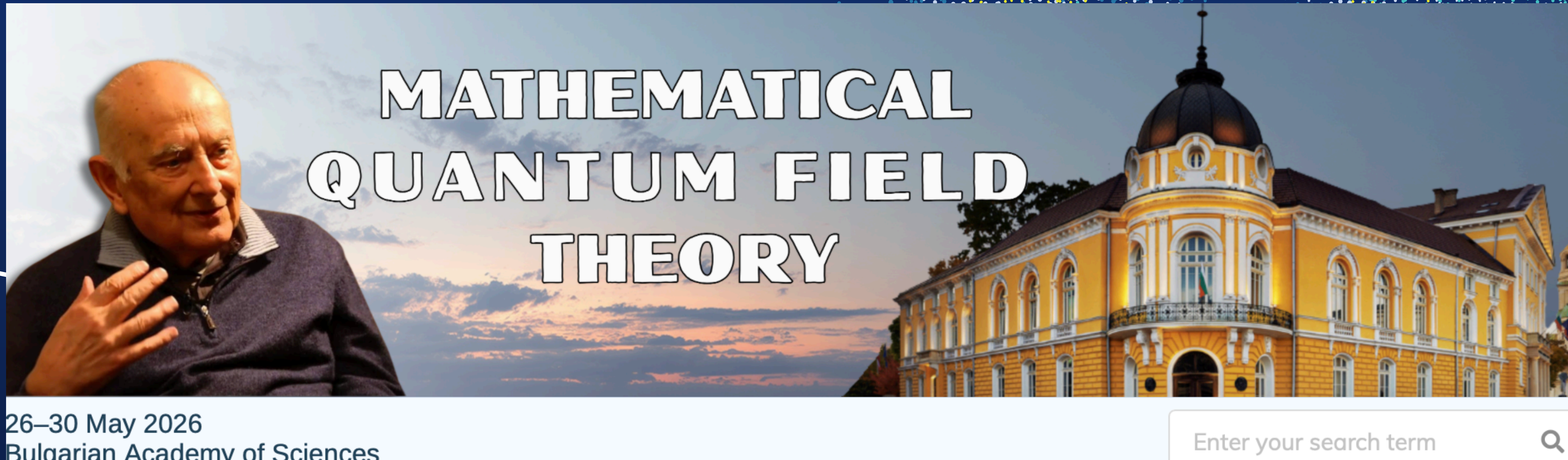

BLTP

Sofia

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The general approach to the renormalization group equations in local quantum field theory

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The foundation:

- Quantum field theory is formulated for all types of interactions independently on renormalizability

- R-operation equally works for NR theories and leads to local counter terms resulting in finite amplitudes

- Local Quantum Field Theory obeying the requirements of causality, unitarity and analyticity has a remarkable property: after applying the R-operation all UV divergent structures are local in coordinate space or are at most polynomial in momentum space (Bogoliubov-Parasiuk theorem).

- This is true for any local QFT irrespectively of its renormalizability or non-renormalizability

- All these statements lead to relations between the subsequent orders of PT resulting in the RG equations which aimed on the summation of infinite series of PT for the asymptotics of the Green functions, amplitudes, potential, etc

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Renormalization

Bogoliubov-Parasiuk-Hepp-Zimmermann R-operation

$$RG = \prod_{\gamma_i}(1 - M_i)G$$

$$RG = (1 - K)R'G$$

G - graph

γ - divergent subgraph

M_i - subtraction operator

K-operation extracts the singular part

Incomplete R-operation A.N.Vasiliev, Green Book

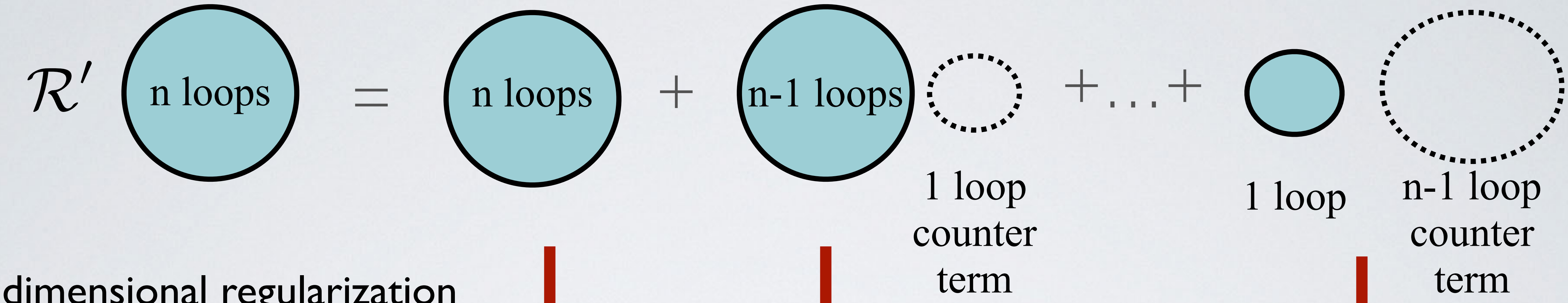
$$R' = 1 - \sum_{\gamma} KR'\gamma + \sum_{\gamma,\gamma'} KR'\gamma\, KR'\gamma' - ...;$$

R-operation is equivalent to the introduction of the counterterms into the Lagrangian

$$\mathcal{L} \Rightarrow \mathcal{L} + \Delta\mathcal{L}$$

Bogolyubov-Parasiuk Theorem: In any local quantum field theory after subtracting the UV divergences in subgraphs the resulting counterterms are always local in coordinate space or at most are polynomials of external momenta in momentum space in each order of perturbation theory

BPHZ R-operation



- dimensional regularization

$$d = 4 - 2\epsilon$$

$$\mathcal{R}'G_n = \frac{A_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + \ldots + \frac{A_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^n}$$

$$+ \frac{B_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^{n-1}} + \frac{B_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^{n-1}} + \ldots + \frac{B_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^{n-1}}$$

$$+ \frac{C_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^{n-2}} + \frac{C_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^{n-2}} + \ldots + \frac{C_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^{n-2}}$$

$$+ \quad \text{lower pole terms,}$$

$A_k^{(n)}$ $B_k^{(n)}$ $C_k^{(n)}$ $(\mu^2)^{k\epsilon}$ terms appear after subtraction of (n-k) loop counter terms

BPHZ R-operation

<u>Bogoliubov-Parasiuk Theorem:</u> <u>(Locality)</u>

$R'G_n$ is local, i.e. terms like $\log^k \mu^2/\epsilon^m$ should cancel for any k and m

- Due to <u>locality</u> all higher order divergences are related to the lower ones

$$A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n},$$ One loop

$$B_n^{(n)} = (-1)^n\left(\frac{2}{n}B_2^{(n)} + \frac{n-2}{n}B_1^{(n)}\right),$$ One and two loops

$$C_n^{(n)} = (-1)^{n+1}\left(\frac{3}{n}C_3^{(n)} + \frac{2(n-3)}{n}C_2^{(n)} + \frac{(n-2)(n-3)}{2n}C_1^{(n)}\right).$$ One, two and three loops

$$\mathcal{K}\mathcal{R}'G_n = \sum_{k=1}^{n}\left(\frac{A_k^{(n)}}{\epsilon^n} + \frac{B_k^{(n)}}{\epsilon^{n-1}} + \frac{C_k^{(n)}}{\epsilon^{n-2}} + \cdots\right) \equiv \frac{A_n^{(n)'}}{\epsilon^n} + \frac{B_n^{(n)'}}{\epsilon^{n-1}} + \frac{C_n^{(n)'}}{\epsilon^{n-2}} + \cdots.$$

$$A_n^{(n)'} = (-1)^{n+1}A_n^{(n)} = \frac{A_1^{(n)}}{n},$$ One loop

$$B_n^{(n)'} = \left(\frac{2}{n(n-1)}B_2^{(n)} + \frac{2}{n}B_1^{(n)}\right),$$ One and two loops

$$C_n^{(n)'} = \left(\frac{2}{(n-1)(n-2)}\frac{3}{n}C_3^{(n)} + \frac{2}{n-1}\frac{3}{n}C_2^{(n)} + \frac{3}{n}C_1^{(n)}\right).$$ One, two and three loops

The Local Counter Terms

Consequence:

Leading divergences: $A_n^{(n)\prime} = \frac{A_1^{(n)}}{n}$ Coefficients of $1/\epsilon^n$

The leading divergences are governed by 1-loop diagrams!

Subleading divergences: $B_n^{(n)\prime} = \left(\frac{2}{n(n-1)} B_2^{(n)} + \frac{2}{n} B_1^{(n)} \right)$ Coefficients of $1/\epsilon^{n-1}$

The subleading divergences are governed by 2-loop diagrams!

Coefficient of the leading logarithms $\log^n \mu^2$

$$\bar{A}_n^{(n)} = A_n^{(n)} = (-1)^{n+1} \frac{A_1^{(n)}}{n}$$

One loop

Coefficient of the subleading logarithms $\log^{n-1} \mu^2$

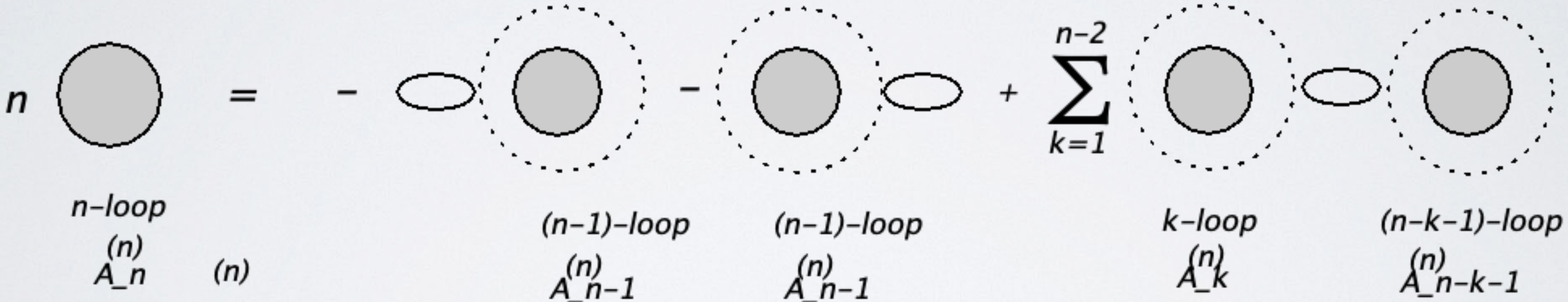
$$\bar{B}_n^{(n)} = (-1)^n \left(\frac{2}{n-1} B_2^{(n)} + B_1^{(n)} \right)$$

One and two loops

The Recurrence Relations

- These properties allow one to write down the recurrence relations connecting the subsequent orders of the counterterms and to evaluate them algebraically without calculating the diagrams. This can be done in renormalizable and non-renormalizable theories. The difference is a more complicated structure of these relations in NR case.

Leading divergences:



- These recurrence relations can be promoted to the RG equations for the scattering amplitudes, effective potential, etc which sum up the leading divergences (logarithms) and to find out the high energy/field behaviour

Loop Expansion (non-renormalizable case)

ϕ_6^4

UV divergences within dim reg

$$\Delta\mathcal{L}_1 \sim \lambda^2(s+t+u)\Phi^4(\frac{1}{\epsilon}+c_{11})$$

Momentum space

$$\Delta\mathcal{L}_1 \sim \lambda^2\partial^2\Phi^2\Phi^2(\frac{1}{\epsilon}+c_{11}),$$

Coordinate space

Increasing power of momentum →

$$\Delta\mathcal{L} = \lambda^2\partial^2\Phi^2\Phi^2 + \lambda^3[\partial^4\Phi^2\Phi^2 + \partial^2\Phi^2\partial^2\Phi^2)] + \lambda^4[...] + \lambda^5[...]$$

$$(\frac{1}{\epsilon}+c_{11}) \quad (\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+c_{12}) \quad (\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+c_{13})$$

$$\searrow \quad \searrow \quad \searrow$$

$$\lambda^3\Phi^6 + \lambda^4[\partial^2\Phi^4\Phi^2 + \partial^2\Phi^2\Phi^4]$$

$$(\frac{1}{\epsilon}+c_{21}) \quad (\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+c_{22}) \quad (\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+c_{23})$$

$$\searrow \quad \searrow$$

$$\lambda^5\Phi^8$$

$$(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+c_{32}),$$

Increasing power of fields ↘

Examples:

- Maximally supersymmetric gauge theory in D=6,8,10 dimensions SYM_D
- Scalar field theory in D=4,6,8,10 dimensions ϕ^4_D
- Four-fermion interaction in D=4
- Supersymmetric Wess-Zumino model with quartic superpotential in D=4 Φ^4_4

Bork,Kazakov,Kompaneets, Tolkachev, Vlasenko, 15

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SYM_D

D=6 N=2

S-channel $S_n(s,t)$ **T-channel** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

Exact all-loop recurrence relation $S_3 = -s/3,\ T_3 = -t/3$

$$nS_n(s,t) = -2s \int_0^1 dx \int_0^x dy\ (S_{n-1}(s,t') + T_{n-1}(s,t'))$$

$n \geq 4$

$t' = t(x-y) - sy$

D=8 N=1

S-channel $S_n(s,t)$ **T-channel** $T_n(s,t)$ $T_n(s,t) = S_n(t,s)$

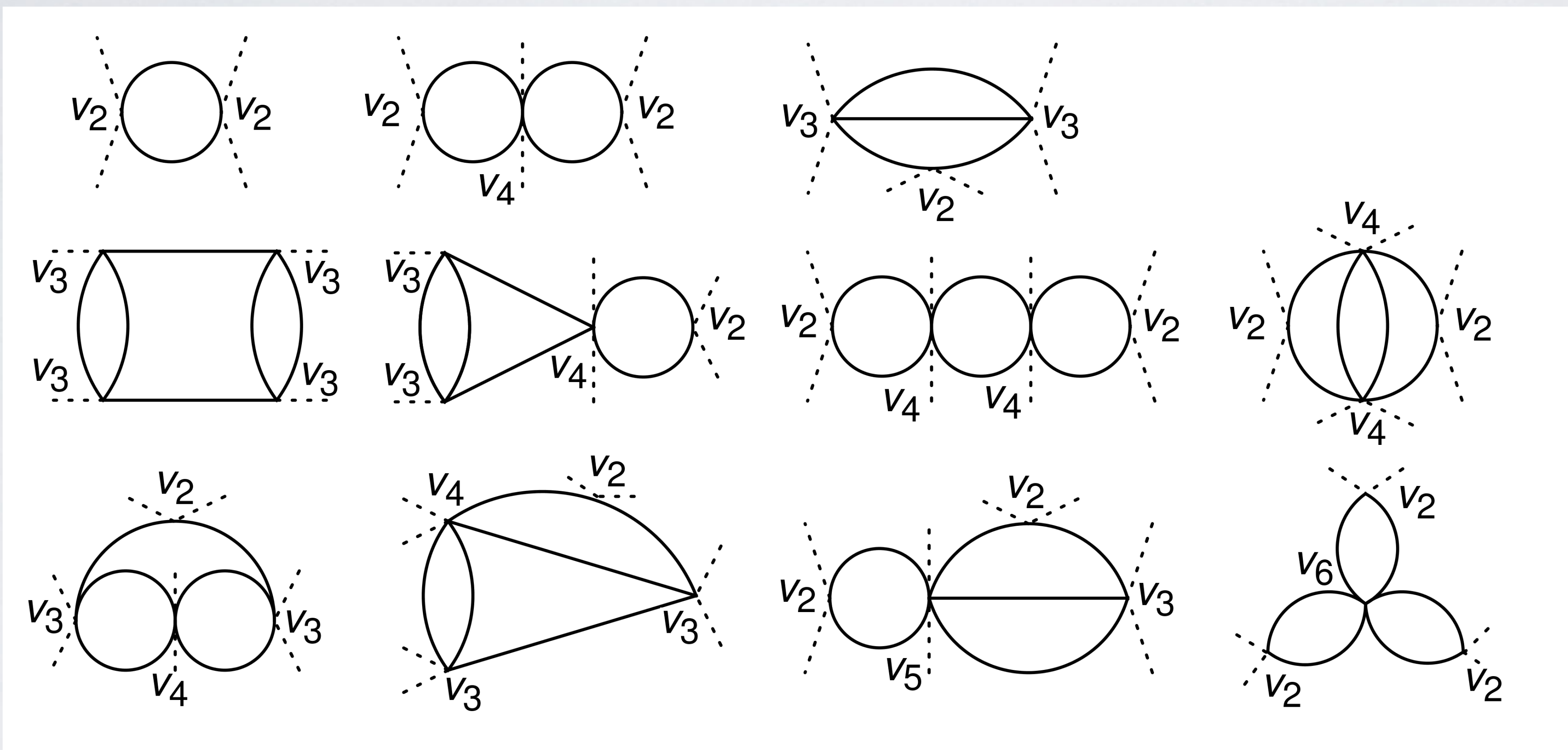
Exact all-loop recurrence relation $S_1 = \frac{1}{12},\ T_1 = \frac{1}{12}$

$$nS_n(s,t) = -2s^2 \int_0^1 dx \int_0^x dy\ y(1-x)\ (S_{n-1}(s,t') + T_{n-1}(s,t'))|_{t'=tx+yu}$$
$$+ \quad s^4 \int_0^1 dx\ x^2(1-x)^2 \sum_{k=1}^{n-2} \sum_{p=0}^{2k-2} \frac{1}{p!(p+2)!} \frac{d^p}{dt'^p}(S_k(s,t') + T_k(s,t')) \times$$
$$\times \frac{d^p}{dt'^p}(S_{n-1-k}(s,t') + T_{n-1-k}(s,t'))|_{t'=-sx}\ (tsx(1-x))^p$$

Effective Potential in Scalar Theory

V_{eff} Is the sum of all vacuum 1PI diagrams

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - g V_0(\phi)$$



$$v_2(\phi) \equiv \frac{d^2 V_0(\phi)}{d\phi^2}$$

$$v_n \equiv d^n V_0 / d\phi^n$$

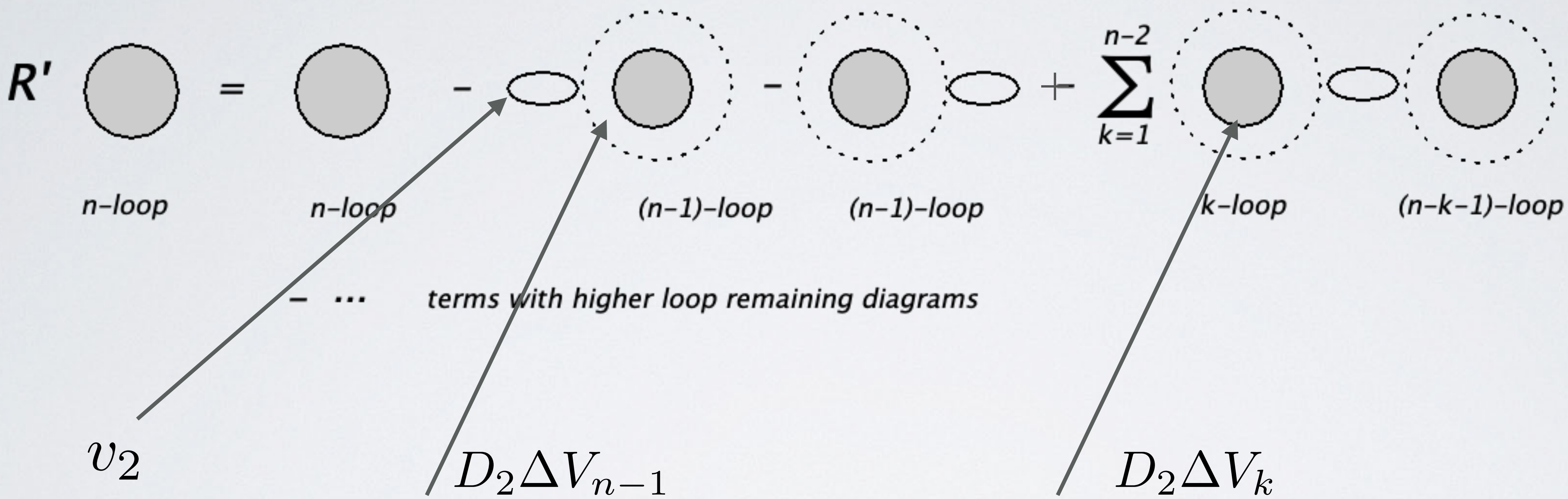
Shown are UV divergent vacuum diagrams in arbitrary scalar theory up to three loops

$$V_{eff} = g \sum_{n=0}^{\infty} (-g)^n V_n.$$

Recurrence relations for the leading poles for effective potential

Kazakov, Iakhibbaev, Tolkachev 22

Action of R'-operation on divergent diagram

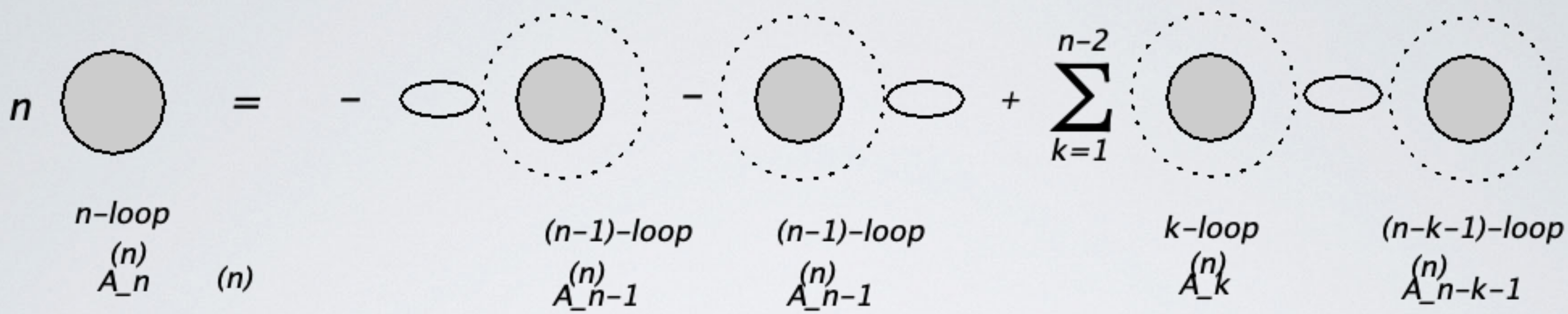


v_2 $\quad D_2 \Delta V_{n-1} \quad D_2 \Delta V_k \quad D_2 = \frac{d^2}{d\phi^2}$

$$n \Delta V_n = \frac{1}{2} v_2 D_2 \Delta V_{n-1} + \frac{1}{4} \sum_{k=1}^{n-2} D_2 \Delta V_k D_2 \Delta V_{n-1-k}, \quad n \geq 2 \quad \Delta V_1 = \frac{1}{4} v_2^2$$

$$n \Delta V_n = \frac{1}{4} \sum_{k=0}^{n-1} D_2 \Delta V_k D_2 \Delta V_{n-1-k}, \quad n \geq 1, \quad \Delta V_0 = V_0$$

From Recurrence Relation to the RG Equation



- This is the general recurrence relation that reflects the locality of counterterms in any theory
- In <u>renormalizable</u> theories A_n is a constant and this relation is reduced to the algebraic one
- In <u>non-renormalizable</u> theories for the scattering amplitudes A_n depends on kinematics and one has to integrate through the one-loop diagrams
- For the effective potential A_n depends on the fields and one has to differentiate

Taking the sum $\sum_n A_n(-z)^n = A(z)$ one can transform the recurrence relation into RG equation

$$\frac{dA(z)}{dz} = -2 \oint\!\!\!\!\!\!\not\;\; A(z) + \oint\!\!\!\!\!\!\not\;\; A(z) \otimes A(z) \qquad \frac{d}{dz} = \frac{d}{d\log\mu^2}$$

This is the generalized RG equation valid in any (even non-renormalizable) theory!

RG Equation

SYM_D

D=6 N=2

$$\Sigma(s,t,z) = z^{-2}\sum_{n=3}^{\infty}(-z)^n S_n(s,t)$$

$$\frac{d}{dz}\Sigma(s,t,z) = s - \frac{2}{z}\Sigma(s,t,z) + 2s\int_0^1 dx\int_0^x dy\ (\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=xt+yu}$$

Linear equation

D=8 N=1

$$\Sigma(s,t,z) = \sum_{n=1}^{\infty}(-z)^n S_n(s,t)$$

$$\frac{d}{dz}\Sigma(s,t,z) = -\frac{1}{12} + 2s^2\int_0^1 dx\int_0^x dy\ y(1-x)\ (\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=tx+yu}$$
$$-s^4\int_0^1 dx\ x^2(1-x)^2\sum_{p=0}^{\infty}\frac{1}{p!(p+2)!}\left(\frac{d^p}{dt'^p}(\Sigma(s,t',z)+\Sigma(t',s,z))|_{t'=-sx}\right)^2\ (tsx(1-x))^p.$$

Non-linear equation

RG pole equation for arbitrary potential

$$\Sigma(z,\phi) = \sum_{n=0}^{\infty}(-z)^n \Delta V_n(\phi) \qquad z = \frac{g}{\epsilon}$$

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RG pole equation

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2 \qquad \Sigma(0,\phi) = V_0(\phi)$$

This a non-linear partial differential equation!

Effective potential

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2}. \qquad v_2(\phi) \equiv \frac{d^2 V_0(\phi)}{d\phi^2}$$

RG Equations in Subleading Order Scattering Amplitude

$$\frac{dS_(s,t,z)}{dz} = s(S_1 + T_1) \otimes (S_1 + T_1)$$

Coefficients of $1/\epsilon^n$

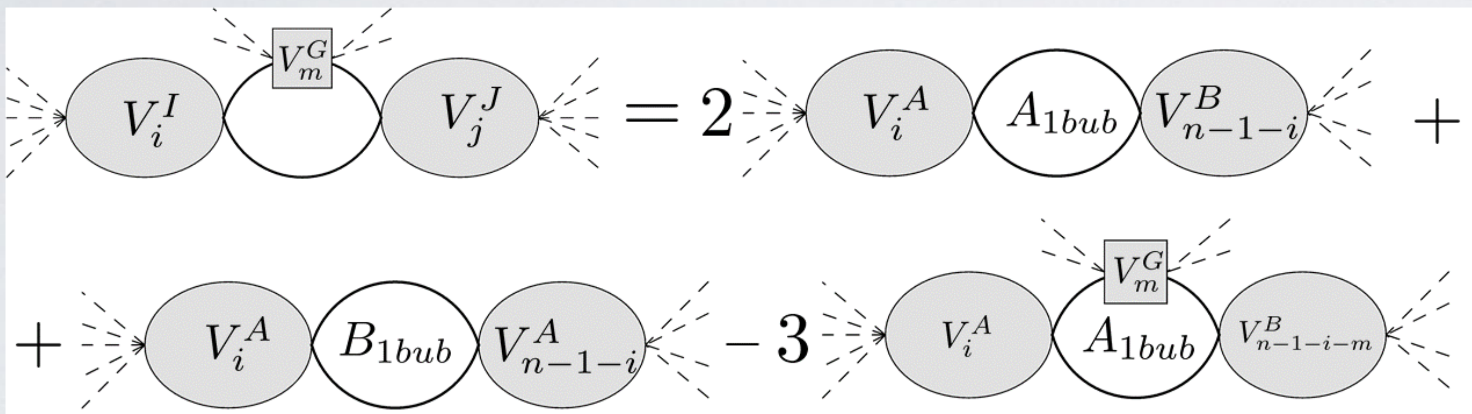
$$\frac{d^2 S_2(s,t,z)}{dz^2} = s\frac{d}{dz}(S_1 + T_1) \otimes (S_2 + T_2)$$
$$+s^2(S_1 + T_1) \otimes (S_2 + T_2) \otimes (S_1 + T_1)$$
$$+s(S_1 + T_1) \otimes \otimes (S_1 + T_1)$$

Coefficients of $1/\epsilon^{n-1}$

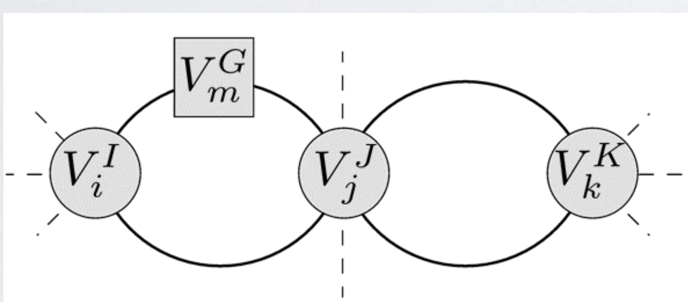
- Equation for the subleading order function as well as for all the subsequent orders is always linear!
- This seems to be in contradiction with the usual RG equation but is not!

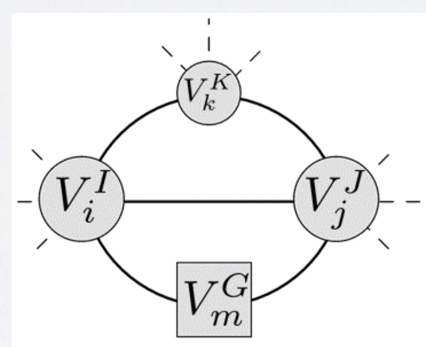
RG Equations in Subleading Order
Effective potential

$$B_n^{(n)\prime} = \left(\frac{2}{n(n-1)}B_2^{(n)} + \frac{2}{n}B_1^{(n)}\right)$$

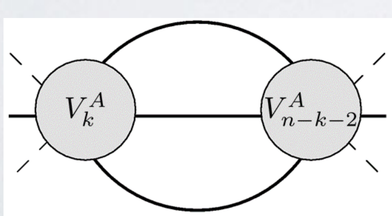


Contribution to $B_1^{(n)}$





Contribution to $B_2^{(n)}$



Contribution to kinetic term

RG Equations in Subleading Order

Effective potential

Σ_A - leading order

Σ_B - subleading order

Σ_G - kinetic term

$$\frac{\partial \Sigma_A}{\partial z} = -\frac{1}{4}(D_2\Sigma_A)^2$$

$$\frac{\partial^2 \Sigma_B}{\partial z^2} = -2\frac{1}{4}\frac{\partial}{\partial z}\left\{2D_2\Sigma_A D_2\Sigma_B - (D_2\Sigma_A)^2 D_2\Sigma_G\right\} -$$

$$-2\left\{\frac{1}{8}(D_3\Sigma_A)^2 D_2\Sigma_B - 2\frac{1}{8}(D_3\Sigma_A)^2 D_2\Sigma_A D_2\Sigma_B + 2\frac{1}{8}D_3\Sigma_A D_3\Sigma_B D_2\Sigma_A + \right.$$

$$+2\frac{1}{8}D_3\Sigma_A D_3\Sigma_B D_2\Sigma_A - \frac{1}{8}(D_3\Sigma_A)^2 D_2\Sigma_A + \frac{1}{8}(D_2\Sigma_A)^2 D_4\Sigma_B -$$

$$\left. -2\frac{1}{8}(D_2\Sigma_A)^2 D_4\Sigma_A D_2\Sigma_G + 2\frac{1}{8}D_2\Sigma_A D_2\Sigma_B D_4\Sigma_A\right\}$$

$$D_n \equiv \frac{\partial^n}{\partial \phi^n}$$

$$\frac{\partial^2 \Sigma_G}{\partial z^2} = \frac{1}{12}(D_3\Sigma_A)^2$$

- Equation for the subleading order function as well as for all the subsequent orders is always linear!
- This seems to be in contradiction with the usual RG equation which is non-linear, however, the proper form of the usual RG equation is also linear!!

RG Equations for the Logarithms

Coefficient of the leading logarithms $\log^n \mu^2$

$$\bar{A}_n^{(n)} = A_n^{(n)} = (-1)^{n+1} \frac{A_1^{(n)}}{n}$$

Coefficient of subleading logarithms $\log^{n-1} \mu^2$

$$\bar{B}_n^{(n)} = (-1)^n \left(\frac{2}{n-1} B_2^{(n)} + B_1^{(n)} \right)$$

Coefficient of subsubleading logarithms

$$\bar{C}_n^{(n)} = (-1)^{n-1} \left(\frac{3}{n-2} C_3^{(n)} + 2C_2^{(n)} + \frac{n-1}{2} C_1^{(n)} \right)$$

- Differential equation for the logarithms in all orders are always linear!
- They contain the known rhs which is calculated from higher order linear differential equation

Demonstration

V-A four fermion Interaction

BLTP

The interaction Lagrangian

$$\mathcal{L}_{int} = -\frac{G_F}{4}\bar{\Psi}\frac{\gamma^\mu(1-\gamma^5)}{2}\Psi\bar{\Psi}\frac{\gamma^\nu(1-\gamma^5)}{2}\Psi = -\frac{G_F}{4}\bar{\Psi}\gamma^\mu P_L\Psi\bar{\Psi}\gamma^\nu P_L\Psi$$

The tree level

$$A_4^{(0)} = \langle 13\rangle[42].$$

The one-loop amplitude

$$A_4^{(1)} = S_1 + T_1 + U_1 = \left(-\frac{16}{3}\frac{s}{\epsilon} - \frac{16}{3}\frac{t}{\epsilon}\right)\langle 13\rangle[42].$$

Recurrence Relations for the Scattering Amplitude

$$nS_n(s,t,u) = -\,4s\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[s(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}} \times$$
$$\times \frac{d^p A_k(s,-s-u',u')}{du'^p}\frac{d^p A_{n-1-k}(s,-s-u',u')}{du'^p}\Big|_{u'\to -sx}$$

$$nT_n(s,t,u) = -\,4t\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[t(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}} \times$$
$$\times \frac{d^p A_k(-t-u',t,u')}{du'^p}\frac{d^p A_{n-1-k}(-t-u',t,u')}{du'^p}\Big|_{u'\to -tx}$$

$$nU_n(s,t,u) = 4u\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[u(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}} \times$$
$$\times \frac{d^p A_k(s',-s'-u,u)}{ds'^p}\frac{d^p A_{n-1-k}(s',-s'-u,u)}{ds'^p}\Big|_{s'\to -ux}$$
$$+\,4u\int_0^1 dx \sum_{k=0}^{n-1}\sum_{p=0}^{k} \frac{[u(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}} \times$$
$$\times \frac{d^p A_k(-t'-u,t',u)}{dt'^p}\frac{d^p A_{n-1-k}(-t'-u,t',u)}{dt'^p}\Big|_{t'\to -ux}$$

$$A_n(s,t,u) = S_n(s,t,u) + T_n(s,t,u) + U_n(s,t,u)$$

RG Equation for the Scattering Amplitude

$$A(s,t,u) = \sum_{n=0}^{\infty} A_n(s,t,u)(-z)^n$$

$$\begin{aligned}
-\frac{dA(s,t,u)}{dz} &= \\
= -4s\int_0^1 dx \sum_{p=0}^{\infty} &\frac{[s(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}} \left(\frac{d^p A(s,-s-u',u')}{du'^p}\right)^2 \\
-\,4t\int_0^1 dx \sum_{p=0}^{\infty} &\frac{[t(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+2)^{-1}} \left(\frac{d^p A(-t-u',t,u')}{du'^p}\right)^2 \\
+4u\int_0^1 dx \sum_{p=0}^{\infty} &\frac{[u(-s-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}} \left(\frac{d^p A(s',-s'-u,u)}{ds'^p}\right)^2 \\
+4u\int_0^1 dx \sum_{p=0}^{\infty} &\frac{[u(-t-u)]^p[x(1-x)]^{p+1}}{p!(p+1)!(p+3)^{-1}} \left(\frac{d^p A(-t'-u,t',u)}{dt'^p}\right)^2
\end{aligned}$$

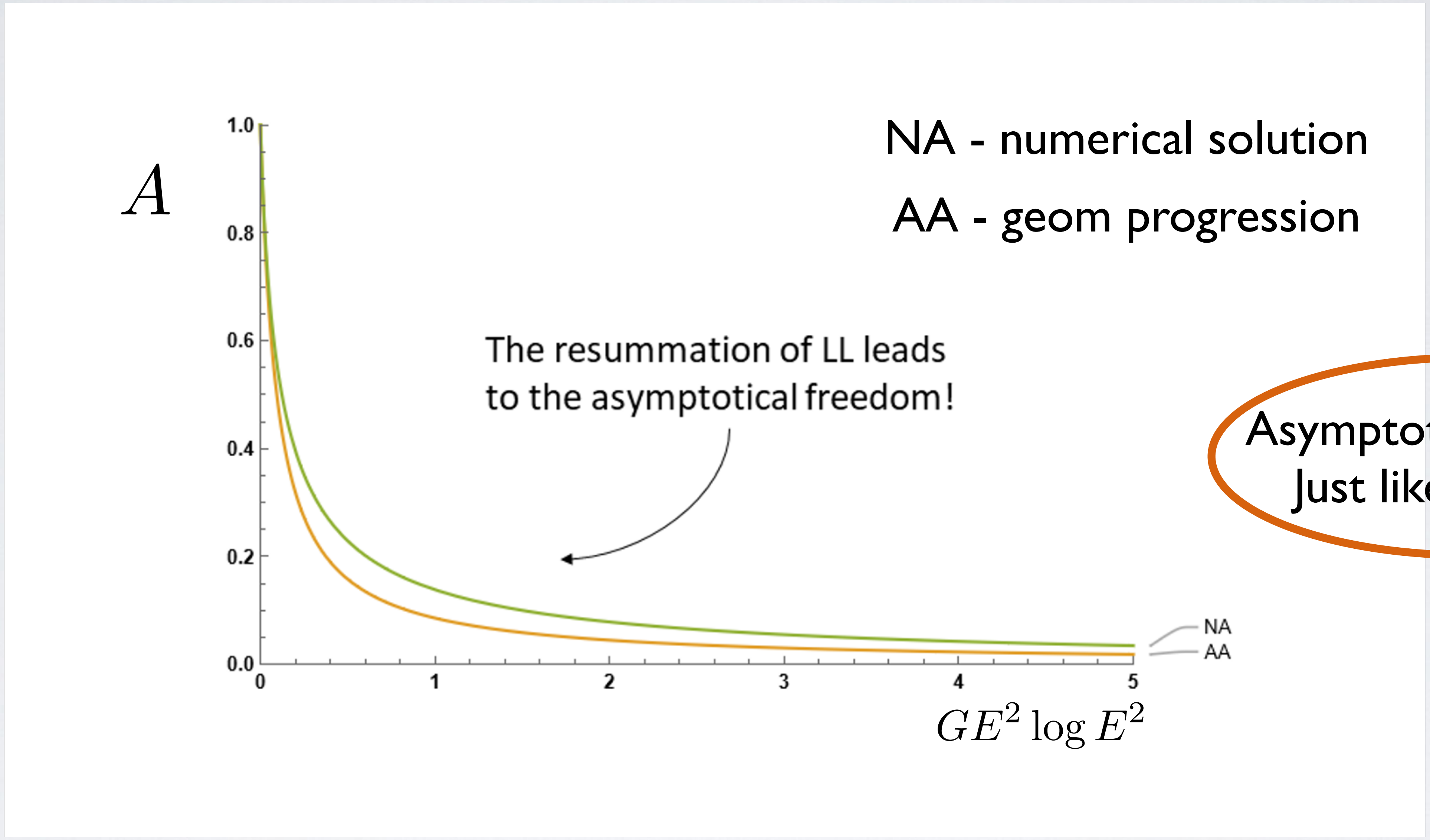
$$z = G/\epsilon \to -G\log Q^2/\mu^2$$

- With the help of this equation one can find the asymptotic behaviour of the amplitude in the high energy regime $s \sim t \sim u \sim E^2 \to \infty$

Numerical Solution of RG Equation for the Scattering Amplitude

Borlakov, Kazakov, 25

$$s = 4E^2, \ t = u = -2E^2, \ E \to \infty \qquad \frac{Amplitude}{A_{Tree}} = A(E^2, G\log E^2/\mu^2)$$



Asymptotic freedom !!
Just like in the SM

$$A_{Tree} \sim GE^2 \qquad A \sim \frac{1}{GE^2 \log E^2} \qquad Amp = A_{Tree} A \sim \frac{1}{\log E^2}$$

Demonstration

α -attractor Inflaton Potential

Kallosh, Linde 13

Inflaton action with hyperbolic geometry

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R(g) + \frac{1}{2}\frac{\partial_\mu\phi\partial^\mu\phi}{1-\frac{\phi^2}{6\alpha}} - V(\phi)\right]$$

Transition to the standard kinetic term

$$\partial\phi/\sqrt{1-\frac{\phi^2}{1-6\alpha}} = \partial\varphi \qquad \phi = \sqrt{6\alpha}\tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right)$$

Inflaton action of α -attractor model

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R(g) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V\left(\sqrt{6\alpha}\tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right)\right)\right].$$

T- model

$$gV_T(\varphi) = g\tanh^n\left(\frac{\varphi}{\sqrt{6\alpha}M_{Pl}}\right)$$

n=2 T_2 - model

RG Equation for the T-model Effective potential

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2$$

Dimensionless variables

$$x = z/M_{Pl}^4 \quad y = \tanh^n(\varphi/\sqrt{6\alpha}M_{Pl})$$

$$\Sigma(z/M_{Pl}^4, \tanh^n(\varphi/\sqrt{6\alpha}M_{Pl})) \equiv S(x,y)$$

$$S_x = -\frac{n^2 y^{2-\frac{4}{n}}\left(y^{2/n}-1\right)^2}{144\alpha^2}\left(\left(y^{2/n}+n\left(y^{2/n}-1\right)+1\right)S_y + ny\left(y^{2/n}-1\right)S_{yy}\right)^2$$

This is a nonlinear partial differential equation!

Boundary conditions

$$S(0,y) = y,\ S(x,1) = 1,\ S_y(x,1) = 0.$$

n=2 case

$$S_x = -\frac{(y-1)^2\left((3y-1)S_y + 2(y-1)yS_{yy}\right)^2}{36\alpha^2}$$

Numerical Solution for T_2 - model

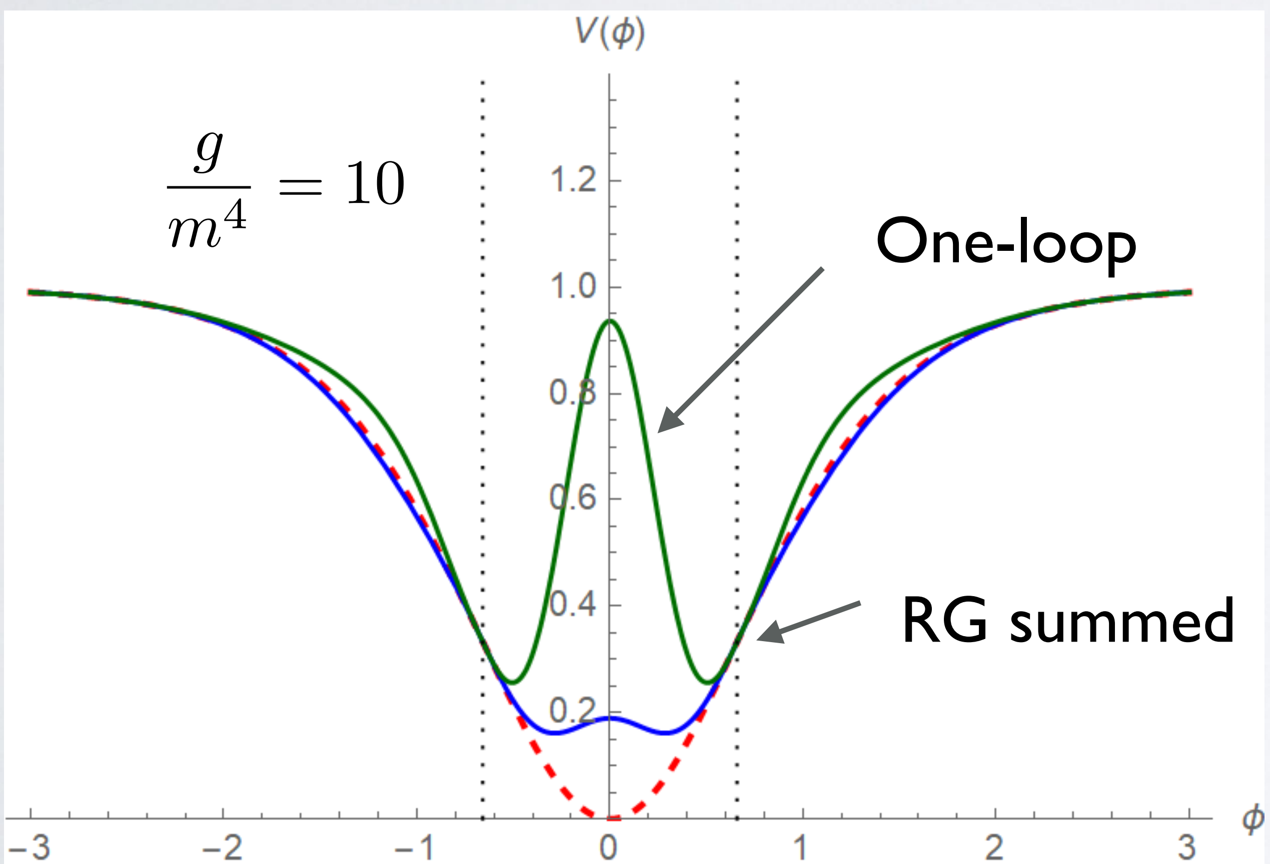
Kazakov, Iakhibbaev, Tolkachev 23

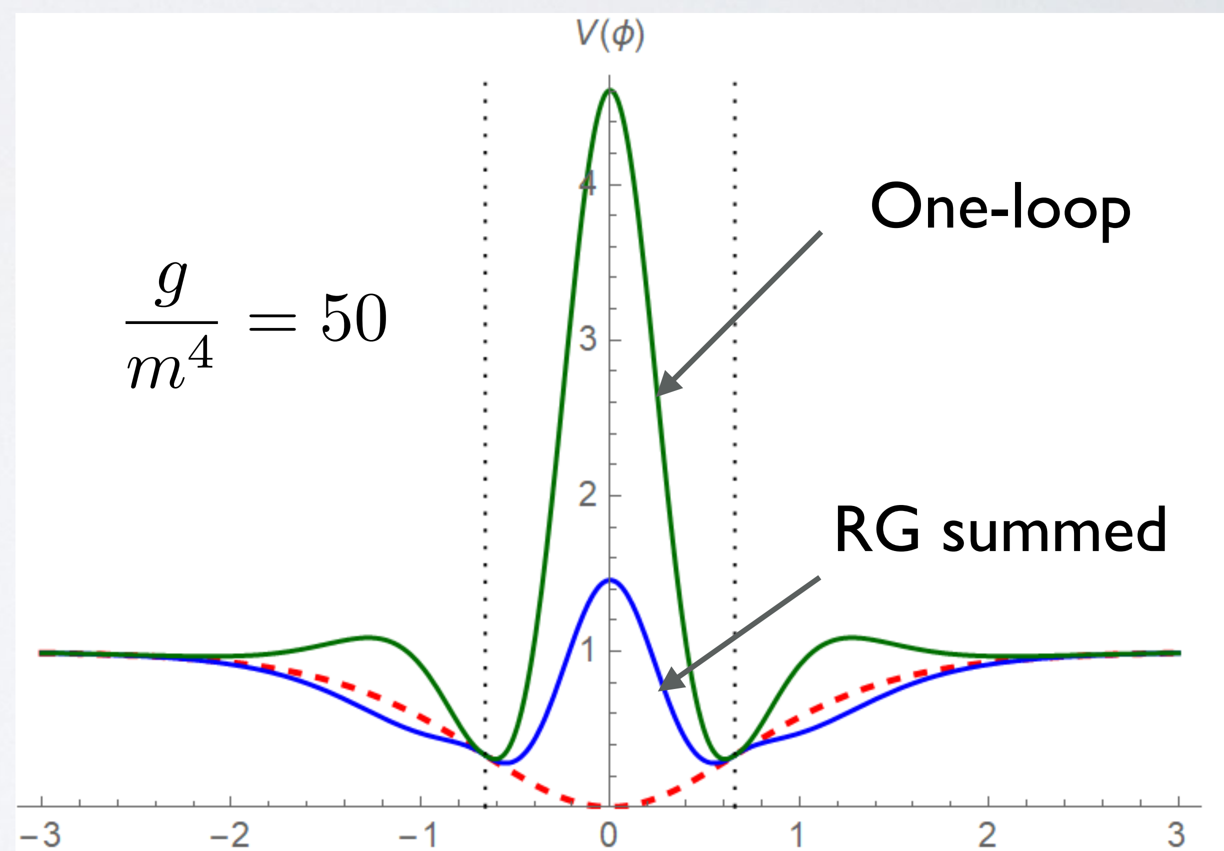
ArXiv: 2308.03872

- JCAP 09 (2023) 049

$$gV_0 = g \tanh^2(\phi/m)$$

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2} \cdot \qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$



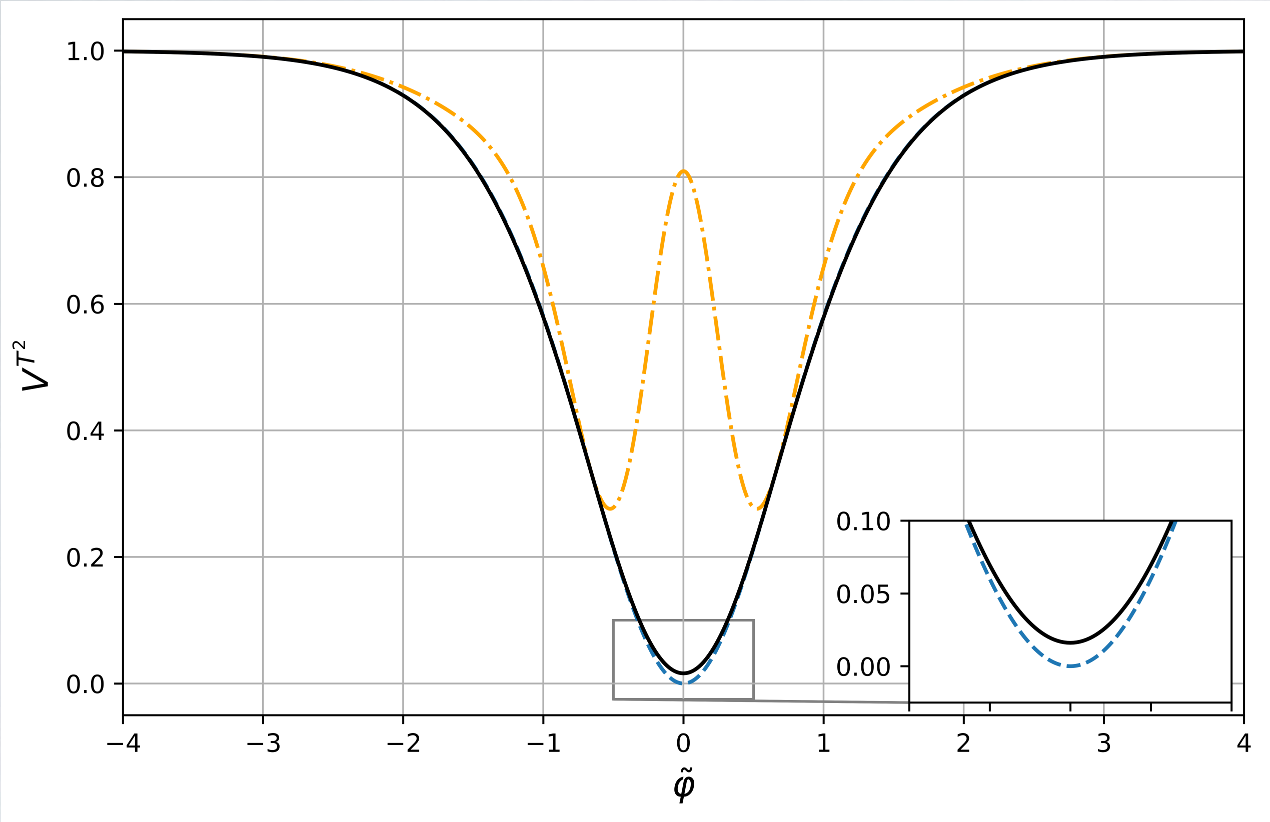


- Peak at the origin
- Additional minima

Lift of the Potential at the Minima - Origin of the Cosmological Constant

Kazakov, Iakhibbaev, Tolkachev, Filippov 24,25

ArXiv: 2405.18818



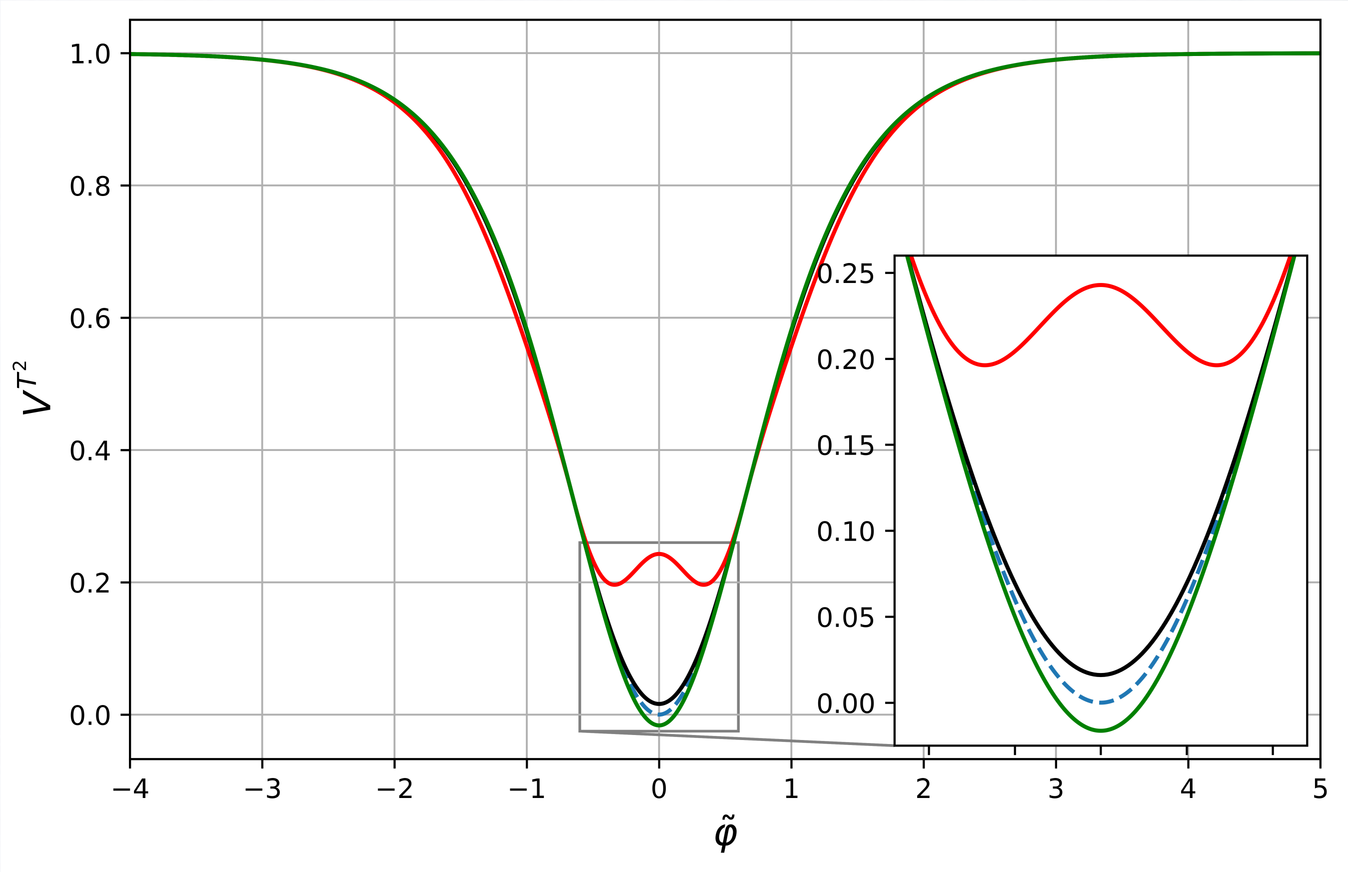
Comparison of the classical T2-model potential (blue dashed line), the one-loop correction (orange dashed line), and the RG summed potential (black solid line) for $g \sim 1, \mu < M_{PL}$

T2-model potential: variation of μ . The classical potential (blue dashed line), the RG summed potential (solid lines) for

$\mu < M_{Pl}$ $\mu \ll M_{Pl}$ $\mu > M_{Pl}$

black line, red line, green line

$$g = 2,\ \alpha = 1$$



Resume

- **The UV divergences in non-renormalizable theories are local and can be removed by local counter terms like in renormalizable ones**

- **Based on locality of the counter terms due to the Bogoliubov-Parasiuk theorem one can construct the recurrence relations for the leading, subleasing, etc divergences in all loops divergences starting from one-, two-, three-, etc loop diagrams**

- **The recurrence relations can be converted into the generalized RG equations just like in renormalizable theories**

  - **The RG equations allow one to sum up the leading (subleading, etc) divergences in all loops and define the high-energy/field behaviour**

  - **The RG equations for the subleading poles, etc are always linear differential equation of increasing order**

  - **The RG equations for logarithms are always first order differential equation with known rhs**