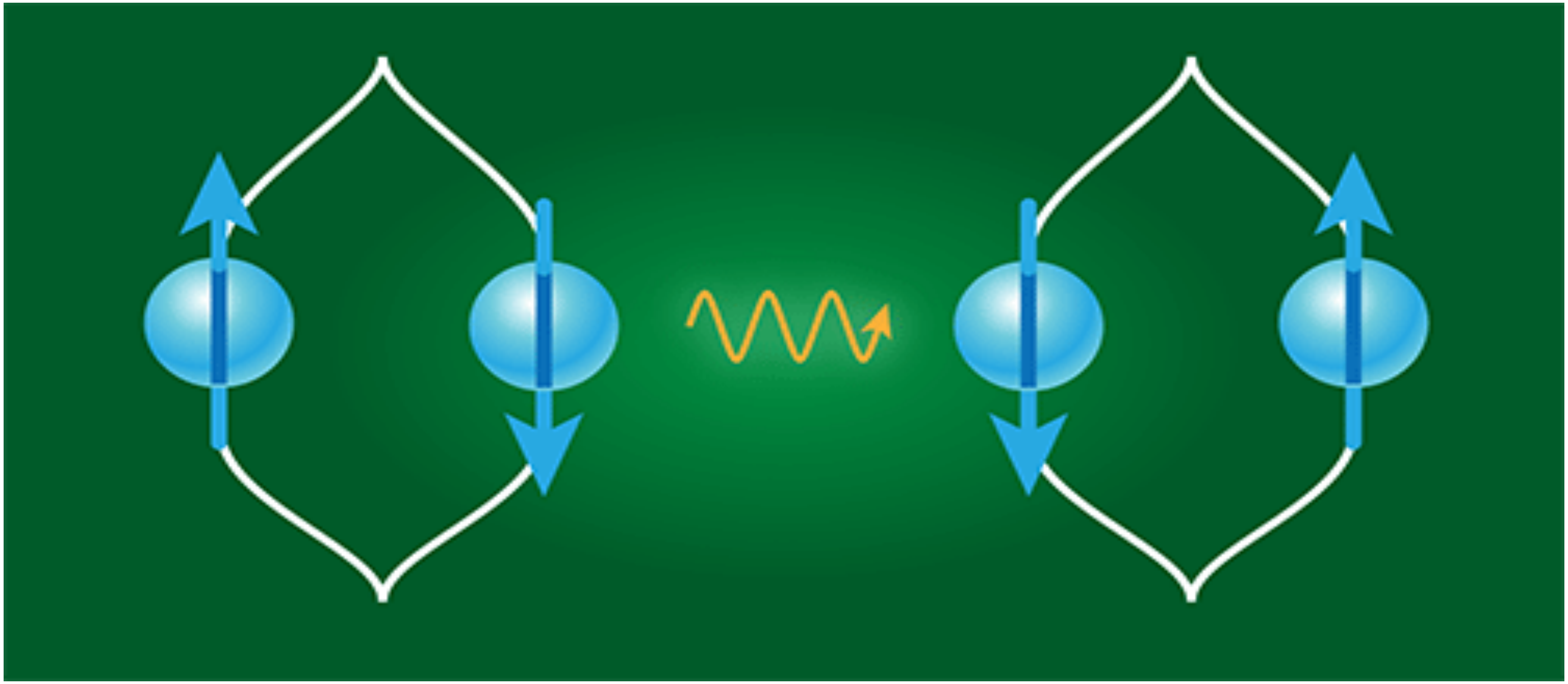
Quantum Features Exploitable in Table Top Fundamental Physics (Superposition, Entanglement, Squeezing Back Action Evasion & more)

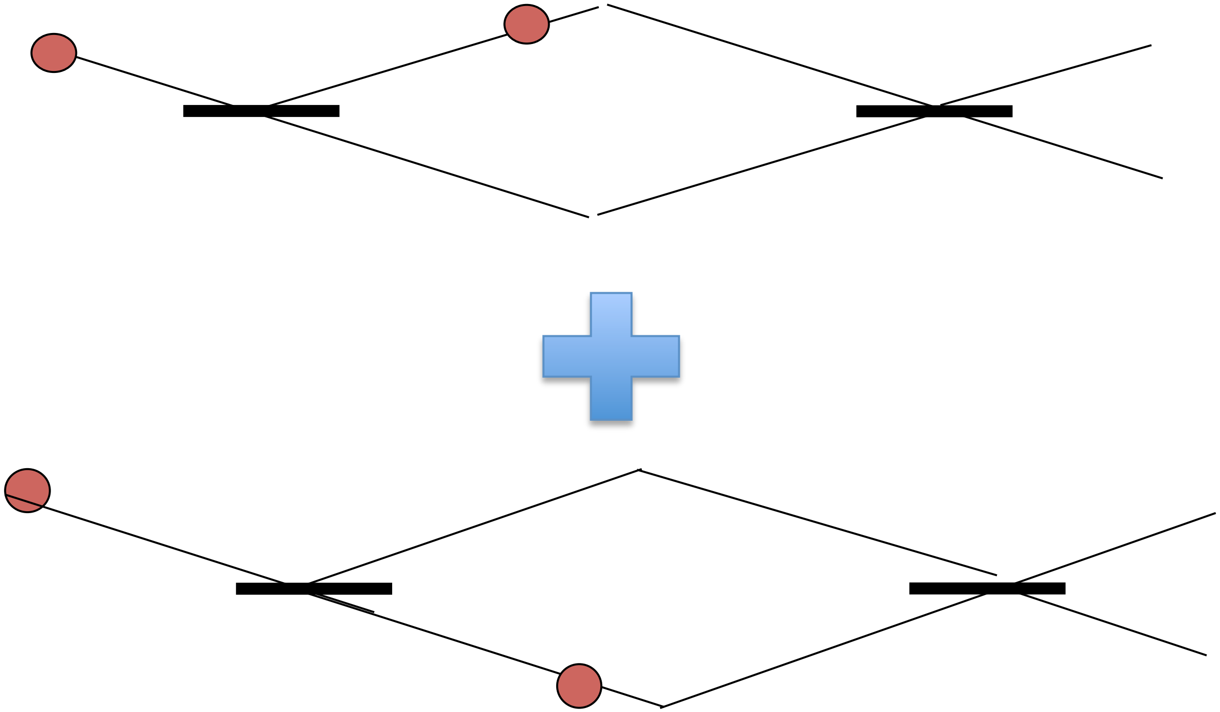
Sougato Bose

University College London



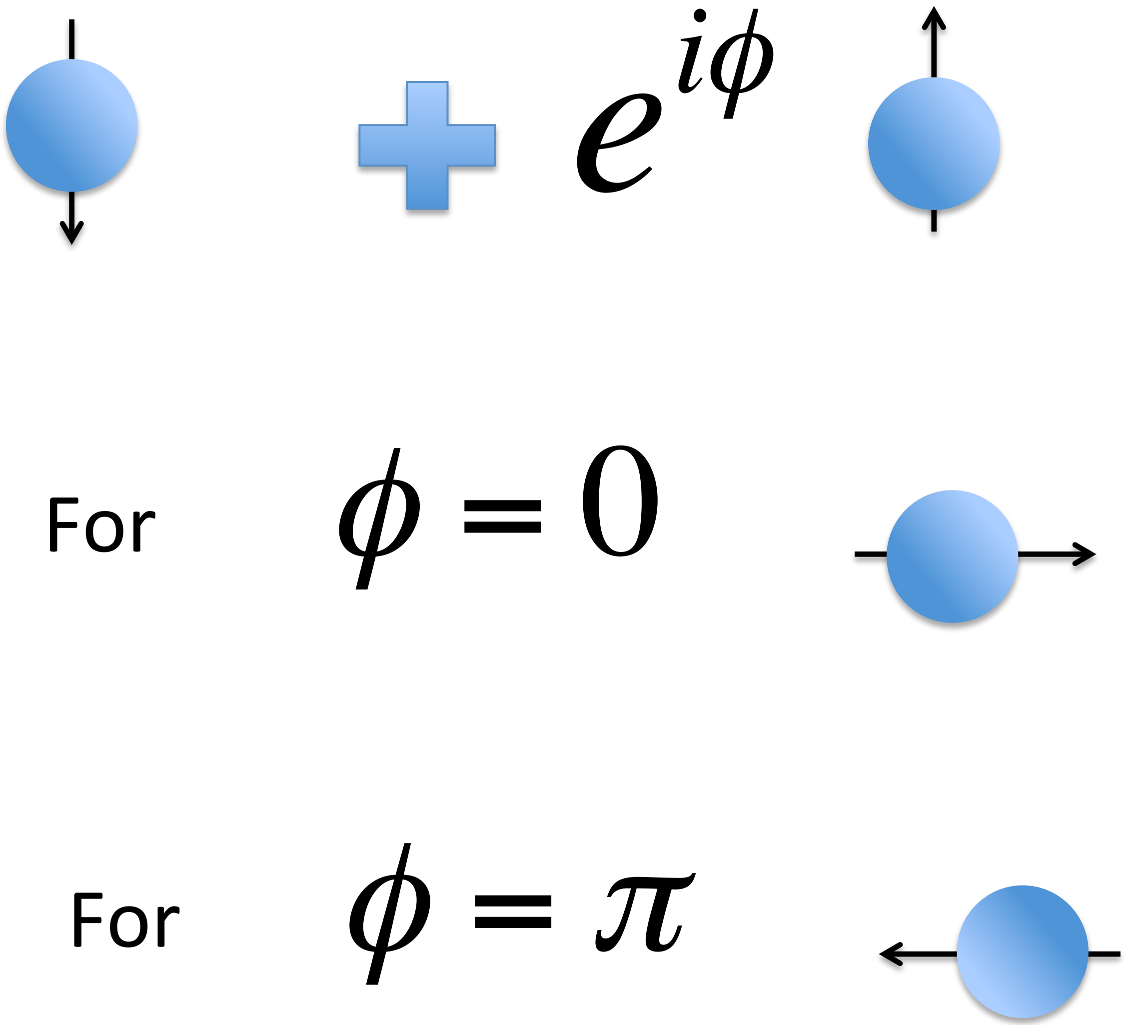
The Superposition Principle **Underpins** Quantum Mechanics

Very familiar
in experiments



If you *decohere* (kill superpositions) nonclassical features of quantum mechanics go away. Even old quantum mechanics: the right difference between energy levels obtained only through a superposition of localized states.

To understand/evidence superposition you have to control the phase

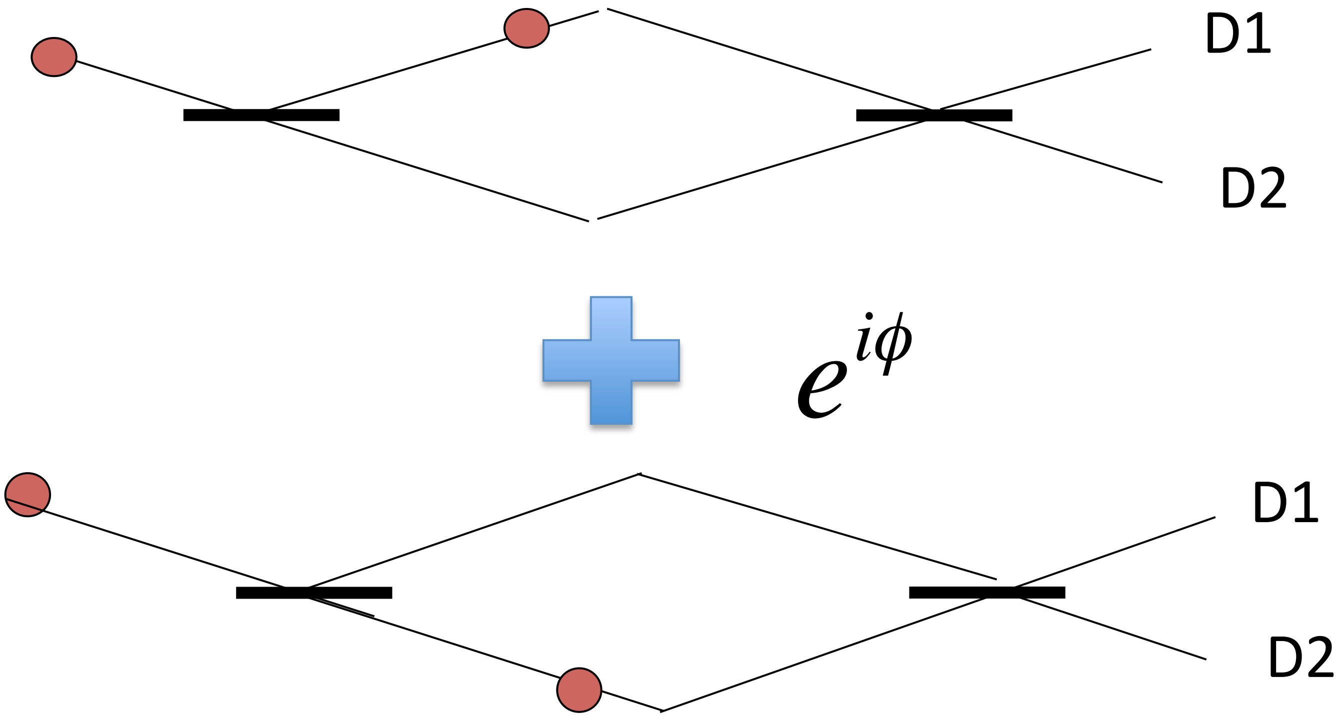


$+ e^{i\phi}$

For $\phi = 0$

For $\phi = \pi$

To understand/evidence superposition you have to control the phase



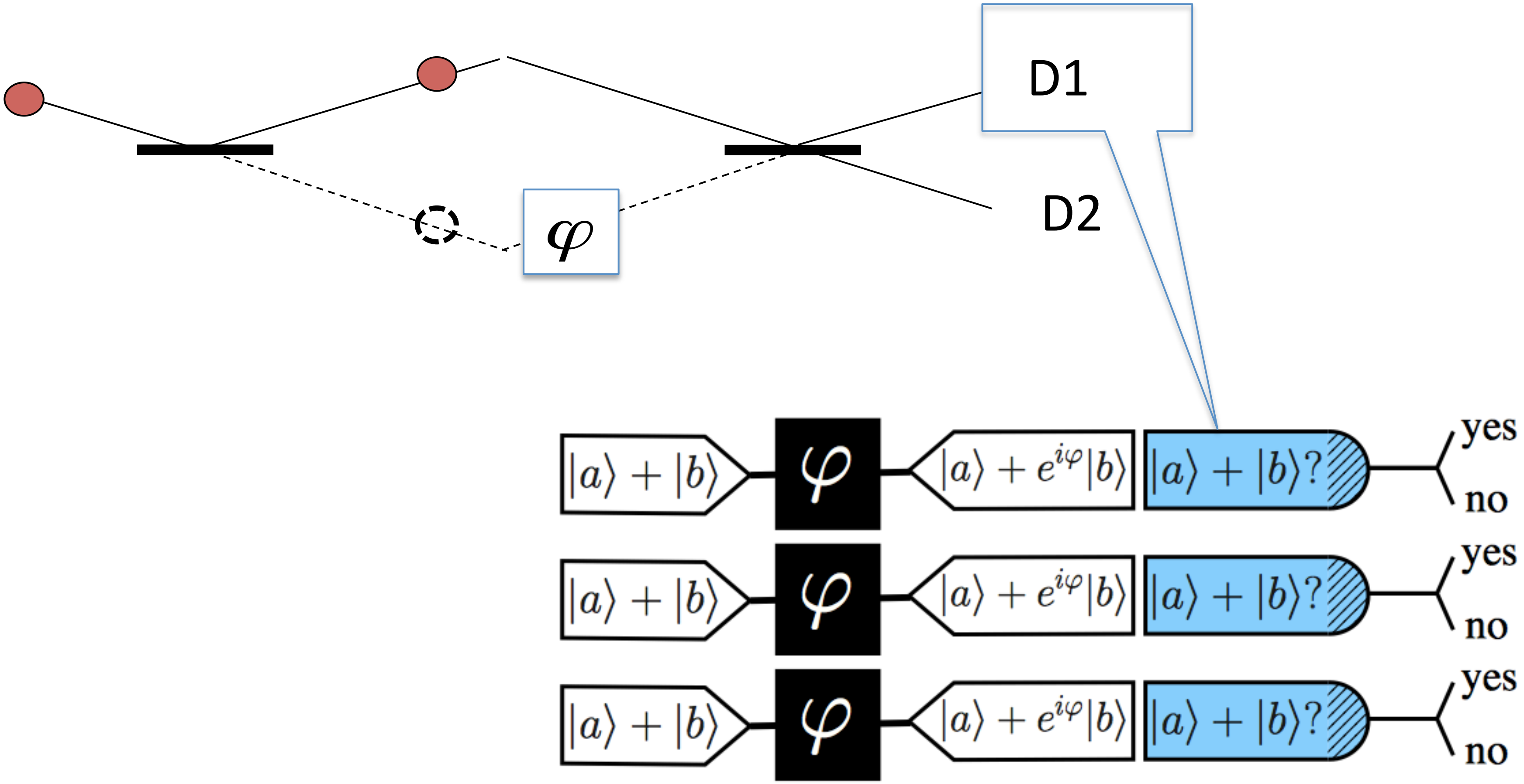
For $\phi = 0$ D1 Clicks

For $\phi = \pi$ D2 Clicks

If the phase is randomized,
all evidence of
Superposition goes
Away: *Decoherence*

e.g., Due to
back-ground atoms,
Black-body radiation
etc.

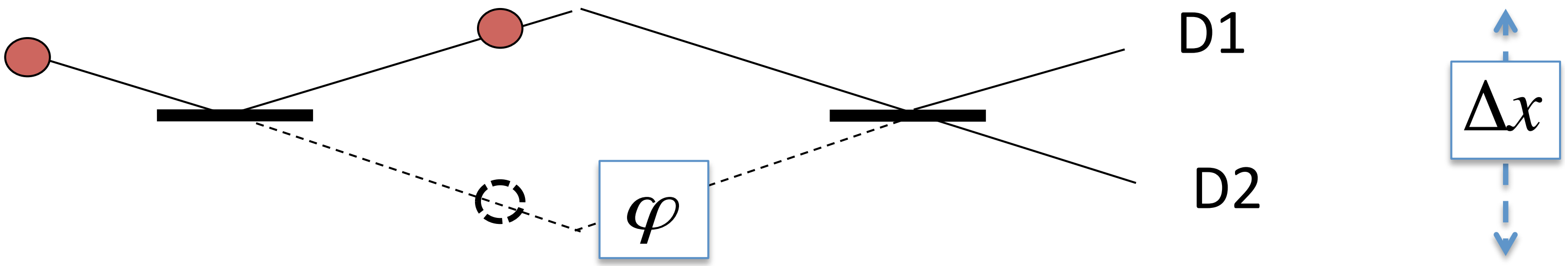
How a force can be sensed by quantum superpositions?



$$p = \frac{\#yes(\text{in } n \text{ repetitions})}{n} \to \frac{1-\cos\varphi}{2}$$

$$\delta\varphi_n = \sqrt{\frac{p(1-p)}{n}} / \left|\frac{\partial p}{\partial\varphi}\right| = n^{-1/2}$$

How a force can be sensed by quantum superpositions?



$$\phi = \frac{F \Delta x \, \tau}{\hbar} \Rightarrow \delta F \sim \delta\phi_n \frac{\hbar}{\Delta x \, \tau} \sim \frac{\hbar}{\sqrt{n} \, \Delta x \, \tau}$$

For micron superpositions, {10^(-28)/sqrt(n)} Newtons/root(Hz)

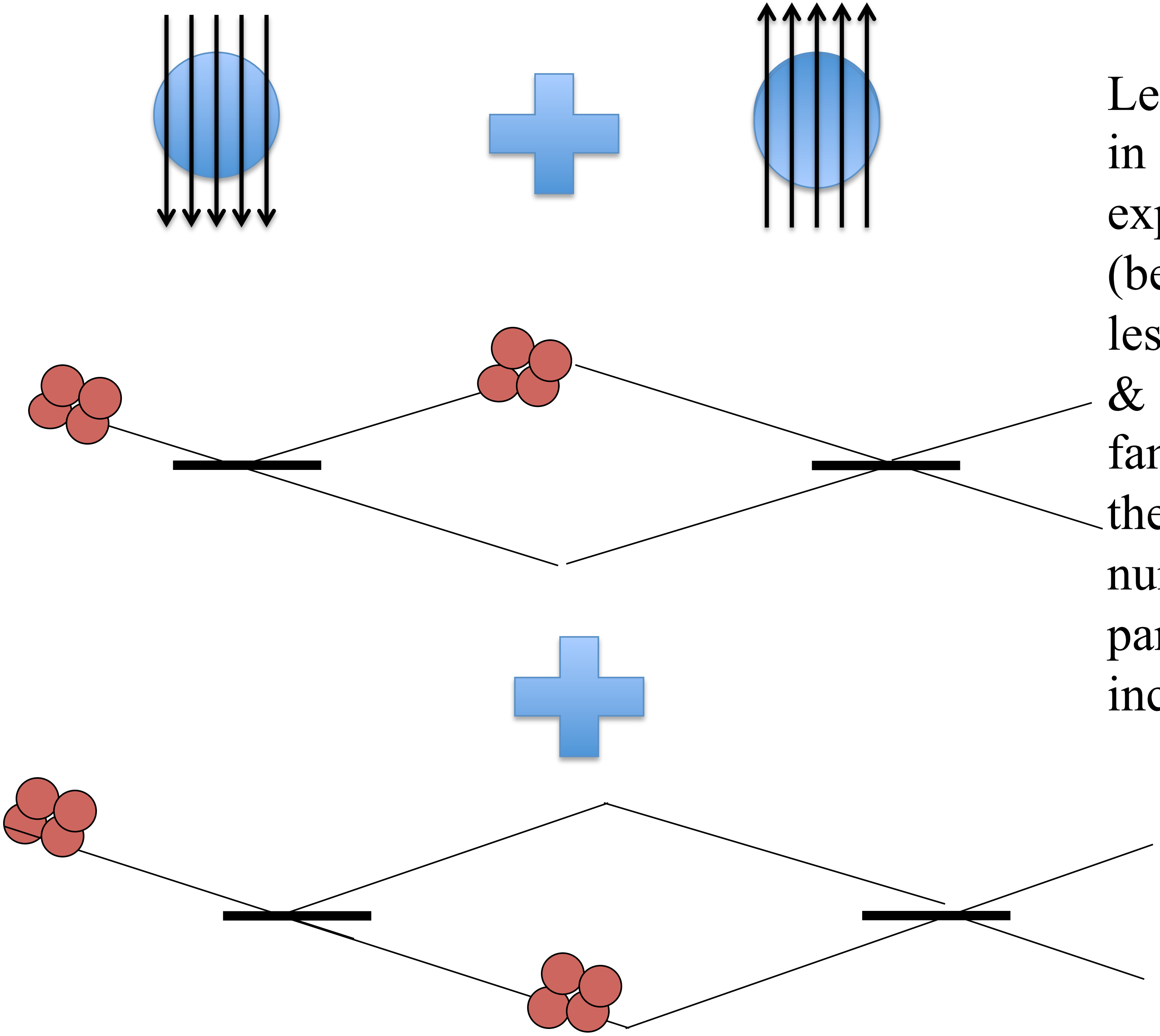
Also **momentum** sensing at the level of the uncertainty principle.

The force to be sensed may depend on an **extensive property** of the system: e.g. Mass, Volume, Surface Area etc. Then,

$$F = mg \Rightarrow \delta g \sim \frac{\hbar}{m\sqrt{n} \, \Delta x \, \tau}$$

For micron superpositions of microspheres, {10^(-14)/ sqrt(n)} m s^(-2)/root(Hz)

This motivates superpositions of larger and larger objects!



Less familiar in experiments (becomes less & less familiar as the number of particles increase)

Such superpositions are also called **GHZ** states or NOON states or **Schroedinger** Cat States

Why do we need to stretch the domain of the superposition principle?

(a) The enhanced sensing application we just pointed out.

(b) We need to understand whether it has any boundaries or whether it holds at all scales & just difficult
to see because of *decoherence* (there are strong beliefs on either side – better to be agnostic and look for experiments).

(c) It is always a winning game: If we can
extend one aspect of the domain e.g. mass, we can extend certain other aspects as well (i.e., use those tools
to stretch quantum attributes further. Eg. Applications to testing **quantum nature of gravity**).

Does quantum mechanics break down when mass becomes large enough? (to explain the Quantum Measurement problem)

Karolyhazy (1967), GRWP (1979), Diosi (1980s), Penrose (1980s)

The general idea is a smooth extrapolation. So larger masses and larger superposition scales collapse faster (superpositions are also a sensor for new fundamental modifications of the Schroedinger equation).

$$d|\psi\rangle_t \;=\; \left[\underbrace{-\frac{i}{\hbar}H\,dt}_{\textbf{quantum}} \;+\; \underbrace{\sqrt{\lambda}(A-\langle A\rangle_t)dW_t-\frac{\lambda}{2}(A-\langle A\rangle_t)^2dt}_{\textbf{collapse}}\right]|\psi\rangle_t$$

$\langle A\rangle_t \;=\; \langle\psi_t|A|\psi_t\rangle \;\longrightarrow$ **nonlinear**

$\gamma = \text{collapse strength}$ $\qquad$ $r_C = \text{localization resolution}$

About 100 nm superpositions, and about ~10^{9} amu masses: strongest collapse models.

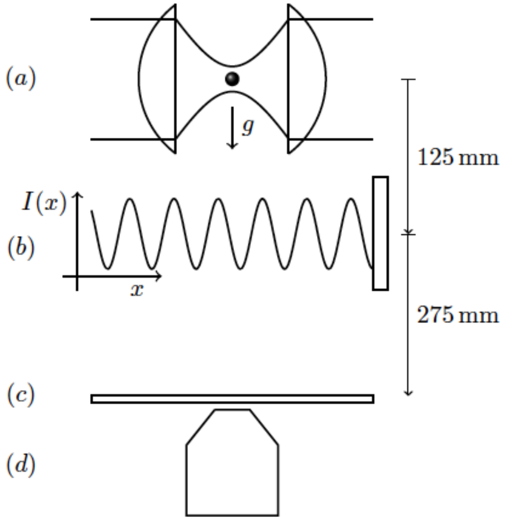
Bassi, Lochan, Satin, Singh, Ulbricht, RMP (2013)

Much of the collapse models are now constrained, and being further constrained by various approaches: e.g., anomalous noise in a cooled trapped object.

M. Bahrami, M. Paternostro, A. Bassi and H. Ulbricht
***Proposal for Non-interferometric Test of Collapse Models in Optomechanical Systems**, PRL* **112**, 210404 *(2014).*

Collaboration between Barker & Ulbricht groups.

What are the ideas being pursued for creating and testing superpositions? Mainly Matter wave interferometry with nano and microspheres:



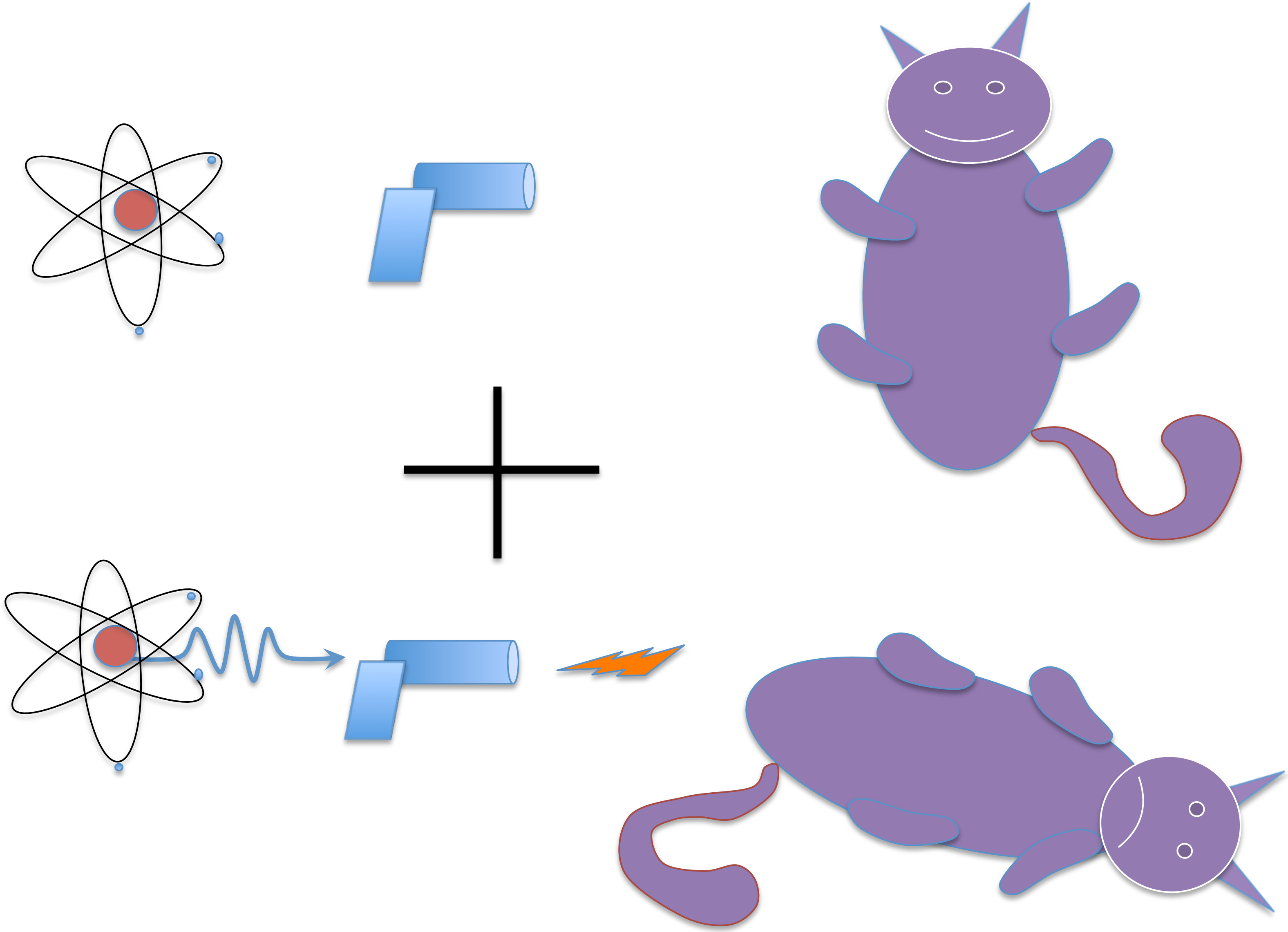
Other approaches: Double slit via x^2 measurements – Romero-Isart; Aspelmeyer; Vanner.

$$\lambda_{dB} \sim \frac{h}{mv} \sim d$$

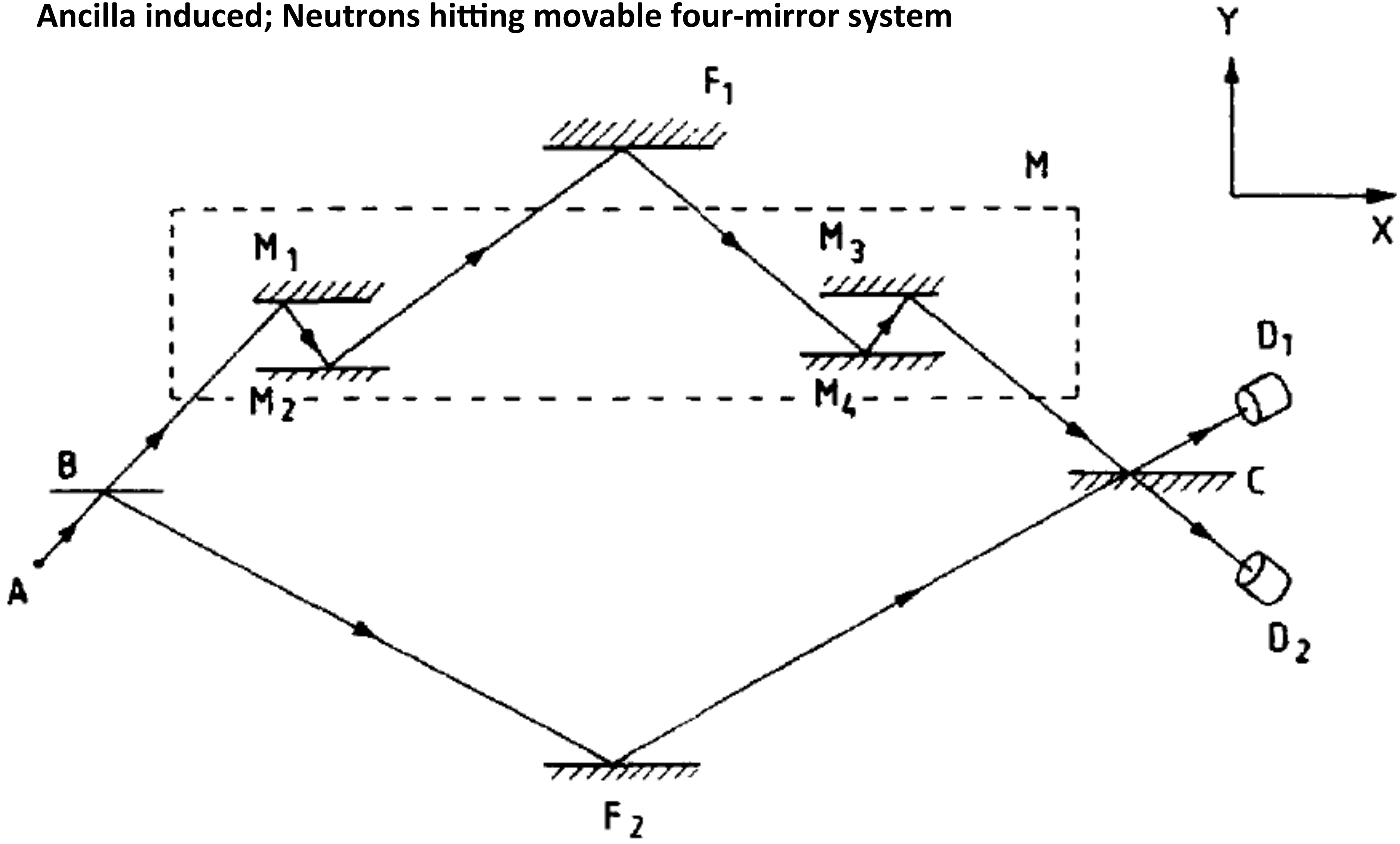
Needed (cooling, Sharp position measurements, Low mass Dispersions)

Bateman, J., S. Nimmrichter, K. Hornberger, and H. Ulbricht
Near-field interferometry of a free-falling nanoparticle from a point-like source
Nature Communications 4, 4788 (2014). (Extending Arndt approach)

How to create the macroscopic superpositions (earliest idea is Schroedinger's Nucleo-Biological mechanism). **Coherent ancilla induced.**

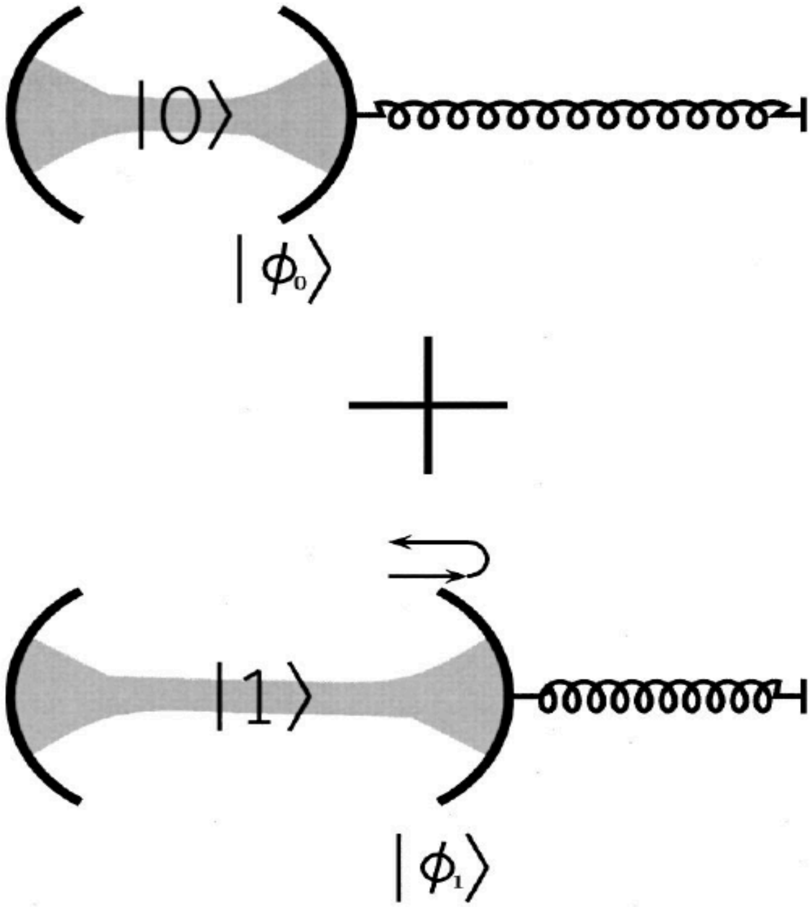


Ancilla induced; Neutrons hitting movable four-mirror system



D. Home & S. Bose, Physics Letters A **217,** 209 (1996); Based on quantum erasure setup of Greenberger and Yasin.

Superpositions of States of a Macroscopic Object using an Ancillary Quantum System:



Ancilla-only probing: Difficult to satisfy a skeptical person: Alternatives --Asadian, Brukner, Rabl. PRL 2013

S. Bose, K. Jacobs, P. L. Knight,
Phys. Rev. A 59 (5), 3204 (1999). [arXiv: 1997].
Decoherence/partial coherence is used to certify superposition.

Armour, Blencowe, Schwab, PRL 2002.
Marshall, Simon, Penrose, Bouwmeester, PRL 2003.
Decoherence & Recoherence is used to certify superpositions

Bose, PRL 2006.

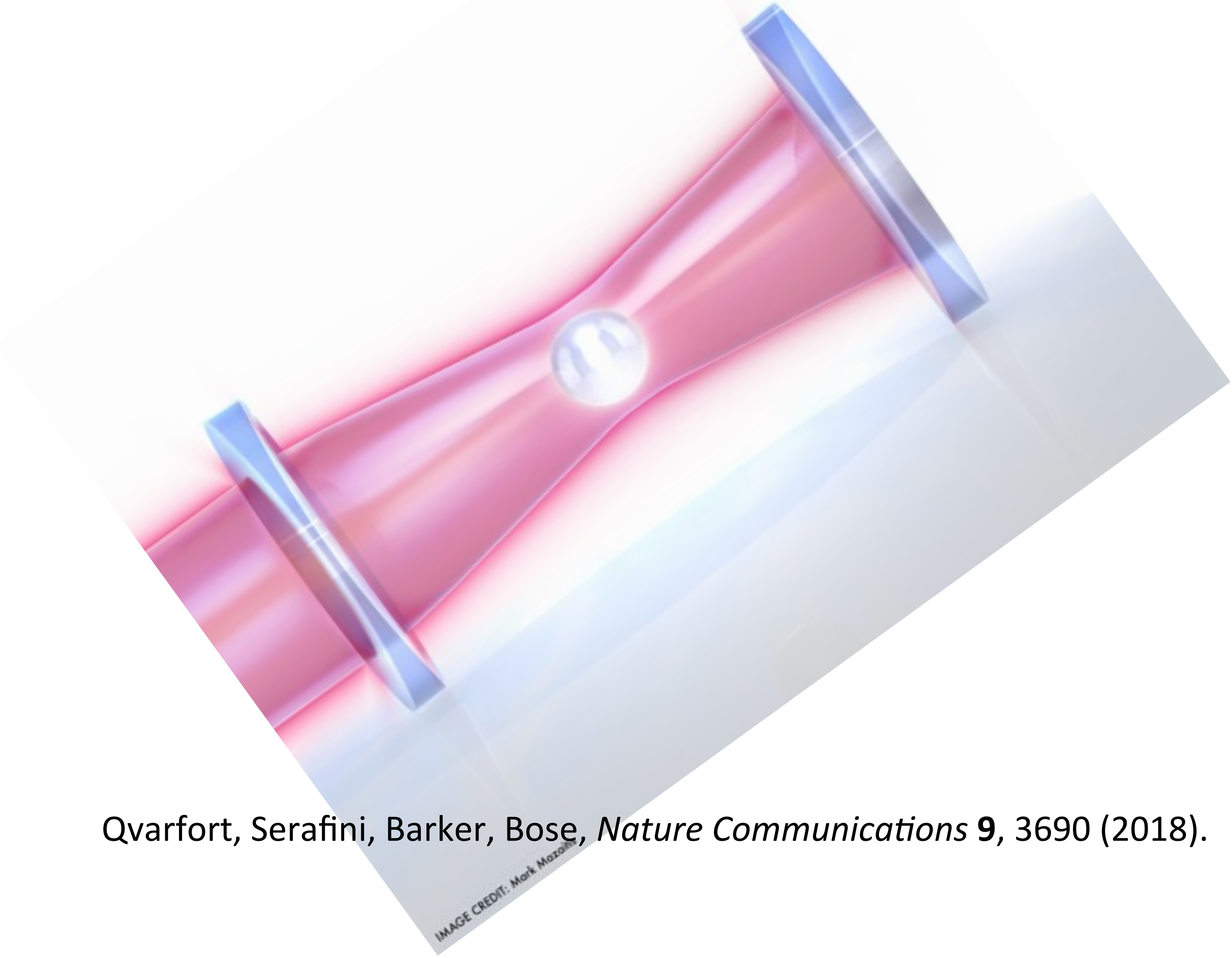
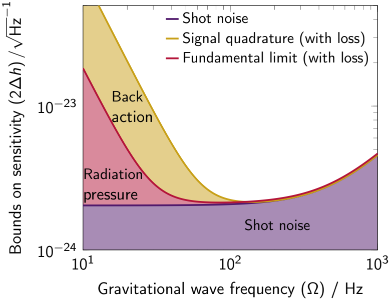


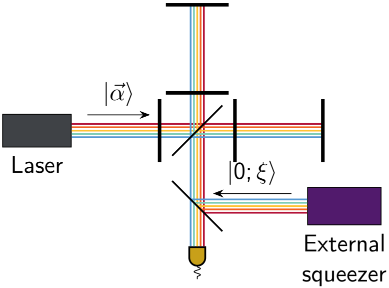
IMAGE CREDIT: Mark Mazaitis

Qvarfort, Serafini, Barker, Bose, *Nature Communications* **9**, 3690 (2018).

Fundamental limits of multi-carrier optomechanical sensors

Optomechanical systems are good at resolving small forces and displacements

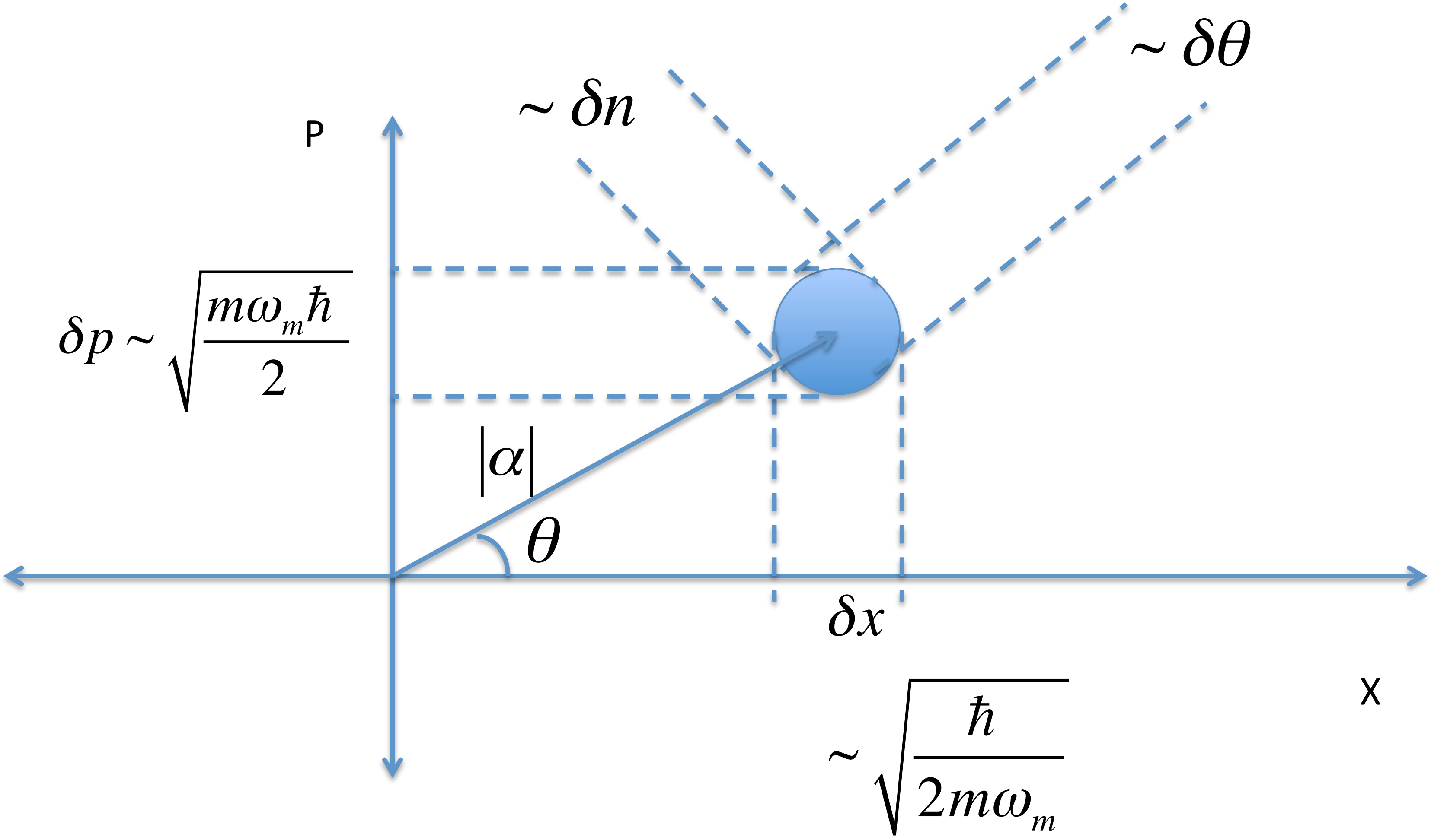




- With loss, noise due to ponderomotive squeezing pervades the fundamental limit
- Optimal interferometer configuration only uses a single-mode

Branford, Miao, and Datta *Phys. Rev. Lett.*, **121** 110505 (2018)

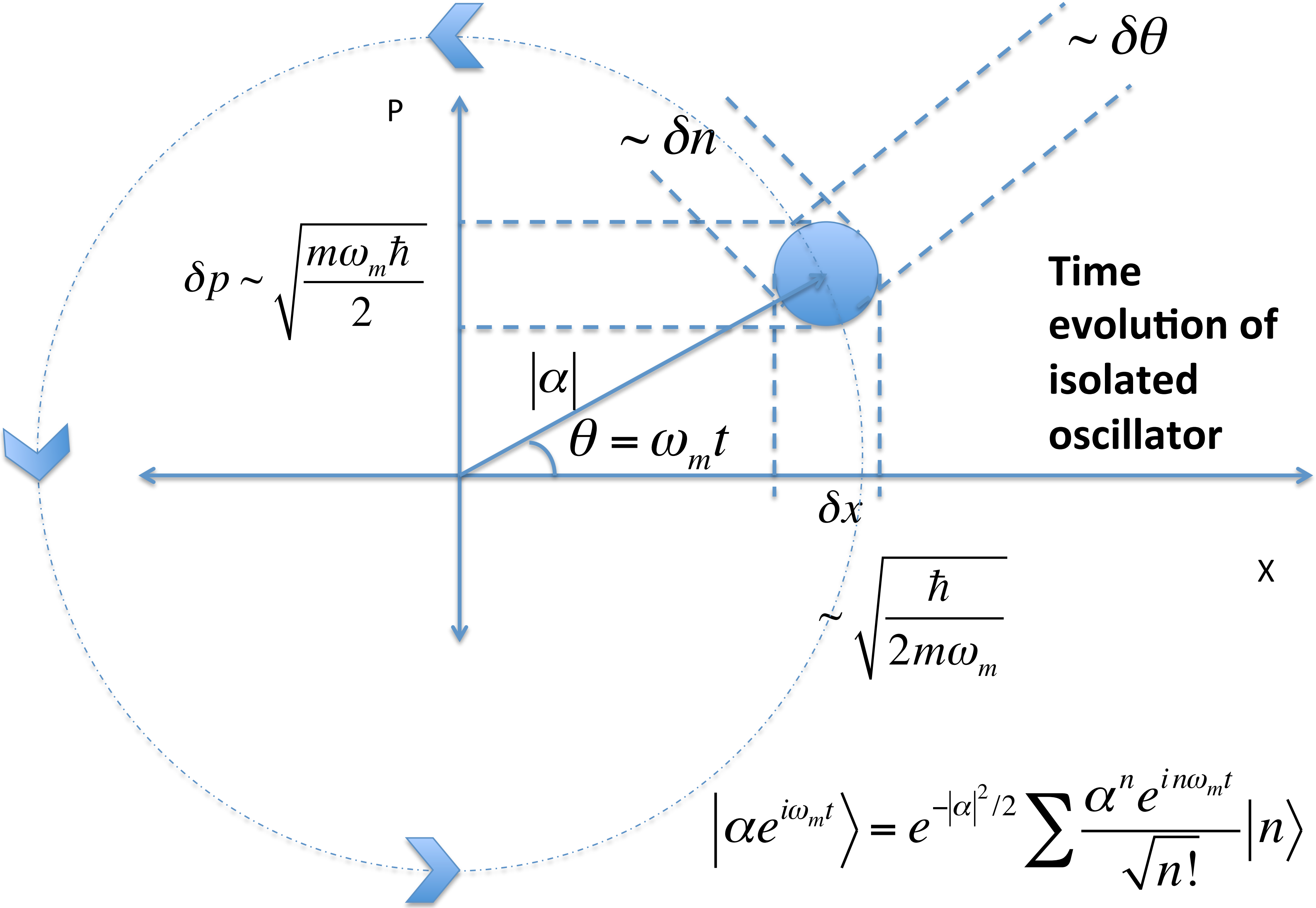
Quantum states of a harmonic oscillator (coherent states)



Represented by complex number: $\alpha = |\alpha| e^{i\vartheta}$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

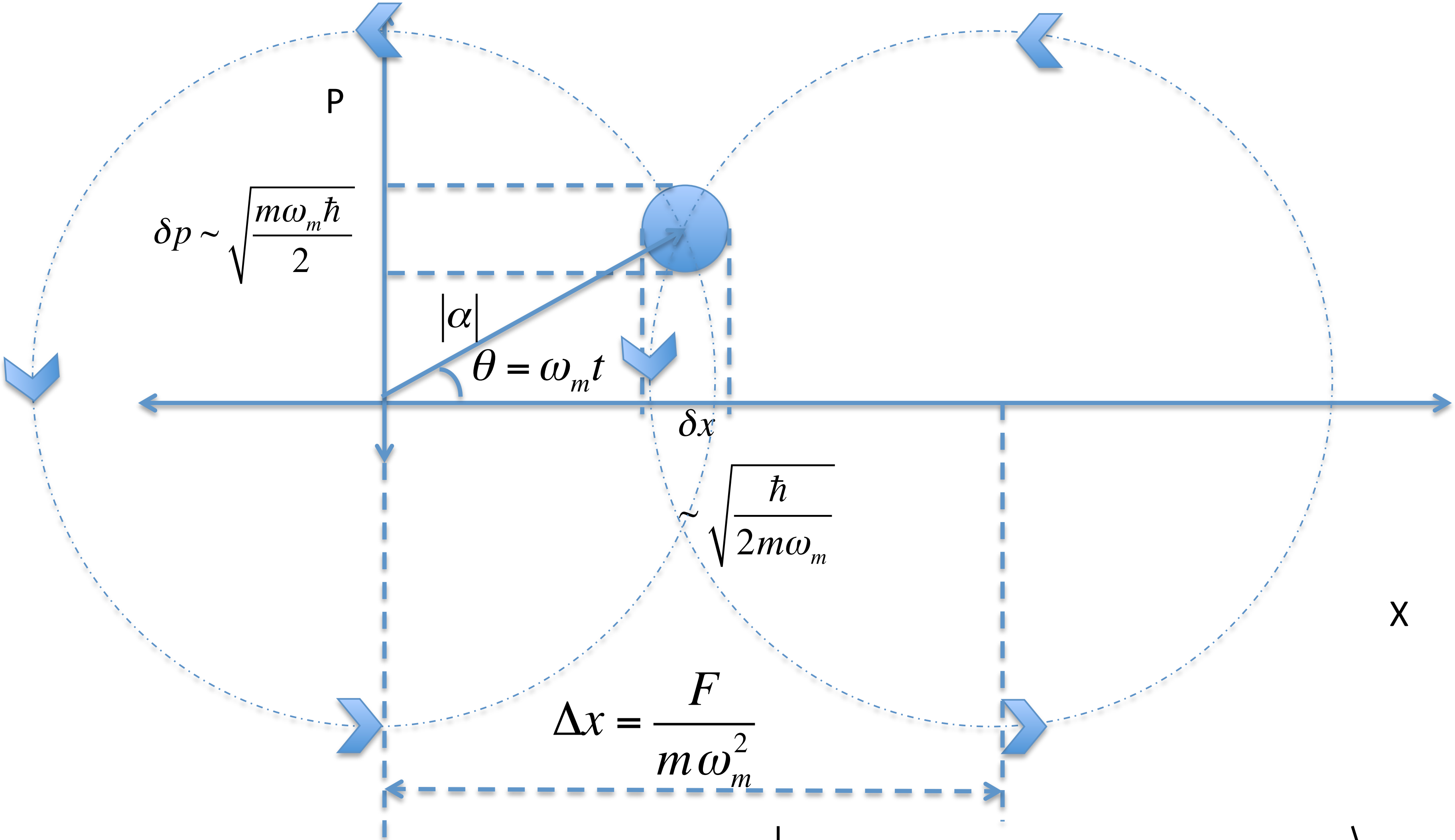
Quantum states of a harmonic oscillator (coherent states)



Time evolution of isolated oscillator

$$\left|\alpha e^{i\omega_m t}\right\rangle = e^{-|\alpha|^2/2} \sum \frac{\alpha^n e^{in\omega_m t}}{\sqrt{n!}} |n\rangle$$

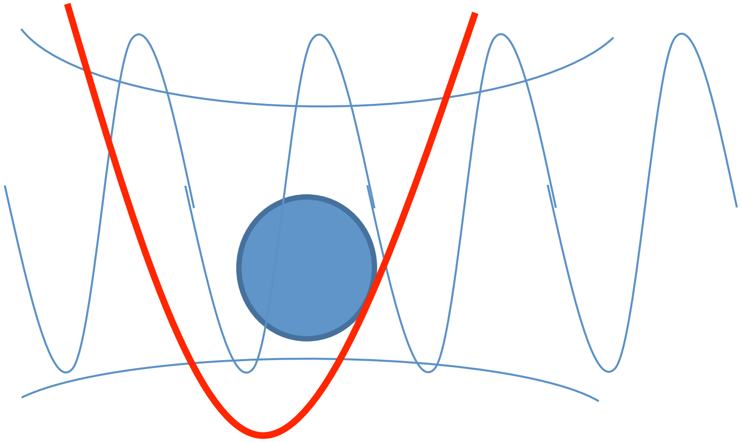
Force sensing by a mechanical harmonic oscillator (coherent states)

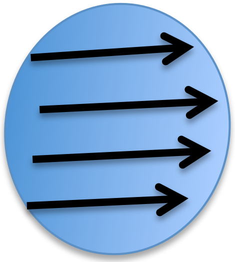


Time Evolution: $$\left|\alpha e^{i\omega_m t} + \frac{F}{m\omega_m^2 \delta x}(1 - e^{i\omega_m t})\right\rangle$$

Ramsey Interferometry with a Levitated Thermal Mesoscopic Object

Diamond bead trapped in an optical trap. The bead contains a spin-1 NV center.



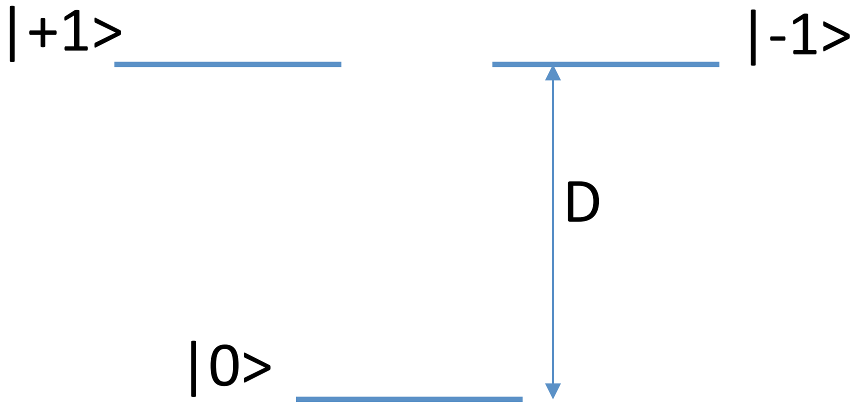


No cavity,
no cooling.

Exploits Spin-Motion coupling mechanism proposed by Rabl et.al. 2009.

Initial State:

$$|\beta\rangle|0\rangle$$



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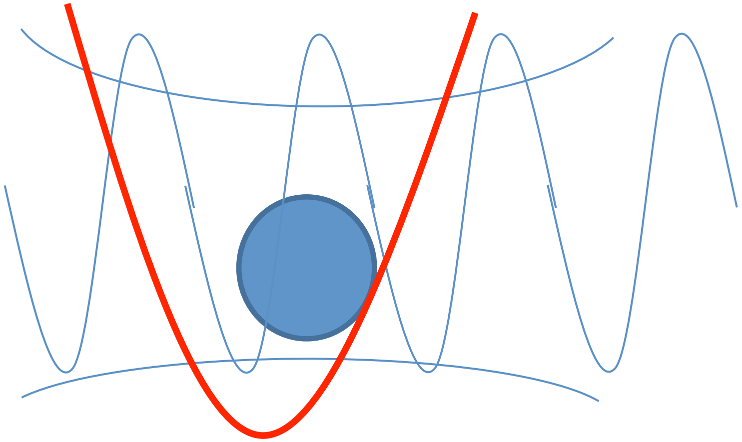
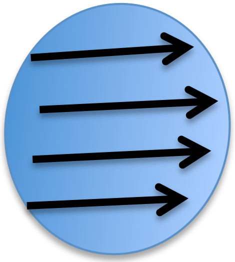
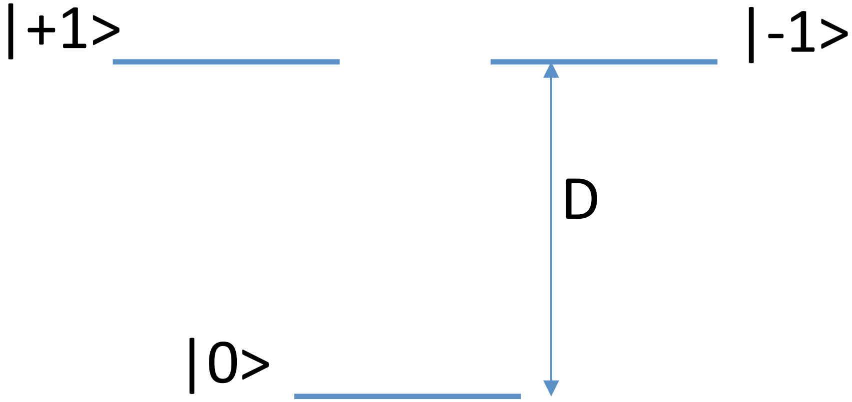




Step 1:

$$|\beta\rangle\Big(|+1\rangle+|+1\rangle\Big)$$



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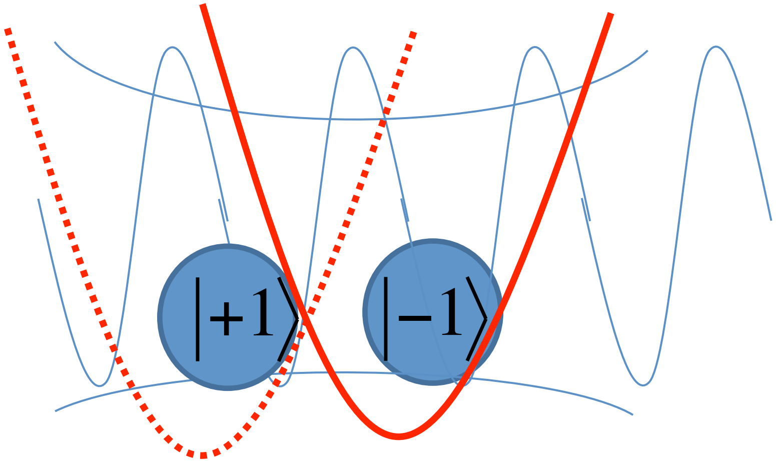
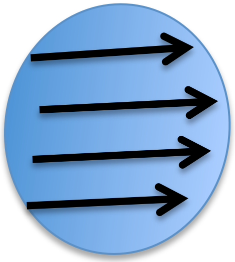
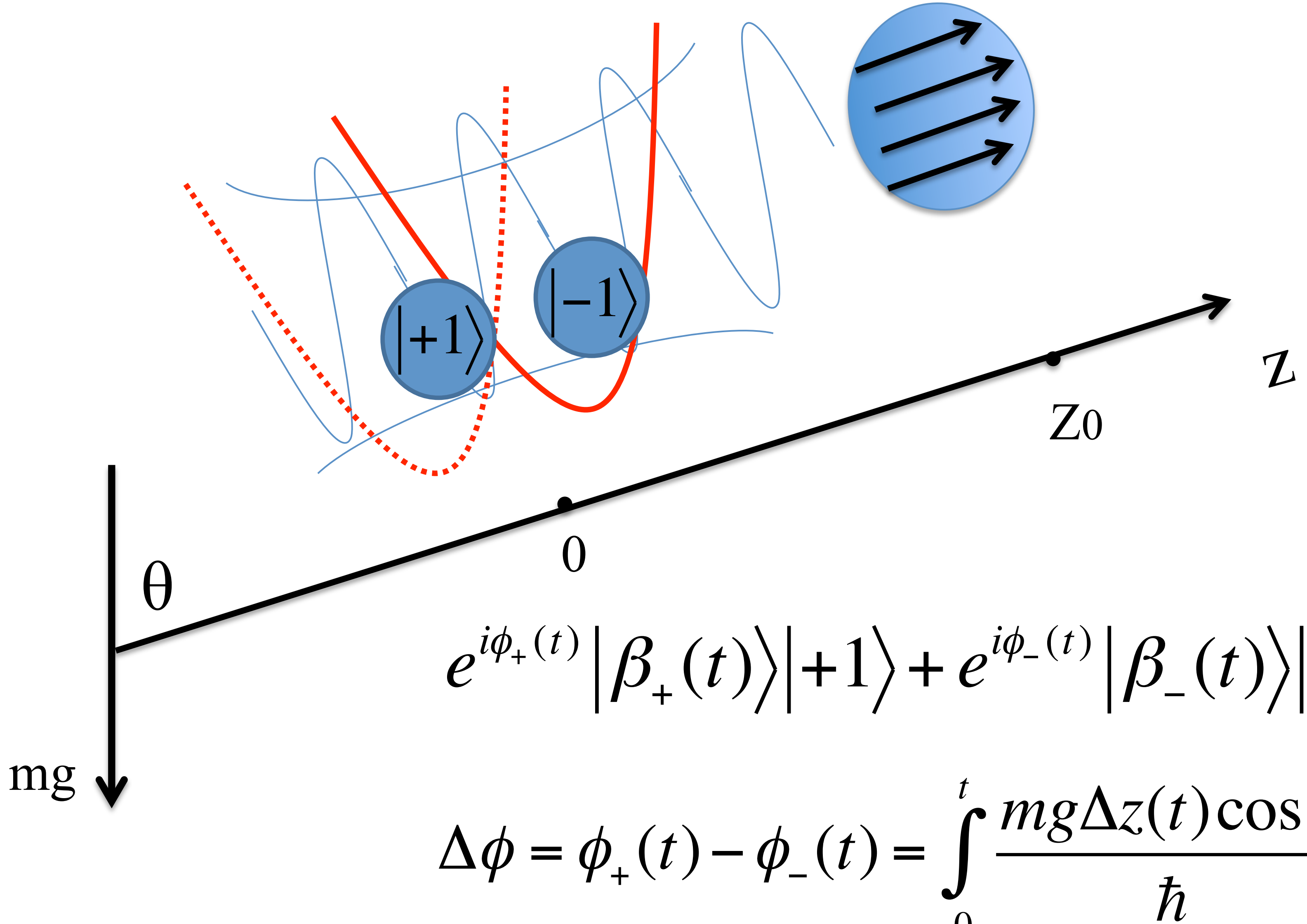




Time Evolution:

$$e^{i\phi_+(t)}\left|\beta_+(t)\right\rangle\left|+1\right\rangle + e^{i\phi_-(t)}\left|\beta_-(t)\right\rangle\left|-1\right\rangle$$

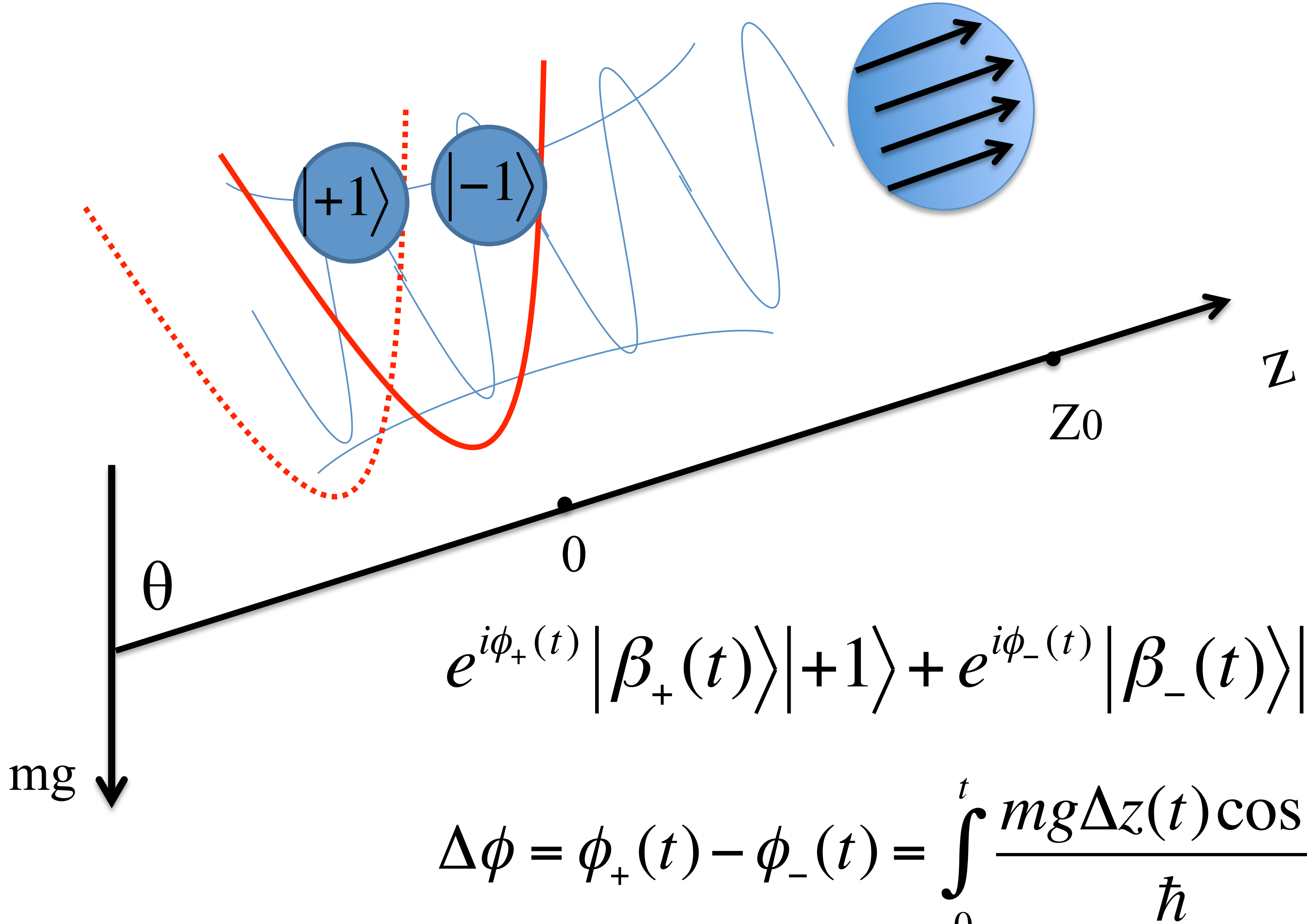
Ramsey Interferometry with a Levitated Thermal Mesoscopic Object



$$e^{i\phi_+(t)}\left|\beta_+(t)\right\rangle\left|+1\right\rangle + e^{i\phi_-(t)}\left|\beta_-(t)\right\rangle\left|-1\right\rangle$$

$$\Delta\phi = \phi_+(t) - \phi_-(t) = \int_0^t \frac{mg\Delta z(t)\cos\theta}{\hbar}dt$$

Ramsey Interferometry with a Levitated Thermal Mesoscopic Object



$$e^{i\phi_+(t)}\left|\beta_+(t)\right\rangle\left|+1\right\rangle + e^{i\phi_-(t)}\left|\beta_-(t)\right\rangle\left|-1\right\rangle$$

$$\Delta\phi = \phi_+(t) - \phi_-(t) = \int_0^t \frac{mg\Delta z(t)\cos\theta}{\hbar}dt$$

Measuring the relative phase shift between superposed components

Step 3: apply the same very rapid mw pulse as in step 1,

The presence of $\Delta\phi$ gives a modulation of the population of $|S_z=0>$ according to:

$$\left|+1\right\rangle + e^{i\Delta\phi}\left|-1\right\rangle \rightarrow \cos\frac{\Delta\phi}{2}\left|0\right\rangle + \ldots$$

For *m*= 10^10 amu (nano-crystal), superposition over 1 pm,
the phase ~ O(1)

- M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, Phys. Rev. Lett. **111**, 180403 (2013).
- Comment: F. Robicheaux, Phys. Rev. Lett. 118, 108901 (2017).
- Response: S. Bose et al, Phys. Rev. Lett. 118, 108902 (2017).

How can we increase the scale of the superposition?

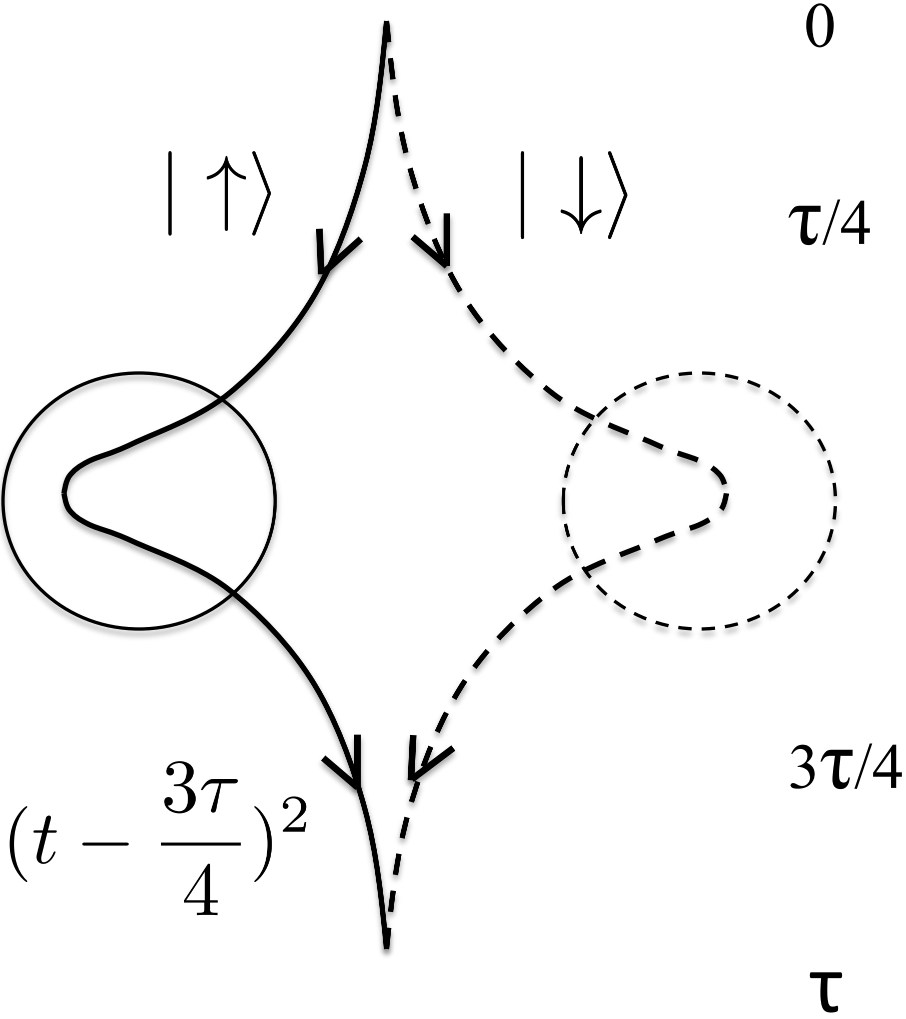
C. Wan, M. Scala, G. W. Morley, ATM. A. Rahman, H. Ulbricht, J. Bateman, P. F. Baker, S. Bose, M. S. Kim, Phys. Rev. Lett. 117, 143003 (2016).

Free particle in an inhomogeneous magnetic field (acceleration +*a* or –*a*)

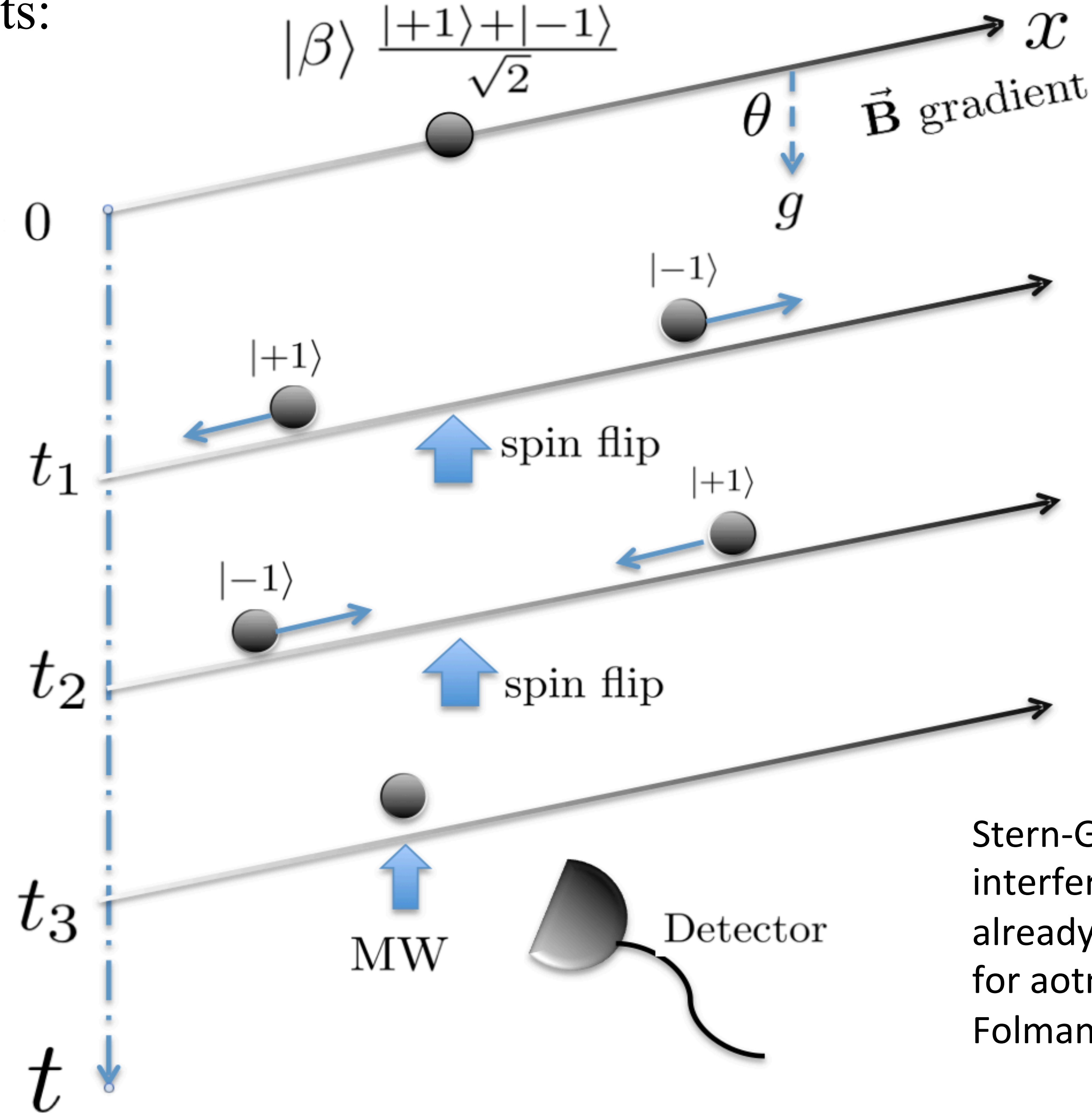
$$x_\sigma(t,j) = x_j(0) \pm \frac{1}{2}at^2$$

$$= \frac{a\tau}{4}(t - \frac{\tau}{4}) \mp \frac{1}{2}a(t - \frac{\tau}{4})^2$$

$$= \frac{1}{2}a(\frac{\tau}{4})^2 \mp \frac{a\tau}{4}(t - \frac{3\tau}{4}) \pm \frac{1}{2}a(t - \frac{3\tau}{4})^2$$



Free flight scheme able to achieve 100 nm separation among superposed components:



Stern-Gerlach interferometry already demonstrated for aotms by Folman et. al.

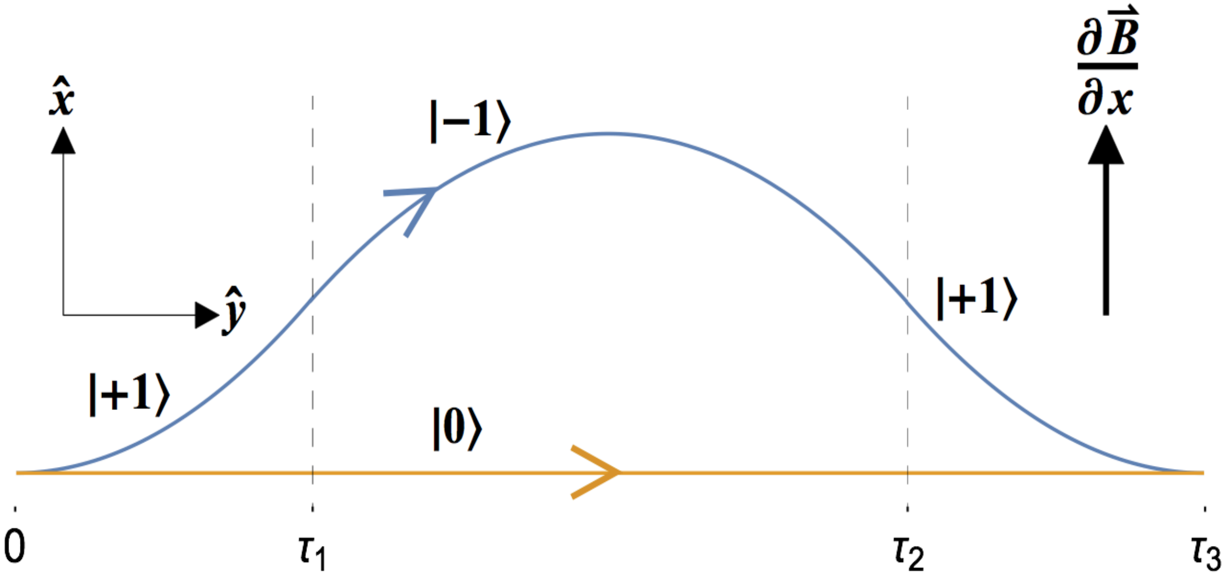
$$|\Psi(t_3)\rangle = \frac{1}{\sqrt{2}}|\psi(t_3)\rangle(|+1\rangle + e^{-i\phi_g}|-1\rangle)$$

$$\langle x|\psi(t_3)\rangle = e^{-ip_0x}e^{-[(x-x_0-p_0t_3/m-g\cos\theta t_3^2/2)^2/2(\sigma')^2]}$$

$$\phi_g = (1/16\hbar)gt_3^3 g_{\mathrm{NV}}\mu_B(\partial B/\partial x)\cos\theta$$

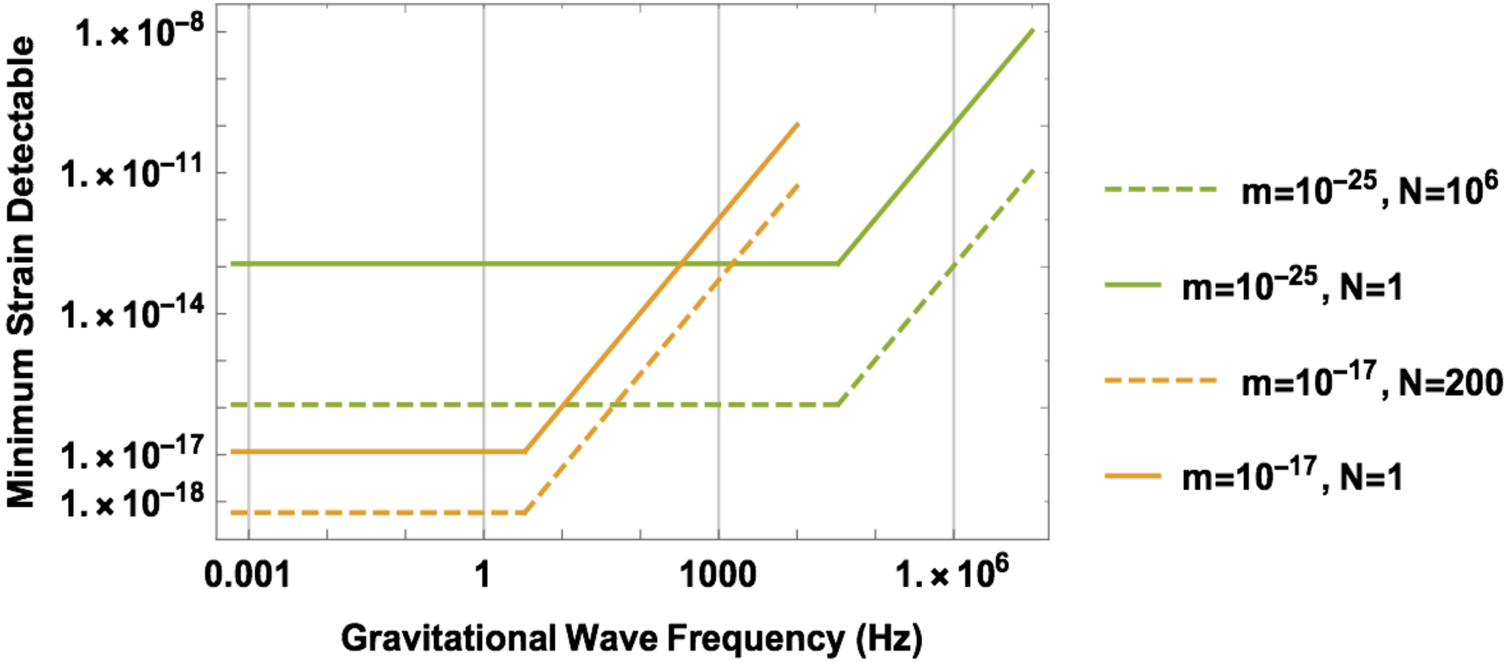
$$\Delta x_M = 2\times\frac{1}{2m}g_{\mathrm{NV}}\mu_B\frac{\partial B}{\partial x}(t_3/4)^2$$

10^10 amu mass can be placed in a superposition of states separated by 100 nm.

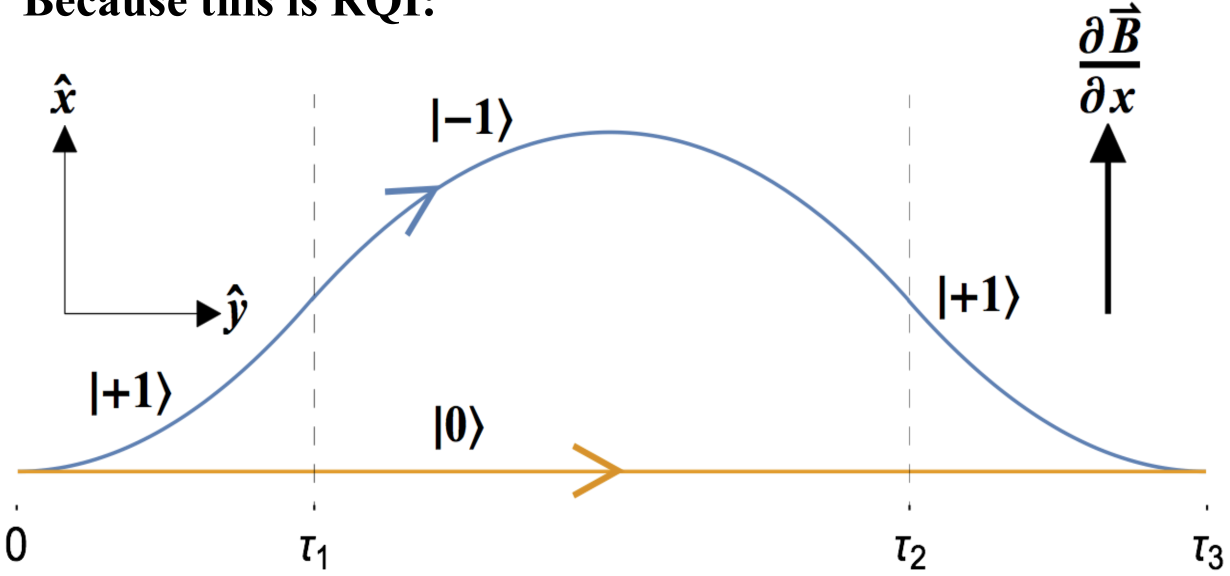


Compact meter scale
detectors for
Gravitational waves:

Ryan J. Marshman,
Anupam Mazumdar,
Gavin W. Morley, Peter F.
Barker, Steven Hoekstra,
Sougato Bose,
arXiv:1807.10830



Because this is RQI:



Compact meter scale detectors for Gravitational waves:

Ryan J. Marshman, Anupam Mazumdar, Gavin W. Morley, Peter F. Barker, Steven Hoekstra, Sougato Bose, arXiv:1807.10830

Mesoscopic Interference for Metric & Curvature (MIMAC)

10^(-17) kg

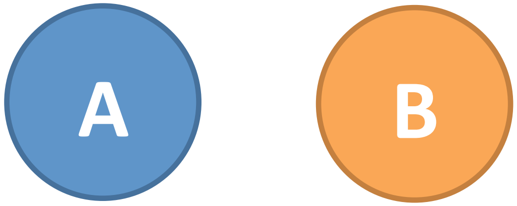
10^(-19)

1 ms^(-1)

$$S \approx m \int \left[c^2 \left(1 - \frac{h_{00}}{2} \right) - c h_{0j} v^j - (\eta_{ij} + h_{ij}) \frac{v^i v^j}{2} \right] dt$$

1 s

Bipartite Systems



Separable pure states: $\left|\psi\right\rangle_{AB} = \left|\alpha\right\rangle_A \otimes \left|\beta\right\rangle_B$

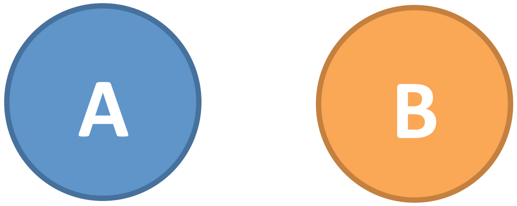
$$\left|\psi\right\rangle_{AB} = \frac{\left|00\right\rangle + \left|01\right\rangle}{\sqrt{2}} = \left|0\right\rangle_A \otimes \left(\frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}\right)_B$$

Non-separable states are called entangled states

$$\left|\psi\right\rangle_{AB} \neq \left|\alpha\right\rangle_A \otimes \left|\beta\right\rangle_B$$

$$\left|\psi\right\rangle_{AB} = \frac{\left|00\right\rangle_{AB} + \left|11\right\rangle_{AB}}{\sqrt{2}}$$

Correlations in multiple bases



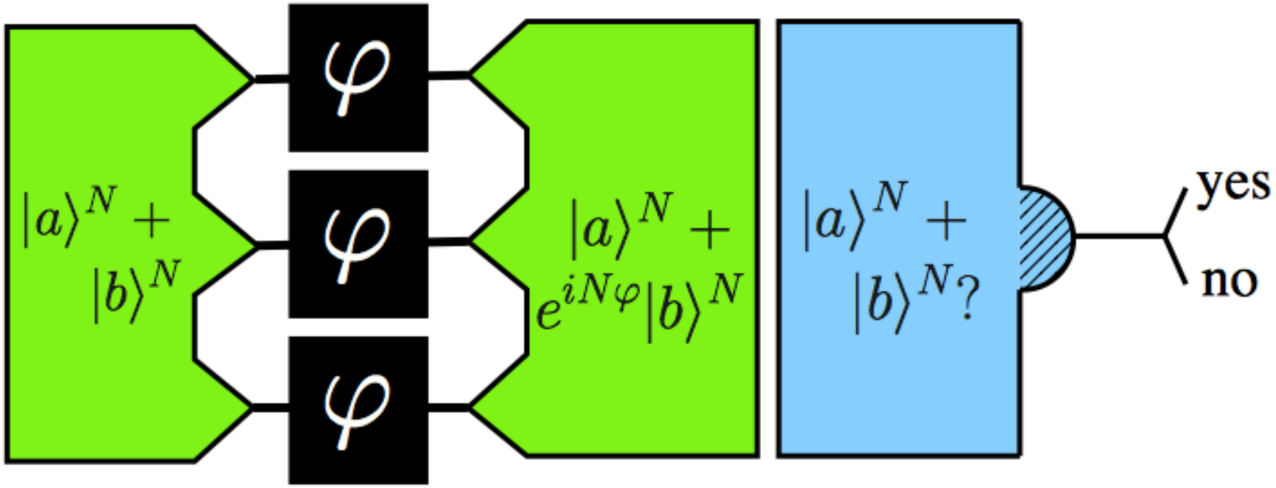
$$|\psi\rangle_{AB} = \frac{|00\rangle_{AB} + |11\rangle_{AB}}{\sqrt{2}} = \frac{|++\rangle_{AB} + |--\rangle_{AB}}{\sqrt{2}}$$

where $$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Entanglement Witness: (all witnesses will not evidence all quantum states!)

$$\langle \sigma_x^A \sigma_x^B \rangle + \langle \sigma_y^A \sigma_y^B \rangle > 1$$

Entangled states in sensing



$$p_{ent} = \frac{\#yes(\text{in } \nu \text{ repetitions})}{\nu} \to \frac{1-\cos N\varphi}{2}$$

$$\delta\varphi_n = \sqrt{\frac{p_{ent}(1-p_{ent})}{\nu}} / \left| \frac{\partial p_{ent}}{\partial \varphi} \right| = (nN)^{-1/2}$$

Review artice: Giovannetti, Lloyd, Maccone, Nat Photonics 2012

Also can get rid of common noise using: $|\psi\rangle_{AB} = \frac{|01\rangle_{AB} + |10\rangle_{AB}}{\sqrt{2}}$

Schmidt Decomposition

The most general state: $|\psi\rangle_{AB} = \sum_{i,j} \alpha_{ij} |i_A, j_B\rangle$

$$\langle i_A | i'_A \rangle = \delta_{ii'}, \qquad \langle j_B | j'_B \rangle = \delta_{jj'}$$

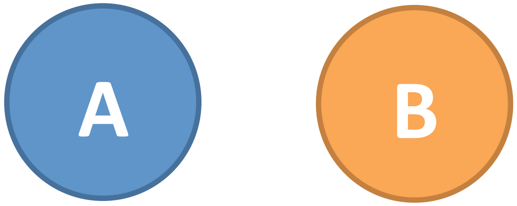
Schmidt basis:

$$|\psi\rangle_{AB} = \sum_{i,j} \alpha_{ij} |i_A, j_B\rangle_{AB} = \sum_{i} \lambda_i |\tilde{i}_A, \tilde{i}_B\rangle$$

Properties of Schmidt decomposition

$$\left\{ \begin{array}{l} \langle \tilde{i}_A | \tilde{i}_A' \rangle = \delta_{ii'}, \quad \langle \tilde{i}_B | \tilde{i}_B' \rangle = \delta_{ii'} \\ \lambda_i 's \text{ are real and positive (Schmidt coefficients)} \end{array} \right.$$

State of the Subsystem



$$\rho_A = Tr_B(\rho_{AB}) = \sum_i \langle i_B | \rho_{AB} | i_B \rangle$$

$$\rho_B = Tr_A(\rho_{AB}) = \sum_i \langle i_A | \rho_{AB} | i_A \rangle$$

Separable Pure States

Separable state: $|\psi\rangle_{AB} = |\alpha\rangle_{A} \otimes |\beta\rangle_{B}$

Subsystems:

$$\left\{\begin{array}{l} \rho_A = |\alpha\rangle\langle\alpha| \\ \\ \rho_B = |\beta\rangle\langle\beta| \end{array}\right. \Rightarrow \left\{\begin{array}{l} P_A = P_B = 1 \\ \\ S(\rho_A) = S(\rho_B) = 0 \end{array}\right.$$

In separable pure states the subsystems are also pure

Entangled Pure States

Entangled states: $|\psi\rangle_{AB} \neq |\alpha\rangle_A \otimes |\beta\rangle_B$

Subsystems are not pure

$$\left\{\begin{array}{l}\rho_A \neq |\alpha\rangle\langle\alpha| \\[2em] \rho_B \neq |\beta\rangle\langle\beta|\end{array}\right. \Rightarrow \left\{\begin{array}{l}P_A = P_B < 1 \\[2em] S(\rho_A) = S(\rho_B) > 0\end{array}\right.$$

Von-Neumann Entropy of the subsystem quantifies the entanglement

Example 1

Maximally entangled states:

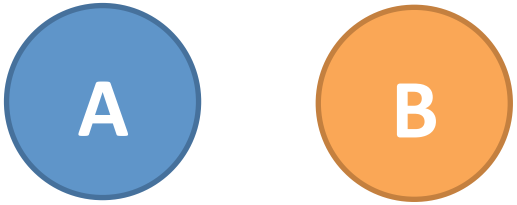
$$|\psi\rangle_{AB} = \frac{|00\rangle_{AB} + |11\rangle_{AB}}{\sqrt{2}} \quad \Rightarrow \quad \rho_A = \frac{I}{2}, \quad \rho_B = \frac{I}{2}$$

$$\Rightarrow \begin{cases} P_A = P_B = \dfrac{1}{2} \\ S(\rho_A) = S(\rho_B) = \log(2) = 1 \end{cases}$$

Maximally entangled states

1. **Subsystems are maximally mixed**
2. **The entropy of subsystems are maximal**
3. **The purity of the subsystems are minimal**

Von Neumann Entropy



Schmidt decomposition: $|\psi\rangle_{AB} = \sum_i \lambda_i \left| \tilde{i}_A, \tilde{i}_B \right\rangle$

$$\left\langle \tilde{i}_A \middle| \tilde{i}_A' \right\rangle = \delta_{ii'}, \quad \left\langle \tilde{i}_B \middle| \tilde{i}_B' \right\rangle = \delta_{ii'}$$

Subsystems:

$$\rho_B = Tr_A(|\psi_{AB}\rangle\langle\psi_{AB}|) = \sum_i \lambda_i^2 \left|\tilde{i}_B\right\rangle\left\langle\tilde{i}_B\right|$$

$$\rho_A = Tr_B(|\psi_{AB}\rangle\langle\psi_{AB}|) = \sum_i \lambda_i^2 \left|\tilde{i}_A\right\rangle\left\langle\tilde{i}_A\right|$$

If AB is pure:

- **Purity:** $P_A = P_B$
- **Von Neumann Entropy:** $S(\rho_A) = S(\rho_B)$

Example 2

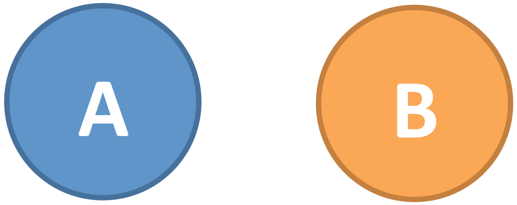
Non-maximal entangled states:

$$|\psi\rangle_{AB} = \sqrt{\frac{1}{3}}|00\rangle_{AB} + \sqrt{\frac{2}{3}}|11\rangle_{AB} \Longrightarrow \rho_A = \rho_B = \frac{1}{3}|0\rangle\langle 0| + \frac{2}{3}|1\rangle\langle 1|$$

$$\Longrightarrow \left\{ \begin{array}{l} P_A = P_B = \frac{5}{9} \\ S(\rho_A) = S(\rho_B) = -\frac{1}{3}\log(\frac{1}{3}) - \frac{2}{3}\log(\frac{2}{3}) \approx 0.9183 \end{array} \right.$$

Entropy of the subsystem can quantify the amount of entanglement

Entanglement of Pure States



Overall state: $$|\psi\rangle_{AB} \Rightarrow \begin{cases} \rho_A = Tr_B(\rho_{AB}) \\ \rho_B = Tr_A(\rho_{AB}) \end{cases}$$

Entanglement between the two subsystems: $E = S(\rho_A) = S(\rho_B)$

$$0 \leq E \leq \log(d)$$

Separable states

Maximally entangled states

All entanglement measures are monotonic functions with respect to the von Neumann entropy

Separable Mixed States

Separable states: $\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$

$$p_i \geq 0, \quad \sum_i p_i = 1$$

With local operations and classical communications Alice and Bob can produce these kind of states

Examples for Separable States

$$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

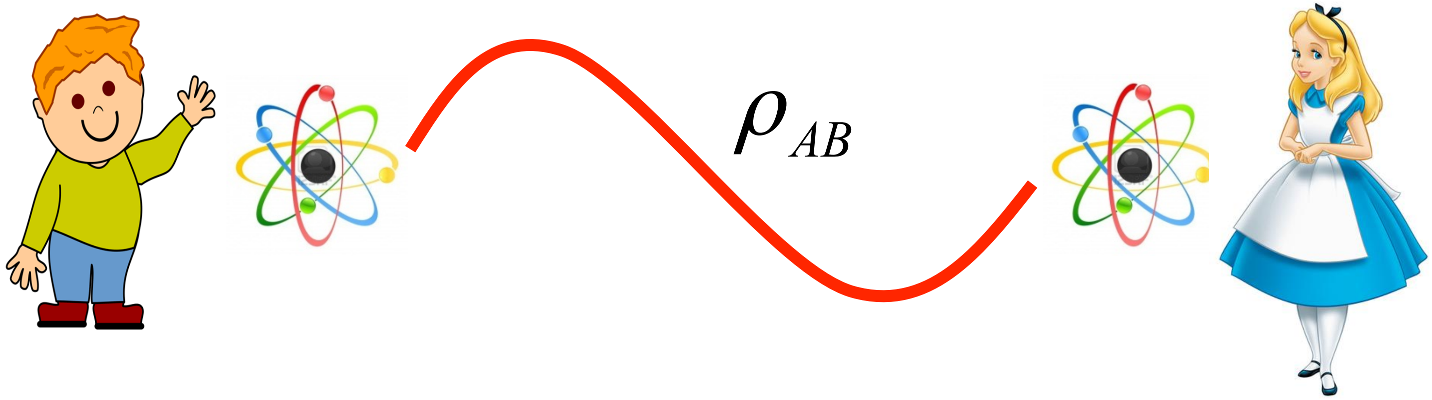
Example 1 (Pure states):

$$|\psi\rangle_{AB} = |\alpha\rangle_A \otimes |\beta\rangle_B \quad \Rightarrow \quad \rho_{AB} = |\alpha_A\rangle\langle\alpha_A| \otimes |\beta_A\rangle\langle\beta_A|$$

Example 2: $\rho_{AB} = \frac{1}{3}|0\rangle\langle 0| \otimes |+\rangle\langle +| + \frac{2}{3}|-\rangle\langle -| \otimes |1\rangle\langle 1|$

Example 3: $\rho_{AB} = \frac{1}{6} I \otimes |0\rangle\langle 0| + \frac{2}{6}|+\rangle\langle +| \otimes (I + \sigma_Z)$

Basic Properties for Entanglement Measures



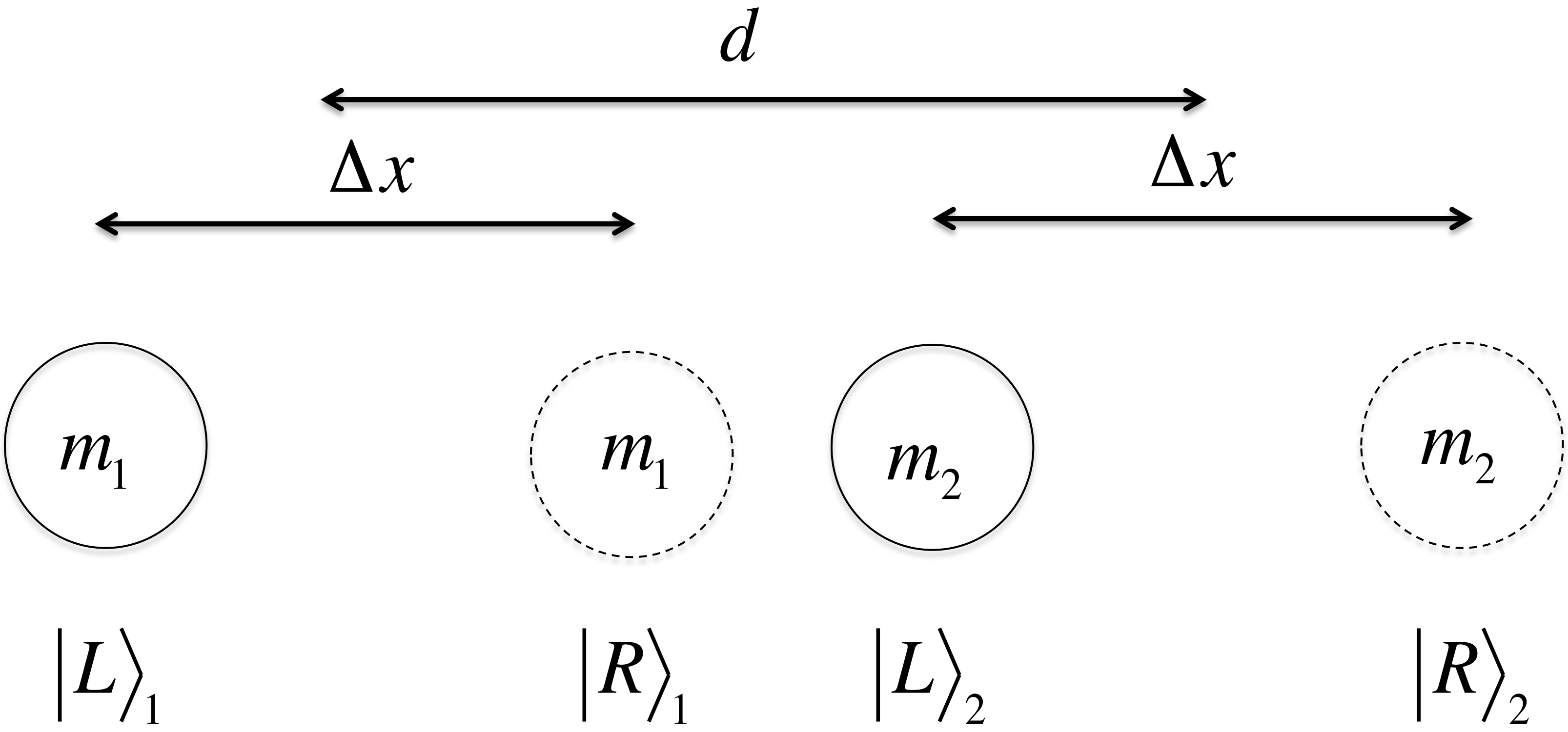
1. $$E(\rho_{AB}) \in R^{+}$$

2. $$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B \Longrightarrow E(\rho_{AB}) = 0$$

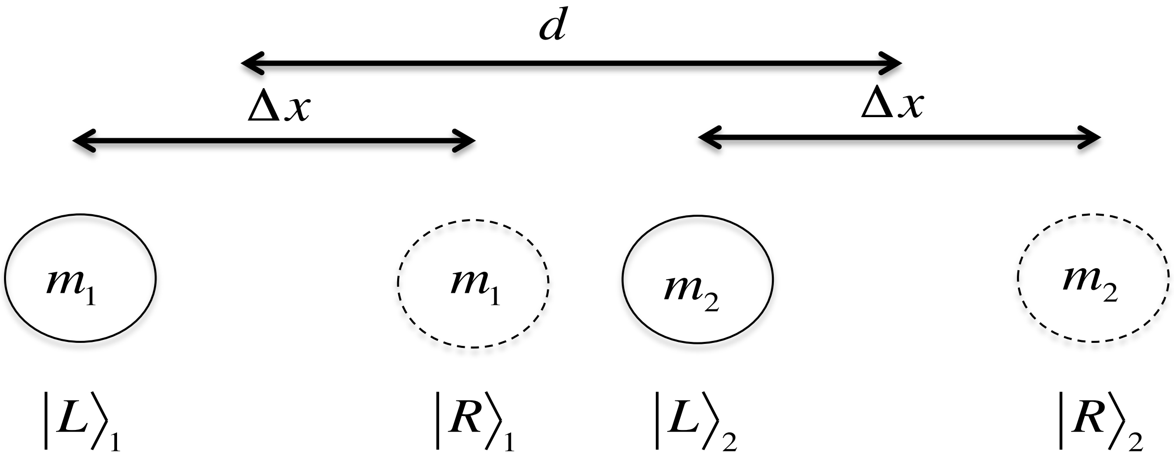
3. $$|\psi_{AB}\rangle = \frac{1}{\sqrt{d}} \sum_i |i_A, i_B\rangle \Longrightarrow E(\rho_{AB}) \text{ is maximum}$$

4. $$\sigma_{AB} = \sum_k A_k \otimes B_k \; \rho_{AB} \; A_k^{+} \otimes B_k^{+} \Longrightarrow E(\rho_{AB}) \geq E(\sigma_{AB})$$

A Schematic of two matter-wave interferometers near each other



Consider two neutral test masses *held* in a superposition, each exactly as a path encoded qubit (states |L> and |R>), near each other.



If they interact *only* through the gravitational force

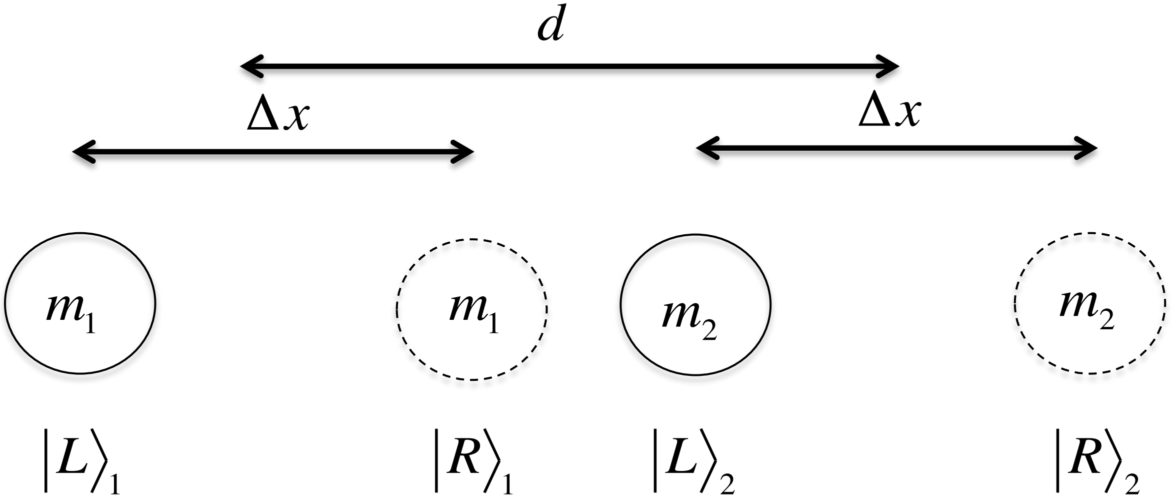
$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2)$$

$$= \frac{1}{2}(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + |R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2)$$

$$\rightarrow |\Psi(t=\tau)\rangle_{12} = \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2$$
$$+ e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2),$$

where

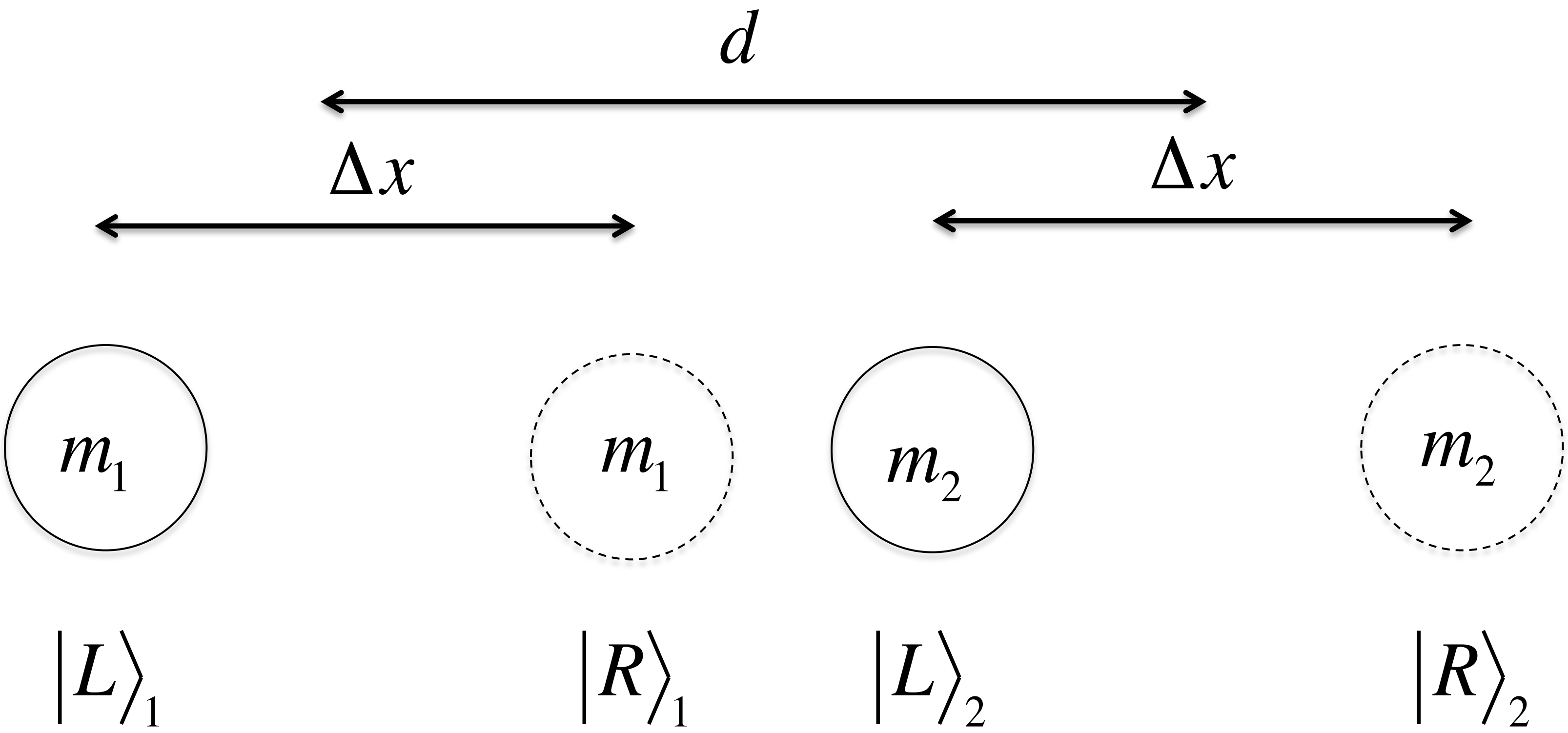
$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)},$$
$$\phi_{LL} = \phi_{RR} \sim \frac{Gm_1m_2\tau}{\hbar d}$$



If they interact *only* through the gravitational force

$$
\begin{aligned}
|\Psi(t=\tau)\rangle_{12} &= \frac{1}{2}(e^{i\phi_{LL}}|L\rangle_1|L\rangle_2 + e^{i\phi_{LR}}|L\rangle_1|R\rangle_2 \\
&+ e^{i\phi_{RL}}|R\rangle_1|L\rangle_2 + e^{i\phi_{RR}}|R\rangle_1|R\rangle_2) \\
&= \frac{e^{i\phi_{RR}}}{\sqrt{2}}\{|L\rangle_1\frac{1}{\sqrt{2}}(|L\rangle_2 + e^{i\Delta\phi_{LR}}|R\rangle_2) \\
&+ |R\rangle_1\frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L\rangle_2 + |R\rangle_2)\}
\end{aligned}
$$

The above state is maximally entangled when $\Delta\phi_{LR} + \Delta\phi_{RL} \sim \pi$.



For

$$d - \Delta x << d, \Delta x,$$

we have

$$\Delta\phi_{RL} \sim \frac{Gm_1 m_2 \tau}{\hbar(d - \Delta x)} >> \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$

For

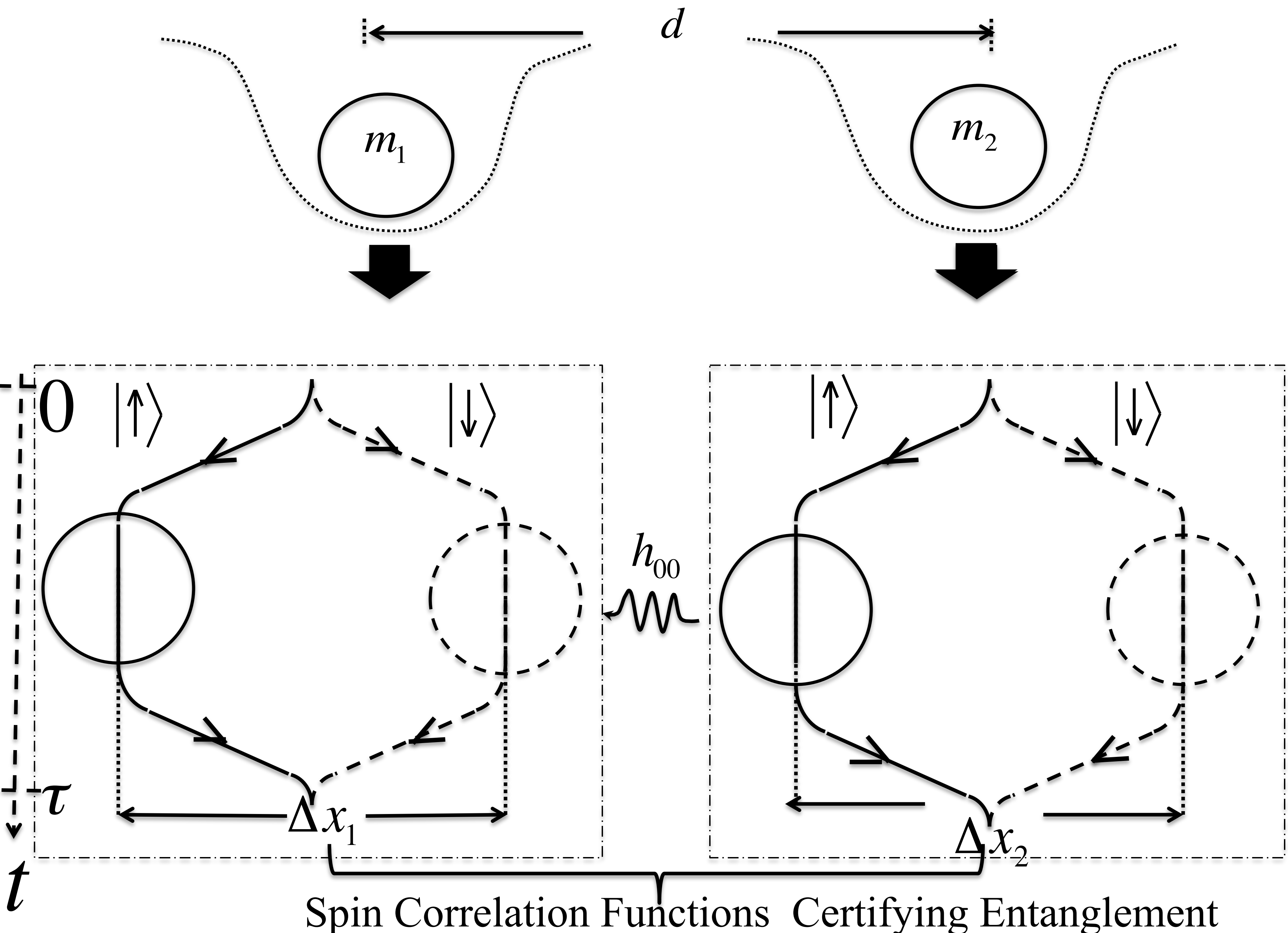
$$d - \Delta x << d, \Delta x,$$

we have

$$\Delta\phi_{RL} \sim \frac{G m_1 m_2 \tau}{\hbar(d - \Delta x)} >> \Delta\phi_{LR}, \Delta\phi_{LL}, \Delta\phi_{RR}$$

For mass ~ 10^(-14) kg (microspheres), separation at closest approach of the masses ~ 200 microns (to prevent Casimir interaction), **time ~ 1 seconds**, gives:
Scale of superposition ~ 100 microns, **Delta phi_{RL} ~ 1**

Planck's Constant fights Newton's Constant!


d
m_1
m_2
0
$|\uparrow\rangle$
$|\downarrow\rangle$
$|\uparrow\rangle$
$|\downarrow\rangle$
h_{00}
τ
t
Δx_1
Δx_2
Spin Correlation Functions Certifying Entanglement

Spin Entanglement Witness:

Step 1: SG splitting:

$$|C\rangle_j \frac{1}{\sqrt{2}}(|\uparrow\rangle_j + |\downarrow\rangle_j) \rightarrow \frac{1}{\sqrt{2}}(|L, \uparrow\rangle_j + |R, \downarrow\rangle_j)$$

Step 2: Gravitational interaction induced phase accumulation on the joint states of masses 1 &2 (*mapped to nuclear spins*)

Step 3: SG recombination: $|L, \uparrow\rangle_j \rightarrow |C, \uparrow\rangle_j, \; |R, \downarrow\rangle_j \rightarrow |C, \downarrow\rangle_j$

Step 4: Witness spin entangled state:

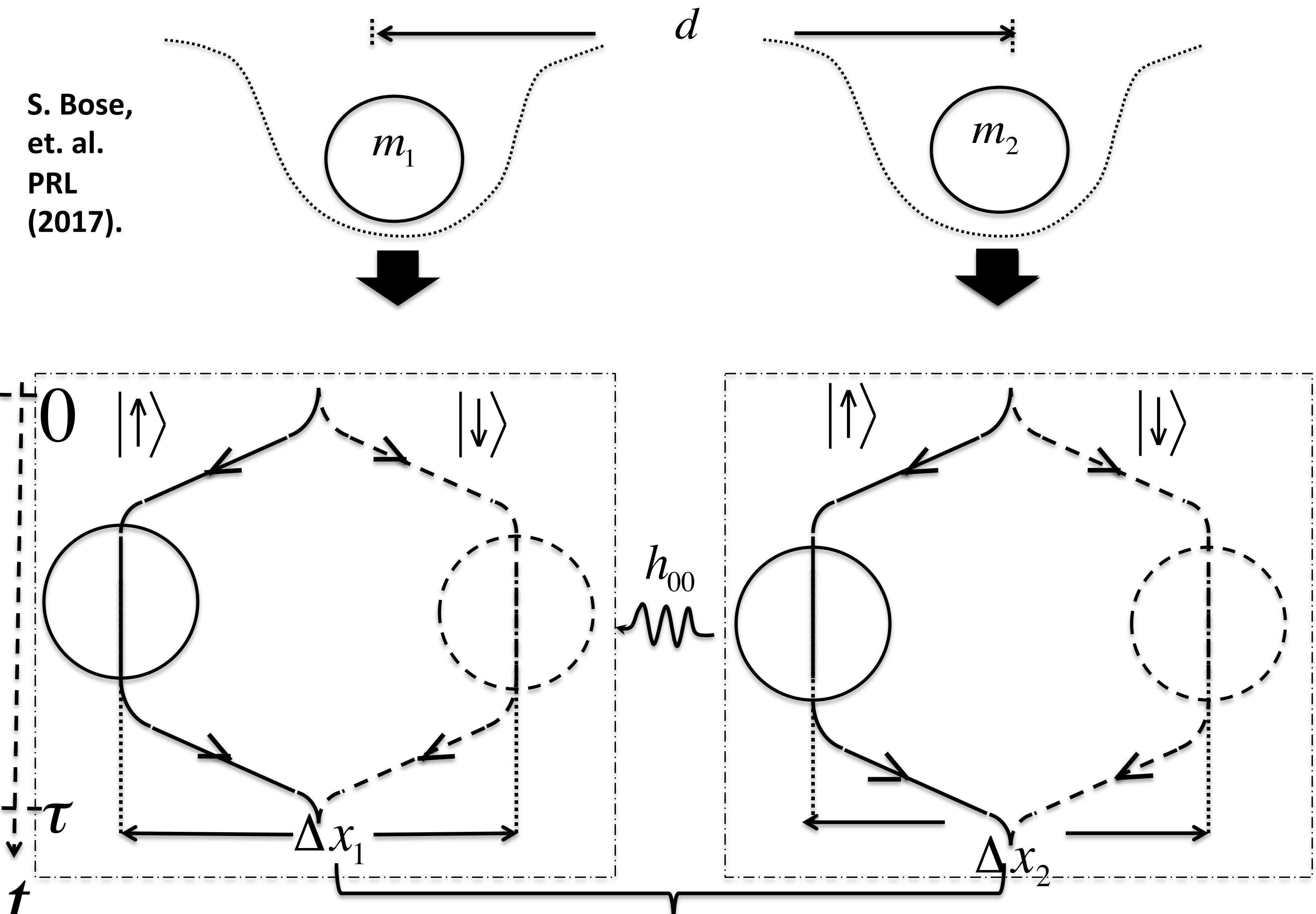
$$|\Psi(t = t_{\text{End}})\rangle_{12} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}|\downarrow\rangle_2)$$

$$+ |\downarrow\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|\uparrow\rangle_2 + |\downarrow\rangle_2)\}|C\rangle_1|C\rangle_2$$

through the correlations:

$$\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle|$$

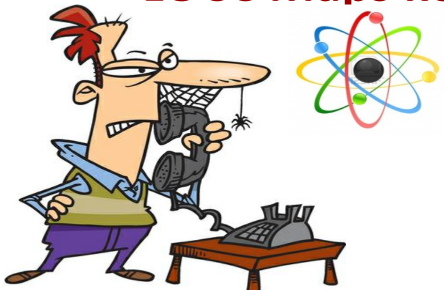
S. Bose, et. al. PRL (2017).

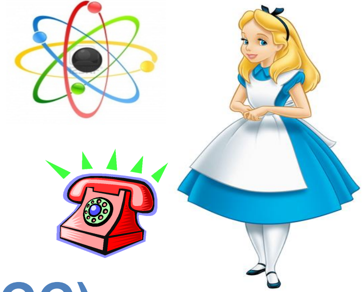


Spin Correlation Functions Certifying Entanglement

How is this related to Quantum Gravity?

LOCC Maps keep separable states separable (*cannot* create entanglement!)

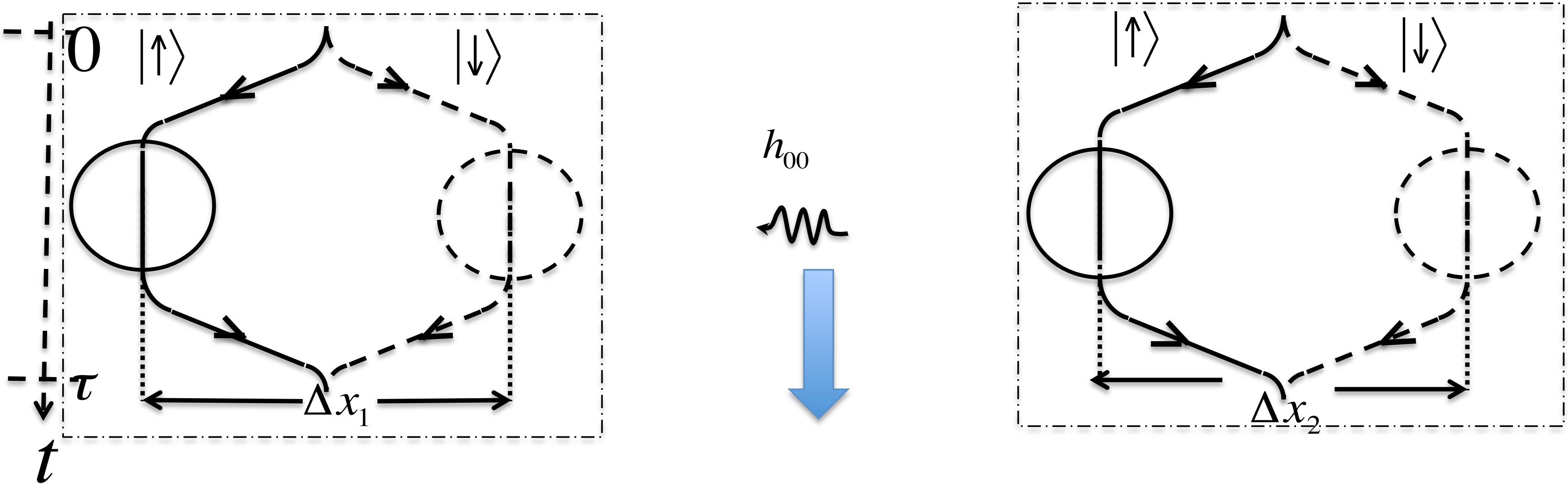




Local Operations and Classical Communication (LOCC)

1. Unitary evolution
2. Measurement

Classical mediator of information/bits



Must be quantum if the spins in the masses get entangled

Quantum Back Action:

For naïve approaches quantum mechanics prevents resolution beyond the standard quantum limit or SQL: *Quantum back action noise*

Measuring the action of a force F over a time τ on a *free* mass m

We imagine doing this by measuring the position x at two times separated by τ

The first position measurement with a precision Δx is necessarily accompanied by a disturbance Δp

Thus the resolution of the second position measurement is

$$\delta x \sim \sqrt{(\Delta x)^2 + \left(\frac{\Delta p\,\tau}{m}\right)^2} \geq \sqrt{\frac{2\Delta x \Delta p\,\tau}{m}} \geq \sqrt{\frac{\hbar\,\tau}{m}} \sim \delta x_{SQL}$$

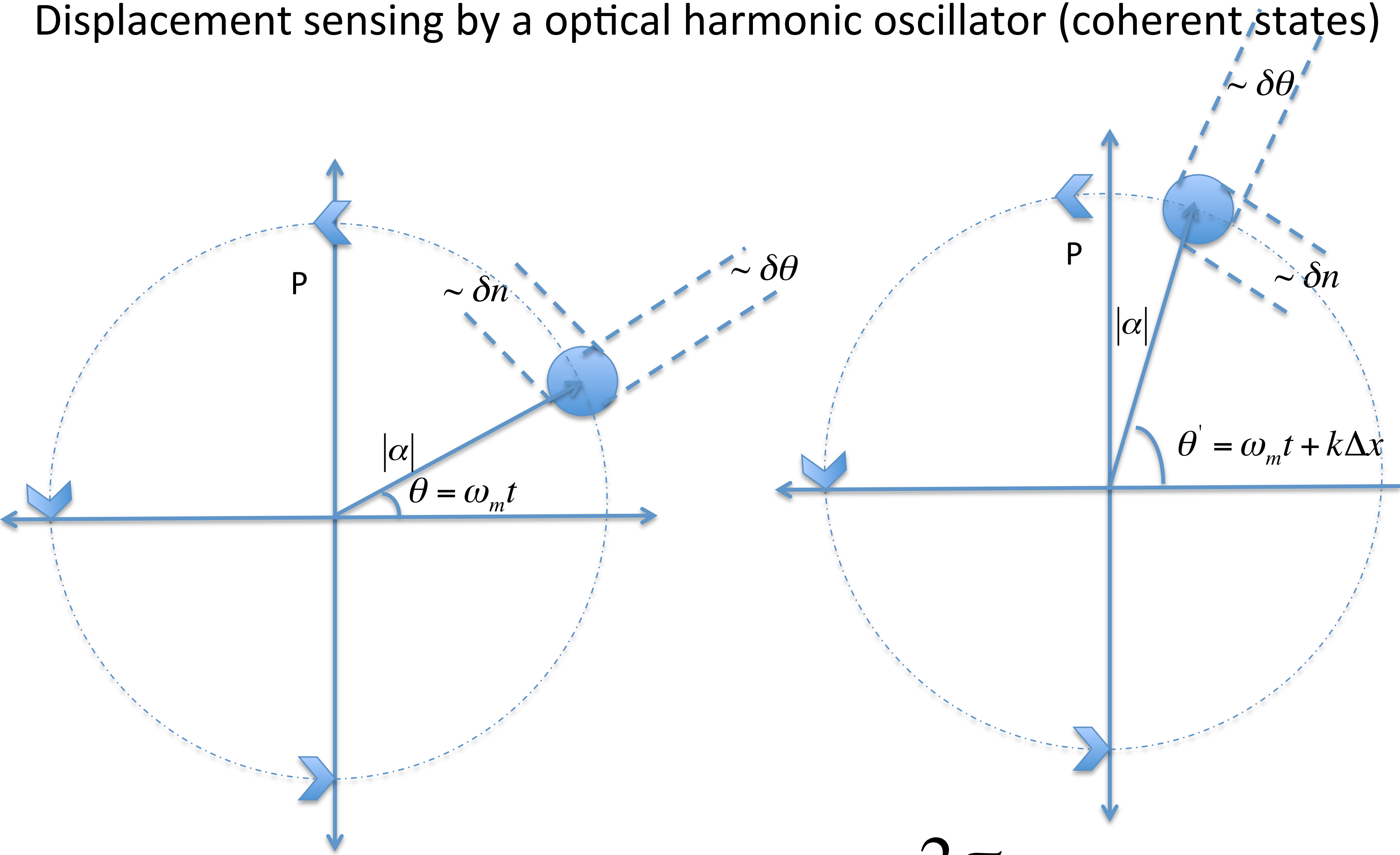
For small free masses, 10^(-14) kgs, an Angstrom in a second

Thus the resolution of force measurement is

$$\delta F_{SQL} \sim \left(\frac{2\,\delta x_{SQL}\ m}{\tau^2}\right) \sim \sqrt{\frac{4\hbar\, m}{\tau^3}}$$

For small free masses, 10^(-14) kgs, 10^(-24) Newtons/root(Hz)

Displacement sensing by a optical harmonic oscillator (coherent states)



$$\theta' = \omega_m t + (finesse)\frac{2\pi}{\lambda}\Delta x$$

Quantum Back Action:

Sometimes quantum mechanics prevents a high resolution: *Quantum back action noise*

Measuring the action of a force F over a time of one oscillation period on an oscillator mass m
and frequency ω_m by continuous position detection;

The limit is
a **coherent state spread**

$$\delta x_{SQL} \sim \sqrt{\frac{\hbar}{2m\omega_m}}$$

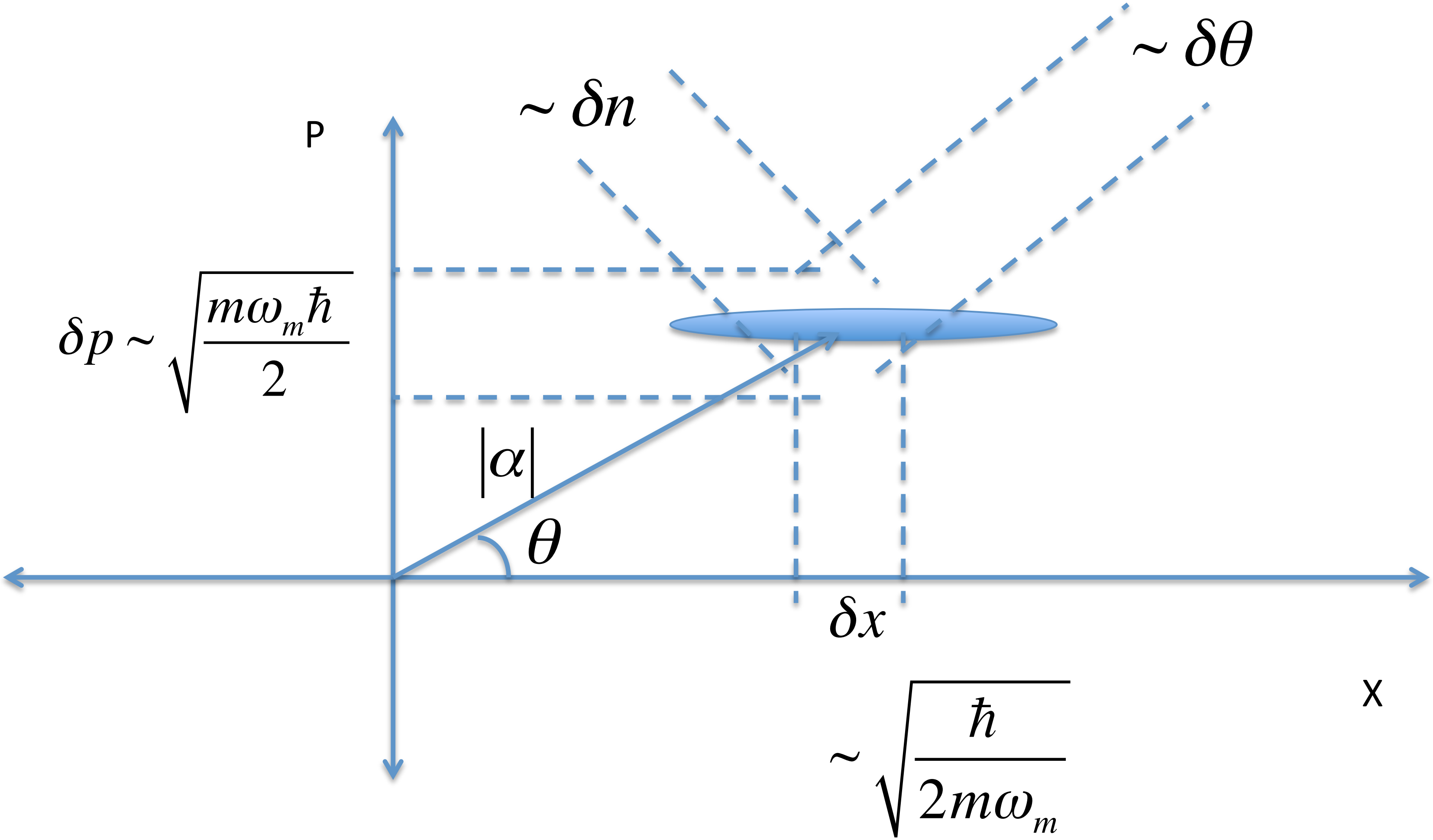
For a small mass, 10^(-14) kgs, and a MHz trap, 0.1 picometer

Thus the resolution of force measurement is

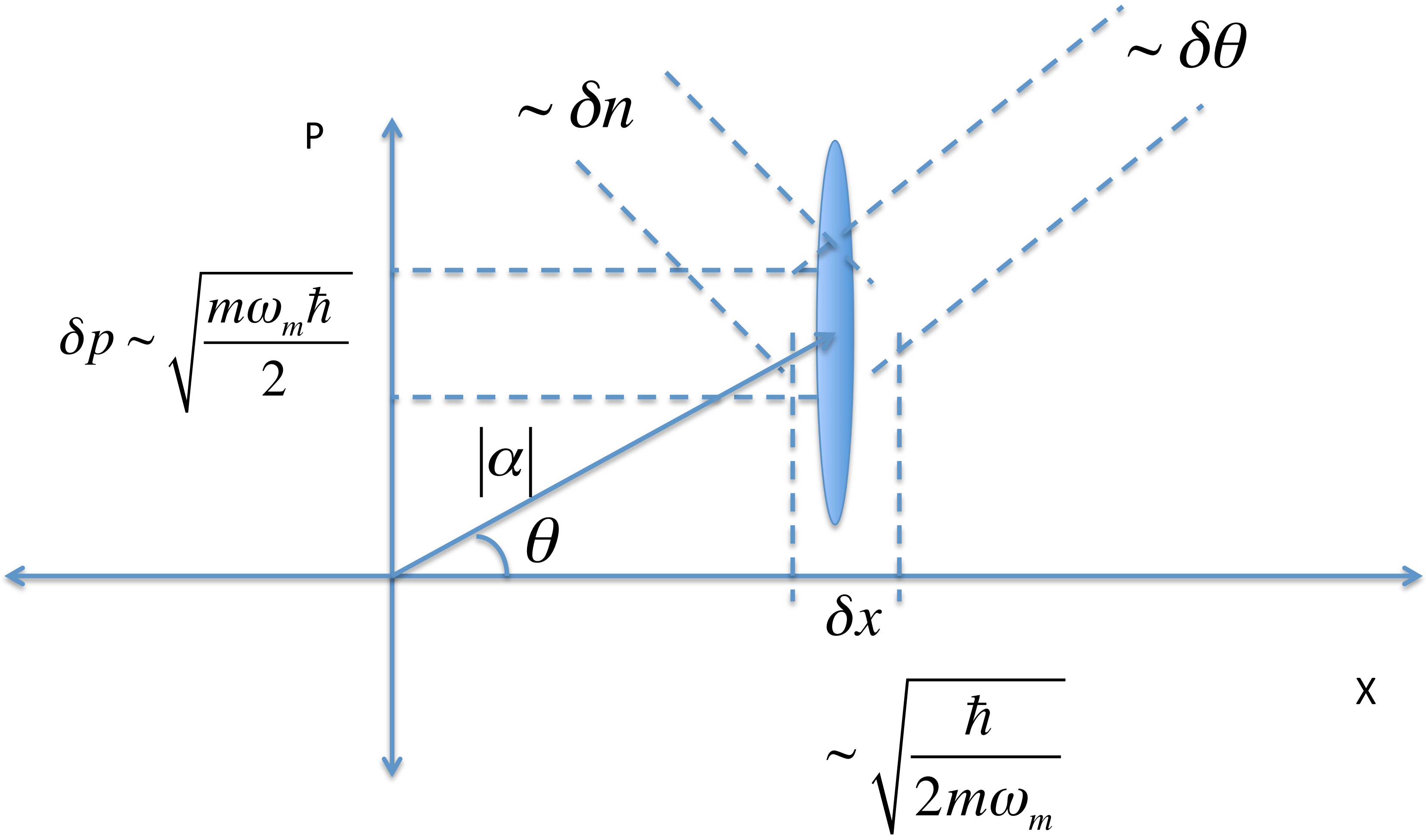
$$\frac{\delta F_{SQL}}{m\omega_m^2} \geq \delta x_{SQL} \Rightarrow \delta F_{SQL} \geq \sqrt{\hbar m \omega_m^3}$$

For small free masses, 10^(-14) kgs, in MHz trap 10^(-18) Newtons/ root(Hz)

Momentum squeezed states



Position squeezed states

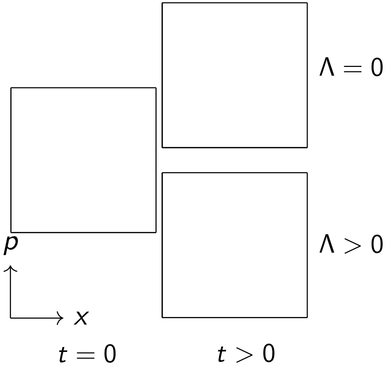


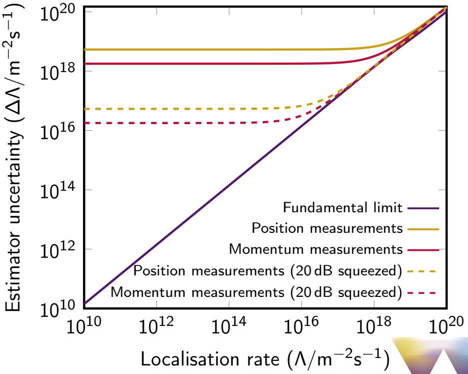
Resolving localisation effects

Fundamental theories require reconciliation between the classical/macro and quantum/micro worlds.
Collapse theories offer one possible explanation.

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \boxed{\Lambda}[x, [x, \rho]]$$

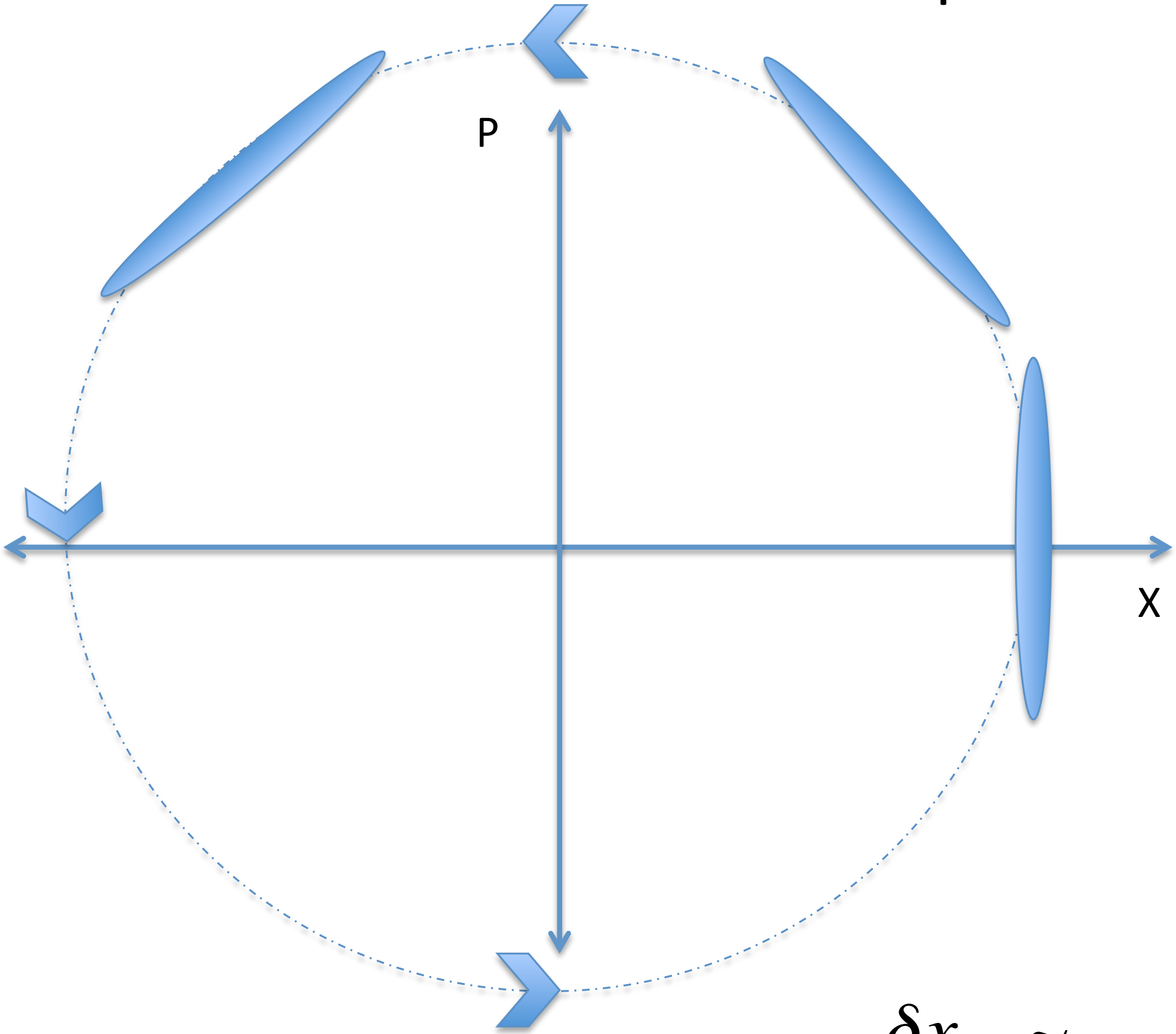
Estimate localisation Λ





Branford, Gagatsos, Grover, Hickey, and Datta (In preparation)

Time evolution of squeezed states



An ideal preparation:

1. Cool to nearly ground state or purify otherwise.

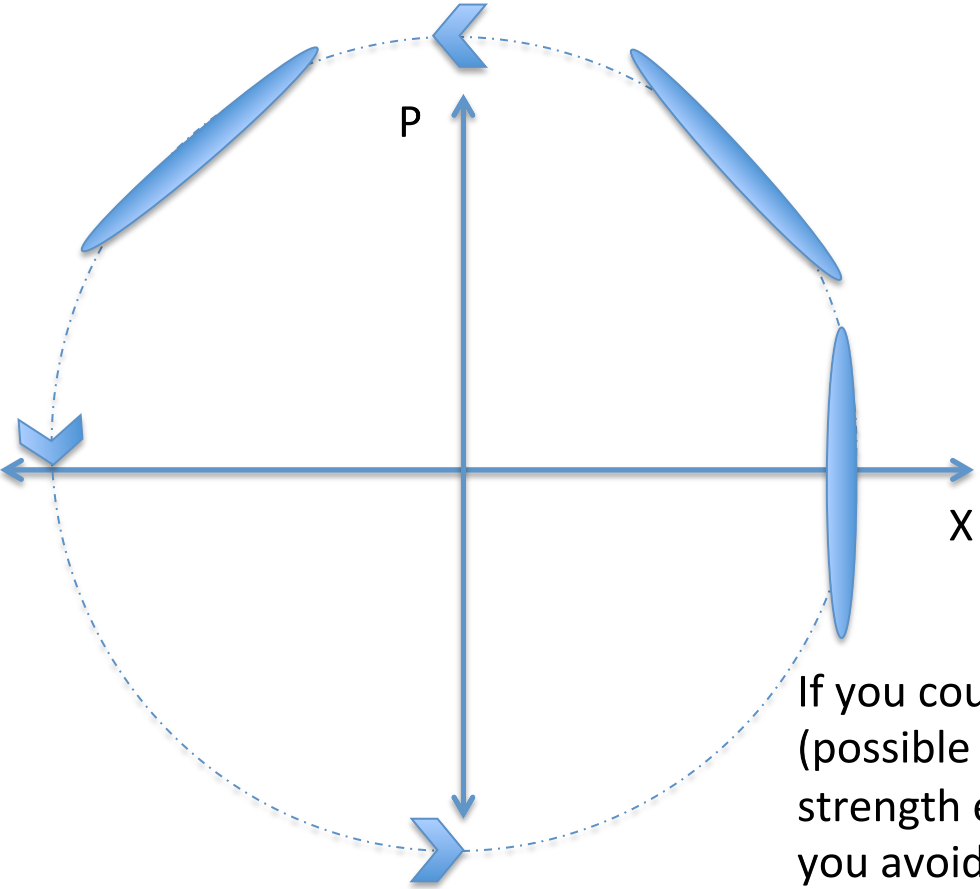
2. Take a snap-shot position measurement, *faster* than the evolution.

Typically optical Interferometric Measurement;
Resolution = position squeezing

$$\delta x_{res} \sim \frac{\lambda}{(finesse)\sqrt{n}} \sim \Delta x_{squeezed}$$

Femtometers per root Hz already possible:
Ulbricht group (Southampton), Barker group (UCL)

Back Action Evasion



Conserved Observables (without external force):

$$\hat{X}_1(\hat{x},\hat{p},t) \equiv \hat{x}\ \cos\omega t - \frac{\hat{p}}{m\omega}\sin\omega t,$$

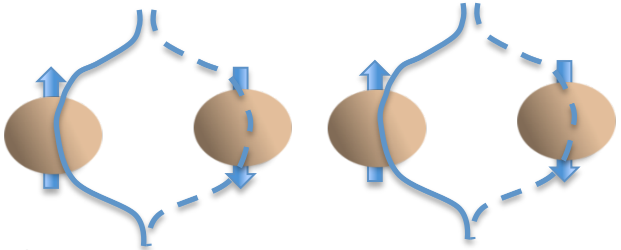
$$\hat{X}_2(\hat{x},\hat{p},t) \equiv \hat{x}\ \sin\omega t + \frac{\hat{p}}{m\omega}\cos\omega t.$$

If you couple a meter to the conserved observables,
(possible through time-modulated coupling
strength even with position coupling!),
you avoid the *back-action affecting
the measurement resolution.*

Braginsky et al. (1970s); Caves, Thorne et. al (1980s); Clerk, Marquardt, Jacobs (2011).

OR you can measure position stroboscopically at
half period intervals (e.g. **Vanner (Imperial)**).

Our papers



- Large mass, small scale of superpositions:

Stern-Gerlach based Ramsey interferometry in a trap:

M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, S. Bose, Phys. Rev. Lett. 111, 180403 (2013). [related work by Tongcang Li et. al.]

- Large mass, large scale superpositions:

Free flight Stern-Grlach based Ramsey interferometry:

C. Wan, M. Scala, G. W. Morley, ATM. A. Rahman, H. Ulbricht, J. Bateman, P. F. Baker, S. Bose, M. S. Kim, Phys. Rev. Lett. 117, 143003 (2016).

- Spin Entanglement Witness for Quantum Gravity:

S. Bose, A. Mazumdar, G. W.Morley, H. Ulbricht, M. Toros, M. Paternostro, P. F. Barker, A. Geraci, M. S. Kim, G. J. Milburn, Phys. Rev. Lett. 119, 240401 (2017). ***Related work:*** C. Marletto and V. Vedral

Phys. Rev. Lett. 119, 240402 (2017)

- Gravitational wave detection with meter scale sensor:

Ryan J. Marshman, Anupam Mazumdar, Gavin W. Morley, Peter F. Barker, Steven Hoekstra, Sougato Bose,

arXiv:1807.10830