Probing the Nucleon Mass from Light to Charm

Srijit Paul with *C. Alexandrou, C. Iona, G. Koutsou, Y. Li, G. Spanoudes*

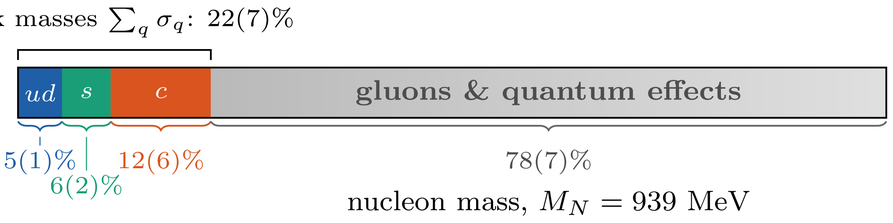
University of Cyprus · AQTIVATE · RESEARCH & INNOVATION FOUNDATION · Extended Twisted Mass Collaboration

Why Sigma Terms?

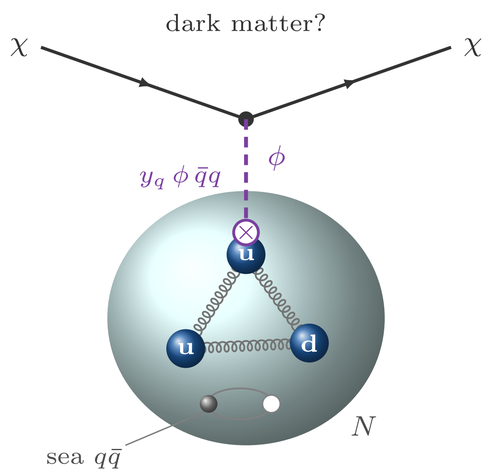
$$\sigma_q = m_q \frac{\partial M_N}{\partial m_q} = m_q \langle N|\bar{q}q|N\rangle$$

m_q alone not enough:
bound-nucleon response = nonperturbative
quark–gluon dynamics ⇒ lattice QCD

quark masses $\sum_q \sigma_q$: 22(7)%



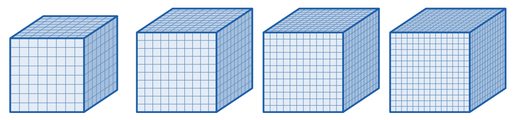
nucleon mass, $M_N = 939$ MeV



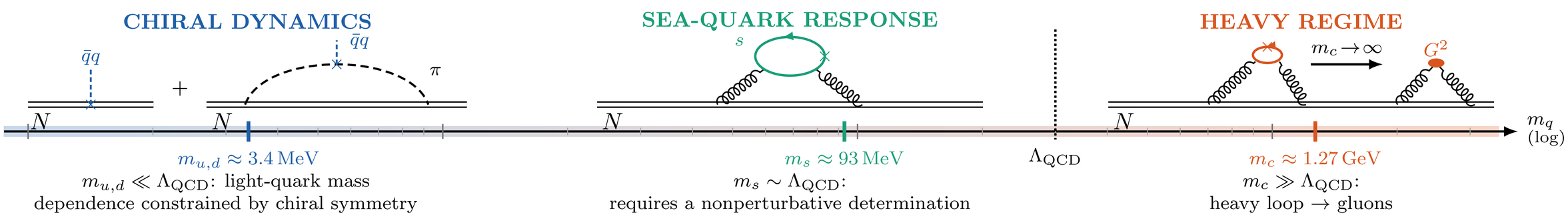
Lattice Setup

ETMC $N_f = 2+1+1$ twisted-mass clover fermions at maximal twist, **physical quark masses**

Ens.	$L^3 \times T$	a [fm]	L [fm]	N_{cfg}
B64	$64^3 \times 128$	0.0795	5.09	725
C80	$80^3 \times 160$	0.0682	5.46	400
D96	$96^3 \times 192$	0.0569	5.46	493
E112	$112^3 \times 224$	0.0489	5.48	502



From Light → Strange → Charm

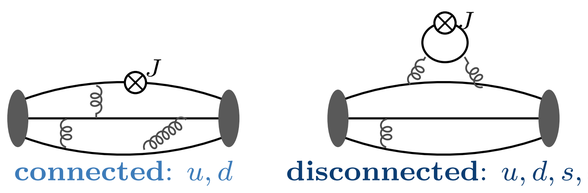


$m_{u,d} \ll \Lambda_{\text{QCD}}$: light-quark mass dependence constrained by chiral symmetry

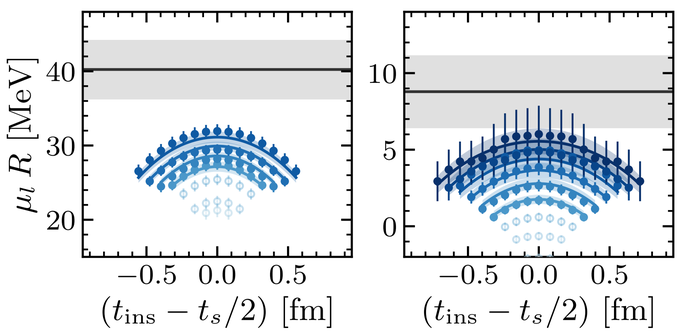
$m_s \sim \Lambda_{\text{QCD}}$: requires a nonperturbative determination

$m_c \gg \Lambda_{\text{QCD}}$: heavy loop → gluons

Excited States: The Central Challenge



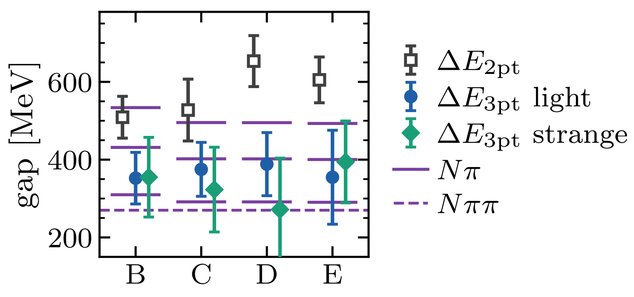
connected: u, d **disconnected**: u, d, s, c



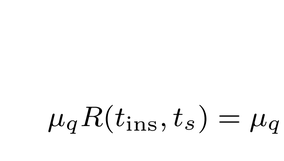
fitted gaps vs. spectrum

$\Delta E_{3\text{pt}} \approx 350$–$390$ MeV at the $N\pi$ / $N\pi\pi$ levels

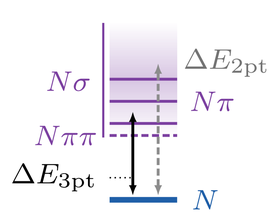
$\Delta E_{2\text{pt}} \approx 510$–$650$ MeV, higher



$$\mu_q R(t_{\text{ins}}, t_s) = \mu_q \times \frac{C_{3\text{pt}}}{C_{2\text{pt}}} = \frac{\sigma_X + a_X\left(e^{-\Delta E_{3\text{pt}}\, t_{\text{ins}}} + e^{-\Delta E_{3\text{pt}}(t_s - t_{\text{ins}})}\right) + c_X\, e^{-\Delta E_{3\text{pt}}\, t_s}}{1 + c_1\, e^{-\Delta E_{2\text{pt}}\, t_s}}$$



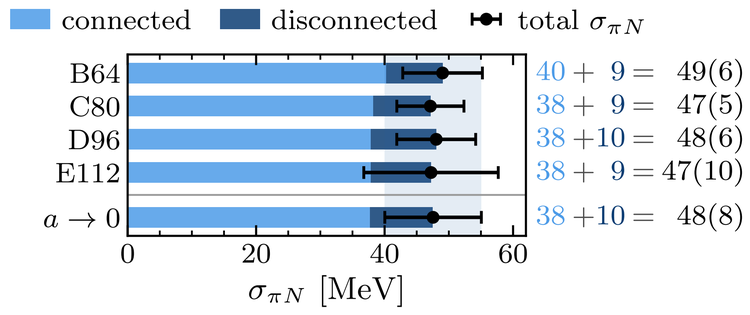
states



σ_X: ground state, X = conn., disc.
$\Delta E_{3\text{pt}}$: **free**, shared by conn. & disc.
c_1, $\Delta E_{2\text{pt}}$: from the 2pt fit
both gaps *effective*: averages over the tower

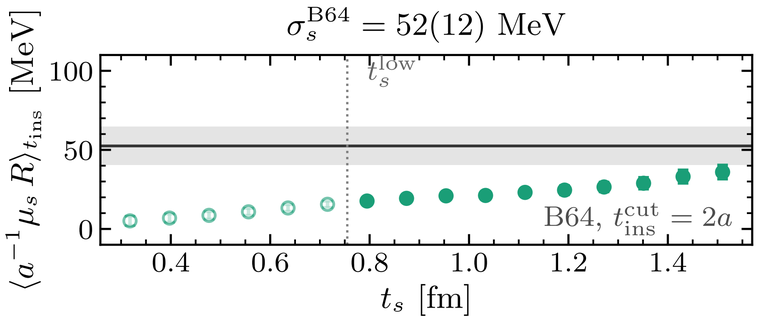
fits with **no priors**; filled points: in fit

Light: $\sigma_{\pi N}$



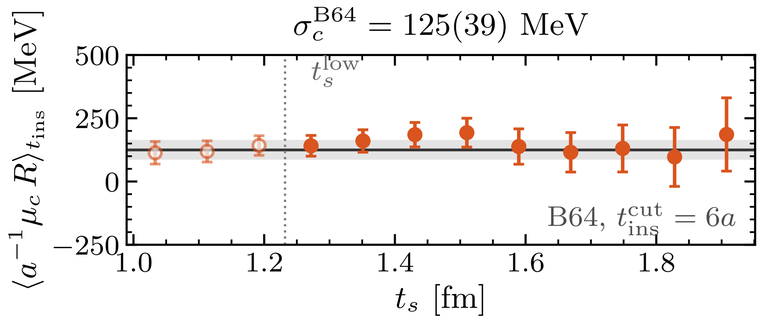
disconnected part ≈ 20% of $\sigma_{\pi N}$

Strange: σ_s



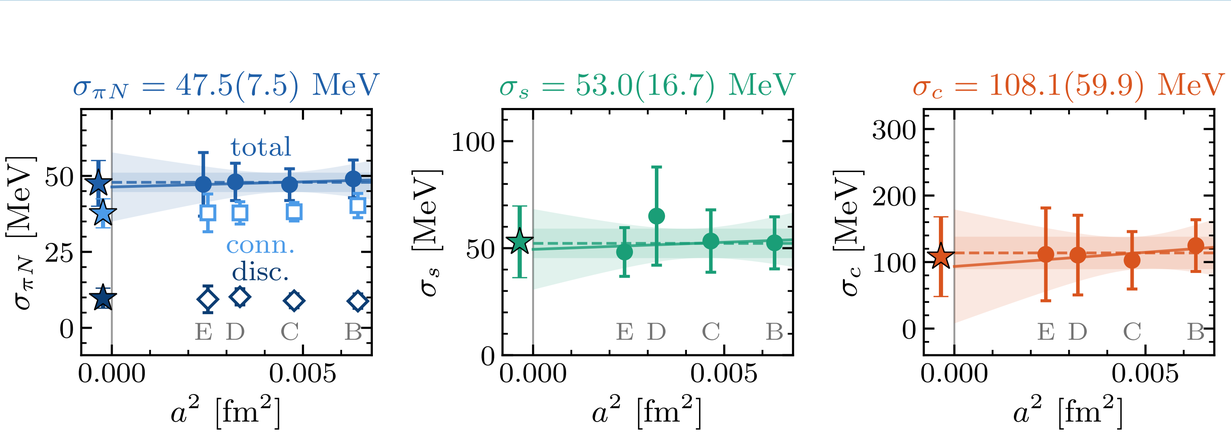
disc. only: strong curvature, two-state fit

Charm: σ_c

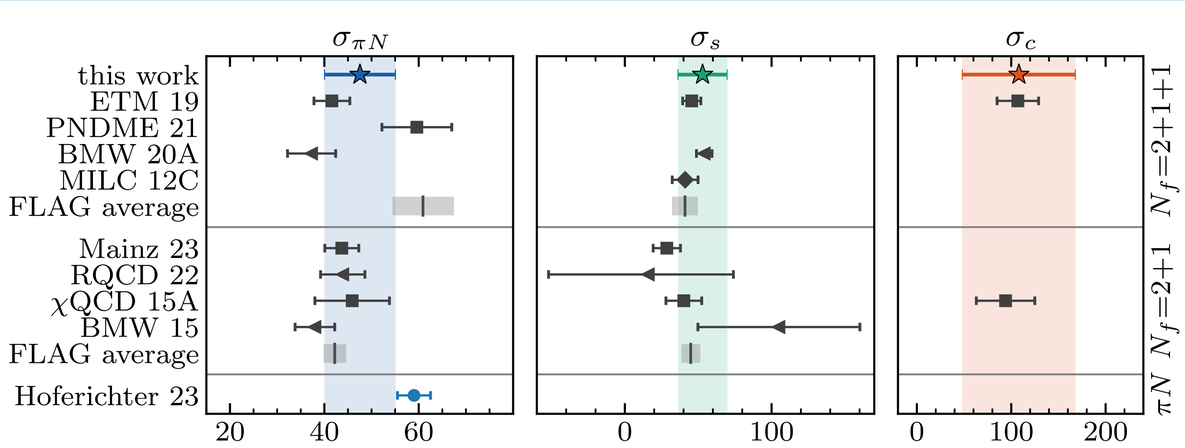


disc. only: flat in t_s, constant fit

Continuum Limit & Comparison



AIC average of constant and linear fits in a^2; ⋆: $a^2 = 0$



■ direct, ◀ Feynman–Hellmann, ♦ hybrid, grey: FLAG 2024, • πN

Take-Home

One calculation: **chiral physics → sea-quark physics → heavy-quark dynamics**: $\sigma_{\pi N} = 47.5(7.5)$, $\sigma_s = 53(17)$, $\sigma_c = 108(60)$ MeV.

References

[1] C. Alexandrou *et al.*, arXiv:2412.01535 (AIC procedure); [2] ETMC ensembles, arXiv:1807.00495; [3] ETMC, arXiv:2411.08852 (bare s, c quark masses); [4] FLAG Review 2024, arXiv:2411.04268; [5] ETM 19, arXiv:1909.00485; [6] PNDME 21, arXiv:2105.12095; [7] BMW 20A, arXiv:2007.03319; [8] MILC 12C, arXiv:1204.3866; [9] Mainz 23, arXiv:2303.08741; [10] RQCD 22, arXiv:2211.03744; [11] χQCD 15A, arXiv:1511.09089 (σ_c: χQCD 13A, arXiv:1304.1194); [12] BMW 15, arXiv:1510.08013; [13] Hoferichter 23, arXiv:2305.07045.