Spin and momentum fraction carried by partons in the nucleon

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A complete parton decomposition at physical quark masses and in the continuum

$$\langle x\rangle_N = 0.995(60)(29) \qquad J_N = 0.507(43)(65)$$

Both momentum and spin sum rules are satisfied within uncertainties.

From form factors to proton spin

How do quarks and gluons share the proton's momentum and spin? Matrix elements of the QCD energy-momentum tensor give the gravitational form factors A_{20} and B_{20}.

$$\langle x\rangle_i = A^i_{20}(0), \qquad J_i = \tfrac{1}{2}\big[A^i_{20}(0) + B^i_{20}(0)\big],$$

where $i = u, d, s, c, g$. Quark angular momentum separates into intrinsic spin and orbital motion:

$$J_q = \tfrac{1}{2}\Delta\Sigma_q + L_q, \qquad \tfrac{1}{2}\Delta\Sigma_q = \tfrac{1}{2}g^q_A.$$

The momentum and spin sum rules are checked without imposing their values:

$$\sum_{i=u,d,s,c,g} \langle x\rangle_i = 1, \qquad \sum_{i=u,d,s,c,g} J_i = \frac{1}{2}.$$

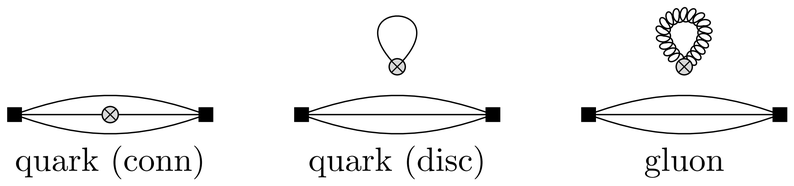
Four physical-point ensembles

ETMC $N_f = 2+1+1$ twisted-mass clover-improved fermions, with automatic $\mathcal{O}(a)$ improvement and similar physical volumes.

Ensemble	$L^3/a^3 \times T/a$	a [fm]	m_π [MeV]
B64	$64^3 \times 128$	0.07948	140.2
C80	$80^3 \times 160$	0.06819	136.7
D96	$96^3 \times 192$	0.05685	140.8
E112	$112^3 \times 224$	0.04892	136.5

$L \simeq 5.1$–5.5 fm; $m_\pi L = 3.62$–3.90. **The continuum limit is taken directly at physical quark masses.**

All quarks and gluons included



Connected quarks Disconnected quarks Gluons

- Extract matrix elements from two- and three-point functions; control excited states with two-state or constant fits, checked against the summation method.
- Determine $A_{20}(0)$ directly; obtain $B_{20}(0)$ by extrapolating finite-Q^2 data.
- Take constant and linear fits in a^2, combined using Akaike information criterion (AIC) model averaging.

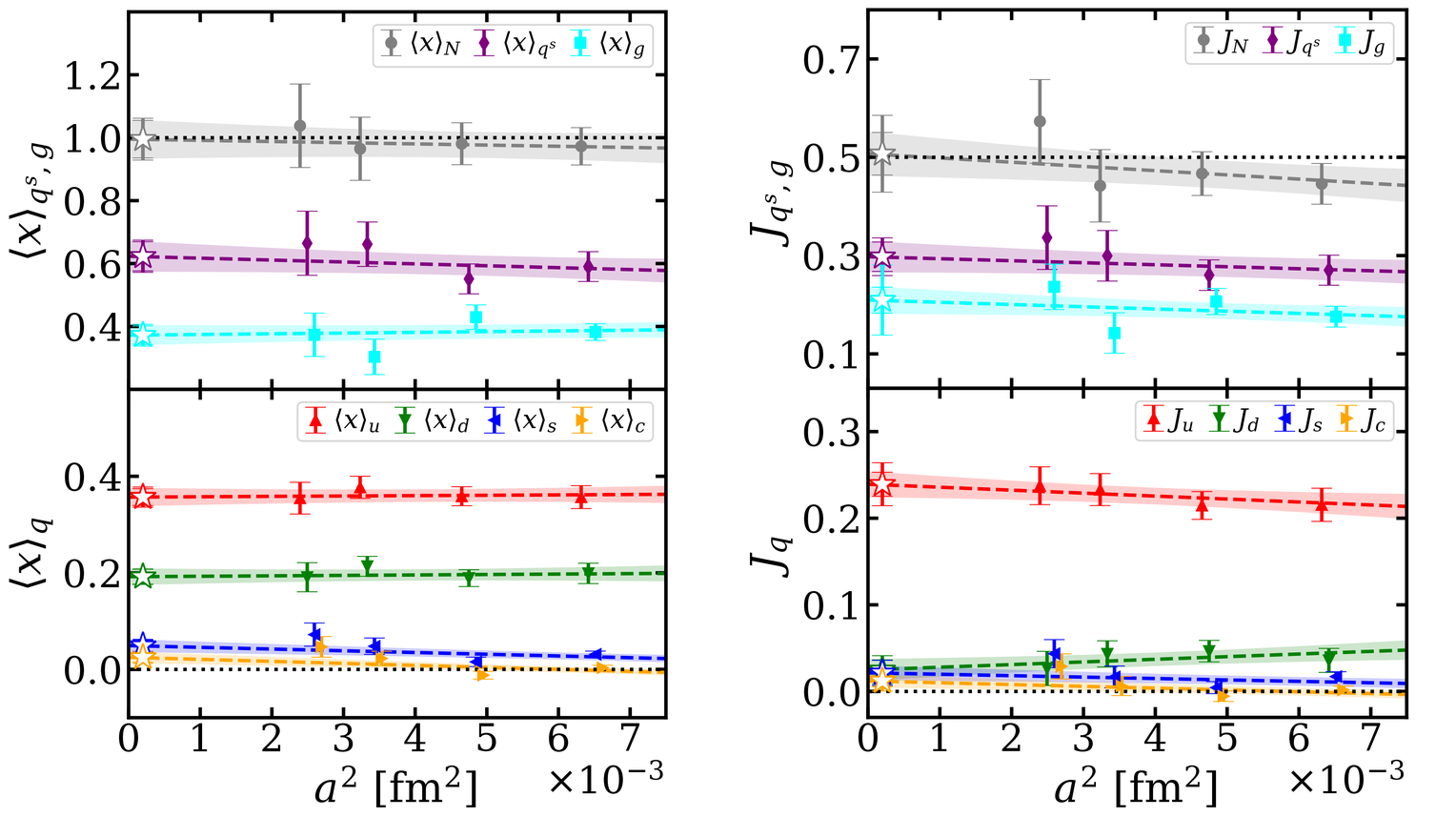
Non-perturbative mixing

The quark singlet $q^s = u + d + s + c$ mixes with the gluon:

$$\begin{pmatrix} T^R_{q^s} \\ T^R_g \end{pmatrix} = \begin{pmatrix} Z^s_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} \begin{pmatrix} T_{q^s} \\ T_g \end{pmatrix}.$$

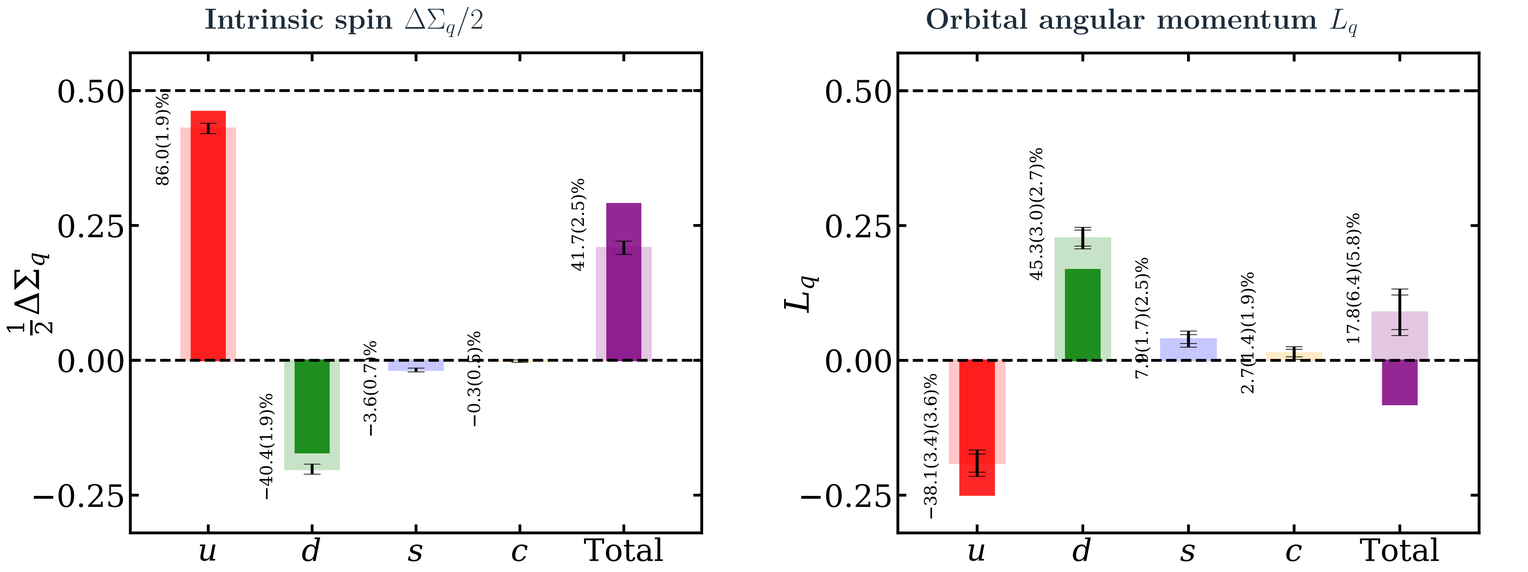
All four coefficients are determined non-perturbatively in RI′-MOM, then perturbatively matched to $\overline{\text{MS}}$ at 2 GeV. Systematic checks vary the excited-state analysis, Q^2 fit range and gluon stout smearing. The disconnected excited-state analysis is a major uncertainty in J_g.

Continuum limit at the physical pion mass



Four lattice spacings: $a \simeq 0.080, 0.068, 0.057, 0.049$ fm. Stars mark the continuum results; bands show AIC-averaged constant and linear fits in a^2. Upper panels: quark singlet, gluon and total. Lower panels: individual quark flavors.

Momentum and angular momentum in the continuum



Outer light bars: full contributions. Inner dark bars: connected quarks. $q^s = u + d + s + c$. Percentages for J refer to the proton spin 1/2. All results are in $\overline{\text{MS}}$ at 2 GeV.

Intrinsic quark spin and orbital angular momentum

L_q is derived as $J_q - \Delta\Sigma_q/2$, using axial charges from the same ensembles. Negative intrinsic down-quark spin is largely canceled by positive L_d, giving a small J_d. Percentages are relative to 1/2.

Conclusions: about half the proton spin comes from sea quarks and gluons

- **First complete determination in the continuum using only physical-quark-mass ensembles:** fully non-perturbative quark–gluon renormalization and four lattice spacings yield both momentum and spin sums directly from QCD.
- **Valence contributions account for only about half the momentum and spin.** Gluons contribute approximately 40% and sea quarks approximately 10%, demonstrating the substantial non-valence origin of proton spin.
- **Strange and charm quarks contribute:** small, statistically nonzero strange contributions appear in all four quantities; the charm momentum fraction is $\langle x\rangle_c = 0.025(10)(7)$.

Papers and uncertainties

[1] C. Alexandrou et al., *Spin and momentum fraction carried by partons in the nucleon*, arXiv:2607.21230.

[2] C. Alexandrou et al., *Nucleon unpolarized second Mellin moments using lattice QCD ensembles with physical quark masses and in the continuum limit*, arXiv:2607.20337.

First error: statistics and continuum AIC model averaging. Second: combined excited-state, Q^2-range and stout-smearing systematics. Axial-charge results have one quoted error. Inner/outer error bars show the first/total uncertainty.

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