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Field Theory for Neutronics

15th Nordic Meeting on Nuclear Physics

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SKC
Swedish Centre for Nuclear Technology

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In collaboration with Ringhals

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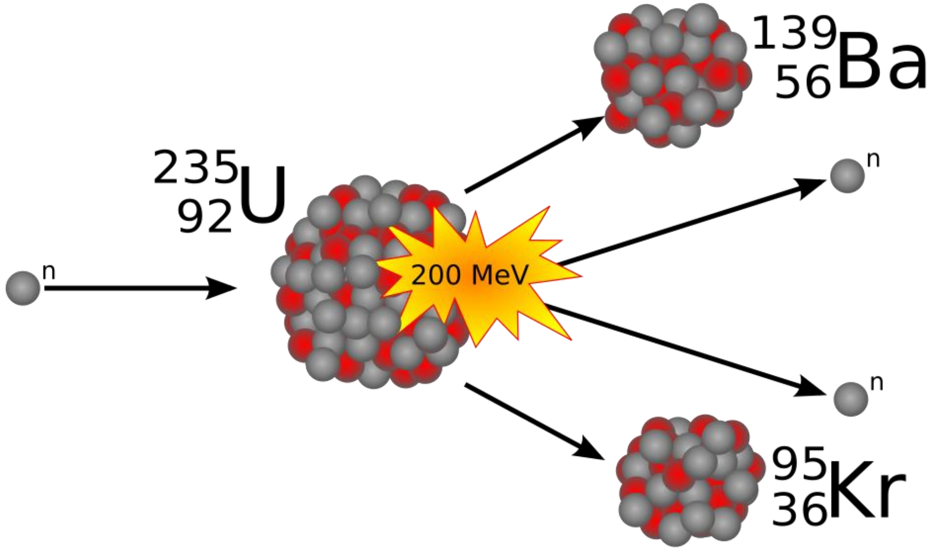
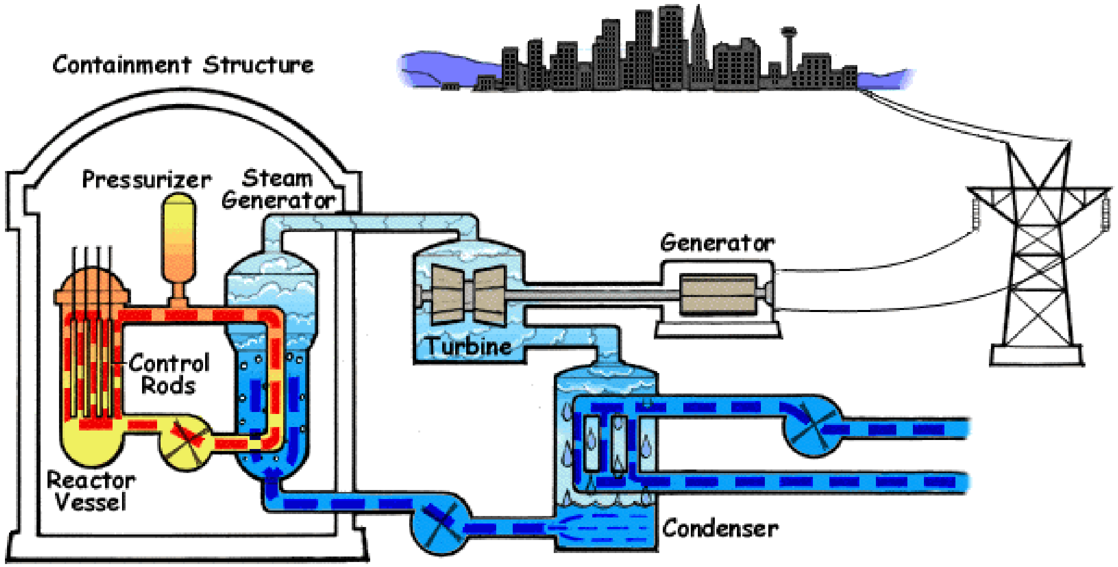
Purpose & Motivation

- Interactions between neutrons and U-235 can cause fission which release energy
- The modelling of neutron transport is important to many aspects of nuclear reactor operation
- We want to improve the model of neutron transport by including fluctuations and feedback effects using methods from quantum field theory



Stefan-Xp. “Nuclear fission of uranium 235” commons.wikipedia.org. Accessed: 29 Jan. 2026. [Online]. Available: https://commons.wikimedia.org/wiki/File:Kernspaltung.svg.



U.S.NRC. “Pressurized Water Reactor.” commons.wikipedia.org. Accessed: 29 Oct. 2025. [Online]. Available: https://commons.wikimedia.org/wiki/File:PressurizedWaterReactor.gif.

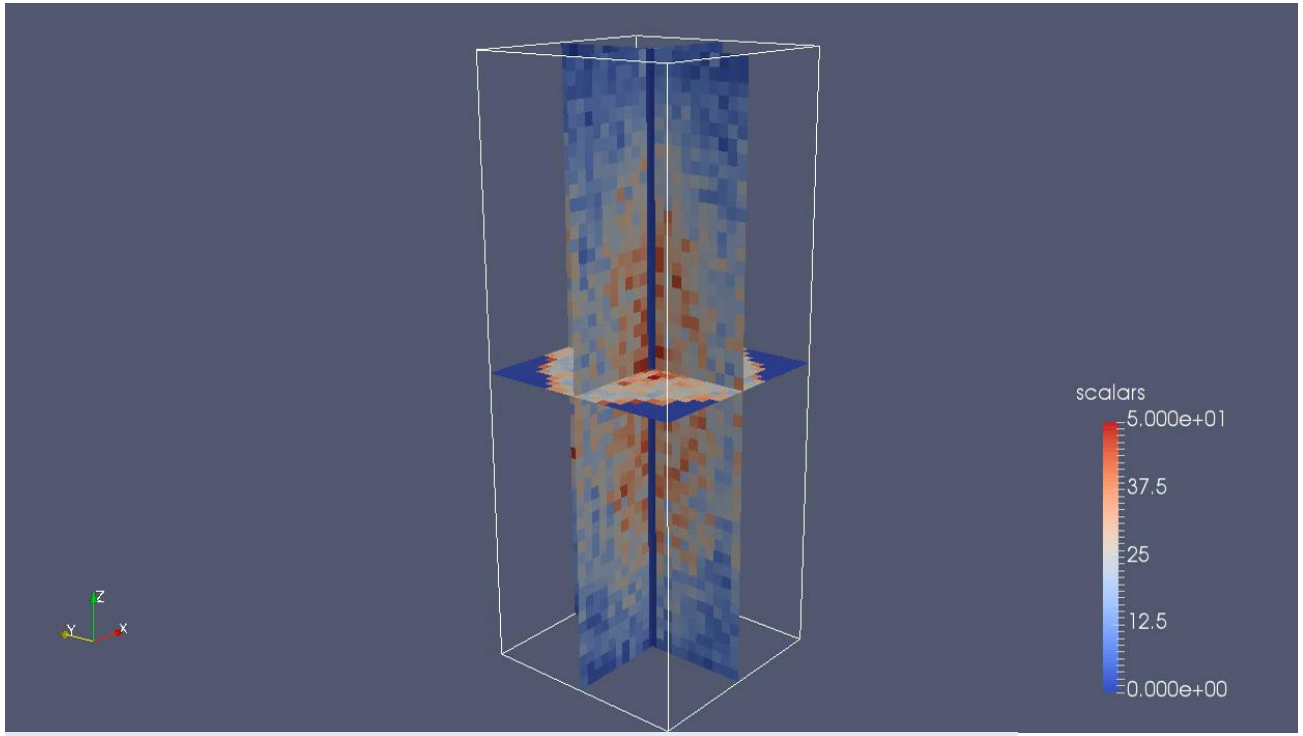
Background

https://doi.org/10.1038/s42005-021-00654-9 OPEN

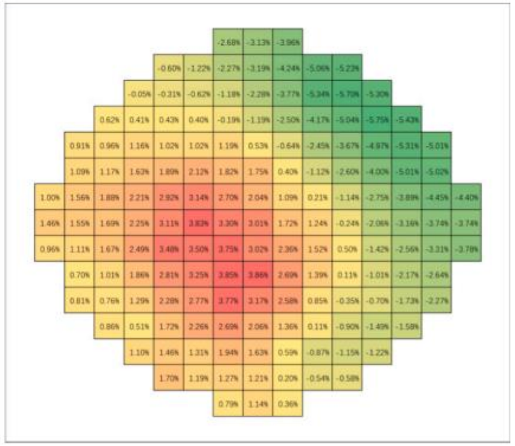
Patchy nuclear chain reactions

Eric Dumonteil[1,4✉], Rian Bahran[2], Theresa Cutler[2], Benjamin Dechenaux[1], Travis Grove[2], Jesson Hutchinson[2], George McKenzie[2], Alexander McSpaden[2], Wilfried Monange[1], Mark Nelson[2], Nicholas Thompson[2] & Andrea Zoia[3]

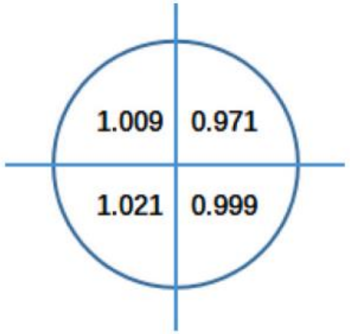
- Experiments have found connections between neutron fluctuations and spatial correlations of the neutron distribution
- This might contribute to the asymmetry of power distribution seen in large reactors



(a) BOL-ARO (0 MWd/tU, All Rods Out)



Radial Power Distribution Deviations*



TILT

Li, X., Cai, D., Wang, X., He, M., Li, Z., Zhao, C., & Wang, X. (2025). Neutronic analysis of PWR fuel-assembly bowing based on measurement data. *Nuclear Engineering and Design*, *443*, 114283.

Neutron transport

- Neutron diffusion equation (approximation of the neutron transport equation):

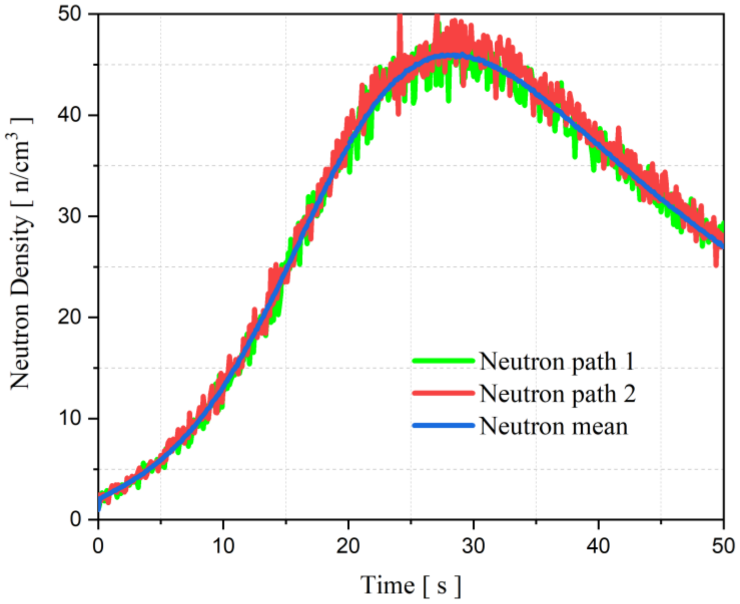
$$\frac{\partial}{\partial t}N(\vec{x},t) = (D\nabla^2 + \rho)N(\vec{x},t)$$

where $N(\vec{x},t)$ is the neutron density (flux) at position $\vec{x}$ and time t. D denotes the diffusion coefficient and ρ the reactivity.

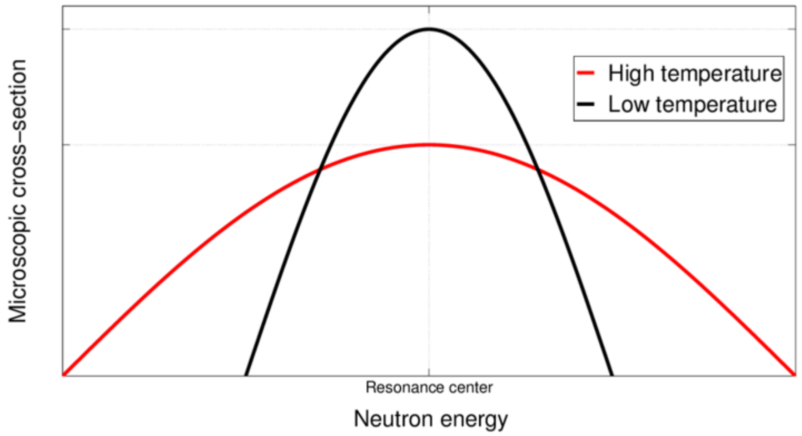
- We want to include fluctuations and feedback effects:

$$\frac{\partial}{\partial t}N(\vec{x},t) = (D\nabla^2 + \rho)N(\vec{x},t) + \eta(\vec{x},t) + \sigma N^2(\vec{x},t)$$

- Doppler effect (example of feedback):
  - Higher neutron flux → more fission → higher temperature → faster vibration of U-238 → broadening of the absorption cross section → more absorption → lower neutron flux



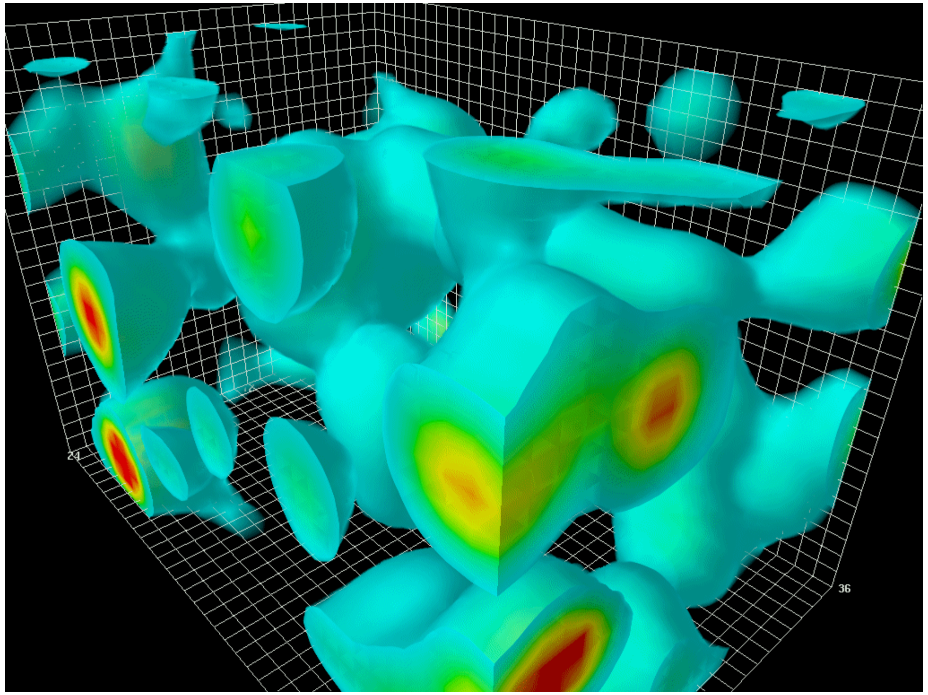
Suescún-Díaz, D., "Stochastic Neutron Population With Temperature Feedback Effects Using The Implicit Runge-Kutta Scheme," *J. Appl. Sci. Eng.*, vol. 28, no. 8, pp. 1795-1803, Sept. 2024, doi: http://dx.doi.org/10.6180/jase.202508_28(8).0016.



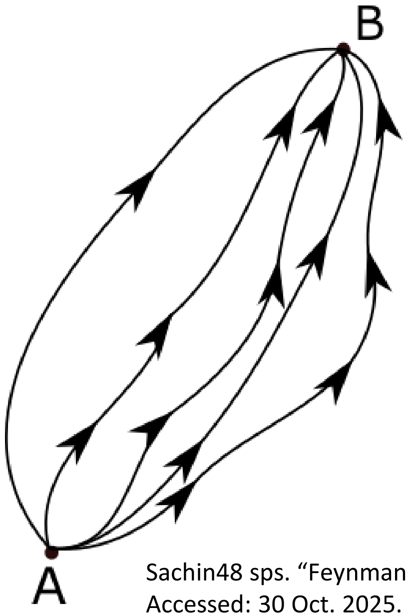
Qvist, S. A. (2013). *Safety and core design of large liquid-metal cooled fast breeder reactors*. University of California, Berkeley.

Quantum Field Theory (QFT)

- QFT is a framework that models the behaviour of fundamental particles
- The fields in QFT fluctuates
  - Tools from QFT can be used to analyse neutron fluctuations



D. Leinweber. “Quantum Fluctuations.” commons.wikipedia.org. Accessed: 30 Oct. 2025. [Online]. Available: https://commons.wikimedia.org/wiki/File:Quantum_Fluctuations.gif.

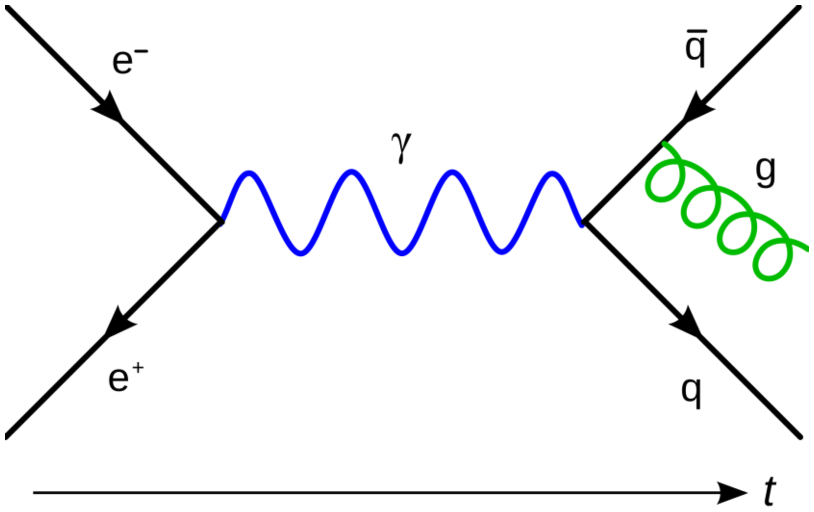


Scattering amplitude from A to B in time T

$$= \sum_{all\ paths} e^{i\cdot(phase)}$$

$$= \int \mathcal{D}x(t)e^{iS[x(t)]/\hbar}$$

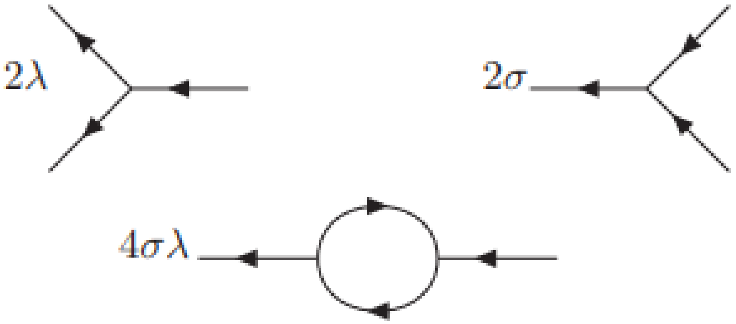
Sachin48 sps. “Feynman paths.” commons.wikipedia.org. Accessed: 30 Oct. 2025. [Online]. Available: https://commons.wikimedia.org/wiki/File:Feynman_paths.png.



J. Holdsworth. “Feynman Diagram Gluon Radiation.” commons.wikipedia.org. Accessed: 3 Feb. 2026. [Online]. Available: https://commons.wikimedia.org/wiki/File:Feynman_Diagram_Gluon_Radiation.svg.

Previous work

$$\mathcal{S} = \int d^d\vec{x} \int dt\,\tilde{N}[\partial_t - D(\Delta + r)]N + \sigma \tilde{N} N^2 - \lambda \tilde{N}^2 N$$



PHYSICAL REVIEW E **106**, 064126 (2022)

Percolation properties of the neutron population in nuclear reactors

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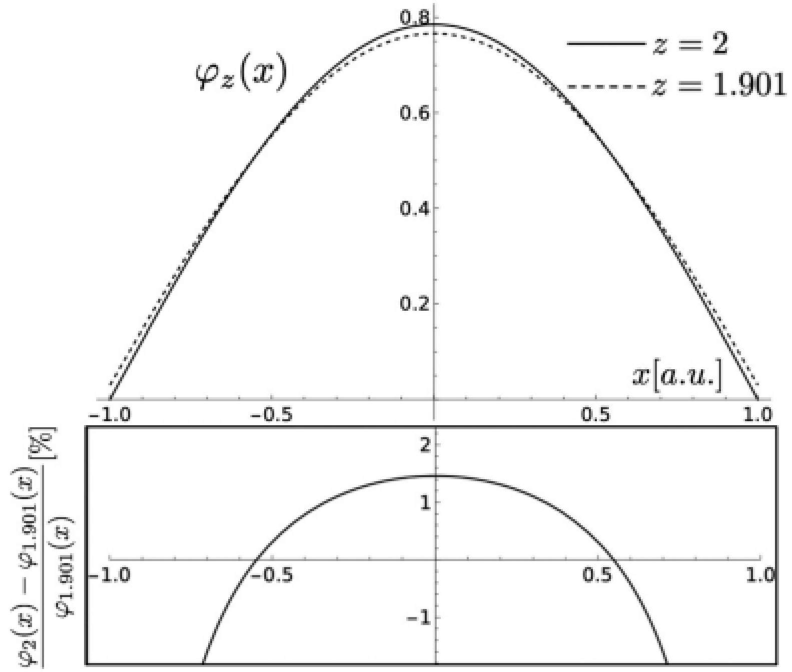
Reactor physics aims at studying the neutron population in a reactor core under the influence of feedback mechanisms, such as the Doppler temperature effect. Numerical schemes to calculate macroscopic properties emerging from such coupled stochastic systems, however, require us to define intermediate quantities (e.g., the temperature field), which are bridging the gap between the stochastic neutron field and the deterministic feedback. By interpreting the branching random walk of neutrons in fissile media under the influence of a feedback mechanism as a directed percolation process and by leveraging on the statistical field theory of birth death processes, we will build a stochastic model of neutron transport theory and of reactor physics. The critical exponents of this model, combined with the analysis of the resulting field equation involving a fractional Laplacian, will show that the critical diffusion equation cannot adequately describe the spatial distribution of the neutron population and shifts instead to a critical superdiffusion equation. The analysis of this equation will reveal that nonnegligible departure from mean-field behavior might develop in reactor cores, questioning the attainable accuracy of the numerical schemes currently used by the nuclear industry.

DOI: 10.1103/PhysRevE.106.064126



Research plan

1. Reproduce/verify the previous work
2. Improve the theoretical model
   - Make the model of feedback more accurate
   - Include delayed neutrons (from nuclear decay) in the same framework
   - Make the reactivity space and time dependent, $\rho \to \rho(\vec{x}, t)$
   - Introduce energy dependence/multiple energy groups to the neutron flux
3. Simulate the system numerically
4. Implement the improvements in established reactor physics code(s)
5. Verify the results experimentally

Backups

Summary

- We want to improve the understanding of neutron transport in nuclear reactors
- Neutron density fluctuates and is subjected to feedback effects
- A toy model (simplistic neutron diffusion equation) has been investigated by using a field theoretic framework
- We want to improve and expand this model, for example by making the reactivity space and time dependent $\rho \rightarrow \rho(\vec{x}, t)$
- Investigate, test and verify the model numerically and experimentally
- Aim is to improve reactor efficiency and safety (margins) to increase power output of current technology

Preliminary results

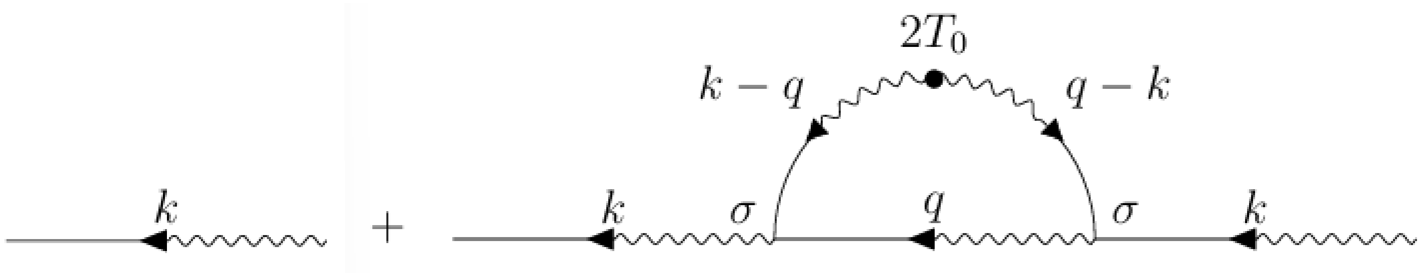
- Neutron diffusion equation with feedback strength σ and noise η

$$\partial_t N(\mathbf{x},t) = D\nabla^2 N(\mathbf{x},t) + \rho N(\mathbf{x},t) + \sigma N^2(\mathbf{x},t) + \eta(\mathbf{x},t)$$

- Assume Gaussian noise with covariance

$$< \eta(\mathbf{x},t)\eta(\mathbf{x}',t') >= 2[T_0 + T_1 N(\mathbf{x},t)]\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

- If $T_1 = 0$, the Feynman diagrams up to second order in σ are



Preliminary results

- Let G be the retarded Green's function solving the neutron diffusion equation, then

$$G_R(\tau_R, \mathbf{k}, \omega)^{-1} \propto D_{phys}|\mathbf{k}|^{2-\eta}\hat{G}_R\left(\frac{\tau_{phys}}{|\mathbf{k}|^{1/\nu}}, \frac{\omega}{D_{phys}|\mathbf{k}|^{z}}\right)^{-1}$$

  (R denotes renormalization, τ the distance from criticality, **k** and ω are momentum and frequency (Fourier transform of ∇^2 and ∂_t))
- η describes spatial decay of correlations
- ν relates the correlation length ξ to the distance from the critical point: $\xi \propto \tau^{-\nu}$
- z relates the characteristic relaxation time t_c of the fluctuations to its spatial size: $t_c \propto \xi^z$
- If $T_1 = 0$ then $\eta = 1/2$, $\nu = 4/3$ and $z = 3/4$ (cf. mean-field values: $\eta = 0$, $\nu = 1/2$, $z = 2$)
- We are currently working on the case $T_1 \neq 0$

Neutron transport

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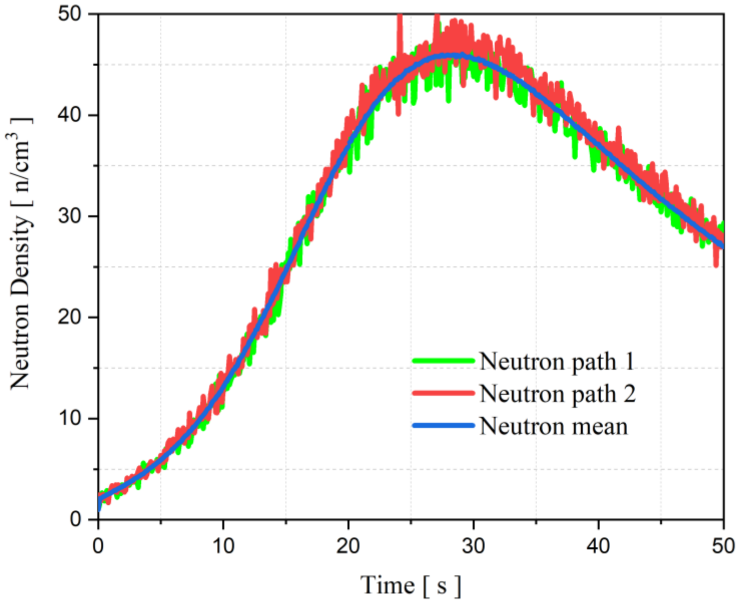
- We want to include fluctuations and feedback effects:

$$\frac{\partial}{\partial t} N(\vec{x}, t) = (D\nabla^2 + \rho) N(\vec{x}, t) + \eta(\vec{x}, t) + \sigma N^2(\vec{x}, t)$$

- Doppler effect (example of feedback):
  - Higher neutron flux → more fission → higher temperature → faster vibration of U-238 → broadening of the absorption cross section → more absorption → lower neutron flux
    - Heat generation: $\dot{q} \sim \Sigma_f N$
    - Heat transfer: $\dot{q} \sim \Delta T$
    - Reactivity feedback: $\partial\rho \sim \partial T$
    - Neutron diffusion: $\partial_t N \sim \rho N$
      → $\partial_t N \sim N^2$



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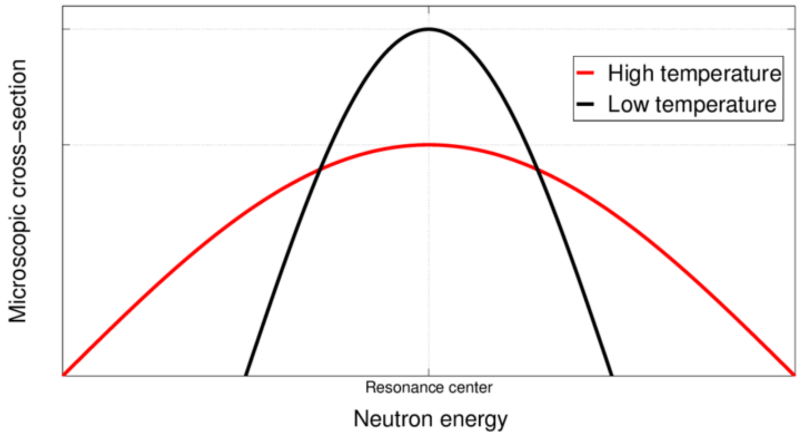

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Neutron transport

How fast the neutron population with direction $\mathbf{\Omega}$ and energy E changes in V

$$\int_V \frac{1}{v(E)}\frac{\partial}{\partial t}\psi(\boldsymbol{r},\mathbf{\Omega},E,t)d^3\boldsymbol{r} =$$

Absorption + scattering

Outgoing - incoming neutrons

$$-\int_V \mathbf{\Omega}\cdot\nabla\psi(\boldsymbol{r},\mathbf{\Omega},E,t)d^3\boldsymbol{r} - \int_V \Sigma_t(\boldsymbol{r},E,t)\psi(\boldsymbol{r},\mathbf{\Omega},E,t)d^3\boldsymbol{r}$$

Scattering

$$+\int_V \int_{(4\pi)} \int_0^\infty \Sigma_s(\boldsymbol{r},\mathbf{\Omega}'\to\mathbf{\Omega},E'\to E,t)\psi(\boldsymbol{r},\mathbf{\Omega}',\mathrm{E}',\mathrm{t})\mathrm{d}^2\mathbf{\Omega}'\mathrm{dE}'\mathrm{d}^3\mathbf{r}$$

Fission

$$+\int_V \frac{\chi^p(\boldsymbol{r},E)}{4\pi}[1-\tilde{\beta}(\boldsymbol{r})]\int_0^\infty \nu(\boldsymbol{r},E')\Sigma_\mathrm{f}(\mathbf{r},\mathrm{E}',\mathrm{t})\phi(\boldsymbol{r},E',t)dE'd^3\boldsymbol{r}$$

Delayed neutrons

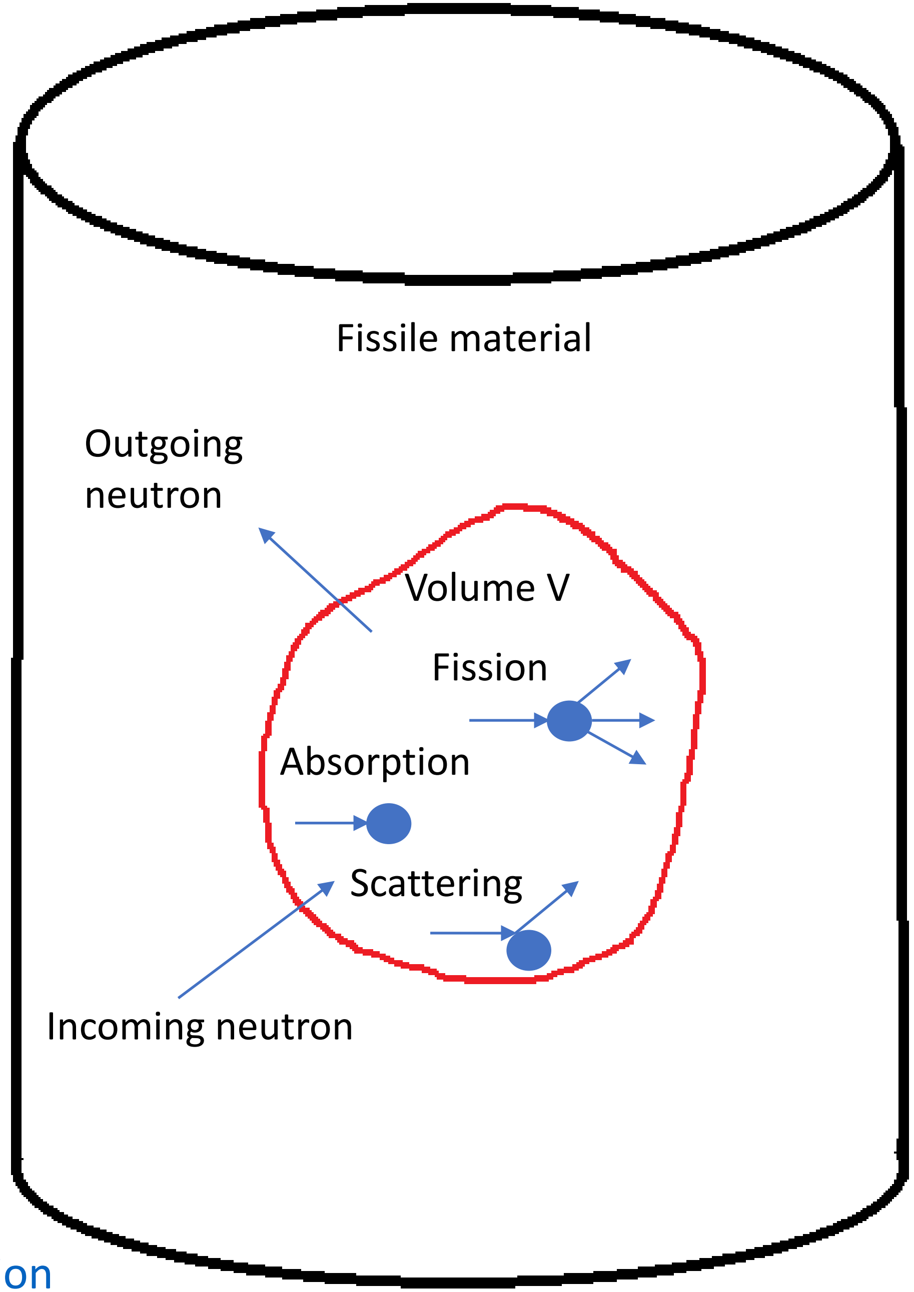
Source

$$+\frac{1}{4\pi}\int_V \sum_{i=1}^{N_d}\chi_i^d(\boldsymbol{r},E)\lambda_i(\boldsymbol{r})C_i(\boldsymbol{r},t)\,d^3\boldsymbol{r} + s(\boldsymbol{r},\mathbf{\Omega},\mathrm{E},\mathrm{t})$$

$$C_i(\boldsymbol{r},t) = \tilde{\beta}_i(\boldsymbol{r})\int_{-\infty}^{\infty}\int_0^\infty \nu(\boldsymbol{r},E)\Sigma_\mathrm{f}(\mathbf{r},E,t)\phi(\boldsymbol{r},E,t)dE - \lambda_i(\boldsymbol{r})C_i(\boldsymbol{r},t),$$

$$i = 1,\ldots,N_d$$

https://en.wikipedia.org/wiki/Neutron_transport#Neutron_transport_equation



Thermal hydraulics

- Conduction:

$$c_\rho(T)\rho(T)\frac{\partial T}{\partial t}(\boldsymbol{r},t) = q'''(\boldsymbol{r},t) + \boldsymbol{\nabla}\cdot[k(T)\boldsymbol{\nabla}\mathrm{T}(\boldsymbol{r},t)]$$

- Convection:

$$\frac{\partial(\rho e)}{\partial t}(\boldsymbol{r},t) + \boldsymbol{\nabla}\cdot(\rho e\boldsymbol{v})(\boldsymbol{r},t)$$
$$= -\boldsymbol{\nabla}\cdot\boldsymbol{q}''(\boldsymbol{r},t) + q'''(\boldsymbol{r},t) + \boldsymbol{\nabla}\cdot(\tau\cdot\boldsymbol{v})(\boldsymbol{r},t) - \boldsymbol{\nabla}\cdot(P\boldsymbol{v})(\boldsymbol{r},t) + (\rho\boldsymbol{g}\cdot\boldsymbol{v})(\boldsymbol{r},t)$$

- Conservation of mass:

$$\frac{\partial\rho}{\partial t}(\boldsymbol{r},t) + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{v})(\boldsymbol{r},t) = 0$$

- Conservation of momentum:

$$\rho(\boldsymbol{r},t)\left(\frac{\partial\boldsymbol{v}}{\partial t}(\boldsymbol{r},t) + \boldsymbol{v}(\boldsymbol{r},t)\cdot\boldsymbol{\nabla}\times\boldsymbol{v}(\boldsymbol{r},t)\right) = \boldsymbol{\nabla}\cdot\tau(\boldsymbol{r},t) - \boldsymbol{\nabla}\mathrm{P}(\boldsymbol{r},t) + \rho(\boldsymbol{r},t)\boldsymbol{g}$$

Derivation – Langevin equation

- Langevin equation: $$\frac{\partial f_i(\mathbf{x},t)}{\partial t} = g_i\left(f_i(\mathbf{x},t)\right) + \eta_i(\mathbf{x},t)$$
  - The noise η varies faster than f
- Expansion of unity: $$\int exp\left(i\int\sum_{i=1}^{N}\xi_i(\mathbf{x})G_i(\mathbf{x})d^n x\right)\prod_{j=1}^{N} det\left(\frac{\delta G_j}{\delta f_j}\right)\mathcal{D}\xi_j\mathcal{D}f_j = 1$$
- Average over noise: $$\langle\mathcal{O}[f_i]\rangle_\eta = \mathcal{N}_i\int\mathcal{O}[f_i]\prod_j\mathcal{P}[\eta_j]\mathcal{D}\eta_j$$
- Assuming a forward discretization scheme and Gaussian noise with zero mean and covariance $< \eta_i(\mathbf{x},t)\eta_j(\mathbf{x}',t') >= 2T_i(f_i)\delta_{i,j}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$ that is independent of $\partial_t f$ →

$$\langle\mathcal{O}[f_i]\rangle_\eta = \mathcal{N}_i\int\mathcal{O}[f_i]\prod_j\tilde{\mathcal{N}}_j e^{-S[f,\xi]}\mathcal{D}[i\xi_j]\mathcal{D}f_j$$

$$S[f,\xi] = \int\sum_k\left(\xi_k(\mathbf{x},t)\left[\partial_t f_k(\mathbf{x},t) - g_k(f_k(\mathbf{x},t))\right] - \xi_k T_k(f_k)\xi_k\right)d^n x dt$$

Derivation – Correlation functions

- Stochastic neutron diffusion equation: $\partial_t N(\mathbf{x},t) = D\nabla^2 N(\mathbf{x},t) + \rho N(\mathbf{x},t) + \eta_1(\mathbf{x},t)$

  → $S[N,\tilde{N}] = \int \tilde{N}(\mathbf{x},t)(\partial_t - D\nabla^2 - \rho)N(\mathbf{x},t) - \tilde{N}(\mathbf{x},t)T(N(\mathbf{x},t))\tilde{N}(\mathbf{x},t)d^3x dt.$

  $N(\boldsymbol{x},t)$ is treated as a scalar field

- Generating functional: $Z[J,\tilde{J}] = \mathcal{N}\int \mathcal{D}N\mathcal{D}[i\tilde{N}]e^{-S[\tilde{N}]+\int \tilde{J}(\mathbf{x},t)^T\tilde{N}(\mathbf{x},t)d^3x dt}$

- Correlation functions:

$$\left\langle \prod_{i,j} N(\mathbf{x}_i,t_i)\tilde{N}(\mathbf{x}_j,t_j)\right\rangle = \mathcal{N}\int \mathcal{D}N\mathcal{D}[i\tilde{N}]\prod_{i,j} N(\mathbf{x}_i,t_i)\tilde{N}(\mathbf{x}_j,t_j)e^{-S[\tilde{N}]}$$

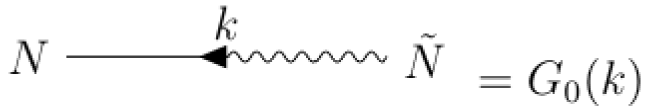
$$= \prod_{i,j}\frac{\delta}{\delta J(\mathbf{x}_i,t_i)}\frac{\delta}{\delta \tilde{J}(\mathbf{x}_j,t_j)} Z[J,\tilde{J}]\Big|_{J=0,\tilde{J}=0}.$$

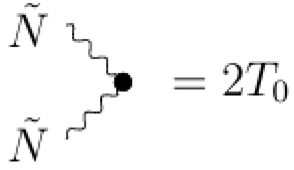
→ $< N(\mathbf{x},t) >_\eta = N_0\theta(t)\frac{1}{(4\pi Dt)^{3/2}}e^{\rho t - \frac{x^2}{4Dt}}$

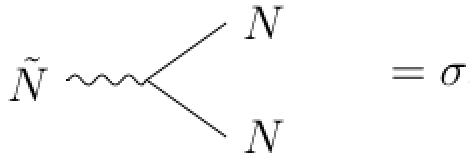
(assuming that the noise is independent of N)

Feynman diagrams

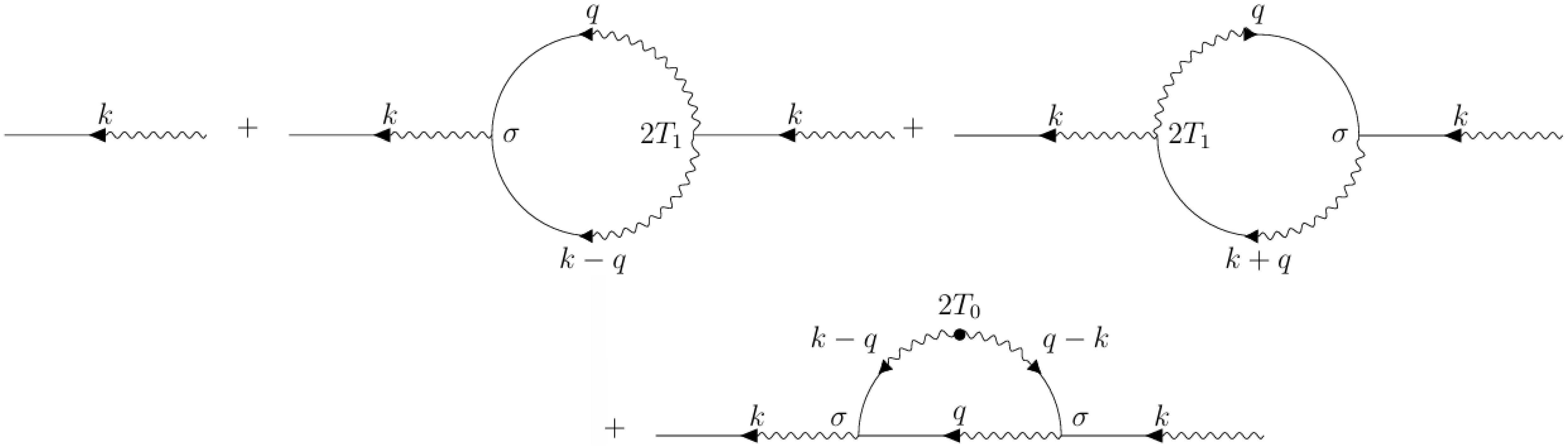
- Components:





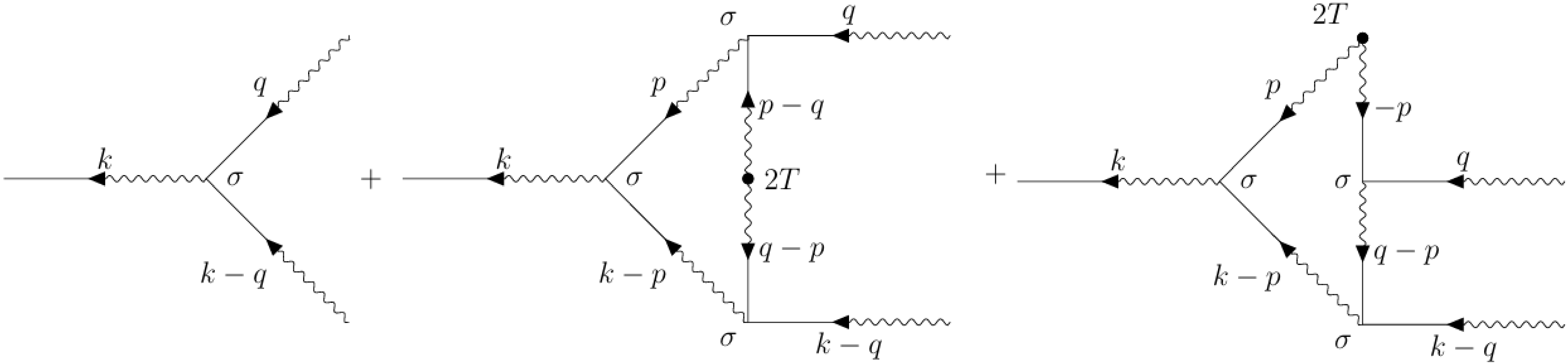


- Propagator + higher order contributions:



Feynman diagrams

- Feedback coupling contributions (if $T_1 = 0$):



Renormalization

- Green's function:

$$LG_0 = \delta \qquad L = (\partial_t - D\nabla^2 - \rho)$$

$$G_0(\mathbf{k},\omega)^{-1} = -i\omega + D\mathbf{k}^2 - \rho$$

- Renormalized parameters:

$$G_R(\mathbf{k},\omega)^{-1} = -i\omega + D_R\mathbf{k}^2 + D_R\mu^2\tau_R$$

$$D_R = Z_D D,\ \tau_R = Z_\tau \tau \mu^{-2} \text{ and } g_R = Z_g g \mu^{d-6}$$

- Universal scaling function:

$$G_R(\tau_R,\mathbf{k},\omega)^{-1} \propto D_{phys}|\mathbf{k}|^{2-\eta}\hat{G}_R\left(\frac{\tau_{phys}}{|\mathbf{k}|^{1/\nu}}, \frac{\omega}{D_{phys}|\mathbf{k}|^z}\right)^{-1}$$

$$\hat{G}_R\left(\frac{\tau_R}{|\mathbf{k}|^{1/\nu}}, \frac{\omega}{D_R|\mathbf{k}|^z}\right)^{-1} \approx \frac{-i\omega}{D_R|\mathbf{k}|^z} + 1 + \frac{\tau_R}{|\mathbf{k}|^{1/\nu}} + \epsilon\int_0^1 \frac{dx}{(2-x)^2}\left(\hat{\Delta}_R(x,\mathbf{k},\omega) ln \hat{\Delta}_R(x,\mathbf{k},\omega) - \frac{\tau_R}{|\mathbf{k}|^{1/\nu}} ln \frac{\tau_R}{|\mathbf{k}|^{1/\nu}}\right)$$

$$\hat{\Delta}_R(x,\mathbf{k},\omega) = \frac{1-x}{(2-x)^2} + \frac{\tau_R}{|\mathbf{k}|^{1/\nu}} - \frac{1-x}{2-x}\frac{i\omega}{D_R|\mathbf{k}|^z} \qquad \epsilon = d - d_c$$

If $T_1 = 0$

$$Z_D \approx 1 - \tfrac{5}{24} A_d g_R \mu^{6-d}$$
$$Z_\tau \approx 1 + \tfrac{1}{12} A_d g_R \mu^{6-d}$$
$$Z_g \approx 1 + \tfrac{1}{2} A_d g_R \mu^{6-d}$$

$$A_d = \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}}\left(2 - \frac{d}{2}\right) r^{-3+\frac{d}{2}}$$

$$r = -\rho/D$$

$$g = \frac{\sigma^2 T}{D^3}$$