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TECHNISCHE UNIVERSITÄT BERLIN

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Funded by the European Union
Grant Agreement No: 101072344

Generalized Parton Distribution Functions of the Nucleon from Twisted Mass Fermions with Physical Pion Masss

Constantia Alexandrou[1,2], Simone Bacchio[1], Minhuan Chu[3], Krzystof Cichy[3], Martha Constantinou[4], Giannis Koutsou[1], Joshua Miller[4], Gabriele Pierini[1,5]

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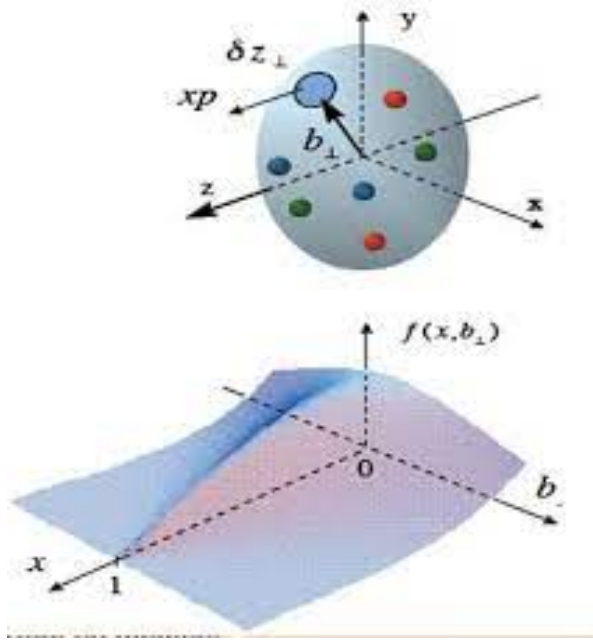
What will I be showing?

- PDFs on lattice;
- Lattice setup;
- Inverse problem: from z space to x space;
- Moments of the PDF;
- Light-cone isovector unpolarized PDF;
- GPDs on lattice;
- Moments of the GPDs;
- Light-cone isovector unpolarized GPDs;

Why GPDs?

- PDFs, and their generalization GPDs, are key quantities for hadron dynamics and for 3D imaging of hadrons.



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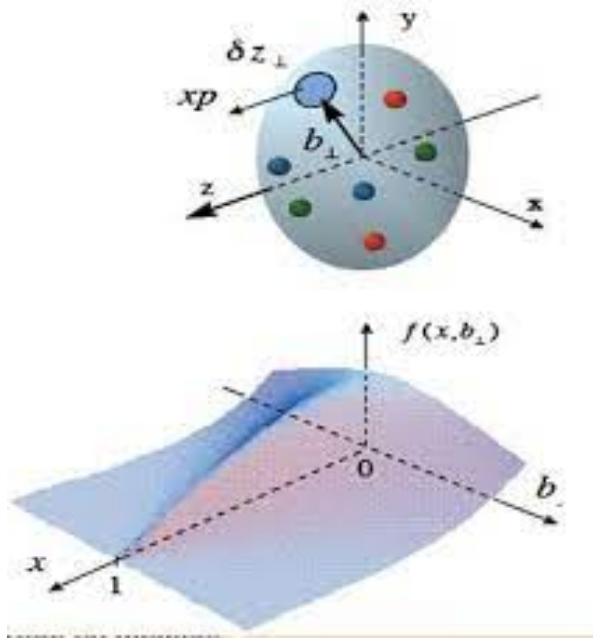

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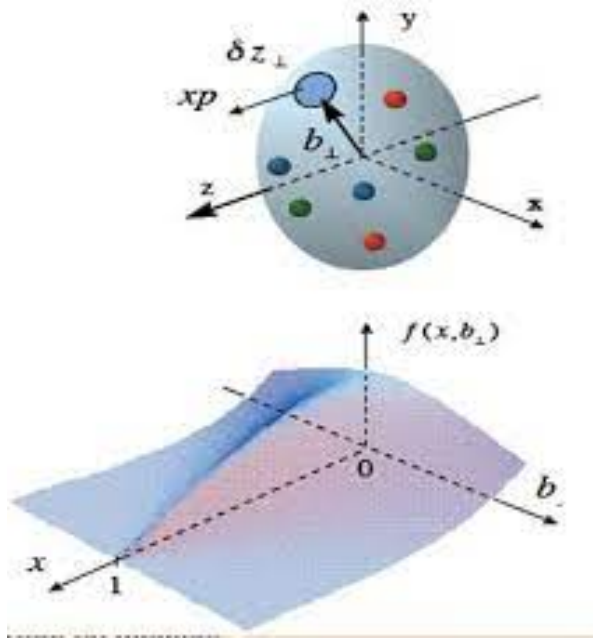
- Non-peturbative quantities, hard to get experimentally;
- Non-perturbative approach to study them: Lattice QCD.



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LaMET approach:

$$F^0(z,P) = \langle N(P)\left|\bar{\psi}(z)\gamma^0\mathcal{W}(z,0)\psi(0)\right|N(P)\rangle = \bar{u}(p,\lambda')\left[\frac{P^0}{m}\mathcal{H}(z)\right]u(p,\lambda)$$

From $\mathcal{H}(z)$ we can reconstruct the light-cone $F^+(x)$ $(H(x))$ PDF.

PDFs can be generalized to GPDs [Bhattacharya et al., arXiv:2209.05373v2]: second part of the talk.

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Compared to local operators, we can access moments higher than the fourth one without Gradient Flow. Can also access anti-quark distribution.

Lattice setup

Ensemble	a [fm]	$(L/a)^3 \times T/a$	m_π [GeV]	L [fm]
cB211.072.64	0.07957(13)	$64^3 \times 128$	140.2(2)	5.093(8)

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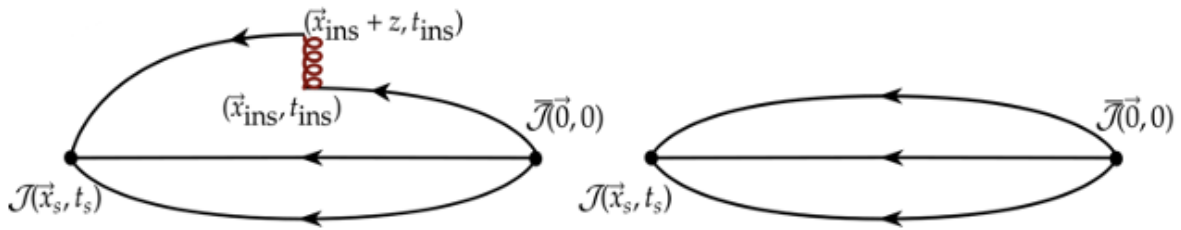
N_{conf}	652	709					
Boosts P_z [GeV]	0	0.24	0.48	0.72	1.21	1.45	1.69
N_{src}	16	1	2	4	12	24	48
Optimized momentum smearing $\vec{\Delta}$ at P_i	$(0, 0, P_z)$				$(2 \cdot 2\pi/L, 0, P_z)$		



Matrix element on lattice QCD

Appropriate ratio of three-point over two-point functions.

For $P_z = 0, 0.24, 0.48, 0.72$ GeV:

$$R(\Gamma_0, \vec{p}; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, \vec{p}; t_s, t_{ins})}{C^{2pt}(\Gamma_0, \vec{p}, \Delta_0; t_s)}$$

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Plateau fit to extract the Matrix element.

Excited states

For $P_z = 1.21$ GeV we computed the three-point function at $t_s = 0.64,\ 0.8,\ 0.96$ fm.

$C^{2pt}(\Gamma_0, \vec{p}; t_s) = \sum_n c_n(\vec{p}) e^{-E_n(\vec{p}) t_s}$

$C^{3pt}(\Gamma_0, \vec{p}; t_s, t_{ins}) = \sum_{i,j} A_0^{ij} e^{-E_i(\vec{p})(t_s - t_{ins}) - E_j(\vec{p})(t_{ins})}$

Two-state analysis

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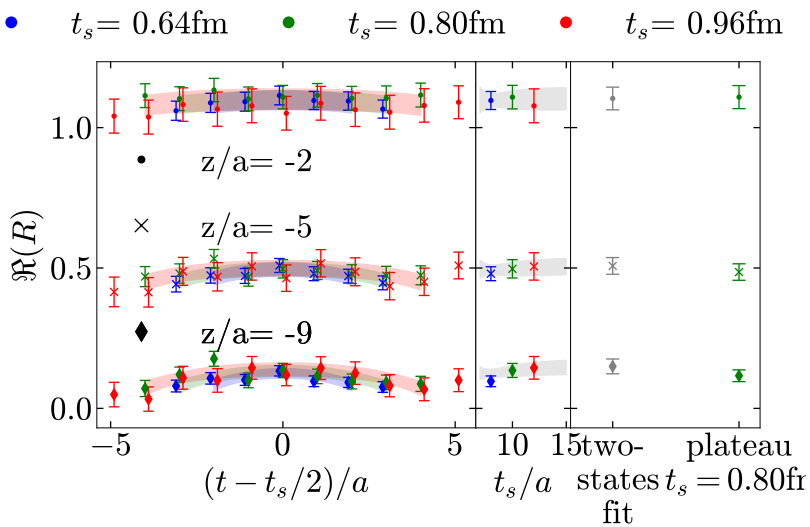
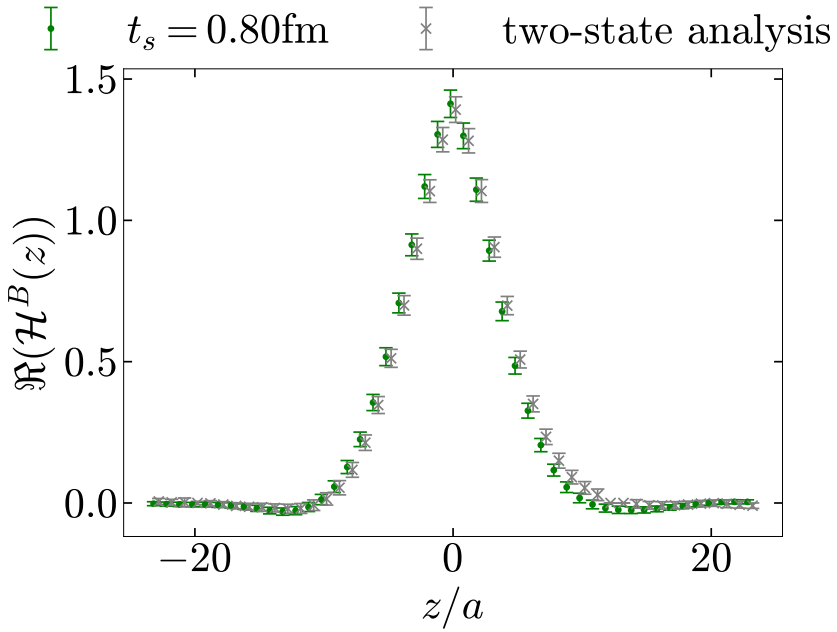
Two-state analysis

$$\Pi^{\mu}(\Gamma_0, \vec{p}) = \frac{A_0^{00}(\vec{p})}{\sqrt{c_0(\Delta) c_0(\Delta_0)}}$$





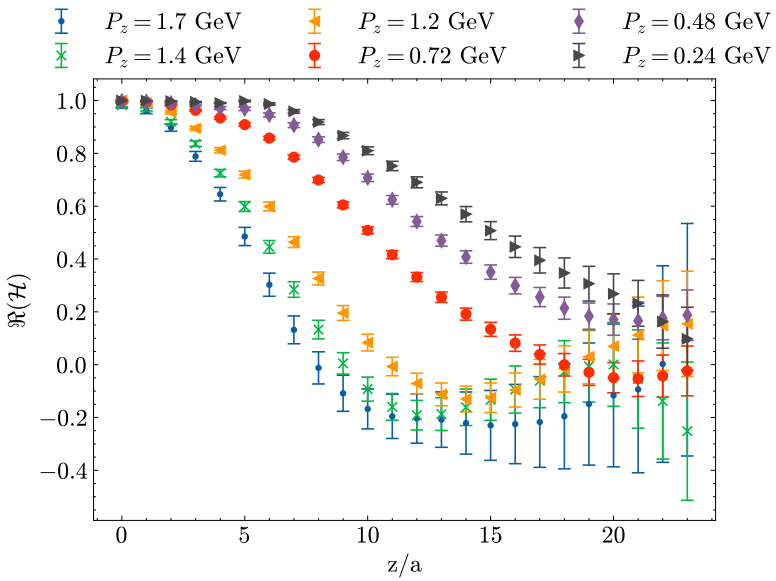
Renormalization

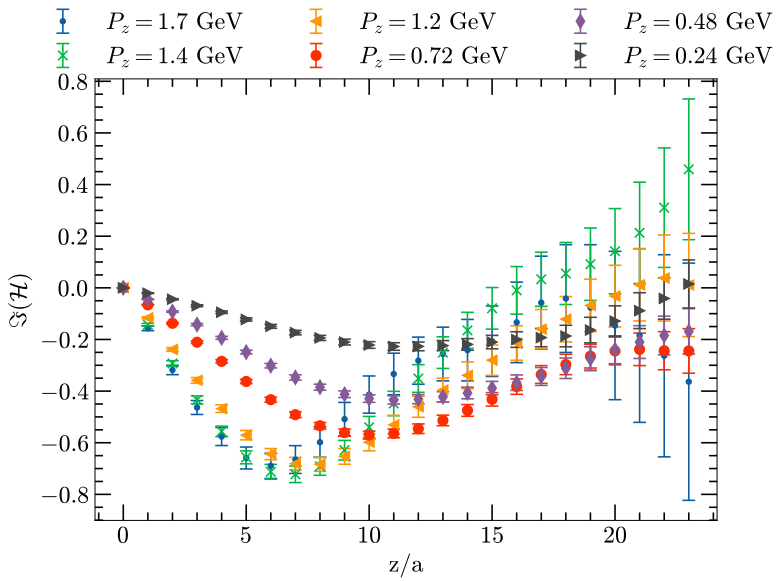
Latest developments in renormalization: Hybrid renormalization [Ding et al., arXiv:2407.03516]

$$\mathcal{H}_{ren}(z, P_z) = \frac{\mathcal{H}_{bare}(z, P_z)}{\mathcal{H}_{bare}(z, 0)} \qquad z \leq z_s$$

$$\mathcal{H}_{ren}(z, P_z) = e^{\delta m (z - z_s)} \frac{\mathcal{H}_{bare}(z, P_z)}{\mathcal{H}_{bare}(z_s, 0)} \qquad z > z_s,$$

where $z_s = 4a$ and $\delta m \cdot a = 0.2660(3)(58)(98)$.





x-reconstruction: Bayes-Gauss-Fourier technique

$$\mathcal{H}(x, P_z, \mu) = 2P_z \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixP_z z}\, \mathcal{H}(z, P_z, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_z^2}, \frac{M_N^2}{P_z^2}\right)$$

We studied both Backus-Gilbert (BG) and a newer approach: Bayes-Gauss-Fourier transformation technique (a gaussian process, GP) [Alexandrou et al., arXiv:2007.13800]. How does it work?

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- Fit ρ with a proper prior function

$$\mu_P(z; I', \sigma', \bar{z}) = I'^2 e^{-\frac{(|z|-\bar{z})^2}{2\sigma'^2}}$$

x-reconstruction: Bayes-Gauss-Fourier technique

- Given a covariance function

$$k_P(z,z') = I^2 e^{-\frac{(z-z')^2}{2\sigma^2}}, \ \tilde{K}_{ij} = k_P(z_i,z_j) + \Delta h_i^2 \delta_{ij}$$

whose parameters are chosen such that they maximize

$$p(\rho) = det(2\pi k_P)^{-1/2} e^{-\frac{1}{2}\sum_{ij}(\rho(z_i)-\mu_P(z_i))k_{Pij}^{-1}(\rho(z_j)-\mu_P(z_j))}$$

x-reconstruction: Bayes-Gauss-Fourier technique

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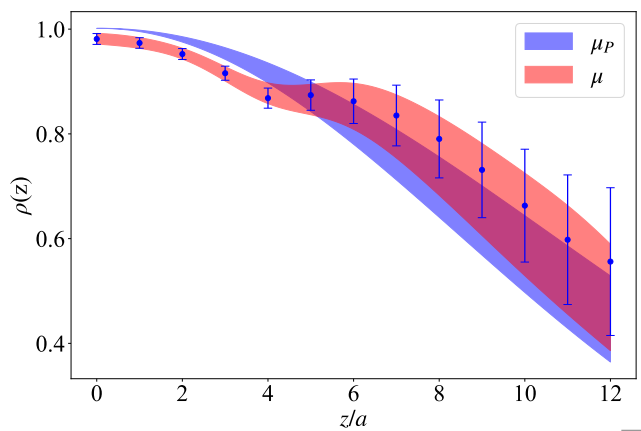
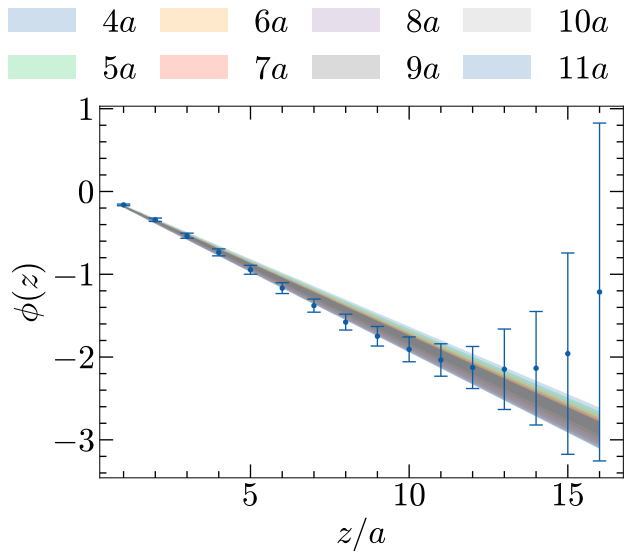
- we can get

$$\mu(z) = \mu_P(z) + \sum_{i,j} k_P(|z|,z_i)\tilde{K}^{-1}_{ij}(\rho(z_j) - \mu_P(z_j))$$



x-reconstruction: Bayes-Gauss-Fourier technique

- The phase $\phi(z)$ is parametrized with a linear function



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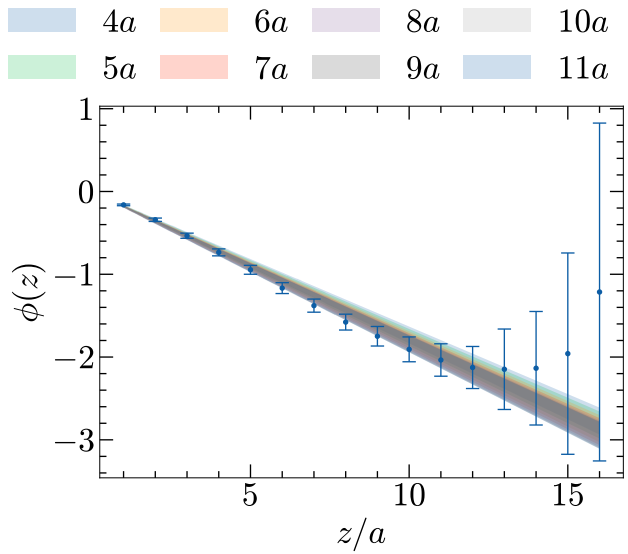

- The reconstructed $\mathcal{H}_{GP}(z)$ is thus

$$\mathcal{H}_{GP}(z) = \mu(z)e^{i\theta z}$$

$\mathcal{H}(x)$ can be now computed in a closed form by doing the Fourier-Transform of $\mathcal{H}_{GP}(z)$.

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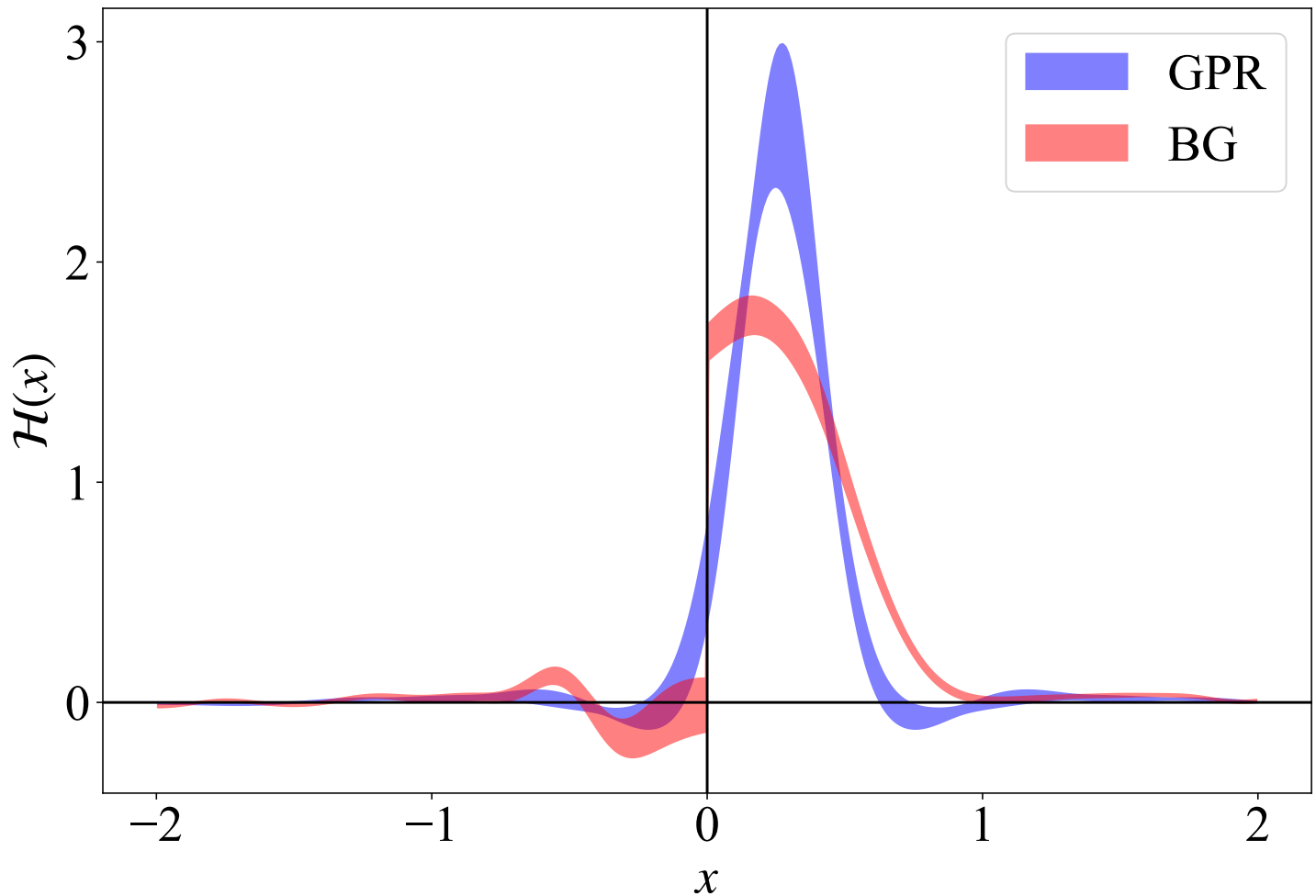

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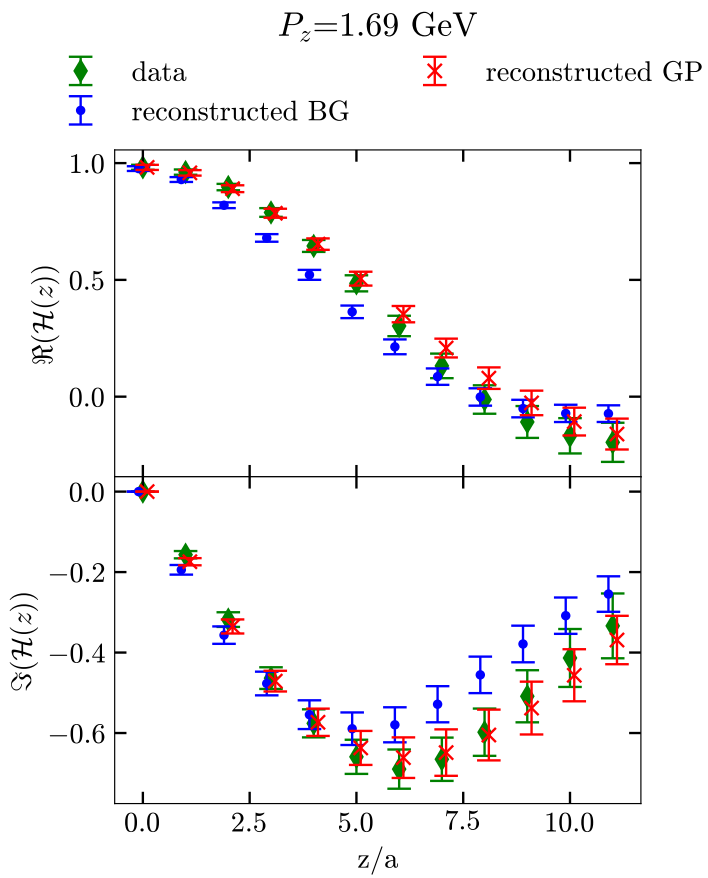

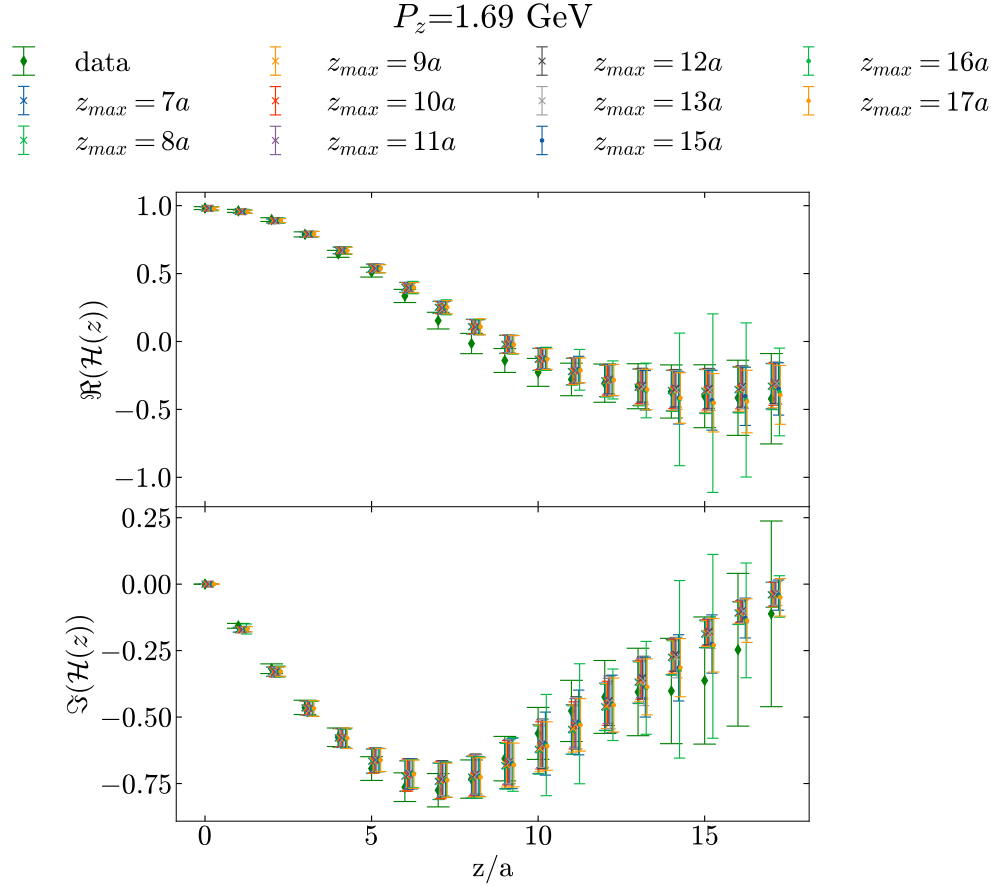

Short-distance factorization

$$\mathcal{M}\left(z,\vec{P}\right)=\frac{\mathcal{H}_{bare}\left(z,\vec{P}\right)}{\mathcal{H}_{bare}\left(z,0\right)}\frac{\mathcal{H}_{bare}\left(0,0\right)}{\mathcal{H}_{bare}\left(0,\vec{P}\right)}=\sum_{n=0}\frac{\left(-izP_z\right)^n}{n!}c_n^{\bar{MS}}\left(\mu^2z^2\right)\langle x^n\rangle+\mathcal{O}\left(\Lambda_{QCD}^2z^2\right),$$

with $c_n^{\bar{MS}}\left(\mu^2z^2\right)$ being the Wilson coefficients at NNLO [Li et al, arXiv:2006.12370v3, Bhat er al, arXiv:2205.07585v1] . We use all the P_z and we set $n_{max}=4$.

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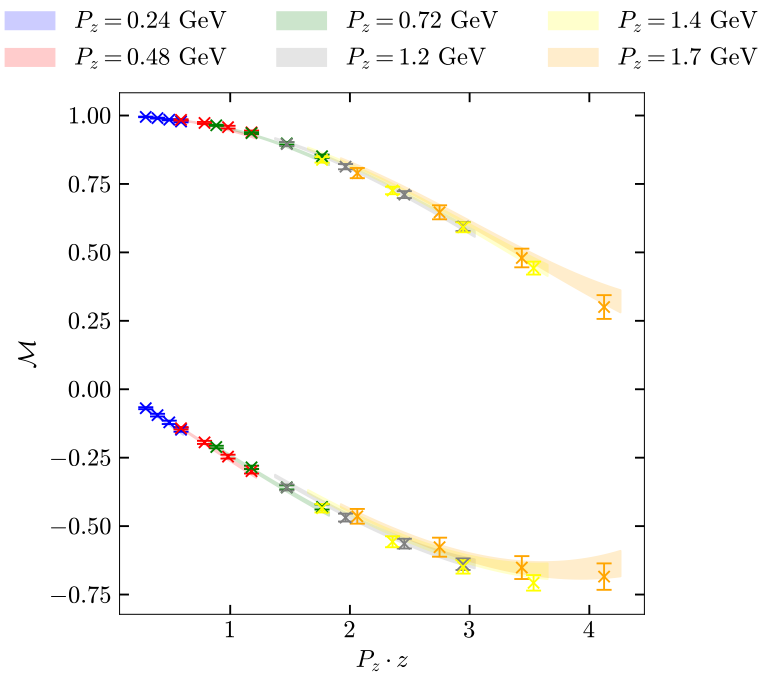

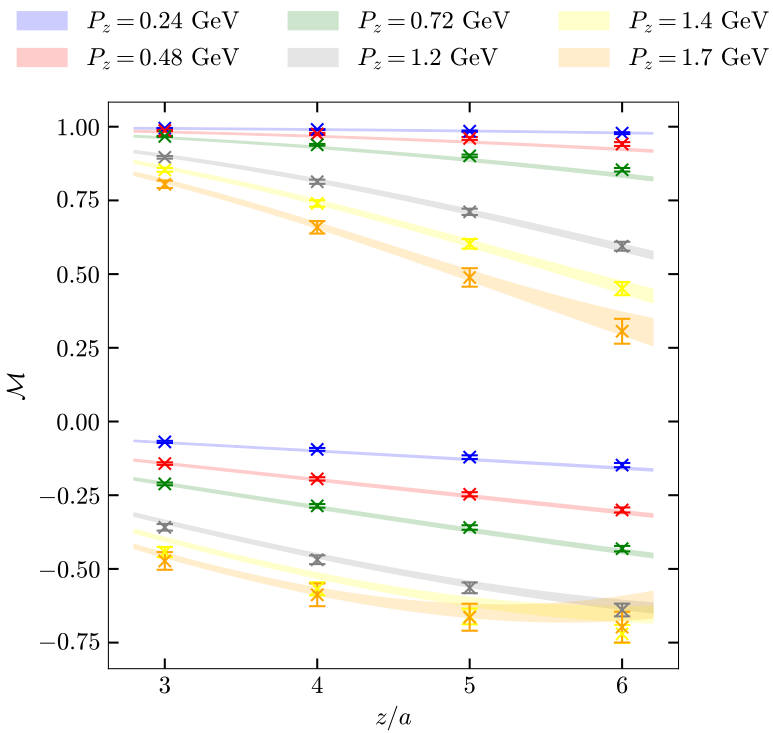

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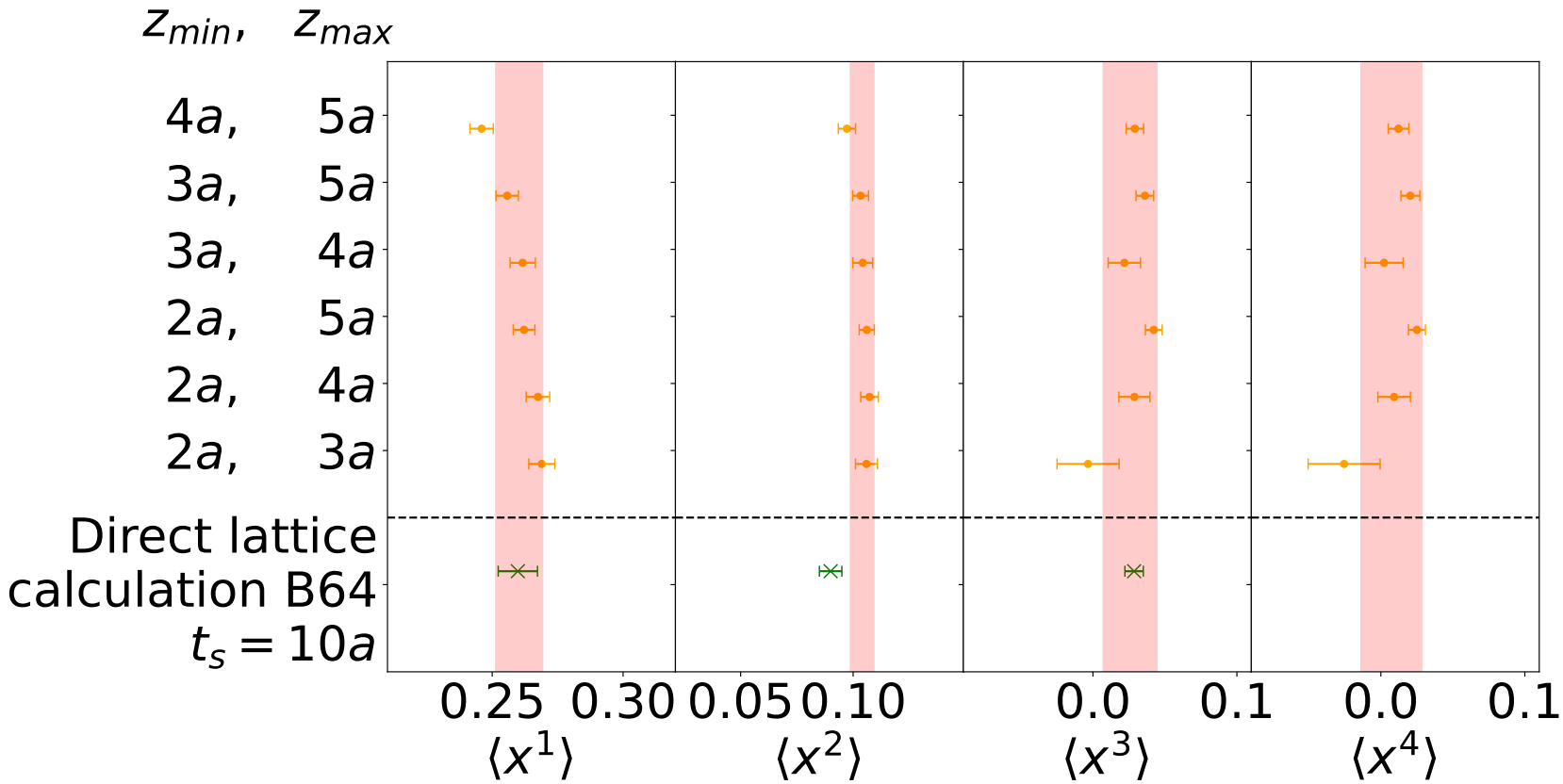
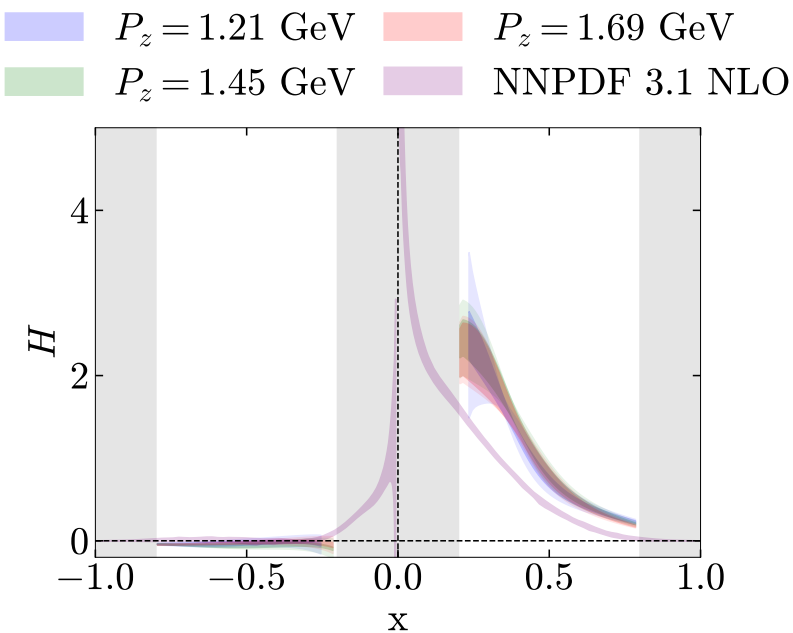


Direct lattice calculation on B64 at $t_s = 10a$ from [Alexandrou et al., arXiv:2607.20337] and [Alexandrou et al., arXiv:2607.03458]. Christian Kummer's talk on Monday at 16.00

Light-cone matching

We computed the matching function $C\left(\frac{x}{y}, \frac{\mu}{P_z}\right)$ [Ding et al., arXiv:2407.03516] and get the light-cone PDF

$$H(x, P_z, \mu^2) = \int \frac{dy}{y} \, C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \mathcal{H}(y, P_z, \mu^2) + \mathcal{O}(\Lambda^2_{QCD}/P_z^2, M_N^2/P_z^2)$$



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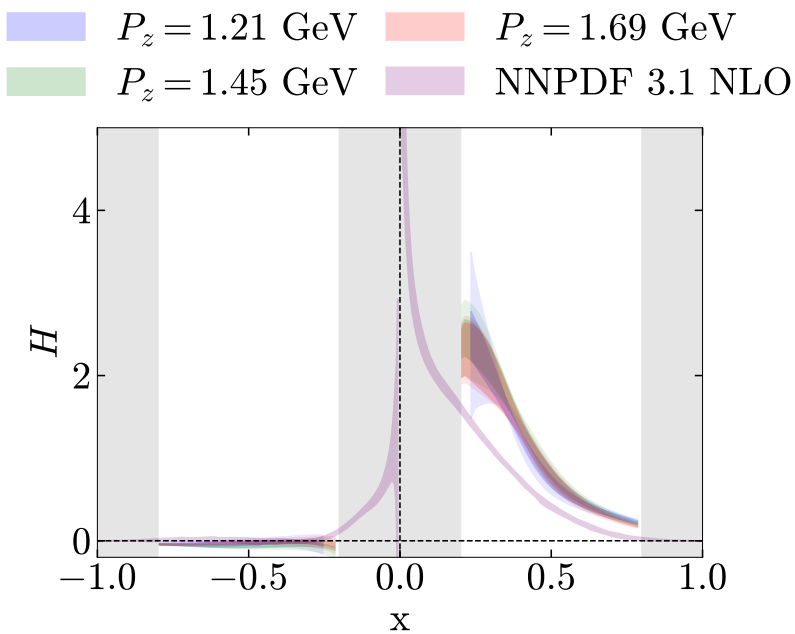
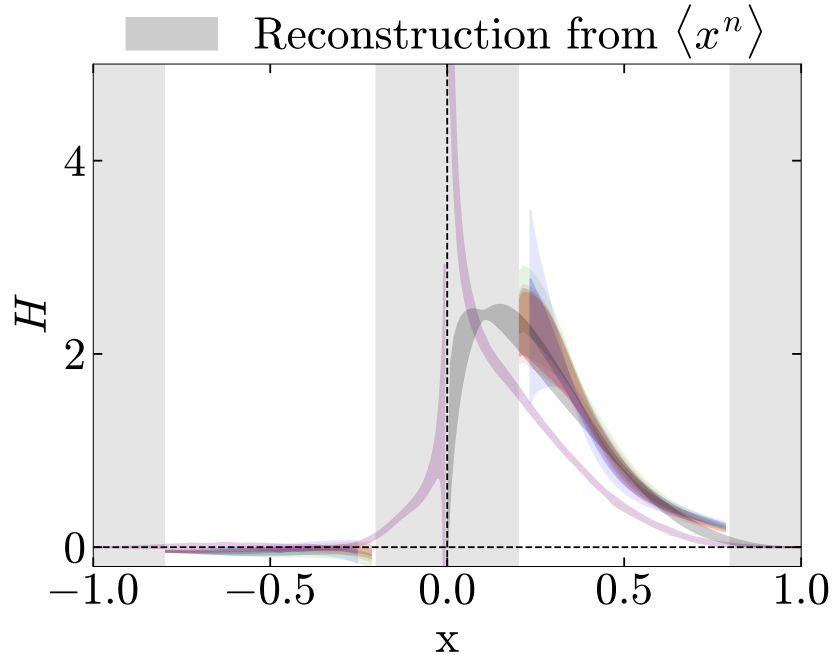
From the moments we can reconstruct the light-cone PDF with the Ansatz

$$H(x) = Nx^\alpha(1-x)^\beta$$





Corrected light-cone PDFs

- As seen from the moments, we have excited state effects;
- Possible way to mitigate these effects on the light-cone: enforcing the second moment of the PDF [Ji and Su, arXiv:2606.2222];
- We use $\langle x \rangle = 0.158(15)$ [Alexandrou et al., arXiv:2607.20337] ;

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$$\hat{\mathcal{H}}(z, P_z) = \mathcal{H}(z, P_z) e^{izP_z \delta y},$$

where δy is chosen by fitting $\hat{\mathcal{H}}$ to

$$\Im(\hat{\mathcal{H}}) = -zP_z \langle x \rangle c_1(\mu^2 z^2),$$

with $\langle x \rangle = 0.158(15)$

Corrected light-cone PDFs

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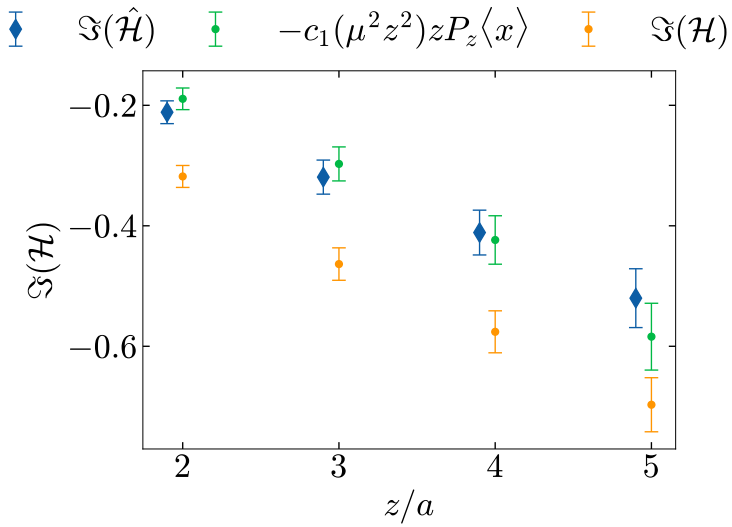

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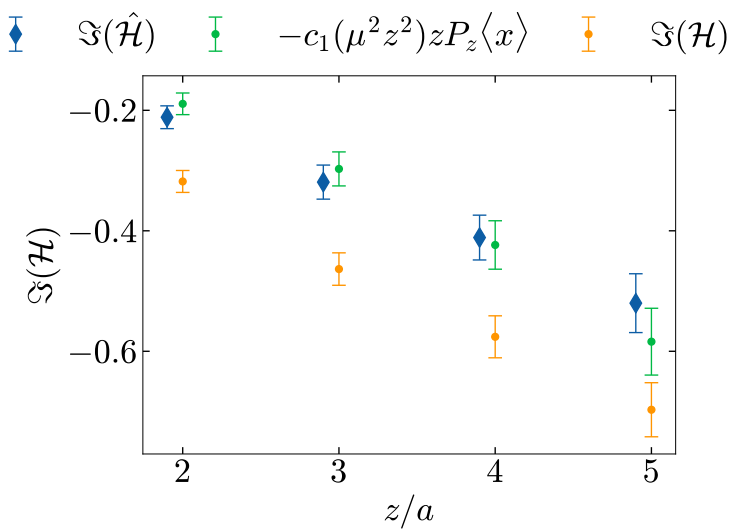

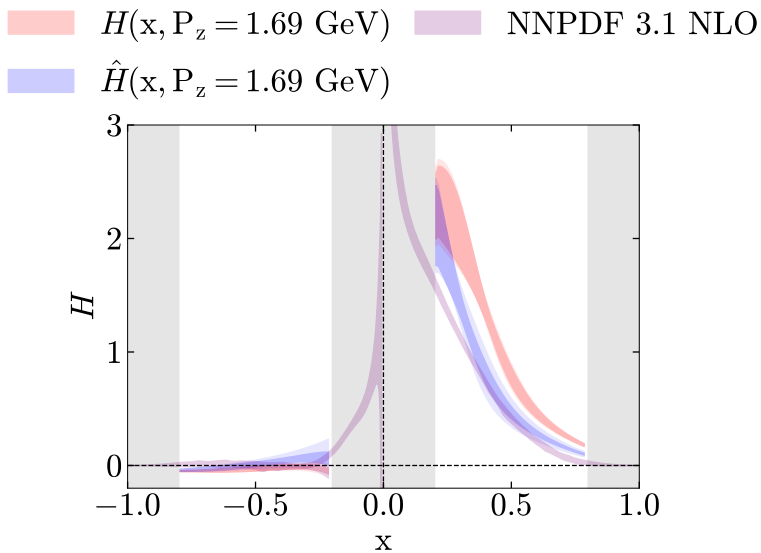

GPDs

- We introduce momentum transfer between source and sink.

GPDs traditionally defined starting from

$$F^0(z,P,\Delta) = \left\langle p_f,\lambda' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^0 \mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\right) q\left(\frac{z}{2}\right)\right| p_i,\lambda\right\rangle =$$

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$$\mathrm{Tr}\left[\Gamma_\kappa \Lambda(p_f) F_\mu \Lambda(p_i)\right], \quad \mu,\kappa = 0,1,2,3,$$

$$\Gamma_0 = \frac{1}{4}(1+\gamma_0), \quad \Gamma_k = \frac{1}{4}(1+\gamma_0) i\gamma_5\gamma_k, \quad k = 1,2,3$$

$$\Lambda(p) = \left(\frac{-i\not{p}+m}{2m}\right).$$

Lattice setup GPDs

N_{conf}	652	762					
Boosts P_z [GeV]	0	0.48	0.72	0.96	1.21	1.45	1.69
N_{src}	16	2	4	6	12	24	48
Optimized momentum smearing $\vec{\Delta}_{smear}$ at P_i	$(0, 0, P_z)$	$(2 \cdot 2\pi/L, 0, P_z)$					

$$R_\mu(\Gamma_\nu, \vec{p}_f, \vec{p}_i, z; t_s, t_{ins}) = \frac{C_\mu^{3pt}(\Gamma_\nu, \vec{p}_f, \vec{p}_i; t_s, t_{ins})}{\sqrt{C_{2pt}(\Gamma_0, \vec{p}_i; t_s) C_{2pt}(\Gamma_0, \vec{p}_f; t_s)}} \times \sqrt{\frac{C_{2pt}(\Gamma_0, \vec{p}_f; t_s - t_{ins}) C_{2pt}(\Gamma_0, \vec{p}_i; t_{ins})}{C_{2pt}(\Gamma_0, \vec{p}_i; t_s - t_{ins}) C_{2pt}(\Gamma_0, \vec{p}_f; t_{ins})}}.$$

$\vec{p}_f = (0, 0, P_z)$, $\vec{p}_i = \vec{p}_f - \vec{\Delta}$, $-t^2 = (p_f - p_i)^2$, asymmetric frame

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Plateau fit to extract the Matrix element.

Short-distance factorization, GPDs

$$\mathcal{M}\left(z,\vec{P},-t\right)=\frac{\mathcal{H}_{bare}\left(z,\vec{P},-t\right)\left(\mathcal{E}_{bare}\right)}{\mathcal{H}_{bare}\left(z,0,0\right)}=$$
$$\sum_{n=0}\frac{\left(-izP_z\right)^n}{n!}c_n^{\bar{M}S}\left(\mu^2z^2\right)A_{n+1,0}(-t)(B_{n+1,0})+\mathcal{O}\left(\Lambda_{QCD}^2z^2\right).$$

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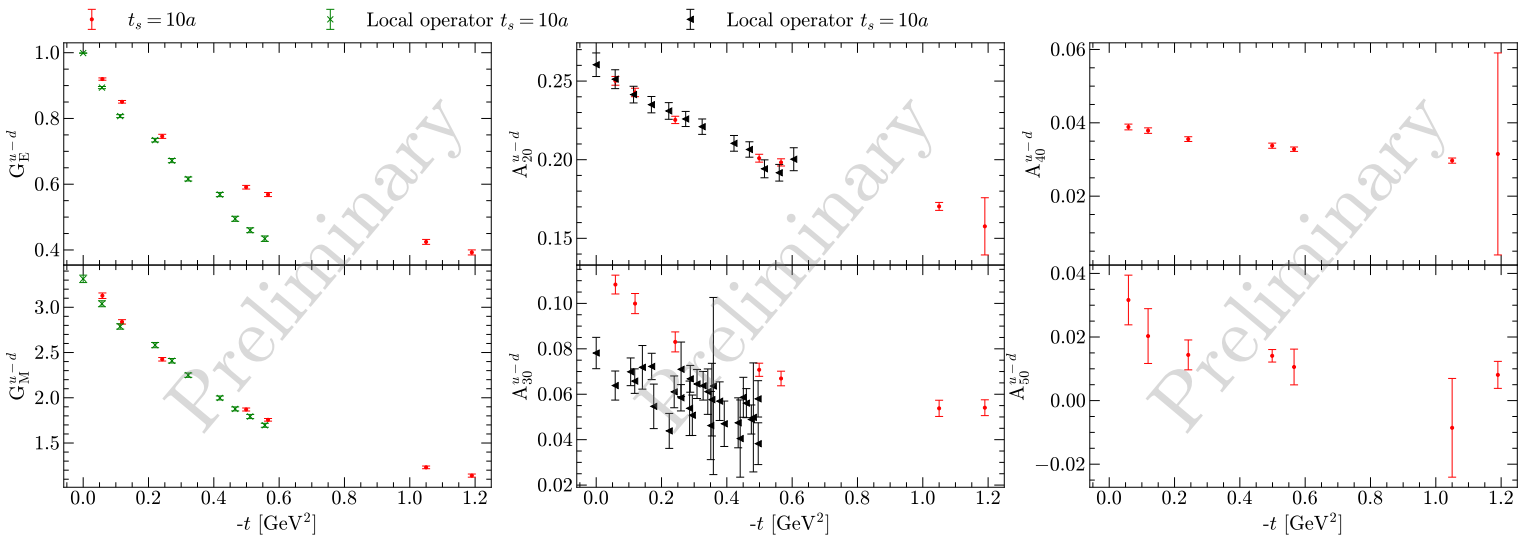

Direct lattice calculation on B64 at $t_s = 10a$ from [Alexandrou et al., 2507.20910] and [Alexandrou et al., arXiv:2607.03458]. Christian Kummer's talk on Monday at 16.00

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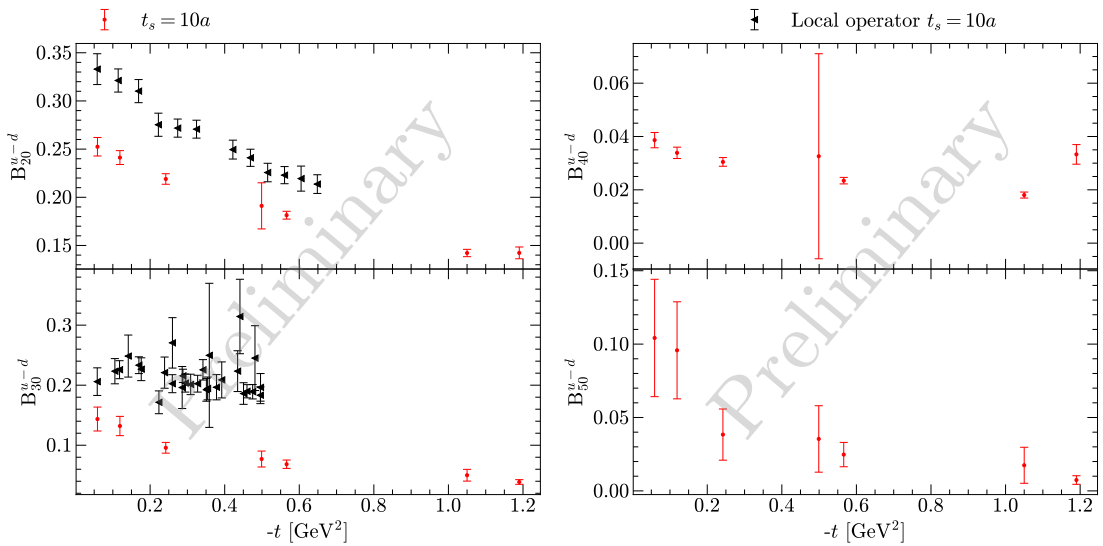

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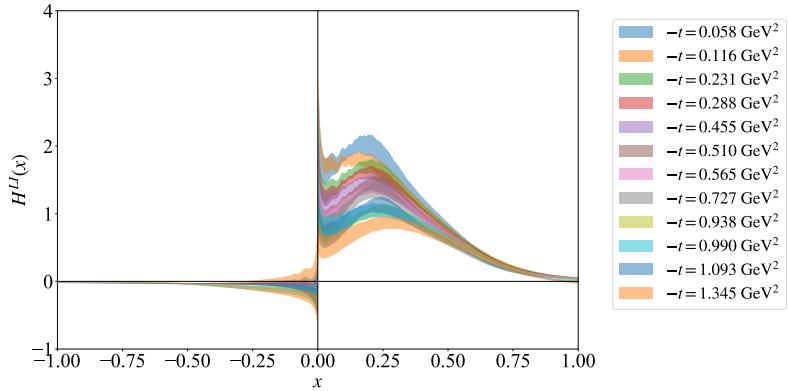
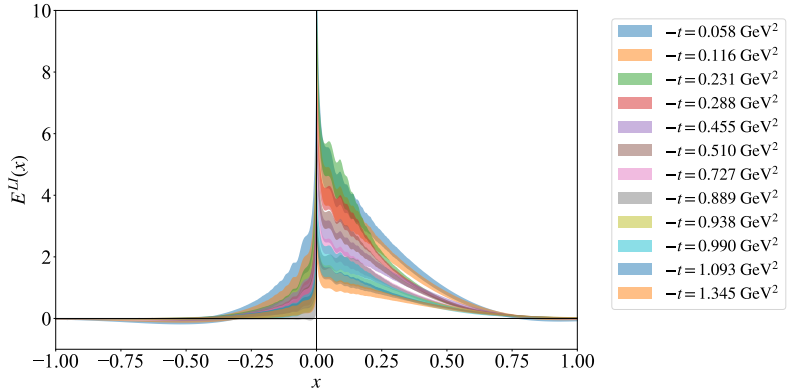
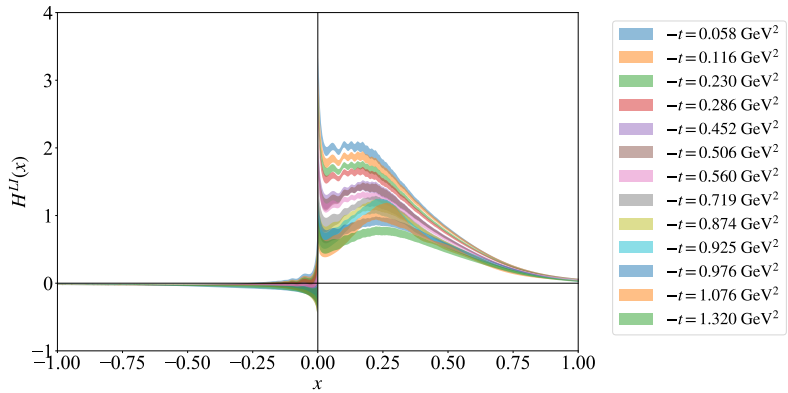
Light-cone matching, GPDs

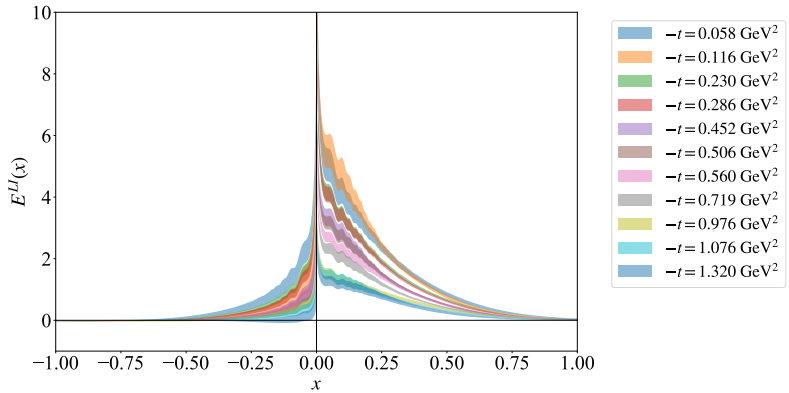
Light-cone GPDs at $P_z = 1.69$ GeV





Light-cone GPDs at $P_z = 1.45$ GeV





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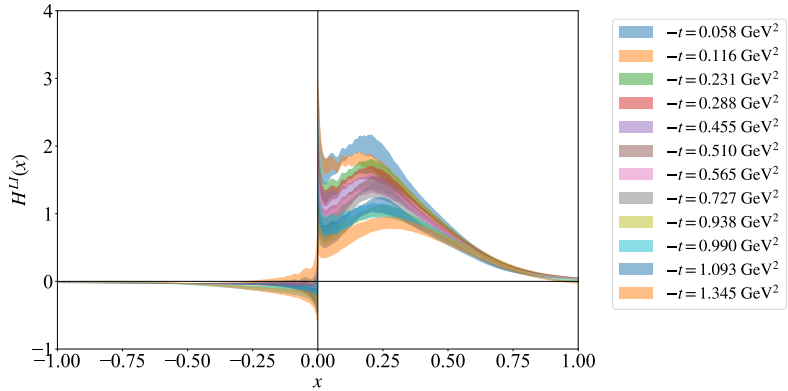

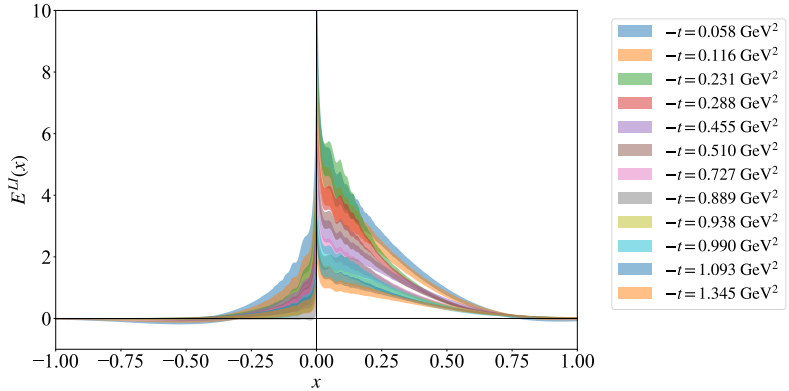

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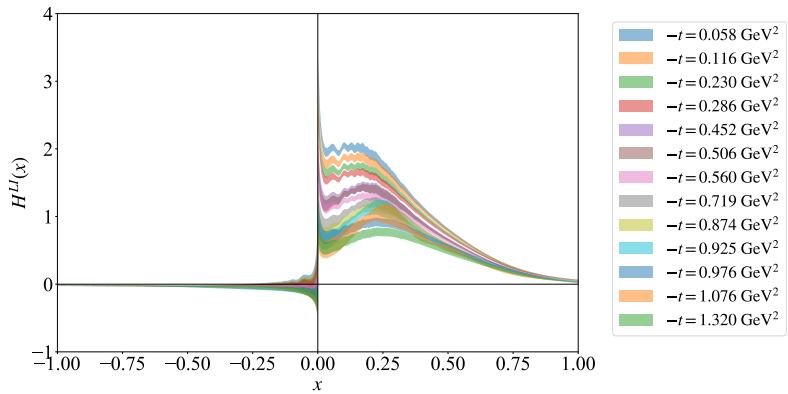

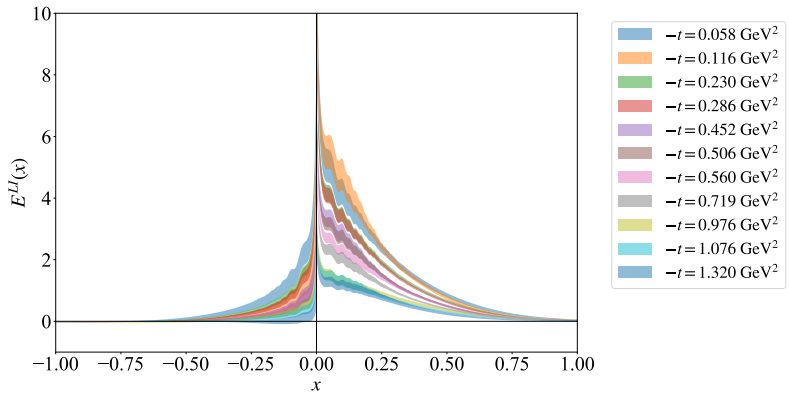

Preliminary results: systematics from renormalization to be still computed

Conclusion and outlook

- Implementation of Bayes-Gauss-Fourier transform;
- Light-cone PDFs and GPDs at physical mass for large boosts;
- Extraction of moments up to $\langle x^4 \rangle$, with agreement with lattice results from direct calculation.

- Finalise GPDs analysis;
- Hybrid renormalization for helicity PDFs and GPDs;
- Two-state analysis for all boosts.


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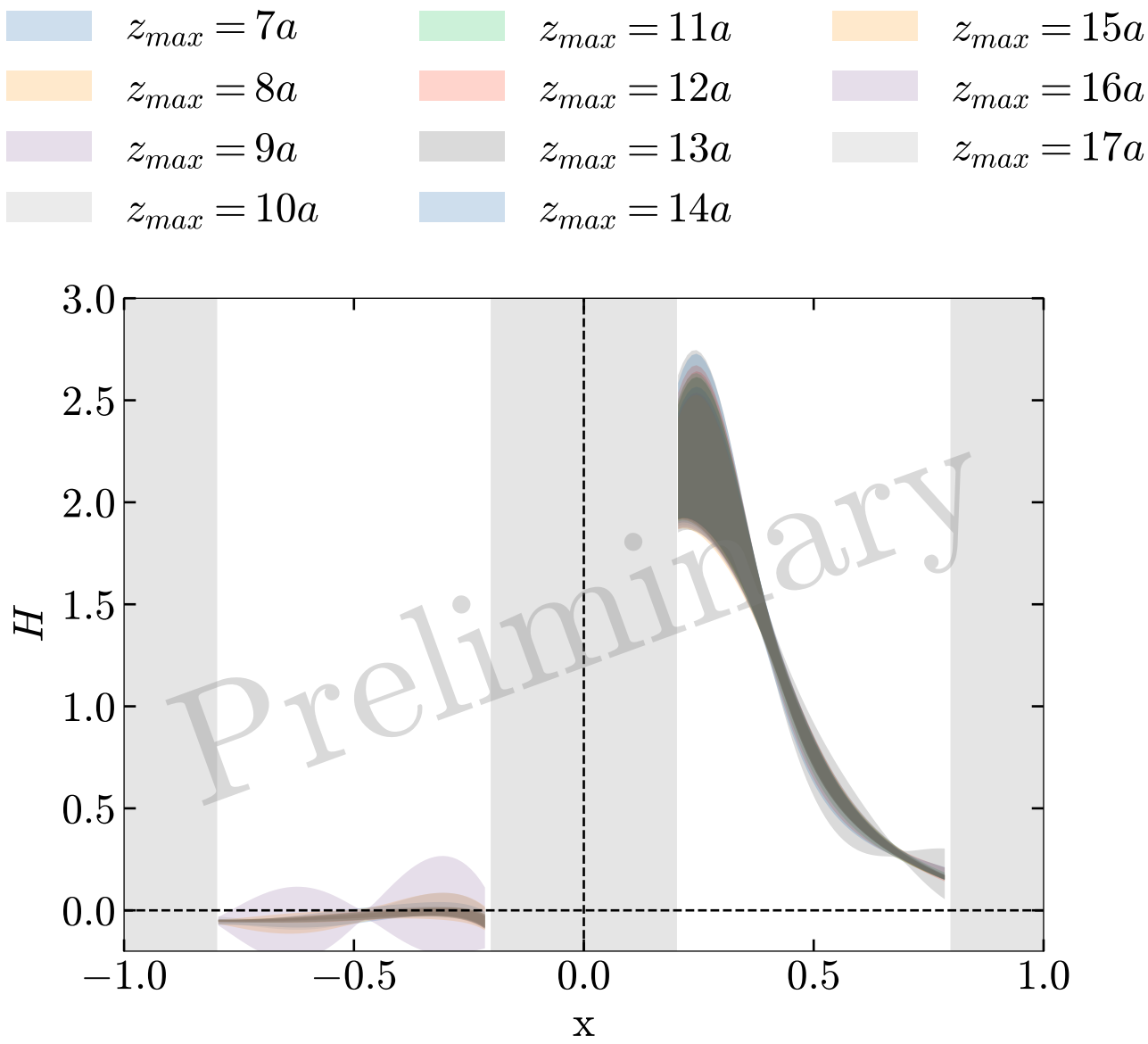
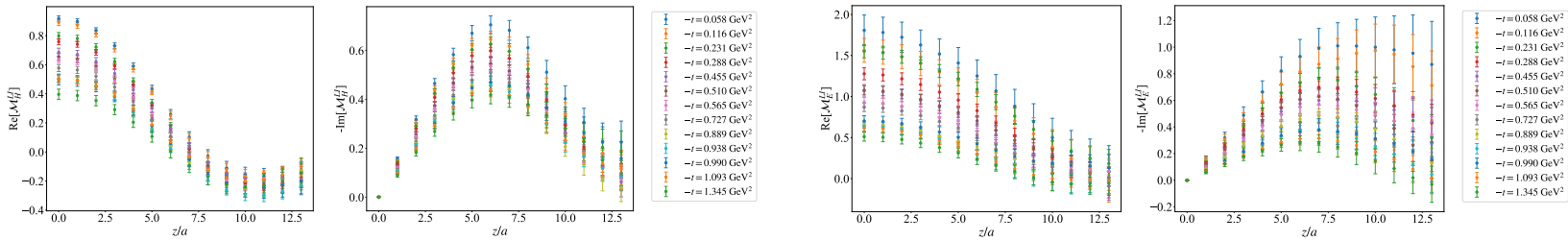
Thank You!

Bayes-Gauss-Fourier stability



Renormalized GPDs

Renormalized quasi-GPDs at $P_z = 1.69$ GeV



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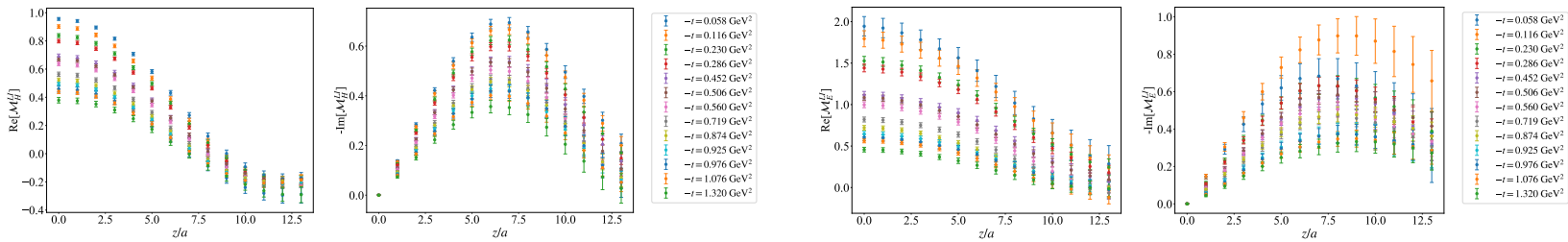

Helicity-PDF moments

$$\mathcal{M}(z,P) = \frac{\tilde{\mathcal{H}}_{bare}\left(\vec{z},\vec{P},\right)}{\tilde{\mathcal{H}}_{bare}(\vec{z},0)} \frac{\tilde{\mathcal{H}}_{bare}(0,0)}{\tilde{\mathcal{H}}_{bare}\left(0,\vec{P}\right)} g_A = \sum_{n=0} \frac{(-izP)^n}{n!} \frac{C_n^{\bar{MS}}\left(\mu^2 z^2\right)}{C_0^{\bar{MS}}\left(\mu^2 z^2\right)} \langle x^n \rangle + \mathcal{O}\left(\Lambda_{QCD}^2 z^2\right),$$

with $C_n^{\bar{MS}}\left(\mu^2 z^2\right)$ being the Wilson coefficients at NLO for the helicity case.

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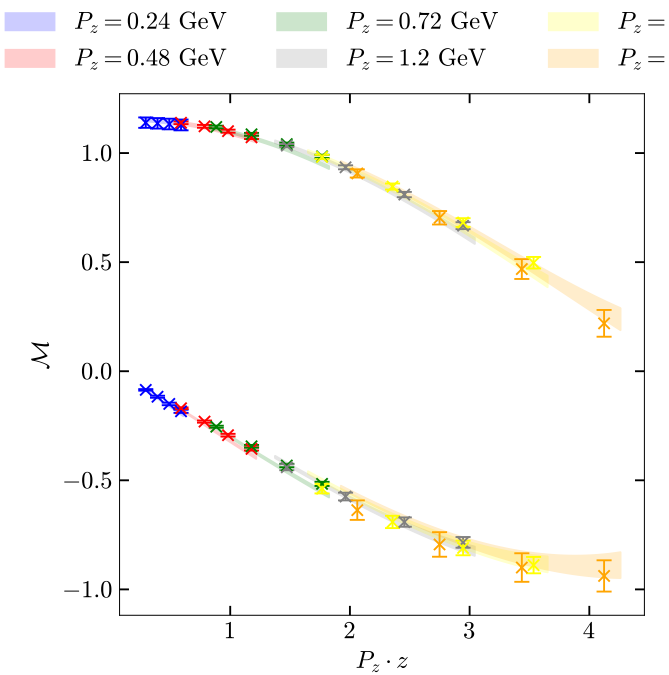

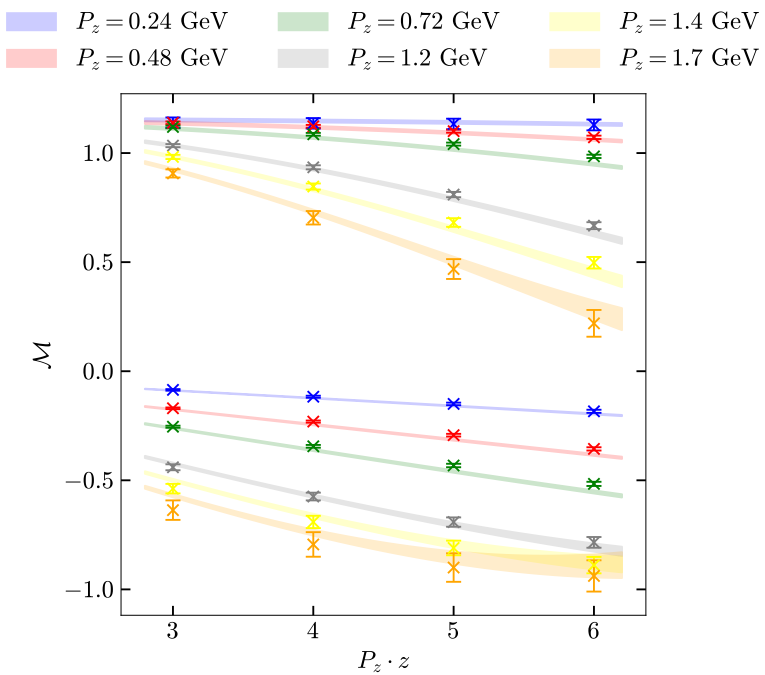

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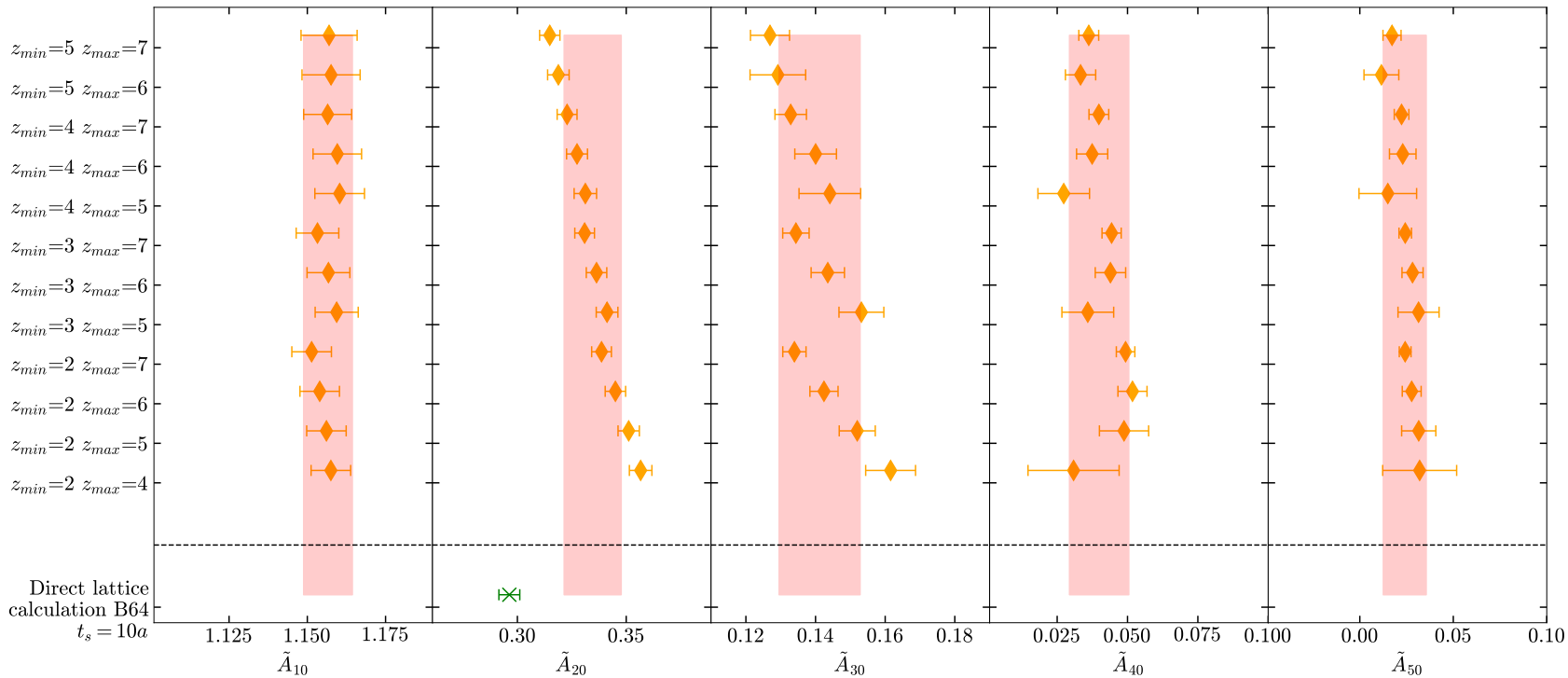
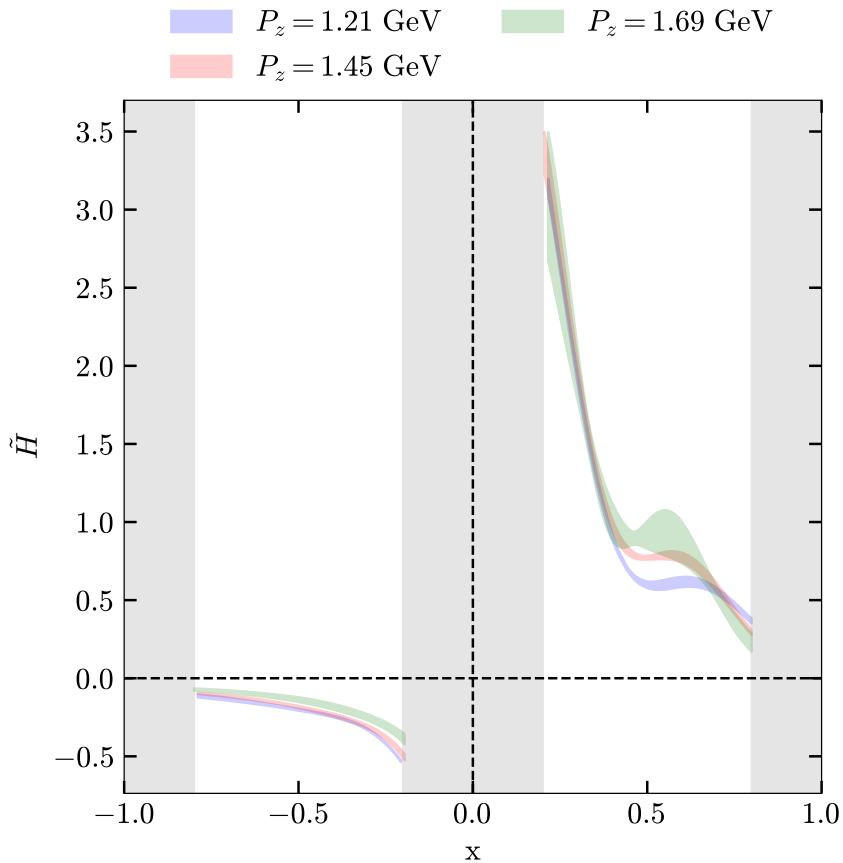


Direct lattice calculation on B64 from [Alexandrou et al., arXiv:2607.20337].

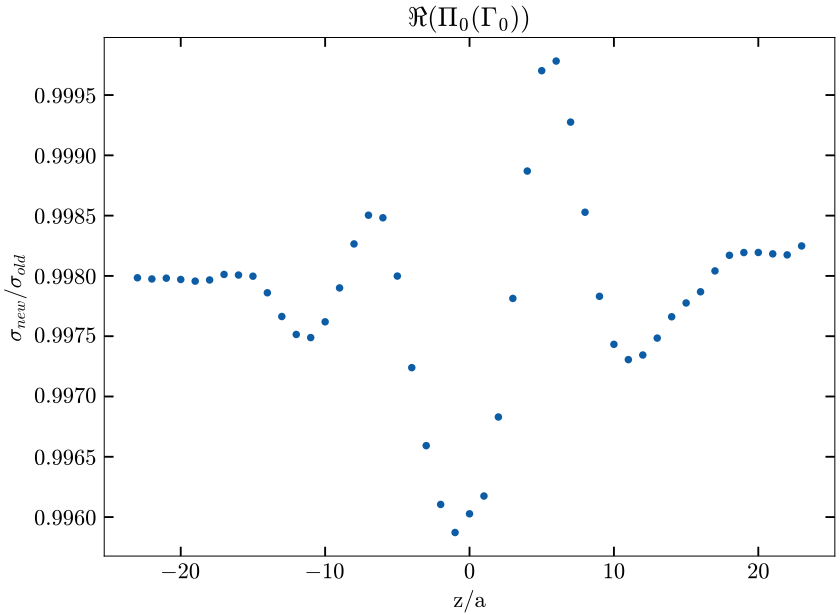
Helicity-PDF light cone

$$\tilde{F}^{\mu}(z,P) = \langle N(P) \left| \bar{\psi}(z)\gamma^{\mu}\gamma^{5}\mathcal{W}(z,0)\psi(0) \right| N(P) \rangle$$

[Bhattacharya et al., arXiv:2310.13114v2] We employ RI/MOM renormalization at $\mu = 2$ GeV. We use Bayes-Gauss-Fourier Transform and then we match them [Liu et al., arXiv:1902.00307] to light-cone helicity PDF.



Different ratios



Ratio of the errors of

$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{\sqrt{C^{2pt}(\Gamma_0, p, \Delta_0; t_s) C^{2pt}(\Gamma_0, p, \Delta; t_s)}}$$

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$$R(\Gamma_0, p; t_s, t_{ins}) = \frac{C^{3pt}(\Gamma_0, p; t_s, t_{ins})}{C^{2pt}(\Gamma_0, p, \Delta_0; t_s)}$$

for $P_z = 1.21$ GeV.