

Dark Matter–Induced UV Radiation and the Formation of Supermassive Black Hole Seeds

McDonald Institute 2026 Annual Community Meeting

Han Wu, Aaron Vincent, Sarah Schoen

July 30, 2026

Outline

- **Background**

Observations, Collapse in Λ CDM Model, Direct Collapse Black Hole (DCBH)

- **Modeling**

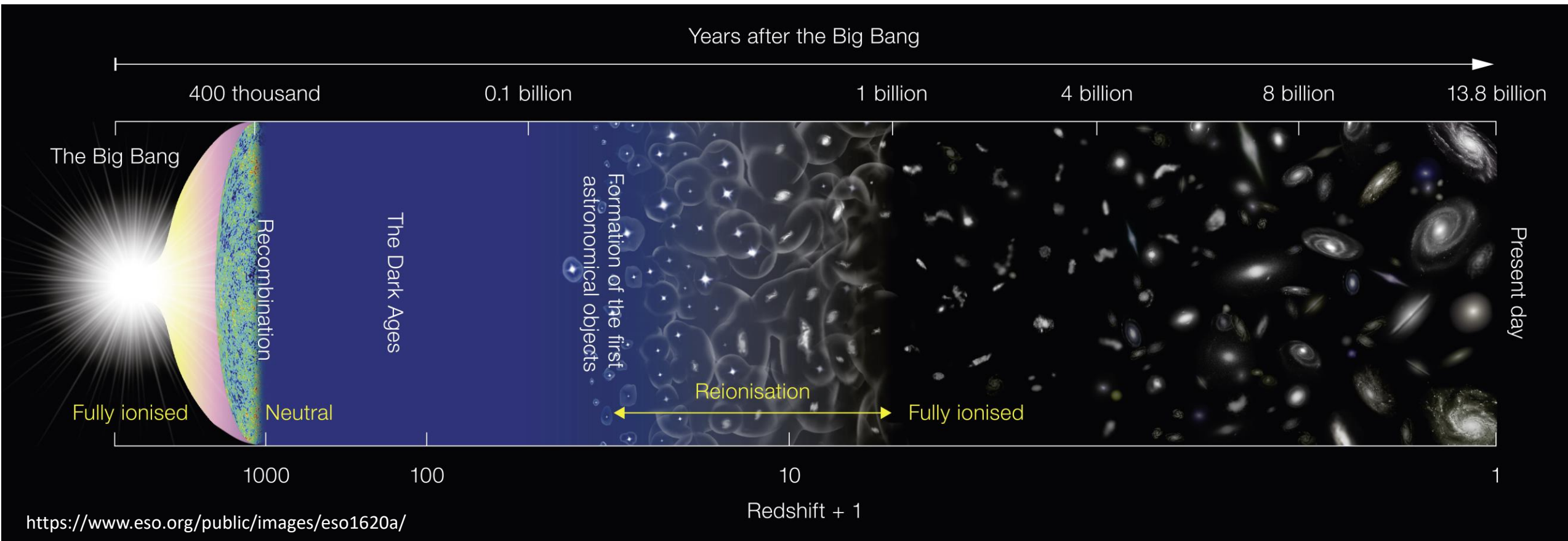
DM Halo, Gas Cloud and DM Decay/ Annihilation, Simulation and Criteria

- **Results**

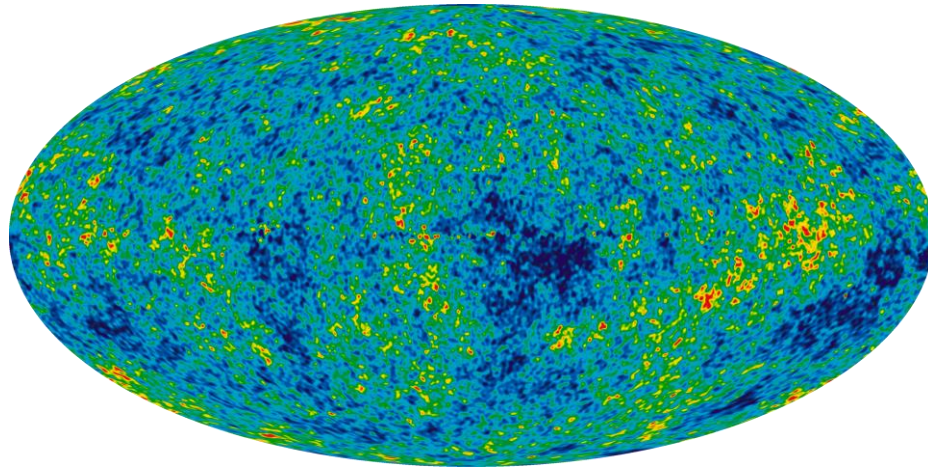
Thermal History, Phase Diagrams

- **Takeaway Information**

Timeline of the Evolution of the Universe



Structure Collapse in Λ CDM Model

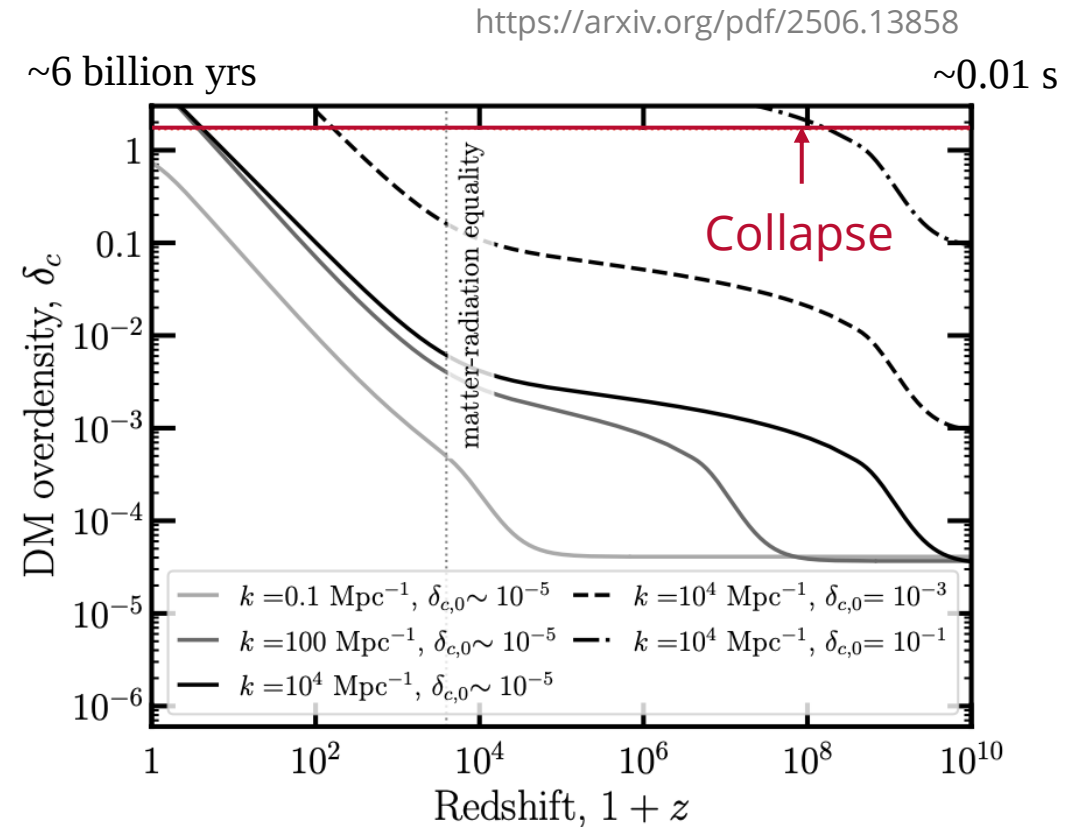


<https://science.nasa.gov/mission/wmap/wmap-overview/>

Initial Density Fluctuation $\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \sim 10^{-5}$

Λ CDM Model:

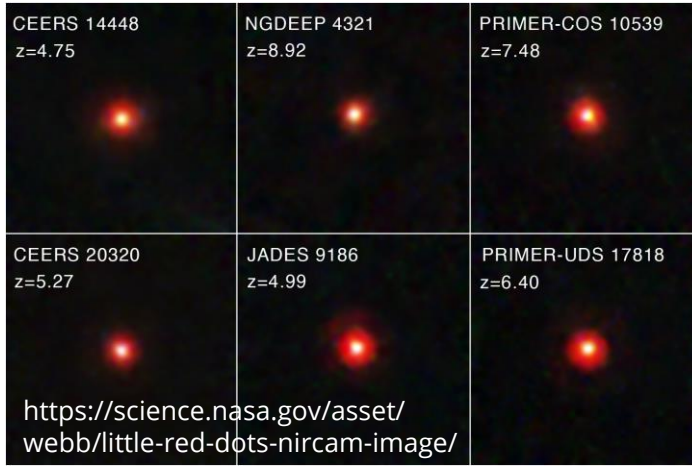
- the large-scale structure in the distribution of galaxies
- the accelerating expansion of the universe observed in the light from distant galaxies and supernovae
- ...



$\delta \gg 1$

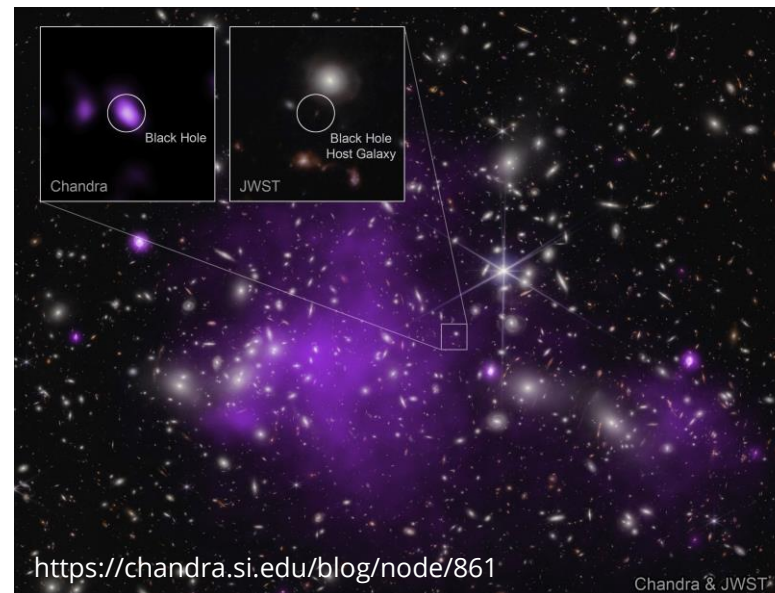
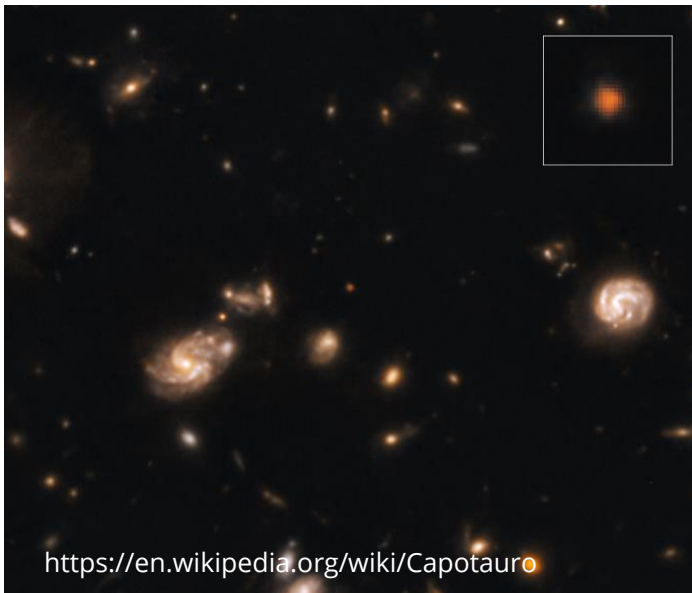
The Hubble Heritage Team (AURA/STScI/NASA)

Observations



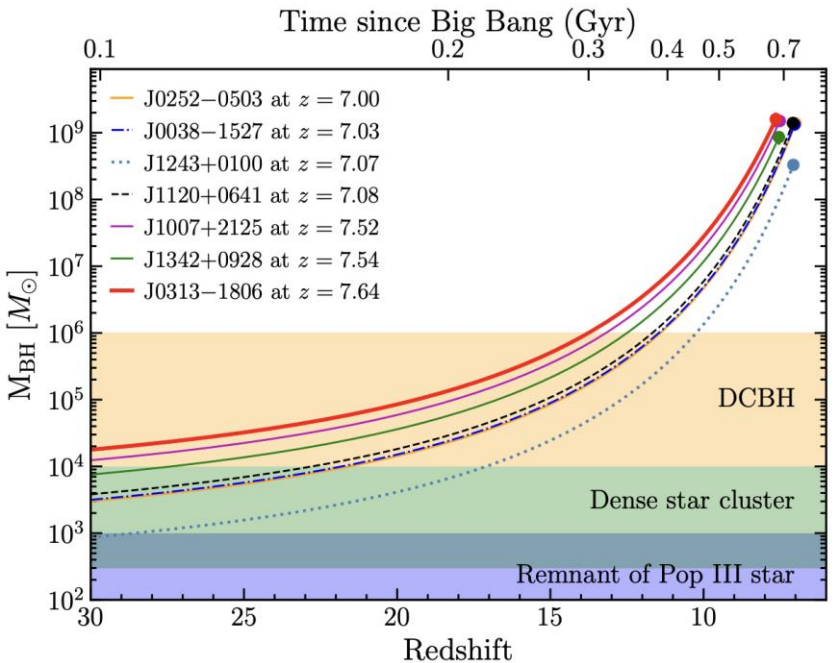
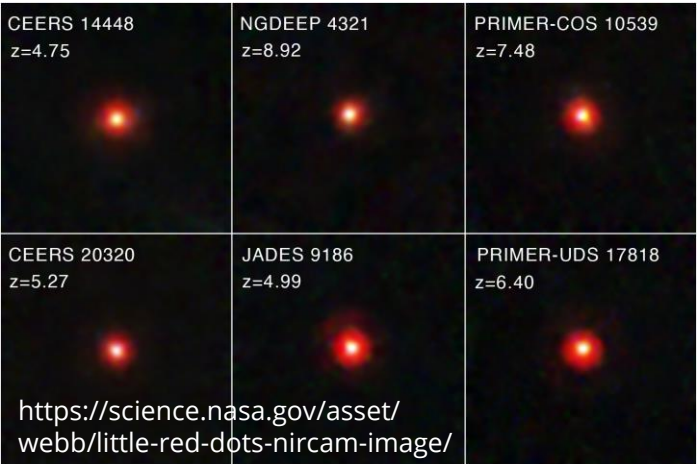
Little red dots (LRDs)

- The original reports identified LRDs as a type of early **active galactic nucleus (AGN)**
- The current leading theory is that the LRDs are a form of **primordial galaxy**
- Early galaxies as result of very slowly spinning dark matter halos
- ...

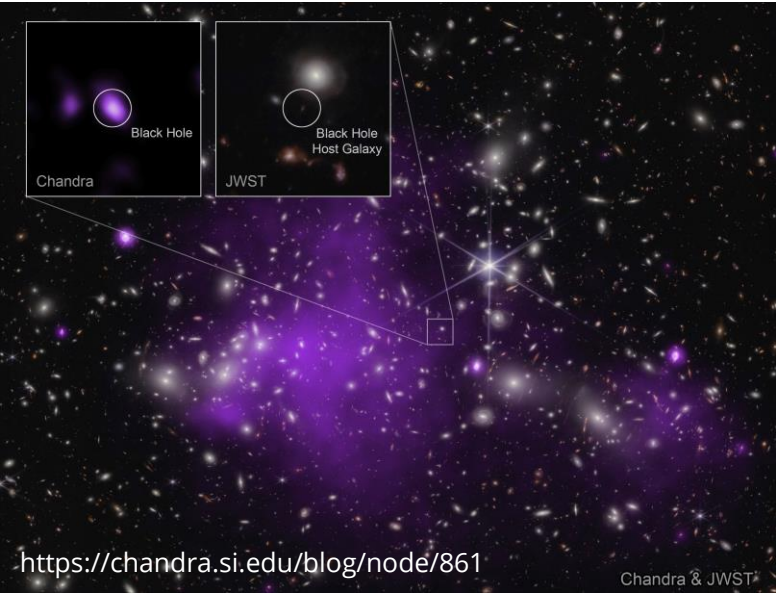
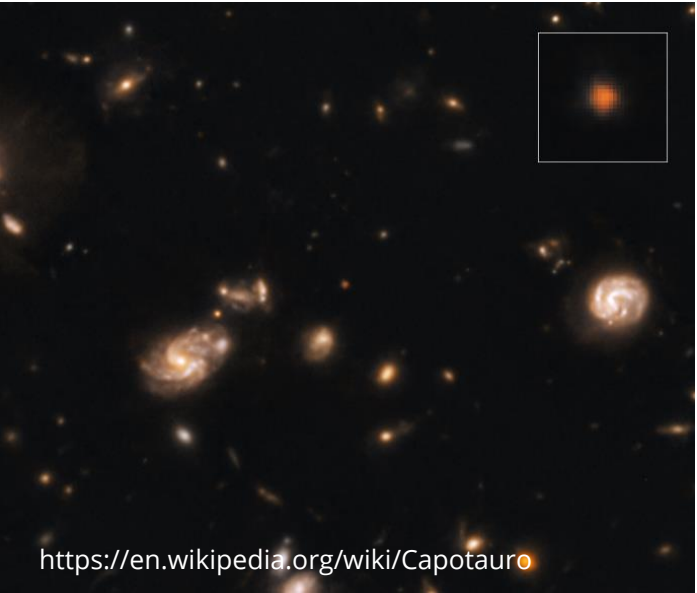


Type	Redshift
Little red dots	$4.75 < z < 8.92$
Galaxy candidates	$z > 10$
Accreting black holes	$z > 17$

Observations

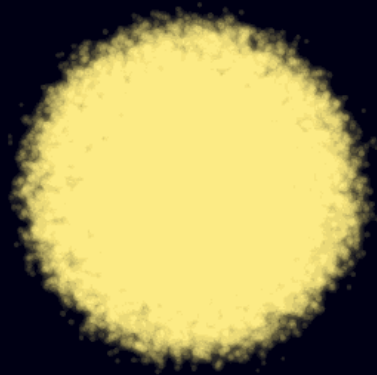


<https://arxiv.org/pdf/2101.03179>



Type	Redshift
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Direct Collapse Scenario

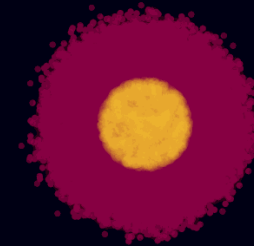


Rapid Cooling



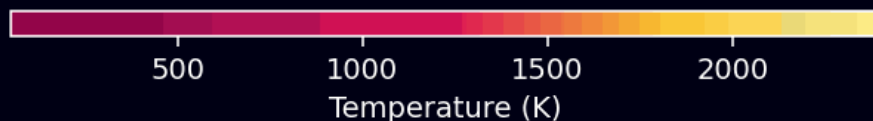
**Star Formation
(Standard Scenario)**

Moderate Cooling



SMBH Formation

Therefore, a moderate cooling rate is required to form a SMBH in the direct collapse black hole scenario.



Let's slow down the cooling

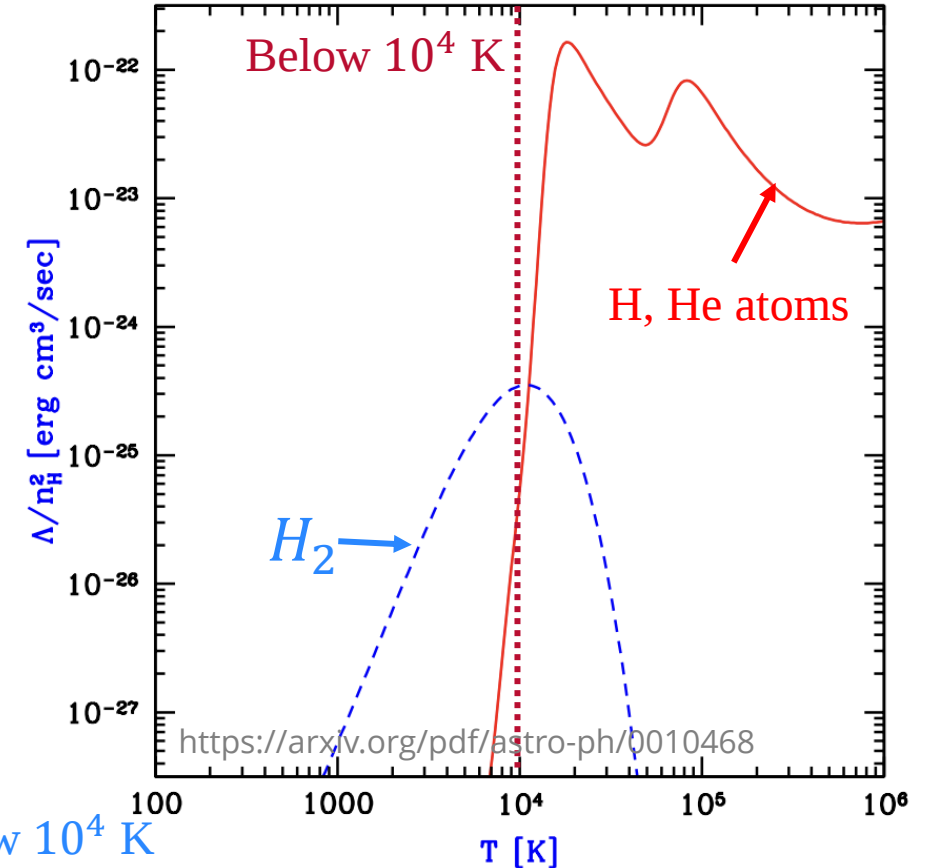
Cooling Channels:

Above 10^4 K

- Collisional excitation cooling ($n_e n_H, n_e^2 n_{He+}, n_e n_{He+}$)
- Collisional ionization cooling ($n_e n_H, n_e n_{He}, n_e n_{He+}, n_e^2 n_{He+}$)
- Recombination cooling ($n_e n_{H+}, n_e n_{He+}, n_e n_{He++}$)
- Bremsstrahlung cooling (All ionized species)
- Photoionization heating (n_H, n_{He}, n_{He+})

- H_2 cooling (Collisions to H, H_2, He, H^+, e^-)
- H_2 radiative cooling (formation or destruction of molecular hydrogen)

- Compton cooling/heating off the CMB



Below 10^4 K



We can slow down the cooling by suppressing the formation of H_2 .

$$M_{gas\ cloud} \sim 10^6 - 10^7 M_{\odot}$$

$$T_{vir} \sim 1000 - 6000\ K$$

Overview of the Model

Cosmic Background



H_2 Formation:



CMB

DM Halo

One zone Model + NFW Profile

Gas Cloud

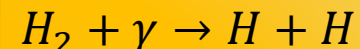
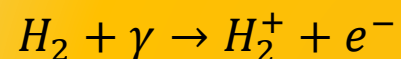
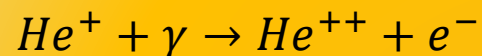
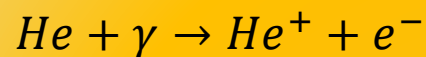
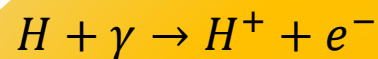
One-zone Model

Photodissociation
(11.2-13.6 eV, LW Band)

Photodetachment
(0.76-13.6 eV)



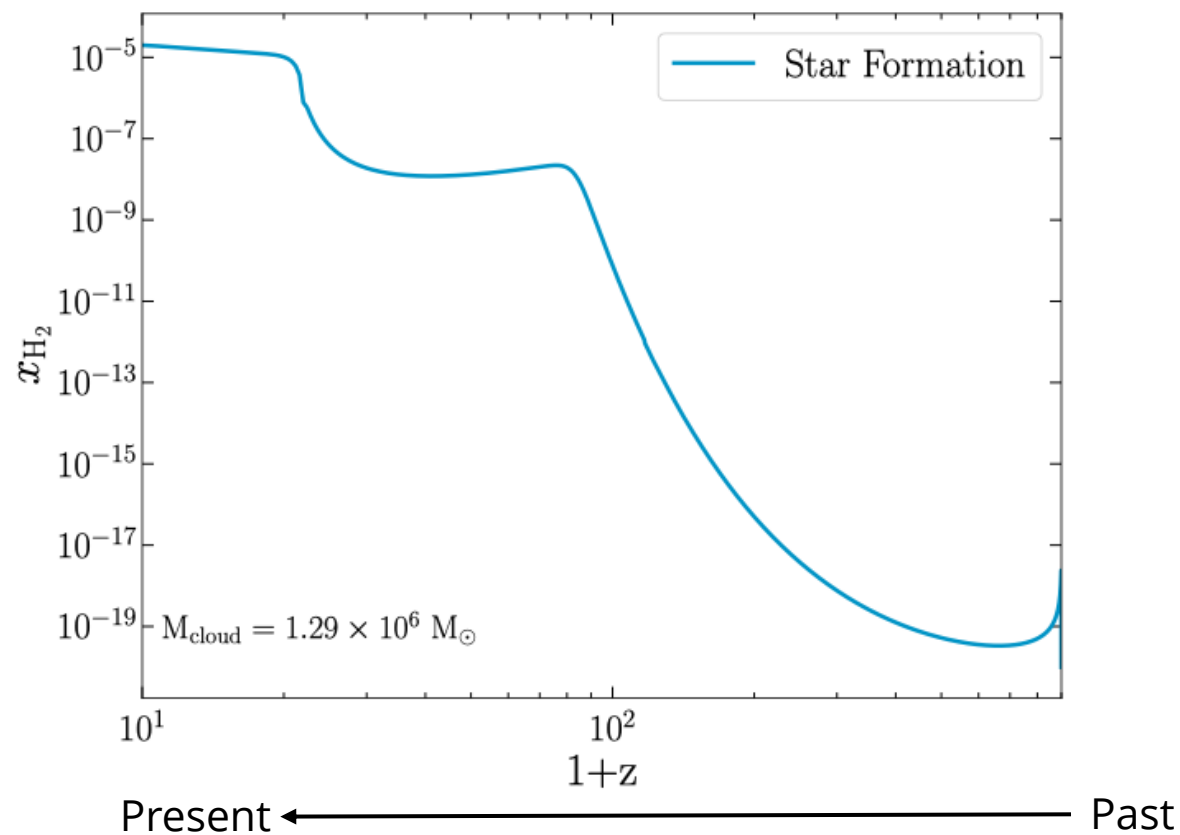
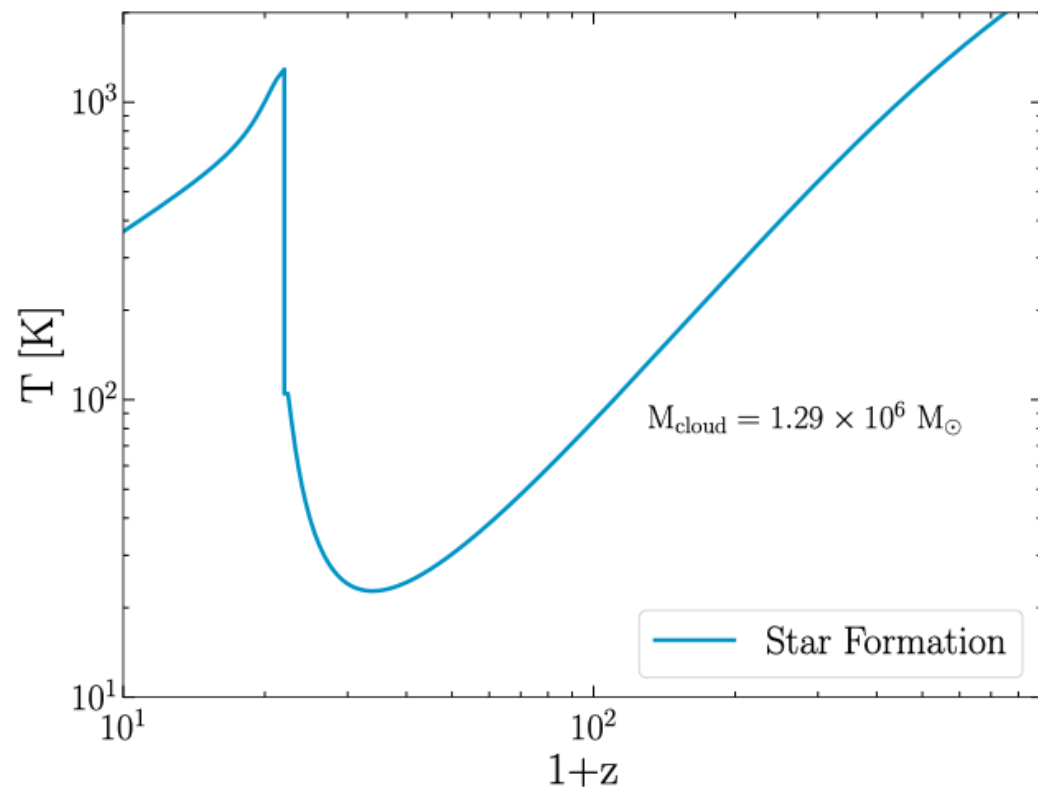
Self-Shielding



...

Chemical Network and Cooling Library: Grackle

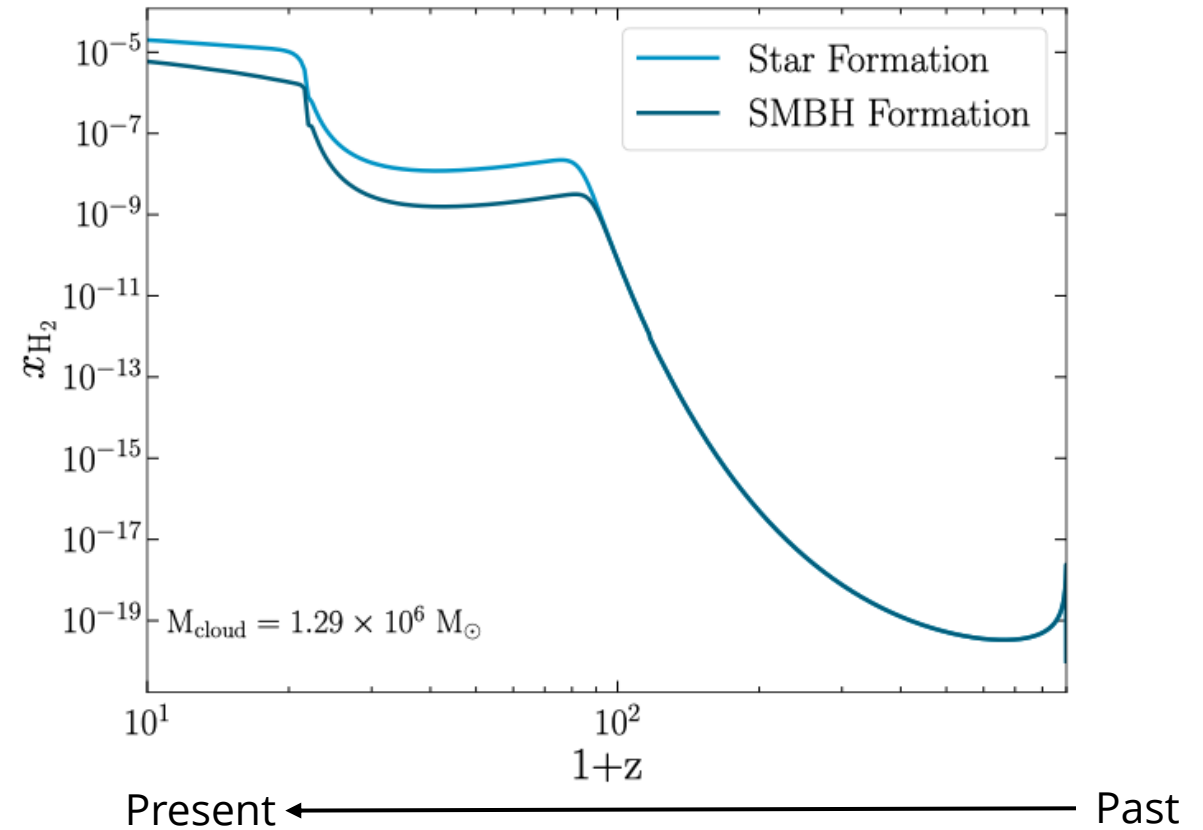
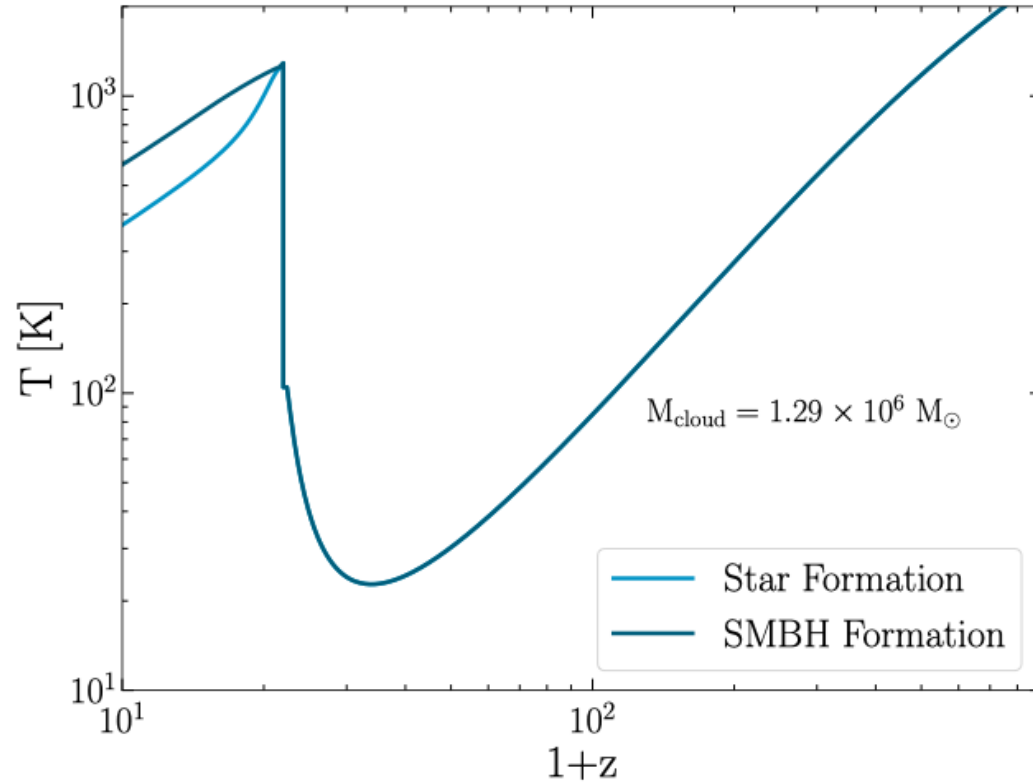
Results: Thermal History



DM Parameter:

- Star Formation: $m_{\chi} = 47.1 \text{ eV}, \tau = 7.74 \times 10^{30} \text{ s}$

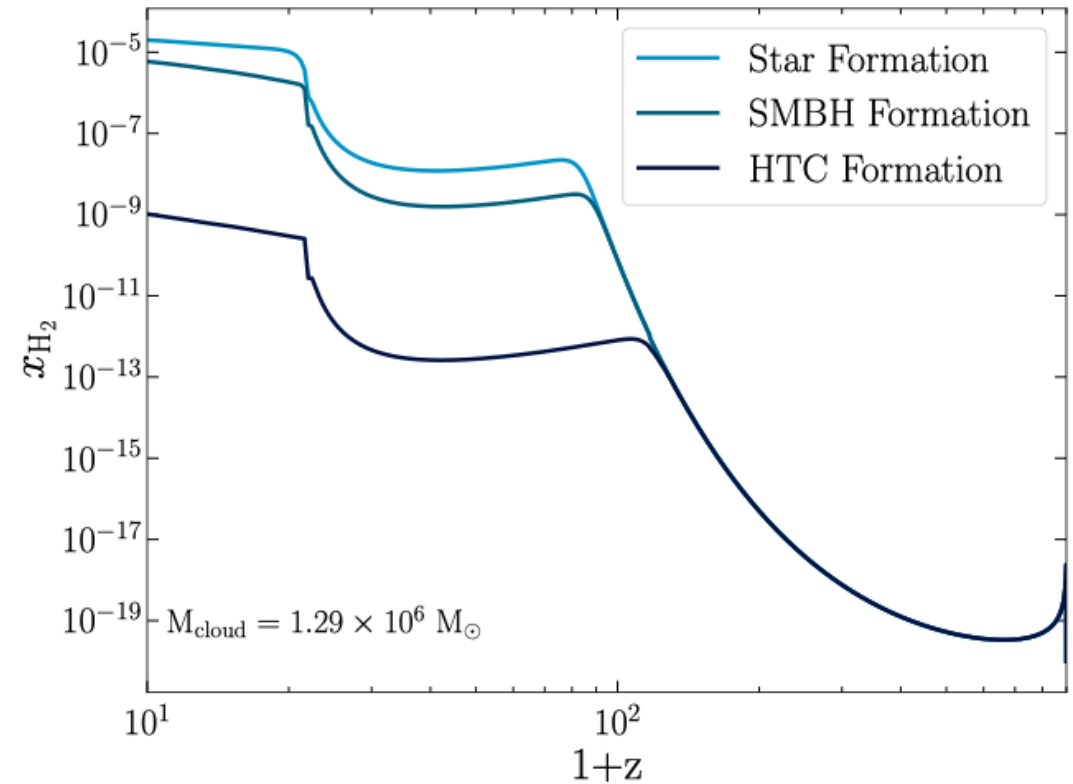
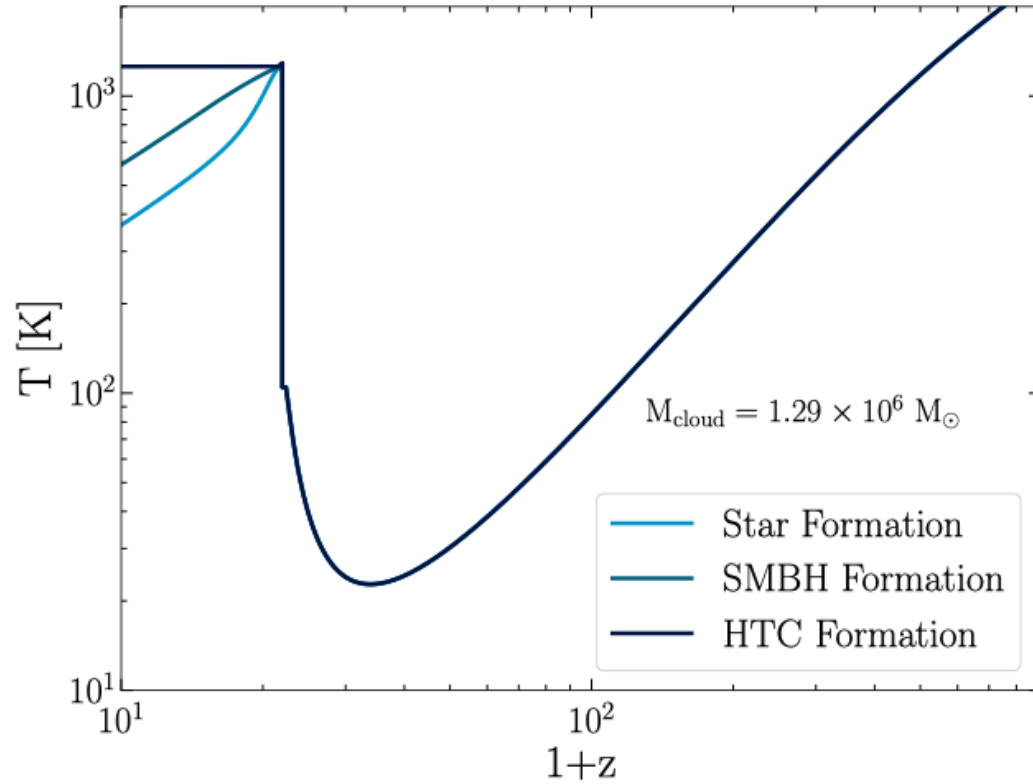
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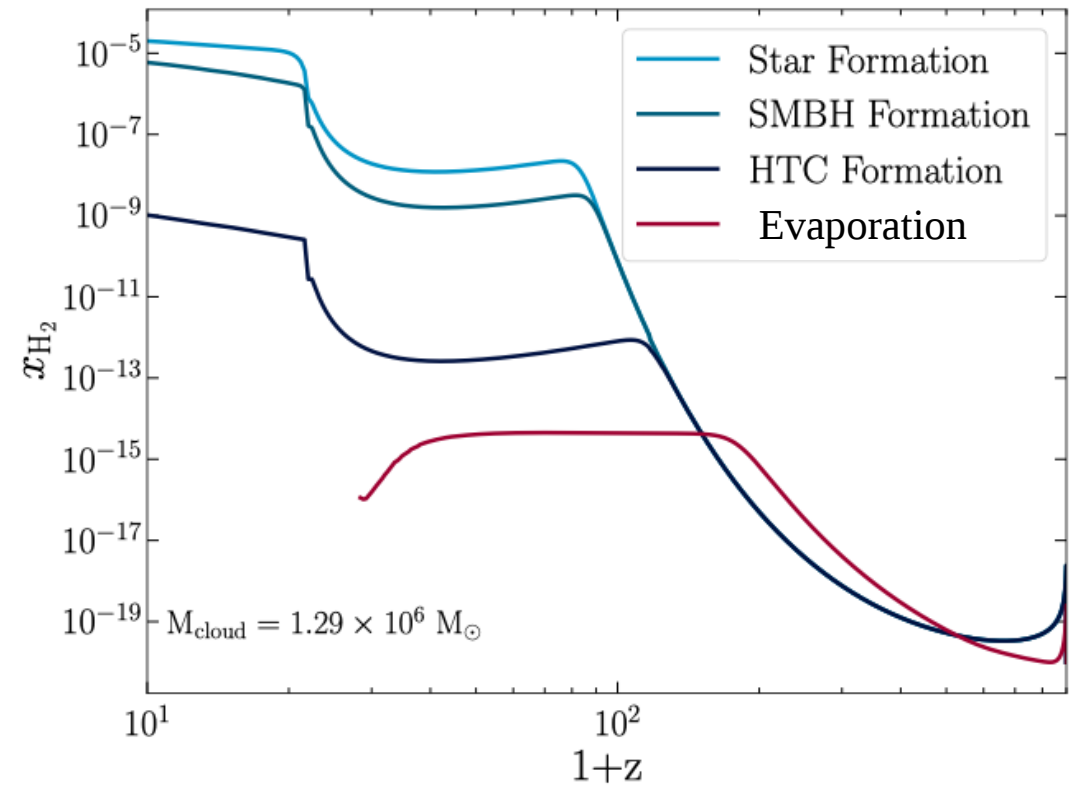
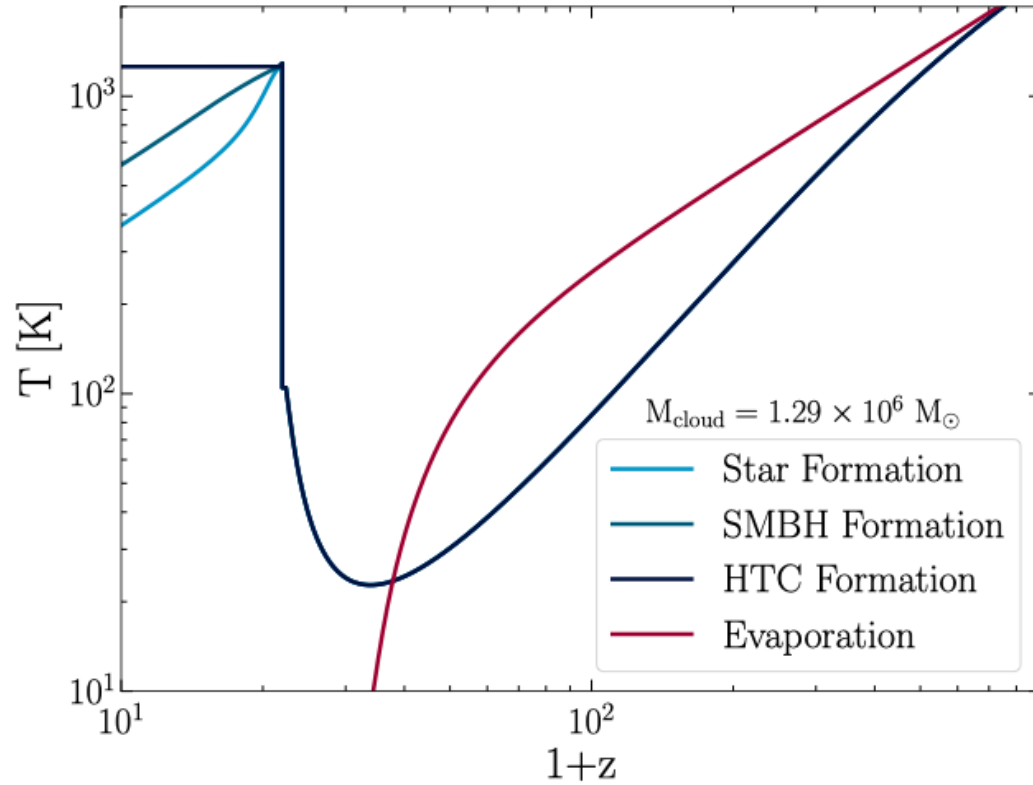
Present ← ————— → Past

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- High Temperature Cloud (HTC) Formation: $m_{\chi} = 22.86 \text{ eV}, \tau = 2.78 \times 10^{26} \text{ s}$

Results: Thermal History



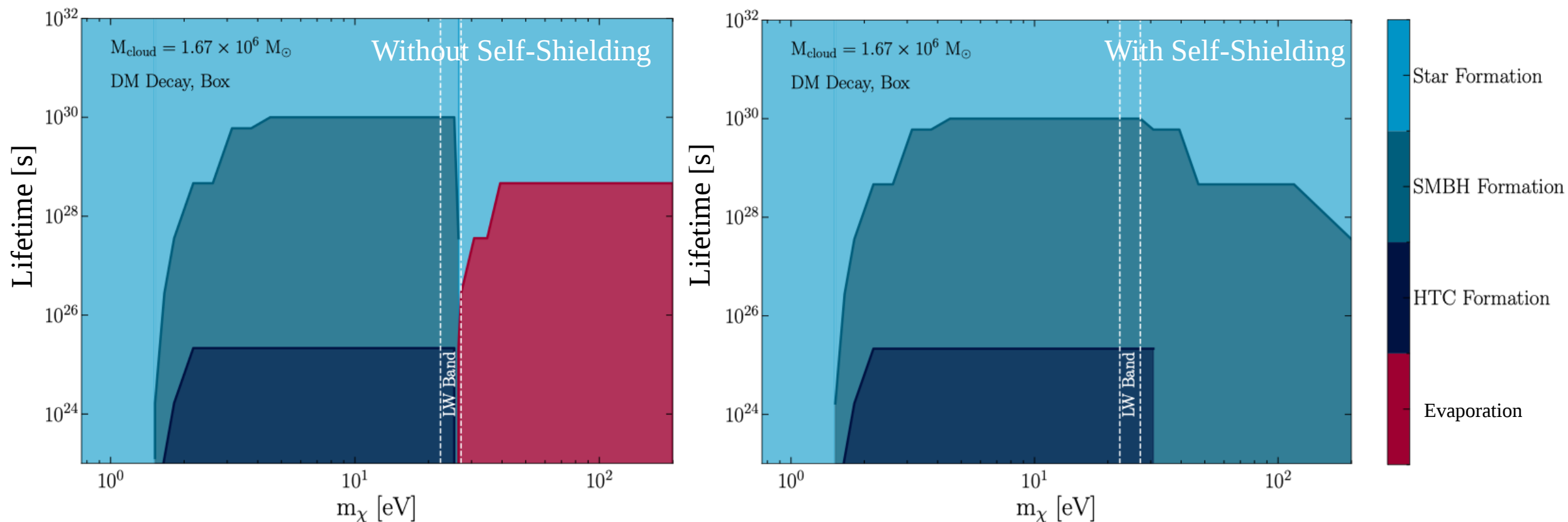
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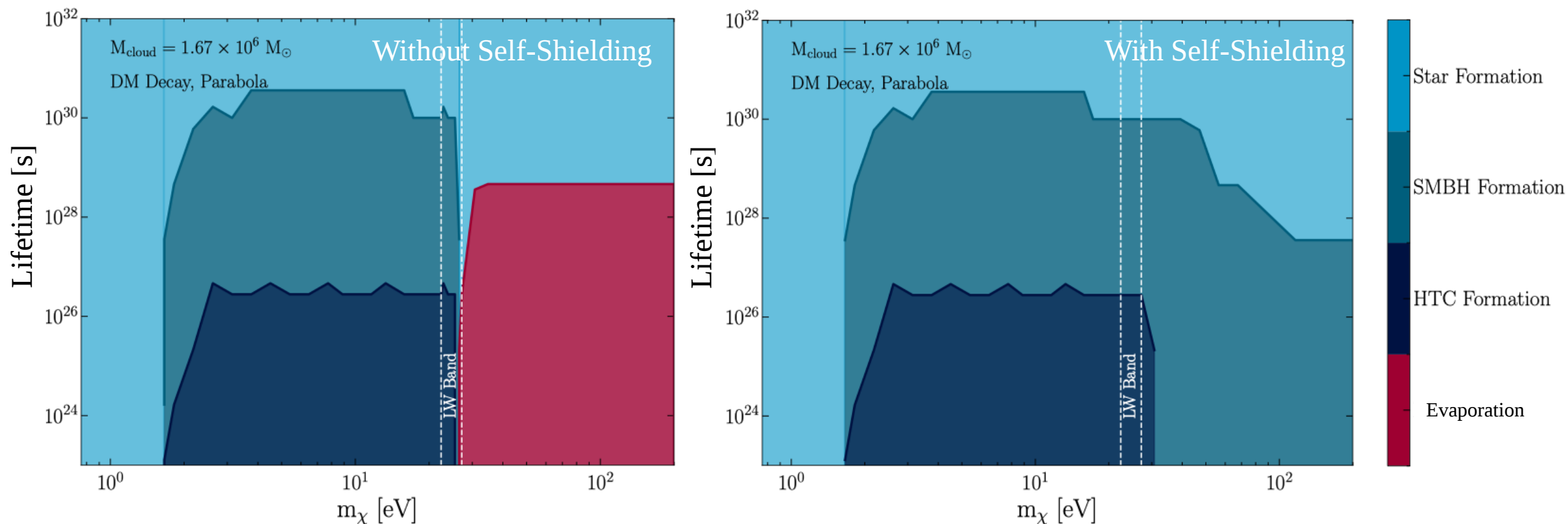
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- High Temperature Cloud (HTC) Formation: $m_{\chi} = 22.86 \text{ eV}, \tau = 2.78 \times 10^{26} \text{ s}$
- Evaporation Formation: $m_{\chi} = 47.1 \text{ eV}, \tau = 1 \times 10^{22} \text{ s}$

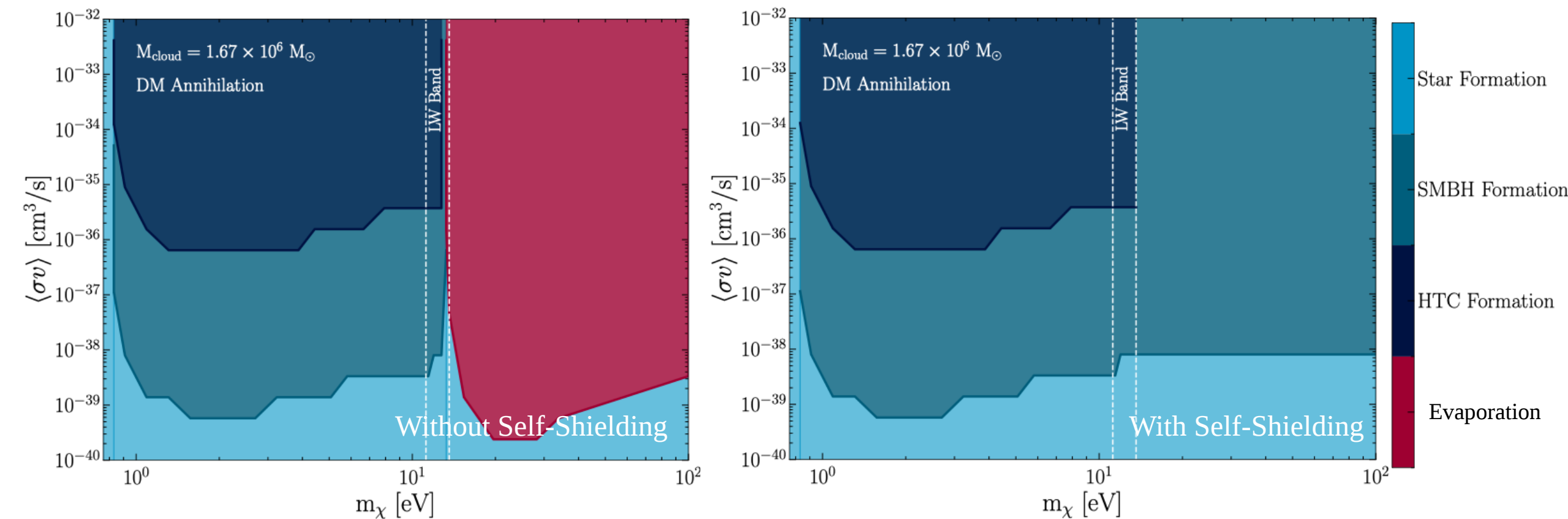
Phase Diagram: Decaying DM with Box Spectrum



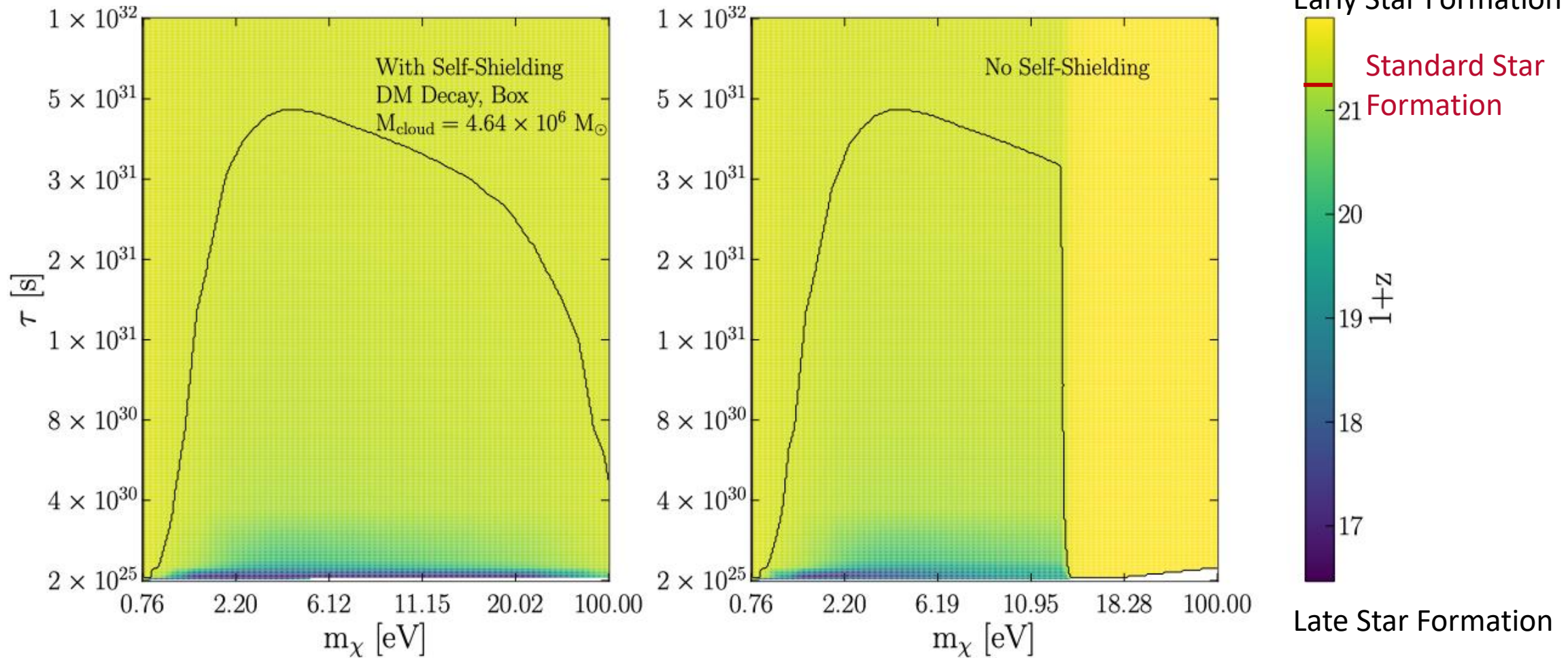
Phase Diagram: Decaying DM with Parabola Spectrum



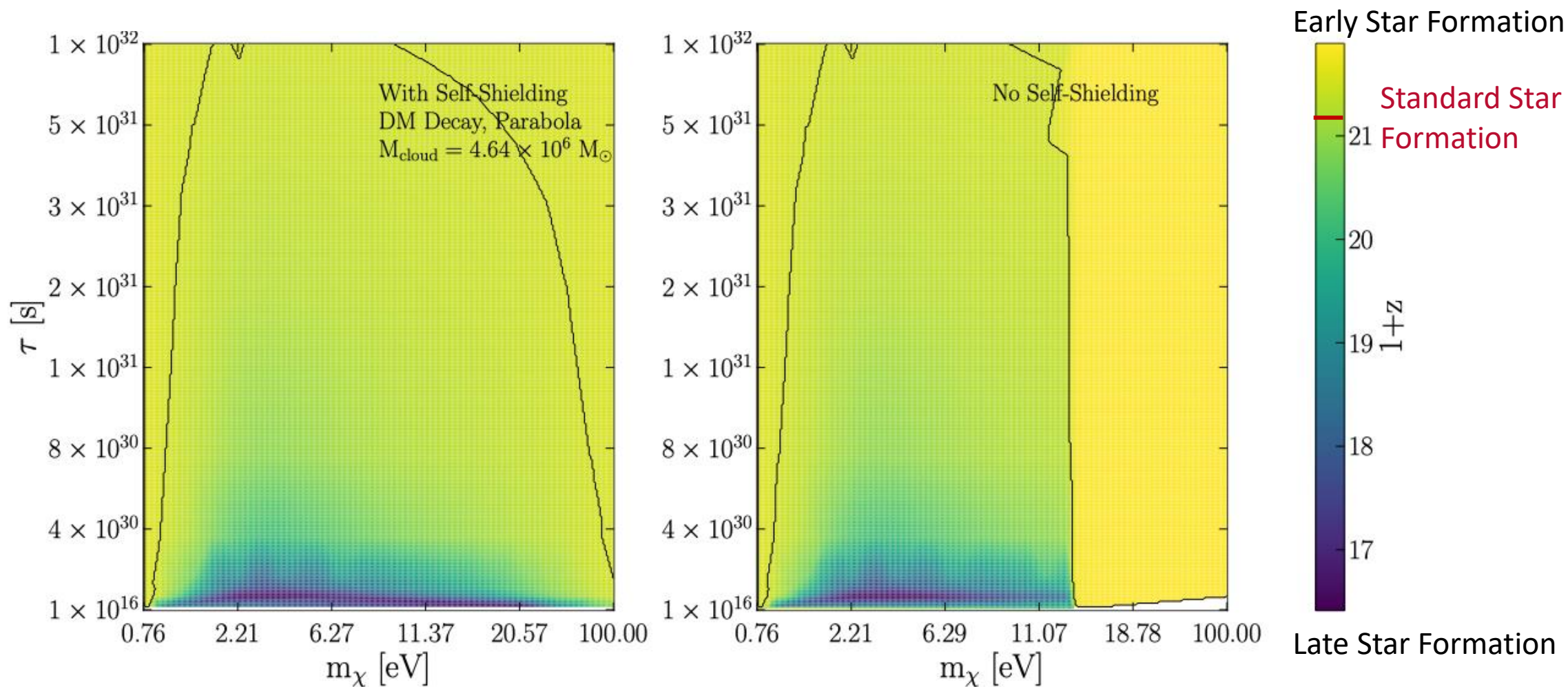
Phase Diagram: Annihilating DM



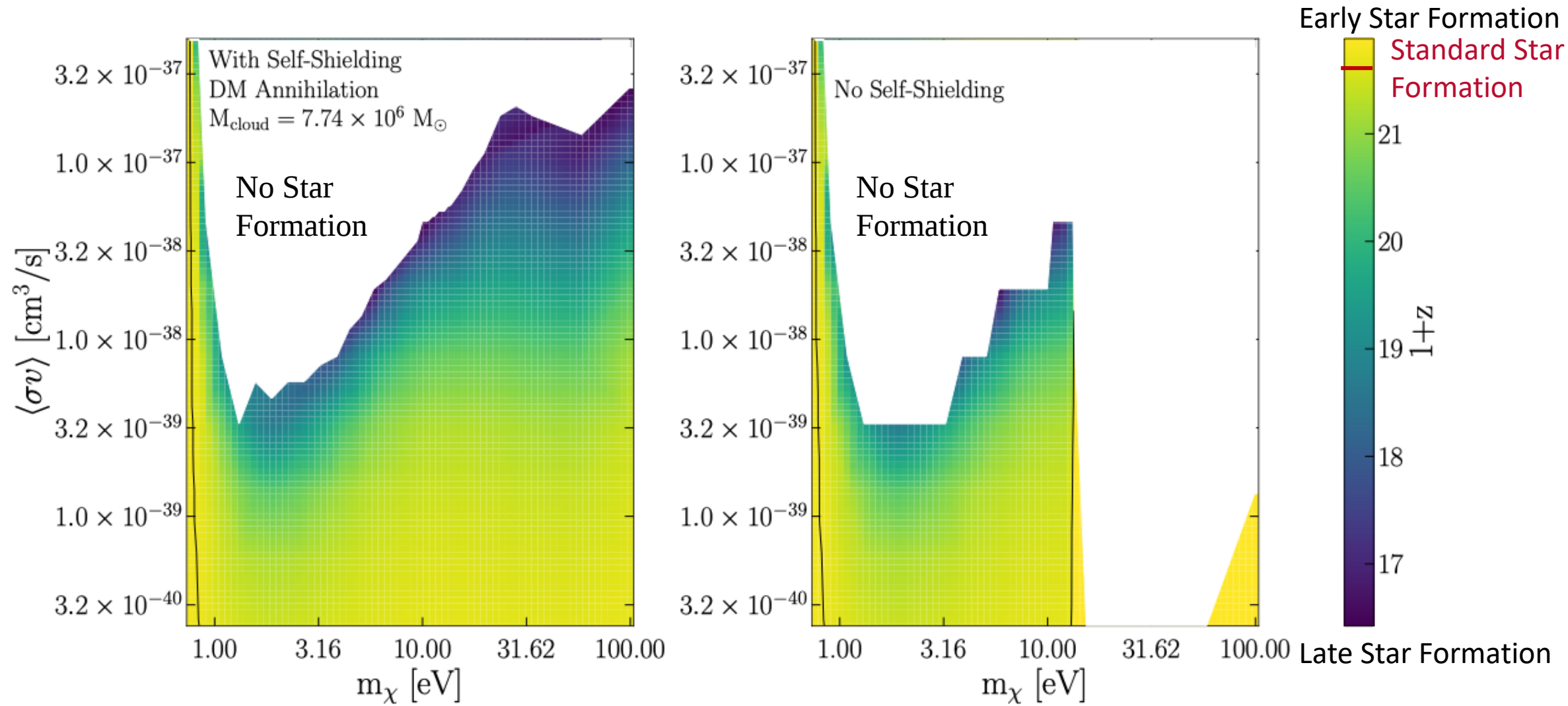
Star Formation Time: Decaying DM with Box Spectrum



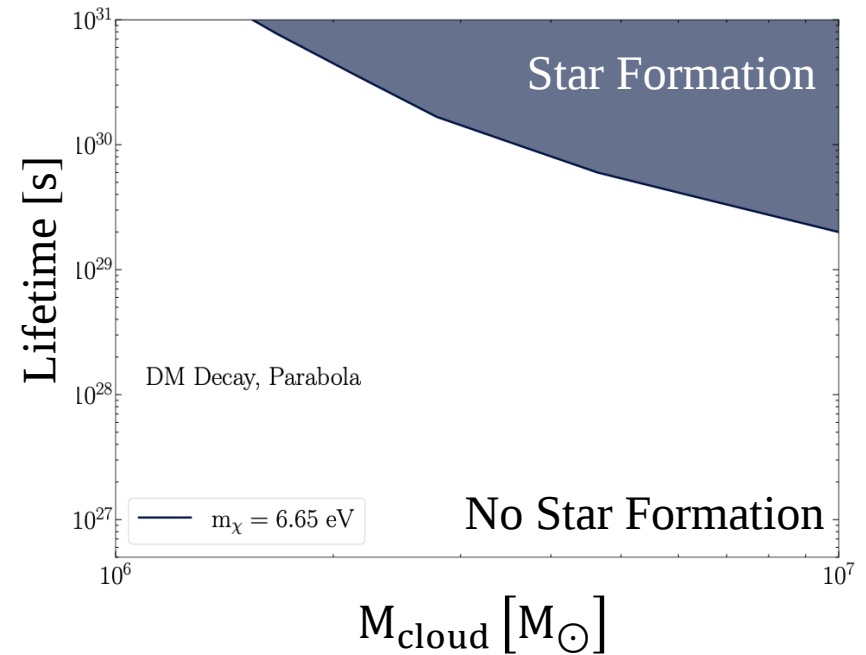
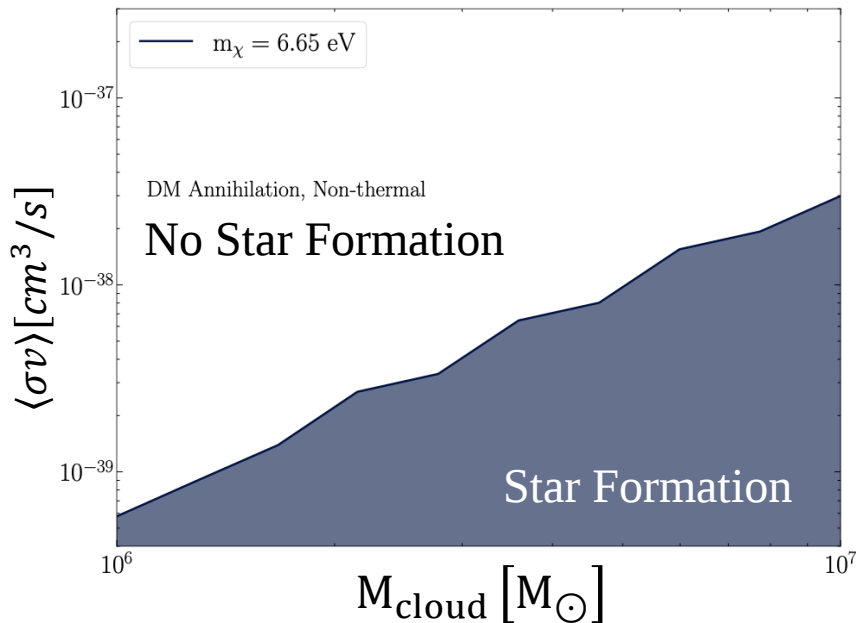
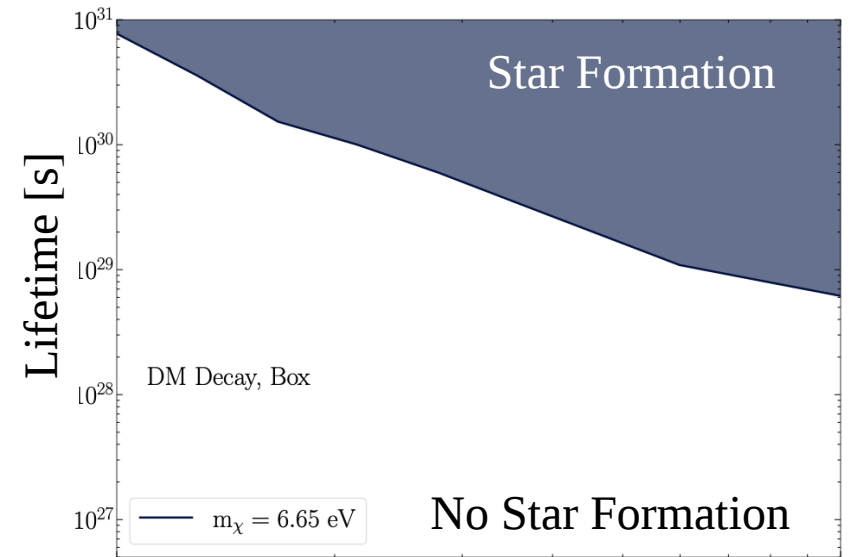
Star Formation Time: Decaying DM with Parabola Spectrum



Star Formation Time: Annihilating DM



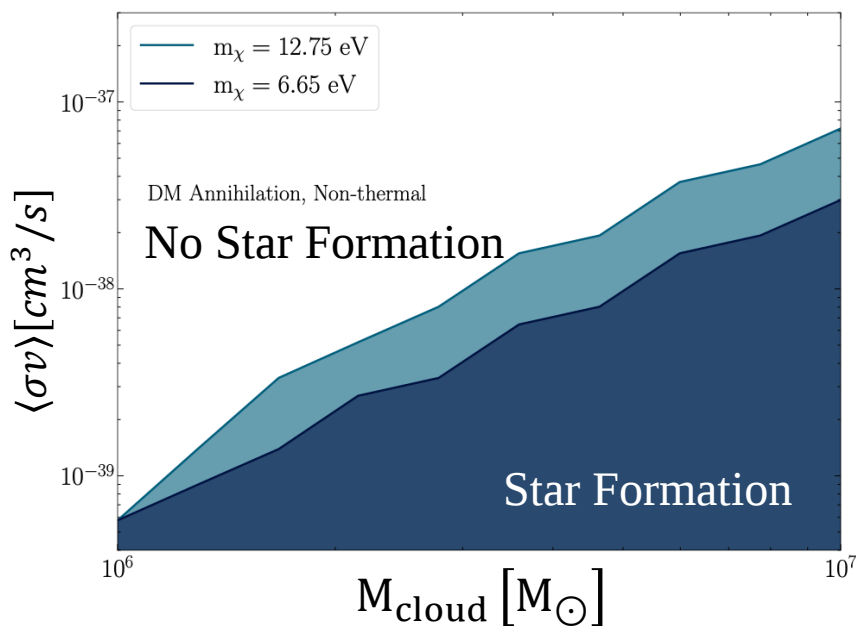
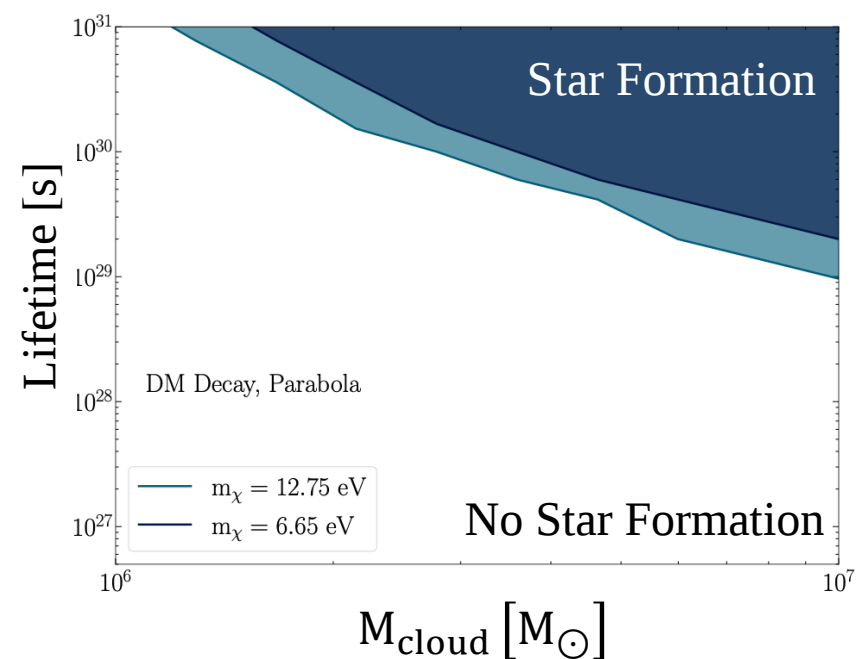
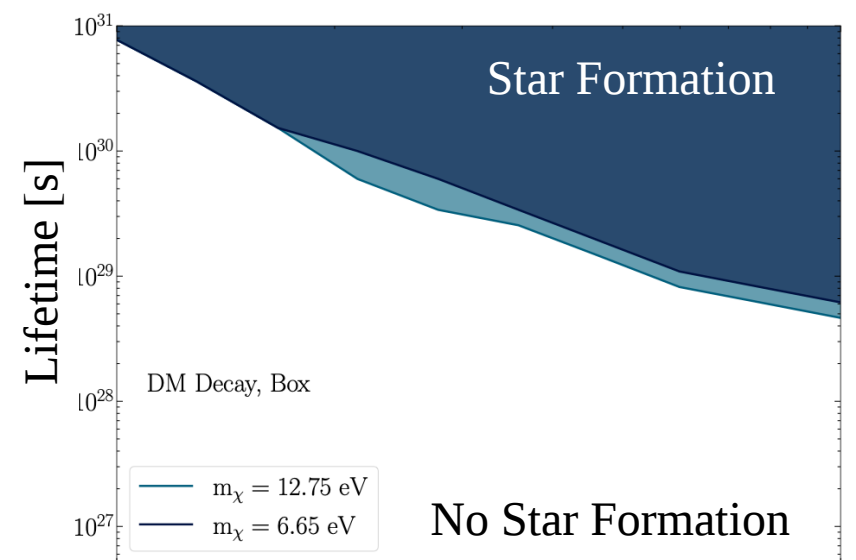
Star Formation: Gas Cloud Mass Critical Curves



$$J_{\text{decay}} \propto (\tau m_\chi)^{-1}$$

$$J_{\text{ann}} \propto \langle\sigma v\rangle m_\chi^{-2}$$

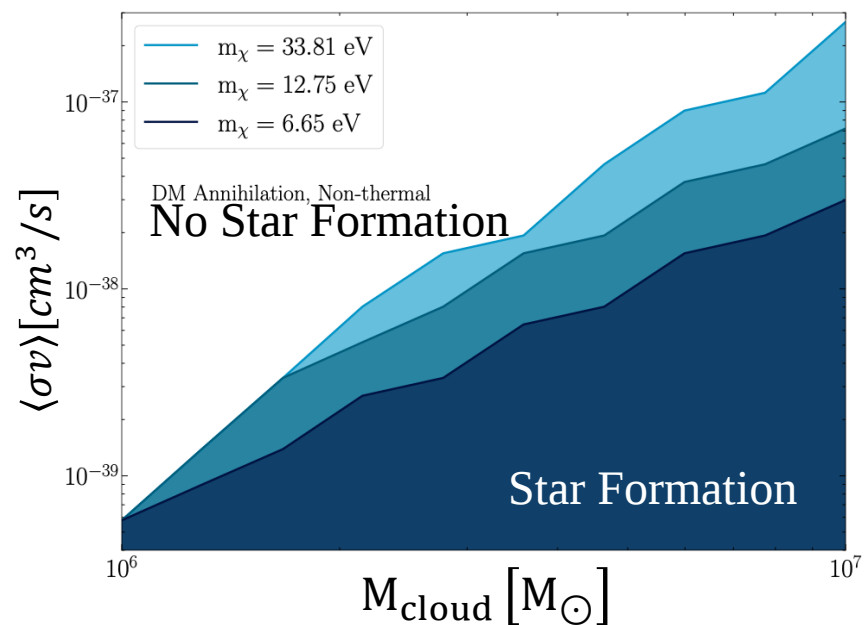
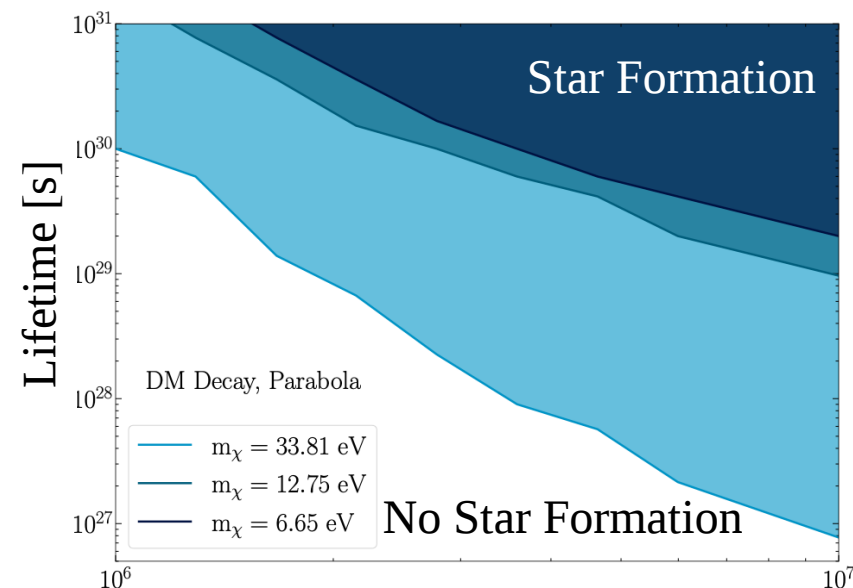
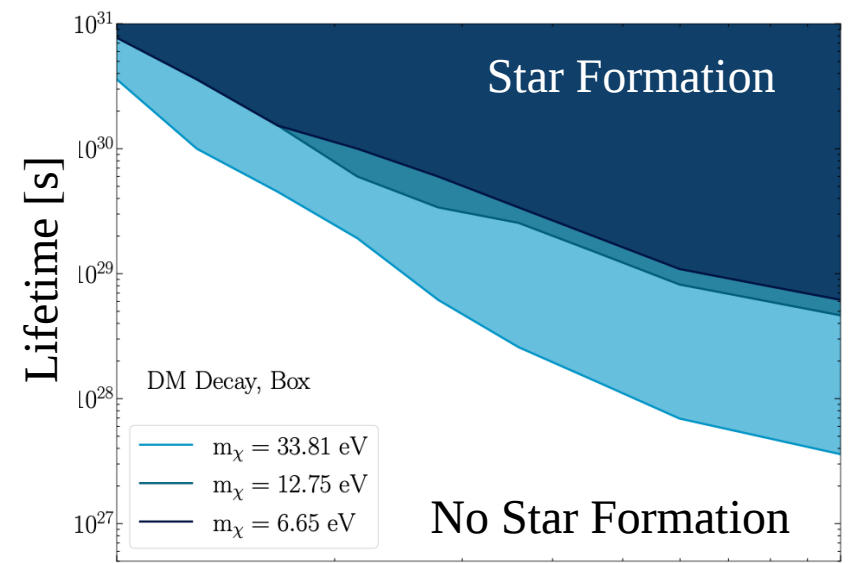
Star Formation: Gas Cloud Mass Critical Curves



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Star Formation: Gas Cloud Mass Critical Curves



$$M_{\text{cloud}} [M_\odot]$$

$$J_{\text{decay}} \propto (\tau m_\chi)^{-1}$$

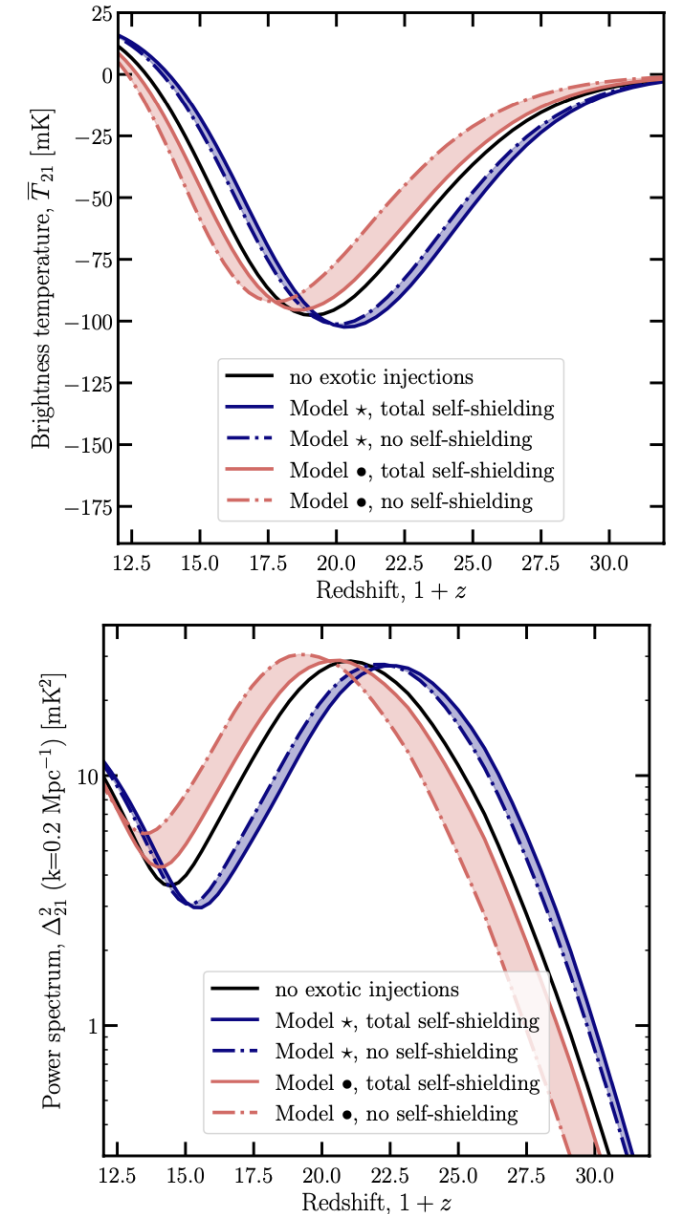
$$J_{\text{ann}} \propto \langle\sigma v\rangle m_\chi^{-2}$$

Ongoing Work

- Next Step:
- How about WIMPs?

Cascade: As propagating through the gas cloud, high-energy photons from heavier DM decay/ annihilation gradually lose energy via interactions with the ambient medium.

- Observation Signals: 21 cm signal, JWST
- Ambitions:
- 1-D hydrodynamic simulation
- 3-D hydrodynamic simulation



Takeaway Information

- Dark matter with different parameter values can alter the evolution of a gas cloud, leading to the formation of the first stars, SMBHs, a high temperature cloud, or simply a cold overdensity region in the universe.
- In the direct-collapse black hole scenario, photons produced by DM decay or annihilation can suppress H_2 formation and slow cooling, promoting SMBH formation.
- In the first-star formation scenario, these photons can either delay or accelerate star formation.
- When self-shielding is included, higher-mass DM can further contribute to H_2 destruction, thereby affecting the thermal and ionization history of the gas cloud.

Thank You!

Takeaway Information

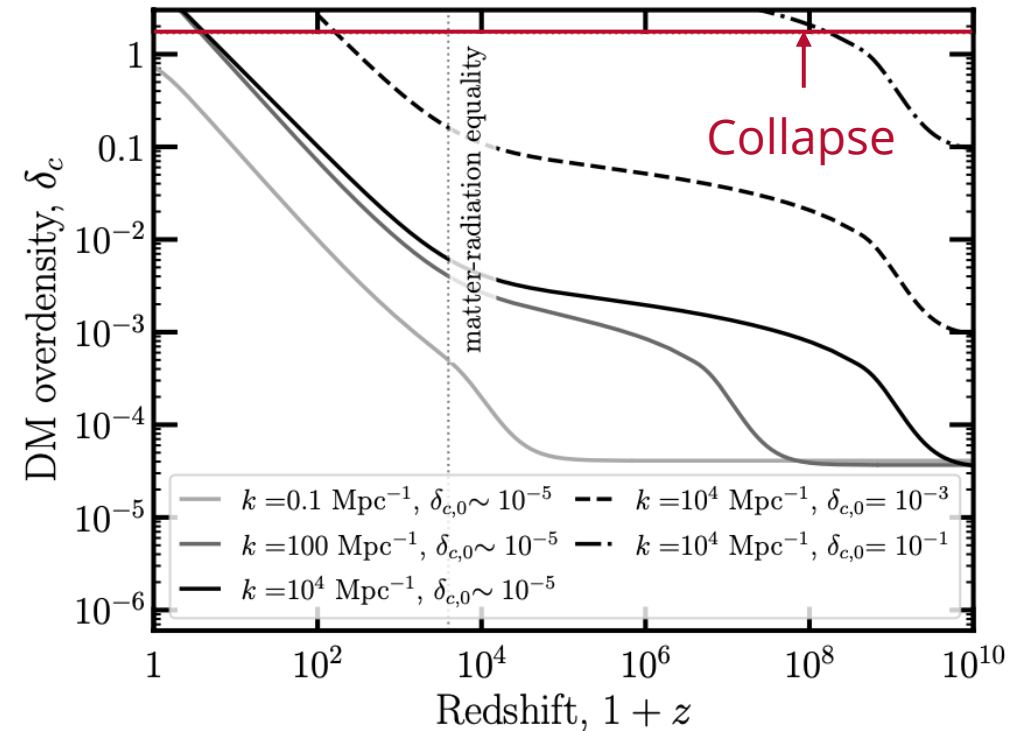
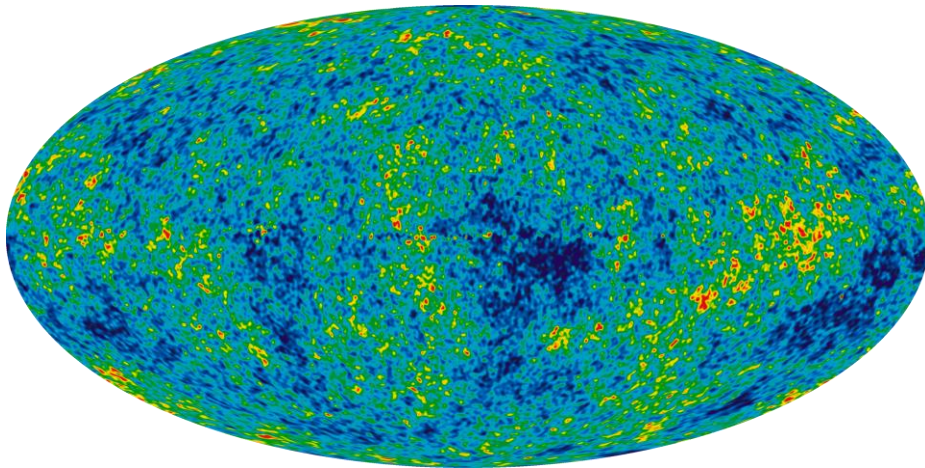
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Backup

Structure Collapse in Λ CDM Model

Λ CDM Model:

- the existence and structure of the cosmic microwave background
- the large-scale structure in the distribution of galaxies
- the observed abundances of hydrogen (including deuterium), helium, and lithium
- the accelerating expansion of the universe observed in the light from distant galaxies and supernovae



<https://arxiv.org/pdf/2506.13858>

Initial Density Fluctuation:

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

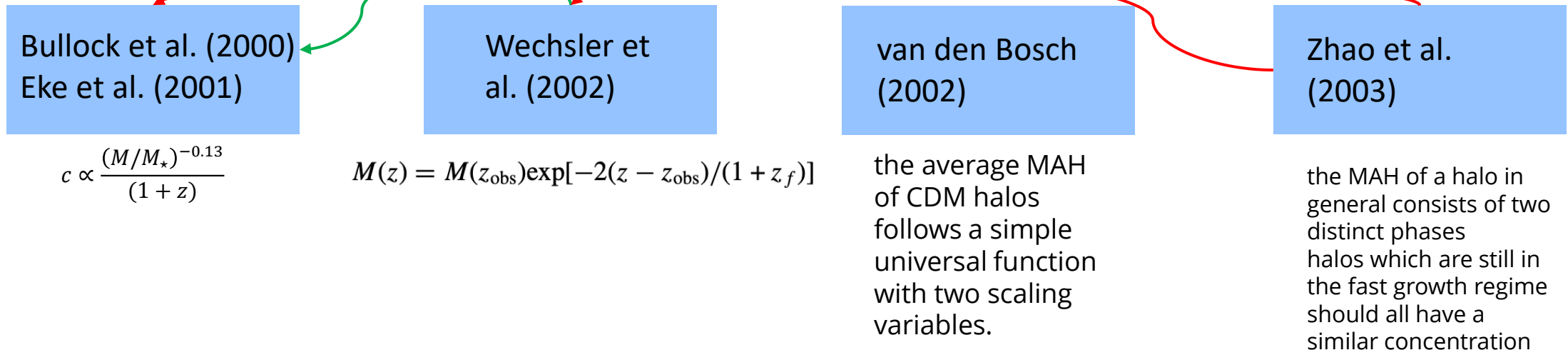
The typical amplitude of an initial fluctuation $\sim 10^{-5}$

All modes collapse at similar redshifts $1+z \lesssim 10$

Backup: Mass Accretion Histories (MAH)

Questions about the fixed mass modeling:

- Bullock et al.(2000) found that the halo concentration (at fixed mass) is systematically lower at higher redshift
- haloes of fixed mass have significant scatter in their c values (Jing 2000), although there is a mean trend of c with M_h
- $c \propto (1+z)^{-1}$ corresponds to the scaling of the virial radius for an object whose mass did not increase with redshift, that is for an isolated system surrounded by a void. Haloes never exist in complete isolation, however; as the virial radius increases, it will enclose more matter, increasing both the total mass and the virial radius further.



Backup: Concentration Parameter

Concentration Parameter Epoch

- **Before NFW**

Power law density profile → Steeper profiles for large n → Galaxy cluster density profiles show no clear dependence on n → Significant departures from power-law behavior

- **NFW and later (slow phase, $c \propto (1+z)^{-1}$)**

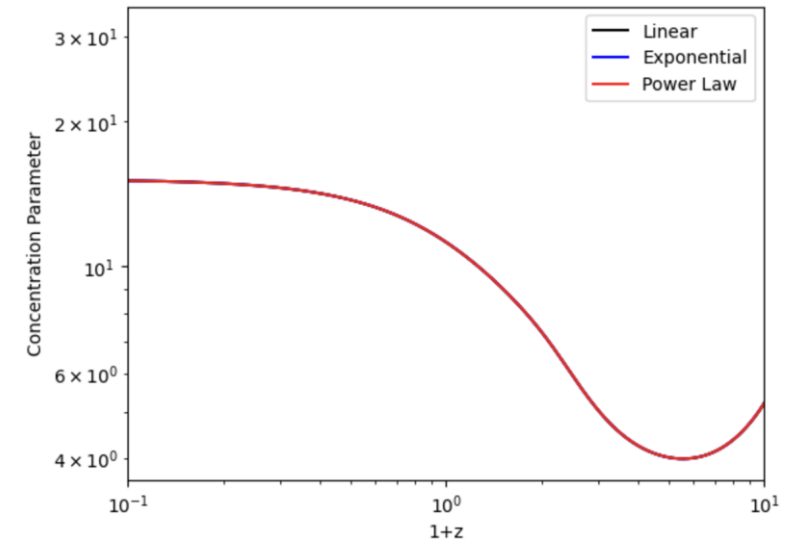
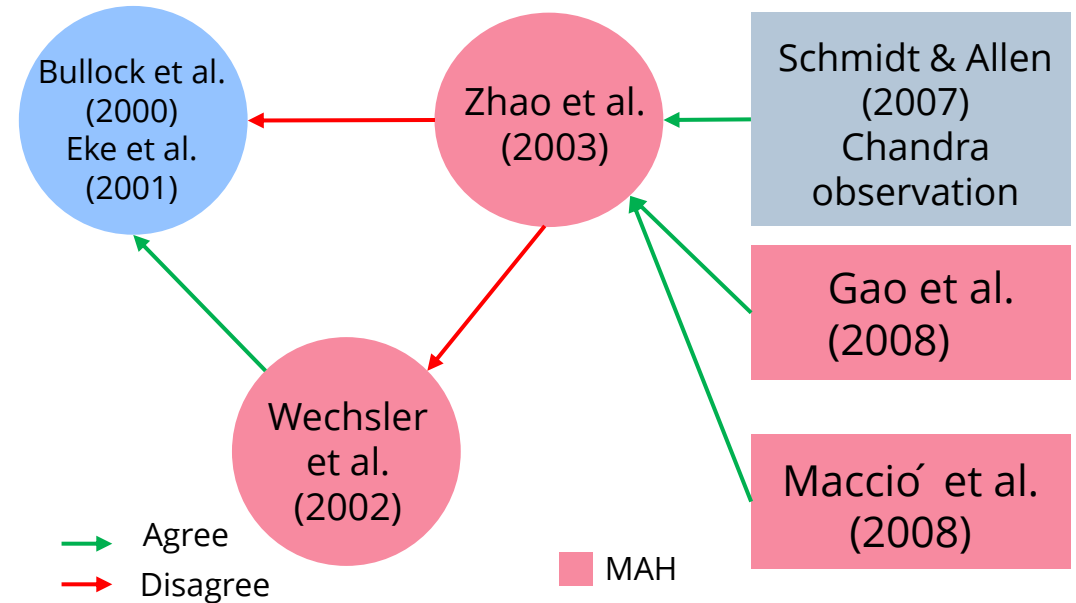
Halo density profiles with a universal shape that does not depend on their mass, on the power spectrum of initial fluctuations, or on the cosmological parameters

- **Mass Accretion Histories (MAH)**

Early rapid phase
Late slow phase

- **Propose and confirm of Rapid Phase ($c \sim 3 - 4$)**

The typical binding energy of the halo is approximately equal to that of a singular isothermal sphere with the same circular velocity



Modeling

- **Gas Cloud and DM Halo**

One-Zone model, NFW Profile

- **DM Decay/ Annihilation**

Photon Spectrum, Photon source

- **Chemical Reaction Rate**

Photodissociation, Photodetachment, Photoionization, Photoheating

- **Thermal and Ionized History Simulation**

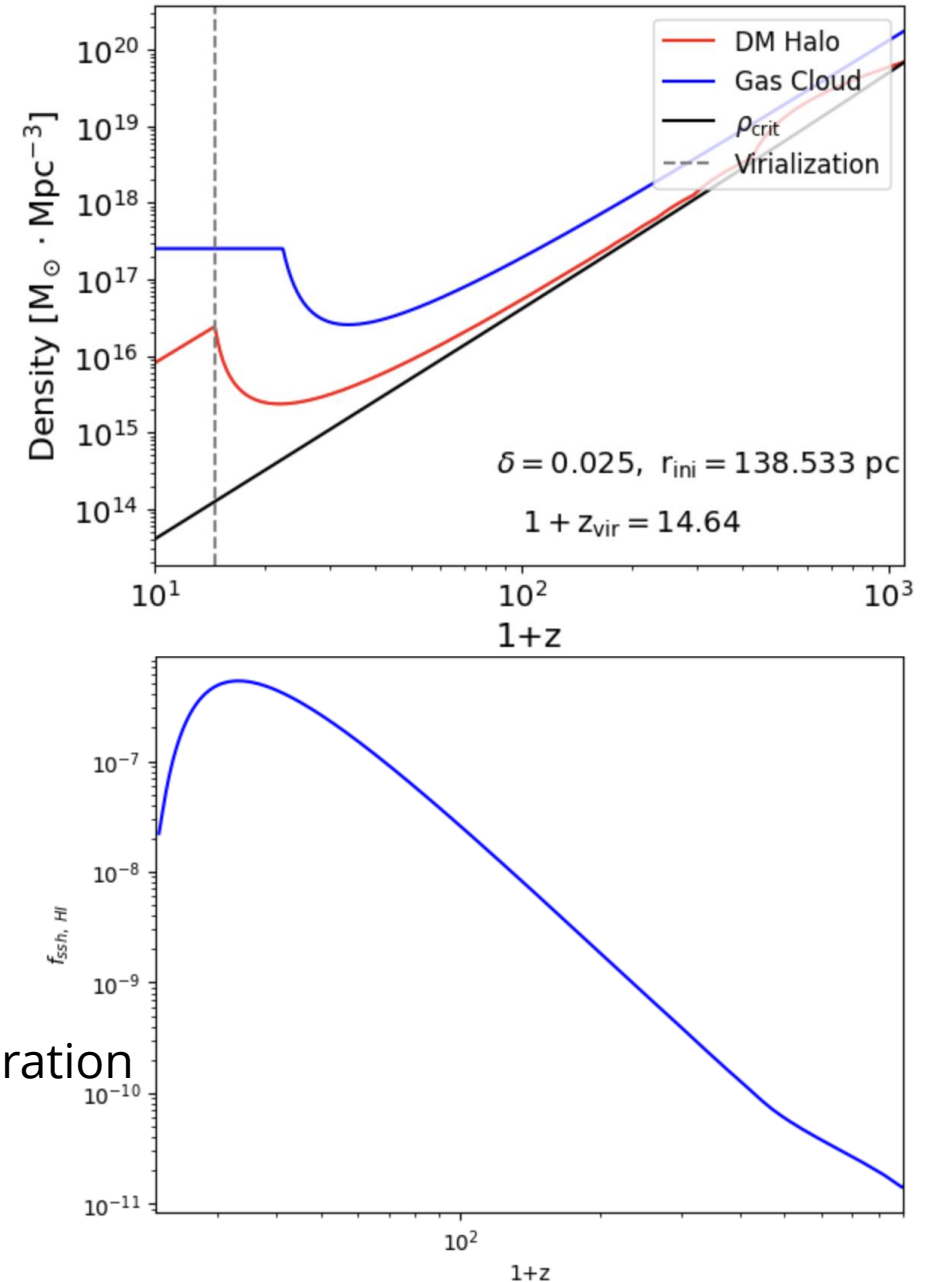
Initial Conditions (CLASS), Grackle

- **Criteria**

Star, SMBH, High Temperature Cloud Formation and Evaporation

- **Self-Shielding**

Fitting Formula from IGM Studies



Backup: B-Factor

Owing to the limited investigation of the mass-redshift-concentration relation and the scarcity of simulations at high redshift ($z \sim 10 - 1100$), its behavior in this regime remains poorly constrained. Although Prada et al. (2012) proposed a fitting formula, it becomes unreliable for redshift above 10.

- **DM Halo**

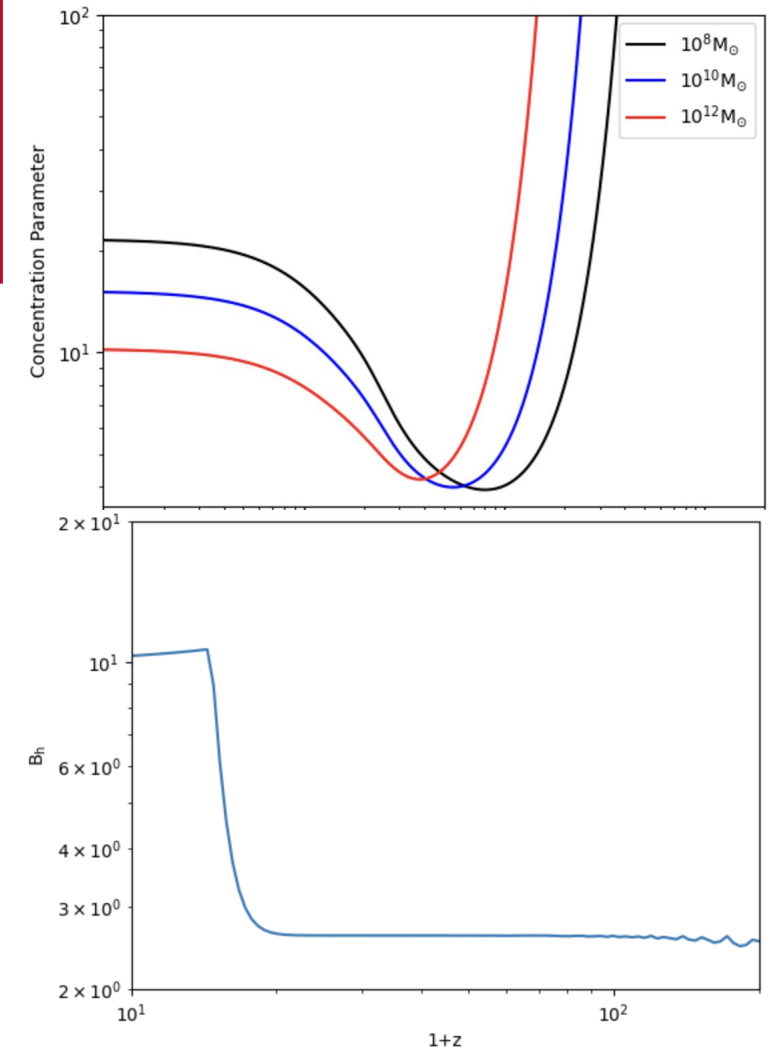
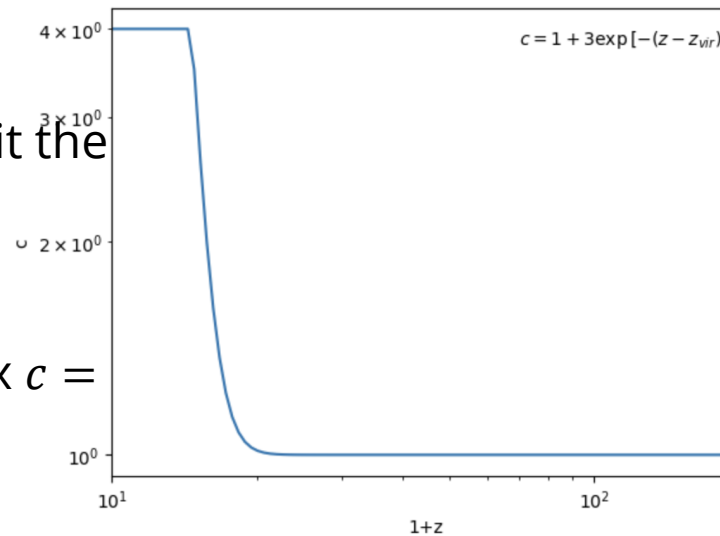
1. Solving ODE with initial conditions:

$$\frac{d^2 r}{dt^2} = -\frac{GM_{halo}}{r^2}$$

2. Using an artificial mass-redshift-concentration relation function, fit the NFW profile

$$c = 1 + e^{-(z-z_{vir})}$$

3. After halo reaches virialization, fix $c = 4$, fit the NFW profile



Modeling: Gas Cloud and DM Halo

Gas Cloud
DM Halo

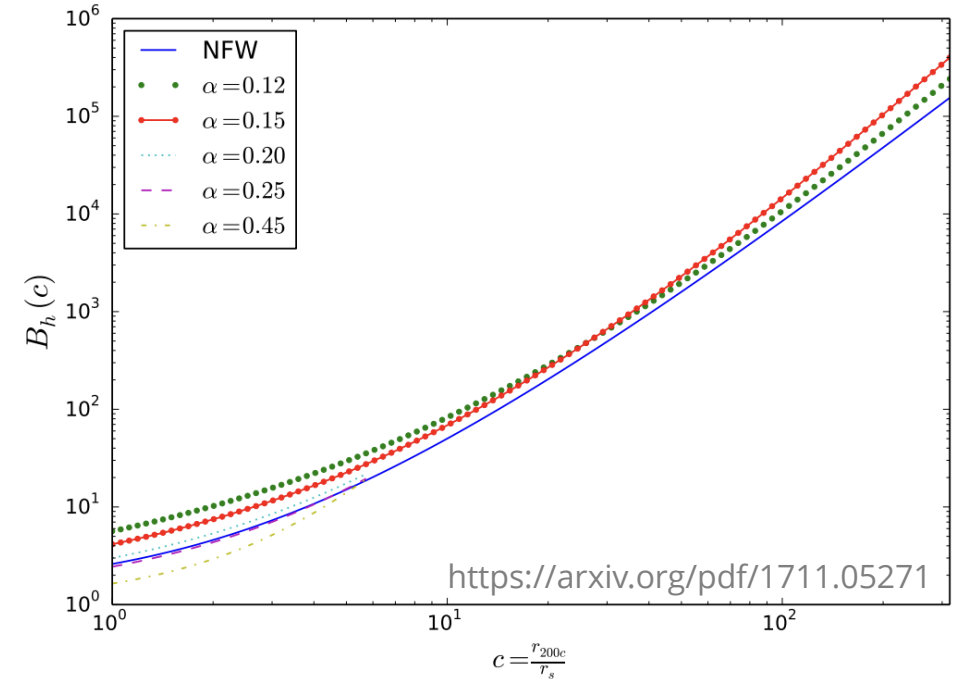
DM Decay
Annihilation

Chemical
Reactions

Simulation
Criteria

Halo Boost Factor

$$B(V) \equiv \frac{1}{\bar{\rho}^2 V} \int \rho^2 dV$$



DM Halo

- NFW Profile:

$$\rho_{NFW} = \frac{4\rho_s}{\left(\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)\right)^2}$$

$$r_s = r_{200}/c \quad \rho_s = \frac{c^3(1+c)M_{200}}{16\pi r_{200}^3[-c + (1+c)\ln(1+c)]}$$

- Halo Radius r_{200}
- Halo Density ρ_{200}
- Concentration parameter c

$$\frac{d^2 r}{dt^2} = -\frac{GM_{halo}}{r^2}$$

$$c \equiv \frac{r_{200}}{r_s}, \quad c = \frac{v_{peak}}{v_{vir}}$$

A measure of the size or density of the central region relative to the size or density of the whole halo.

$c = ?$

B characterizes the inhomogeneity of the particle distribution

Modeling: Gas Cloud and DM Halo

Gas Cloud
DM Halo

DM Decay
Annihilation

Chemical
Reactions

Simulation
Criteria

- **Concentration Parameter**

- Origin Paper of NFW Profile $\rightarrow c \propto (1+z)^{-1}$
- Later Simulations agree with $c \propto (1+z)^{-1}$
- Simulations of fixed mass halo underestimate the $c \rightarrow$ **Mass Accretion History (MAH)**
- MAH Halo Evolution:

Rapid Phase

- Halos start as rare density peaks and experience fast and nearly radial infall that brings mass closer to the centre.
- halos slide into the plateau regime where the accretion becomes less radial
- halos move into the domain of declining concentration-mass relation

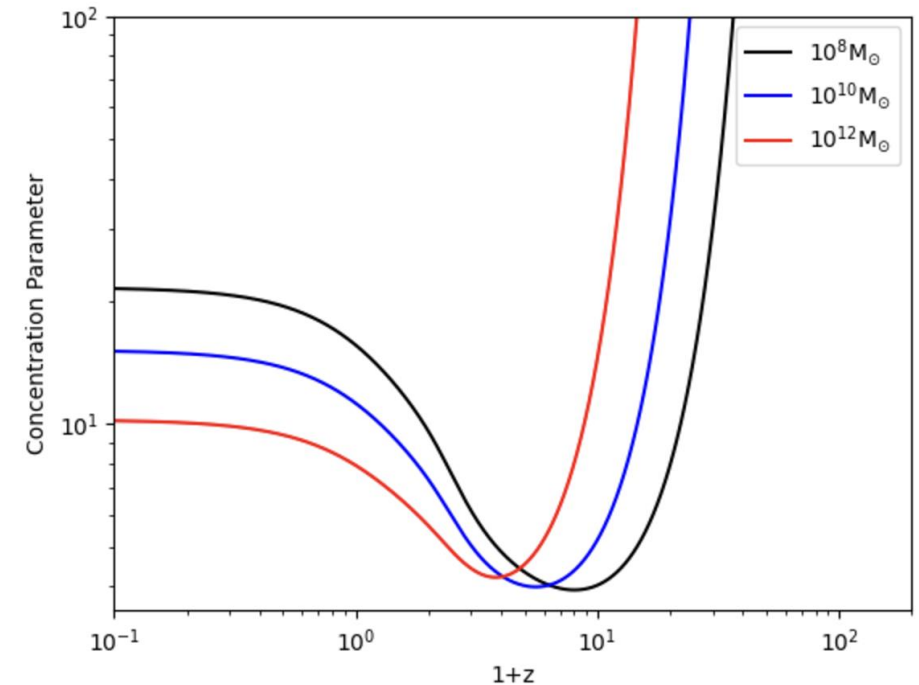
Slow Phase

Rapid Phase



How long it will take?

Slow Phase



Modeling: Gas Cloud and DM Halo

Gas Cloud
DM Halo

DM Decay
Annihilation

Chemical
Reactions

Simulation
Criteria

• Concentration Parameter

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Slow Phase

• Interpolating Concentration Function

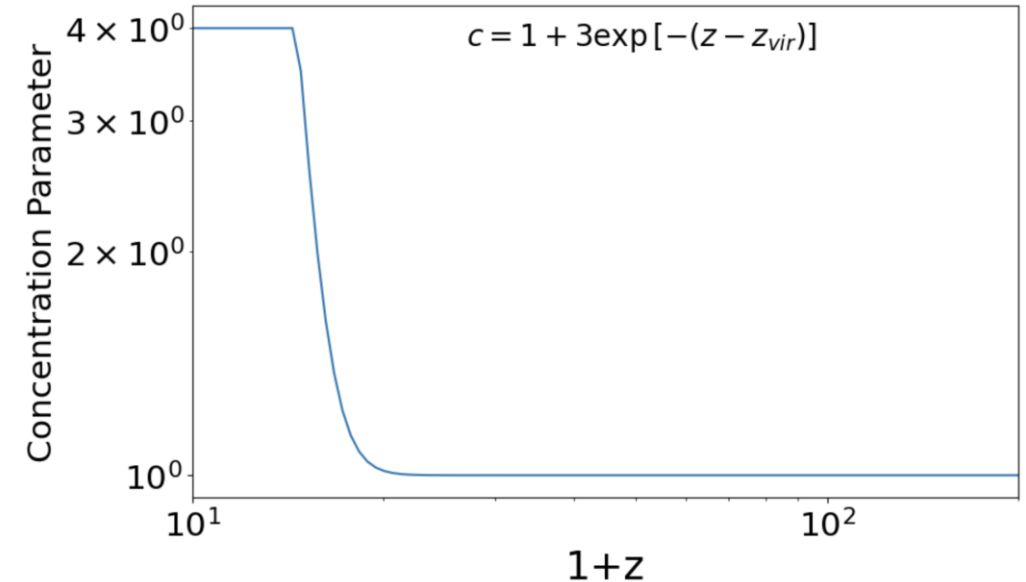
$$c = \begin{cases} 1 + e^{-(z-z_{vir})} & \text{Before virialization} \\ 4 & \text{After virialization} \end{cases}$$

Rapid Phase



How long it will take?

Slow Phase



Julio F. Navarro (1996)

Ewa L. Lokas (1999)

Bullock et al.(2000)

Zhao et al. (2003)

Gao et al. (2008)

Modeling: Gas Cloud and DM Halo

Gas Cloud
DM Halo

DM Decay
Annihilation

Chemical
Reactions

Simulation
Criteria

- DM Halo: NFW Profile**

- Solving ODE with initial conditions:

$$\frac{d^2 r}{dt^2} = -\frac{GM_{halo}}{r^2}$$

- Using an modified mass-redshift-concentration relation function, fit the NFW profile

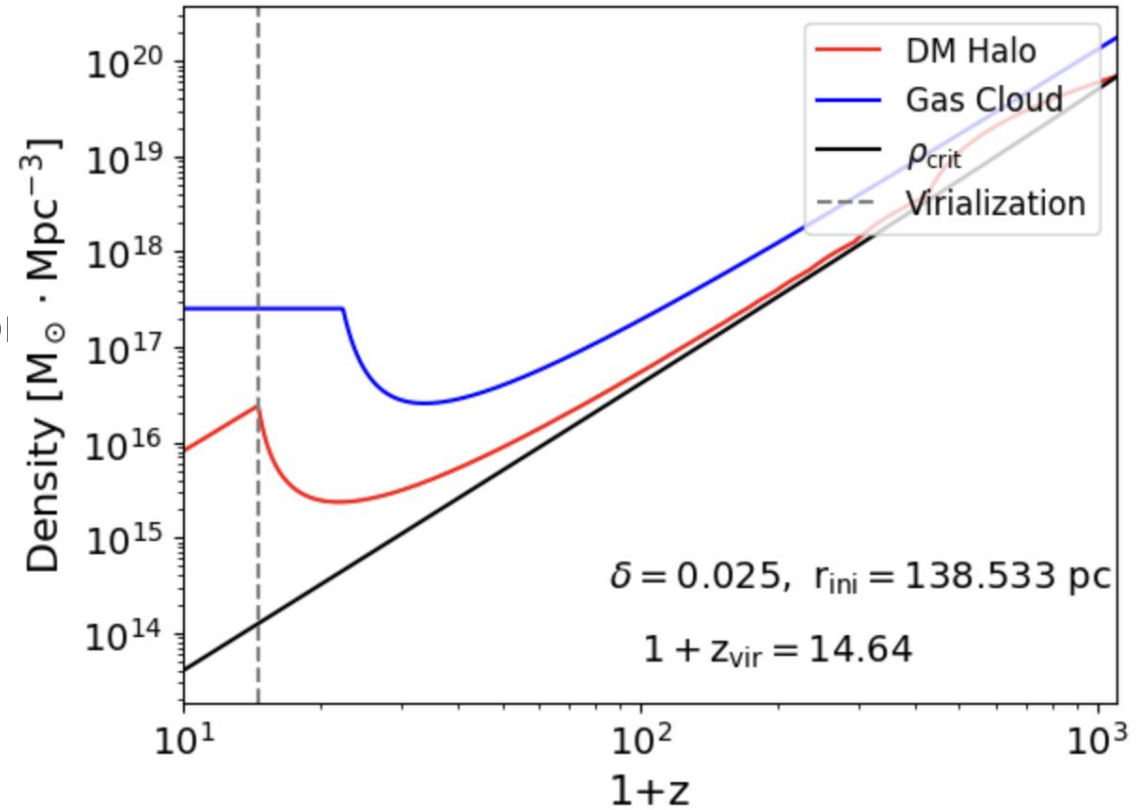
$$c = 1 + 3e^{-(z-z_{vir})}$$

- 3. After halo reaches virialization, fix $c = 4$, fit the NFW profile

- Gas Cloud: One Zone Model**

$$\rho(z) \approx \rho_0(1+z)^3 \exp\left(\frac{1.9A}{1-0.75A^2}\right)$$

$$A = \frac{1+z_{vir}}{1+z}$$



$$z_{vir} = 20$$

$$M_{halo} = M_{cloud}/f_b \quad f_b = \Omega_{baryon}/\Omega_m$$

Modeling: DM Decay/ Annihilation and Source of Photon



- DM Decay ($\chi \rightarrow \gamma\gamma \dots$)

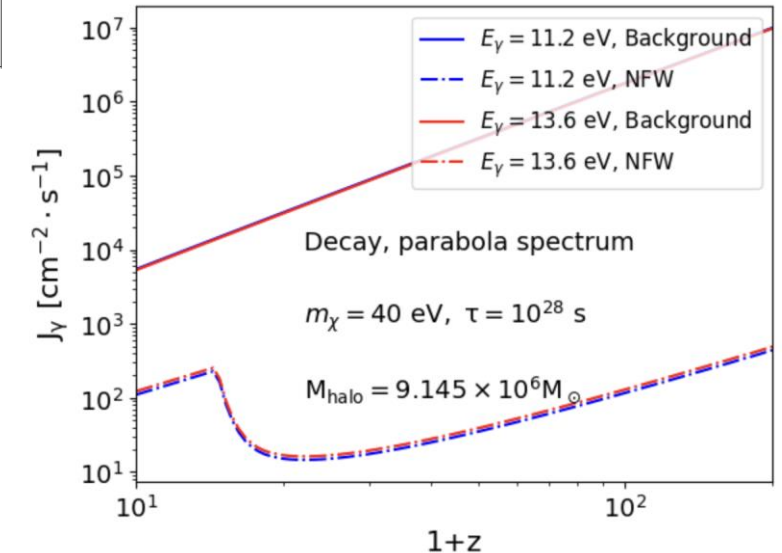
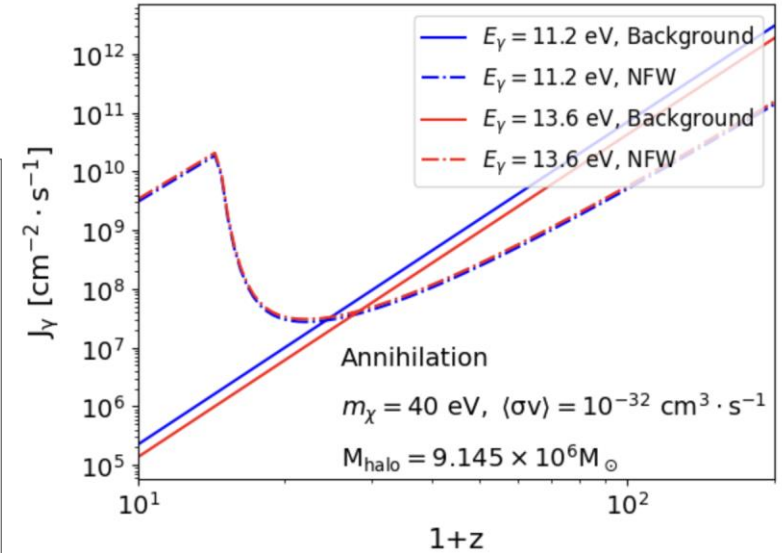
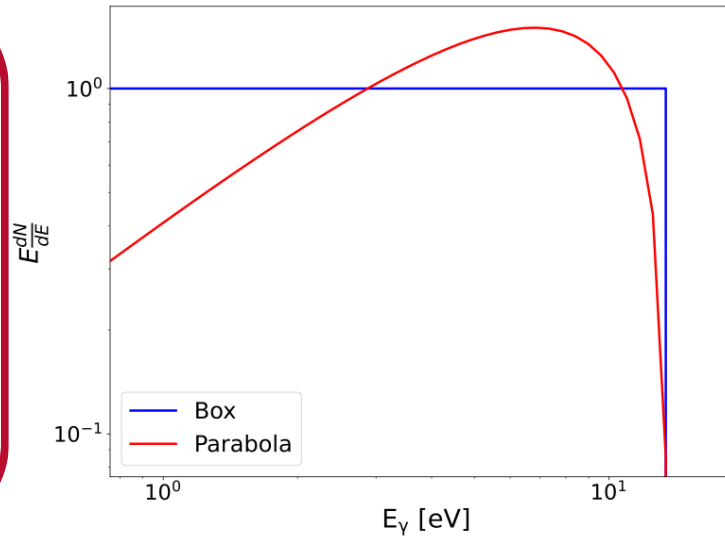
- Box Spectrum

$$E \frac{dN}{dE} \Big|_{\text{box}} = \Theta \left(1 - \frac{E}{m_{dm}} \right)$$

- Parabola Spectrum

$$E \frac{dN}{dE} \Big|_{\text{parabola}} = 6 \left[\frac{E}{m_{dm}} - \left(\frac{E}{m_{dm}} \right)^2 \right] \Theta \left(1 - \frac{E}{m_{dm}} \right)$$

- DM Annihilation ($\chi\chi \rightarrow 2\gamma$)



UV Photon Source

- CMB (high redshift)
- Background DM
- DM Halo

Modeling: Chemical Reaction Rate

Gas Cloud
DM Halo

DM Decay
Annihilation

**Chemical
Reactions**

Simulation
Criteria

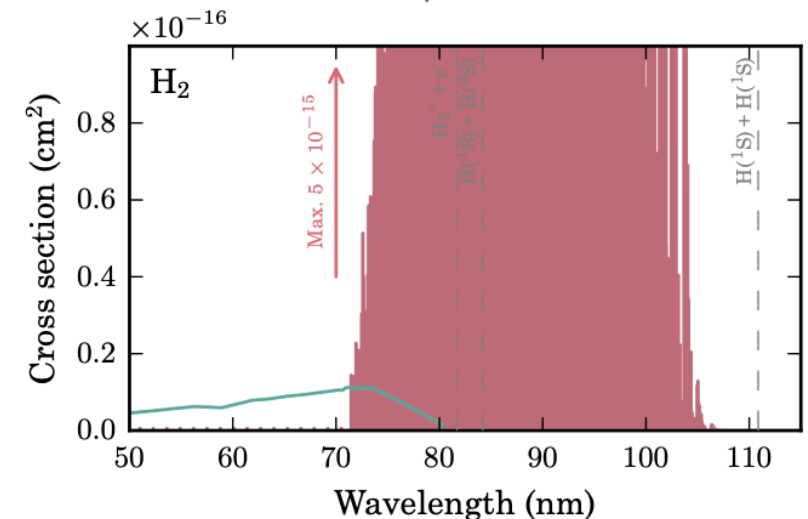
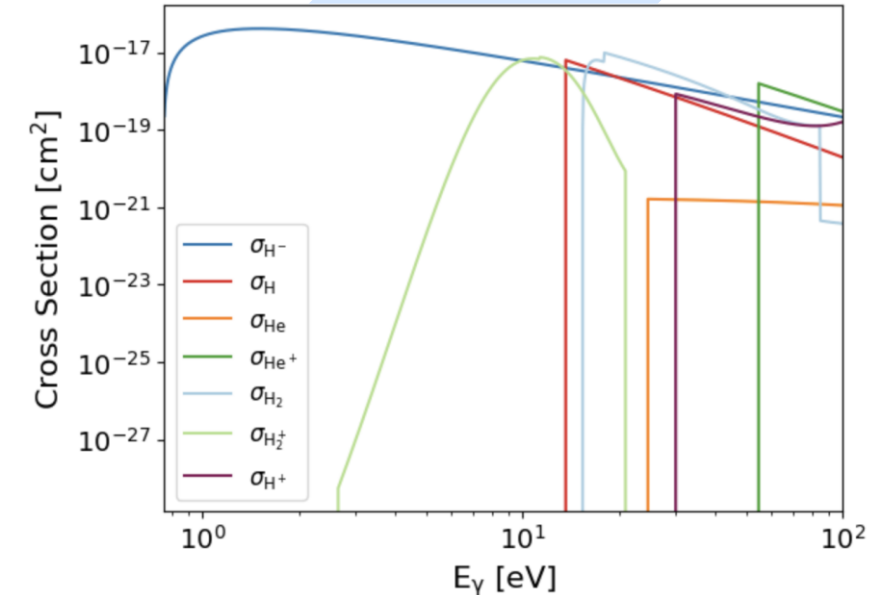
Reaction		Reaction	
$\text{H} + \text{e}^-$	$\rightarrow \text{H}^- + \gamma$	$\text{H} + \text{e}^-$	$\rightarrow \text{H}^+ + \text{e}^- + \text{e}^-$
$\text{H}^- + \text{H}$	$\rightarrow \text{H}_2 + \text{e}^-$	$\text{H}^+ + \text{e}^-$	$\rightarrow \text{H} + \gamma$
$\text{H} + \text{H}^+$	$\rightarrow \text{H}_2^+ + \gamma$	$\text{He} + \text{e}^-$	$\rightarrow \text{He}^+ + \text{e}^- + \text{e}^-$
$\text{H}_2^+ + \text{H}$	$\rightarrow \text{H}_2 + \text{H}^+$	$\text{He}^+ + \text{e}^-$	$\rightarrow \text{He} + \gamma$
$\text{H}_2 + \text{H}^+$	$\rightarrow \text{H}_2^+ + \text{H}$	$\text{He}^+ + \text{e}^-$	$\rightarrow \text{He}^{++} + \text{e}^- + \text{e}^-$
$\text{H}_2 + \text{e}^-$	$\rightarrow \text{H} + \text{H} + \text{e}^-$	$\text{He}^{++} + \text{e}^-$	$\rightarrow \text{He}^+ + \gamma$
$\text{H}_2 + \text{H}$	$\rightarrow \text{H} + \text{H} + \text{H}$	$\text{H} + \text{H}$	$\rightarrow \text{H}^+ + \text{e}^- + \text{H}$
$\text{H}^- + \text{e}^-$	$\rightarrow \text{H} + \text{e}^- + \text{e}^-$	$\text{H} + \text{He}$	$\rightarrow \text{H}^+ + \text{e}^- + \text{He}$
$\text{H}^- + \text{H}$	$\rightarrow \text{H} + \text{e}^- + \text{H}$	$\text{H} + \gamma$	$\rightarrow \text{H}^+ + \text{e}^-$
$\text{H}^- + \text{H}^+$	$\rightarrow \text{H} + \text{H}$	$\text{He} + \gamma$	$\rightarrow \text{He}^+ + \text{e}^-$
$\text{H}^- + \text{H}^+$	$\rightarrow \text{H}_2^+ + \text{e}^-$	$\text{He}^+ + \gamma$	$\rightarrow \text{He}^{++} + \text{e}^-$
$\text{H}_2^+ + \text{e}^-$	$\rightarrow \text{H} + \text{H}$		
$\text{H}_2^+ + \text{H}^-$	$\rightarrow \text{H}_2 + \text{H}$		
$\text{H} + \text{H} + \text{H}$	$\rightarrow \text{H}_2 + \text{H}$		
$\text{H} + \text{H} + \text{H}_2$	$\rightarrow \text{H}_2 + \text{H}_2$		
$\text{H}^- + \gamma$	$\rightarrow \text{H} + \text{e}^-$		
$\text{H}_2^+ + \gamma$	$\rightarrow \text{H} + \text{H}^+$		
$\text{H}_2 + \gamma$	$\rightarrow \text{H}_2^+ + \text{e}^-$		
$\text{H}_2^+ + \gamma$	$\rightarrow \text{H}^+ + \text{H}^+ + \text{e}^-$		
$\text{H}_2 + \gamma$	$\rightarrow \text{H} + \text{H}$		

Neutralization

Photodetachment (0.76– 13.6 eV)

Photodissociation (11.2 – 13.6 eV)

31 reactions; 8 require manual input.



Modeling: Thermal and Ionized Simulation and Criteria

- **Initial Conditions** (Temperature, f_X , x_e ...)

CLASS (<http://class-code.net/>)
(the Cosmic Linear Anisotropy Solving System)

CLASS

the Cosmic Linear Anisotropy Solving System

- **Chemistry and Cooling Library for Astrophysics**
Grackle (<https://github.com/grackle-project/grackle>)



Grackle: a Chemistry and Cooling Library for Astrophysics



Criteria:

- Star Formation

$$T_{cloud}\Big|_{z=\eta z_{vir}} \leq \eta T_{cloud}\Big|_{z=z_{vir}}$$

Rapid Cooling

- SMBH Formation

$$T_{cloud}\Big|_{z=\eta z_{vir}} > \eta T_{cloud}\Big|_{z=z_{vir}}$$

Moderate Cooling

$$M_{cloud} > M_{Jeans}$$

- High Temperature Cloud (HTC) Formation

$$M_{cloud} > M_{Jeans}$$

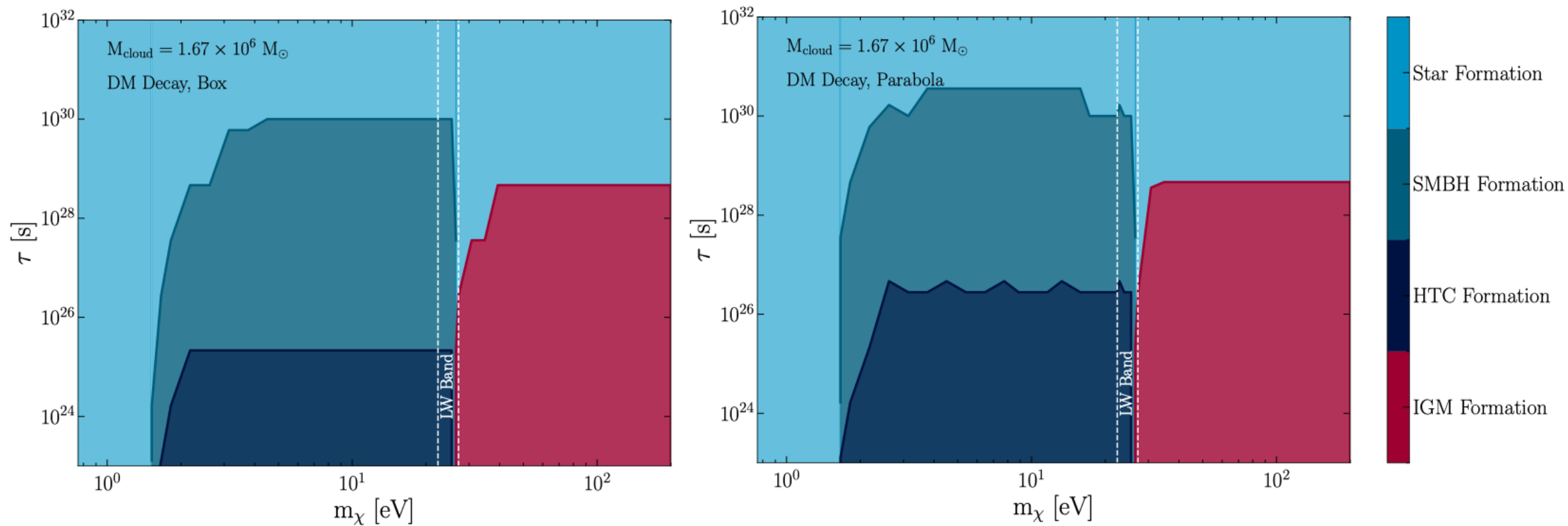
$$T_{cloud} \sim T_{cloud}\Big|_{z=z_{vir}}$$

Very slow Cooling

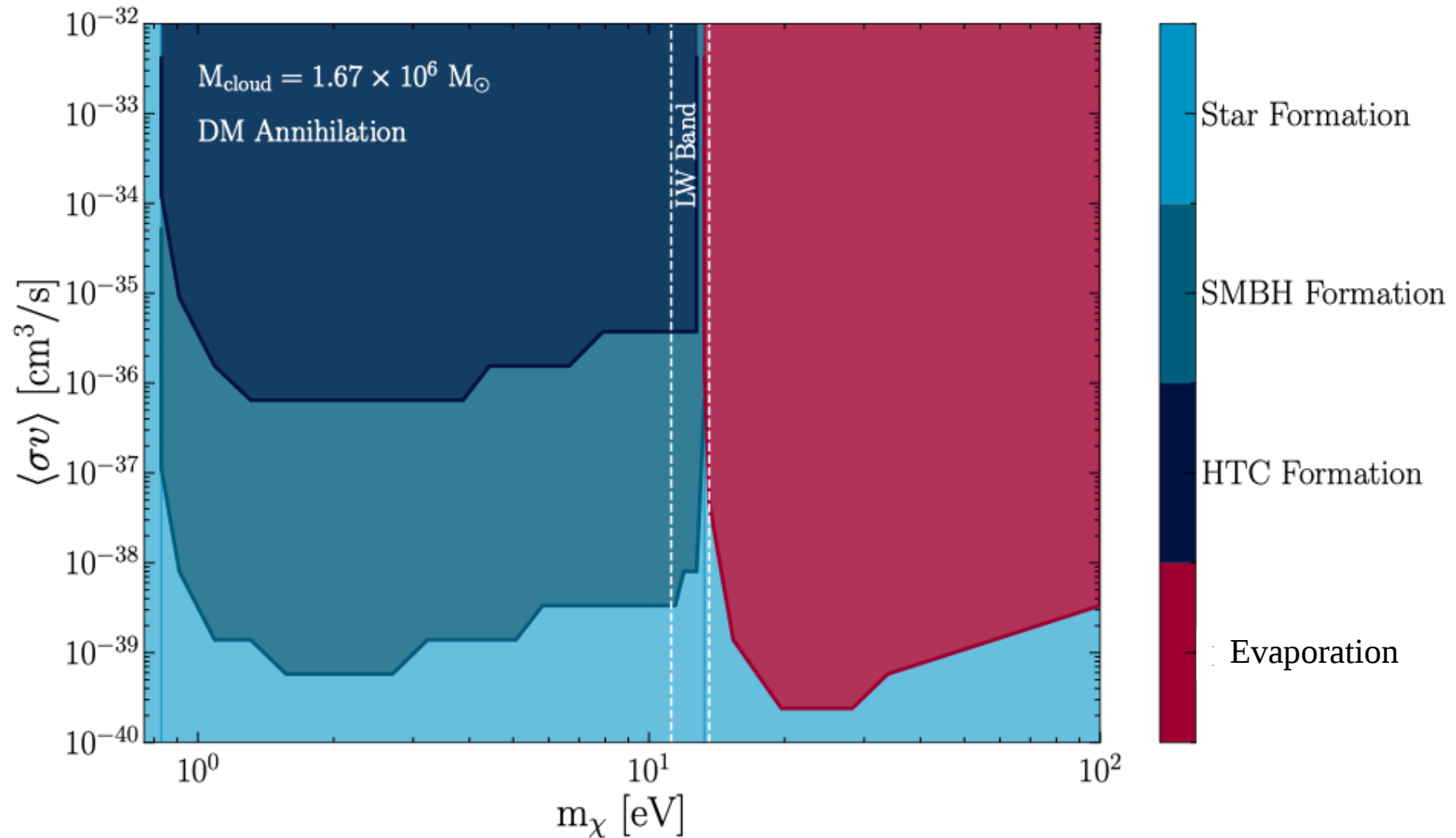
- Evaporation

Gas cloud becomes too cold as high redshift

Phase Diagram: DM Decay



Phase Diagram: Non-Thermal DM Annihilation



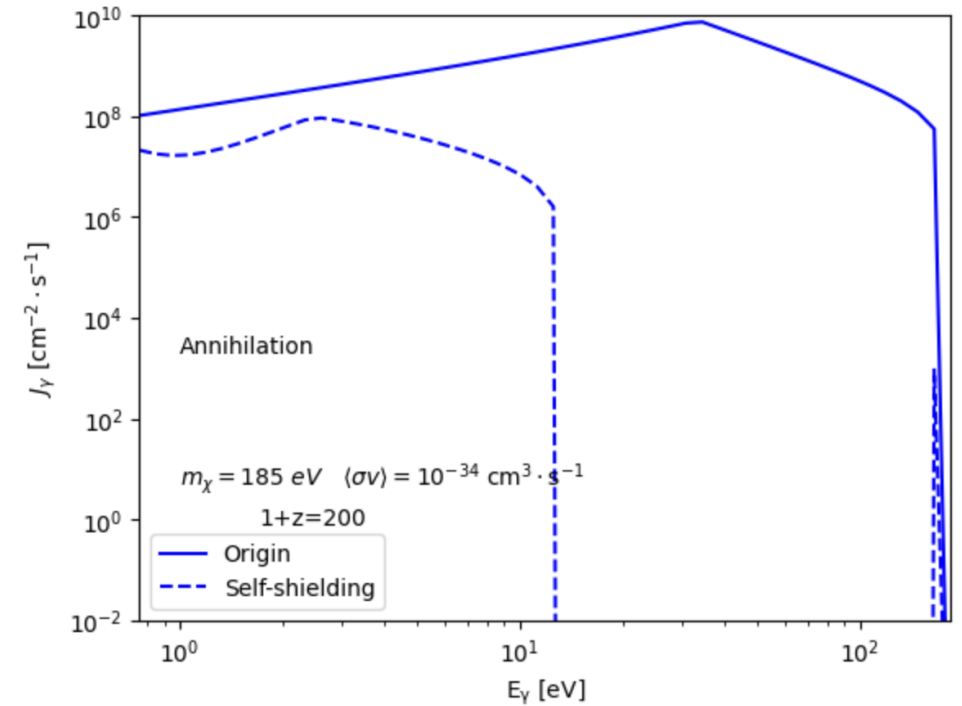
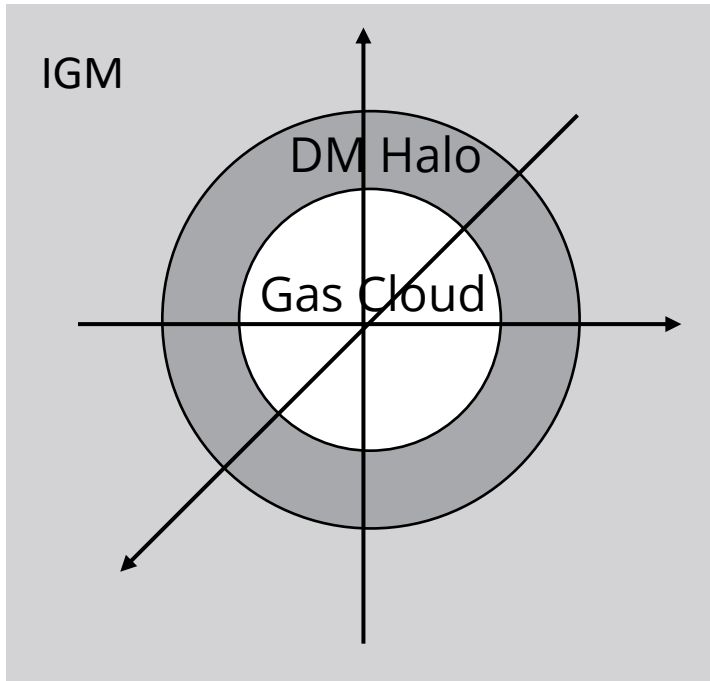
Self-Shielding

- Distance**

$$L(\vec{r}', \vec{r}) = \begin{cases} |\vec{r}' - \vec{r}|, & |\vec{r}'| \leq R_{\text{cloud}} \\ \ell(\vec{r}', \vec{r}) + s(\vec{r}', \vec{r}), & |\vec{r}'| > R_{\text{cloud}} \end{cases}$$

- Optical Depth**

$$\tau(\vec{r}', \vec{r}) = \begin{cases} n_{\text{cloud}} \sigma_{\nu} |\vec{r}' - \vec{r}|, & |\vec{r}'| \leq R_{\text{cloud}} \\ n_{\text{cloud}} \sigma_{\nu} s(\vec{r}', \vec{r}) + n_{\text{background}} \sigma_{\nu} \ell(\vec{r}', \vec{r}), & |\vec{r}'| > R_{\text{cloud}} \end{cases},$$



- Photons from DM Decay/ Annihilation in the DM Halo

$$J_{\text{total}}(E) = \pi E \int_0^{R_{\text{cloud}}} r^2 dr \int_0^{R_{\text{halo}}} r'^2 dr' \int_0^{\pi} d\theta \frac{\sin \theta}{r^2 + r'^2 - 2rr' \cos \theta} \frac{dn}{dE dt} e^{-\tau(r, r', \theta)}.$$

- Photons from Background DM Decay/ Annihilation

$$J_{\text{eff}}(E) = (1 + z_{\text{obs}})^3 \int_{z_{\text{obs}}}^{\infty} \frac{dz}{H(z)(1 + z)^3} E' \frac{dn'_{\gamma}}{dE' dt'} e^{-\tau(E, z)}$$

Self-Shielding Fitting Formula

- H_2 Self-Shielding Fraction

$$f_{sh}(N_{H_2}, T) = \frac{0.965}{(1 + x/b_5)^{1.1}} + \frac{0.035}{(1 + x)^{0.5}} \exp[-8.5 \times 10^{-4}(1 + x)^{0.5}]$$

— H_2 self-shielding fraction

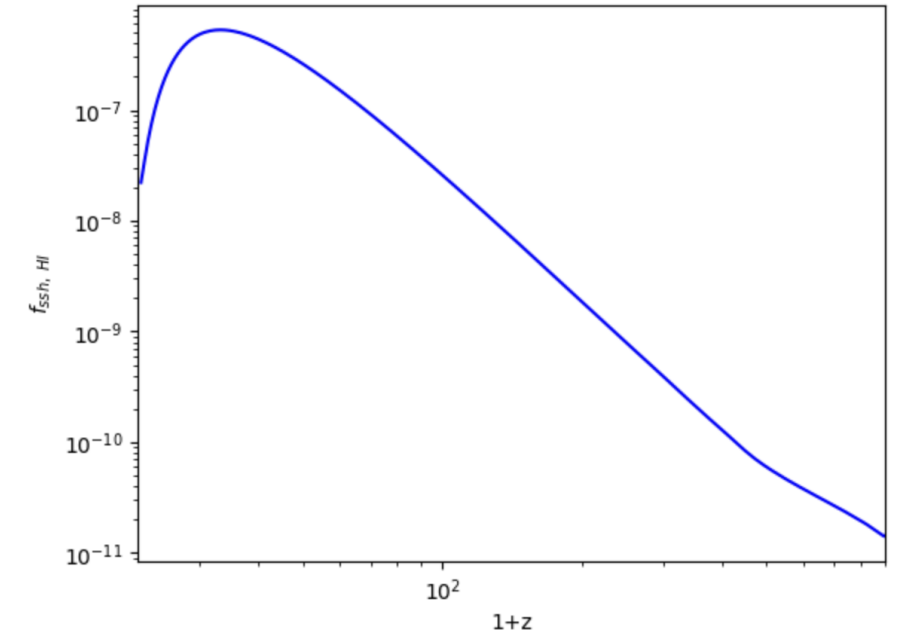
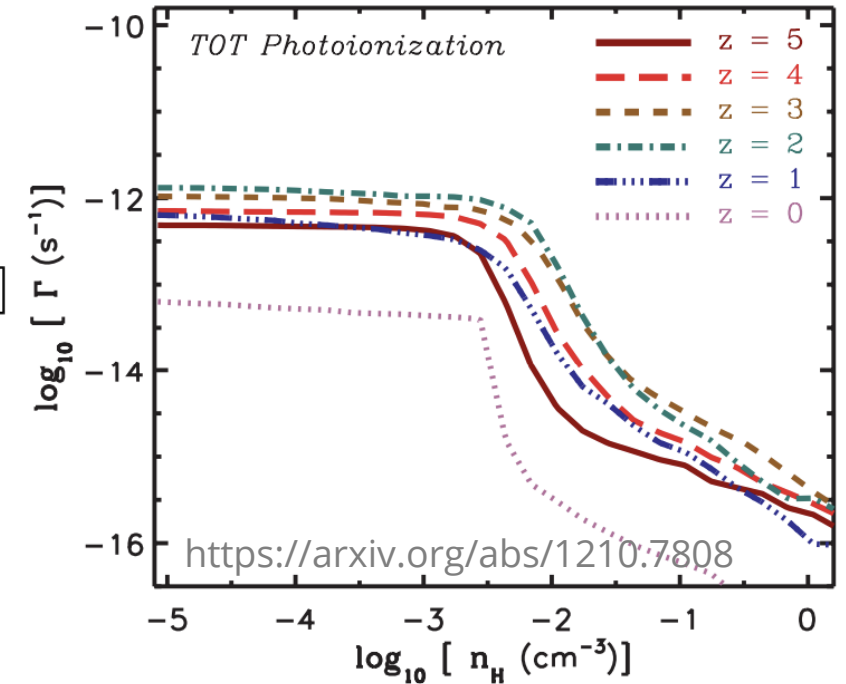
— HI self-shielding fraction

and b is the Doppler broadening parameter

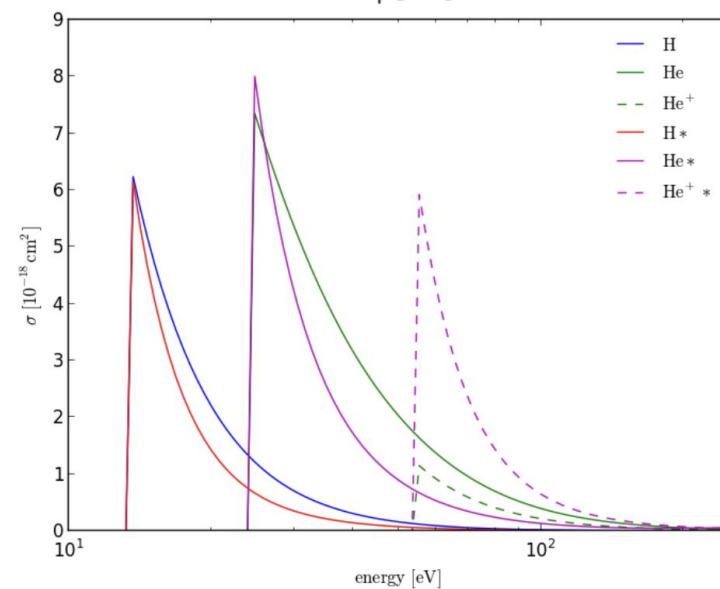
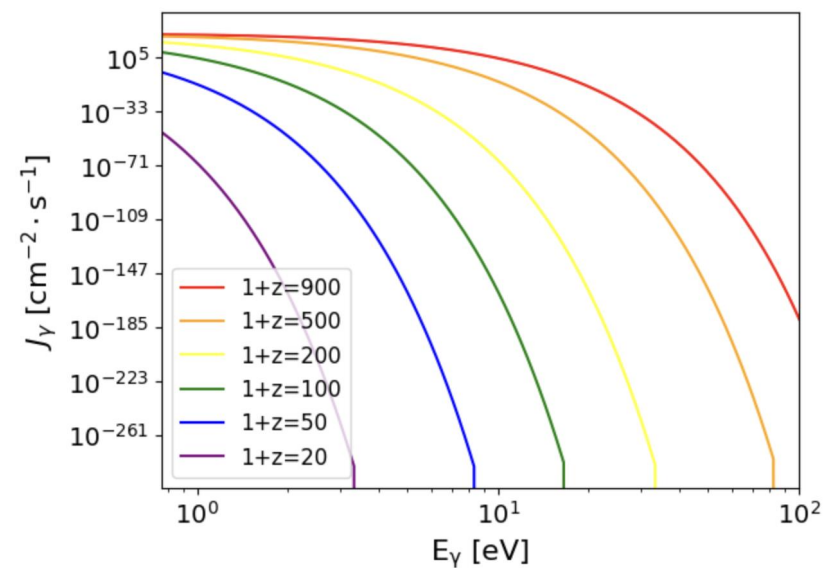
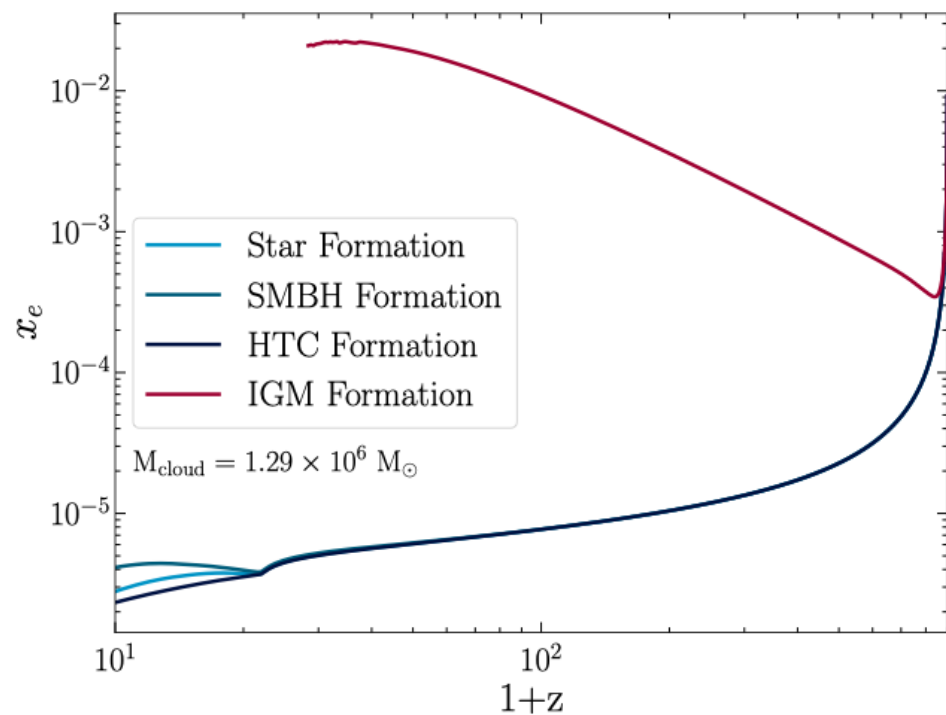
- HI Self-Shielding Fraction

$$\frac{\Gamma}{\langle \Gamma \rangle_{HI}} = (1 - f) \left[1 + \left(\frac{n_H}{n_0} \right)^\beta \right]^{\alpha_1} + f \left[1 + \frac{n_H}{n_0} \right]^{\alpha_2}$$

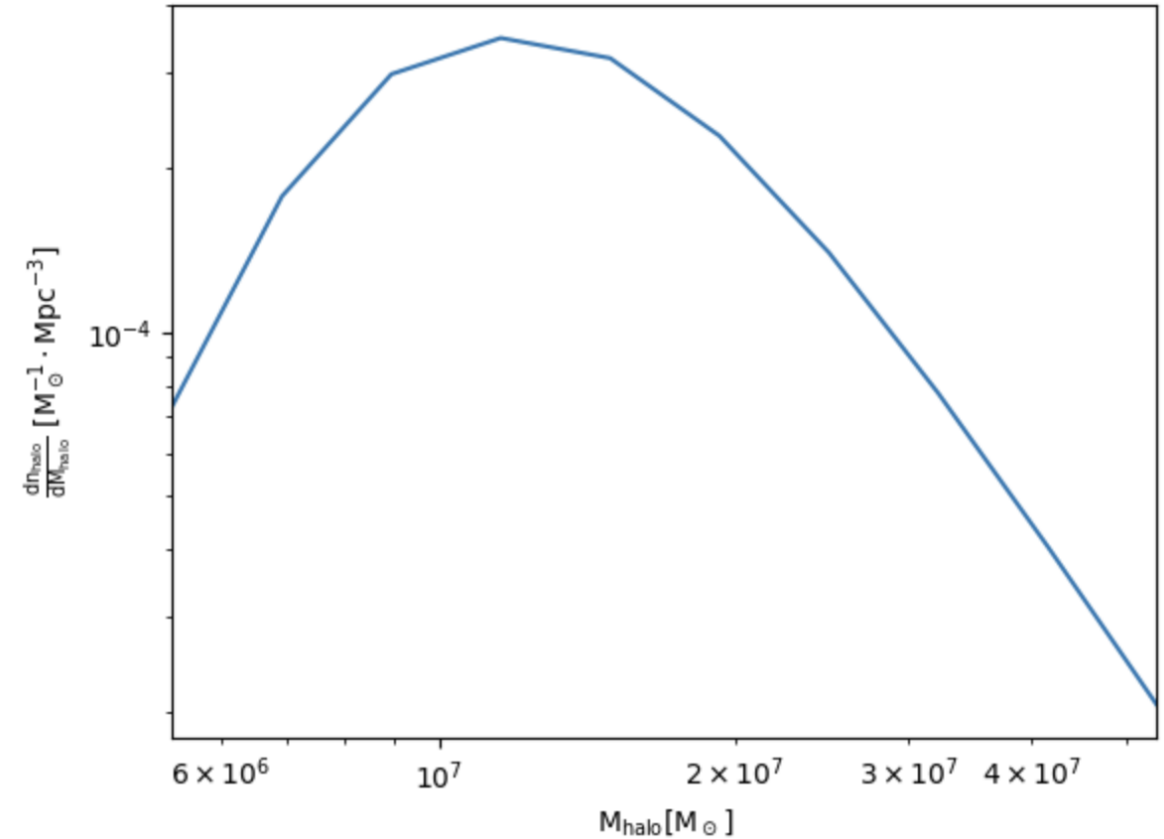
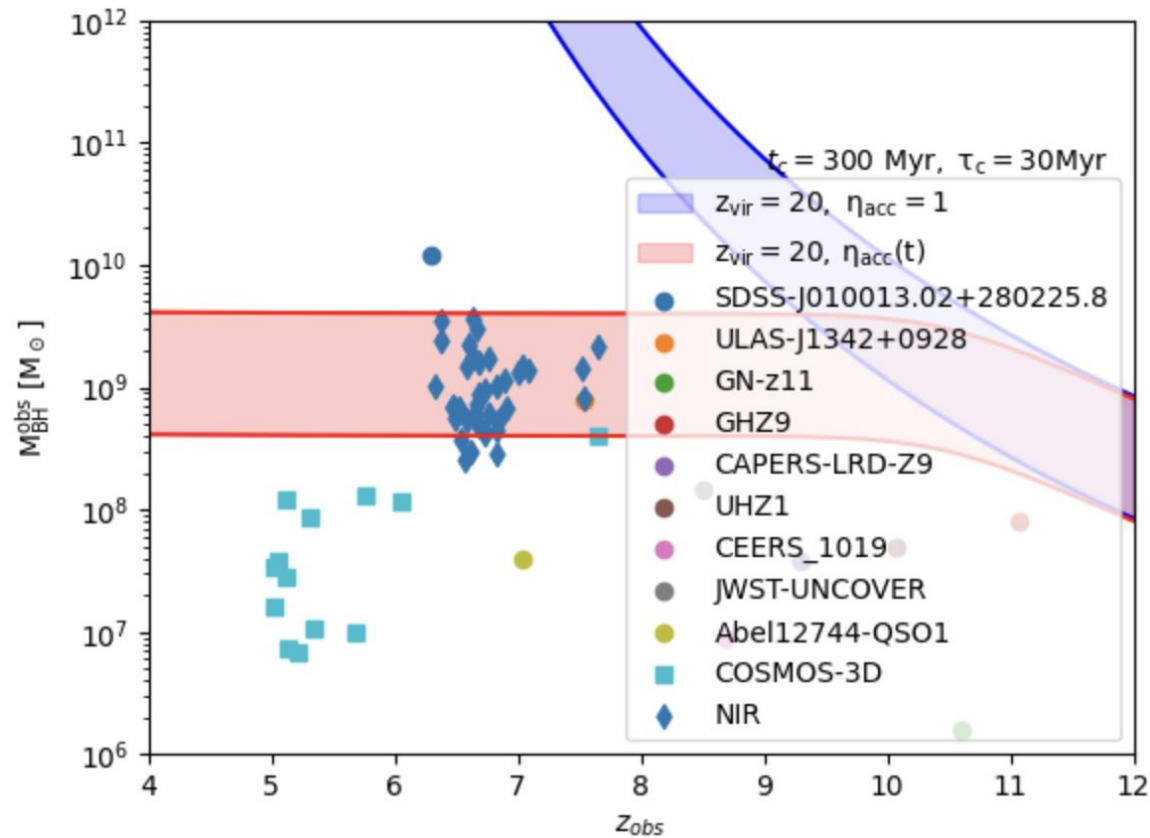
z	Γ_{HI} (10^{-12} s^{-1})	n_0 (cm^{-3})	α_1	α_2	β	f
3.0	1.13	0.0090	-1.12	-1.65	5.32	0.018
4.0	1.05	0.0093	-0.95	-1.50	5.87	0.015
5.0	0.90	0.0103	-1.29	-1.60	5.06	0.024
6.0	0.34	0.0070	-0.94	-1.51	6.11	0.029
7.0	0.07	0.0027	-0.86	-1.27	7.08	0.041
8.0	0.10	0.0040	-0.74	-1.40	7.12	0.041
9.0	0.14	0.0046	-0.64	-1.21	9.99	0.029
10.0	0.20	0.0047	-0.39	-0.86	12.94	0.006



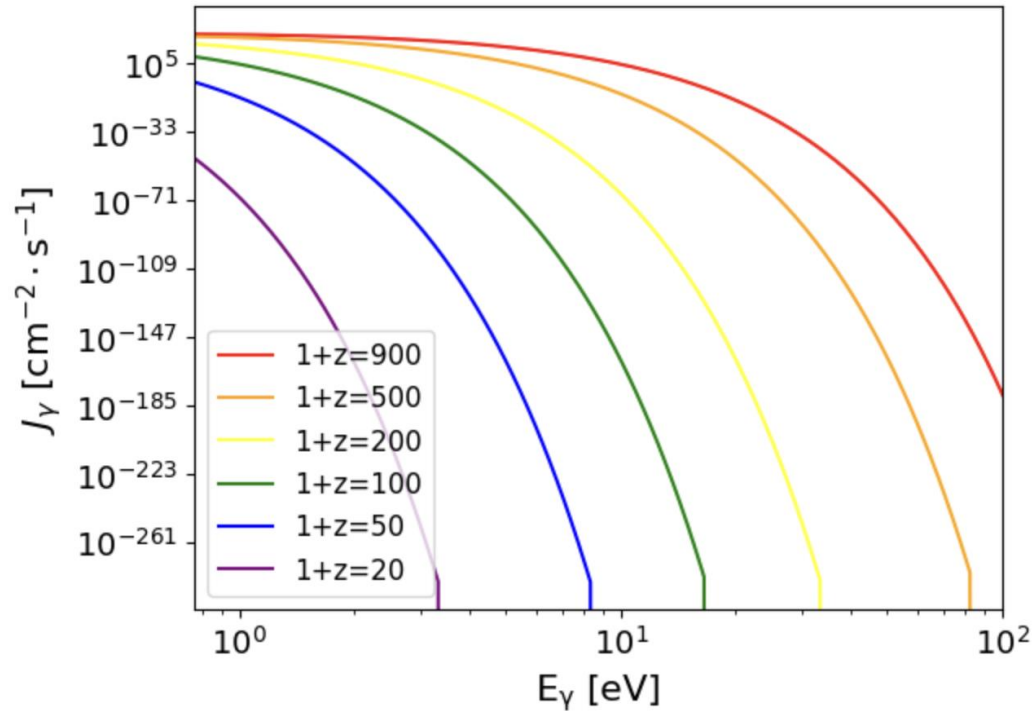
Backup: Ionized History



Backup: Accretion and Number Density of the Halo



Backup: CMB and Photon Flux



Decay DM

$$J(E) = \frac{f_\gamma f_{dm}}{\tau m_{dm}} E \frac{dN}{dE} \int dr' \rho_{dm}(r')$$

$$E \frac{dN}{dE} \Big|_{\text{box}} = \Theta \left(1 - \frac{E}{m_{dm}} \right)$$

$$E \frac{dN}{dE} \Big|_{\text{parabola}} = 6 \left[\frac{E}{m_{dm}} - \left(\frac{E}{m_{dm}} \right)^2 \right] \Theta \left(1 - \frac{E}{m_{dm}} \right)$$

$$\frac{d\Phi_\gamma}{dE_\gamma} = \frac{1}{4\pi} \frac{f_\gamma f_\chi \Omega_{DM} \rho_c}{\tau m_\chi} \int_{z_{step}}^{\infty} \frac{1}{H(z)} \frac{2}{E_\gamma} \delta \left(\frac{m_\chi}{E_\gamma} - \frac{1+z}{1+z_{step}} \right) dz$$

Annihilation DM

$$\frac{d\Phi_\gamma}{dE_\gamma} = \frac{1}{4\pi} \frac{\Omega_{DM}^2 \rho_c^2 \langle \sigma v \rangle}{2m_\chi^2} \int_{z_{step}}^{\infty} \frac{(1+z)^3}{H(z)} \frac{2}{E_\gamma} \delta \left(\frac{m_\chi}{E_\gamma} - \frac{1+z}{1+z_{step}} \right) dz$$

$$J_{bg}(1+z, E_\gamma) = E_\gamma \frac{d\Phi_\gamma}{dR_\gamma}$$

$$J_{ann}(E_\gamma) = \frac{3 \langle \sigma v \rangle}{m_\chi^2} \left[\frac{E_\gamma}{m_\chi} - \left(\frac{E_\gamma}{m_\chi} \right)^2 \right] \int_0^r \rho_{dm}^2(r') dr'$$

Backup: DM Halo Modeling

At very high redshift (e.g. $z \sim 1100$), before the overdensity becomes nonlinear, the mass shell is still moving with the Hubble flow. So the initial velocity of the mass shell is

$$v_{ini} = r_{ini} \times H(z), \quad (8)$$

However, we still need to consider the correction of this initial velocity since too fast initial velocity will delay the virialization of the Halo. From the Eq.5.43 in Baumann's book

$$\nabla \cdot \mathbf{v} = -a\dot{\delta}, \quad (9)$$

where $\mathbf{v} = a\dot{\mathbf{x}}$ is the peculiar velocity, δ is the density contrast. For spherically symmetric, the peculiar velocity is radial:

$$\mathbf{v} = v(r)\hat{\mathbf{r}}, \quad (10)$$

the divergence of $\mathbf{v}$ is

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{d}{dr} (r^2 v(r)). \quad (11)$$

Assume $v(r) = Ar$, where A is a constant, then

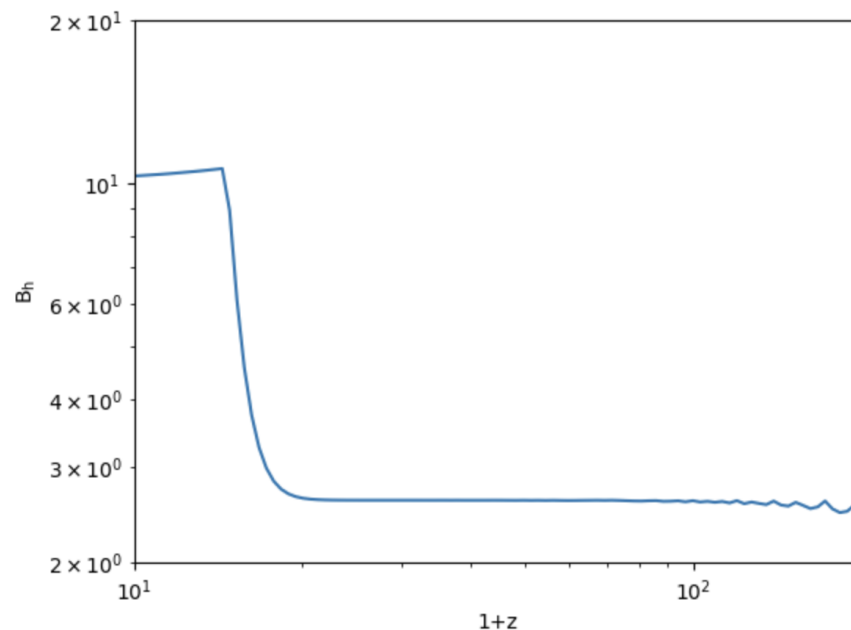
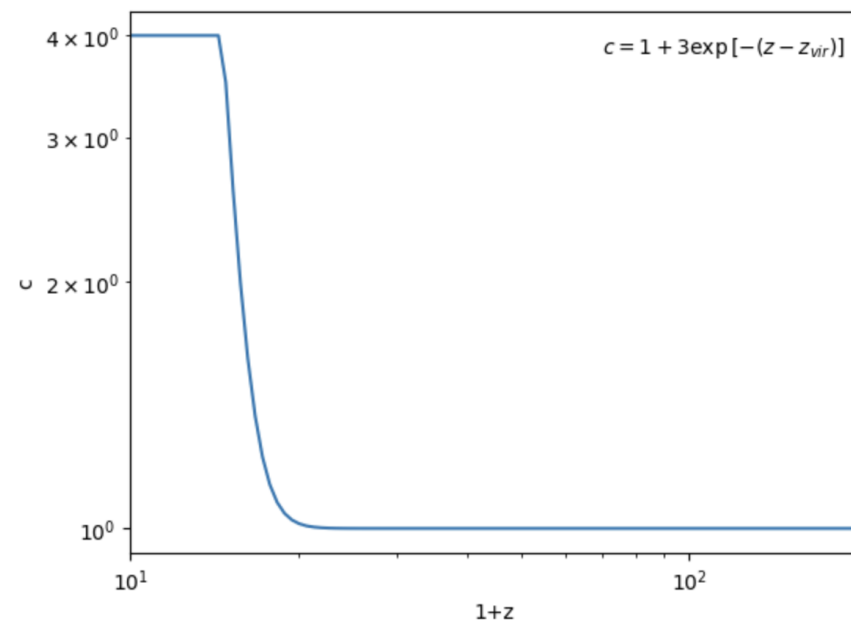
$$\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{d}{dr} (Ar^3) = 3A \Rightarrow A = -\frac{a\dot{\delta}}{3}, \quad v(r) = -\frac{a\dot{\delta}}{3}r. \quad (12)$$

We should be careful with the notation r in the above calculation, actually it is in the comoving coordinate. Since the physical velocity is

$$\mathbf{v}_{physical} = \dot{a}r_{phy} + \dot{r}_{phy}a,$$

meanwhile, it is reasonable to estimate the time derivative of density contrast as $\dot{\delta} \approx H\delta$, therefore, the velocity of halo is

$$v_{phy} = Hr_{phy} + v = Hr_{phy} \left(1 - \frac{\delta}{3}\right). \quad (14)$$



Backup: DM Halo Modeling

the external photon flux from the background dark matter decay/annihilation is

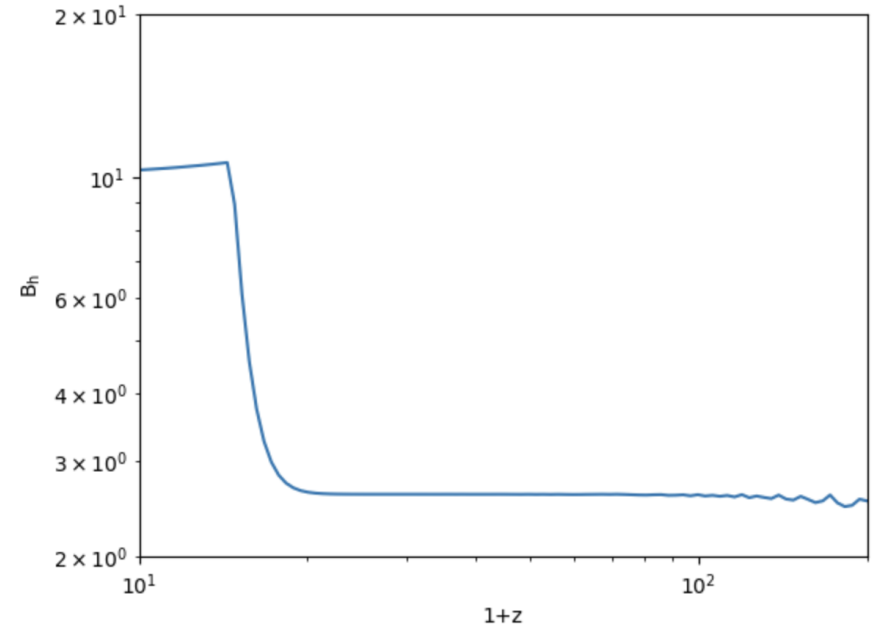
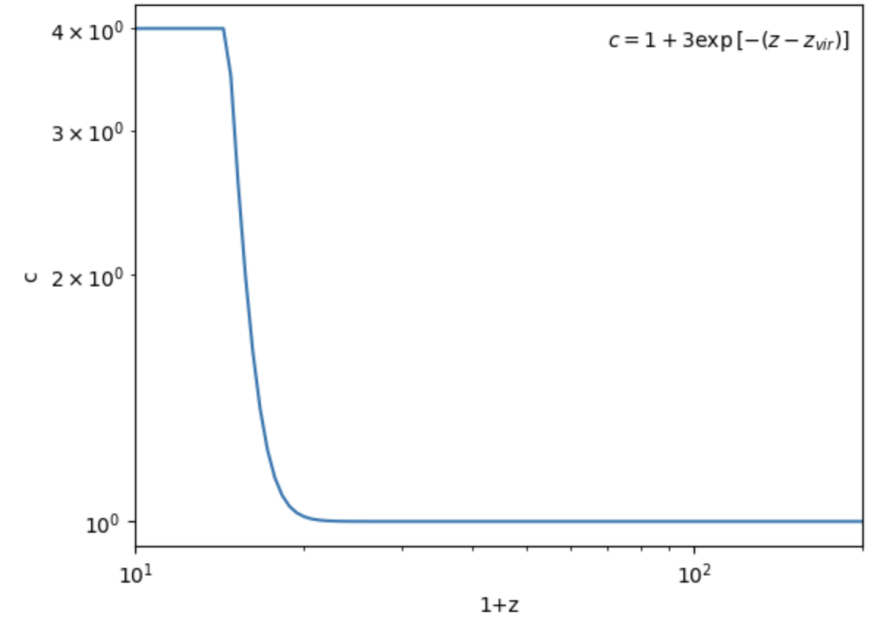
$$J(E) = (1 + z_{obs})^3 \int_{z_{obs}}^{\infty} \frac{dz}{H(z) (1 + z)^3} E' \frac{dn'_{\gamma}}{dE' dt'},$$

$$J_{decay, halo}(E, z) = \frac{f_{\gamma} f_{dm} \bar{\rho}_{dm}(z) R_{dm}(z)}{\tau m_{dm}} E \frac{dN}{dE},$$

$$J_{decay, background}(E, z_{obs}) = \frac{(1 + z_{obs})^3}{4\pi} \frac{f_{\gamma} f_{dm} \Omega_{dm} \rho_{crit}}{\tau m_{dm}} \int_{z_{obs}}^{\frac{m_{dm}(1+z_{obs})}{E} - 1} \frac{dz}{H(z)}$$

$$J_{ann, halo}(E, z) = \frac{6 f_{\gamma} f_{dm}^2 \bar{\rho}_{dm}^2(z) R_{dm}(z) \langle \sigma v \rangle}{2^p m_{dm}^2} \left[\frac{E}{m_{dm}} - \left(\frac{E}{m_{dm}} \right)^2 \right] \Theta \left(1 - \frac{E}{m_{dm}} \right)$$

$$J_{ann, background} = \frac{3}{2\pi} \frac{f_{\gamma} f_{dm}^2 \Omega_{dm}^2 \rho_{crit}^2 \langle \sigma v \rangle}{2^p m_{dm}^2} \int_{z_{obs}}^{\frac{m_{dm}(1+z_{obs})}{E} - 1} dz \left[\frac{(1 + z_{obs})^2 E (1 + z)^4}{m_{dm} H(z)} - \frac{(1 + z_{obs}) E^2 (1 + z)^5}{m_{dm}^2 H(z)} \right]$$



Backup: Gas Cloud Mass

