The Muon anomalous magnetic moment

the HVP solved (?) puzzle

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Extended Twisted Mass Collaboration

AQTIVATE

AQTIVATE Conference

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the anomalous magnetic moment of the muon



- magnetic moment of a spin-$1/2$ particle parametrises its response to an external magnetic field

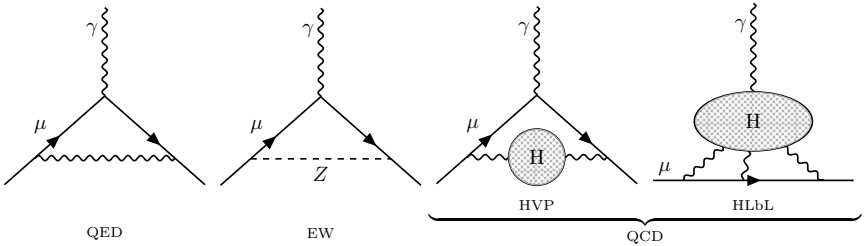
$$\boldsymbol{\mu} = g\frac{qe}{2m}\boldsymbol{S} \qquad \text{classically} \quad g = 2$$

- within the QFT framework, the response of fermions to an electromagnetic field

$$\left\langle \mu(p_2)\middle| J^{\mu}_{\text{em}} \middle|\mu(p_1)\right\rangle = -ie\bar{u}(p_1)\Gamma^{\mu}\left(p_1, p_2\right) u(p_2)$$

$$\Gamma^{\mu}\left(p_1, p_2\right) = \gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu}\, q_{\nu}}{2m} F_2(q^2) \quad \rightarrow \quad g = 2\left[F_1(0) + F_2(0)\right]$$

$$a_{\mu} = \frac{g_{\mu} - 2}{2} =$$



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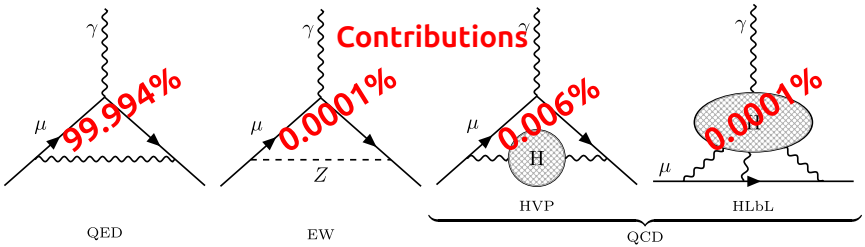

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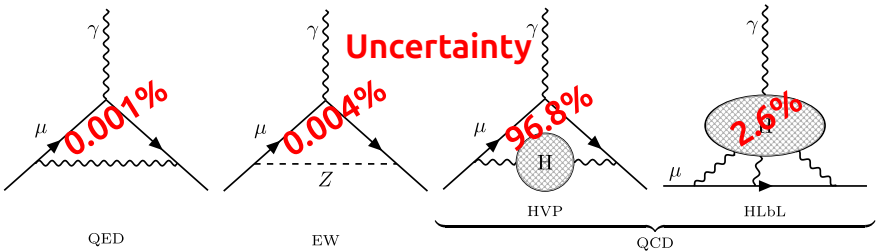
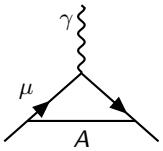


why $g_\mu - 2$?

sensitivity to BSM physics



$$a_\ell^A \simeq R_{\text{BSM}} \left(\frac{m_\ell}{m_A} \right)^2$$

$$= R_{\text{BSM}} \left(\frac{1\ \text{TeV}}{m_A} \right)^2 \cdot \begin{cases} \ell = e : 1.7 \times 10^{-15} \\ \ell = \mu : 7.1 \times 10^{-11} \\ \ell = \tau : 2.0 \times 10^{-8} \end{cases}$$

τ would be even better
but porely measured

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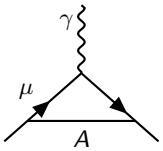

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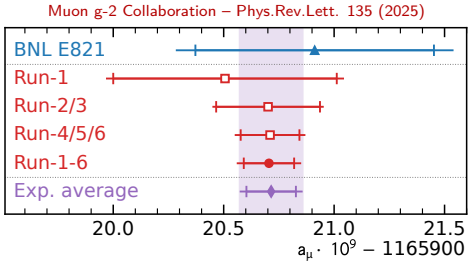
experimental precision



$$a_\mu^{\text{exp}} = 116\ 592\ 071.5(14.5) \times 10^{-11}$$

a two-decade effort to **reach a precision of 127pb**

a new experiment E-034 set up at J-PARK

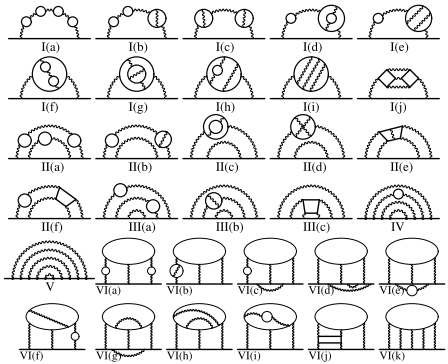
the contributions to a_μ

the QED contribution

$$a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + \sum_{n=2}^{\infty} C_\mu^{(n)} \left(\frac{\alpha}{\pi}\right)^n$$

- 1-loop contribution computed by Schwinger in 1948
  dominant one
- Feynman diagrams number quickly rises with n
  more than 10000 diagrams contribute to the 5-loops
- sixth-order contributes at $\simeq C_\mu^6 \times 10^{-16}$

$$a_\mu^{\text{QED}} = 116\,584\,718.8(2) \times 10^{-11}$$



heroic effort in T. Aoyama – Phys.Rev.Lett. 109 (2012)

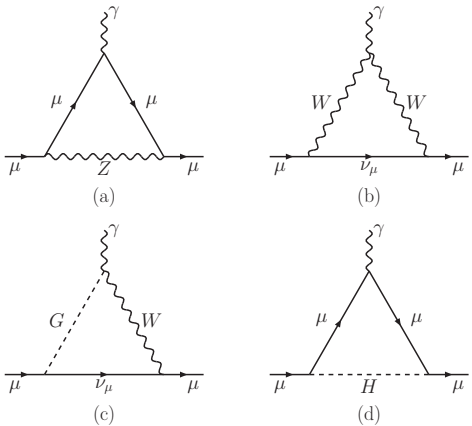
the EW contribution

- loops with Z, $W^{\pm}$ and Higgs bosons exchange
- Fermi scale suppression

$$a_\mu^{\mathrm{EW}} \propto \alpha_W^2 \left(\frac{m_\mu}{M_W}\right)^2$$

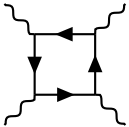
- two loops are enough to reach 3ppb precision

$$a_\mu^{\mathrm{EW}} = 154.4(4) \times 10^{-11}$$



the QCD contribution: HLbL

- occurs at α^3
  related to $2 \to 2$ photon scattering

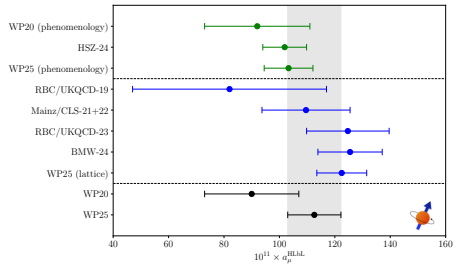


$$\mathrm{T}\left\langle 0\middle|J^\mu\, J^\nu\, J^\rho\, J^\sigma\middle|0\right\rangle = \Pi^{\mu\nu\rho\sigma}$$

- dispersion relation parametrised by transition form factors
  isolating the dominant intermediate states

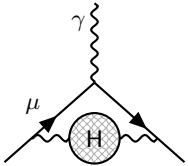
- general agreement between lattice and phenomenology
  below 10% final uncertainty

WP25 – Phys.Rept. 1143 (2025)



$$a_\mu^{\mathsf{HLbL}} = 115.5(9.9) \times 10^{-11}$$

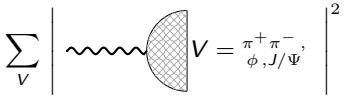
the QCD contribution: HVP



optical theorem



$$\propto$$



$$\sum_V \left| \quad \right|^2$$

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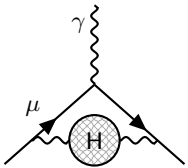

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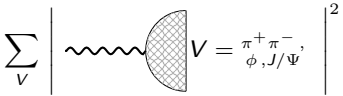


dispersive approach

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha_{\mathrm{EM}}}{\pi}\right)^2 \int_0^\infty \mathrm{d}s\; R_{\mathrm{had}}(\sqrt{s})\, \tilde{K}(s)$$

$e^+e^- \to$ had normalised cross section

$$R_{\mathrm{had}} = \frac{\sigma\left(e^+e^- \to \mathrm{had}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

lattice TMR approach

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha_{\mathrm{EM}}}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\; C(t)\, \hat{K}(t)$$

$C(t)$ related to the R-ratio through a Laplace transform

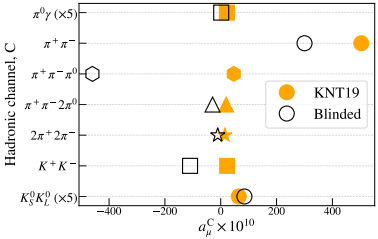
dispersive approach to the HVP

- inclusive R-ratio needed
  combination of all possible exclusive channels
  dipion channel is the largest

- control of all systematics
  radiative corrections are a difficult systematic

- no BSM contribution is assumed

KNTW – Phys.Rev.D 111 (2025)



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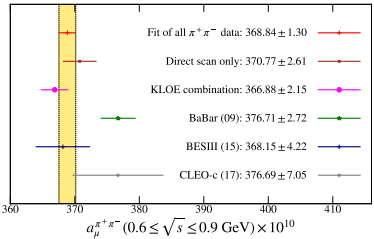
KNTW – Phys.Rev.D 111 (2025) **ρ peak region**



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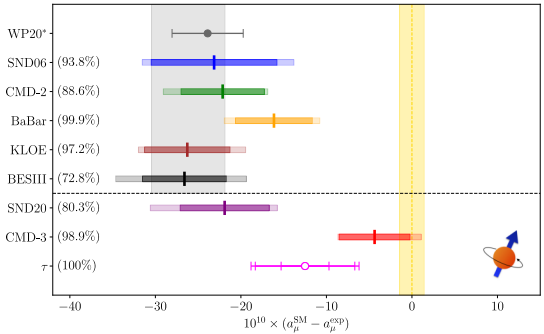
WP25 – Phys.Rept. 1143 (2025)



KNTW – Phys.Rev.D 111 (2025)

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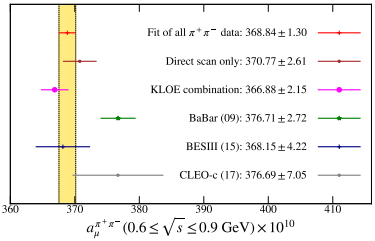
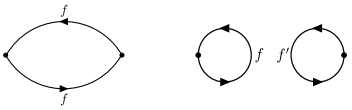


- more than 5σ tension with experiment
- unexplained tension between e^+e^- experiments
  mainly in the around the ρ peak

lattice approach to the HVP

$$C(t) = -\frac{1}{3}\sum_{k=1,2,3}\int \mathrm{d}^3x\ \langle 0|J^k(x)\,J^k(0)|0\rangle$$

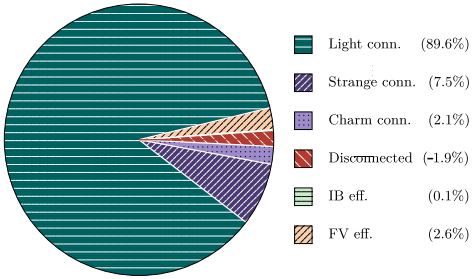
$$J^\mu = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \frac{1}{3}\bar{s}\gamma^\mu s + \frac{2}{3}\bar{c}\gamma^\mu c$$



flavour decomposition

$$a_\mu^{\mathrm{HVP}} = a_\mu^{\mathrm{HVP}}(\ell) + a_\mu^{\mathrm{HVP}}(s) + a_\mu^{\mathrm{HVP}}(c) + a_\mu^{\mathrm{HVP}}(\mathrm{disc.})$$

BMW20 – Nature 593 (2021)



- ℓ dominate both signal and error (72%)
- FV effects are the second source of error (20%)
- strong cancellations in IB effects (6%)
- s and c errors are irrelevant ($< 0.1\%$)

lattice approach to the HVP

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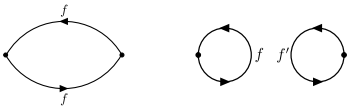

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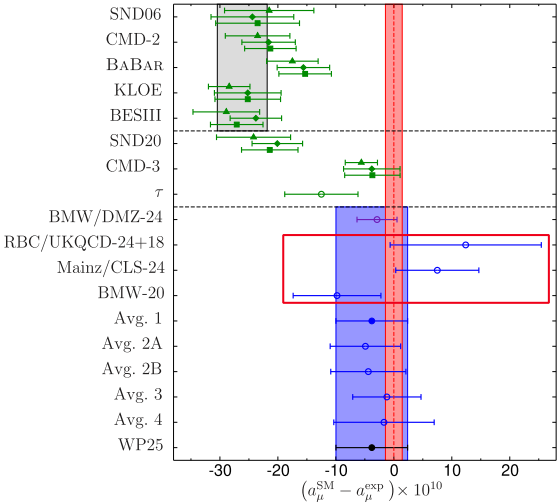


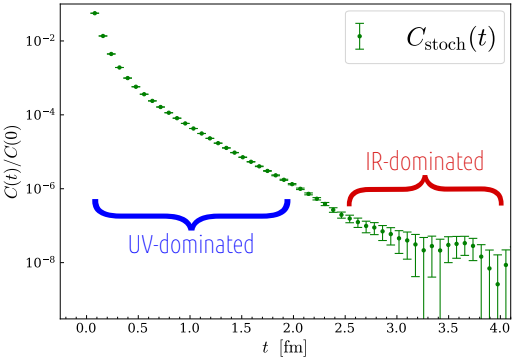
the signal to noise problem

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha_{\mathrm{EM}}}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\; C(t)\,\hat{K}(t)$$

$\hat{K}(t) \propto t^2$

$C(t) = e^{-Et}$

- exponential growth of N/S ratio
- $t \geq 2.5$ fm became challenging
- integral saturates within errors



- UV part computable at high precision stochastically
- IR part can be treated *exactly*
  - **a) spectrum reconstraction**
  - **b) low modes of the Dirac operator**

solutions to the S/N problem

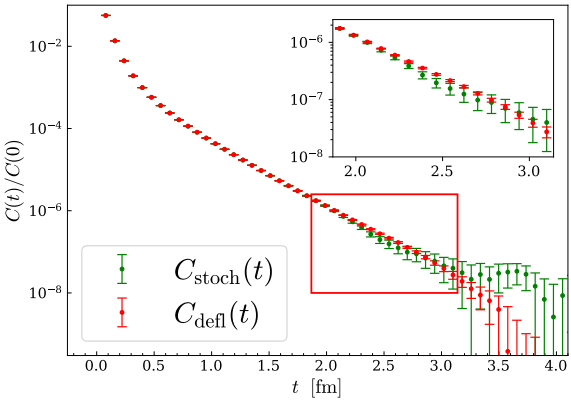
- large-time separations $\longrightarrow$ IR modes

  Low Mode Averaging approaches

$$S = S^{\mathrm{UV}} + S^{\mathrm{IR}}$$

$$= S_\eta^{\mathrm{UV}} + \sum_{k=1}^{N_{\mathrm{ev}}^{\mathrm{IR}}} \frac{1}{\Lambda_k} v_k v_k^\dagger$$

$$C_\eta^{\mathrm{LMA}}(t) = C_\eta(t) - C_\eta^{\mathrm{IR}}(t) + C^{\mathrm{IR}}(t)$$



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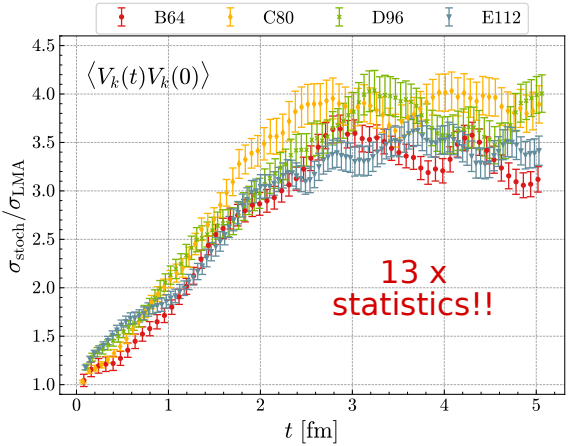

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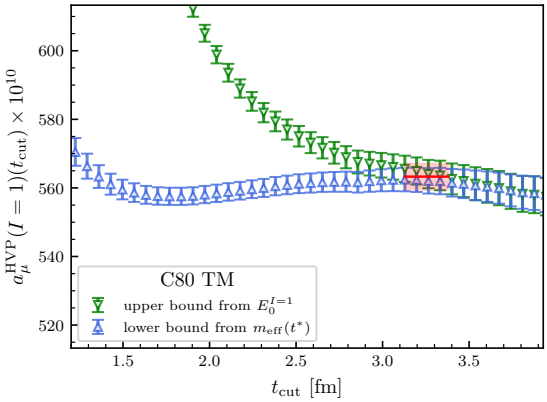
- bounds on the correlators

$$C_{\mathrm{low.}}(t) \leq C(t) \leq C_{\mathrm{upp.}}(t)$$

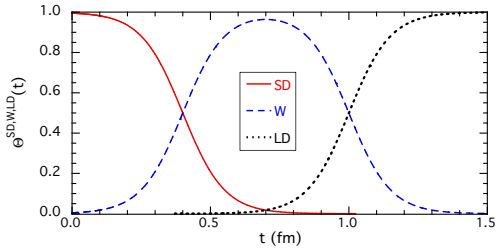
through expected exponential behaviour



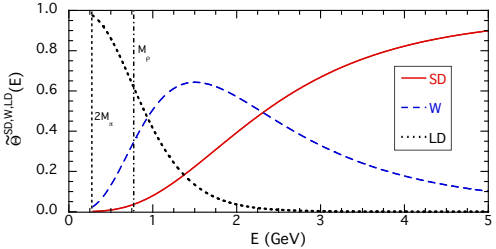
the windows approach

RBC/UKQCD – Phys.Rev.Lett. 121 (2018)

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha_{\mathrm{EM}}}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\; C(t)\,\hat{K}(t)\,\Theta^w(t) \qquad \Theta^{\mathrm{W}}(t) \equiv \underbrace{\frac{1}{1+e^{-2(t-t_0)/\Delta}}}_{1-\Theta^{\mathrm{SD}}} - \underbrace{\frac{1}{1+e^{-2(t-t_1)/\Delta}}}_{\Theta^{\mathrm{LD}}}$$



LQCD $\longleftrightarrow$ e^+e^-

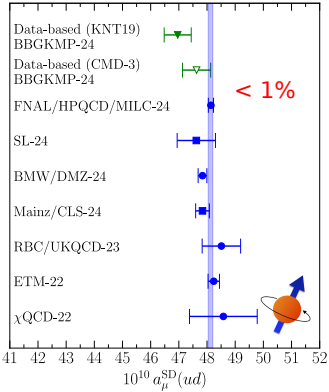


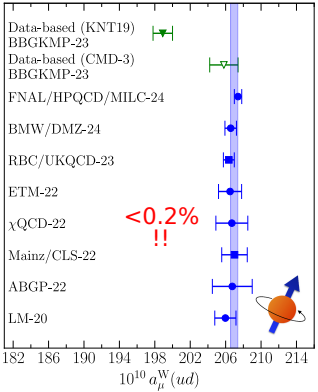
SD and W allow avoiding noisy data at large t

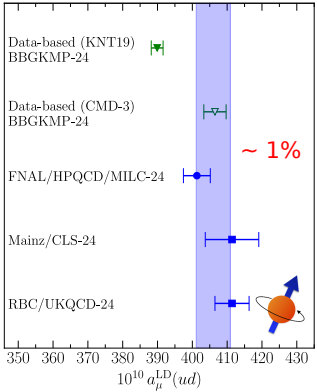
allows comparison with e^+e^- experiments for each window

lattice results comparison

different collaborations use different discretisations of the theory → different systematics







relatively compatible with e^+e^- | most precise from LQCD | deserve more investigation from LQCD

***ud* sector is dominated in e^+e^- experiments by 2π states**

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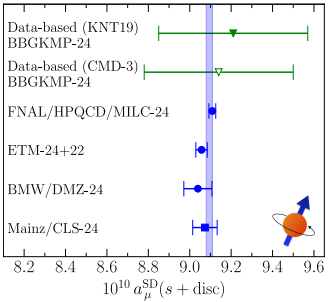
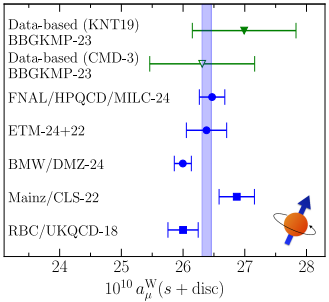
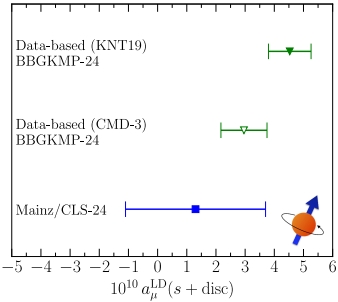




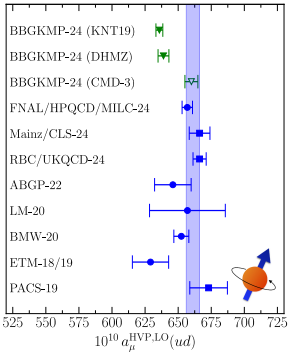


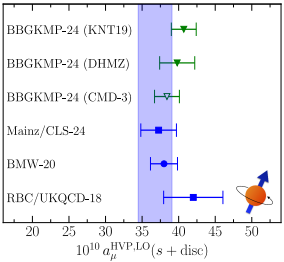
good agreement with e^+e^- experiments

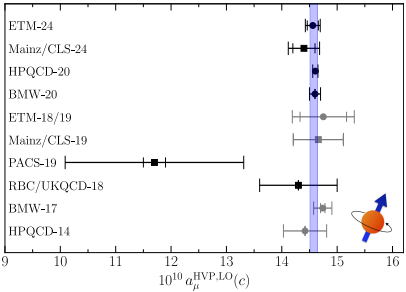
further hint of some problem in the $\pi^+\pi^-$ channel

lattice results comparison

different collaborations use different discretisations of the theory → different systematics

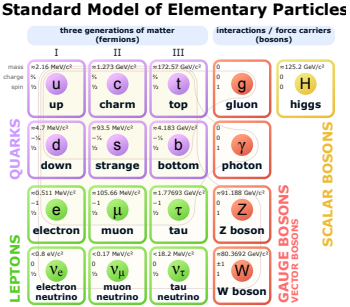






very good agreement between different lattice collaborations

QCD + QED on the lattice



what do we mean by QCD+QED?

1. SM theory

$$\mathcal{H}_{\rm SM} = \mathcal{H}_{\rm QCD} + \mathcal{H}_{\rm QED} + \mathcal{H}_{\rm W}$$

2. we "integrate out" the particles heavier than some energy

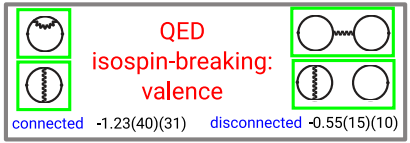
Higgs boson, $W^{\pm}$ and Z bosons, t- and b-quark $\rightarrow$ **Fermi's weak theory**

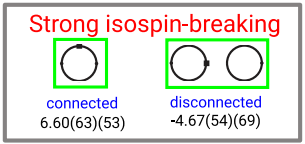
$$\mathcal{H}_{\rm QCD+QED} + \int \mathrm{d}^4 x \, h_W^F(x)$$

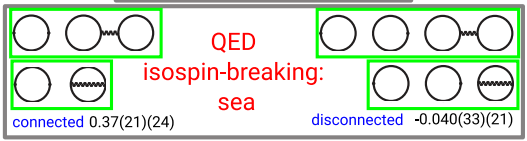
3. physical observables computable as **QCD+QED correlators** with insertion of $h_W^F(x)$

$$\langle \mathcal{O} \rangle_{\rm QCD+QED} = \langle \mathcal{O} \rangle_{\rm QCD} + e^2 \left. \frac{\partial \langle \mathcal{O} \rangle}{\partial e^2} \right|_{\rm QCD} + \sum_f (m_f - m_f^0) \left. \frac{\partial \langle \mathcal{O} \rangle}{\partial m_f} \right|_{\rm QCD}$$

the HVP isospin breaking effects

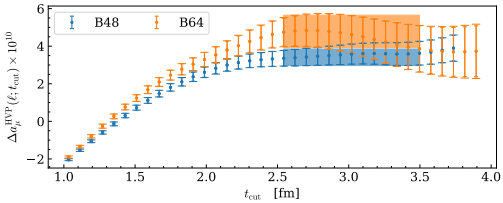








- computing them is very challenging
  only BMW compute all contribution so far
- strong cancellations between contributions
  important to control all sources of errors
- as a collaboration, in **ETMC** we are working on it



A. Evangelista et al. – PoS LATTICE2024 (2025) 260

the R-ratio on the lattice

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can we compare the R-ratio directly?

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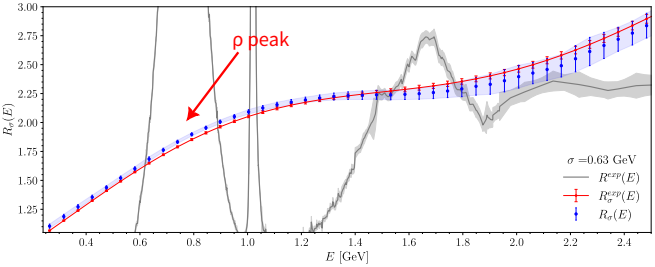
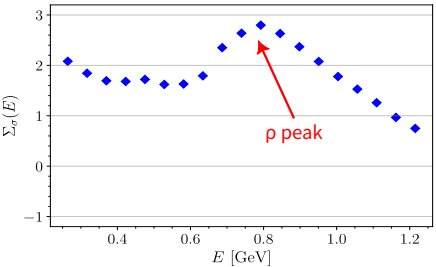
C. Alexandrou *et al.* – Phys.Rev.Lett. 130 (2023)





first-ever LQCD result by ETMC via the HLT method! HLT – Phys.Rev.D 99 (2019)

same hint of discrepancies around the ρ peak as $a_\mu^{\rm W}$

conclusions and outlooks

- computing theoretically the full a_μ is an heroic effort
- QED, Electroweak, and HLbL contributions under control
- the HVP contribution has the largest impact on the total error
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figures/HVP/Fig1_iso.pdf

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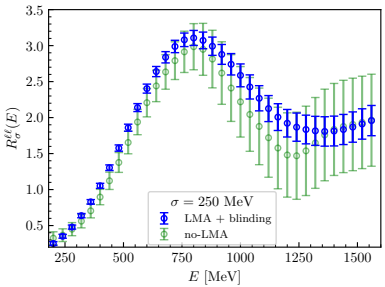
F. Margari *et al.* – PoS LATTICE2024 (2025) 446



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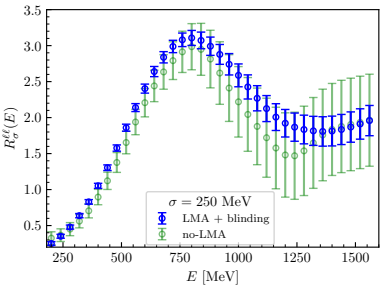

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Thank you for the attention!

EXCELLENCE/0524/0459, POST-DOC/0925-ROANDOTHER/0148