QCD Vacuum $\rightarrow$ Galactic Halos
An Unorthodox Dark Matter Proposal

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Mathematical Quantum Field Theory
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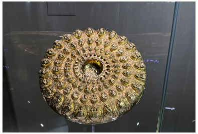


Outline of this alternative interpretation of dark matter

Motivation

Empirical puzzles: rotation curves, the RAR, and the universal surface density Σ_0.

Part I

Gluonic BEC survival after hadronisation: from the QCD trace anomaly to the dark-to-baryon ratio.

Part II

From a scale to a protected spectral gap: $\mathrm{SO}(2,3)$ UIR and the effective AdS spectrum.

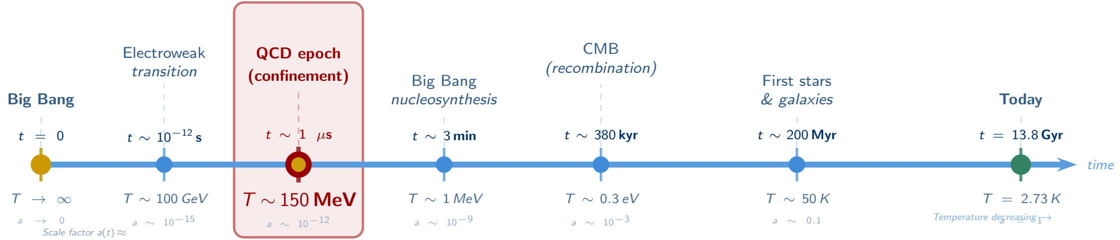
Part III

Cored halo profiles, finite total mass, and $g_\star = \pi^2\, G\Sigma_0 \simeq 1.9 \times 10^{-10}\,\mathrm{m\,s^{-2}}$.

Part IV

Comparisons with ΛCDM, MOND, fuzzy DM; conclusion and open questions.

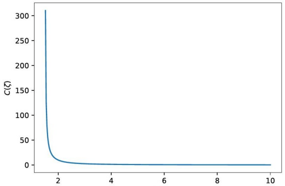
The Universe Through Time: Key Epochs and Temperatures



Causal chain (this talk)

$$\underbrace{\text{Trace anomaly}}_{\Lambda_{\mathrm{QCD}}} \longrightarrow \underbrace{\text{Di-gluonic BEC}}_{\text{colour-singlet, decoupled}} \longrightarrow \underbrace{\text{AdS} - \text{harmonic trap}}_{\mathrm{SO}(2,3)\ \text{UIR}} \longrightarrow \underbrace{\text{Cored halos}}_{\rho_c(1+x^2)^{-\zeta}\ \ \zeta>3/2} \longrightarrow \underbrace{g_\star = C(\zeta) G\Sigma_0}_{\sim g_\dagger}$$

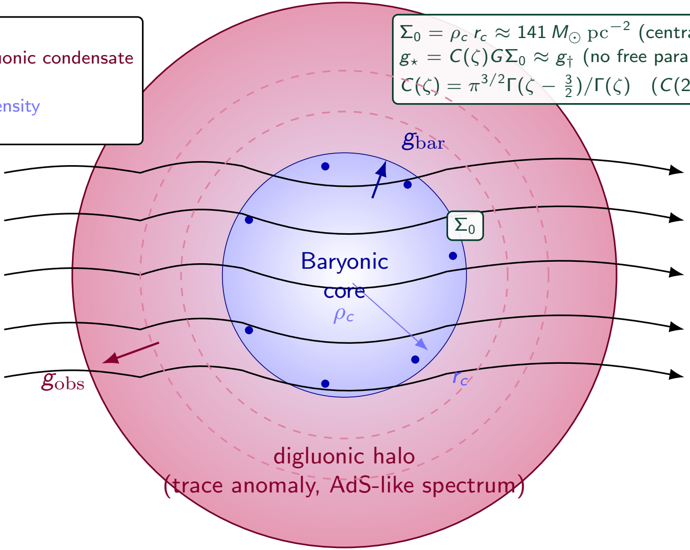
$$C(\zeta) = \pi^{3/2}\Gamma(\zeta - \tfrac{3}{2})/\Gamma(\zeta) \quad (= \pi^2 \text{ for } \zeta = 2)$$



- Baryons (stars/gas)
- Halo: AdS-like, trace anomaly / digluonic condensate
- Light rays deflected (lensing)

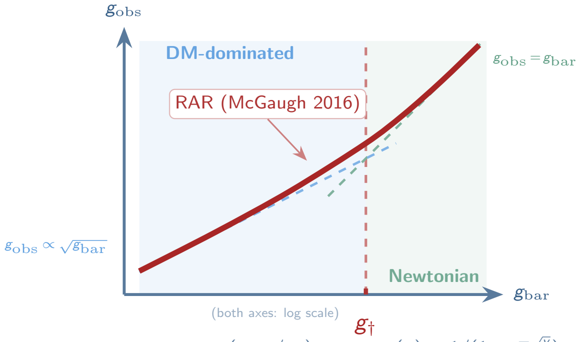
r_c: halo core radius; ρ_c: central DM density

$\Sigma_0 = \rho_c\, r_c \approx 141\, M_\odot\, \mathrm{pc}^{-2}$ (central surface density)
$g_\star = C(\zeta) G \Sigma_0 \approx g_\dagger$ (no free parameter)
$C(\zeta) = \pi^{3/2}\Gamma(\zeta - \frac{3}{2})/\Gamma(\zeta)$ $\quad (C(2) = \pi^2)$



Schematic (not to scale). Concept: dark matter as an effective digluonic Bose-Einstein condensate surrounding baryonic regions.

Four Key Quantities: $g_{\rm obs}$, $g_\dagger$, Σ_0, $g_\star$, + core radius r_c and central density ρ_c



$g_{\rm obs} = g_{\rm bar}\,\nu(g_{\rm bar}/g_\dagger)$, $\nu(y) = 1/(1-e^{-\sqrt{y}})$

RAR *Radial Acceleration Relation*: empirical tight correlation $g_{\rm obs} = f(g_{\rm bar})$ established by McGaugh, Lelli & Schombert (PRL 2016) from 153 rotationally supported galaxies.

SPARC *Spitzer Photometry & Accurate Rotation Curves*: catalogue of 175 late-type disk galaxies combining *Spitzer* 3.6 μm stellar-mass maps with high-quality HI/Hα rotation curves (Lelli, McGaugh, Schombert, AJ 2016).

$g_{\rm obs}(r) = v^2_{\rm obs}(r)/r$ *observed total acceleration*

Centripetal acceleration at radius r from rotation curves. Includes all contributions (baryonic + DM).

$g_\dagger \approx 1.5 \times 10^{-10}\,\mathbf{m\,s^{-2}}$ *empirical RAR scale*

Transition scale in the RAR (SPARC, 175 galaxies; McGaugh 2016). Origin is unexplained in ΛCDM.

$\Sigma_0 = \rho_c r_c \approx 141\,M_\odot\,{\rm pc}^{-2}$ *central surface density*

Near-universal across ∼1000 galaxies, 14 magnitudes in luminosity (Donato 2009). Input to this work.

$g_\star = GM_h/r_c^2 = C(\zeta)G\Sigma_0$ *predicted acceleration scale*

Derived from the SO(2, 3) UIR $U(\zeta)$ (this work). The r_c dependence cancels exactly; $g_\star$ is set by Σ_0 alone.

Central result

$g_\star = C(\zeta)G\Sigma_0 \simeq 1.9\times10^{-10}\,{\rm m\,s^{-2}} \approx g_\dagger$ **(no free parameter)** $C(2) = \pi^2$

The Logical Chain — Roadmap of the Talk

Quark–Gluon Plasma era (QCD epoch) → **Gluonic BEC survival after hadronisation** → **AdS Infrared Spectrum $\mathrm{SO}(2,3)$ UIR** → **Cored halo finite M_h $\rho_c(1+x^2)^{-\zeta}$** → **Observed $\Sigma_0 \simeq 141\, M_\odot\, \mathrm{pc}^{-2}$ Donato 2009**

§I §II §III §IV–V

↓

$g_\star = C(\zeta) G \Sigma_0 \simeq 1.9 \times 10^{-10}$ $\mathbf{m\,s^{-2}} \sim g_\dagger$

Central claim

The galactic acceleration scale $g_\dagger \sim 10^{-10}\,\mathrm{m\,s^{-2}}$ equals $C(\zeta)G\Sigma_0$, where $C(\zeta) = \pi^{3/2}\Gamma(\zeta - \frac{3}{2})/\Gamma(\zeta)$ ($C(2) = \pi^2$), and Σ_0 is the observed near-universal central DM surface density. This equality is **representation-theoretically exact**, and **scale-independent**: r_c drops out. **No modified gravity. No new elementary particles.**

r_c: core radius

ρ_c : central density

Galactic Rotation Curves — The Core-vs-Cusp Puzzle

- Observed circular velocities $v_{\rm obs}(r)$ far exceed the baryonic prediction $v_{\rm bar}(r)$ at large radii $\Rightarrow$ **dark matter halo**.
- ΛCDM N-body simulations predict cuspy NFW profiles: $\rho_{\rm NFW}(r) \propto r^{-1}$ at $r \to 0$.
- Observations of dwarf and low-surface-brightness galaxies strongly favour **cored** profiles: $\rho(0) < \infty$. *Core–cusp tension.*
- Standard NFW/Burkert profiles diverge in total mass without an ad-hoc cutoff radius.

Profile comparison

Model	$\rho(0)$	$M(<\infty)$
NFW	∞	∞
Burkert	finite	∞
This work	**finite**	**finite**

Key feature: our halo has a **finite total mass** for $\zeta > 3/2$, without any external cutoff. Cored profile $\Rightarrow$ no core–cusp problem.

NFW *Navarro–Frenk–White* profile: $\rho_{\rm NFW}(r) \propto r^{-1}(1 + r/r_s)^{-2}$, cuspy ($\rho \to \infty$ as $r \to 0$) and with infinite total mass; standard ΛCDM prediction.

Burkert profile (Burkert 1995): $\rho_B(r) = \dfrac{\rho_0\, r_0^3}{(r + r_0)(r^2 + r_0^2)}$, cored (finite $\rho(0) = \rho_0$) but still infinite total mass without an external truncation.

Two Empirical Facts to Explain Simultaneously

Fact 1 — RAR (McGaugh et al. 2016):

$$g_{\rm obs} = g_{\rm bar}\,\nu\!\left(\frac{g_{\rm bar}}{g_\dagger}\right), \quad \nu(y) = \frac{1}{1 - e^{-\sqrt{y}}}$$

Single acceleration scale (SPARC, 175 galaxies):

$$g_\dagger \sim 1\text{–}2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

Why is this scale universal? ΛCDM has no fundamental parameter of this size.

Fact 2 — Universal central surface density:

$$\Sigma_0 = \rho_c\, r_c \simeq 141\, M_\odot\, \mathrm{pc}^{-2}$$

Observed across ∼1000 galaxies spanning 14 magnitudes in luminosity [4, 5, 6].

Our answer in one formula ($\zeta = 2$)

$$g_\star = \frac{GM_h}{r_c^2} = \pi^2 G\,\Sigma_0 \simeq 1.9 \times 10^{-10}\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

- r_c drops out: result is **scale-independent**.
- Depends only on the **observed** Σ_0.
- The π^2 factor comes from the choice $\zeta = 2$ mass integral $M_h = \pi^2 \rho_c r_c^3$.
- $g_\star \sim g_\dagger$ within the empirical range **without fine-tuning**.

(Not a first-principles derivation of the RAR itself; the linear response to baryons is left for future work.)

Step I — QCD Timeline and the Birth of the Dark Sector

- At the QCD epoch ($T \sim 150\,\mathrm{MeV}$, scale factor $a \sim 10^{-12}$): **colour confinement**.
- Quarks and gluons bind into colour-singlet hadrons.
- **Key question:** can a gluonic component remain distinct from the hadronic thermal bath and survive as a *coherent dark sector*?
- Classical Yang–Mills is scale-invariant. Quantum renormalization breaks this via the **trace anomaly**, seeding Λ_{QCD}.
- Two competing channels in $T^{\mu}{}_{\mu}$:
  - Gluonic (antiscreening): $\propto 11N_c$
  - Fermionic (screening): $\propto 2N_f$
- We posit: gluonic channel $\to$ dark sector; fermionic channel $\to$ baryonic sector.

Trace anomaly structure

$$T^{\mu}{}_{\mu} = \frac{\beta(g_s)}{2g_s} G^a_{\mu\nu} G_a^{\mu\nu} + \cdots$$

$$\beta(g_s) \sim -\frac{g_s^3}{48\pi^2}\,(\underbrace{11N_c}_{\text{gluons}} \underbrace{-2N_f}_{\text{quarks}})$$

One-loop ratio of anomaly weights ($N_c = N_f = 3$):

$$\mathcal{R}_{\mathrm{QCD}} = \left.\frac{11N_c}{2N_f}\right|_{N_c=N_f=3} = \frac{11}{2} = 5.5$$

Key assumption

Colour-singlet scalar di-gluonic bound states decouple from the thermal plasma at confinement and evolve as a coherent dark sector.

Why the Gluonic BEC Survives Hadronisation

Dark-to-baryonic abundance ratio:

The gluonic/fermionic anomaly-weight ratio predicts:

$$\mathcal{R}_{\rm QCD} = 5.5$$

The Planck-2018 cosmological value is:

$$\mathcal{R}_{\rm obs} = \frac{\Omega_{\rm DM}}{\Omega_b} \simeq 5.36$$

Numerical proximity

$\mathcal{R}_{\rm QCD}/\mathcal{R}_{\rm obs} \simeq 1.03$ — within 3% without any free parameter.

We treat this as a *phenomenological anomaly-sector matching ansatz*, not as a derivation.

Physical reasons for survival:

1. **Colour neutrality.** Di-gluonic $|gg\rangle$ states are singlets; they do not interact with the hadronic plasma via colour forces.
2. **Long mean free path.** After confinement, colour-singlet hadronic matter has cross-sections $\sigma \sim 1/\Lambda^2_{\rm QCD}$; the gluonic condensate is below this threshold.
3. **Bose statistics.** Scalar bosons can macroscopically occupy the lowest-weight state once $T < T_c$.
4. **No subsequent QCD interactions.** Post-confinement, colour charges are screened; the dark sector evolves gravitationally.

Detailed cosmological justification (entropy production, baryogenesis, hadronisation, sector dilution) is left for future work.

Step II — QCD Trace Anomaly: Generating an Infrared Scale

Classical Yang–Mills theory is scale-invariant:

$$T^{\mu}{}_{\mu} = 0$$

No intrinsic mass or length scale is present.

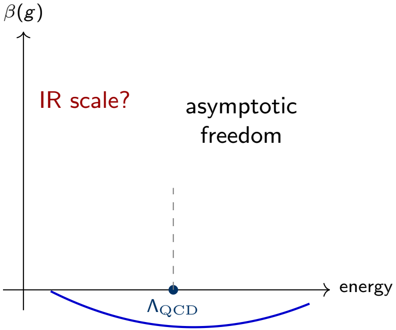
Quantum effects break this:

$$\langle T^{\mu}{}_{\mu} \rangle \neq 0 \qquad \Leftarrow \ \text{renormalization}$$

The **trace anomaly** generates via *dimensional transmutation*: $\Lambda_{\mathrm{QCD}} \sim 200\,\mathrm{MeV}$.

What the anomaly does and does not do

- ✓ **Guarantees** a dimensionful IR scale.
- ✓ Sets $\mathcal{R}_{\mathrm{QCD}} \approx \mathcal{R}_{\mathrm{obs}}$.
- × Does **not** determine spectral structure (gap, discreteness, boundedness).



The anomaly is the **microscopic seed**; additional structure (lowest-weight UIR of $\mathrm{SO}(2,3)$) promotes it to a **spectrally protected infrared length** r_c.

Step III — From a Scale to a Protected Spectral Gap

What we need: an infrared spectrum that is

1. Bounded from below (positive energy).
2. Gapped (isolated ground state $\Rightarrow$ finite correlation length).
3. Spectrally rigid (gap cannot be removed continuously $\Rightarrow$ universality).

The representation-theoretic solution:
In 4D, requiring *Lorentz covariance* + *positive-energy lowest-weight unitary structure* strongly restricts the admissible symmetry algebras. The simplest choice:

$$\mathrm{SO}(2,3) \;\cong\; \text{isometry group of AdS}_4$$

Its maximal compact subalgebra $\mathfrak{so}(2) \oplus \mathfrak{so}(3)$ contains a compact $\mathfrak{so}(2)$ generator — the **Hamiltonian bounded from below**.

Resulting discrete spectrum (schematic)

$$E_k = E_0 + k\hbar\omega_{\mathrm{AdS}}, \quad k = 0, 1, 2, \dots, \quad E_0 > 0 \qquad \omega_{\mathrm{AdS}} = \kappa c = c/r_c$$

$\Rightarrow$ correlation length $r_c \sim \kappa^{-1}$ is **representation-theoretically protected**.

Important: "AdS" here is *strictly representation-theoretic*. The cosmological background remains Friedmann–Robertson–Walker.

The SO(2,3) Representation — Embedding and Casimir

AdS_4 realized as the hyperboloid in $\mathbb{R}^{2,3}$:

$$(X^0)^2 - \|\mathbf{X}\|^2 + (X^5)^2 = \kappa^{-2} \sim r_c^2$$

Quadratic Casimir:

$$Q^{(1)}_{\mathrm{AdS}} = -\tfrac{1}{2} L_{AB} L^{AB}$$

Fixed-Casimir (Klein–Gordon type) equation on AdS:

$$\left[\Box_{\mathrm{AdS}} + \kappa^2 \zeta(\zeta - 3)\right] \phi(x) = 0$$

Parameter ζ labels the UIR and sets the lowest weight:

$$E^{\mathrm{rest}}_{\mathrm{AdS}} = \hbar \kappa c\, \zeta$$

Focus on $\zeta = 2$

Energy-density integrability requires $\zeta > 3/2$. $\zeta = 2$ is the **smallest admissible value** giving:

- A normalizable scalar lowest-weight UIR.
- A **finite total halo mass**.
- Conformally coupled scalar $[\zeta(\zeta - 3) = -2]$.
- The exact result $M_h = \pi^2 \rho_c r_c^3$.

The $\zeta = 2$ and $\zeta = 1$ modes share the same Casimir eigenvalue: the conformal pair $D(2,0) \oplus D(1,0)$.

Step IV — Fronsdal Modes and Bose–Einstein Condensation

Fronsdal normal modes (Phys. Rev. D, 1974) for $\zeta > 3/2$:

$$\phi_{nlm}(x) = C_{nl}\, Y_{lm}(\theta, \varphi)\, e^{-i(\zeta+2n+l)\omega_{\mathrm{AdS}} t} \times (\kappa r)^{2n+l} \frac{1}{(1+(\kappa r)^2)^{\frac{\zeta+2n+l}{2}}} \cdots$$

Discrete spectrum:

$$E_k = \hbar\omega_{\mathrm{AdS}}(k+\zeta), \quad G_k = \frac{(k+1)(k+2)}{2}$$

Equivalent to 3D isotropic harmonic oscillator $\Rightarrow$ ideal Bose gas in grand-canonical ensemble.

BEC without an external trap

The harmonic structure arises **intrinsically** from the effective AdS geometry — not from an externally imposed potential.

Critical temperature:

$$T_c \sim \frac{\hbar\omega_{\mathrm{AdS}}}{k_B} \left(\frac{N_G}{\zeta(3)}\right)^{1/3}$$

For large occupation $N_G \gg 1$, the system condenses into the lowest-weight state.

Key point

Condensation is a **statistical consequence** of the representation-theoretic spectral organization, not of additional binding dynamics.

Ground State and Excited Modes

Ground state $(n = 0, l = 0)$, for $\zeta = 2$:

$$|\phi_{000}|^2 \propto \frac{1}{(1 + (r/r_c)^2)^\zeta} \quad \Rightarrow \quad \rho_{\rm DM}(r) = \frac{\rho_c}{(1 + x^2)^\zeta}$$

- Constant-density **core** for $r \ll r_c$.
- Power-law decay $\rho \sim r^{-2\zeta}$ for $r \gg r_c$.
- **Finite total mass** (for $\zeta > 3/2$).

First excited modes couple to baryonic perturbations:

- $(n, l) = (0, 1)$: **dipolar** deformation $\rightarrow$ DM polarization around baryonic concentrations.
- $(n, l) = (1, 0)$: **breathing mode** $\rightarrow$ radial pulsation of the core.

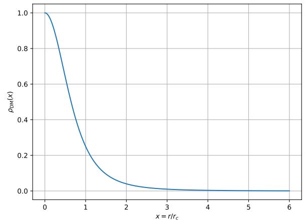
Mode structure, $\zeta = 2$

Breathing mode has a radial node at $r \approx r_c$.

The condensate near baryons:

$$\Psi_{\rm DM} \sim a_{000}\phi_{000} + \sum_m a_{01m}\phi_{01m} + a_{100}\phi_{100} + \cdots$$

Baryons probe the lowest modes of the AdS spectrum.



Density profile $\rho_{\rm DM}(x)$, $\zeta = 2$. Rapid falloff guarantees finite M_h.

Step V — Halo Mass Profile

Enclosed DM mass ($x = r/r_c$):

$$M_{\rm DM}(< x, \zeta) = 4\pi\rho_c r_c^3 F(x,\zeta)$$

$$F(x,\zeta) = \int_0^x u^2(1+u^2)^{-\zeta}\,du$$

Closed form via incomplete beta function:

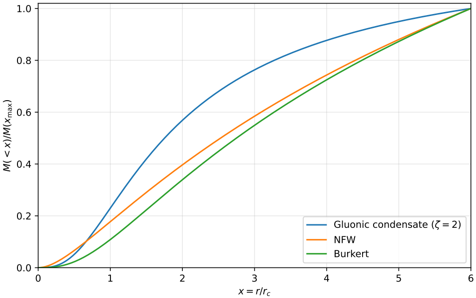
$$F(x,\zeta) = \tfrac{1}{2} B_{\frac{x^2}{1+x^2}}\left(\tfrac{3}{2}, \zeta - \tfrac{3}{2}\right)$$

For $\zeta = 2$:

$$F(x,2) = \tfrac{1}{2}\left(\arctan x - \frac{x}{1+x^2}\right)$$

$$\boxed{M_h = \pi^2 r_c^3 \rho_c} \quad \Rightarrow \quad \rho_c = \frac{M_h}{\pi^2 r_c^3}$$

The π^2 comes directly from
$F(\infty, 2) = \pi/4 \Rightarrow M_h = 4\pi\rho_c r_c^3 \cdot \pi/4$.



Normalized enclosed mass $M(<x)/M(x_{\rm max})$.
NFW/Burkert: $M(<r) \to \infty$.
Gluonic condensate: $M(<\infty) = M_h < \infty$.

Rotation Curves and Radial Accelerations

Circular velocity:

$$v^2(x,\zeta) = 4\pi G\rho_c r_c^2 \, \frac{F(x,\zeta)}{x}$$

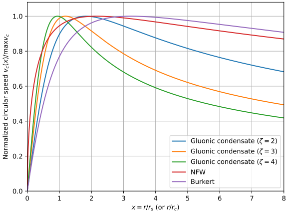
- $x \ll 1$: $v \propto r$ (solid-body, **core**)
- $x \gg 1$: $v \propto r^{-1/2}$ (Keplerian)

Peak velocity at $x_{v;\max}$; e.g. $\zeta = 2$: $x_{v;\max} \approx 1.83$.

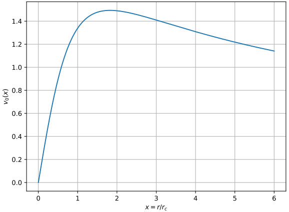
Radial acceleration:

$$g(x,\zeta) = 4\pi G\rho_c r_c \, \frac{F(x,\zeta)}{x^2}$$

Peak at $x_{g;\max} < x_{v;\max}$ ($\zeta = 2$: $x_{g;\max} \approx 0.97$).



Normalized $v_c(x)/\max v_c$. Each gluonic curve peaks earlier than NFW/Burkert; larger ζ gives faster Keplerian decline.



Dimensionless velocity $v(x)/\sqrt{4\pi G\rho_c r_c^2}$ for various ζ.

The Key Step: $g_\star = \pi^2 G\Sigma_0$ ($\zeta = 2$)

Step 1 — Intrinsic halo acceleration:

$$g_\star = \frac{GM_h}{r_c^2}$$

Step 2 — Use $\zeta = 2$ mass formula:

$$M_h = C(\zeta = 2) r_c^3 \rho_c = \pi^2 r_c^3 \rho_c \quad \Rightarrow \quad g_\star = \pi^2 G\, \rho_c r_c$$

Step 3 — Identify the surface density:

$$\Sigma_0 \equiv \rho_c\, r_c$$

Step 4 — Remarkable cancellation:

$$g_\star = \pi^2 G\, \Sigma_0$$

r_c drops out completely. $g_\star$ is determined by Σ_0 alone.

Observational input

$\Sigma_0 = \rho_c r_c$ is **approximately universal** across $\sim$1000 galaxies spanning 14 magnitudes in luminosity:

$$\Sigma_0 \simeq 141\, M_\odot\, \mathrm{pc}^{-2}$$

Donato et al. (2009); Gentile et al. (2009); Kormendy & Freeman (2004).

Predicted acceleration scale

$$g_\star = \pi^2\, G \times 141\, M_\odot\, \mathrm{pc}^{-2} \simeq 1.9 \times 10^{-10}\, \mathrm{m\, s^{-2}}$$

This lies well within the empirical RAR range:

$$g_\dagger \sim (1\text{–}2) \times 10^{-10}\, \mathrm{m\, s^{-2}}$$

Why This Result Is Non-Trivial

Without the finite-mass condition ($\zeta > 3/2$):

- NFW, Burkert, pseudo-isothermal profiles all have $M(< x = \infty) = \infty$.
- $g_\dagger = GM_h/r_c^2$ requires an ad-hoc truncation radius R_{200} to make M_h finite.
- $\Sigma_0 = \rho_c r_c$ then depends on the arbitrary choice of R_{200}.

With the finite-mass profile:

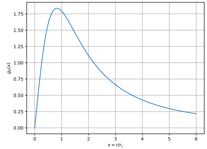
- M_h is **intrinsically finite**: $C(\zeta)\rho_c r_c^3$.
- $g_* = \pi^2 G\Sigma_0$ requires **no cutoff**.
- The $C(\zeta)$ prefactor is **exact**, e.g. $C(2) = \pi^2$, a consequence of the $\mathrm{SO}(2,3)$ UIR structure.
- Plugging in the observed Σ_0 gives $g_\star \sim g_\dagger$.

Why Σ_0 is universal

Both observations suggest Σ_0 varies by less than one order of magnitude despite baryonic mass varying by 14 magnitudes.

Possible explanation within this framework: $\Sigma_0 = \rho_c r_c$ is constrained by the BEC ground state + self-gravity equilibrium — an intrinsic property of the condensate, not of the baryonic environment.

(Theoretical derivation of the near-universality of Σ_0 from first principles is an open problem.)



$g_0(x)$: dimensionless acceleration profile, $\zeta = 2$.

Comparison with Other Dark Matter Scenarios

vs. Ultralight/Fuzzy DM (axions, FDM):

- FDM: coherence from tiny particle mass $m \sim 10^{-22}$ eV; de Broglie wavelength $\sim$ kpc.
- **This work:** coherence from spectrally protected IR structure of the gluonic vacuum. *No new particle.*
- RAR connection is **direct** here (intrinsic $g_\star = C(\zeta)G\Sigma_0$; indirect in FDM.

vs. MOND / modified gravity:

- MOND modifies the force law.
- **This work:** standard gravity throughout; the scale $g_\dagger$ is a DM property.

vs. standard ΛCDM (NFW halos):

- NFW: cuspy, infinite total mass, no intrinsic $g_\dagger$.
- **This work:** cored, finite mass, $g_\star \sim g_\dagger$ from first principles.

Summary table

	Core	Fin. M	$g_\dagger$
NFW	$\times$	$\times$	$\times$
Burkert	$\checkmark$	$\times$	$\times$
FDM	$\checkmark$	$\times$	indirect
This work	$\checkmark$	$\checkmark$	$\checkmark$

Conclusion — The Logical Chain Revisited

QGP era
colour confinement
δ-point
§I

→ **BEC survival**
$\mathcal{R}_{\rm QCD} \simeq \mathcal{R}_{\rm obs}$
di-gluonic singlets
§II

→ **AdS spectrum**
$\mathrm{SO}(2,3)$ UIR
gap $\hbar\omega_{\rm AdS}$
§III

→ **Cored halo**
$\rho_c(1+x^2)^{-\zeta}$
$M_h = \pi^2 \rho_c r_c^3$
§IV–V

→ **Observed**
$\Sigma_0 \simeq 141\, M_\odot\, \mathrm{pc}^{-2}$

↓ **RAR**
$g_\star = \pi^2 G \Sigma_0$
$\simeq 1.9 \times 10^{-10}\, \mathrm{m\,s^{-2}}$

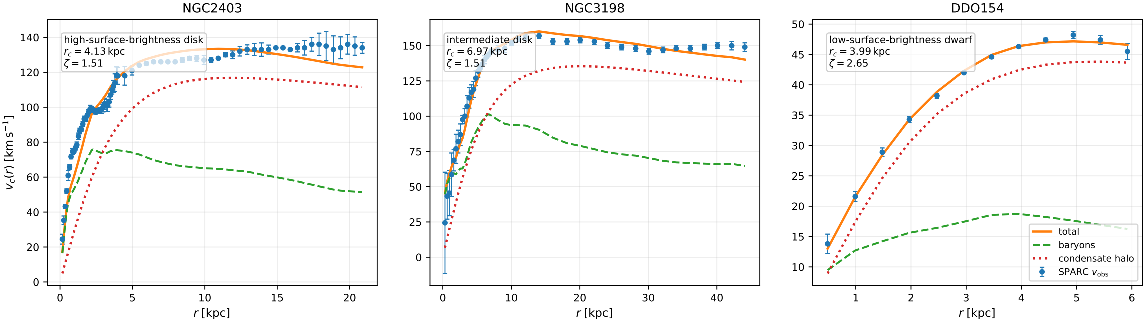
What we have shown:

- ✓ Gluonic BEC survives confinement: $\mathcal{R}_{\rm QCD} = 5.5 \approx \mathcal{R}_{\rm obs} = 5.36$.
- ✓ Cored, finite-mass halo from Fronsdal ground state.
- ✓ $g_\star = \pi^2 G\Sigma_0 \simeq 1.9 \times 10^{-10}\, \mathrm{m\,s^{-2}}$: scale-independent, no fine-tuning.
- ✓ Newtonian dynamics fully consistent with AdS microphysics.

Open frontiers:

- ☐ Linear-response: derive $g_\dagger(\kappa, \zeta)$ from baryonic perturbations.
- ☐ Derive near-universality of Σ_0.
- ☐ Systematic SPARC rotation-curve fits.
- ☐ Large-scale multi-halo and weak lensing.
- ☐ GW signatures (LISA/ET/CE).

Compared with SPARC. Here one explore with different $\zeta > 3/2$



Illustrative comparison between the finite-mass gluonic-condensate halo profile and representative systems from the SPARC database. Points with error bars denote the observed circular velocities. Dashed curves show the baryonic contribution inferred from the SPARC decomposition into gas and stellar components, dotted curves represent the condensate-halo contribution, and solid curves show the total circular velocity. The examples span representative high-surface-brightness, intermediate, and low-surface-brightness systems. The comparison is intended as a representative consistency check rather than a precision statistical fit of the full SPARC sample.

Summary — Two Key Results

1. Gluonic BEC survives hadronisation:

$$\mathcal{R}_{\rm QCD} = 11N_c/(2N_f)|_{N_c=N_f=3} = 5.5 \quad \text{vs.} \quad \mathcal{R}_{\rm obs} = \Omega_{\rm DM}/\Omega_b \simeq 5.36$$

Dark-to-baryonic ratio explained by the QCD trace anomaly, within 3%.

2. Scale-independent acceleration, e.g., $\zeta = 2$:

$$\boxed{g_\star = \frac{GM_h}{r_c^2} = \pi^2 G\Sigma_0 \simeq 1.9 \times 10^{-10}\,\mathrm{m\,s^{-2}}}$$

r_c drops out; depends only on the observed universal surface density $\Sigma_0 \simeq 141\,M_\odot\,\mathrm{pc}^{-2}$. No modified gravity. No new particles.

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Backup — Breitenlohner–Freedman Window and $\zeta = 2$

For scalar fields in AdS_4, stability requires the Breitenlohner–Freedman (BF) bound:

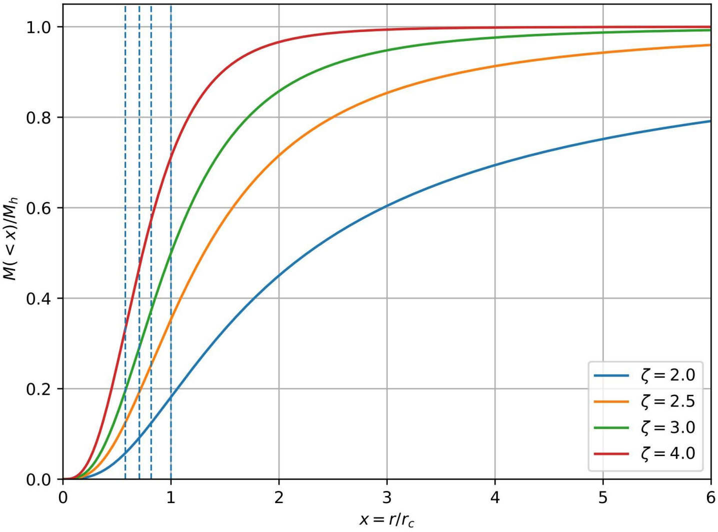
$$\kappa^2\zeta(\zeta-3) \geq -\frac{9\kappa^2}{4} \quad \Leftrightarrow \quad \zeta \geq \tfrac{1}{2}$$

Double quantization window $1/2 < \zeta \leq 3/2$: both fall-offs $r^{-\zeta}$ and $r^{-(3-\zeta)}$ are admissible (Robin boundary conditions).

Why $\zeta = 2$ is special:

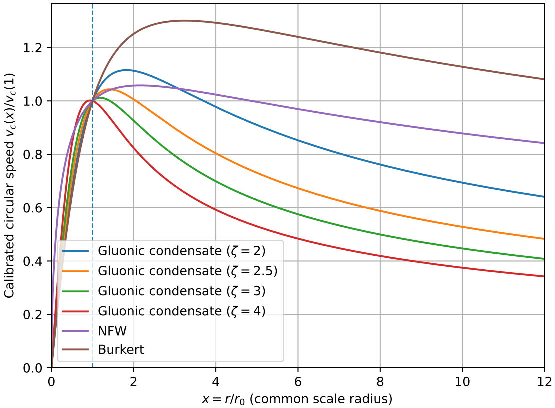
- Smallest value above the double-quantization window.
- Total mass integral $\propto \int_0^\infty r^{2-2\zeta} dr$ converges iff $\zeta > 3/2$; $\zeta = 2$ is the **minimal normalizable scalar UIR** with finite mass.
- Conformal scalar: $\zeta(\zeta-3) = -2$, same Casimir as $\zeta = 1 \to$ pair $D(2,0) \oplus D(1,0)$.
- Gives the **exact** factor $M_h = \pi^2 \rho_c r_c^3$, hence $g_\star = \pi^2 G \Sigma_0$.

Backup — Mass Profiles for Various ζ



Normalized enclosed mass $M(< x, \zeta)/M_h$ for $\zeta = 2, 2.5, 3, 4$. Vertical dashed lines indicate the inflection radii

Backup — Calibrated Rotation Curves



Calibrated circular speed $v_c(x)/v_c(1)$ vs. $x = r/r_0$ (common scale radius). For $\zeta = 2$ the curve is nearly flat out to $x \approx$ 3–4 before declining; this is the closest analogue to the observed extended flat rotation curves. NFW peaks later, Burkert stays high for longest.

Backup — Spectral Degeneracy and Level Structure

The discrete spectrum depends only on $k = l + 2n$:

$$\omega_k = \omega_{\mathrm{AdS}}(k + \zeta), \quad k = 0, 1, 2, \ldots$$

Uniform level spacing:

$$\Delta\omega = \omega_{\mathrm{AdS}} = c/r_c, \quad \Delta E = \hbar c/r_c$$

Degeneracy at level k: $\lfloor k/2 \rfloor + 1$ (not counting the $(2l + 1)$ magnetic degeneracy).

Although ΔE is tiny in absolute units (galactic r_c), the condensate physics is **collective, not microscopic**: baryonic perturbations induce controlled admixtures of degenerate/near-degenerate levels, deforming the ground-state halo profile.

Backup — Effective Curvature Scale vs. Cosmological Constant

The effective AdS curvature:

$$|\Lambda_{\rm AdS}^{\rm eff}| = \frac{3}{r_c^2} \simeq 10^{-40}\text{–}10^{-38}\ \mathrm{m}^{-2} \quad (\text{for } r_c \sim 0.6\text{–}5\ \mathrm{kpc})$$

versus the observed cosmological constant:

$$\Lambda_{\rm obs} \sim 10^{-52}\ \mathrm{m}^{-2}$$

$\Lambda_{\rm AdS}^{\rm eff}$ is 10^{12}–10^{14} times larger: not a cosmological constant of spacetime, but an **effective parameter controlling spectral organization of the dark sector on galactic scales**.

The background Universe remains well described by a spatially flat FRW geometry; the AdS structure is strictly internal to the dark sector.

A logarithmic ladder of physical scales

Object	**Size in metres**	**Natural unit**
Proton	10^{-15} m	1 fm
Atom	10^{-10} m	1 Å
Human	10^{0} m	1 m
Stellar black hole	$10^{4} - 10^{5}$ m	$10 - 100$ km
Planet	10^{7} m	10^{4} km
Star	10^{9} m	10^{6} km
Supermassive black hole	$10^{10} - 10^{13}$ m	$0.1 - 100$ AU
Solar system / QGP Universe	10^{15} m	10^{4} AU
Galaxy	10^{21} m	30 kpc
Galactic dark-matter halo	10^{22} m	$100 - 300$ kpc
Cluster dark-matter halo	10^{23} m	$1 - 5$ Mpc
Observable Universe	10^{26} m	10 Gpc

$10^{-15}\ \mathrm{m} \longrightarrow 10^{26}\ \mathrm{m}$ about 41 orders of magnitude.