

Data driven approaches to learning about nuclear structure

David Jenkins



PLANET

A doctoral training centre in applied nuclear physics



- ▶ Radiation Detection and Measurement
- ▶ Nuclear Data and Nuclear Reactions
- ▶ Reactor Physics and Advanced Nuclear Systems
- ▶ AI, Machine Learning & Computational Modelling
- ▶ Medical Isotope Production
- ▶ Waste, Decommissioning & Environmental Monitoring
- ▶ Nuclear Security and Nuclear Forensics
- ▶ Nuclear Theory and Quantum Computing
- ▶ Nuclear Fusion



Baseline training (M1/2)

Data-driven
nuclear structure

Hands-on
theory

Nuclear
models

Measurement
and metrology

Instrumentation (M3/4)

Instrumentation

Radiation
detector theory

AMS

Hands-on
detector
work

Environmental
research

Simulation and Modelling (M3/4)

Reactor
physics

Neutron
transport

LUNar
simulator

GEANT4

High-performance
computing

Nuclear data pipeline (M4/5)

Nuclear Data

FISPACT

TALYS

ANSWERS

Statistics

Data
science

Nuclear
forensics

Consolidation (M6)

Neutron beam
experiment (NPL)

TRIGA reactor
experiment

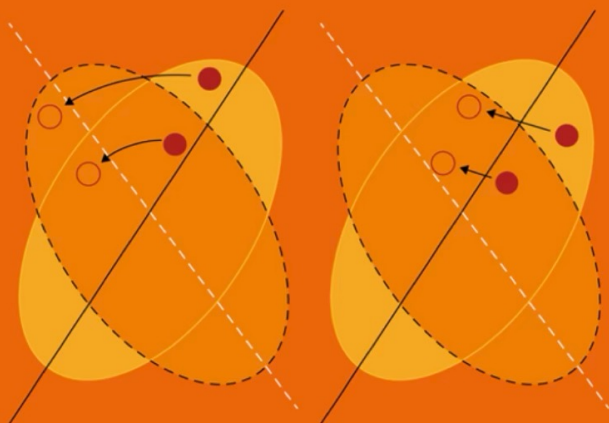


IOP Series in Nuclear Spectroscopy and Nuclear Structure

Nuclear Data

A primer

David G Jenkins
John L Wood



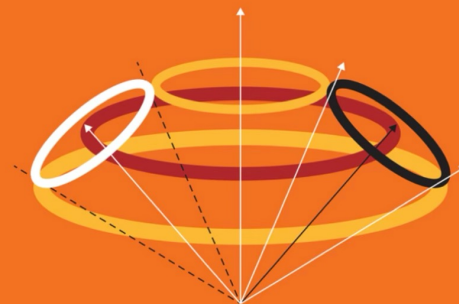
IOP ebooks

IOP Series in Nuclear Spectroscopy and Nuclear Structure

Nuclear Data

A collective motion view

David Jenkins
John L Wood



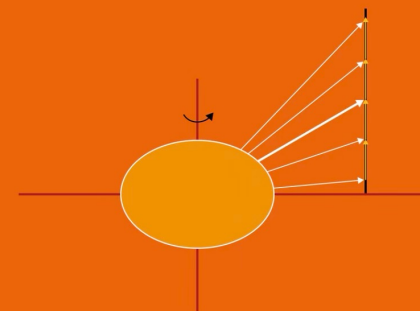
IOP ebooks

IOP Series in Nuclear Spectroscopy and Nuclear Structure

Nuclear Data

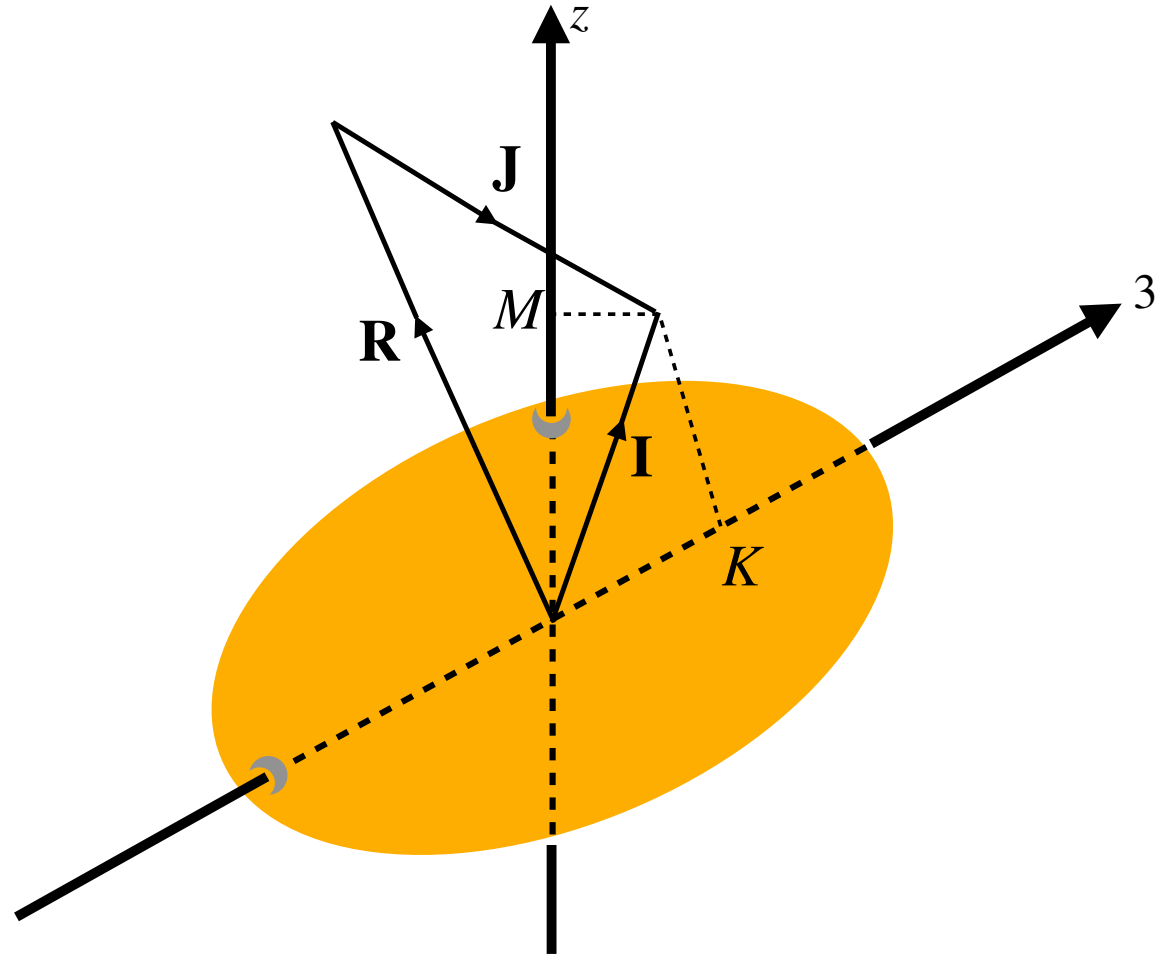
An independent-particle motion view

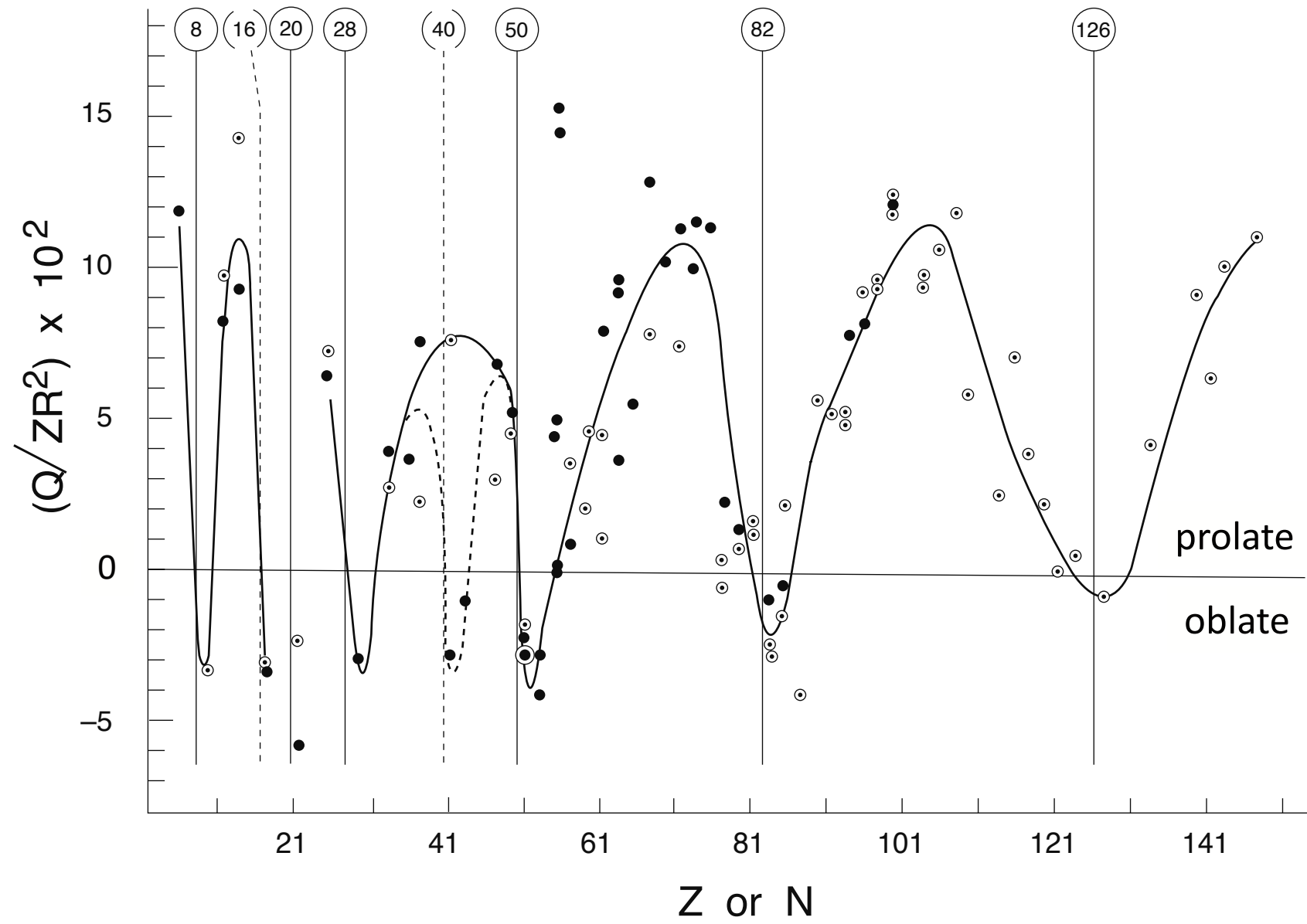
David Jenkins
John L Wood



IOP ebooks

How well defined are rotations in nuclei?





• NB Q is only defined for states with $J \geq 1$

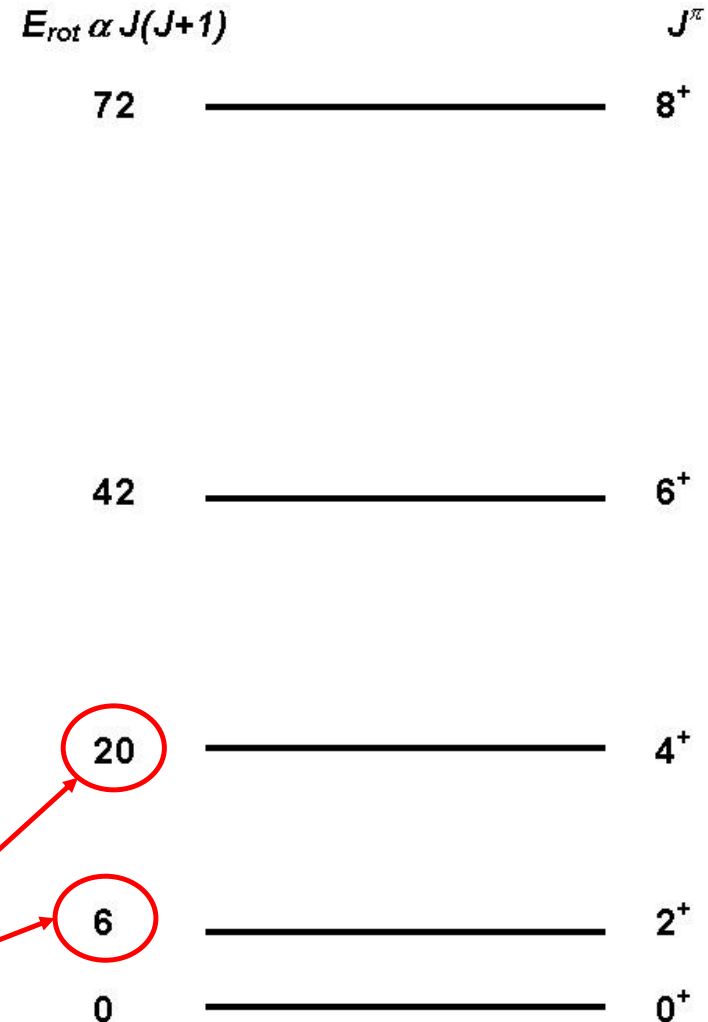
Rotational bands even-even nuclei

For the special case of even-even nuclei only even J values are allowed in a rotational band built upon the $J^\pi=0^+$ ground states (due to some symmetry properties)

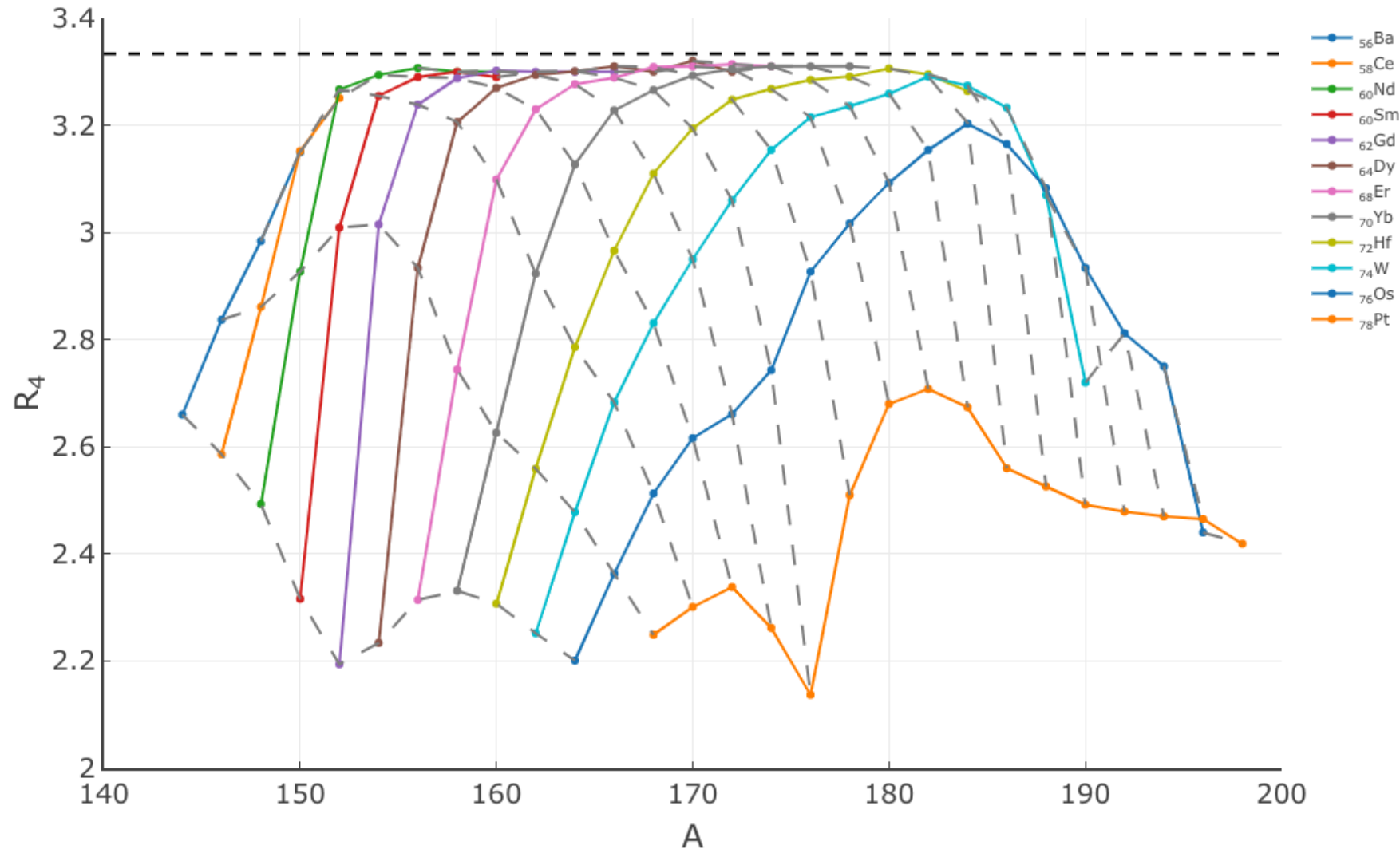
Thus, for an even-even nucleus with a fixed deformation (i.e. a fixed moment of inertia) we expect to find a level scheme like this

A structure of excited states such as this is known as a **rotational band, states within the band are known as “band members” and the lowest-E state is the “band head”**

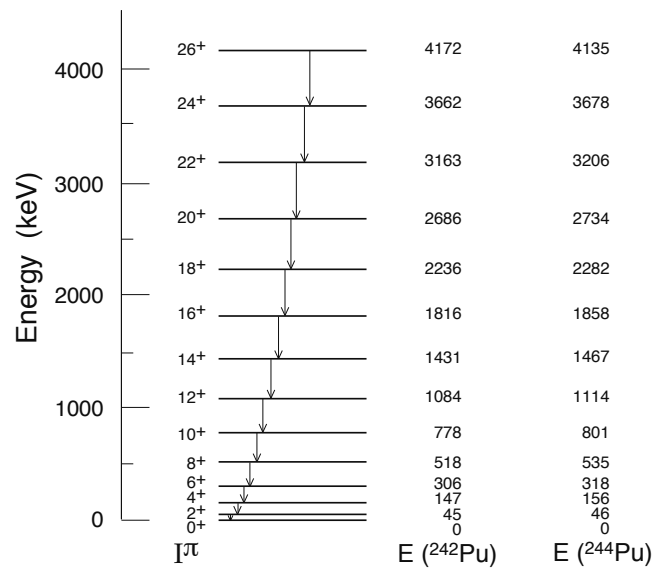
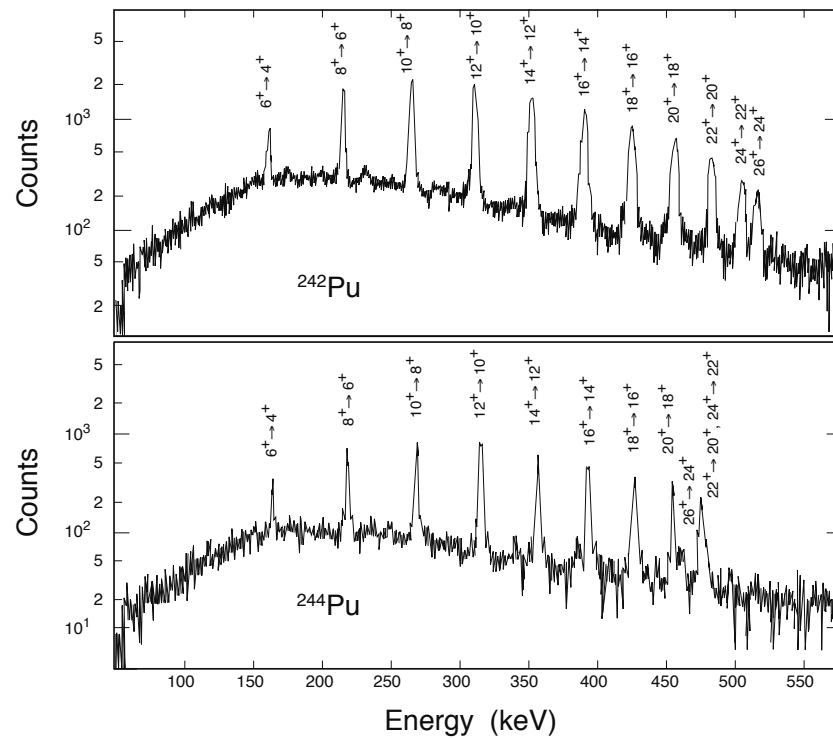
So we expect $E(4_1^+)/E(2_1^+) \approx 3.33$



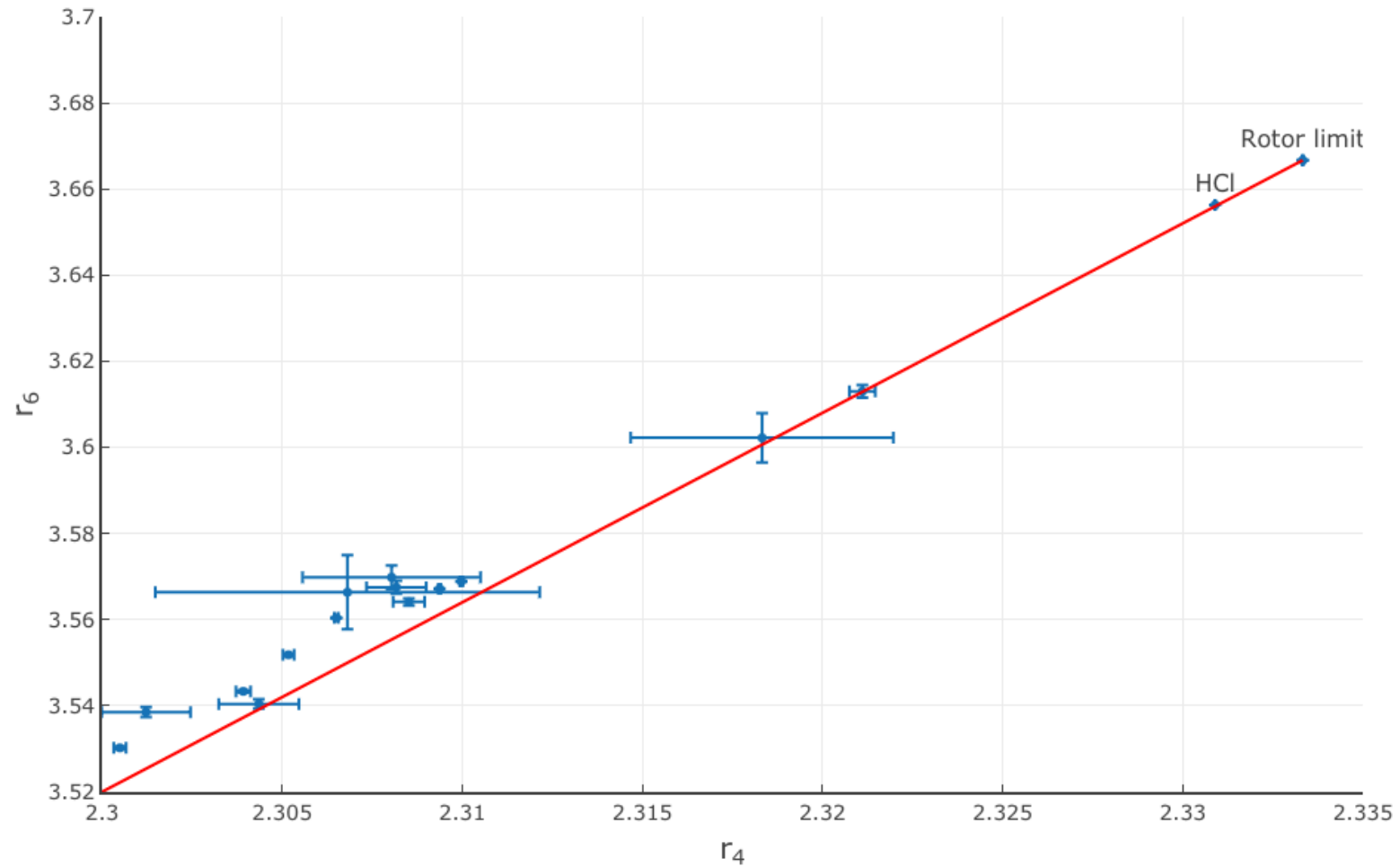
$$R_4 = E(4^+)/E(2^+)$$



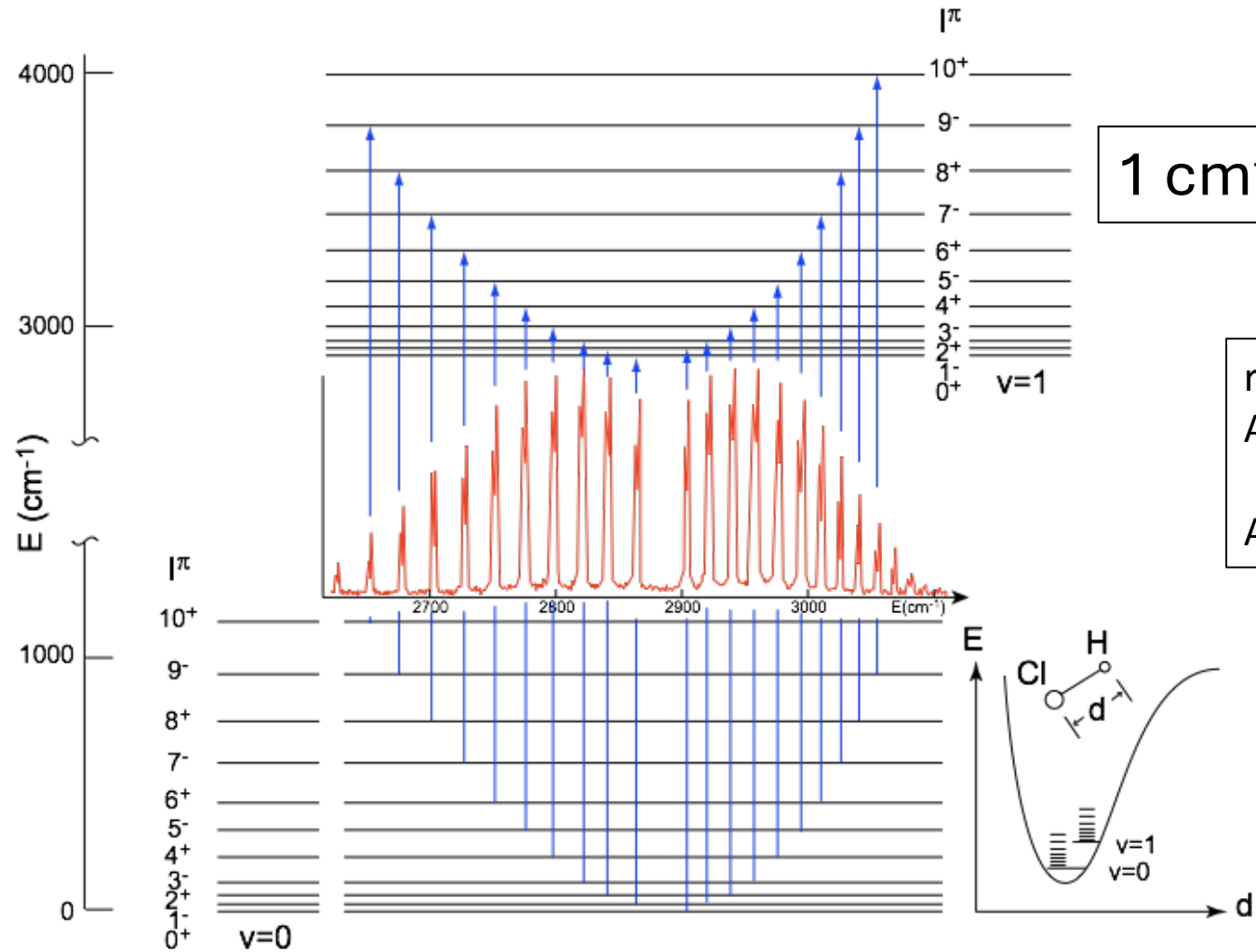
$$E = \frac{\hbar^2 I(I+1)}{2 \mathcal{J}}$$



$$r_6 := \frac{E(6_1^+) - E(4_1^+)}{E(2_1^+)} \text{ vs. } r_4 := \frac{E(4_1^+) - E(2_1^+)}{E(2_1^+)},$$



The infrared absorption spectrum of HCl reveals molecular vibrations and rotations.



$$1 \text{ cm}^{-1} = 1.24 \times 10^{-4} \text{ eV}$$

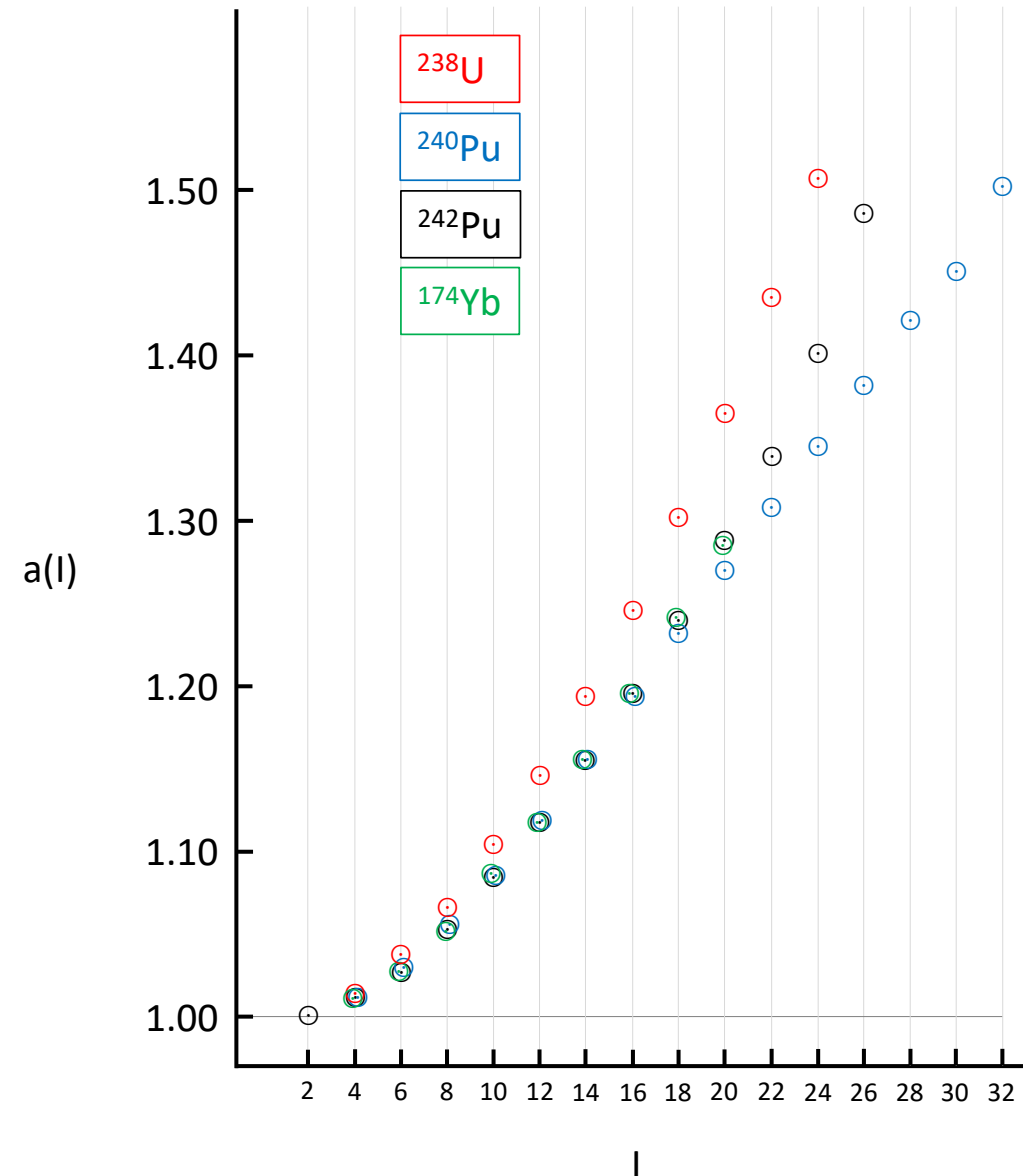
rotational const.
 $A_1 = 10.134 \text{ cm}^{-1}$
 $A_0 = 10.437 \text{ cm}^{-1}$

We can also explore the deviation from rotor behaviour as a function of I to very large values of angular momentum using the rotational parameter:

$$a(I) := [E_\gamma(I \rightarrow I - 2)/(4I - 2)]/[E_\gamma(2 \rightarrow 0)/6]$$

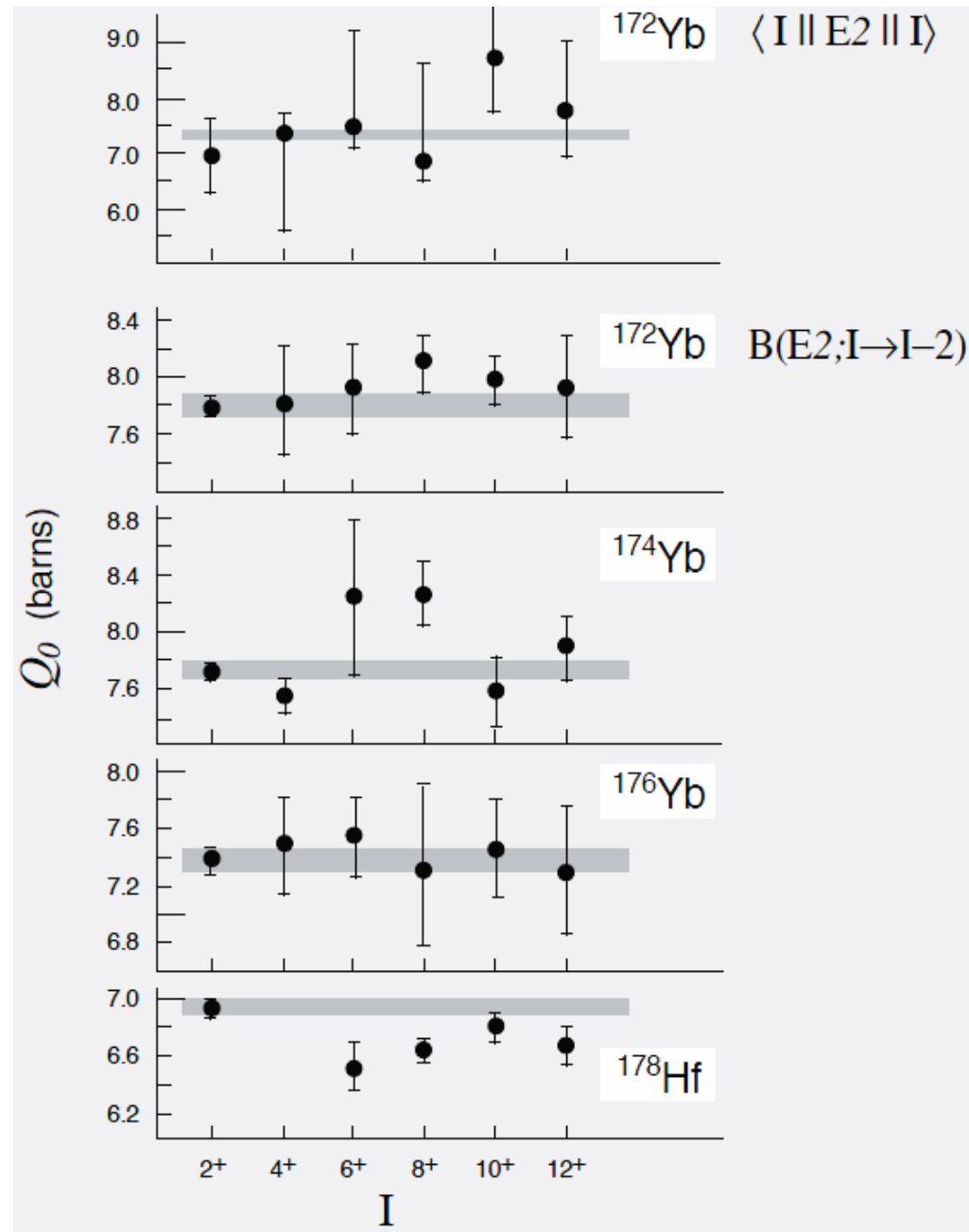
This parameter should be unit for all values of I if the rotor model were perfectly respected.

For discussion: what is fascinating is that the rotational parameter for ^{174}Yb and ^{242}Pu are almost identical despite the very large difference in number of protons and neutrons making up the nucleus!



See just how similar ^{242}Pu and ^{174}Yb are when energies are scaled

| I_i | $E(^{242}\text{Pu})$ (keV) | $E(^{174}\text{Yb})$ (keV) $\times 0.5824$ | $E(^{174}\text{Yb})$ (keV) | % dev. |
|-------|----------------------------|--|----------------------------|---------------|
| 2 | 44.54 ² | 44.54 [norm.] | 76.471 ¹ | - |
| 4 | 102.8 ¹ | 102.9 | 176.645 ² | +0.098 |
| 6 | 159.0 ¹ | 158.9 | 272.918 ⁶ | -0.063 |
| 8 | 211.7 ⁴ | 211.8 | 363.64 ⁵ | +0.047 |
| 10 | 260.5 ⁶ | 260.4 ⁶ | 447.2 ¹⁰ | -0.038 |
| 12 | 305.8 ⁸ | 305.4 ⁸ | 524.4 ¹³ | -0.131 |
| 14 | 347.3 ¹⁰ | 347.1 ¹⁰ | 595.9 ¹⁷ | -0.058 |
| 16 | 385.0 ¹¹ | 384.4 ¹¹ | 660 ² | -0.156 |
| 18 | 419.3 ¹² | 418.7 ¹⁷ | 719 ³ | -0.143 |
| 20 | 450.2 ¹³ | 450.8 ²⁹ | 774 ⁵ | +0.133 |
| | | | | -0.035 (avg.) |



An experimental moment of inertia can be derived from the excitation energy of the 2+ state. Theoretical moments of inertia can be extracted for two different scenarios of rigid rotor and irrotational flow and compared to experiment.

$$\mathcal{I}_{\text{expt}} = 0.2080 \times 10^{-54} E(2_1^+)^{-1} [\text{MeV}^{-1}],$$

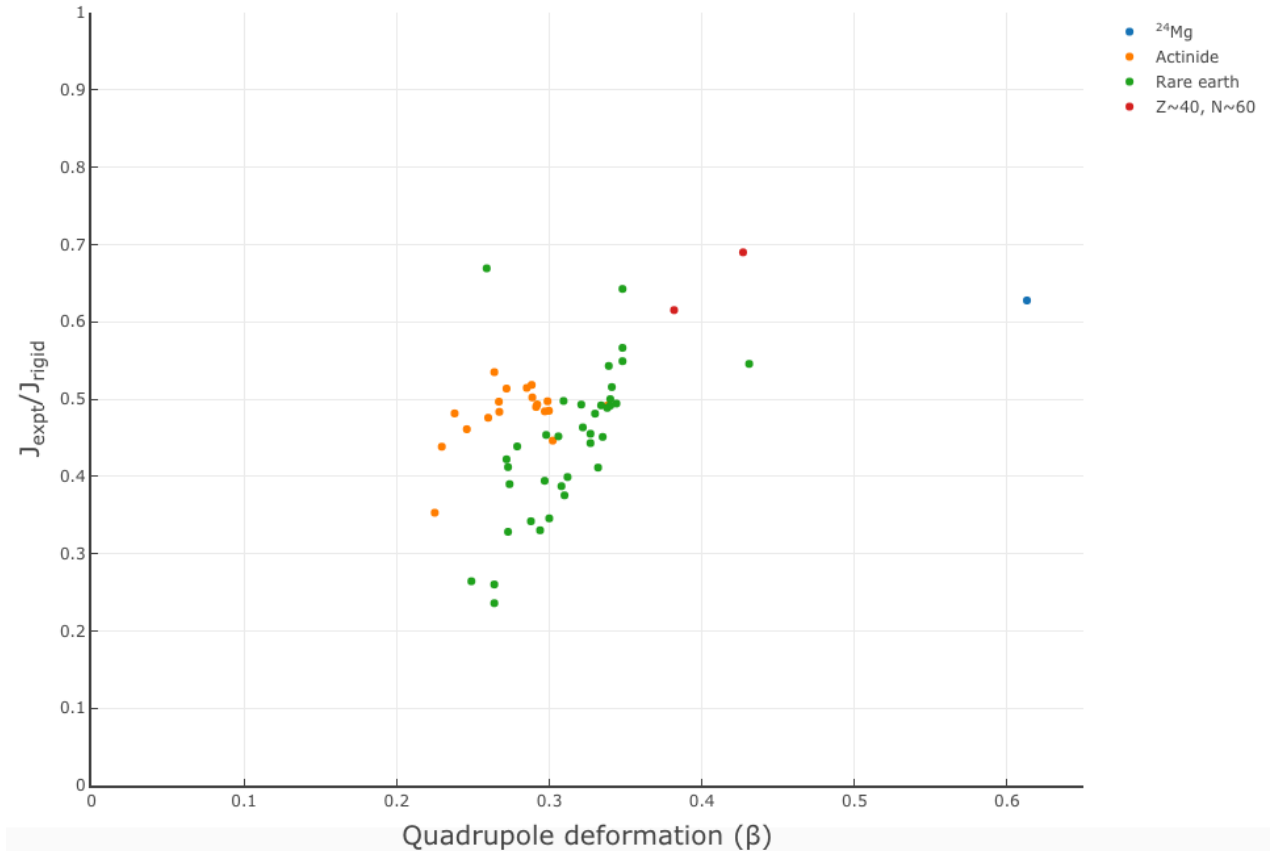
$$\mathcal{I}_{\text{rigid}} = 0.8864 \times 10^{-57} A^{5/3} (1 + 0.3154\beta + 0.44\beta^2),$$

$$\mathcal{I}_{\text{irrot}} = 0.8864 \times 10^{-57} A^{5/3} 0.8951\beta^2,$$

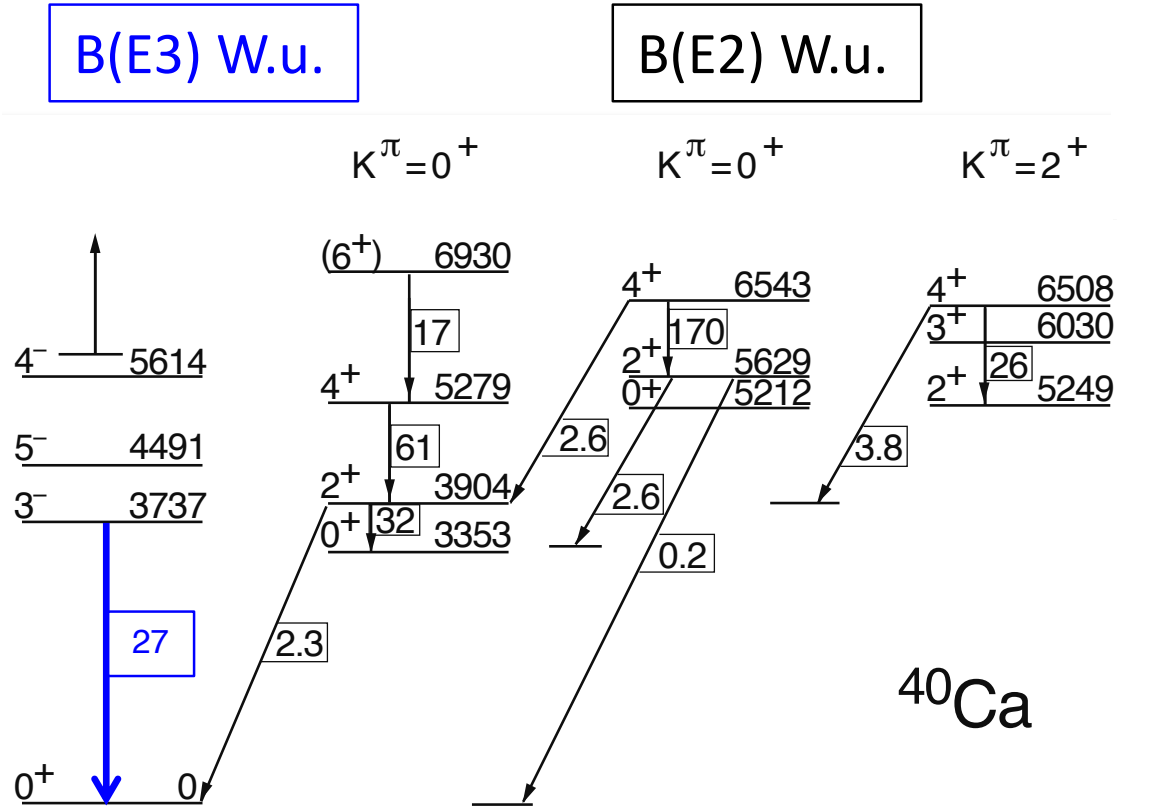
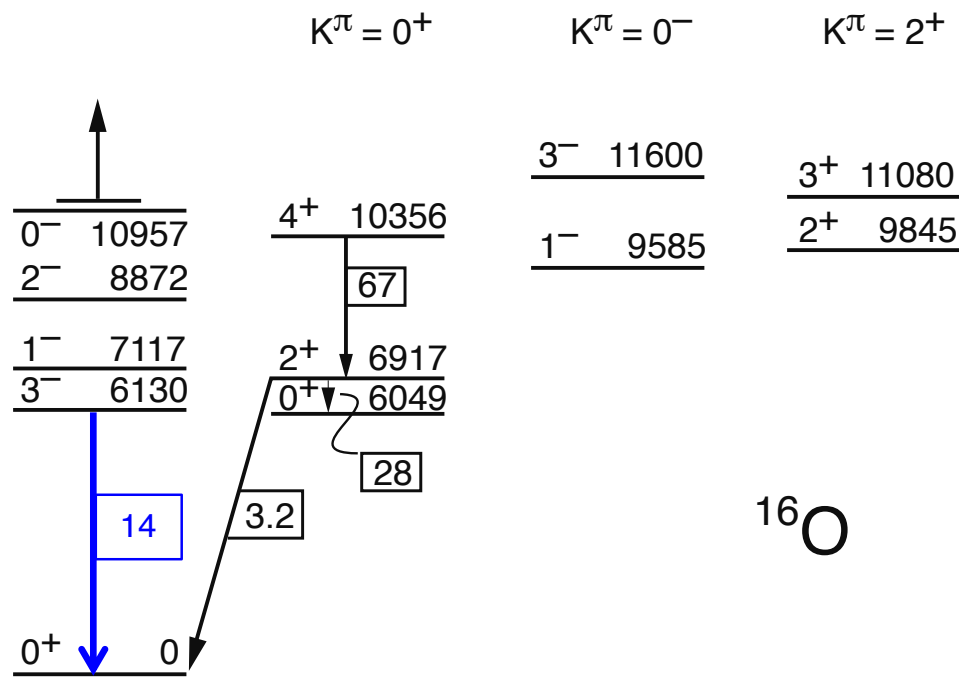
$$\beta = Q_0 \frac{\sqrt{5\pi}}{3ZR^2},$$

The figure shows that the experimental value is never more than about 0.3-0.7 of the rigid rotor value.

CONCLUSION: Everything looks like rotation but it's nothing like a classical rotating rigid body. What is actually rotating is a very good question!



| | Z | Q ₀ (b) | A ^{2/3} | β | $\mathcal{I}_{\text{rigid}}$ × 10 ⁻⁵⁴ (kg.m ²) | E(2 ₁ ⁺) (keV) | $\mathcal{I}_{\text{expt}}$ (kg.m ²) | $\frac{\mathcal{I}_{\text{expt}}}{\mathcal{I}_{\text{rigid}}}$ |
|-------------------|----|-----------------------|------------------|--------|--|--|---|--|
| ¹⁷⁴ Yb | 70 | 7.82 ⁵ | 31.167 | 0.3081 | 5.955 × 10 ⁻⁵⁴ | 76.471 ¹ | 2.723 × 10 ⁻⁵⁴ | 0.4573 |
| ²⁴² Pu | 94 | 11.90 ⁶ | 38.834 | 0.2823 | 10.184 × 10 ⁻⁵⁴ | 44.54 ² | 4.675 × 10 ⁻⁵⁴ | 0.4591 |
| ¹⁵² Dy | 66 | 17.5 ² | 28.482 | 0.7076 | 6.025 × 10 ⁻⁵⁴ | 33.75 | 6.170 × 10 ⁻⁵⁴ | 1.024 |

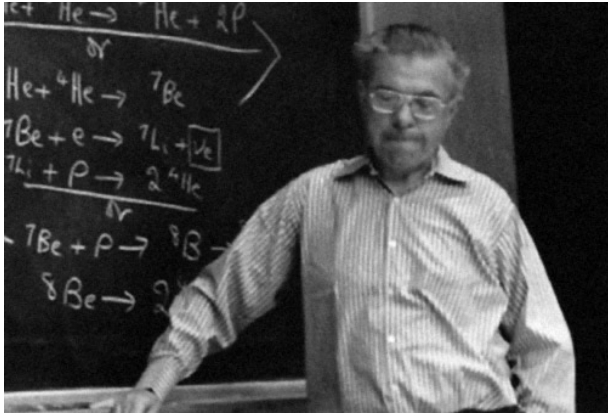


Independent particle plus 3^- collective

collective (deformed)

Independent particle plus 3^- collective

collective (deformed)

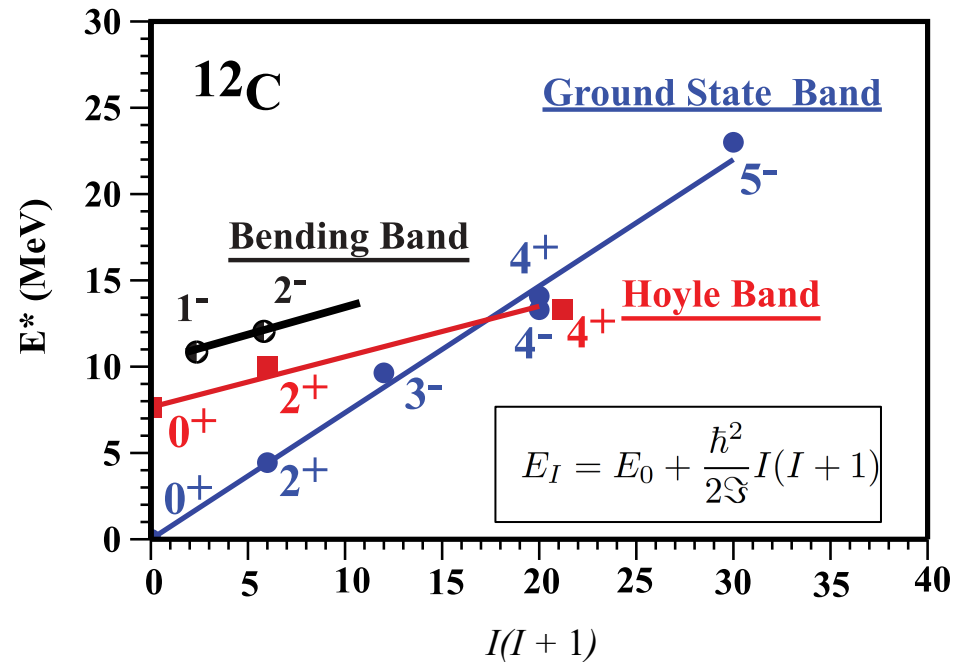
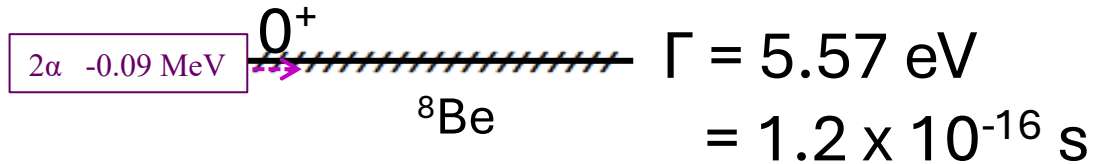
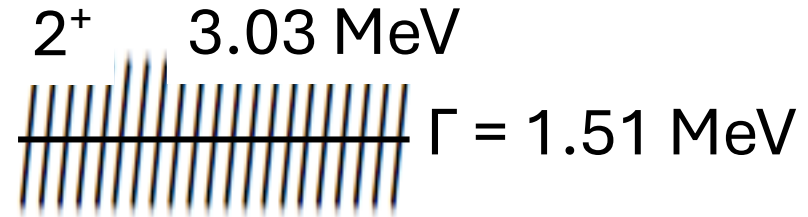
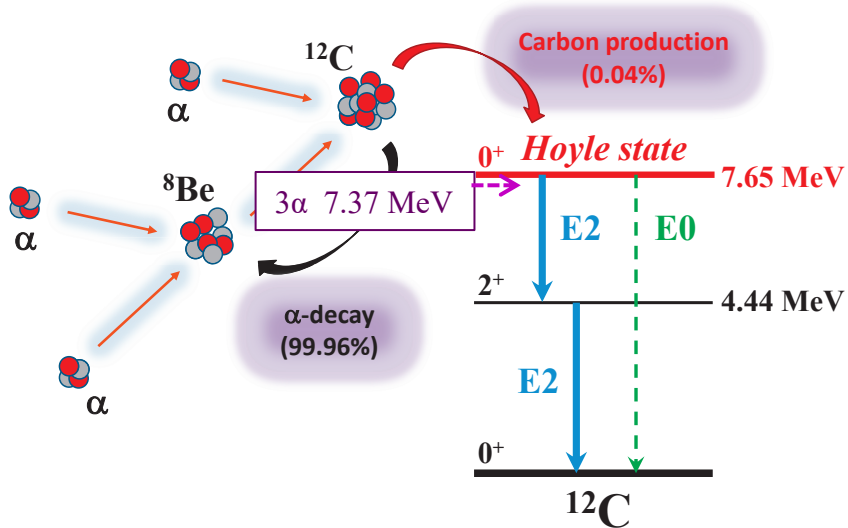


Sir Fred Hoyle (1915-2001)

Helium fusion in stars

F. Hoyle, *Astrophysical J. Suppl.*

Ser. 1 121 1954



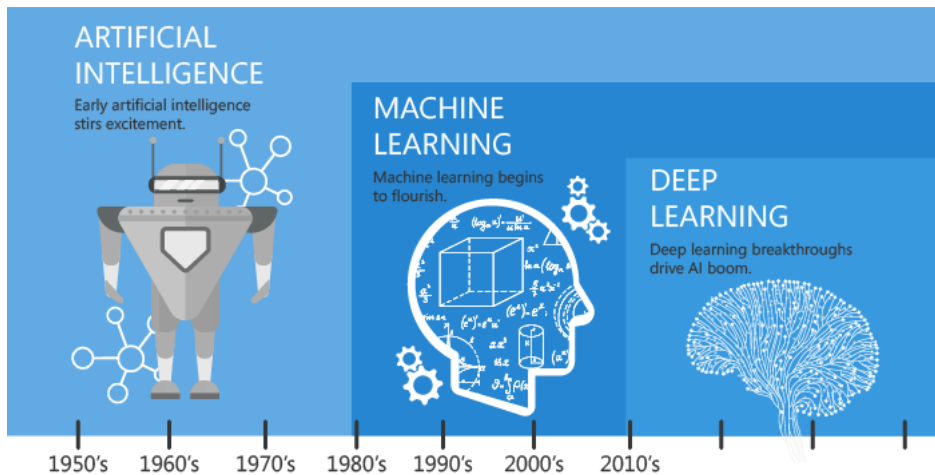
A few parting questions....

1) Do we need more data?

2) If so, what kind of data do we need?

3) Are we doing enough with the data we have?

How to use the amazing resources on NNDC?



Adopted Levels, Gammas

| Type | Author | History | Citation | Literature Cutoff Date |
|-----------------|--|---------|-------------------|------------------------|
| Full Evaluation | M. Shamsuzzoha Basunia, Anagha Chakraborty | | NDS 186, 2 (2022) | 31-Mar-2022 |

$Q(\beta^-) = -13884.77$ 23; $S(n) = 16531.22$ 3; $S(p) = 11692.69$ 1; $Q(\alpha) = -9316.56$ 1 2021Wa16
 $S(2n) = 29676.23$ 16, $S(2p) = 20486.805$ 22 (2021Wa16).

Other reactions:

2004Be18, 2004Be08: $^{12}\text{C}(^{24}\text{Mg}, ^{12}\text{C})$, $E = 130$ MeV; measured E_γ , (particle) γ -coin.

2011Fr14: $^{12}\text{C}(^{13}\text{C}, n)$ $E = 12, 13.5, 20$ MeV; measured reaction products ^{25}Mg ; deduced ^{24}Mg excited states and reported resonance energies at 13.25 MeV 20 and 14.25 MeV 20.

2001Di12: $^{11}\text{B}(^{13}\text{N}, X)$, ($^{13}\text{N}, ^{12}\text{C}$), $E = 29.5, 45$ MeV. Measured particle spectra, fusion σ . Deduced ^{24}Mg $6-\alpha$ decay features, isospin purity/mixing in ^{24}Mg at excitation energy ~ 47 MeV, GDR γ -emission features.

2006Va20: $^{28}\text{Si}(p, p'X)^{24}\text{Mg}$, $E = 1$ GeV; measured E_γ ; deduced σ .

^{24}Mg Levels

Cross Reference (XREF) Flags

| | | | | | |
|----------|---|----------|--|----------------|---|
| A | $^{24}\text{Na} \beta^-$ decay (14.956 h) | N | $^{20}\text{Ne}(\alpha, \gamma)$:Resonances | Others: | |
| B | $^{24}\text{Na} \beta^-$ decay (20.18 ms) | O | $^{20}\text{Ne}(\alpha, \alpha'), (\alpha, \alpha')$:Resonances | AA | Coulomb excitation |
| C | $^{24}\text{Al} \varepsilon$ decay (2.053 s) | P | $^{20}\text{Ne}(^6\text{Li}, d), (^7\text{Li}, t)$ | AB | $^{24}\text{Mg}(\alpha, \alpha' \gamma)$ |
| D | $^{24}\text{Al} \varepsilon$ decay (130.7 ms) | Q | $^{22}\text{Ne}(^3\text{He}, n)$ | AC | $^{24}\text{Mg}(^6\text{Li}, ^6\text{Li}')$ |
| E | $^{25}\text{Si} \varepsilon p$ decay | R | $^{23}\text{Na}(p, \gamma), (p, p'), (p, X)$, | AD | $^{24}\text{Mg}(^{16}\text{O}, ^{16}\text{O}')$ |
| F | $^{26}\text{P} \varepsilon 2p$ decay | S | $^{23}\text{Na}(^3\text{He}, d), (^3\text{He}, d\gamma)$ | AE | $^{25}\text{Mg}(p, d)$ |
| G | $^{28}\text{P} \varepsilon \alpha$ decay | T | $^{24}\text{Mg}(\gamma, \gamma')$ | AF | $^{25}\text{Mg}(^3\text{He}, ^4\text{He})$ |
| H | $^{12}\text{C}(^{12}\text{C}, \gamma)$ | U | $^{24}\text{Mg}(e, e')$ | AG | $^{27}\text{Al}(\mu^-, \nu 3n\gamma)$ |
| I | $^{12}\text{C}(^{12}\text{C}, p)$:Resonances | V | $^{24}\text{Mg}(\pi^+, \pi^+'), (\pi^-, \pi^-')$ | AH | $^{27}\text{Al}(p, \alpha)$ |
| J | $^{12}\text{C}(^{14}\text{N}, d)$ | W | $^{24}\text{Mg}(p, p'), (\text{pol } p, p')$, | AI | $^{28}\text{Si}(d, ^6\text{Li})$ |
| K | $^{12}\text{C}(^{24}\text{Mg}, ^{12}\text{C}\gamma)$ | X | $^{24}\text{Mg}(n, n' \gamma)$ | AJ | $^{28}\text{Si}(^{28}\text{Si}, X\gamma)$ |
| L | $^{12}\text{C}(^{16}\text{O}, \alpha), (^{16}\text{O}, \alpha\gamma)$ | Y | $^{24}\text{Mg}(^3\text{He}, ^3\text{He}')$ | | |
| M | $^{12}\text{C}(^{24}\text{Mg}, 2^{12}\text{C}), (^{20}\text{Ne}, 2^{12}\text{C})$ | Z | $^{24}\text{Mg}(\alpha, \alpha')$ | | |

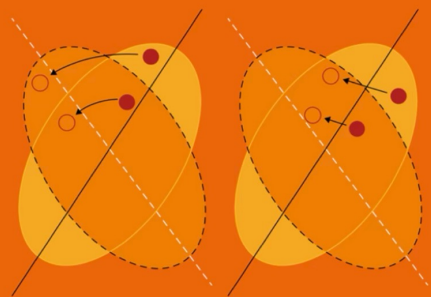
| E(level) [†] | J^π | $T_{1/2}$ or Γ^J | XREF | Comments |
|-------------------------|---------|-------------------------|---------------------------|--|
| 0^P | 0^+ | stable | ABCDEFGHIJKL N PQRSTUWXYZ | XREF: Others: AA, AB, AD, AE, AF, AG, AH, AI, AJ $\delta \langle r^2 \rangle (^{26}\text{Mg}, ^{24}\text{Mg}) = +0.140 \text{ fm}^2$ 5 (stat) 25 (syst) (2012Yo01). $\langle r^2 \rangle^{1/2} (^{24}\text{Mg}) = 3.0570$ 16 (charge radius) (2013An02 evaluation). Others: 3.0570 fm 7 (stat) 48 (syst) (2012Yo01), 3.030 fm 30 (1971Li26 - (e, e')). |
| 1368.667 ^P 5 | 2^+ | 1.36 ps 3 | A CDEF H JKL N PQRSTUWXYZ | XREF: Others: AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ $\mu = +1.08$ 3; $Q = -0.29$ 3 |

IOP Series in Nuclear Spectroscopy and Nuclear Structure

Nuclear Data

A primer

David G Jenkins
John L Wood



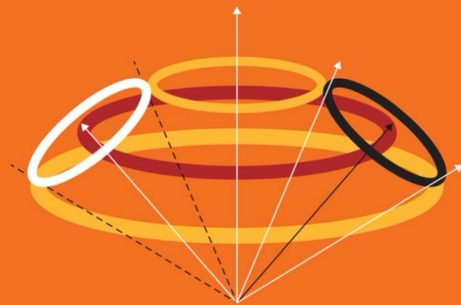
IOP ebooks

IOP Series in Nuclear Spectroscopy and Nuclear Structure

Nuclear Data

A collective motion view

David Jenkins
John L Wood



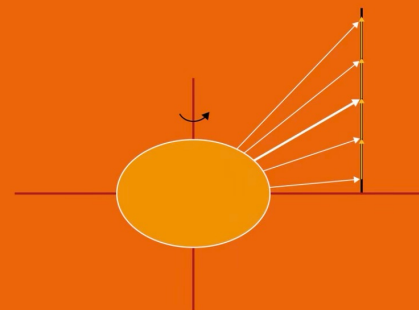
IOP ebooks

IOP Series in Nuclear Spectroscopy and Nuclear Structure

Nuclear Data

An independent-particle motion view

David Jenkins
John L Wood



IOP ebooks

IOP Series in Nuclear Spectroscopy and Nuclear Structure

Radiation Detection for Nuclear Physics

Methods and industrial applications

David Jenkins



IOP ebooks

Finis

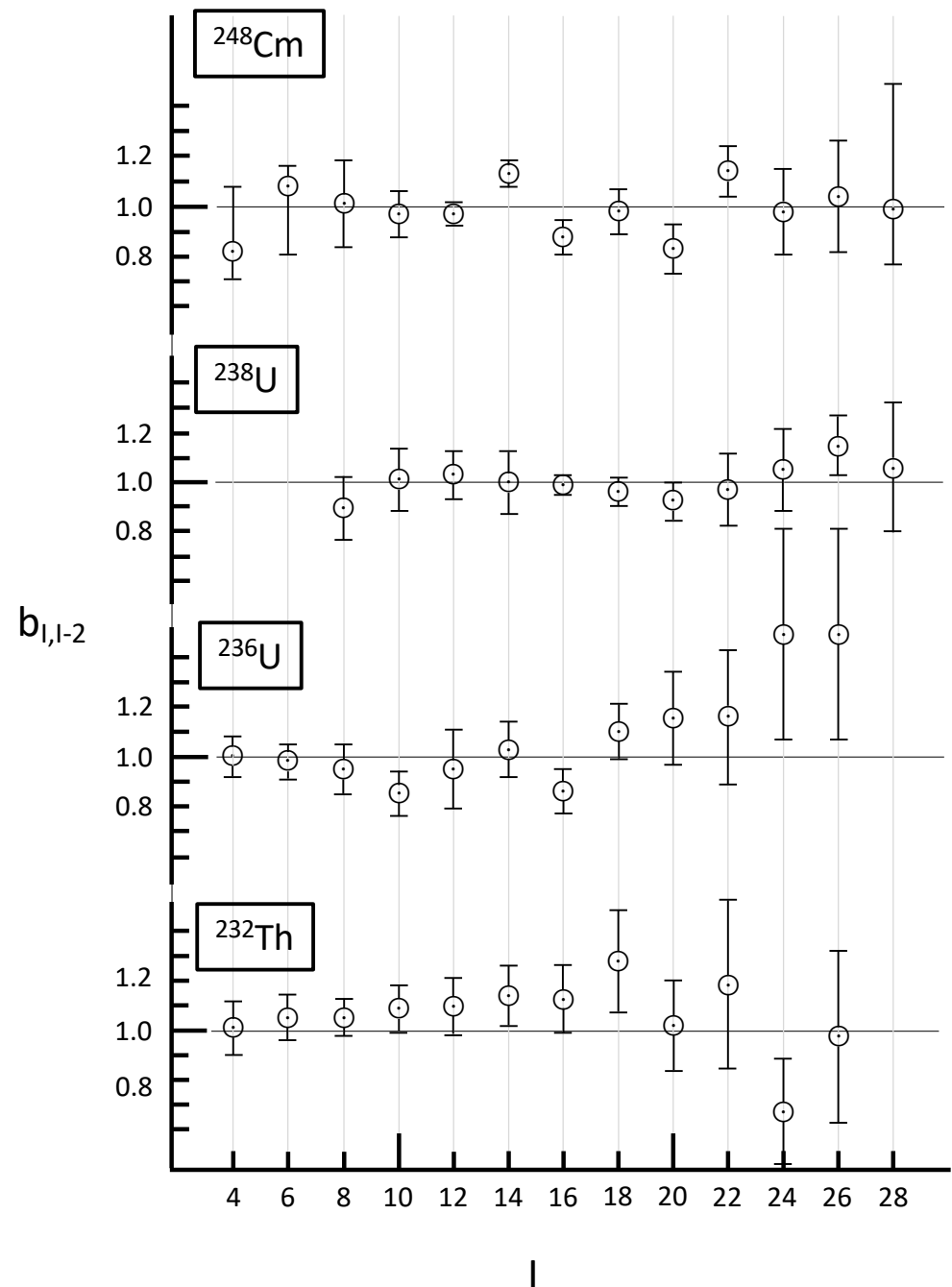
There should be a fixed relationship between B(E2) transition strengths up a rotational band

$$\frac{B_{I,I-2}}{B_{20}} = \frac{15I(I-1)}{2(2I-1)(2I+1)} := f(I)$$

We then define

$$b_{I,I-2} := B_{I,I-2}/B_{20}f(I)$$

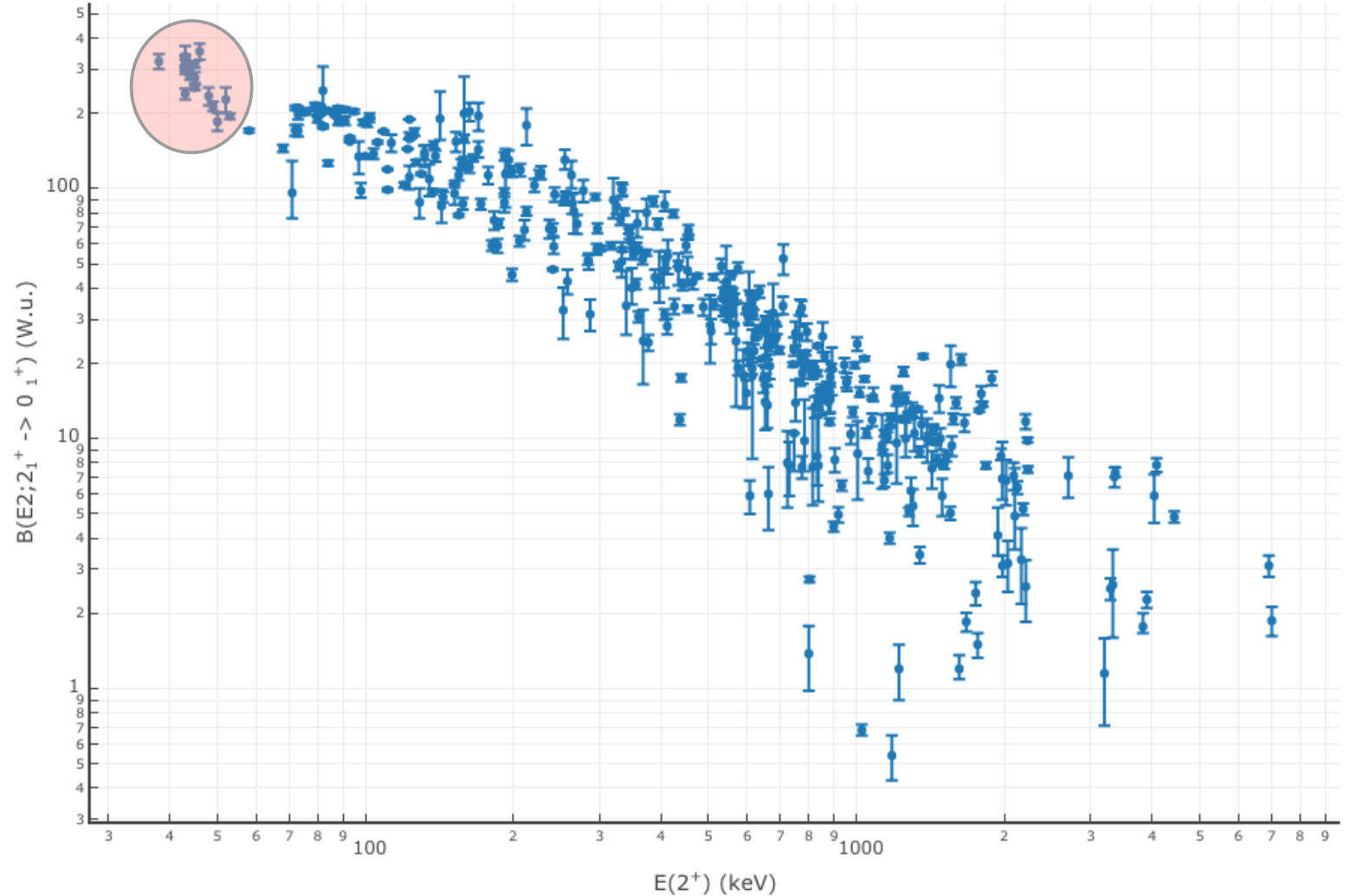
which should be unity for all values of I if the rotor model is exactly observed. The plot of $b_{I,I-2}$ shows that this is well reproduced for actinide nuclei.



There is a general trend between transition strength and transition energy for $2^+ \rightarrow 0^+$ transitions in even-even nuclei. NOTE: This is a log-log plot.

On this plot, our actinide nuclei have the highest transition strengths and lowest transition energies.

Q: Why should there be such a trend?



Transition strengths indicate how probable an electromagnetic decay is. Weisskopf made an estimate for the decay strength for a single proton transition. This is our yardstick for transition strengths - the Weisskopf unit (W.u.).

We can calculate transition strengths in W.u. for E2 transitions from the following formula:

$$B(E2) = \frac{9527}{E_\gamma^5 T_{1/2}(\gamma) A^{4/3}}$$

where E_γ is in MeV and $T_{1/2}$ in ps.

