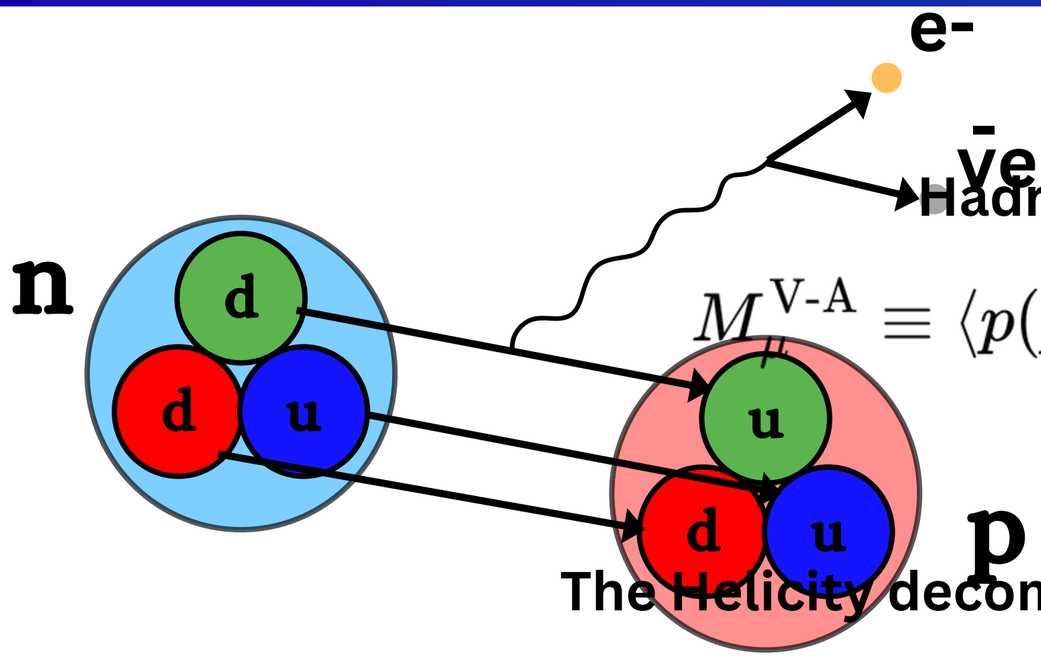


Our estimation of the Vus matrix element is obtained by

$$\mathcal{B}^{\mathrm{PDG}}_{pe\bar{\nu}_e} = 8.34(14)\cdot 10^{-4}$$

$$|V_{us}| \equiv \left(\frac{\Gamma_{pe\bar{\nu}_e}}{|V_{us}|^2} \frac{1}{\Gamma^{\mathrm{PDG}}_{pe\bar{\nu}_e}} \right)^{-1/2}$$

Form Factors



Hadronic matrix element

$$M^{\text{V-A}} \equiv \langle p(p_p, s_p)|\bar{u}\,\Gamma_\mu^{\text{V-A}} s|\Lambda(p_\Lambda, s_\Lambda)\rangle$$

The Helicity decomposition of the matrix elements

$$M_\mu^{\text{V}} = \bar{u}_p \left[\frac{(m_\Lambda - m_p)q^\mu}{q^2}F_0(q^2) + \frac{m_\Lambda + m_p}{s_+}\left(p_\Lambda^\mu + p_p^\mu - \frac{m_\Lambda^2 - m_p^2}{q^2}q^\mu\right)F_+(q^2) + \left(\gamma^\mu - \frac{2m_N p_\Lambda^\mu}{s_+} - \frac{2m_\Lambda p_p^\mu}{s_+}\right)F_\perp(q^2)\right]u_\Lambda\,,$$

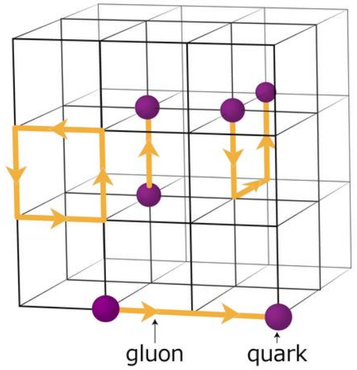
$$M_\mu^{\text{A}} = -\bar{u}_p\gamma_5\left[\frac{(m_\Lambda + m_N)q^\mu}{q^2}G_0(q^2) + \frac{m_\Lambda - m_N}{s_-}\left(p_\Lambda^\mu + p_p^\mu - \frac{m_\Lambda^2 - m_N^2}{q^2}q^\mu\right)G_+(q^2) + \left(\gamma^\mu + \frac{2m_N p_\Lambda^\mu}{s_-} - \frac{2m_\Lambda p_p^\mu}{s_-}\right)G_\perp(q^2)\right]u_\Lambda$$

Content

Motivation

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Lattice QCD



Data Analysis

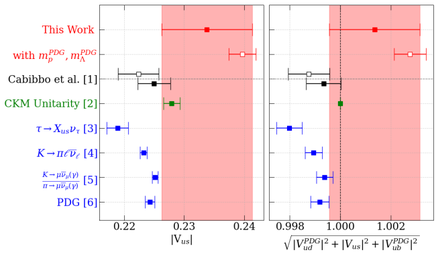
Form Factors

$$\bar{u}_p \Big[\gamma_\mu \left(F_1(q^2) - G_1(q^2)\gamma_5 \right) + -i\sigma_{\mu\nu}q^\nu \frac{F_2(q^2) - G_2(q^2)\gamma_5}{m_\Lambda} + q_\mu \frac{F_3(q^2) - G_3(q^2)\gamma_5}{m_\Lambda} \Big] u_\Lambda$$

Compare to Experimental data



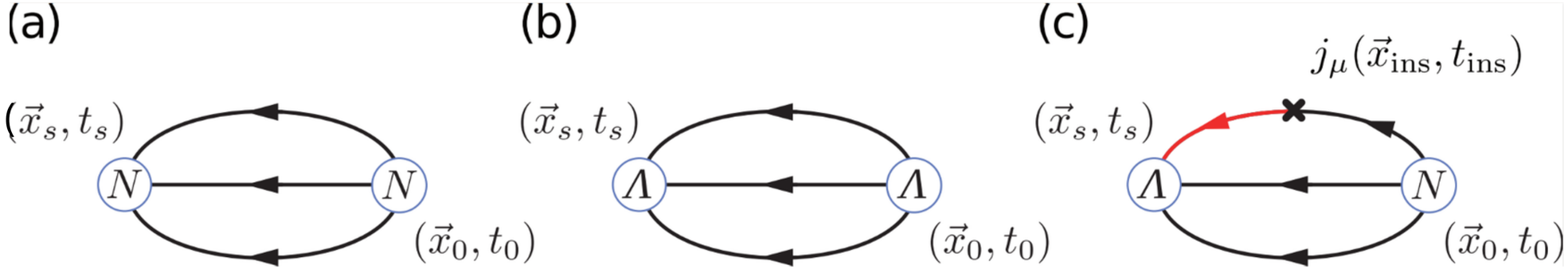
Main Results



Summary & Conclusions

- FFs vs q2
- Decay Rates
- |Vus| value

Two- and Three- point functions



Two-point functions (a & b)

$$C_B(\vec{p}_B, t_s) = \sum_{\vec{x}_s} Tr\left[P_0 \langle \chi_B(x_s)\bar{\chi}_B(0)\rangle e^{-i\vec{x}_s\cdot\vec{p}_B}\right]$$

Three-point function (c)

$$C^{\text{V-A}}_{\mu\nu}(\vec{p}_N, \vec{p}_\Lambda, t_s, t_{\rm ins}) = \sum_{\vec{x}_{\rm ins}, \vec{x}_s} e^{i\vec{x}_{\rm ins}\cdot(\vec{p}_\Lambda - \vec{p}_N) - i\vec{x}_s\cdot\vec{p}_\Lambda}$$

$$\times Tr\left[P_\nu \langle \chi_\Lambda(x_s)\bar{u}(x_{\rm ins})\, \Gamma^{\text{V-A}}_\mu s(x_{\rm ins})\bar{\chi}_N(0)\rangle\right]$$