How the failure of Ward identities determines particle interactions

–

The Standard Model of particles without indefinite state spaces

Karl-Henning Rehren
ITP, Uni Göttingen

"Mathematical Quantum Field Theory"
Conference in the memory of Ivan T. Todorov

BAS, Sofia, May 27, 2026

Abstract:

In contrast to standard procedures, **photons** and **massive vector bosons** can be quantized with their physical masses in the Fock space over their **unitary** Wigner representations, and one can find renormalizable interaction densities $L_{\rm int}(x)$. There is only a "tiny" but potentially disastrous flaw: the fields are **localized along some "string"** extending from x to infinity. Naively, this flaw should be ineffective because it changes local interactions only by a total derivative which is classically irrelevant; but the failure of Ward identities in quantum perturbation theory causes **obstructions**. The condition that all obstructions can be cancelled, strongly **constrains the admissible interactions**. The outcome are precisely the Standard Model interactions.

This new selection criterion is "purely quantum": classical Lagrangians and canonical quantization are not used. "Gauge symmetry" is neither invoked, nor spontaneously broken.

See pertinent recent work:

KHR: **Hidden gauge invariance. arXiv.2605.01453.**
I. Hemprich, KHR: Dressed fields for **quantum chromodynamics**. Lett. Math. Phys. 115 (2025) 112.
KHR: On the effect of derivative interactions in quantum field theory. Lett. Math. Phys. 115 (2025) 3.
J.M. Gracia-Bondía, KHR, J.C. Várilly: The full **electroweak interaction**: an autonomous account. Ann. H. Poincaré 26 (2025) 4529–4574.
J.M. Gracia-Bondía, J. Mund, J.C. Várilly: The chirality theorem. Ann. H. Poincaré 19 (2018) 843–874.

IVAN TODOROV and AXIOMATIC QFT

Commitment to axiomatic approaches

ICMP 1994:

- Wightman's **axiomatic QFT (fields)** vs Haag's **Algebraic QFT (algebras)**

Ivan (rooted in the field-theoretic approach) asked me about conformal extensions in 2D CFT, which seemed to clash with results from AQFT. We eventually understood the issue (joint paper with Yassen Stanev).

Commun. Math. Phys. 174, 605–633 (1996)

Communications in Mathematical Physics

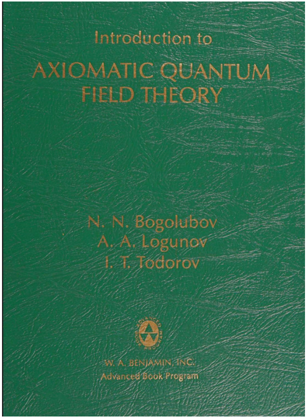
Characterizing Invariants for Local Extensions of Current Algebras

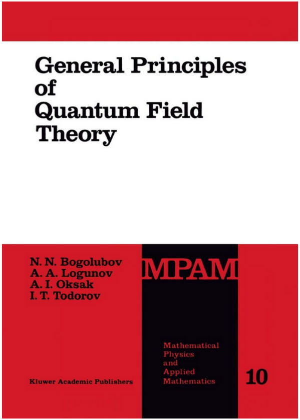
Karl-Henning Rehren[1], Yassen S. Stanev[2, 3], Ivan T. Todorov[2, 3]

For Ivan, the important part was the **juxtaposition of two complementary approaches**.

- This memory illustrates Ivan's life-long striving, not for mere "results" in the first place, but for the underlying **conceptual underpinnings**.

Remember Ivan as a coauthor of the reference textbooks on axiomatic QFT, with Bogoliubov, Logunov (and Oksak).





Emphasis of Hilbert space

- Ivan was always interested in, and worried about, **Hilbert space positivity** (and how restrictive it is in the "space of all QFTs").

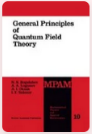
Chapter devoted to "Quantum fields in indefinite metric", addressing the pitfalls in this setting.

Home › General Principles of Quantum Field Theory › Chapter

Fields in an Indefinite Metric

Chapter

pp 417–449 | Cite this chapter



General Principles of Quantum Field Theory

N. N. Bogolubov, A. A. Logunov, A. I. Oksak, I. T. Todorov & G. G. Gould

Access this chapter

Log in via an institution →

Part of the book series: Mathematical Physics and Applied Mathematics ((MPAM,volume 10))

Trouble with **indefinite state spaces:**

- No interpretation of “negative probabilities”
- Severe mathematical problems by the failure of the **Cauchy-Schwarz Inequality**.
- Many operator algebra theorems in axiomatic QFT break down: **Reeh-Schlieder theorem** (“state-field correspondence”), **Spin-Statistics theorem**, ...
- “**Null fields**”: zero correlations with all observables, but nonzero commutators may contribute to the perturbation theory.

BRST (the operator version of “Faddeev-Popov ghosts”) and its more abstract interpretation by Kugo and Ojima supplied **solid footing in shaky grounds** – provided there is a “BRST symmetry”. In fact, ghost fields can only exists if Spin-Statistics is violated.

2005–2009:

Joint project with Ivan and Nikolay Nikolov (and Bojko Bakalov) on classifying 4D conformal QFTs with "Global Conformal Invariance" (absence of covering representations). The guiding principle was **Hilbert space positivity**, that must be reflected in correlation functions.

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 194002 (12pp) doi:10.1088/1751-8113/41/19/194002

Infinite-dimensional Lie algebras in 4D conformal quantum field theory*

Bojko Bakalov[1], Nikolay M Nikolov[2], Karl-Henning Rehren[2,3] and Ivan Todorov[2]

Analysis of positivity constraints.

- Hope of bootstrapping interesting interacting theories from Hilbert space positivity plus locality: GCI seems to admit only free fields.

The topic of my talk

- **Avoiding indefinite metric from the outset** in SM particle physics

is one of the main motivations for the program that I am busy with since a decade, with collaborators B. Schroer, J. Mund, J.M. Gracia-Bondía, J.C. Várilly, and others.

Background literature:
J. Mund, B. Schroer, J. Yngvason: String-localized quantum fields and modular localization. Commun. Math. Phys. 268 (2006) 621–672.
J. Mund: String–localized quantum fields, modular localization, and gauge theories. Proc. of Sci. (2006) 028.
B. Schroer: An alternative to the gauge theoretic setting. Found. of Phys. 41 (2011) 1543–1568.
B. Schroer: A Hilbert space setting for interacting higher spin fields and the **Higgs** issue. Found. of Phys. 45 (2015) 219–252.
B. Schroer: Peculiarities of massive vector mesons and their zero mass limits. Eur. Phys. J. C 75 (2015) 365.
J. Mund, E.T. de Oliveira: String-localized free vector and tensor potentials for massive particles with any spin: I. Bosons. Commun. Math. Phys. 355 (2017) 1243–1282.
J. Mund, KHR, B. Schroer: Helicity decoupling in the massless limit of massive tensor fields. Nucl. Phys. B924 (2017) 699–727.
KHR: Pauli-Lubanski limit and stress-energy tensor for **infinite-spin** fields. JHEP 11 (2017) 130.
J. Mund, KHR, B. Schroer: Gauss' Law and string-localized quantum field theory. JHEP 01 (2020) 001.
Ch. Gass, J.M. Gracia-Bondía, J.C. Várilly: Revisiting the Okubo-Marshak argument. Symmetry 13 (2021) 1645.
Ch. Gass: Renormalization in string-localized field theories: a microlocal analysis. Ann. H, Poincaré 23 (2022) 3493–3523.
J. Mund, KHR, B. Schroer: **Infraparticle quantum fields** and the formation of **photon clouds**. JHEP 04 (2022) 083.
J. Mund, KHR, B. Schroer: How the **Higgs potential** got its shape. Nucl. Phys. B 987 (2023) 116109.
Ch. Gass, J.M. Gracia-Bondía, KHR: Quantum **General Covariance**. Class. Qu. Grav. 40 (2023) 195016.
KHR, L.T. Cardoso, Ch. Gass, J.M. Gracia-Bondía, B. Schroer, J.C. Várilly: sQFT: an autonomous explanation of the interactions of quantum particles. Found. of Phys. 54 (2024) 57.

WARD IDENTITIES

Ward identities (if they hold) assert that classical differential equations, like current conservation, are preserved in the quantum theory.

- **Ward identites may fail** in quantum perturbation theory,

because the latter proceeds in terms of time-ordered or retarded correlation functions = propagators, and

- time-ordering in general does not commute with time derivatives.

The quantative failure of a Ward identity is called an “obstruction”.

- Typically: **obstructions** against current conservation: $\partial_\mu T[j^\mu X'] \neq 0$.
- Here: **against some expression being a total derivative**:

$$\int dx\, T[\partial_\mu V^\mu(x) L(x_2) \dots L(x_n)] \neq \int dx\, \partial^x_\mu T[V^\mu(x) L(x_2) \dots L(x_n)] = 0$$

need not vanish, because of an obstruction.

The example is a contribution to the S-matrix

$$S = Te^{i \int L_{\rm int}(x)} = \sum \frac{i^n}{n!} \int \cdots \int T[L_{\rm int}(x_1) \dots L_{\rm int}(x_n)],$$

when a total derivative $\partial_\mu V^\mu$ is added to $L_{\rm int}(x)$.

Thus, adding a total derivative (even rapidly decaying) to an interaction will change the dynamics, unlike in classical theory.

Cancellation of obstructions (schematically)

- Interaction $L_{\text{int}} = gL_1 + \partial_\mu V_1^\mu$
- Contribution to S-matrix at order $n = 2$:

$$(ig)^2 \int dx_2 \int dx \underbrace{\left[T[\partial_\mu V_1^\mu(x)L_1(x_2)] - \partial_\mu^x T[V_1^\mu(x)L_1(x_2)]\right]}_{\sim i\delta(x-x_2)X(x_2) \quad \neq 0: \quad \text{obstruction}}$$

$$= -ig^2 \int dx_2\, X(x_2) \stackrel{!}{=} -ig^2 \int dx\, [L_2(x) - \partial_\mu V_2^\mu(x)] = -ig^2 \int dx\, L_2(x)$$

The obstruction is cancelled by adding $g^2 L_2$ to L_{int}, contributing a leading term $ig^2 \int dx\, L_2(x)$ to the S-matrix S.

Thus, the obstruction X must be of a special form, allowing a split $X = L_2 - \partial V_2$, where L_2 qualifies as a second-order (quartic) interaction. This *determines* L_2 (provided it exists).

Scheme must work out similarly at all orders (involved combinatorics).

PARTICLE INTERACTIONS and GAUGE INVARIANCE

Well-known motto (C.N. Yang):

symmetry dictates interactions.

All SM interactions can be derived from some assumed gauge symmetries, including “spontaneously broken” ones.

- But gauge symmetries are not really symmetries of Nature.

They rather assert that something is independent of an **unphysical redundance** used for its formulation. One may ask the “philosophical” question how an unphysical principle can play a major physical role.

Worse: the attempt to maintain classical gauge invariance in **quantum** field theory notoriously forces us to **violate quantum principles**. Most prominently, the **Hilbert space** is sacrificed by canonical quantization of gauge potentials, with all the trouble that comes along with it. It has to be restored by BRST methods, or similar.

The BRST treatment of gauge theories starts with an

- interaction that is **BRST-invariant only up to a total derivative**[1]

This is subject to obstructions, as explained above.

For the S-matrix to be defined on the BRST Hilbert space, the obstructions must be cancelled – which they do thanks to gauge invariance of the underlying classical Lagrangian. In other words:

- **gauge invariance "protects" the BRST cohomology** against violation of Ward identities by perturbative interactions.

G. Scharf and M. Dütsch have undertaken great efforts to show the **converse**: without assuming any gauge structure, they analyzed **which cubic interactions admit a BRST invariance up to at most a total derivative**.

[1] E.g., in QED: $\delta_{\mathrm{BRST}}(A_\mu j^\mu) = \partial_\mu u j^\mu = \partial_\mu(u j^\mu)$.

Studying the obstructions occurring at higher order of perturbation theory, they discovered that certain **"induced" higher interactions** must be present in order to cancel the obstructions.

Thus, the BRST cohomology survives in higher orders of perturbation theory **only for very special interactions**, typically cubic plus quartic.

These interactions turn out to be the non-free parts of gauge invariant Lagrangians. Somehow, **gauge invariance does not like Hilbert space**, but **Hilbert space wants gauge invariant interactions.**

- **Gauge invariance is not a principle**, but an **emergent feature**.

It arises as a necessity from the need to preserve BRST, which in turn is required to restore a Hilbert space that was lost with canonical quantization of massless gauge potentials.

M. Dütsch (2005) Proof of perturbative gauge invariance for tree diagrams to all orders.
M. Dütsch, J.M. Gracia-Bondía, F. Scheck, J.C. Várilly (2010) Quantum gauge models without classical Higgs mechanism.
G. Scharf (2015) Gauge Field Theories: Spin One and Spin Two.

THE NEW STORY:

GAUGE INVARIANCE from HILBERT SPACE (without BRST)

I want to present now **yet a different rationale**, explaining the interactions of the SM, exactly as we know them.

Again, it explains them without a gauge principle, and gauge invariance appears as a **(possibly hidden) emergent feature**, necessary in order to cancel obstructions.

- But it is possible to work in the **Hilbert space from the outset**.

- The problem of “canonical quantization” of massless vector fields is circumvented by introducing new potentials that can be expressed in terms of the local quantum Maxwell fields associated to the **unitary Wigner representation** of massless particles of helicity 1. In these Hilbert spaces, local gauge potentials do not exist, only $F_{\mu\nu}$ exists.

The new potentials look like this:

$$A_\mu(x,c) = \int dy\, F_{\mu\nu}(x+y)c^\nu(y),$$

where $c^\mu(y)$ satisfies $\partial_\mu c^\mu = \delta$. It follows

$$\partial_\mu A_\nu(x,c) - \partial_\nu A_\mu(x,c) = F_{\mu\nu}(x).$$

There are many such functions c satisfying $\partial_\mu c^\mu = \delta$, but they all have necessarily **non-compact support**.

- $A(x,c)$ are localized (in the sense of quantum commutators) on the support of c, which extends to infinity.

Thinking of it as a **narrow cone**, one calls it a "string", and $A(x,c)$ is called "**string-localized**".[2]

This feature is potentially disastrous: If one replaces, say, the canonical local QED interaction

$$L_{\rm int} = e A_\mu j^\mu$$

by a string-localized Hilbert space interaction

$$L_{\rm int}(c) = e A_\mu(c) j^\mu,$$

the **locality of the perturbed quantum fields is at stake.**

Usually, this would end the story.

[2] Yet, c^μ may also be Coulomb-like (rotation invariant).

However, one can see that the dependence of the string function is a total derivative. Varying $c \to c + \delta c$,

$$\delta_c A_\mu(x,c) = \partial_\mu w(x,\delta c).$$

- Consequently,

  $$\delta_c L_{\rm int}(c) = \partial_\mu w\, j^\mu = \partial_\mu \big(w\, j^\mu\big)$$

  is a derivative. Varying c amounts to adding an infinitesimal $\partial_\mu Q^\mu$ to $L_{\rm int}(c)$. S-matrix is invariant at first order.

Can this property **secure string-independence of the S-matrix**?

$$\delta_c\Big(e^{i\int L_{\rm int}(c)}\Big) \stackrel{?}{=} 0.$$

Potential obstructions at second order must be cancellable, as in p. 13.

The trouble is "only" a **possible failure of Ward identities**, and a sufficient **control of obstructions** might save the day.

- This is precisely what happens!

For **QED**, one can show that all pertinent obstructions vanish.

For other theories, like YM, one would first search cubic self-interactions of string-localized gluons **for which $\delta_c(L_1(c))$ is a derivative**, and then ask **whether obstructions again vanish**. This is the case only provided one adds also a quartic **"induced" interaction**. The resulting interaction is exactly the same as local YM, but with A_μ replaced by $A_\mu(c)$.

- "Prediction of YM theory" without invoking gauge invariance.

A similar reasoning applies to **massive vector bosons**, where not the Hilbert space is at stake, but renormalizability:

- The massive Proca field B_μ is **too singular in the UV to construct renormalizable interactions** (like $B_\mu j^\mu$) with it.

As you know, this problem motivated the "Higgs mechanism". It does the renormalization by invoking gauge invariance with canonical quantization of massless potentials and indefinite metric, and then uses SSB to "generate mass", and BRST.

This (successful but physically misleading[3]) narrative can be **avoided altogether with massive string-localized potentials $A_\mu(c)$**.

[3] suggesting a physical process, where only artificial gauge degrees of freedom are eliminated. J. Earman: "A genuine physical property like mass cannot be gained by eating descriptive fluff."

- String integration **improves the UV-behaviour**, such that renormalizable interactions are possible:

The same formula as in the Maxwell case:

$$A_\mu(x,c) = \int dy\, F_{\mu\nu}(x+y)c^\nu(y),$$

but now $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ the Proca tensor, produces string-localized potentials $A_\mu(c)$ on the Wigner Hilbert space of a spin 1 massive particle. They satisfy

$$\partial_\mu A_\nu(x,c) - \partial_\nu A_\mu(x,c) = F_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x).$$

Again, one may first search cubic self-interactions of MVBs using string-localized Proca potentials, for which $\delta_c(L_1(c))$ is a derivative, and then ask whether obstructions vanish.

This is indeed possible provided

1. the MVBs also couple to a **scalar field H**,
2. one adds various **quartic interactions**.

The resulting renormalizable string-localized interaction “looks like” the non-renormalizable local GSW interaction **after SSB**, with B fields replaced by $A(c)$, some extra terms involving a field $\phi(c)$ such that $A(c) - B = \partial\phi$; and the total self-interaction of the H field is the well-known “Higgs potential” (around one of its minima).

With more effort one can prove that the S-matrix of the string-localized SM interactions is not only **string-independent**, but actually **equals the S-matrix of the gauge-theory approach** in physical states.

Is this again (as in YM) a “prediction of gauge invariance” ?

- **NO**, because the (local but non-renormalizable) GSW Lagrangian $L^{\text{after SSB}}[A_{\text{QED}}, B_{\text{MVB}}, H]$ **after SSB** has only the electromagnetic $U(1)$ gauge invariance.
- **YES**, because a miraculous identity, called “**hidden gauge invariance**”, allows to write the string-localized interaction $L_{\text{int}}(c)$ in terms of the local GSW Lagrangian $L^{\text{before SSB}}[A, \Phi]$ **before SSB**:

$$(L_0 + L_{\text{int}}(c))[A(c), \phi(c), H] = L^{\text{before SSB}}\left[\widehat{A}, \widehat{\Phi}\right],$$

where $\widehat{A} = \widehat{A}[A(c)]$ involves a Weinberg rotation dictated by the mass ratio of the MVBs, and $\widehat{\Phi} = \widehat{\Phi}[\phi(c), H]$ is a complex doublet built from the scalar H and the three MVB fields $\phi(c)$.

Classical gauge fields in L^{GSW} **need no canonical quantization** with indefinite metric, because Hilbert space quantum fields take their place.

- The hidden gauge invariance is not only “just there”.

Its presence can be used to establish the cancellation of all obstructions at all order:

Certain maps constructed from obstructions (quantifying the violation of Ward identities) **act on $A(c)$ and $\phi(c)$ and H** precisely in such a way, that they **become “gauge transformations” on $\widehat{A}$ and $\widehat{\Phi}$**.

KHR: Hidden gauge invariance. arXiv.2605.01453.

- Gauge invariance is no longer a “first principle”, but rather a feature of obstruction-less renormalizable Hilbert space interactions.
- Responsible for the cancellation of all obstructions against string-independence.

AUTONOMOUS QFT

We call the approach based on string-localized vector potentials for massless and massive vector bosons "**autonomous**" because it is **based on quantum principles** (notably the Hilbert space axiom) rather than a gauge principle.

It beneficially departs from the "usual" axioms.

- Perturbation theory produces local observable fields, and (typically) **string-localized charged fields**.

The latter is **most desirable**, because the electron cannot be a local field: that would be at variance with Gauss' Law (the electron field **must not commute** with the electric flux at infinity)!

There are other features of string-localized perturbation theory for **QED**, that cannot be obtained with local perturbation theory, but are in accord with previous axiomatic results for QED: **Fröhlich, Morchio, Strocchi, Buchholz,... ± 1980**

- The electron is an "**infraparticle**" because the photon cloud contributes to its energy, so that (even in the scattering asymptotics) it is no longer a sharp mass eigenstate.
- The "profile" of its asymptotic electric field is **superselected**.

The autonomous approach **predicts several details** of the electroweak interactions that are otherwise assumptions of the GSW model: maximal chirality of the weak interaction, the shape of the Higgs potential, $m_Z > m_W$, Yukawa couplings $\propto$ fermion masses, ...

Summary

- "Autonomous QFT" does not rely on gauge invariance but on (most prominently) Hilbert-space positivity by means of string-localized potentials, used to set up interactions.

- From this input, one can derive all SM interactions, and show that the S-matrix is the same as if it was computed in indefinite metric, with ghosts and BRST.

Two conceptual **take-home messages**:

- One is not forced to use indefinite metric in particle interactions.
- A **physically most desirable amount of non-locality for charged fields** arises naturally by the autonomous perturbation theory.