LUND UNIVERSITY

Neutron reactions from Hamiltonian solutions in deformed nuclei

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Nordic Meeting on Nuclear Physics
Visby, 07 May 2026

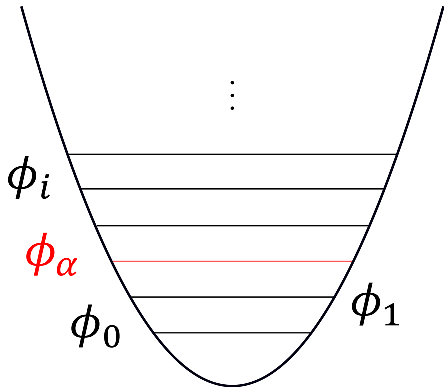
A tale of two spectroscopic factors

components of many-body states

$$S = \left|\langle \psi_0^A | c_\alpha | \psi_n^{A+1} \rangle\right|^2$$

α s.p. (basis) component of
A+1 manybody state

basis



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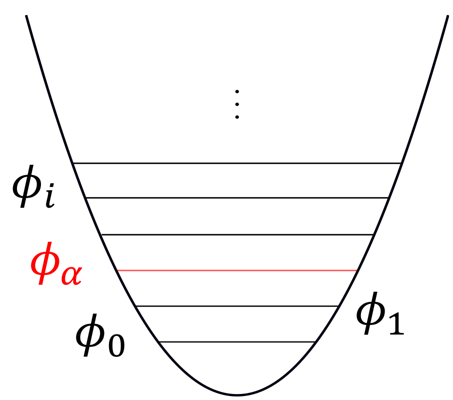
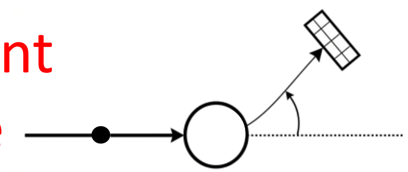
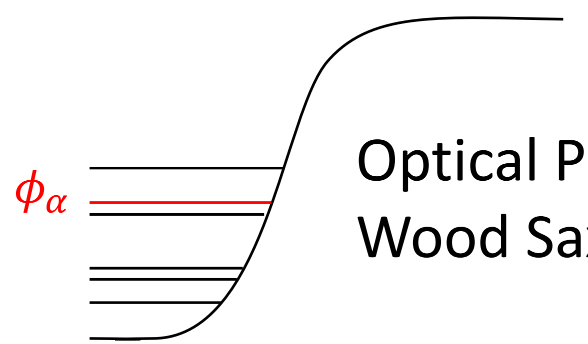


single particle states + “rest”

$$(C^2)S = \frac{\sigma^{exp}}{\sigma^{Indip.Part.Model}}$$

α independent particle state

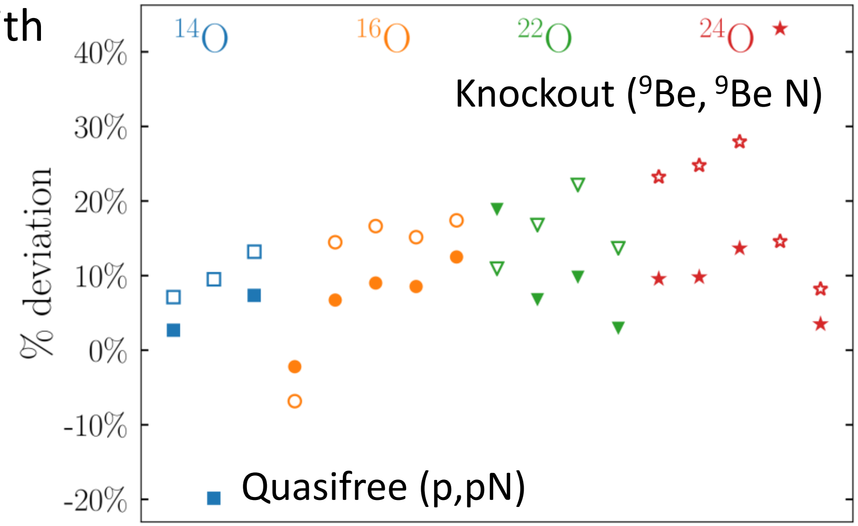




Optical Potential
Wood Saxon

Different reactions probe the correlated wavefunction

% Deviation of σ between analysis with Independent particle model vs. Manybody Green's function model



Bertulani, AI, Barbieri, Phys. Rev. C 104 (6), L061602 (2021)

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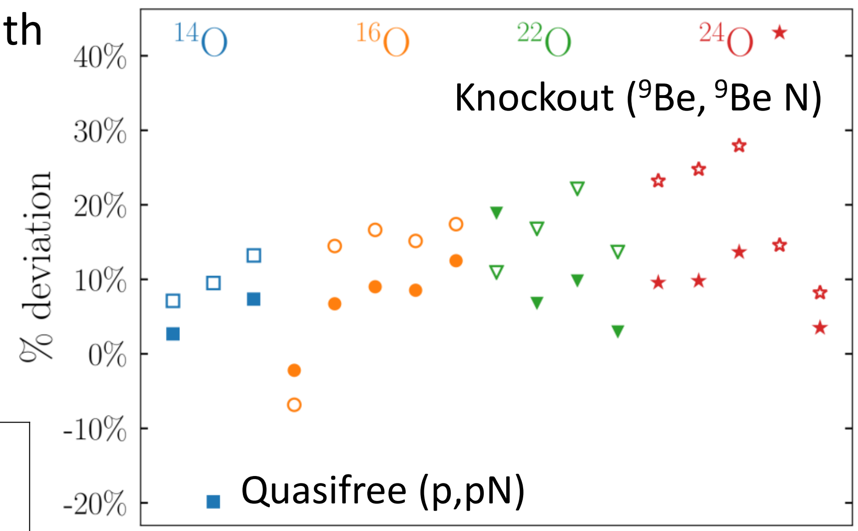
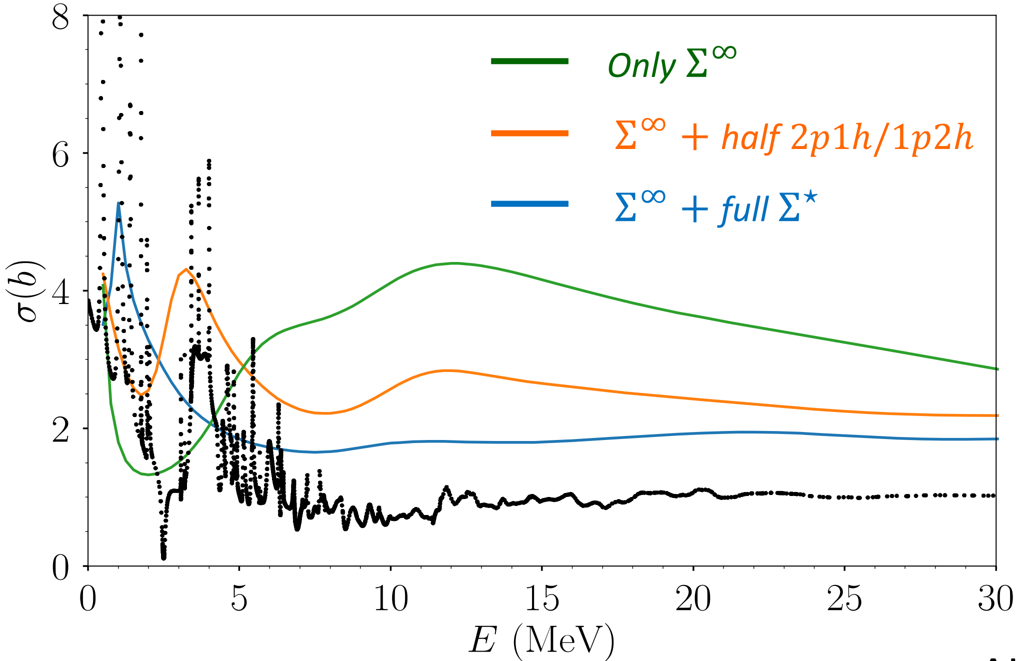


Neutron elastic scattering, ^{16}O+n



Many-body correlations reduce the calculated cross section

This calculation is still ~ twice the experiment

AI, Barbieri, Navratil, PRL 123, 092501 (2019)
Bertulani, AI, Barbieri, PRC 104, L061602 (2021)

Generator Coordinate Method

weight function

generating states

$$|\Psi\rangle = \int \mathrm{d}a\, f(a) |\Phi(a)\rangle$$

generator Coordinate

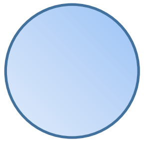
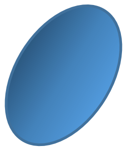
G. Carlsson's Talk Tuesday

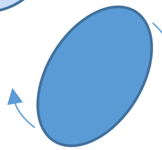
In our case

$$|\psi\rangle = \sum f(\cdot) |\Phi(\beta, \gamma, g_n, g_p, j_x)\rangle$$

Generating states as constrained HFB

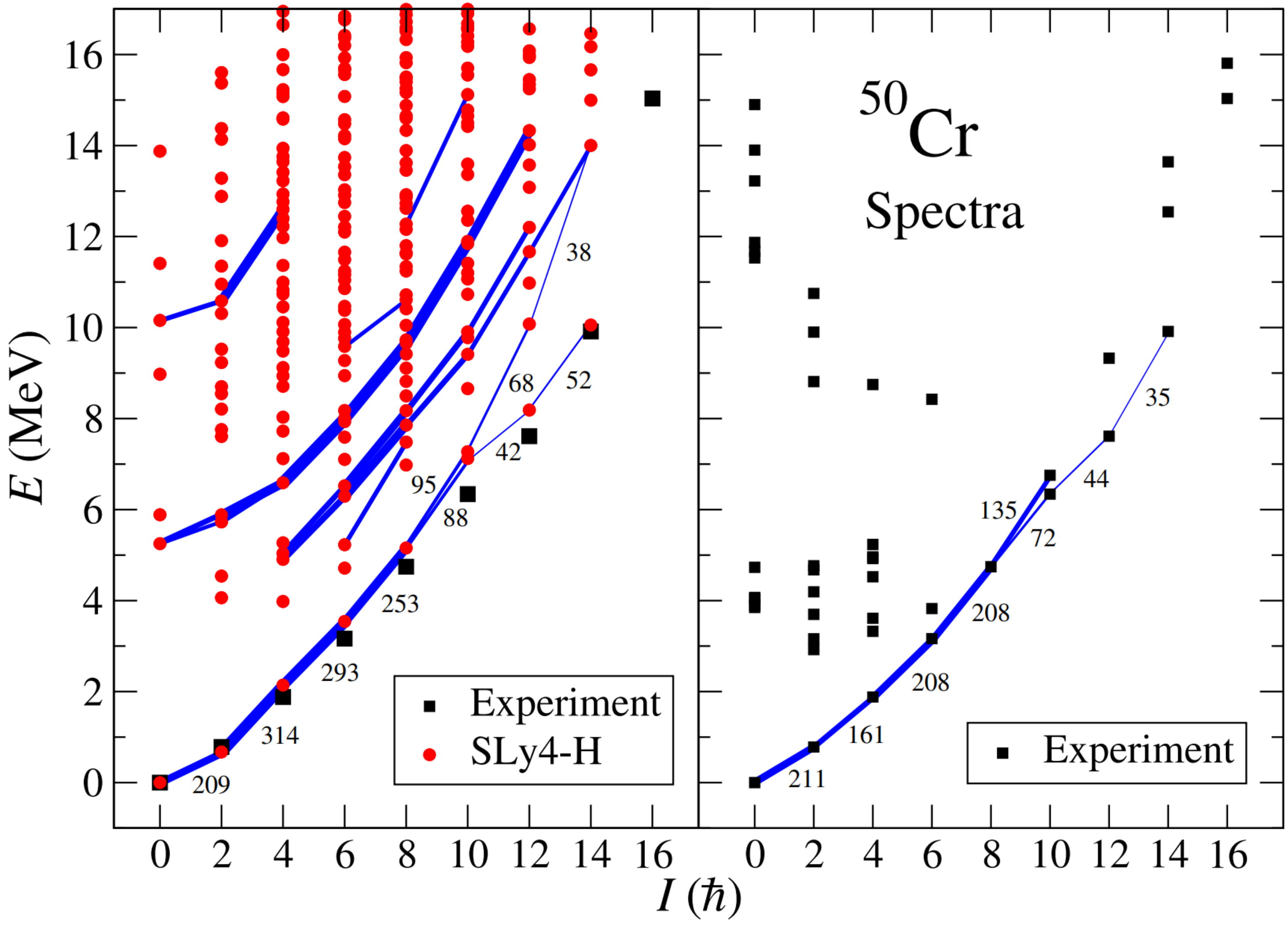
Project to good number of particles and angular momentum

$|\Psi\rangle = P^N P^Z P^I_{MK} (h_1 |\Phi($  $)\rangle + h_2 |\Phi($  $)\rangle$

$+ h_3 |\Phi($  $)\rangle + \ldots)$

It works!

G. Carlsson's Talk
Tuesday



Challenge 1:
Spectroscopic amplitude for projected states

$$\sigma^{+}_{J,i,\alpha} = \left\langle \Psi^{+\,I,M}_{\ \ i} \middle| a^{\dagger}_{\alpha} \middle| \Psi^{0,0}_{0} \right\rangle$$

$$= \sum_{a,a',K} \left(h^{I}_{ia,K}\right)^{*} h^{0}_{0a',0} \left\langle \phi_a \middle| \beta_\gamma \hat{P}^{A+1} \hat{P}^{I}_{KM} a^{\dagger}_{\alpha} \hat{P}^{0}_{00} \hat{P}^{A} \middle| \phi_{a'} \right\rangle .$$

Brute force: evaluate an operator 100k for each quasiparticle α times

Solution: Modify overlap formula from [Carlsson, Rotureau, PRL126, 172501 (2021)]

$$\langle \phi | \beta_\gamma \beta_\delta | \phi' \rangle = (-1)^{n/2}\,\mathrm{pf}\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^{\mathrm{T}} & \mathcal{C} \end{pmatrix} = (-1)^{n/2}\ \mathrm{pf}\left(\mathcal{A}\right) \mathrm{pf}\left(\mathcal{C} + \mathcal{B}^{\mathrm{T}} \mathcal{A}^{-1} \mathcal{B}\right)$$

An expensive part representing the even state, to evaluate once

A 2x2 matrix that depends on the blocked quasiparticles

Obtained a formula to calculate spectroscopic factors of Bogoliubov states efficiently.

Boström, Carlsson, AI arXiv:2604.0103 (2026)
Boström, AI, et al. J. Phys. Conf. 2586, 012080 (2023)

Challenge 2:
Not a complete set of states

$$\delta_{\alpha,\beta} = \left\langle \Psi_0 \right| \left\{ a_\alpha, a_\beta^\dagger \right\} \left| \Psi_0 \right\rangle = \sum_{i,s=\pm} {\sigma^s}^*_{J,i,\alpha} \sigma^s_{J,i,\beta}.$$

Sum rule 1: Sum of spectroscopic factors (probabilities) should be 1.
Problem: it isn't: they are separate $A, A \pm 1$ calculations. We would need the whole Fock space!

Solution: Derive new sum rule

$$\left\langle \Psi_0 \right| \left\{ a_\alpha, \left[\hat{H}, a_\beta^\dagger \right] \right\} \left| \Psi_0 \right\rangle = \sum_i {\sigma^+}^*_{J,i,\alpha} ({E^+}^J_i - E_0) \sigma^+_{J,i,\beta} - \sum_i {\sigma^-}^*_{J,i,\alpha} ({E^-}^J_i - E_0) \sigma^-_{J,i,\beta},$$

many-body single-particle field (generalized mean field)

Use it to "complete" the spectroscopic states on the basis of the generalized mean field

$$\sigma^\dagger \sigma + c^\dagger c = \mathrm{I}$$

$$\sigma^\dagger \epsilon \sigma + c^\dagger \epsilon_c c = H_A.$$

Boström, Carlsson, AI arXiv:2604.0103 (2026)
Boström, AI, et al. J. Phys. Conf. 2586, 012080 (2023)

Construct the optical potential

$$G^{J\pi}_{\alpha,\beta}(E) = \lim_{\eta\to 0^+} \sum_{i,s=\pm} \frac{{\sigma^{s*}}_{J,i,\alpha}{\sigma^{s}}_{J,i,\beta}}{E - (\bar{E}^s)^{J\pi}_i(\eta)}$$

We have spectroscopic amplitudes and energies for A, $A \pm 1$ states: ingredients of the Green's function

We know how a neutron particle (hole) moves in the correlated A particle nucleus,
we can construct the potential it feels.
Inversion of Dyson Equation:

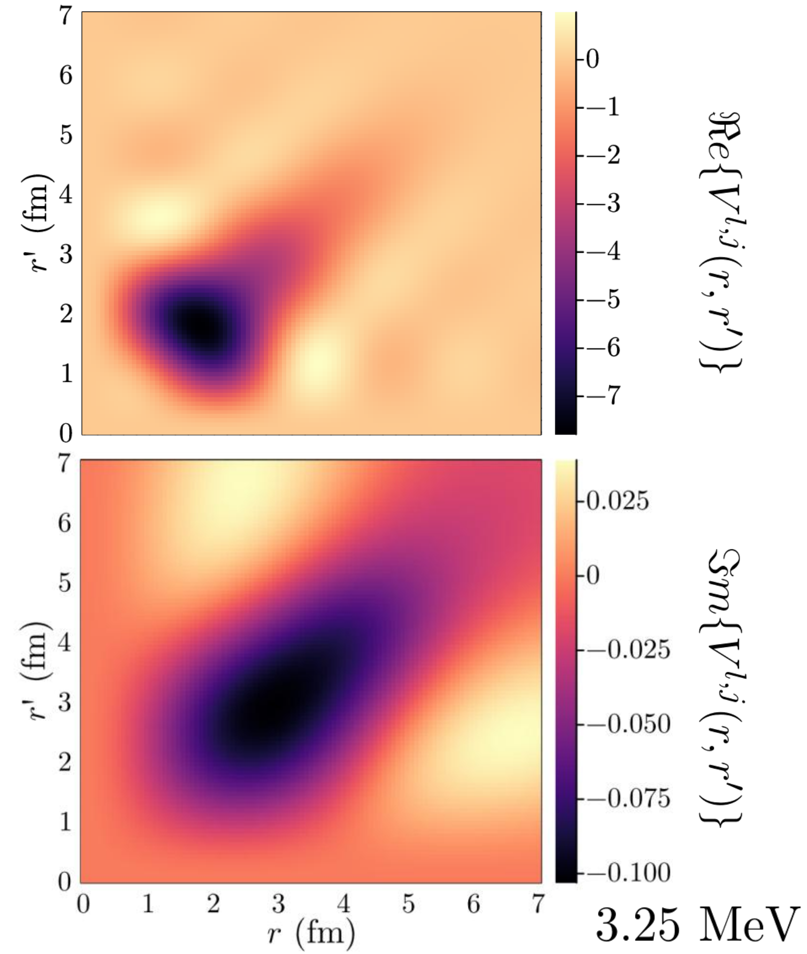
$$\Sigma = G_0^{-1} - G^{-1}$$

We get the Optical Potential:

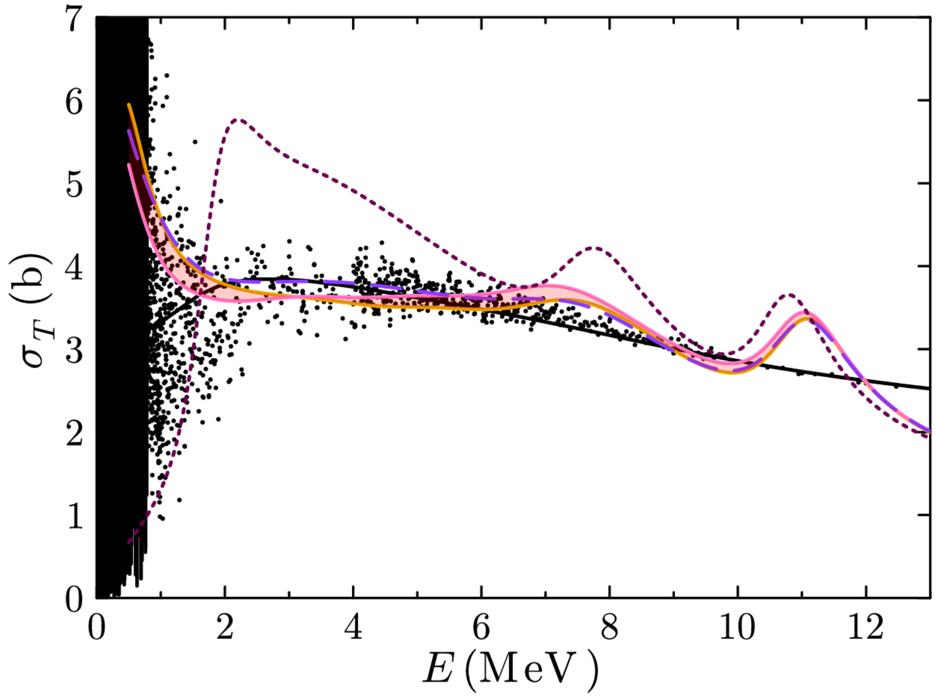
$$V^{l,j}(r,r';E) = V_0 + \Sigma^{l,j}(r,r';E)$$

Nonlocal, partial wave, energy dependent optical potential obtained directly from Hamiltonian solutions for deformed nuclei.
<u>No tuning or data from the isotope</u>

Boström, Carlsson, AI arXiv:2604.0103 (2026)



Results: n + ^{50}Cr



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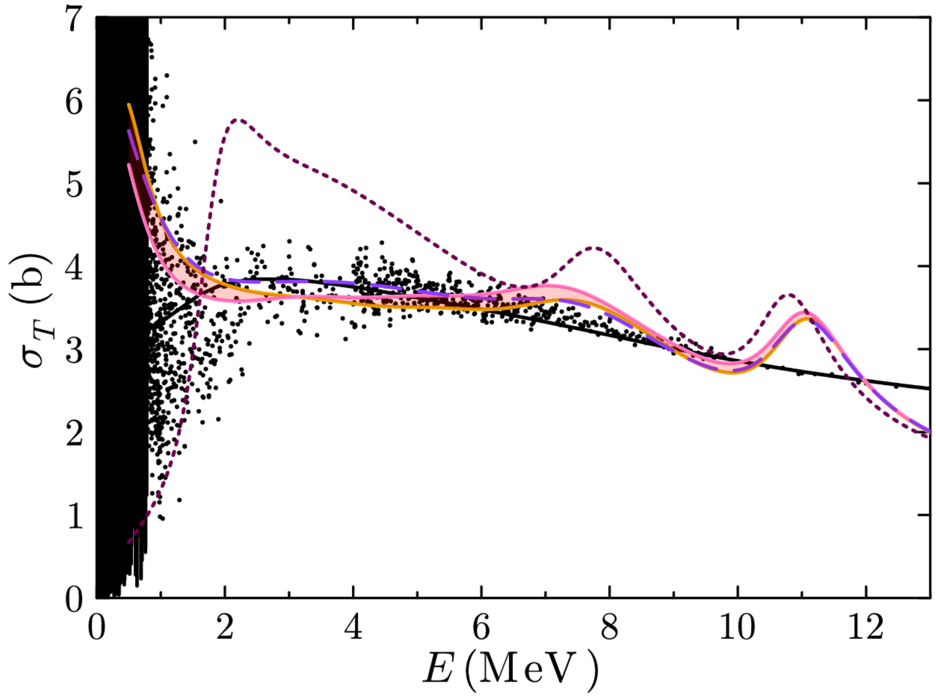
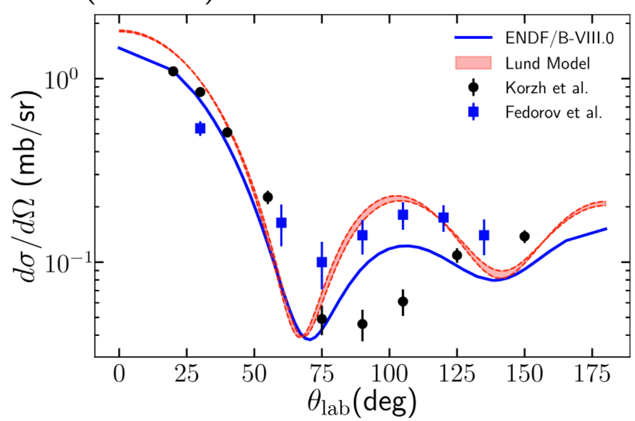
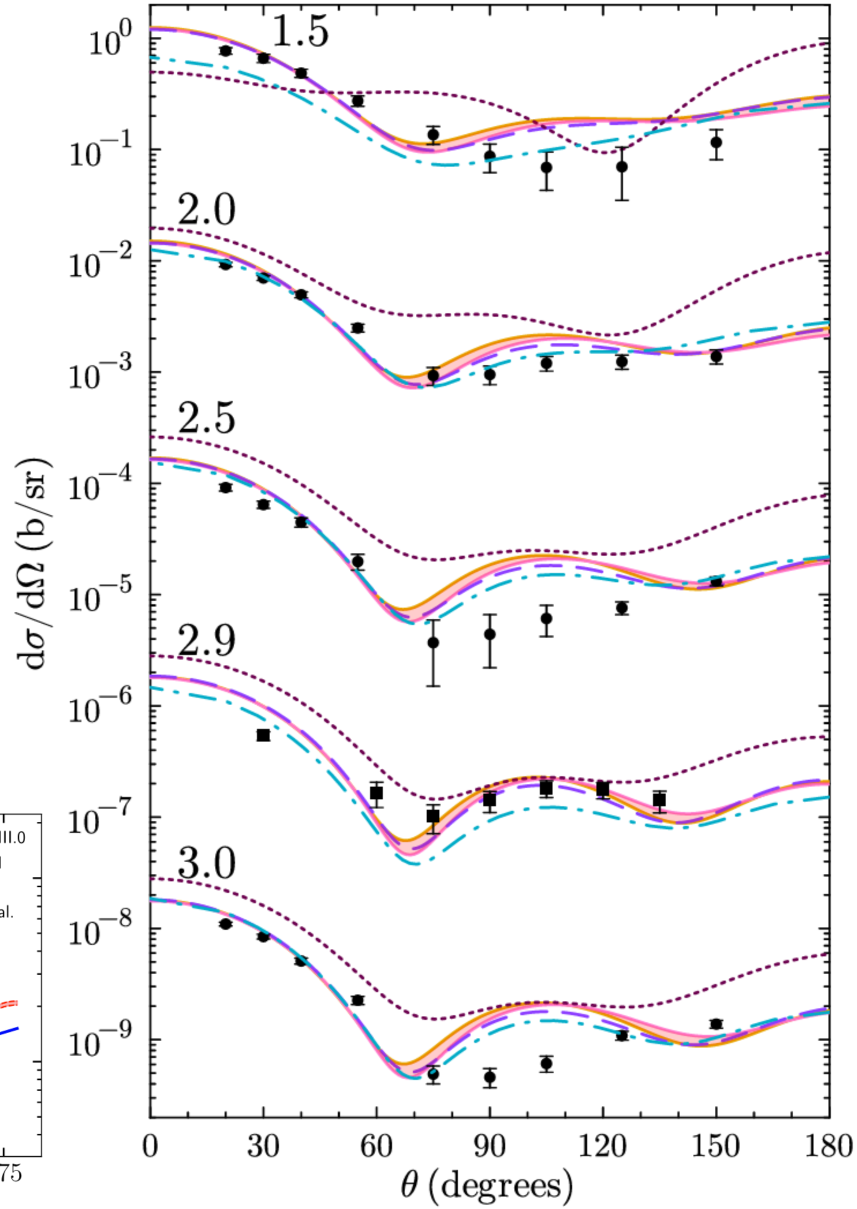


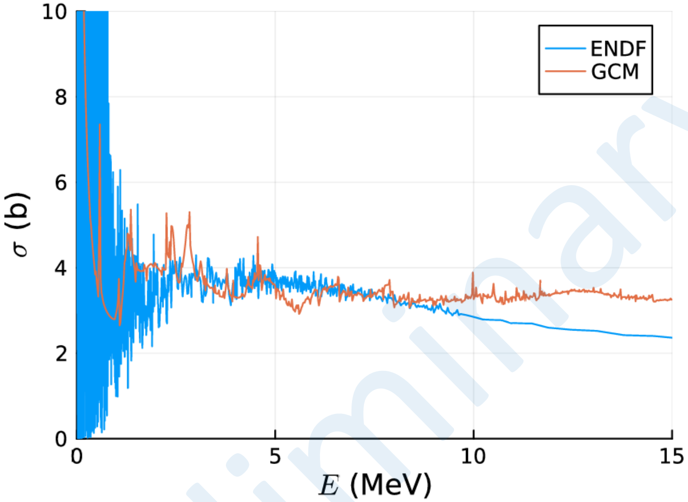




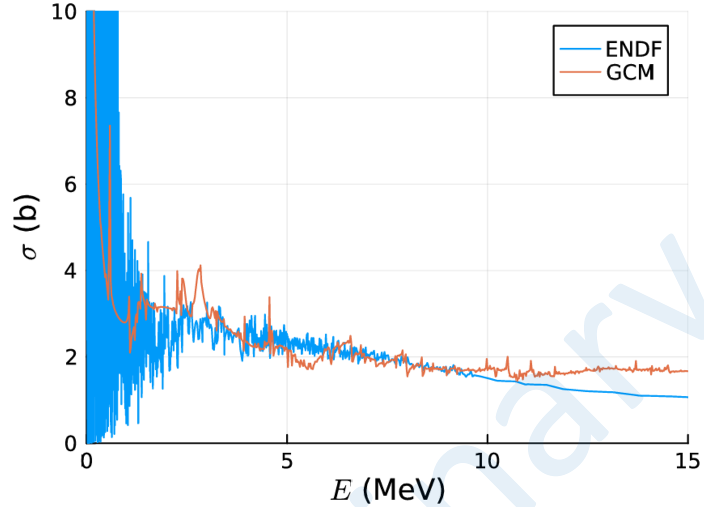
Boström, Carlsson, AI arXiv:2604.0103 (2026)

Preliminary results: n + ^{50}Cr with J-matrix method

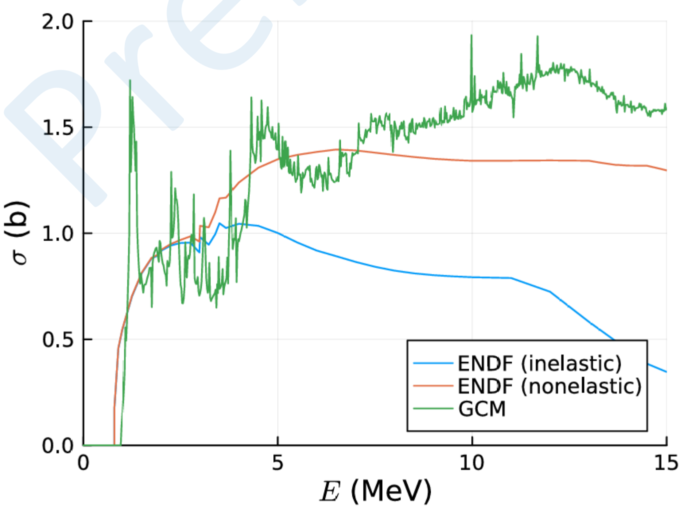
total



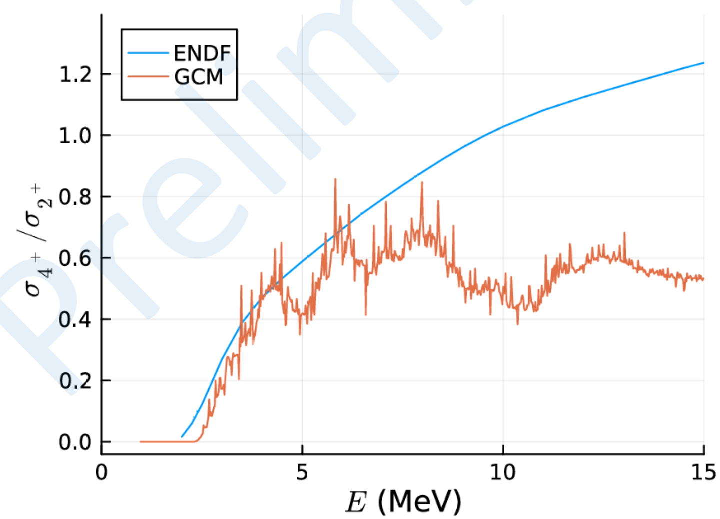
elastic



inelastic

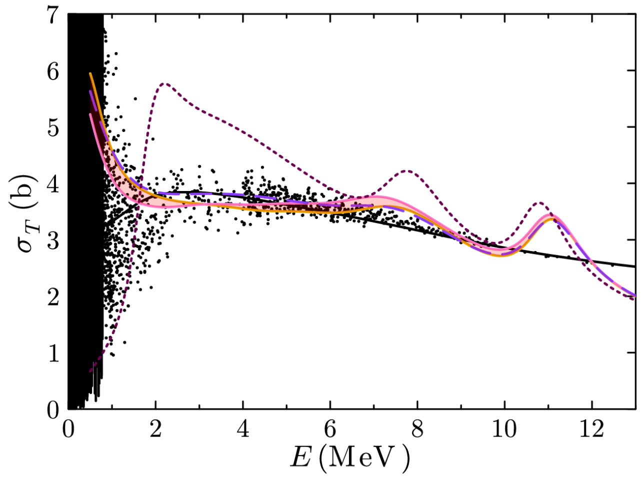


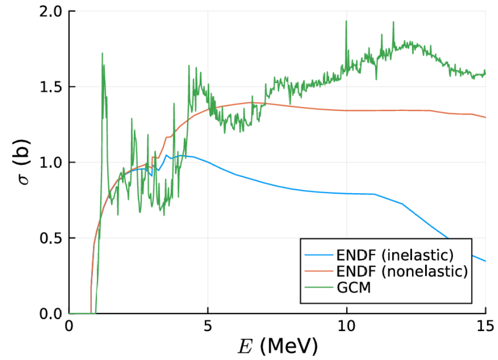
Ratio inelastic to 2+ and 4+

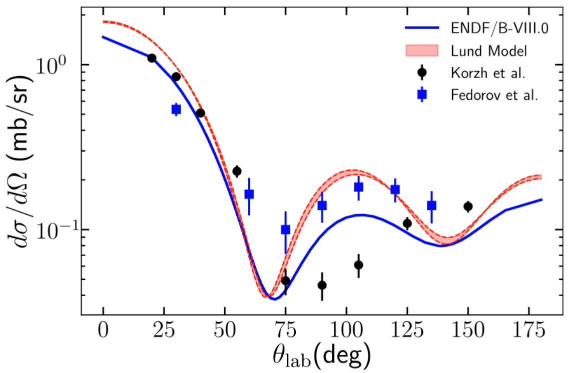


Conclusions & Outlook

- GCM can describe low energy states with a great degree of collectivity within a consistent framework.
- This was used to construct the first optical potential based on exploiting symmetry breaking, without any tuning, with exceptional results.
- Working towards:
  - Generalizing the effective Hamiltonian
  - Systematic studies: uncertainty analysis and isotopic chains.
  - Additional reactions

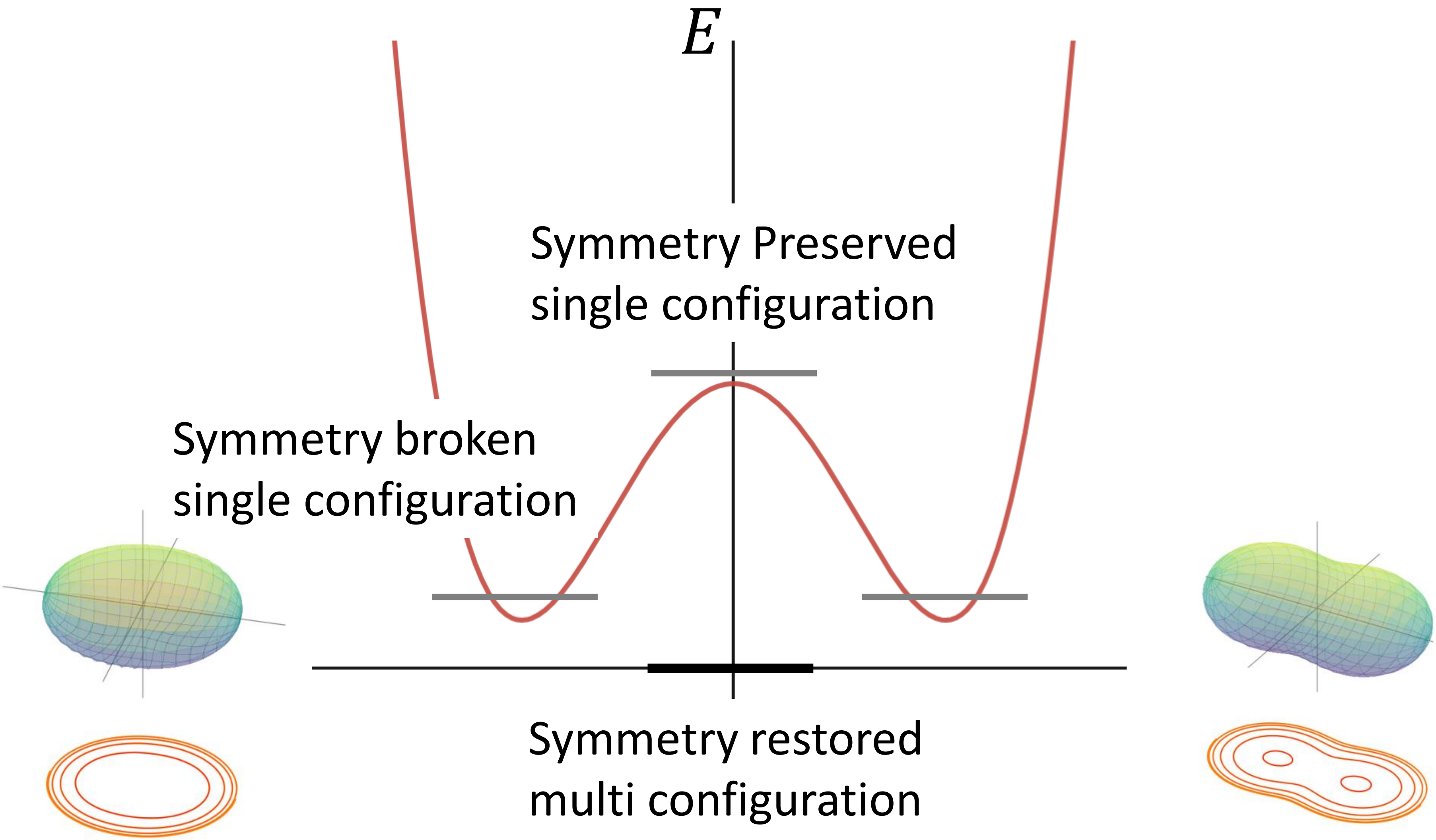






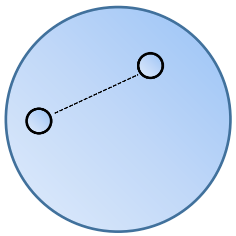
[Ljungberg et al., Phys. Rev. C 106, 014314 (2022)]()
[Boström, Rotureau, Carlsson, AI, PRC 112, L051602 (2025)]()
[Boström, Carlsson, AI arXiv:2604.0103 (2026)]()

Symmetry Breaking



Nuclear Correlations

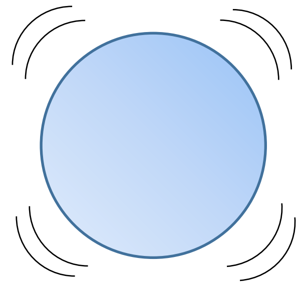
Pairing



Bogoliubov basis

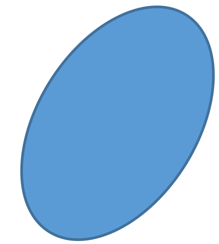
Particle number symmetry breaking

Vibrations



RPA

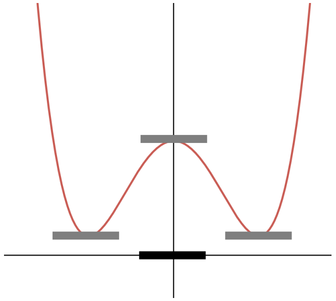
Rotations



Constrained variation

Dynamic and static deformation symmetry breaking

Projection to restore “good” symmetries of the Hamiltonian

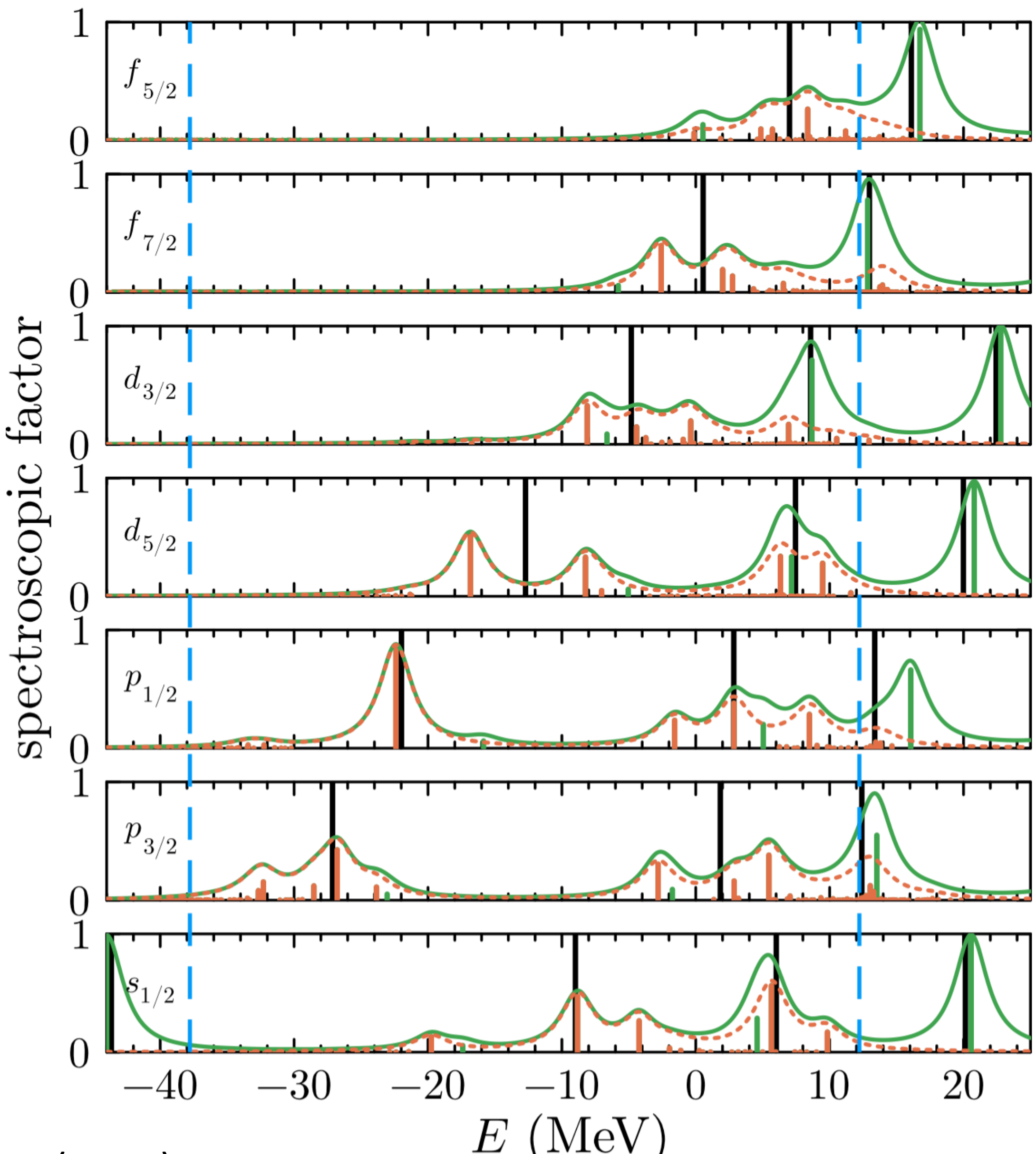


Challenge 2:
Not a complete set

— H_A

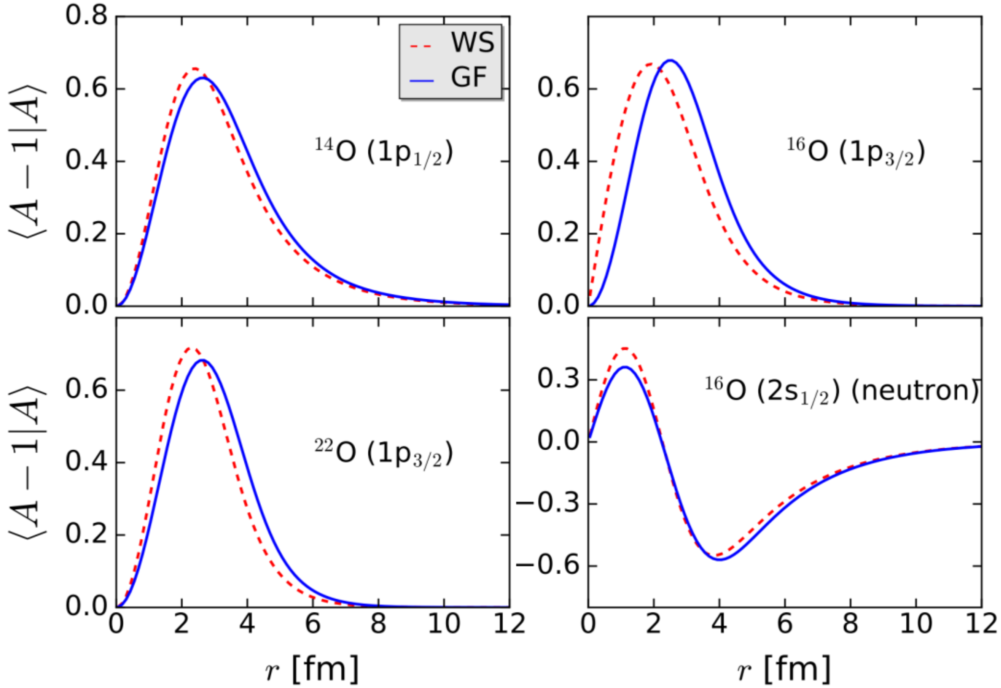
— GCM

— c (+GCM)

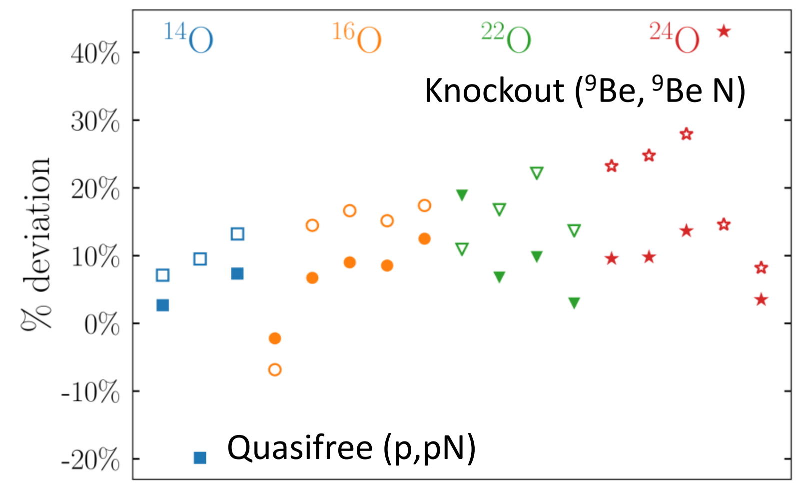


GCM is covering a good range of the Fock space reproducing most low-energy single particle excitations

Boström, Carlsson, AI arXiv:2604.0103 (2026)

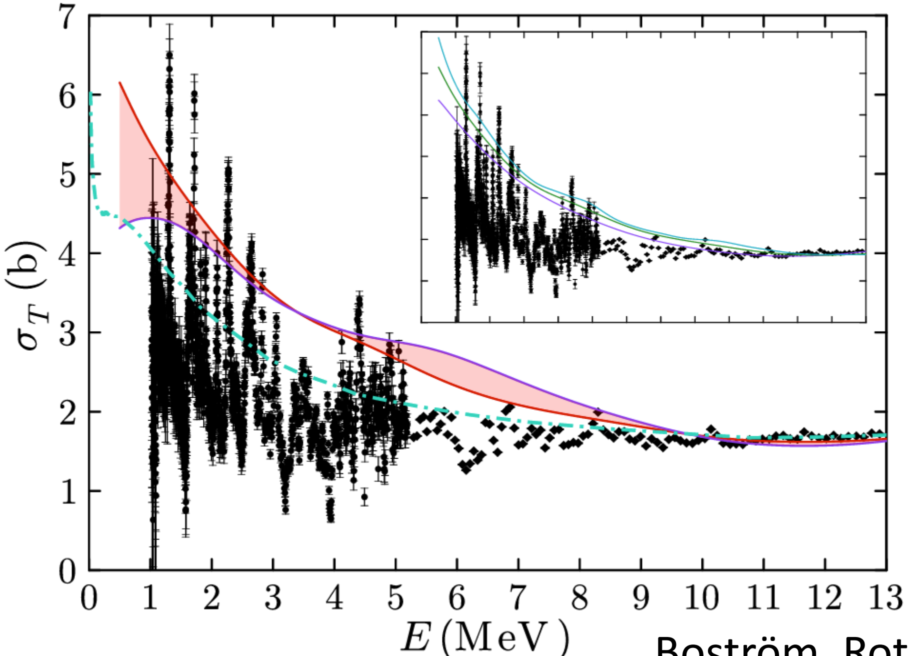


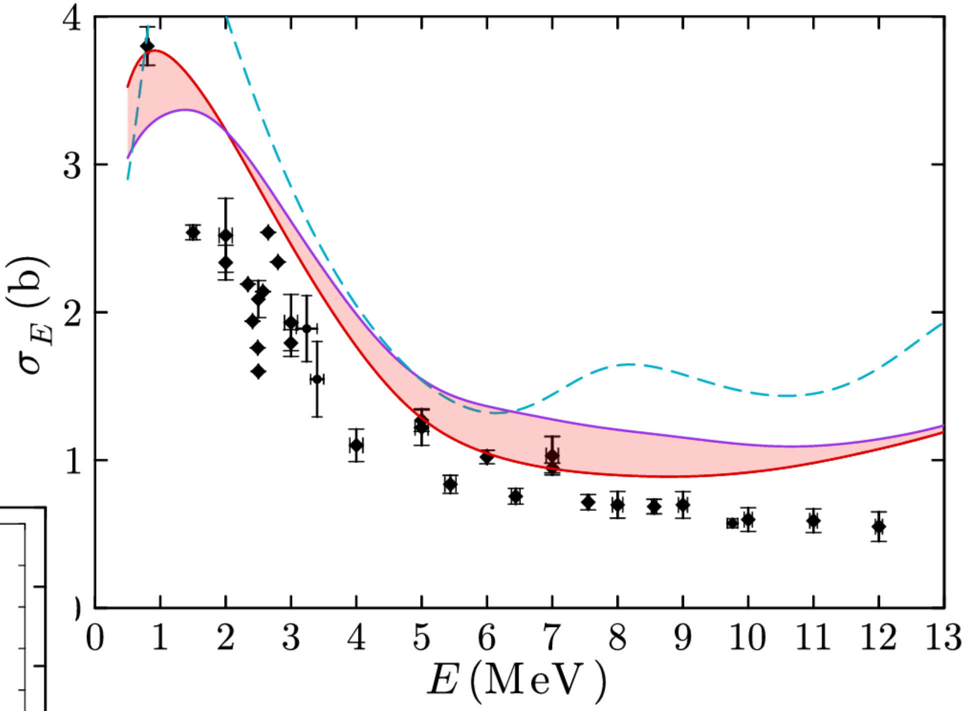
Overlap function for
Correlated or independent particle



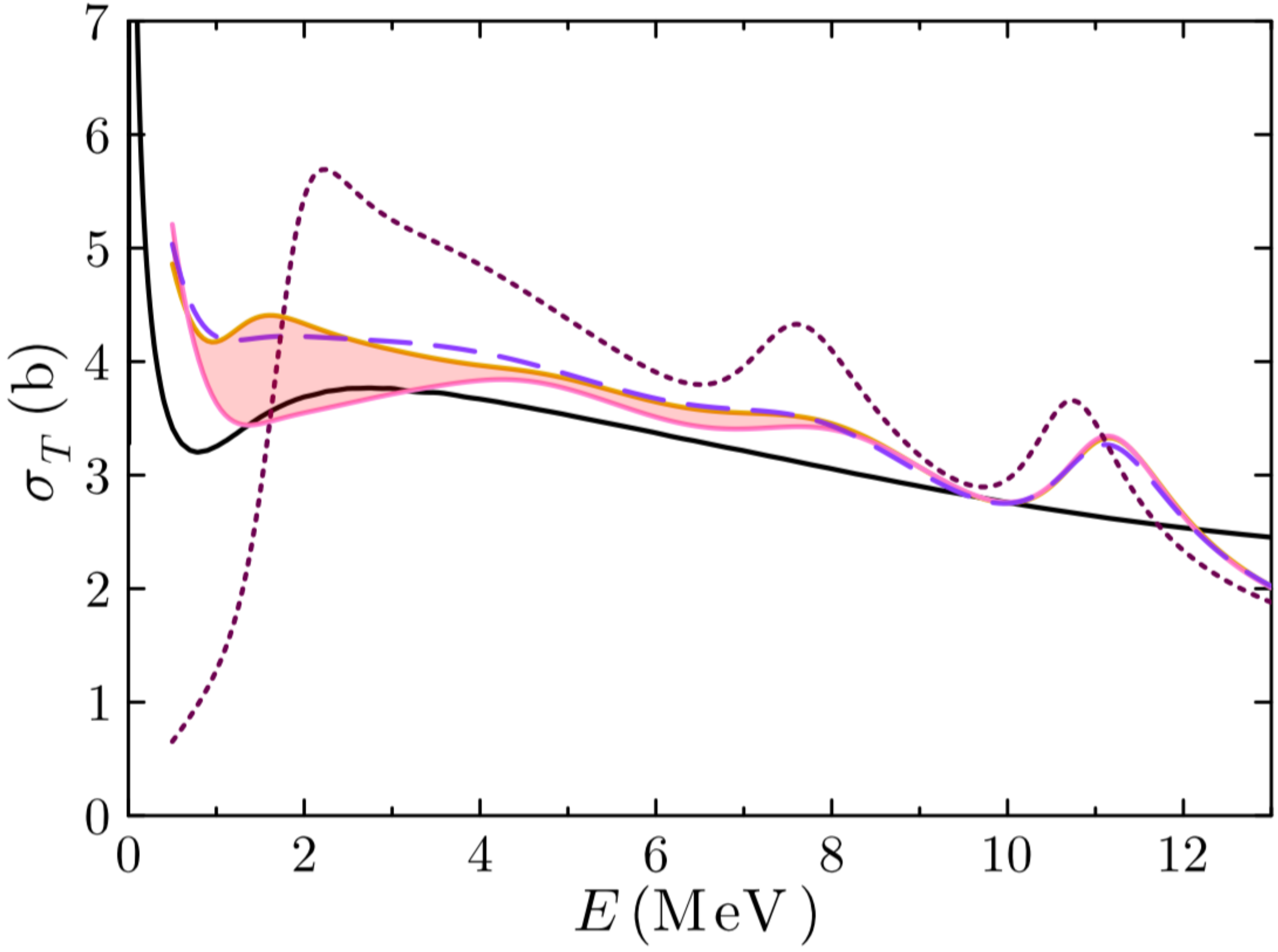
$$\sigma = S(lj)\frac{2\pi}{2j+1}\sum_m \left\langle \frac{d\sigma_{pN}}{d\Omega}\right\rangle_{o.s.} |\mathcal{C}_{lm}|^2$$

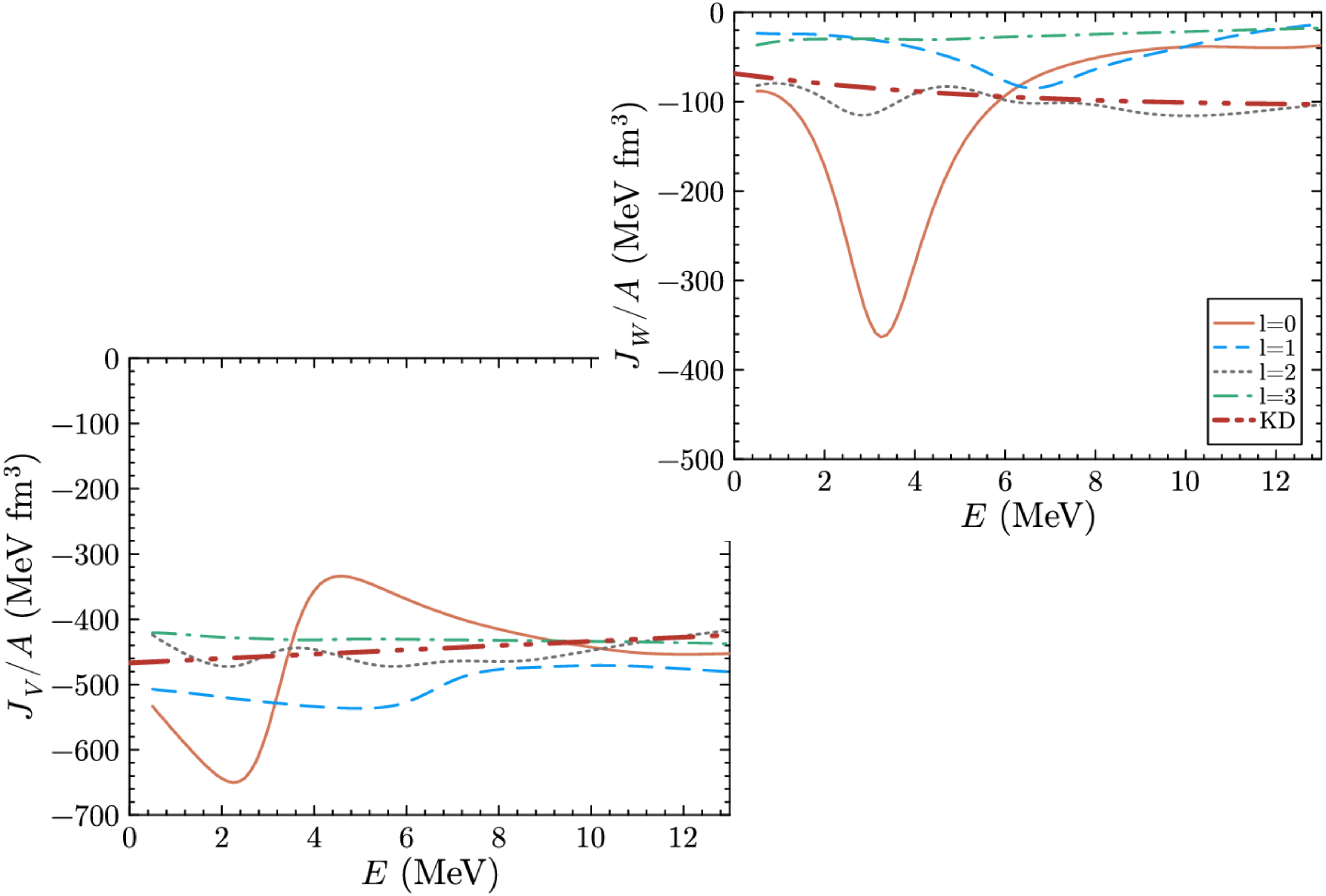
$$\times \int_0^\infty db\, b |\langle \mathcal{S}(b)\rangle_{o.s.}|^2 \int_{-\infty}^{\infty} dz \left| \frac{u_{lj}(r)}{r} P_{lm}(b,z)\right|^2 .$$





Boström, Rotureau, Carlsson, AI, PRC 112, L051602 (2025)


7
6
5
4
3
2
1
0
σ_T (b)
0
2
4
6
8
10
12
E (MeV)


0
−100
−200
−300
−400
−500
J_W/A (MeV fm^3)
0
2
4
6
8
10
12
E (MeV)
l=0
l=1
l=2
l=3
KD
0
−100
−200
−300
−400
−500
−600
−700
J_V/A (MeV fm^3)
0
2
4
6
8
10
12
E (MeV)