# Finite-temporal-extent effects in lattice-QCD meson three-point functions

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$\sim 5\%$
shift in the physical-pion momentum fraction

## The issue

Lattice calculations extract hadron structure from three-point correlation functions. Standard analyses usually assume an effectively infinite Euclidean-time direction. On a finite periodic lattice, however, particles may propagate around the temporal boundary, producing additional cyclic time orderings.

These terms become least suppressed when the source–sink separation approaches half of the temporal extent, $t_s \simeq T/2$—precisely where meson data can remain statistically precise.

The effect is not an ordinary forward excited state. It is a distinct finite-$T$ propagation channel with its own spectrum and amplitudes.

## What changes at finite $T$?

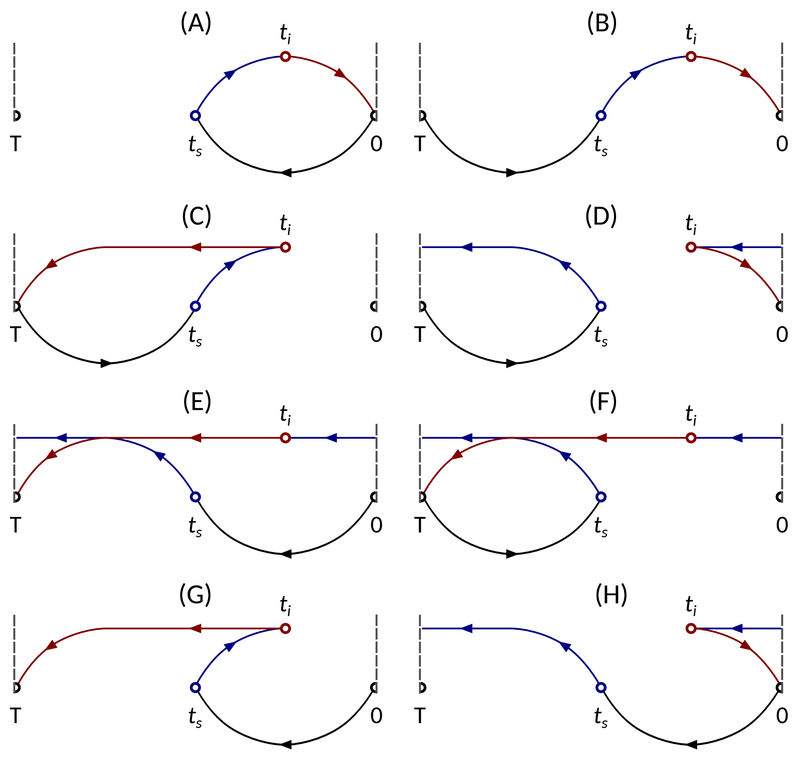



**Figure 1:** Finite-$T$ temporal orderings. A is the usual contribution; C and D are the relevant alternative cyclic orderings, while the remaining terms are more strongly thermally suppressed.

After normalizing by the forward two-point ground state, the fit has the schematic form

$$\widehat{C}_3(t_i, t_s) = \mathcal{M}_{00}\left[\mathcal{F}(t_i, t_s) + e^{-m_H(T-2t_s)}\mathcal{B}(t_i, t_s)\right],$$

where $\mathcal{F}$ contains the conventional forward spectrum and $\mathcal{B}$ contains the finite-$T$ channel,

$$\mathcal{B} \sim \sum_{n,m} \rho^W_{nm} e^{-\Delta^{H,\mathrm{bwd}}_n (T-t_s)} \left[e^{-E^J_m t_i} + e^{-E^J_m (t_s - t_i)}\right].$$

For the pion one-derivative operator, the current can create a low-energy two-pion state, $E^J_0 \simeq 2m_\pi$. At $t_i = t_s/2$, $t_s = T/2$, and $T = 2L$,

$$\frac{C_{\mathrm{bwd}}}{C_{\mathrm{fwd}}} \simeq 2\rho^W_{00} e^{-m_\pi L}.$$

With $m_\pi L \simeq 3.6$–$3.9$ and $|\rho^W_{00}| \sim 1$, a few-percent effect is expected.

## Analysis in one glance

- Three physical-pion ensembles with $a \simeq 0.057$–$0.080\,\mathrm{fm}$ and $L \simeq 5.1$–$5.5\,\mathrm{fm}$.
- One correlated fit describes all source–sink separations and both time orderings with a common ground-state matrix element.
- We average over resolved forward and backward spectral truncations using the Akaike information criterion; uncertainties are propagated with jackknife samples.
- Stability is checked against the source/sink exclusion, the minimum resolvable energy gap, and the two-point fit range.

## Direct evidence in the physical-pion data

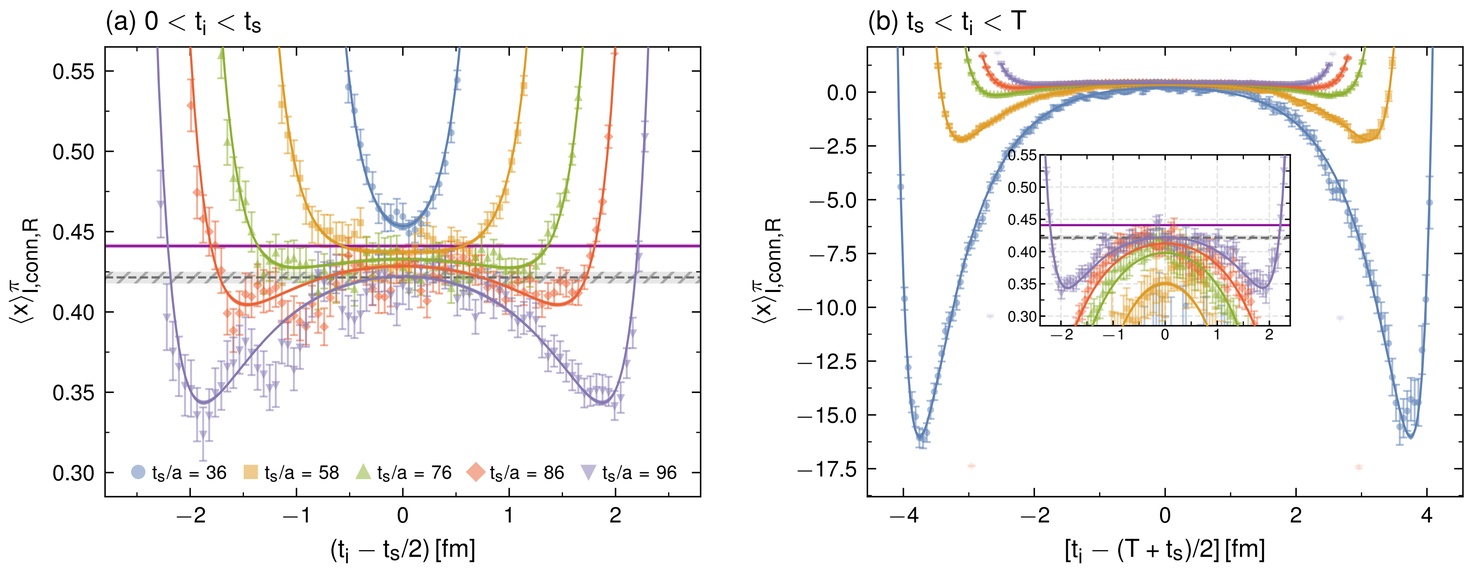



**Figure 2:** D96 pion data and the finite-$T$ fit. A single fit describes both temporal orderings. The purple band is the extracted ground-state momentum fraction; the gray hatched band is the earlier forward-only plateau determination.

On D96, the result changes from $0.4215(35)$ to $0.4411(13)$ when the finite-$T$ terms are included.

## Impact on the continuum result

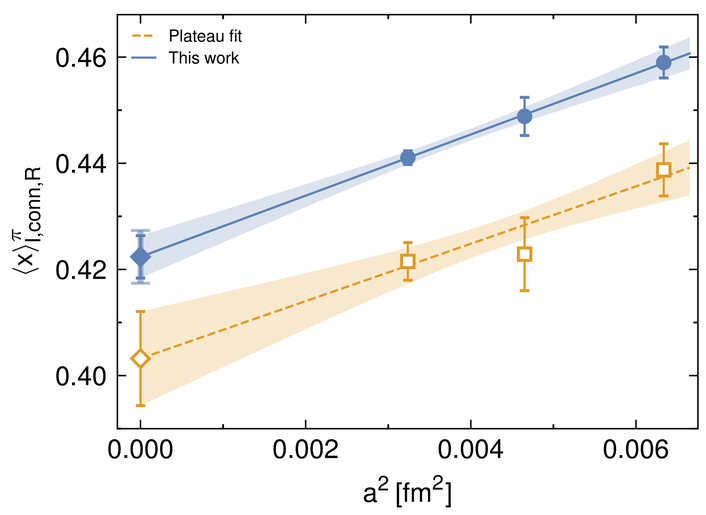



**Figure 3:** Continuum extrapolation of the renormalized connected-light pion momentum fraction.

### Final physical-pion result

$$\langle x \rangle^{\pi}_{l,\mathrm{conn},R} = 0.4224(40)(30)$$

The first uncertainty combines statistics and spectral-model averaging; the second covers residual analysis choices.

The previous forward-only continuum value is $0.4032(89)$.

## What we learn

- Finite temporal extent generates distinct contributions that can survive at the few-percent level near $t_s = T/2$.
- Forward excited states can mimic the shape through cancellations, but the ground-state matrix element becomes unstable.
- The fitted current-channel ground-state energy is consistent with $E^J_0 \simeq 2m_\pi$, supporting its interpretation as a low-energy two-pion state.
- The same insertion-time dependence is also observed at $m_\pi \simeq 265\,\mathrm{MeV}$, so the effect is not restricted to the physical pion mass.
- The present physical-mass kaon data are too noisy to resolve a significant shift.
- The same finite-$T$ analysis can be applied to higher Mellin moments, including matrix elements constructed from Wilson-flowed operators.

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**References:** C. Alexandrou *et al.*, PRL **134**, 131902 (2025); N. Jay and E. Neil, PRD **103**, 114502 (2021).