Finite temperature and density effects and BSM phenomenology

Hugo Schérer

In collaboration with Nirmalya Brahma, Saniya Heeba, Katelin Schutz (arXiv:2507.14277)

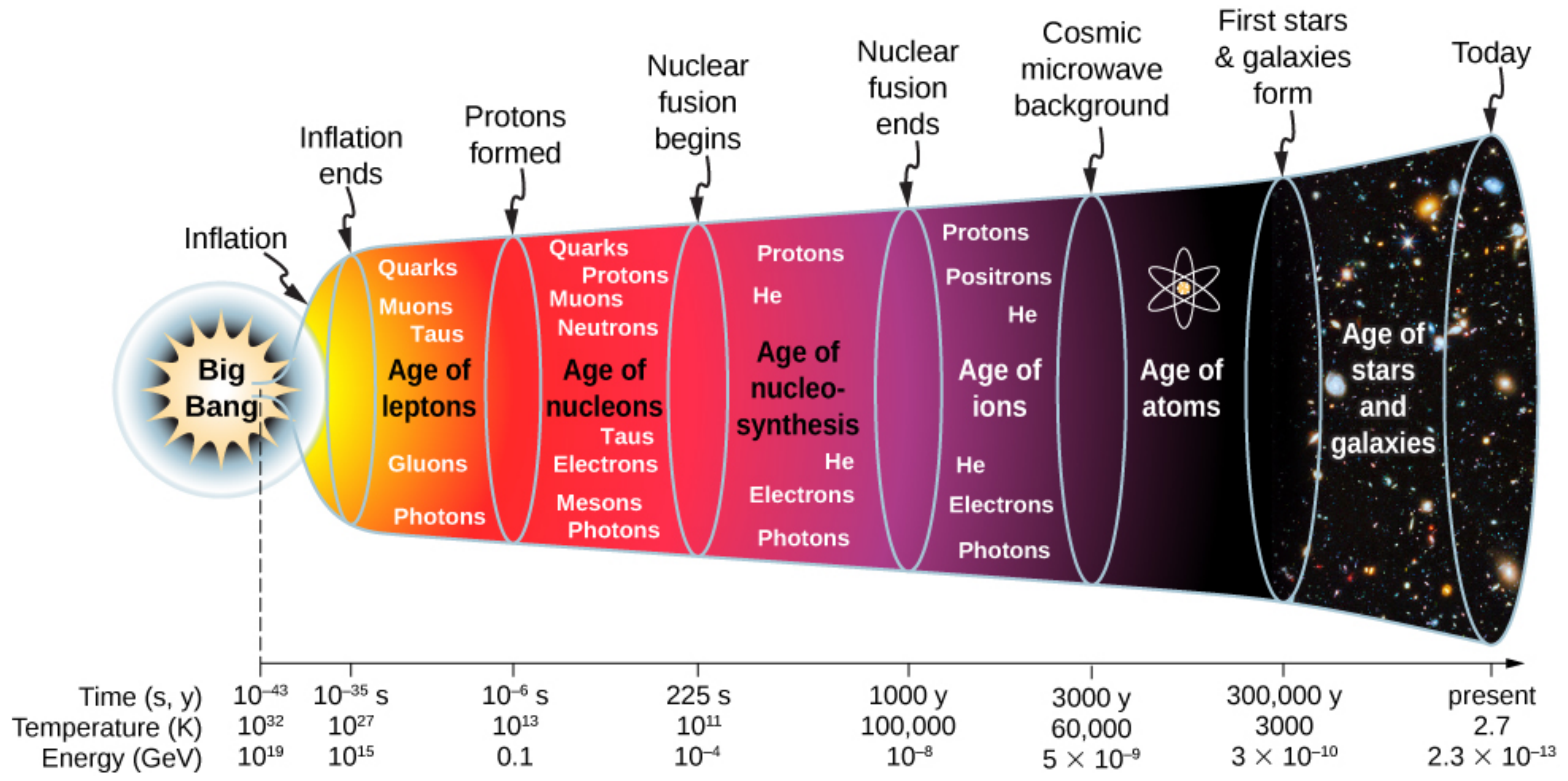
Light Dark World (July 30th 2026)

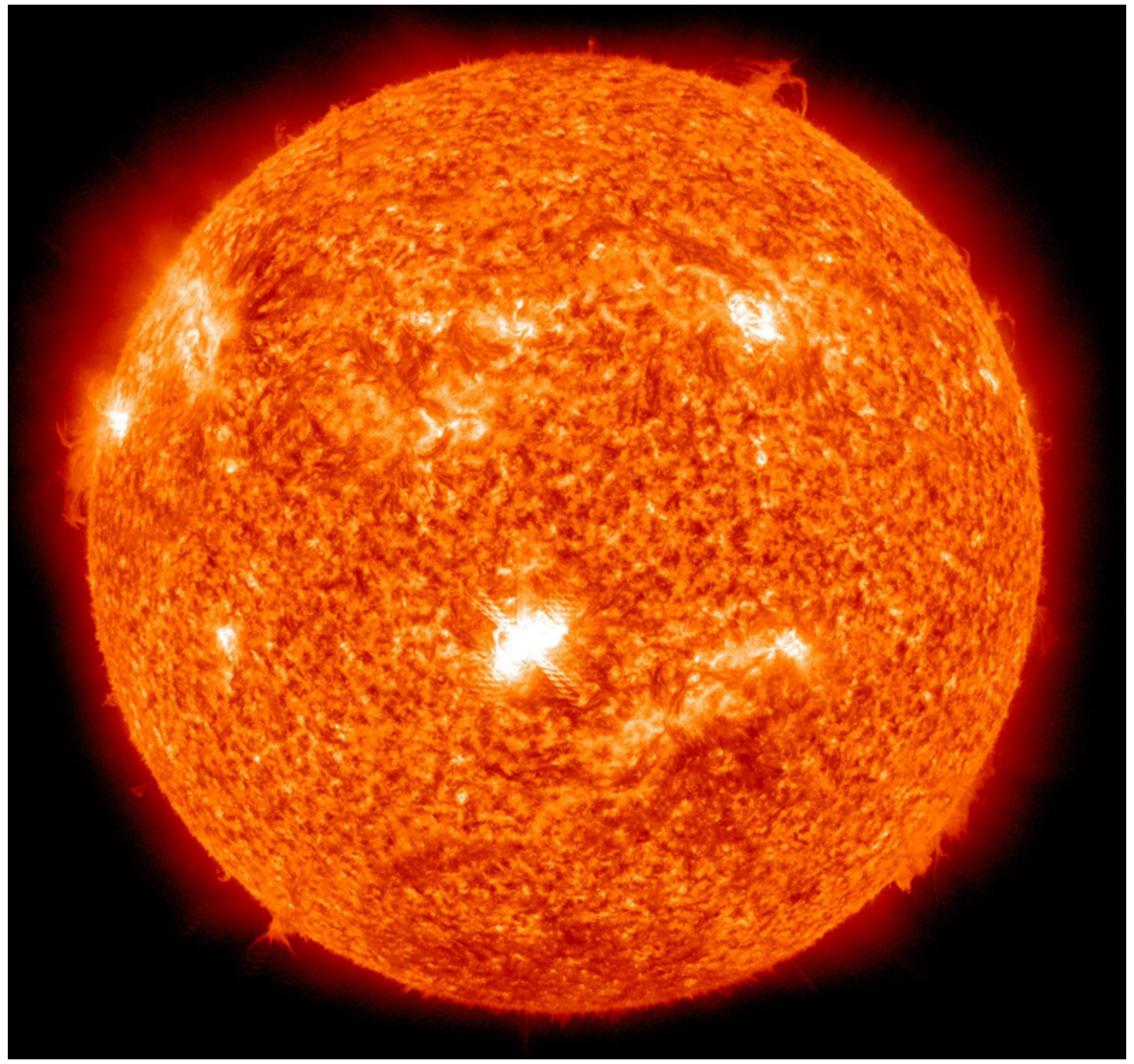
☞ We should not neglect a thermal effect

Thermal effects are *essential* for accurate BSM predictions

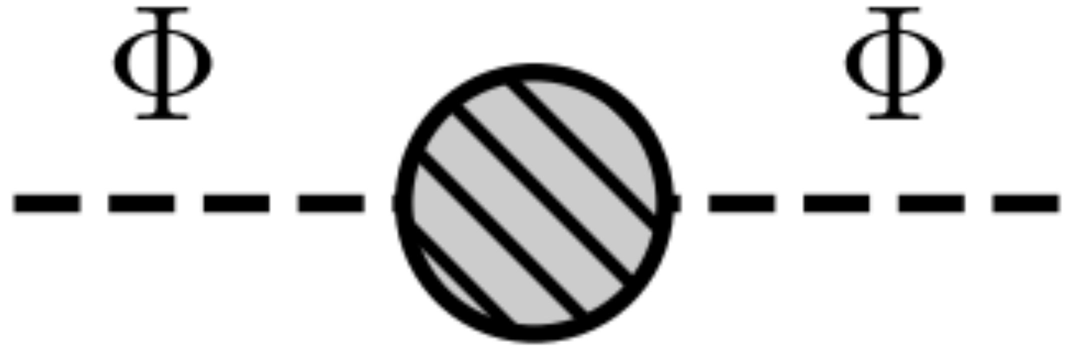
Finite temperature and density effects

- The universe is not a vacuum!
- Many environments where we look for BSM are hot and dense medium





The self-energy Π



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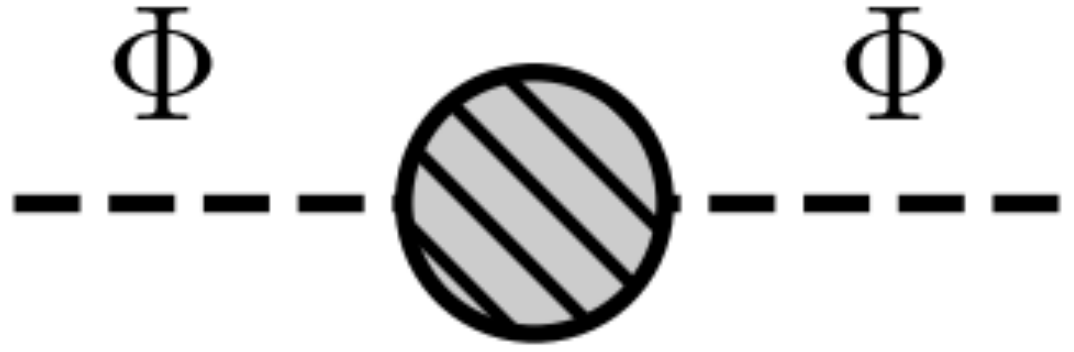

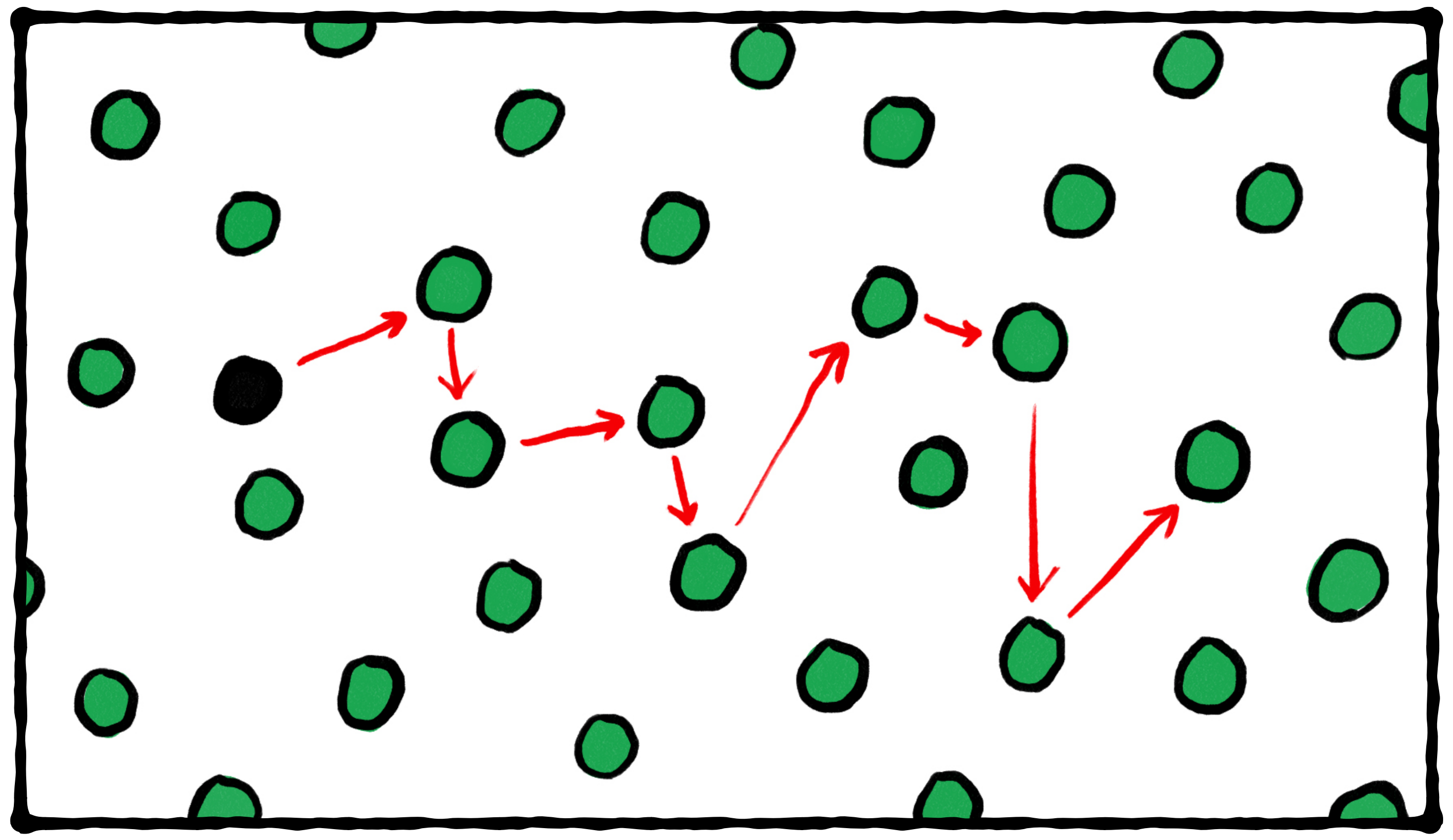

- **Real part**: mass (squared) correction to the dispersion

$$\omega^2 - \mathbf{k}^2 = m_\Phi^2 + \mathrm{Re}\,\Pi_\Phi$$

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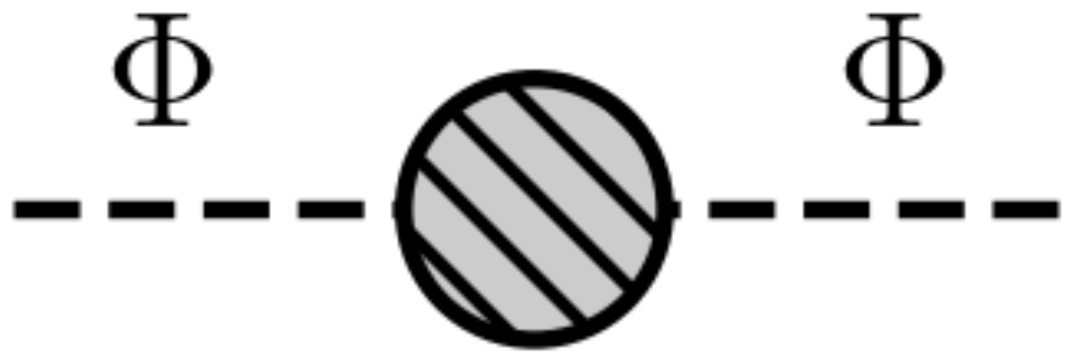

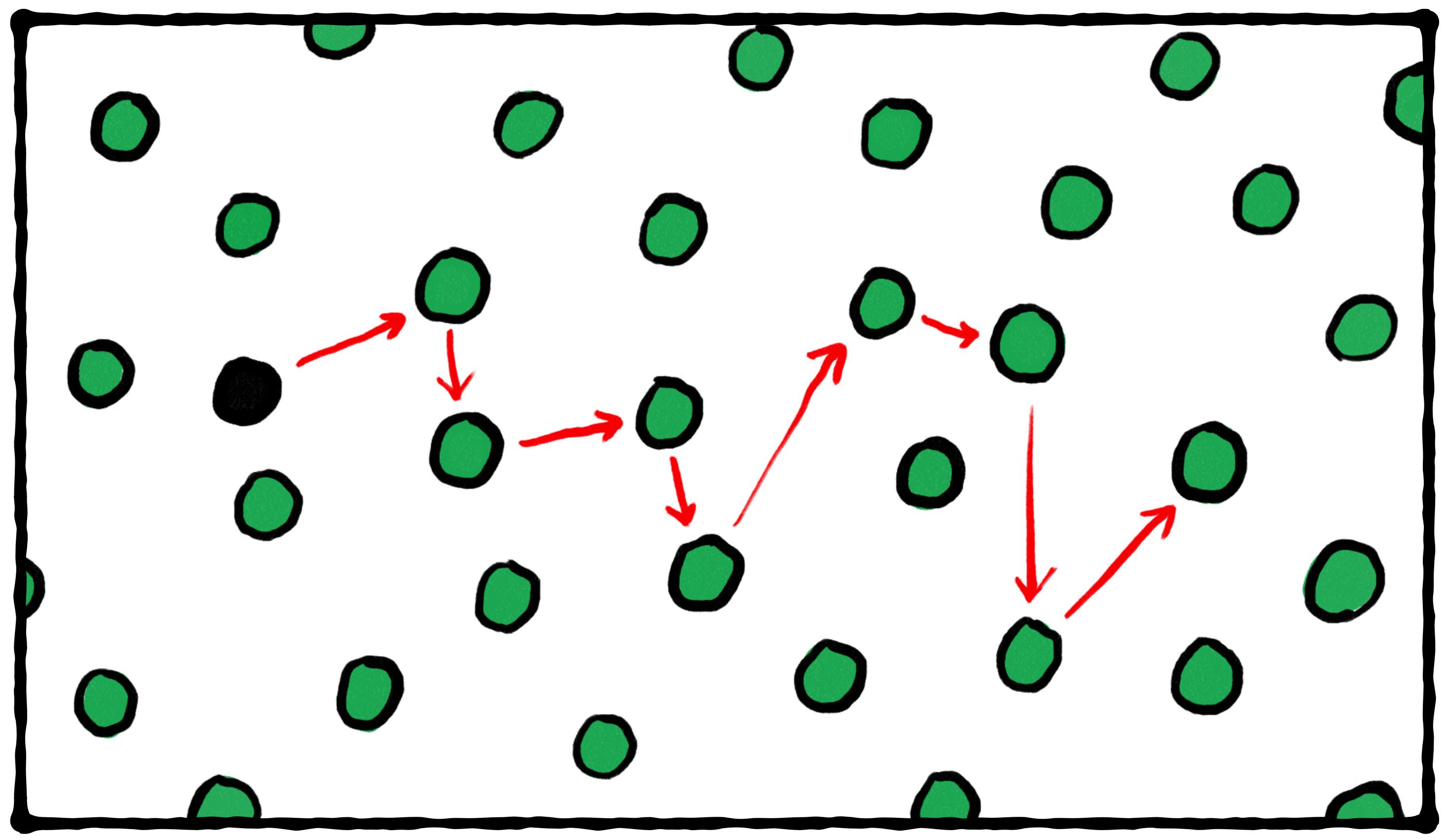

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- **Imaginary part**: particle production and absorption rate

$$\frac{\mathrm{d}n_\Phi}{\mathrm{d}t} = -\int \frac{\mathrm{d}^3k}{(2\pi)^3 2\omega} \left(f_\Phi^{\mathrm{eq}} - f_\Phi\right) 2\,\mathrm{Im}\,\Pi_\Phi$$

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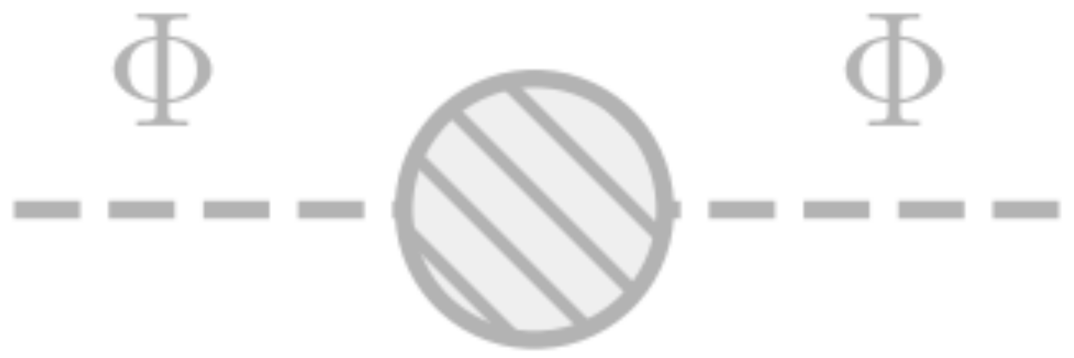

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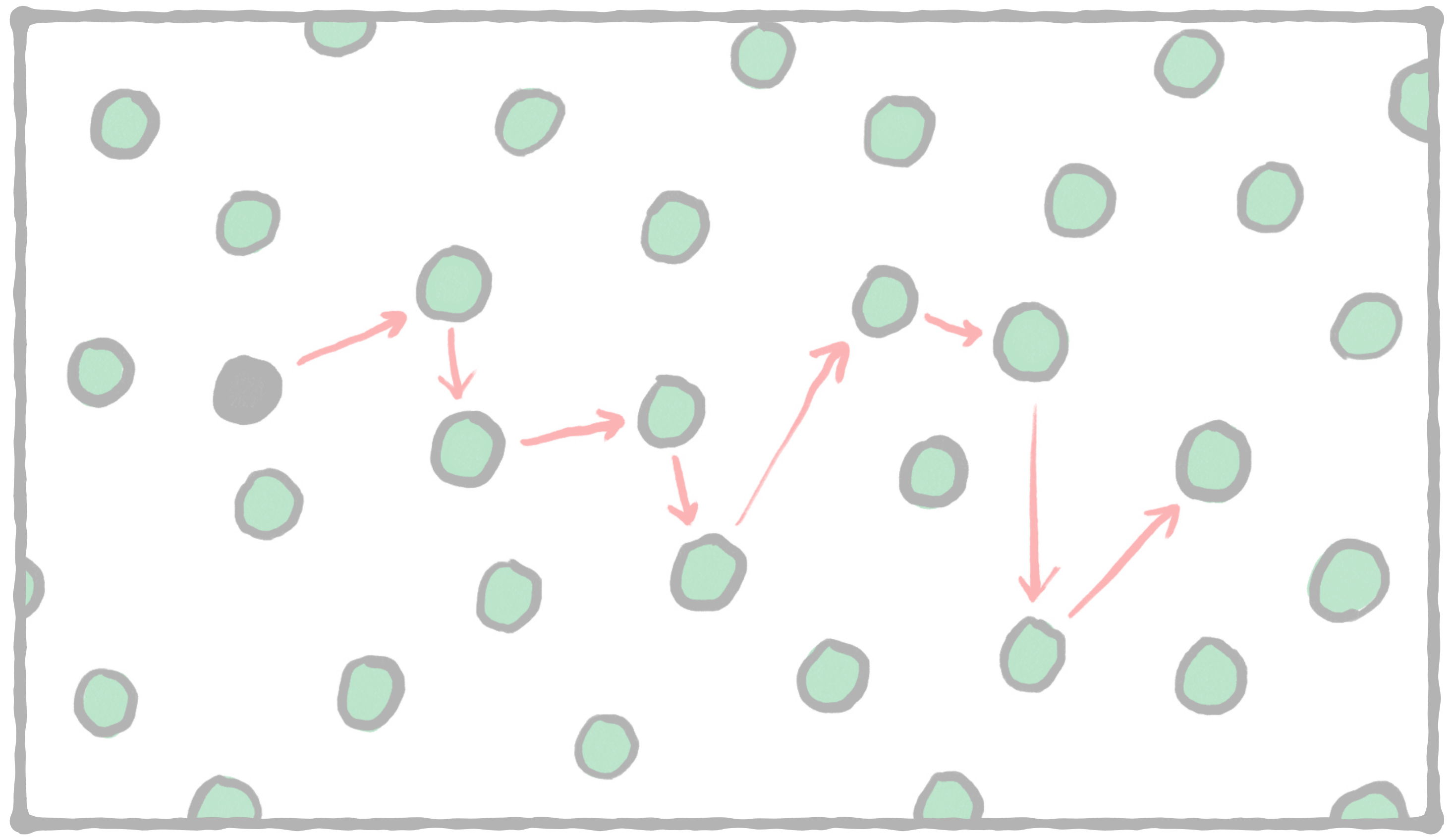

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Similar to optical theorem —
weighted by thermal phase space
distributions

Finite Temperature Field Theory (FTFT)

- **What is FTFT**: Quantum Field Theory at non-zero temperature and/or density
- **Equilibrium**: imaginary-time formalism

$$\rho(T) = e^{-H/T} \quad \leftrightarrow \quad U(t) = e^{-itH}$$

- **How to do FTFT**:

Same diagrammatic rules, but discrete imaginary frequencies

$\omega_n = i2n\pi T$ (boson) $\qquad \omega_n = i(2n+1)\pi T$ (fermion)

$$\int \frac{d^4p}{(2\pi)^4} f(p_0, \mathbf{p}) \to T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} f(\omega_n + \mu, \mathbf{p})$$

Finite Temperature Field Theory (FTFT)

- **What** density
- **Equilil**
- **How t**

☞ ***Spectral surgery in a heat bath:***

user-friendly cutting rules
that cut complexity, but not corners!

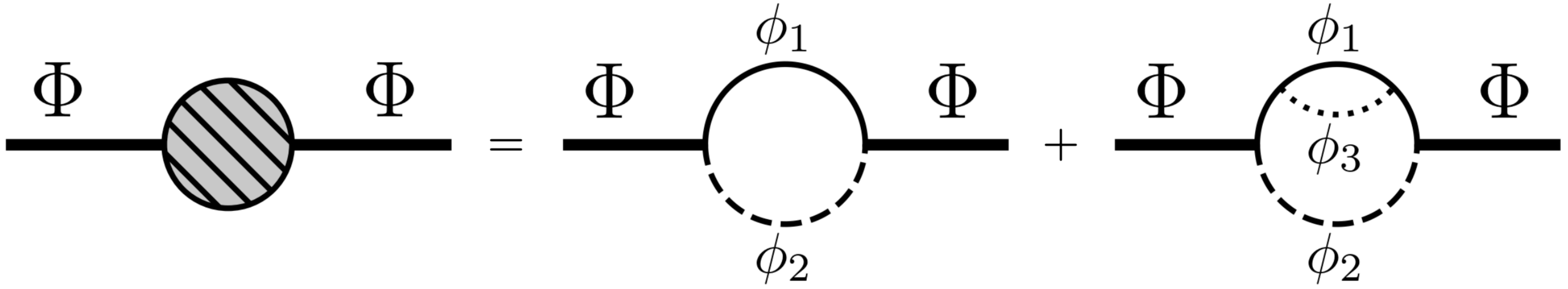
2507.14277

$\mathrm{Im}\,\Pi$: Spectral Surgery in a Heat Bath (SSHB)

- Toy model with four scalar particles

$$\mathcal{L}_{\mathrm{int}} = g\Phi\phi_1\phi_2 + \frac{1}{2}\lambda\phi_1^2\phi_3$$

- Self-energy up to two loops



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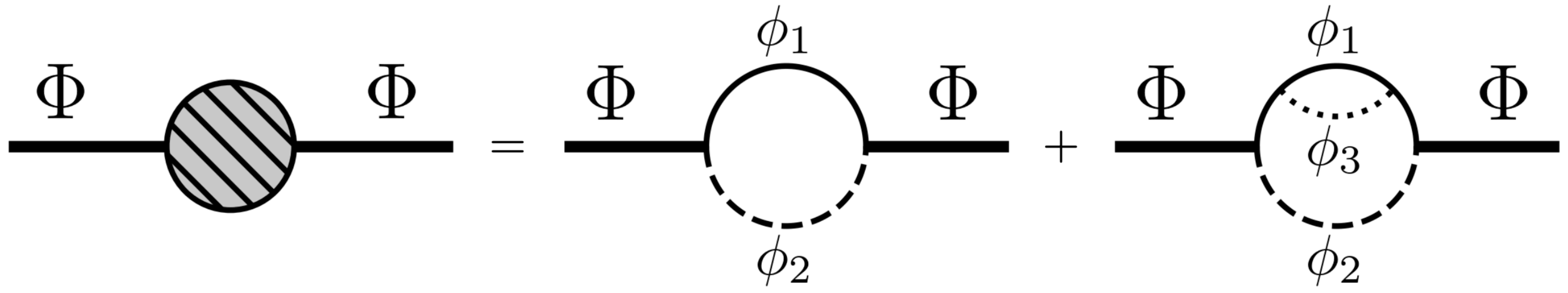
$\mathrm{Im}\,\Pi$: Spectral Surgery in a Heat Bath (SSHB)

- Step 0: Self-energy diagrams



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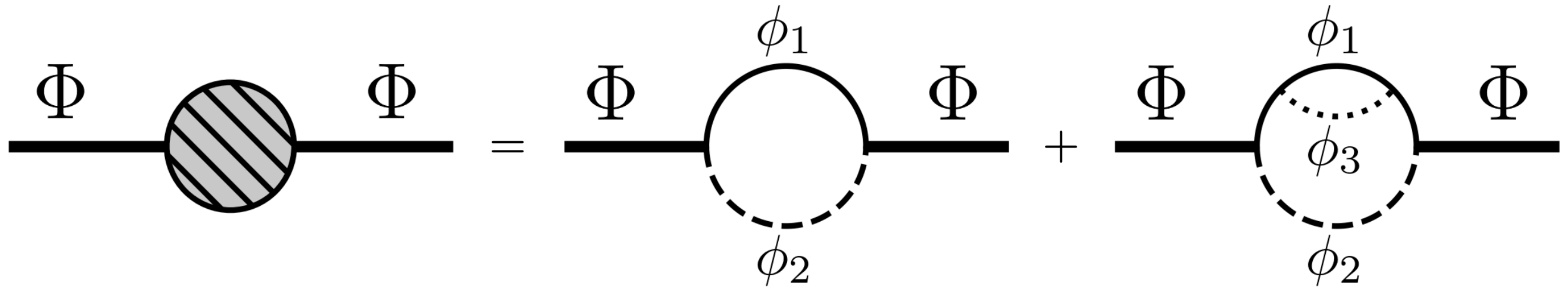

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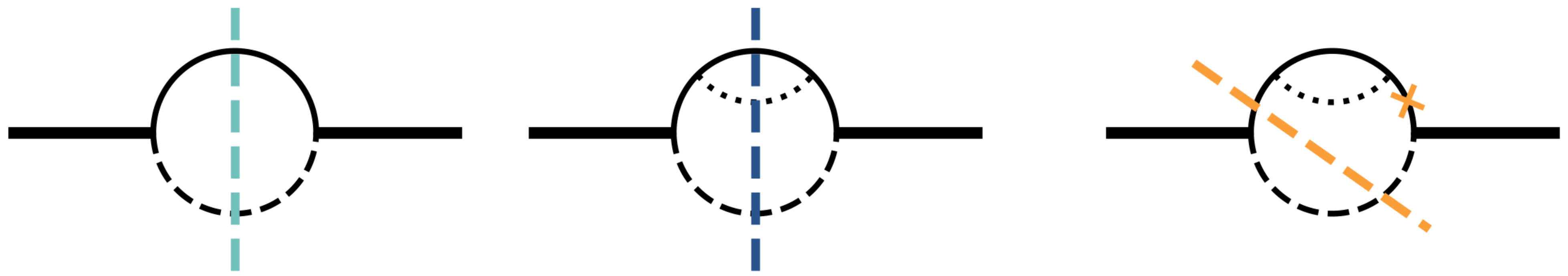

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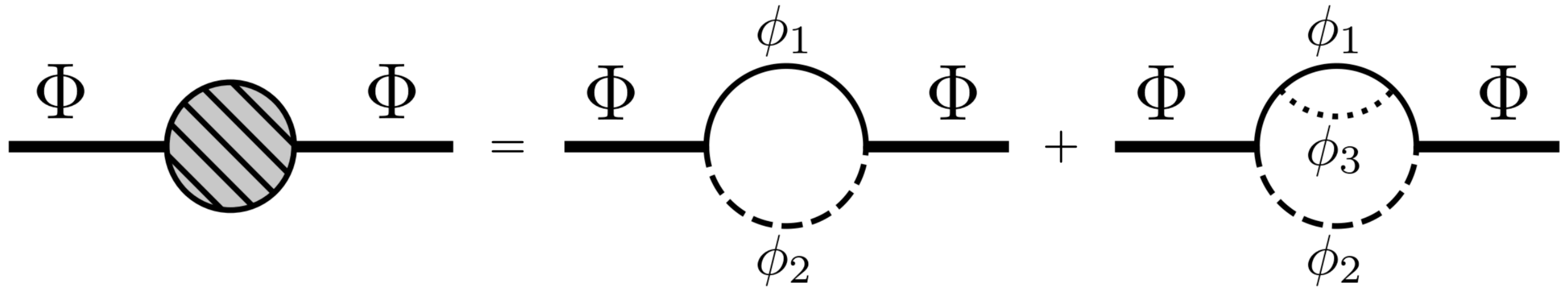

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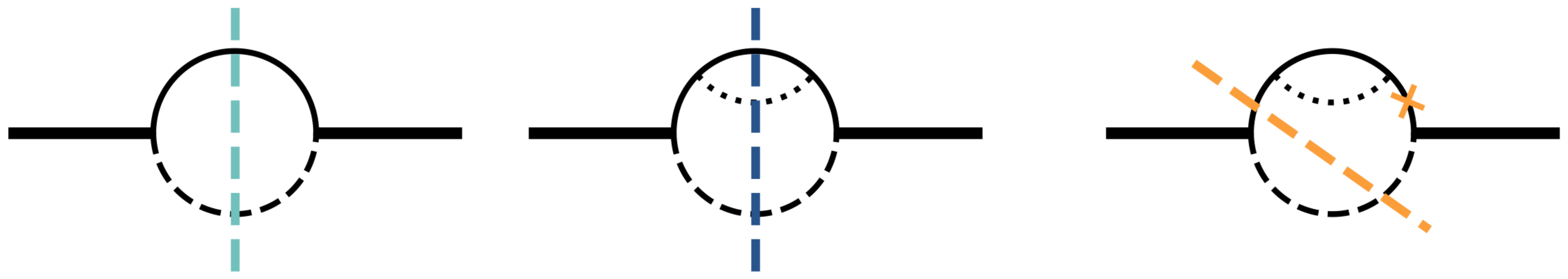
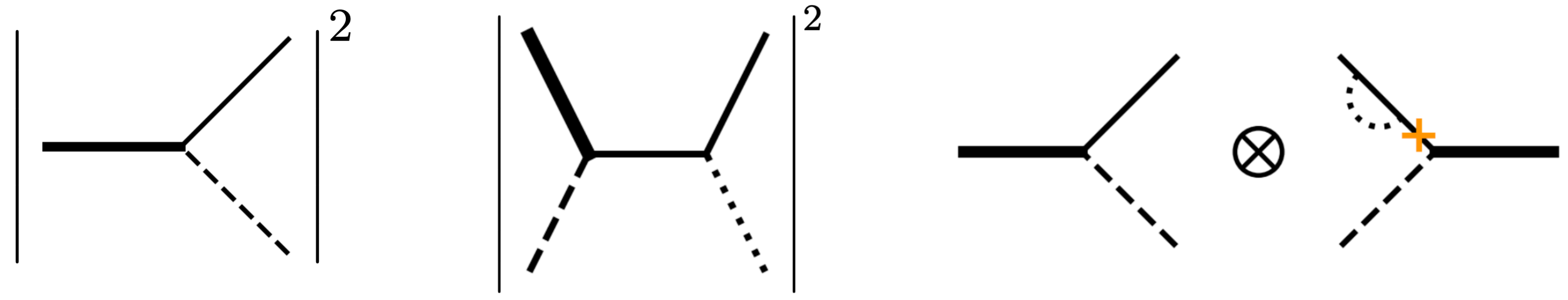


- Step 2: Converting bisections into processes



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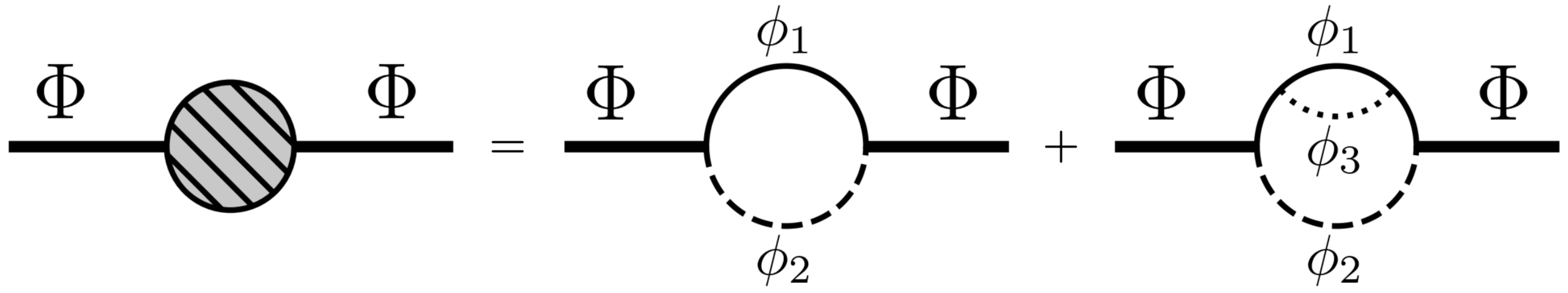

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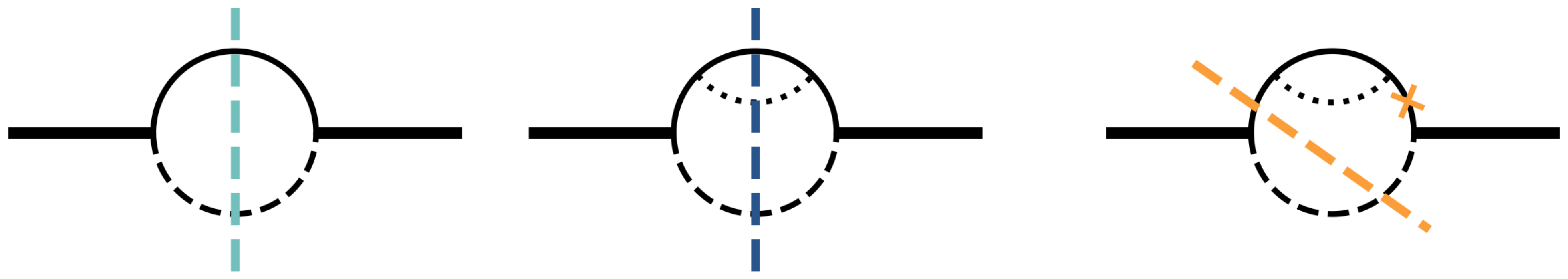

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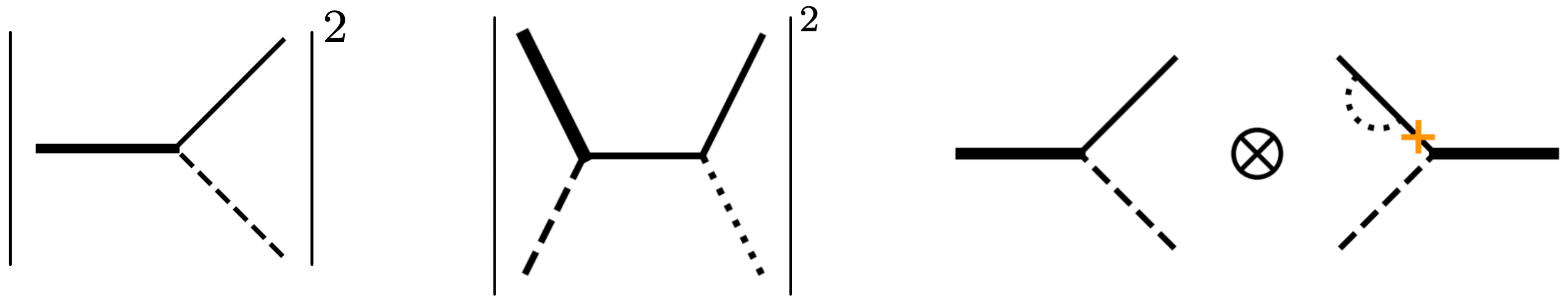
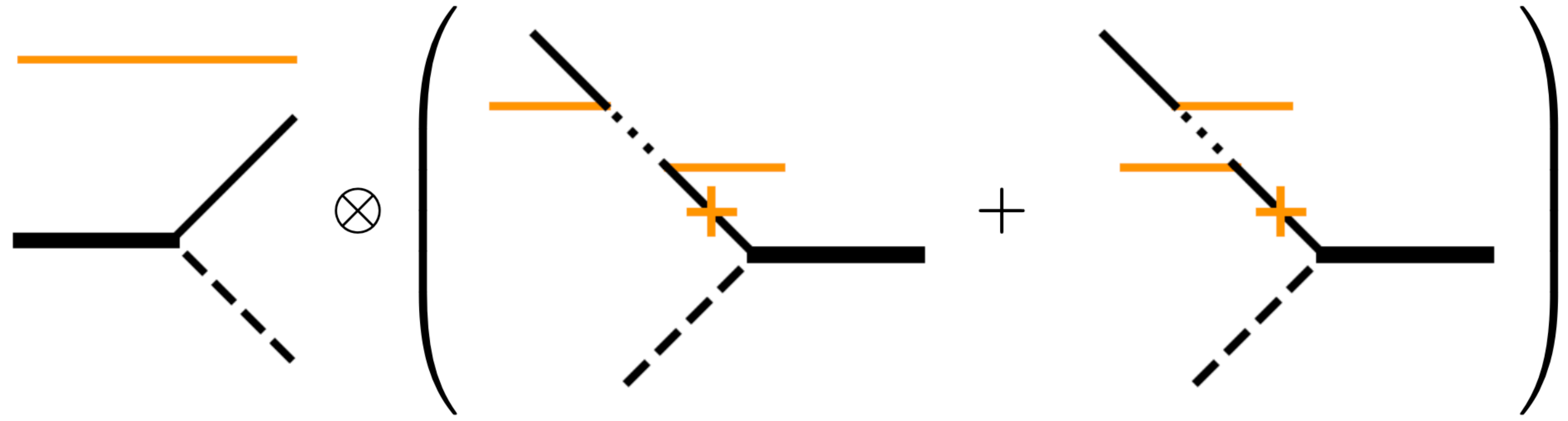


- Step 3: Loopectomy (internal loop partial cuts)



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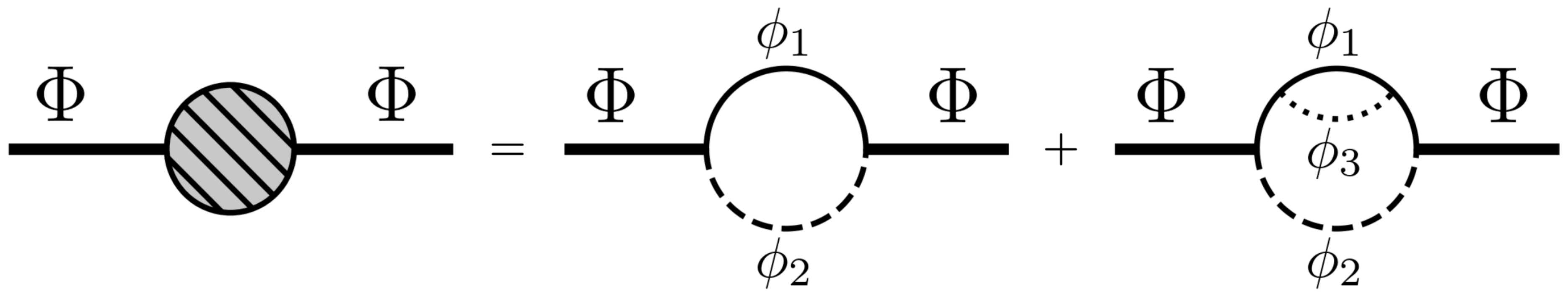

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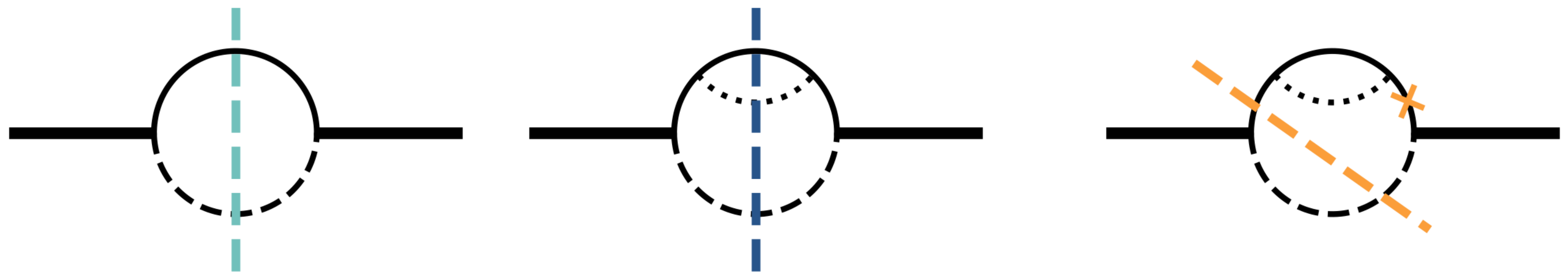

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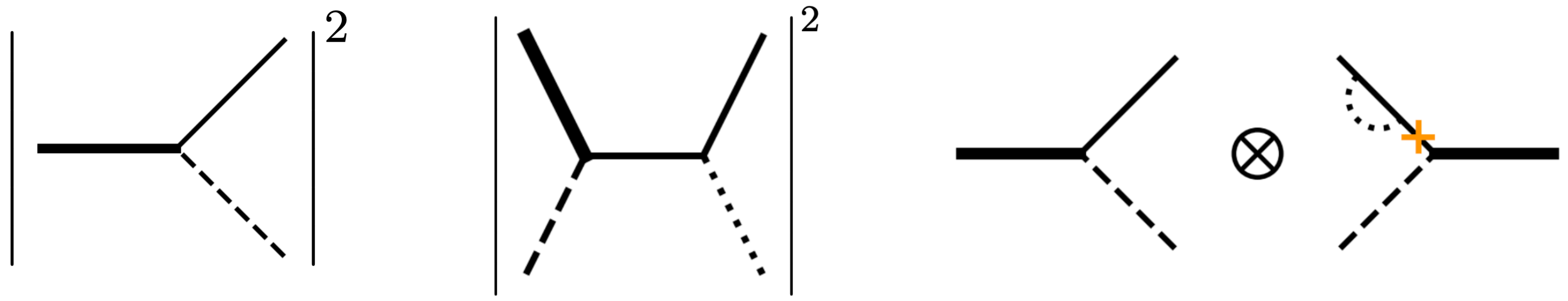

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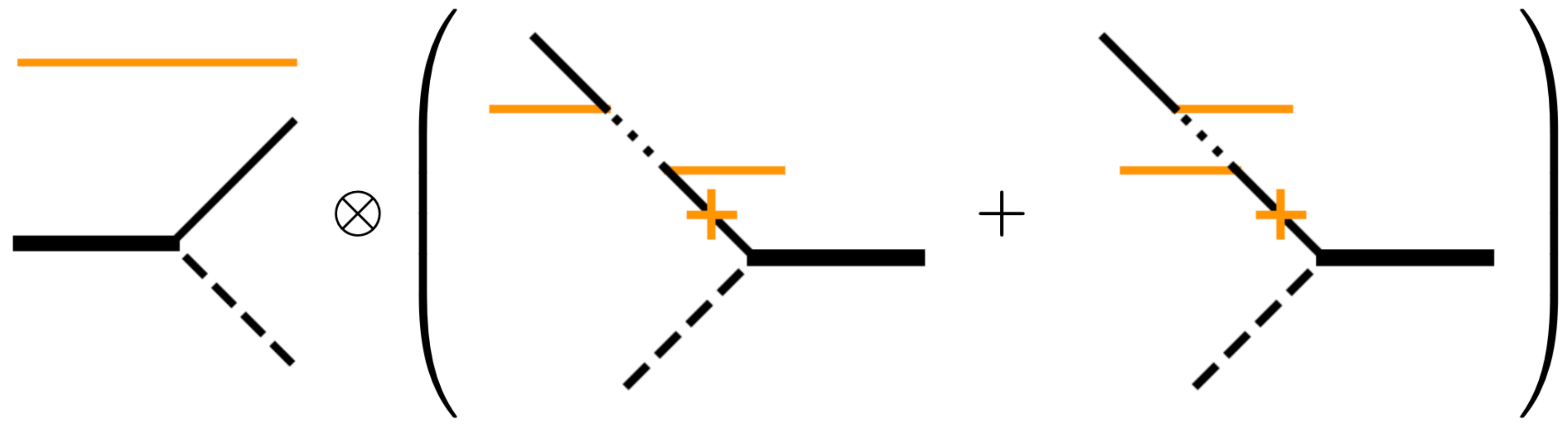

- Step 4: Sum over all these tree processes

$\boxed{\text{Cut = on-shell}}$ $\boxed{\text{Phase space distributions}}$

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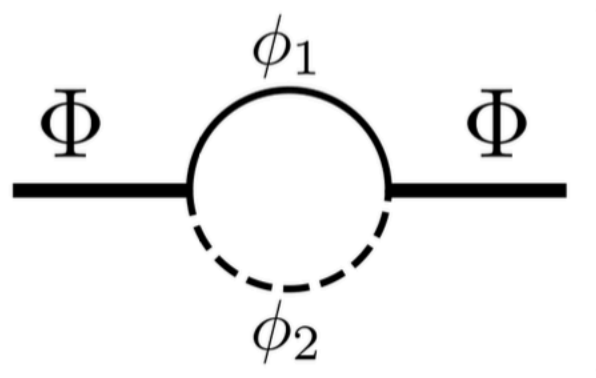
$\mathrm{Im}\,\Pi$: Spectral Surgery in a Heat Bath (SSHB)

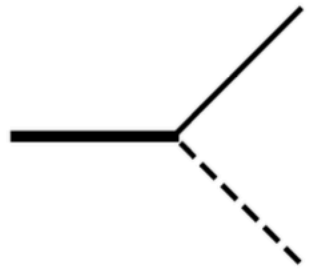
- Example: one-loop contribution

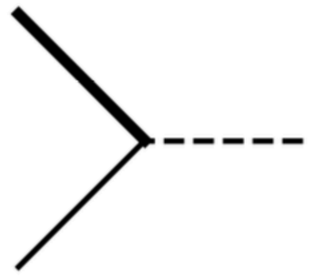
Cut propagators are on-shell

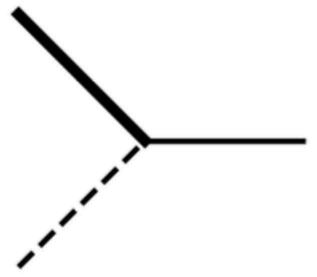
Phase space factors, absorption - emission

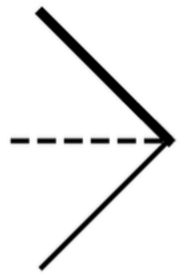
Usual momentum conservation

$\mathrm{Im}\Bigg($  $\Bigg) = -\frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}$

$\times \Big\{ (2\pi)^4\delta^{(4)}(k-p_1-p_2) \Big|$  $\Big|^2 [(1+f_1)(1+f_2)-f_1 f_2]$

$+ (2\pi)^4\delta^{(4)}(k+p_1-p_2) \Big|$  $\Big|^2 [f_1(1+f_2)-f_2(1+f_1)]$

$+ (2\pi)^4\delta^{(4)}(k-p_1+p_2) \Big|$  $\Big|^2 [f_2(1+f_1)-f_1(1+f_2)]$

$+ (2\pi)^4\delta^{(4)}(k+p_1+p_2) \Big|$  $\Big|^2 [f_1 f_2-(1+f_2)(1+f_1)] \Big\}$

$\mathrm{Im}\,\Pi$: Spectral Surgery in a Heat Bath (SSHB)

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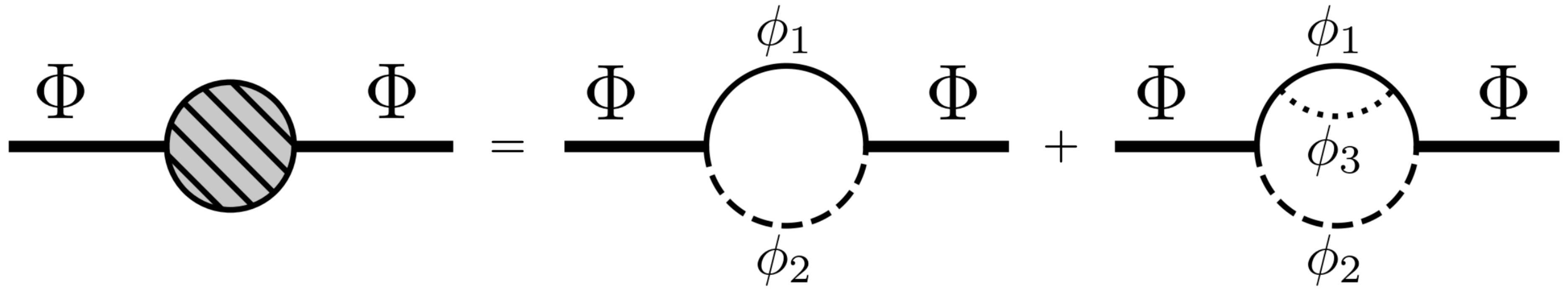

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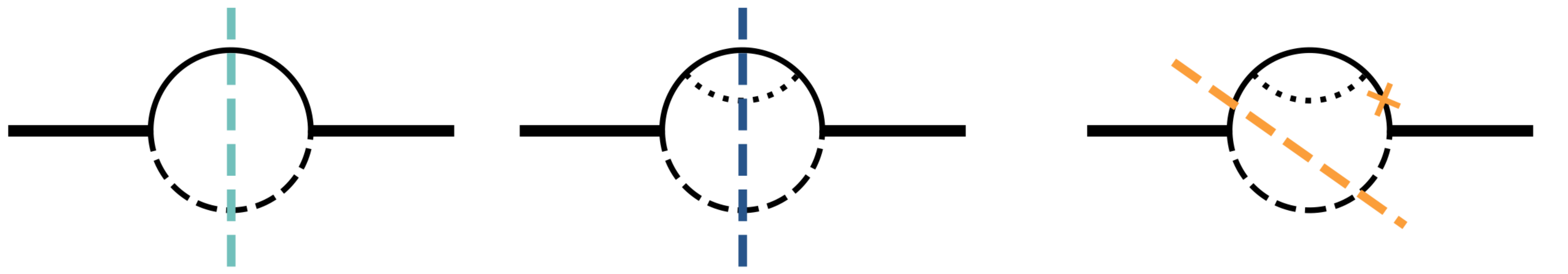

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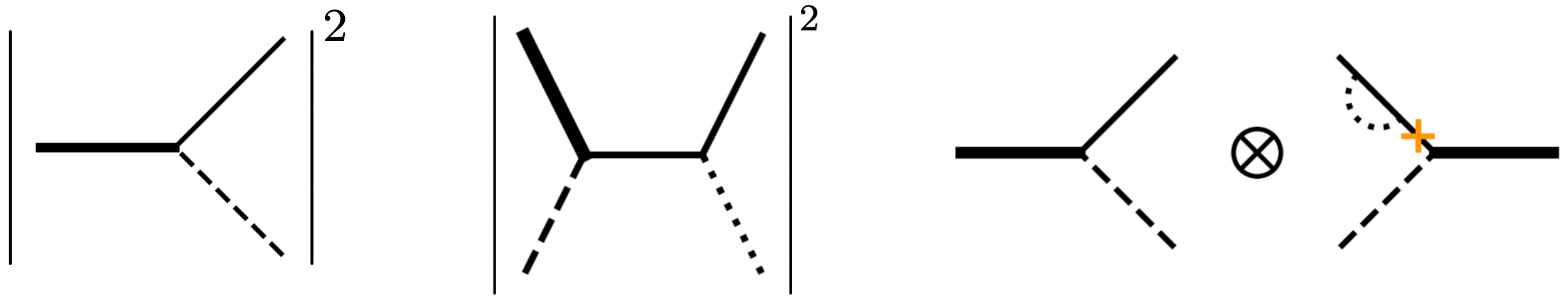

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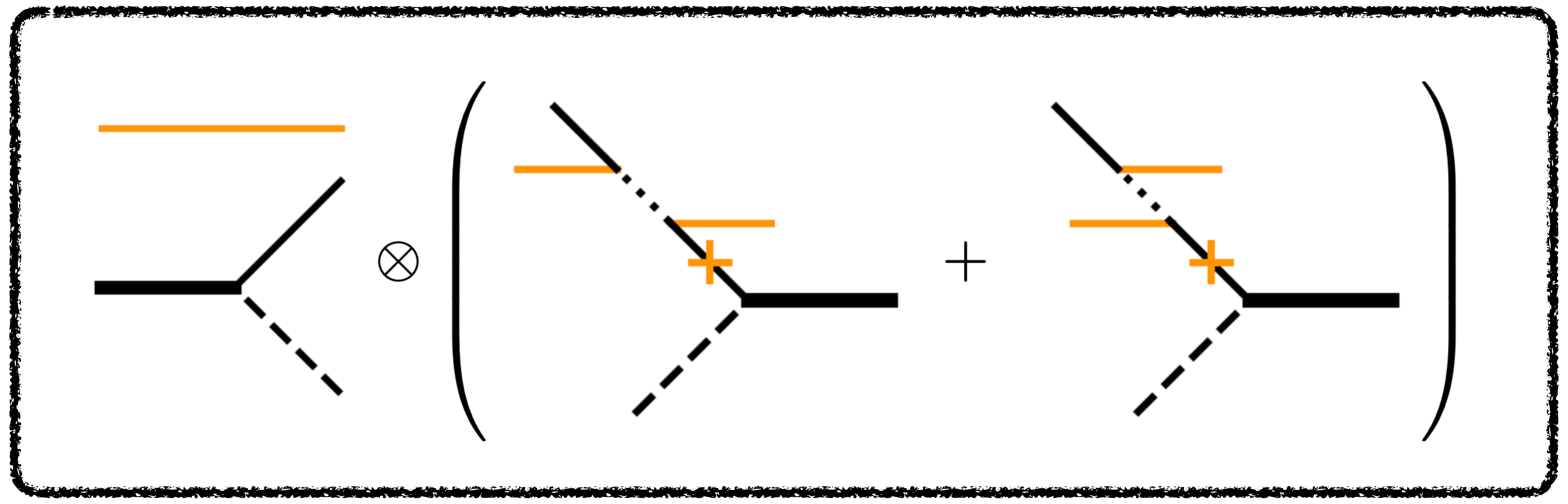

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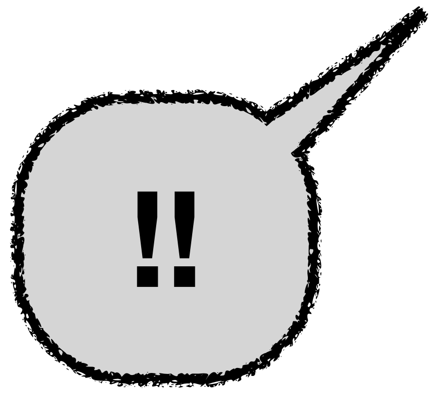
Cut = on-shell

Phase space distributions



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☞ Thermal *interference* processes,

from ***asymmetric*** cuts,

can significantly impact

production rates

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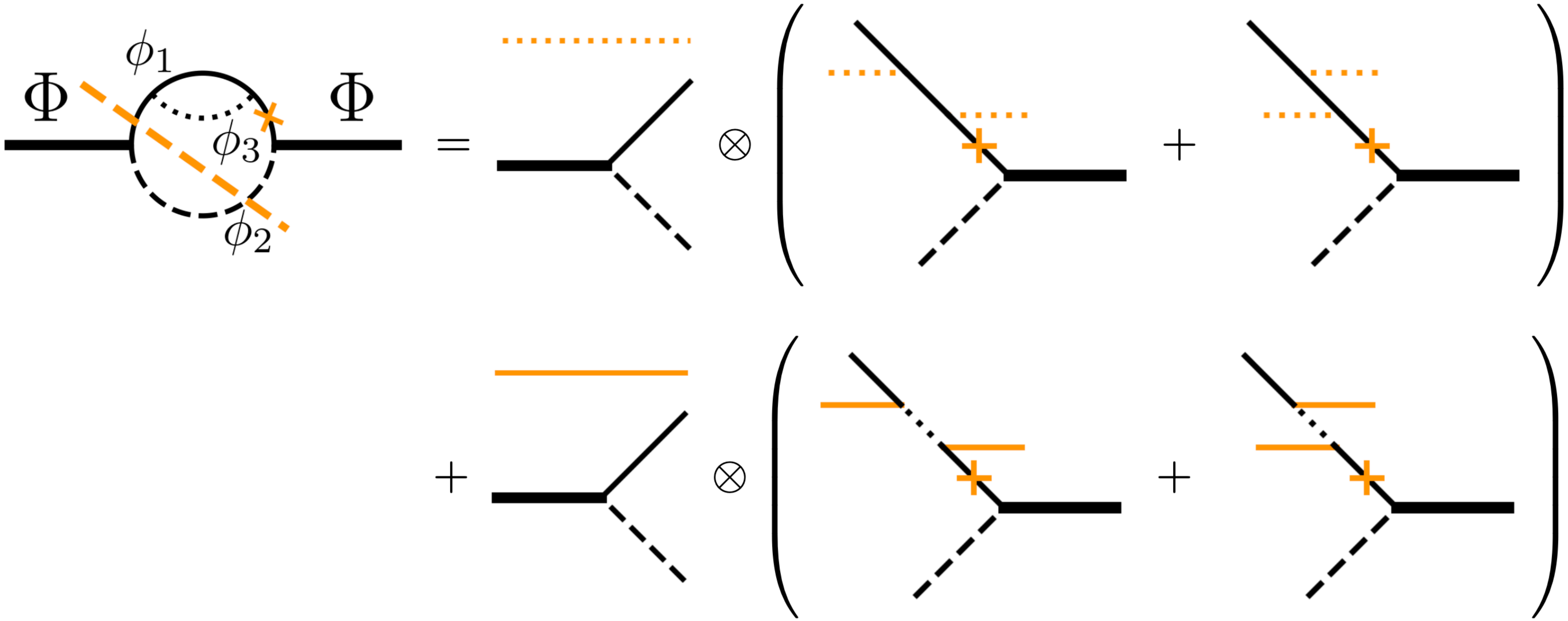
$\mathrm{Im}\,\Pi$: Spectral Surgery in a Heat Bath (SSHB)

- Two-loop contribution, asymmetric cuts
- Interference: $1 \to 2$ times $2 \to 3$ amplitudes



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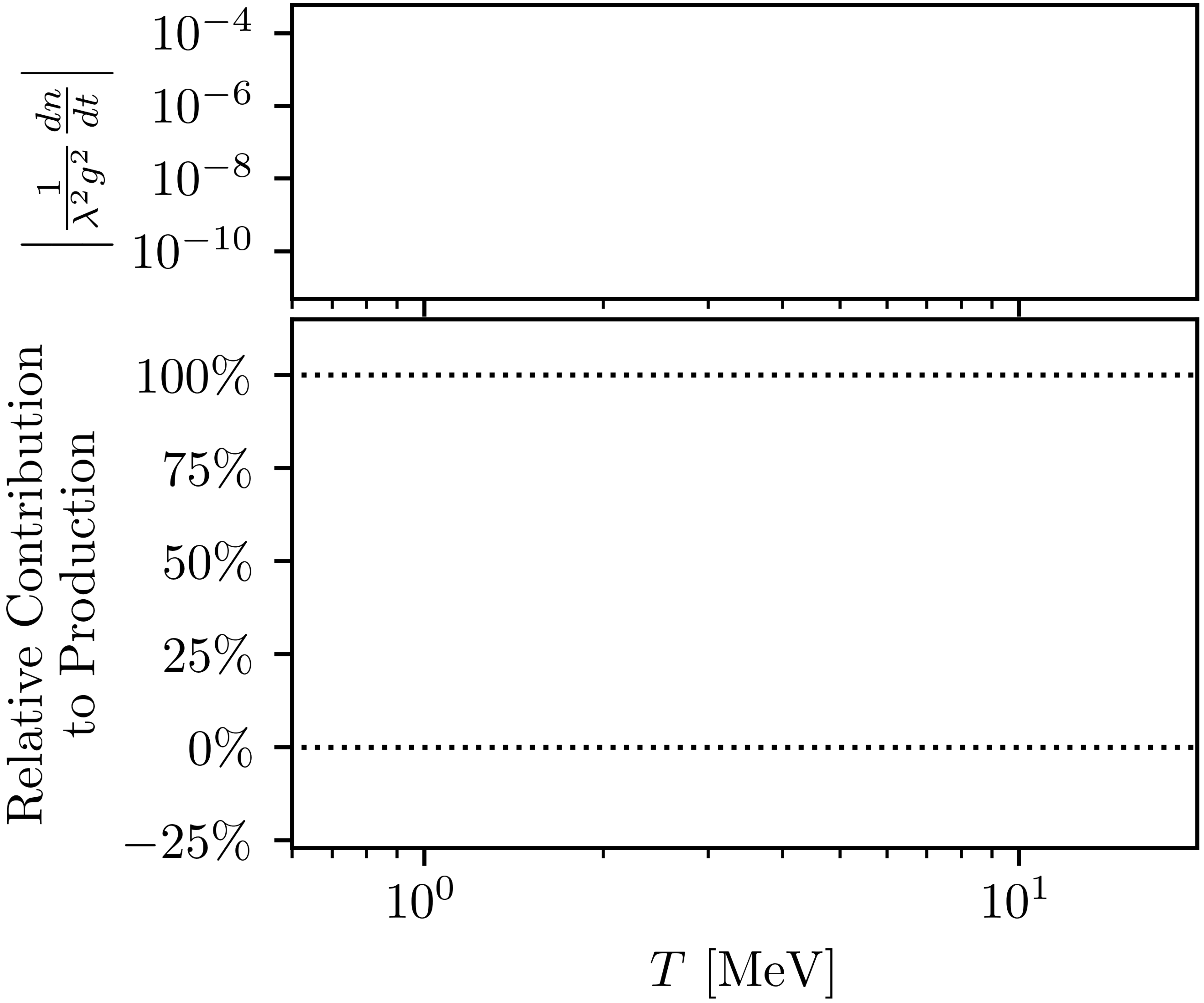
All-scalar model production

$$\mathcal{L}_{\rm int} = g\Phi\phi_1\phi_2 + \tfrac{1}{2}\lambda\phi_1^2\phi_3$$

$m_\Phi = 9$ MeV
$m_{\phi_1} = 4$ MeV
$m_{\phi_2} = 3$ MeV
$m_{\phi_3} = 6$ MeV
$\lambda = 100$ MeV



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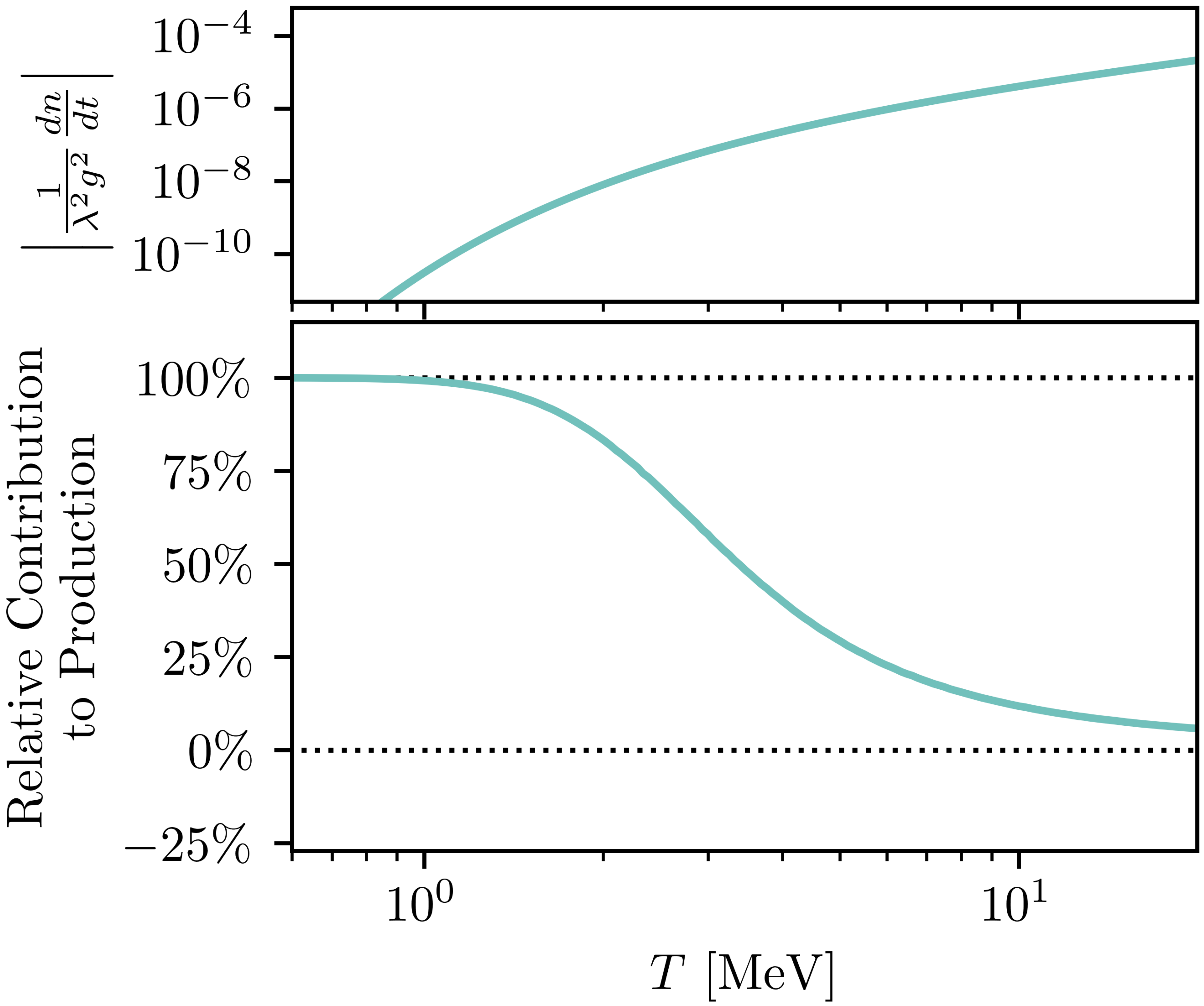

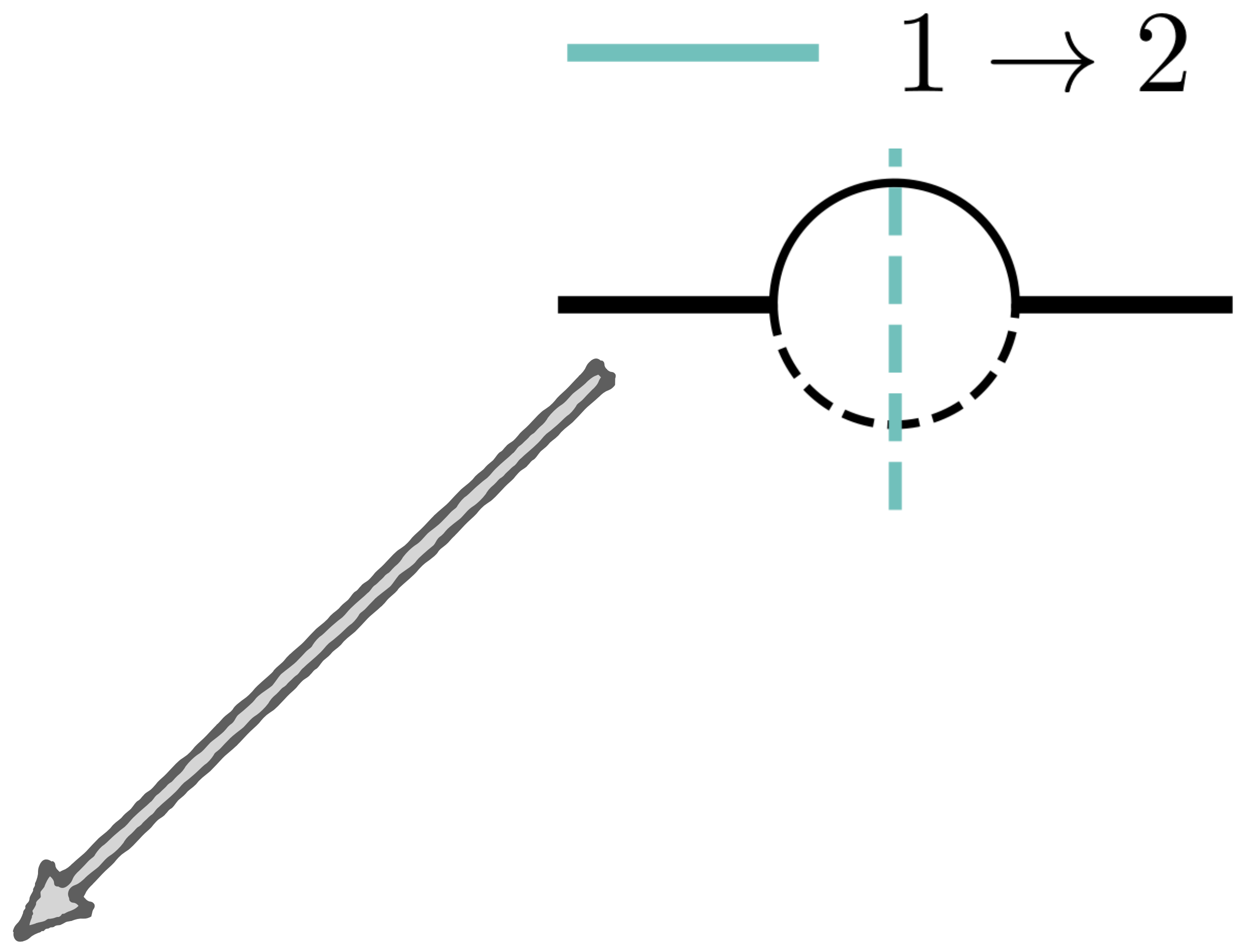

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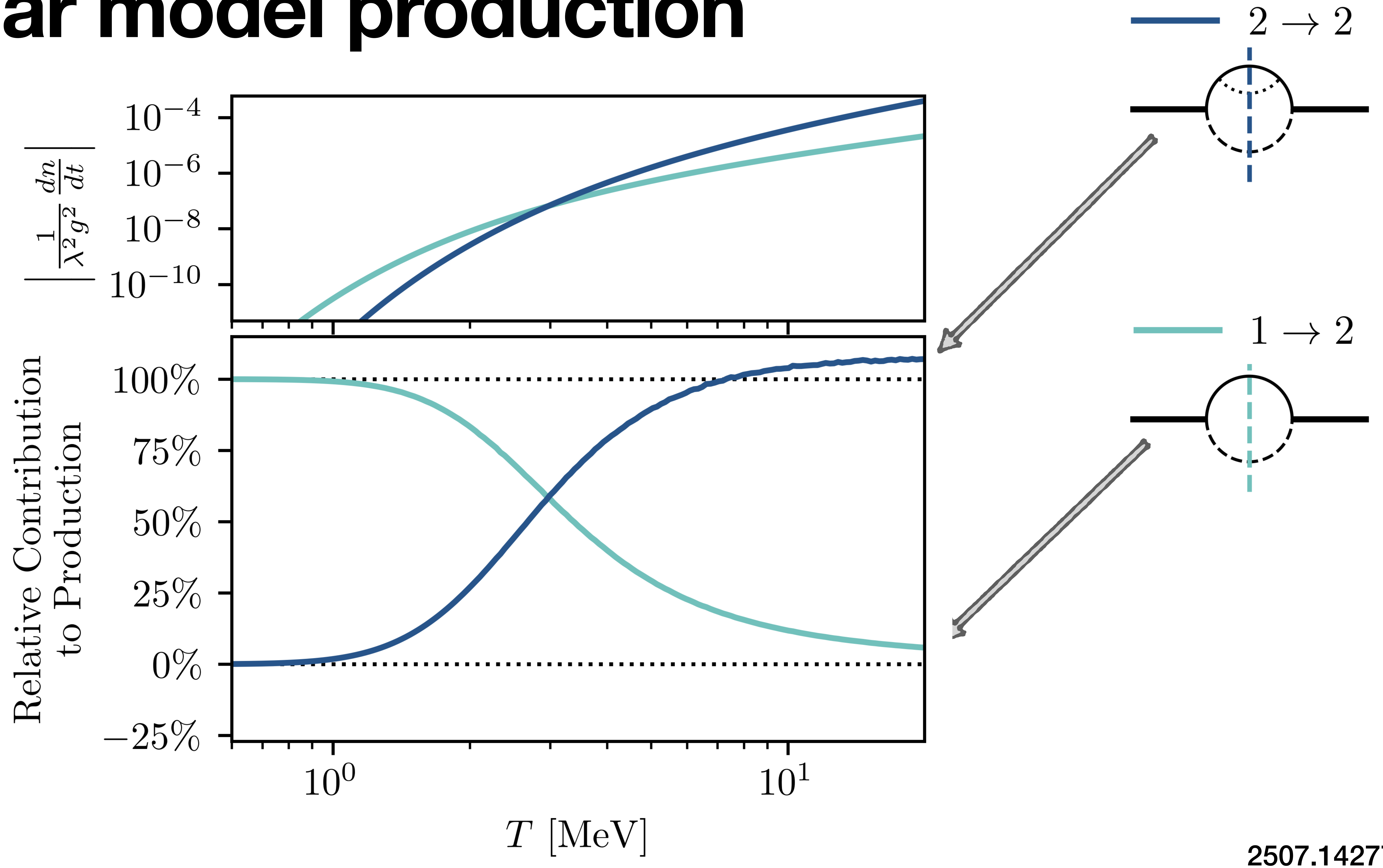

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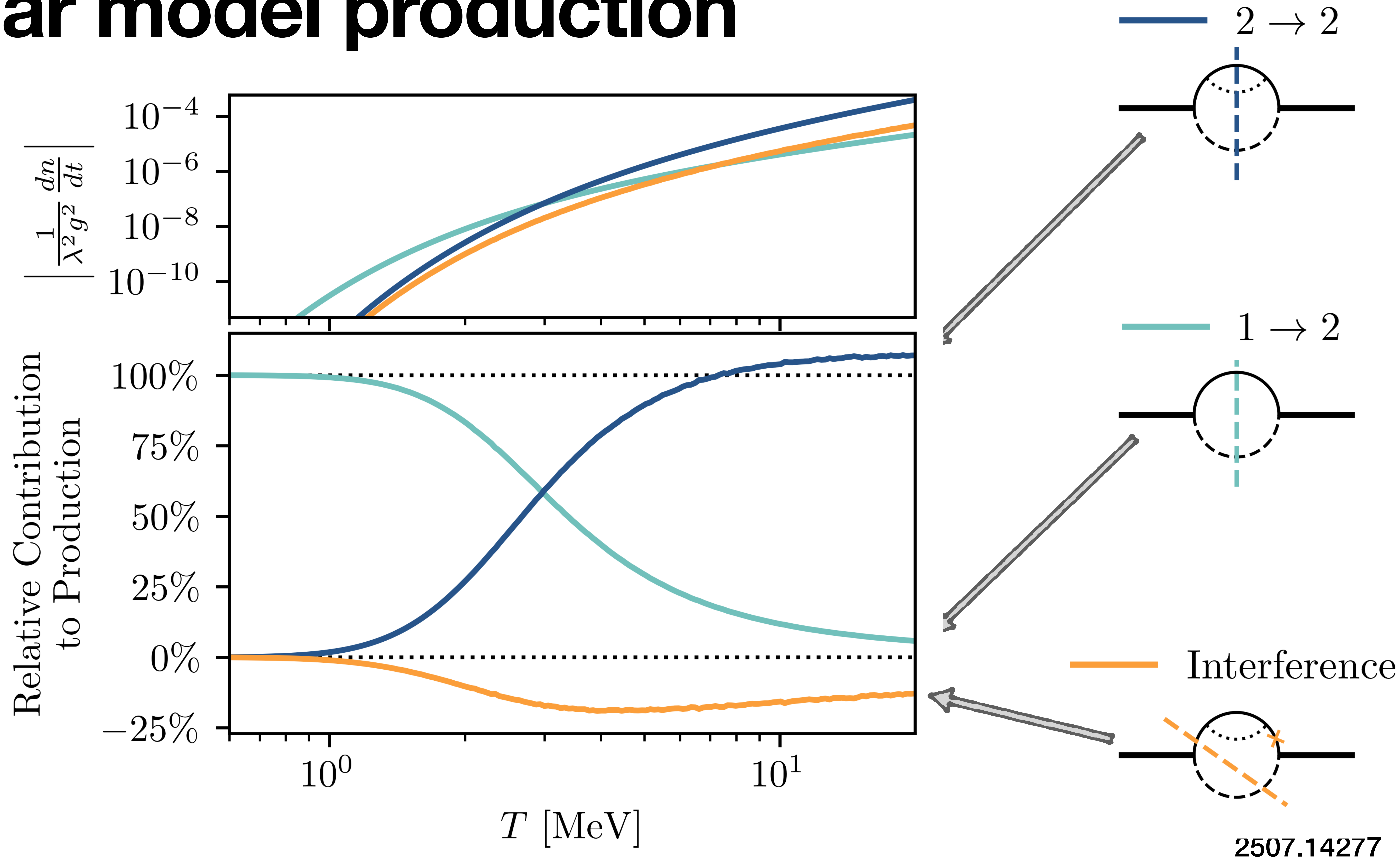

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Using these cutting rules

☞ Axion/Dark Photon production
in magnetic white dwarfs

see the talk from Dr. Nirmalya Brahma

2509.07085

☞ Light scalar production
in the core of red giants

see the talk from Natnael Debru

26XX.XXXXX

Takeaways

☞ We should not neglect a thermal effect:
Thermal effects are ***essential*** for accurate BSM predictions

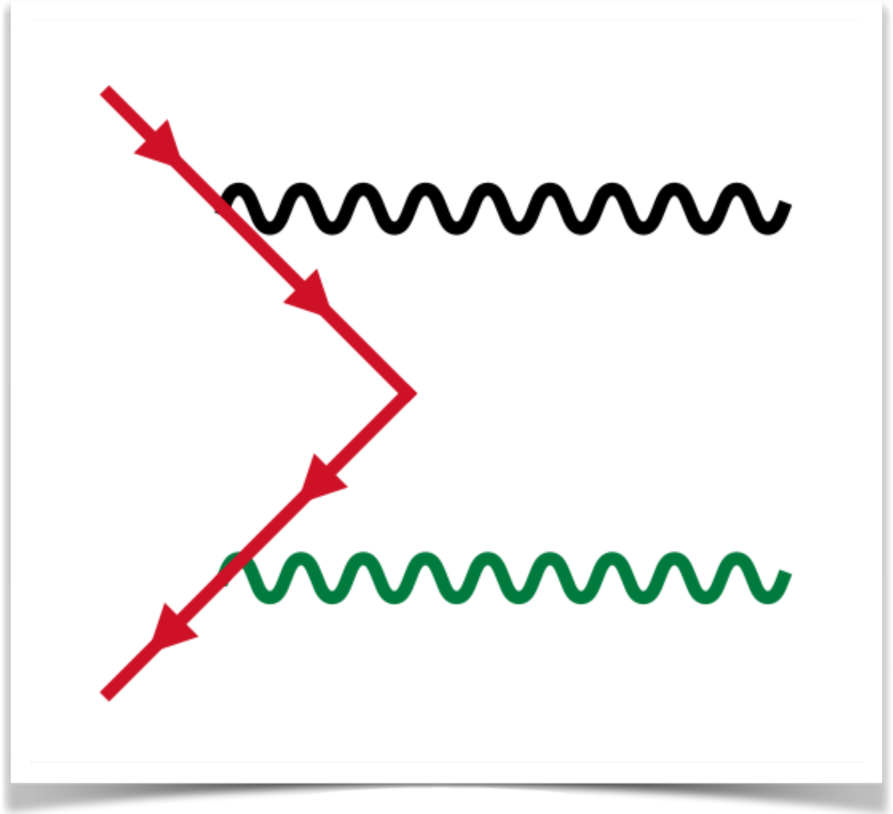
☞ ***Spectral surgery in a heat bath:***
user-friendly cutting rules that cut complexity, but not corners!

☞ Thermal *interference* processes can significantly impact production rates

Want to draw some Feynman diagrams:

```
> pip install feyndraw
```

(or: https://github.com/schhug/feyndraw)



More at
arXiv:2507.14277

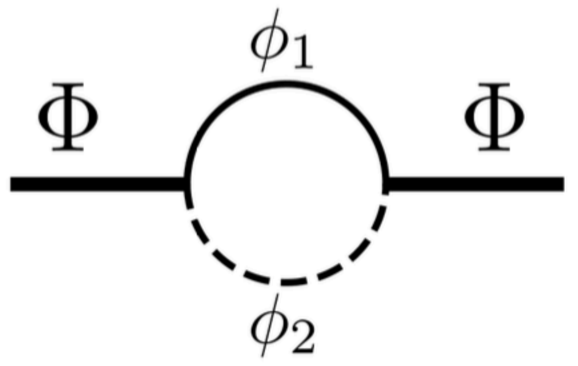
Back up slides

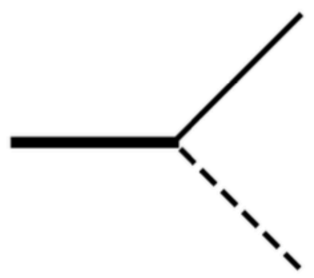
All-scalar model production

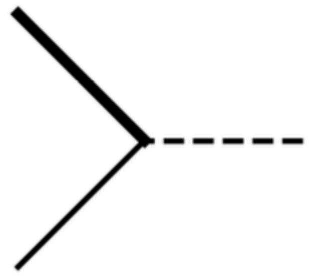
- One-loop contribution

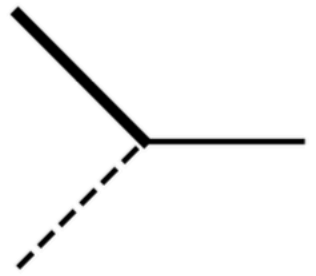
Cut propagators are on-shell

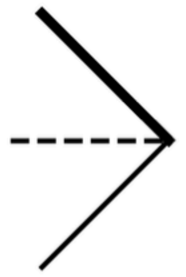
Phase space factors, absorption - emission

$\mathrm{Im}\Big($  $\Big) = -\frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}$

$\times \Big\{ (2\pi)^4\delta^{(4)}(k-p_1-p_2)\Big|$  $\Big|^2 [(1+f_1)(1+f_2)-f_1 f_2]$

$+\,(2\pi)^4\delta^{(4)}(k+p_1-p_2)\Big|$  $\Big|^2 [f_1(1+f_2)-f_2(1+f_1)]$

$+\,(2\pi)^4\delta^{(4)}(k-p_1+p_2)\Big|$  $\Big|^2 [f_2(1+f_1)-f_1(1+f_2)]$

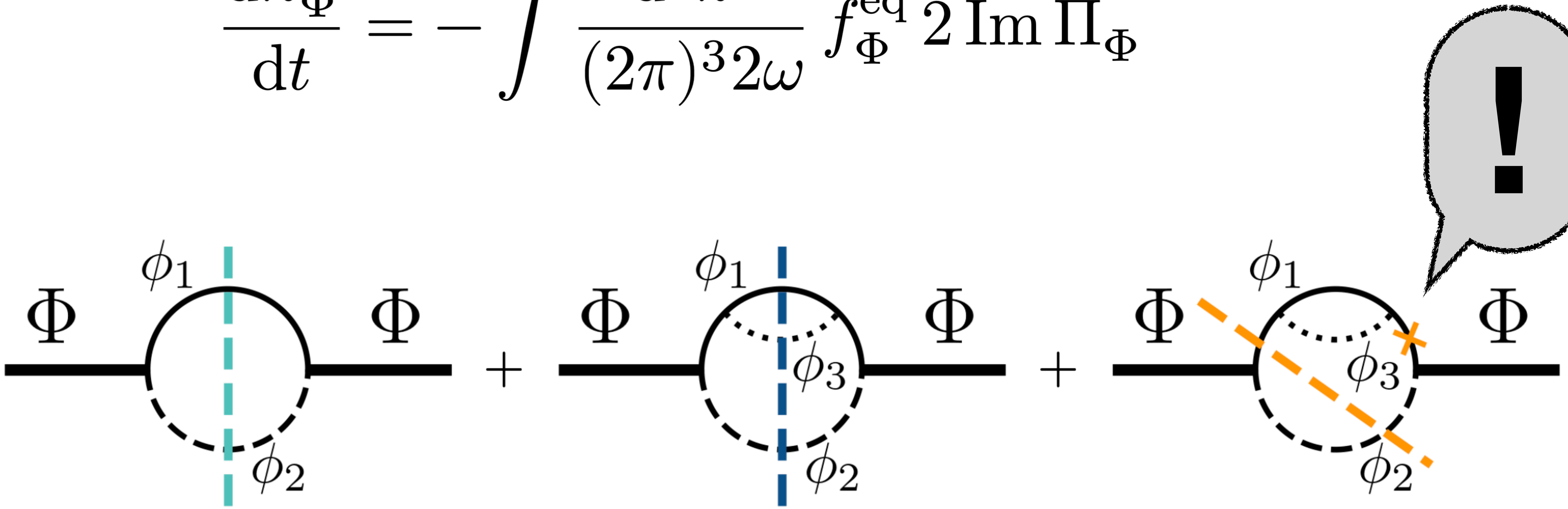
$+\,(2\pi)^4\delta^{(4)}(k+p_1+p_2)\Big|$  $\Big|^2 [f_1 f_2-(1+f_2)(1+f_1)]\Big\}$

All-scalar model production

- Assume Maxwell-Boltzmann distributions
- Assume freeze-in

$$\frac{\mathrm{d}n_\Phi}{\mathrm{d}t} = -\int \frac{\mathrm{d}^3k}{(2\pi)^3 2\omega} f_\Phi^{\mathrm{eq}}\, 2\,\mathrm{Im}\,\Pi_\Phi$$

$\mathrm{Im}\,\Pi_\Phi =$



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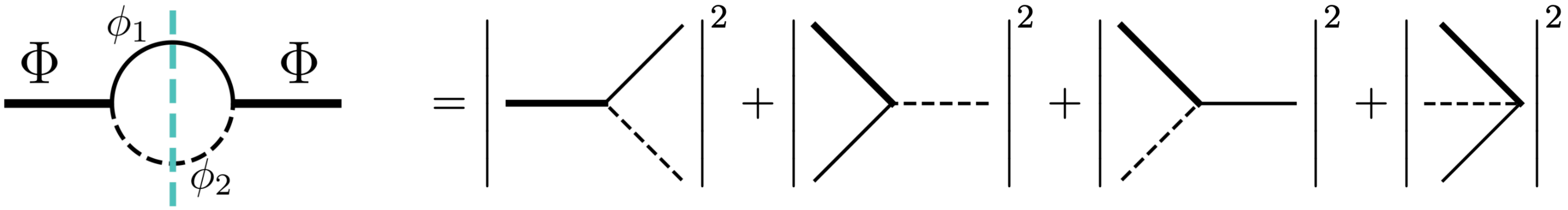
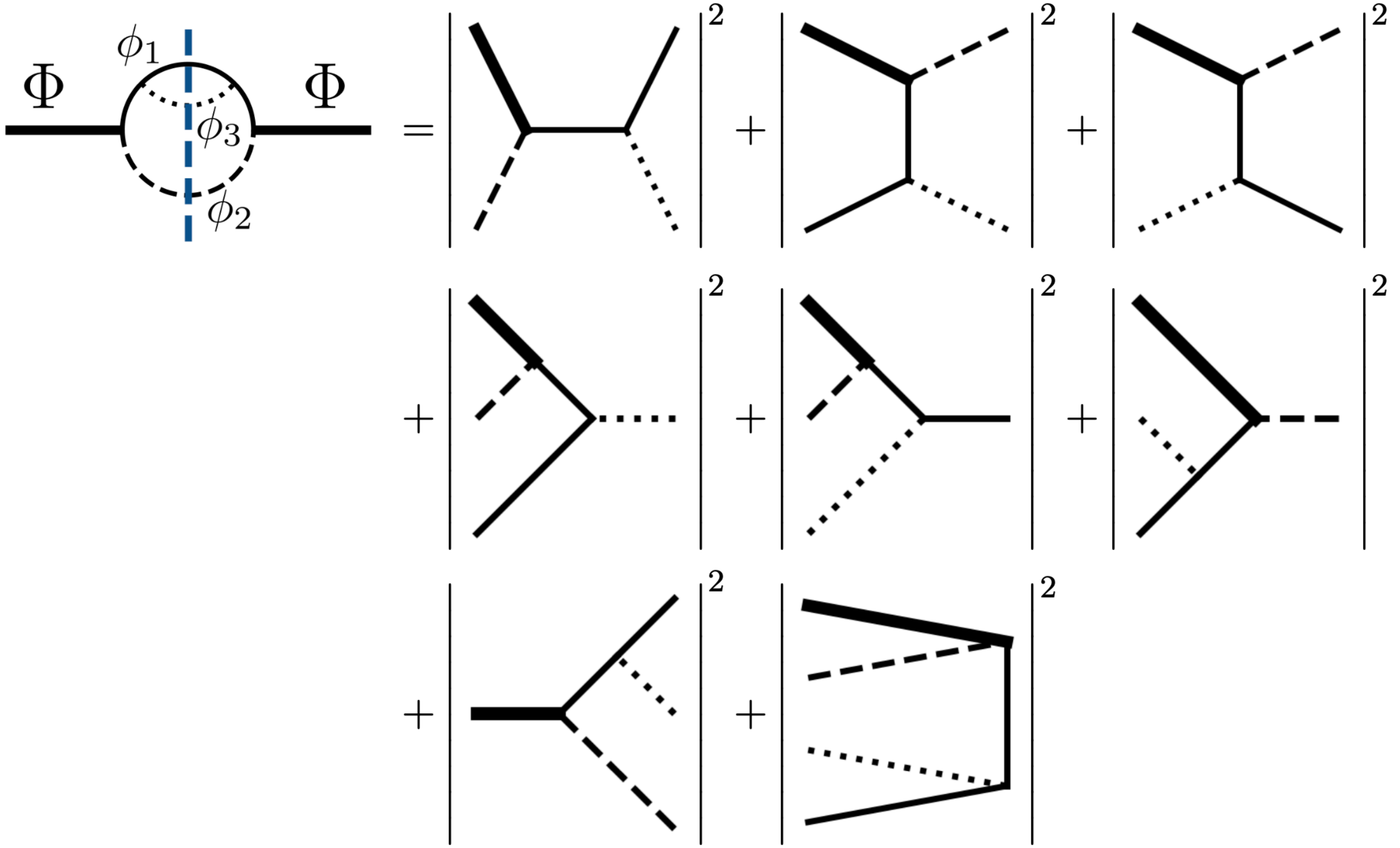
All-scalar model production

- One-loop contribution



All-scalar model production

- Two-loop contribution, symmetric cuts



All-scalar model production

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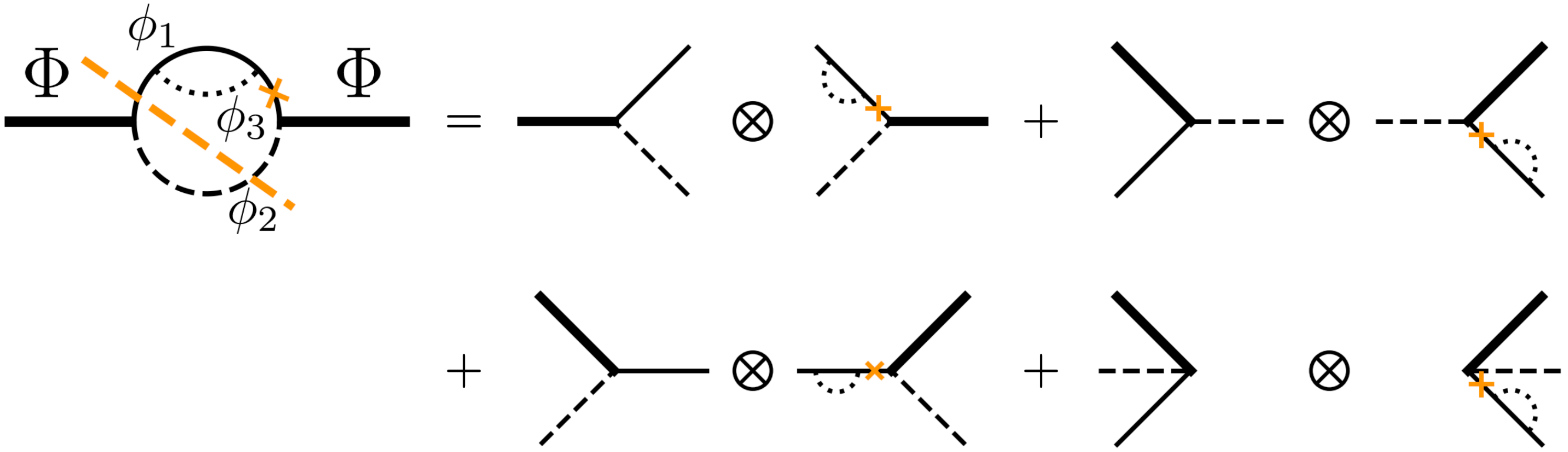

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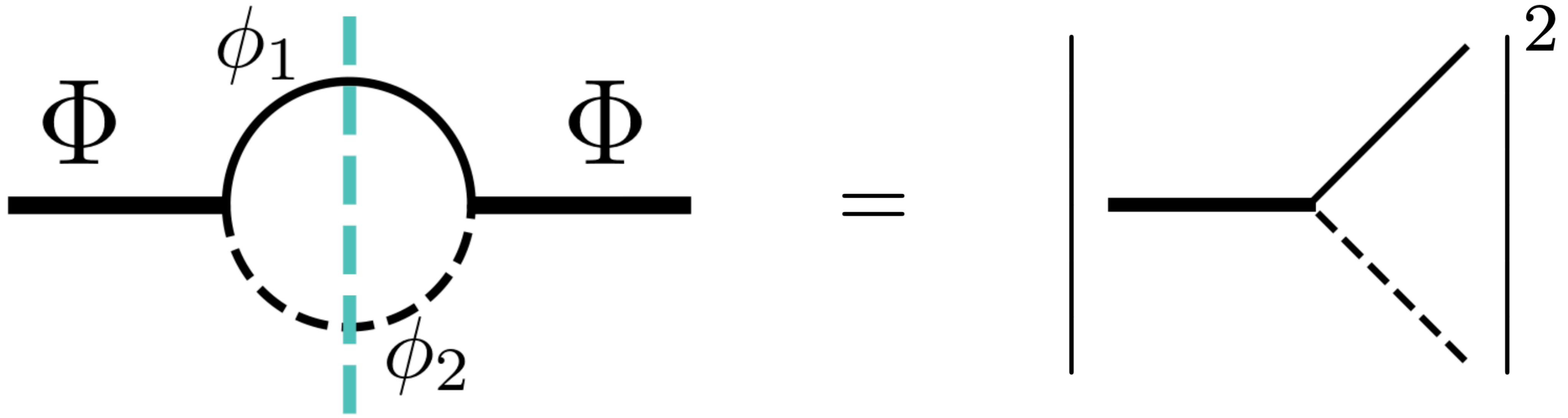
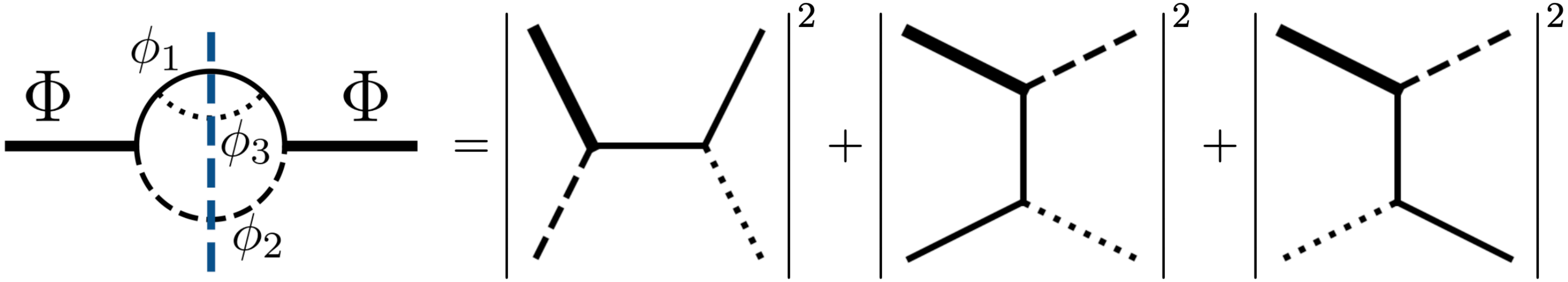
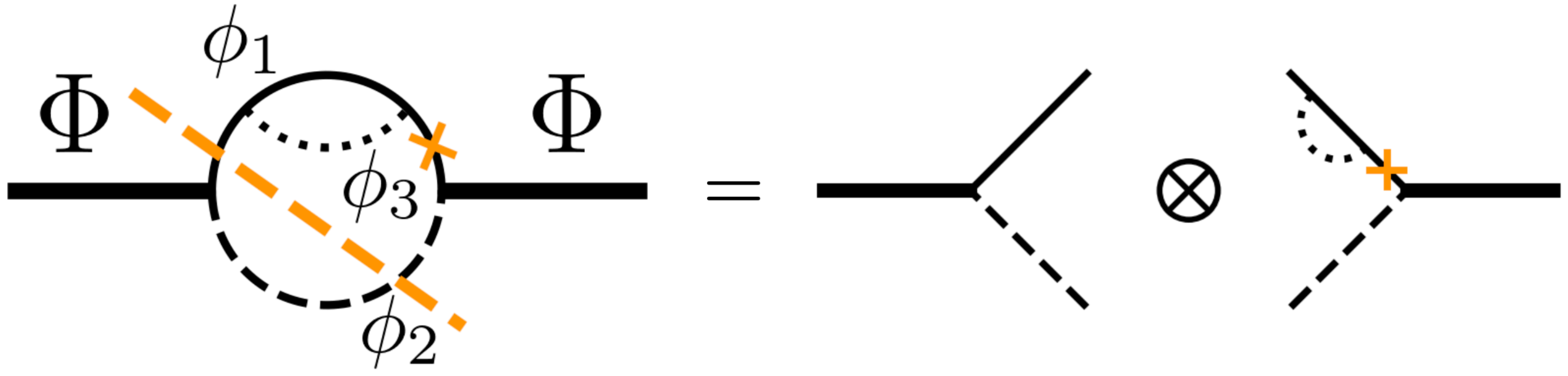


$$m_{\phi_1} + m_{\phi_2} < m_\Phi$$

$$m_\Phi < m_{\phi_1} + m_{\phi_2} + m_{\phi_3}$$

$$m_{\phi_3} < m_{\phi_1} + m_{\phi_2} + m_\Phi$$

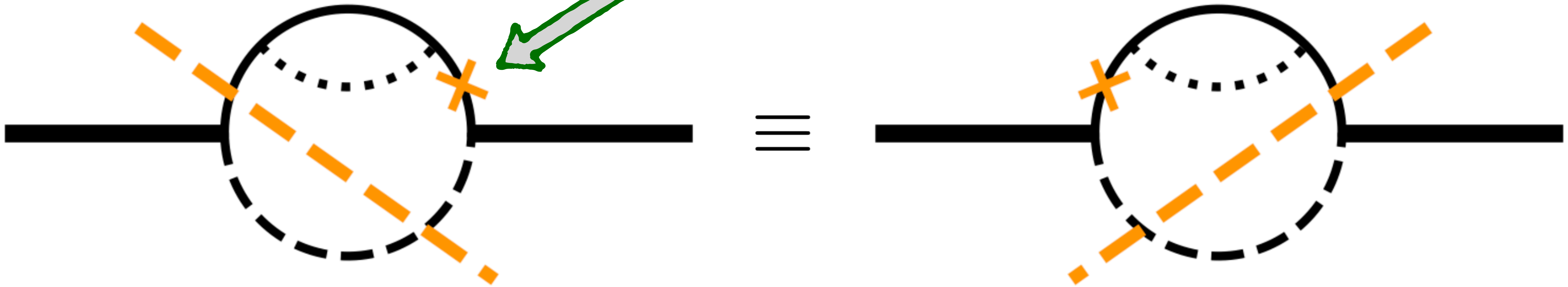




$\mathrm{Im}\,\Pi$: Spectral Surgery in a Heat Bath (SSHB)

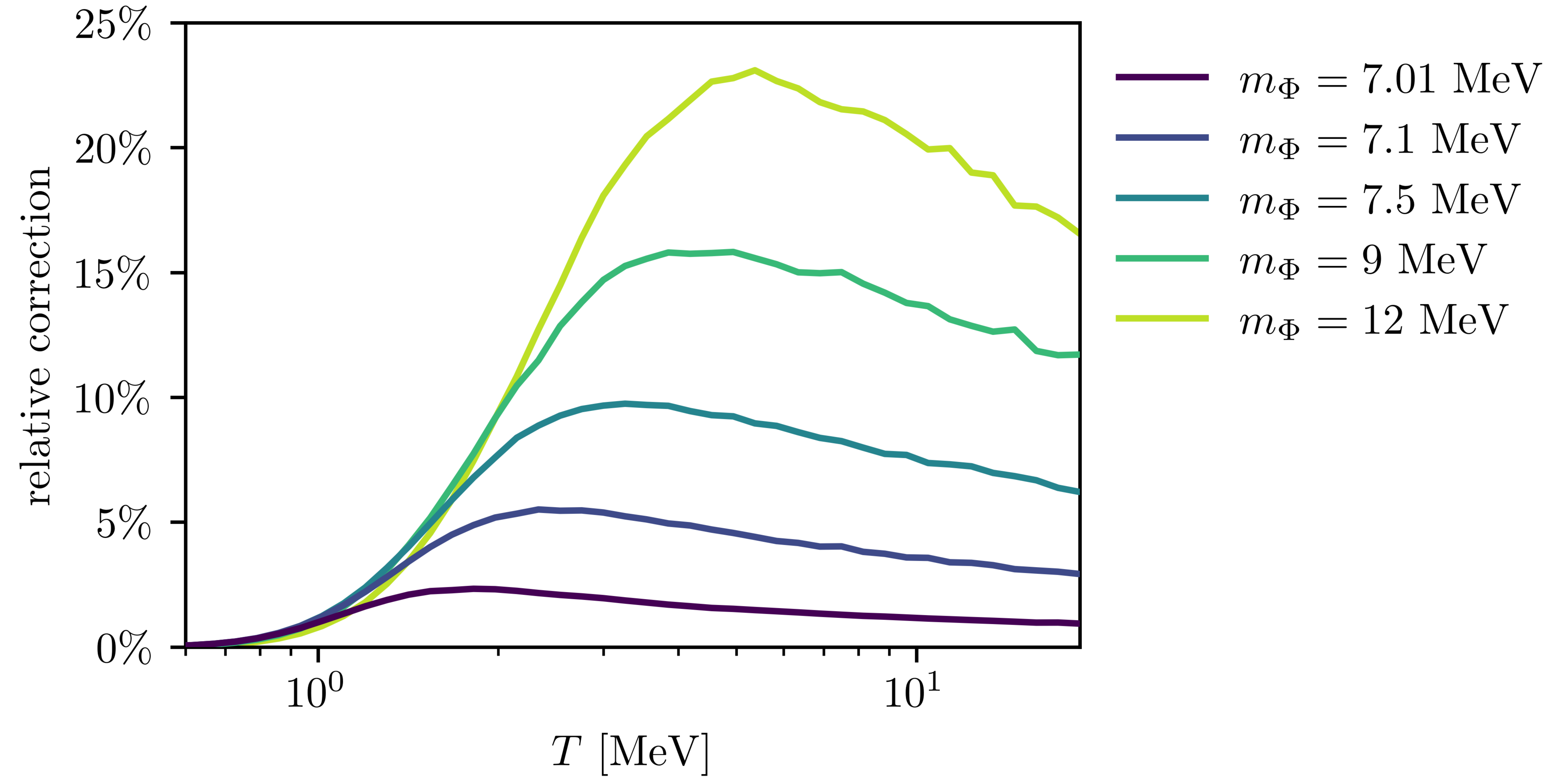
- Those are the same bisection (full cut)

$$\theta(p^0_{\phi_1})\delta'(p^2_{\phi_1} - m^2_{\phi_1})$$



2507.14277

All-scalar model production



$m_{\phi_1} = 4$ MeV
$m_{\phi_2} = 3$ MeV
$m_{\phi_3} = 6$ MeV
$\lambda = 100$ MeV

2507.14277

Scalar-fermion-vector Toy Model

- Toy model with a fermion, a massive scalar and a massive boson

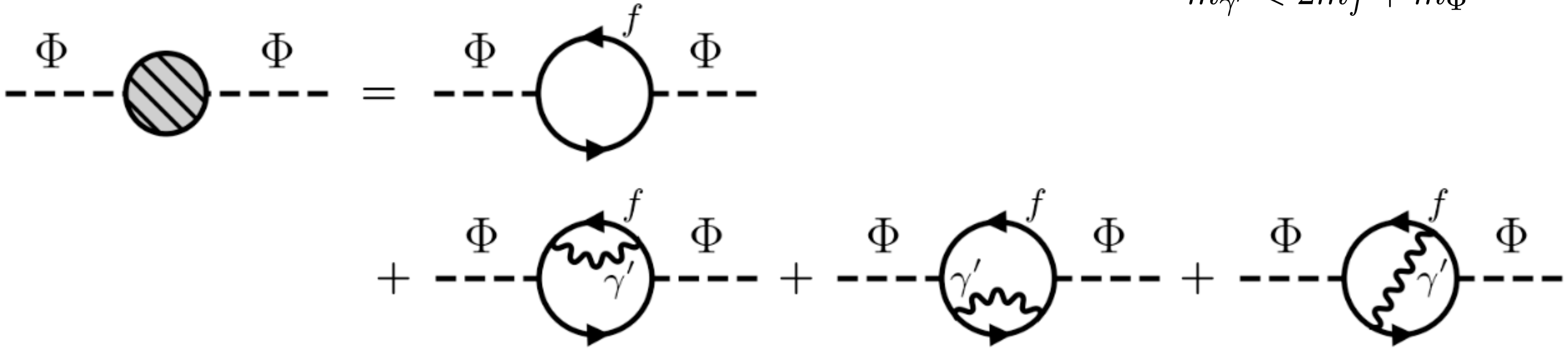
$$\mathcal{L}_{\text{int}} = g\Phi \bar{f} f - eA'_{\mu} \bar{f} \gamma^{\mu} f$$

$$2m_f < m_\Phi$$

$$m_\Phi < 2m_f + m_{\gamma'}$$

$$m_{\gamma'} < 2m_f + m_\Phi$$

- Self-energy up to two loops

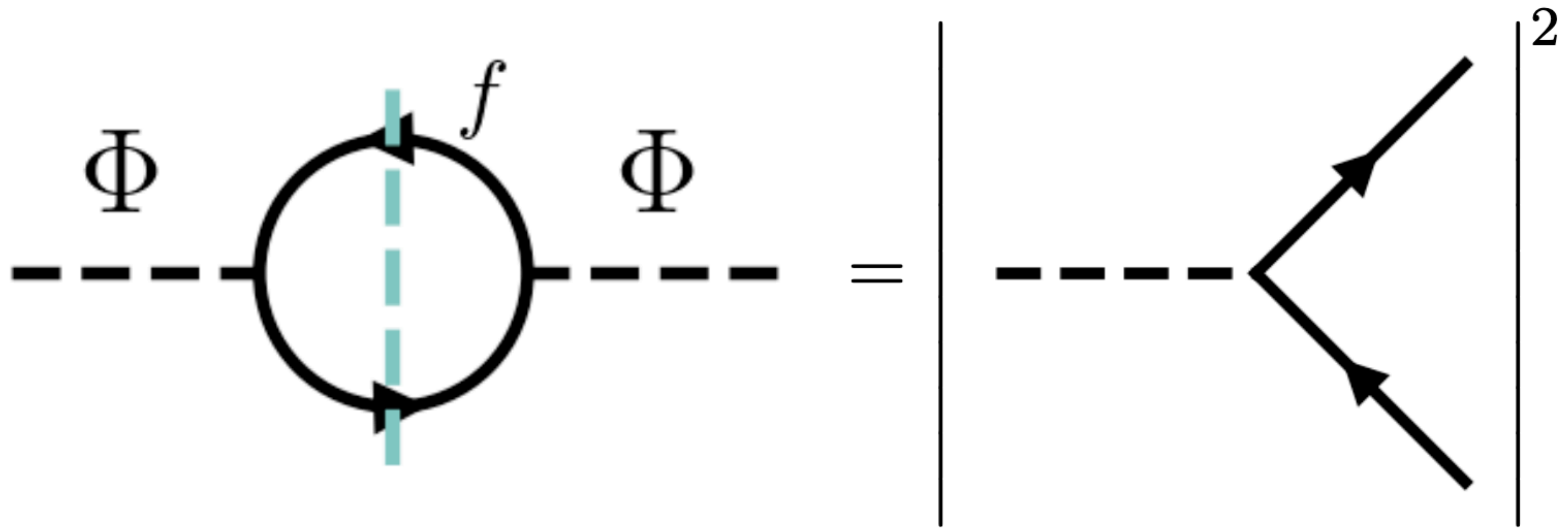


2507.14277

Scalar-fermion-vector Toy Model

$$2m_f < m_\Phi$$
$$m_\Phi < 2m_f + m_{\gamma'}$$
$$m_{\gamma'} < 2m_f + m_\Phi$$

- One-loop contribution

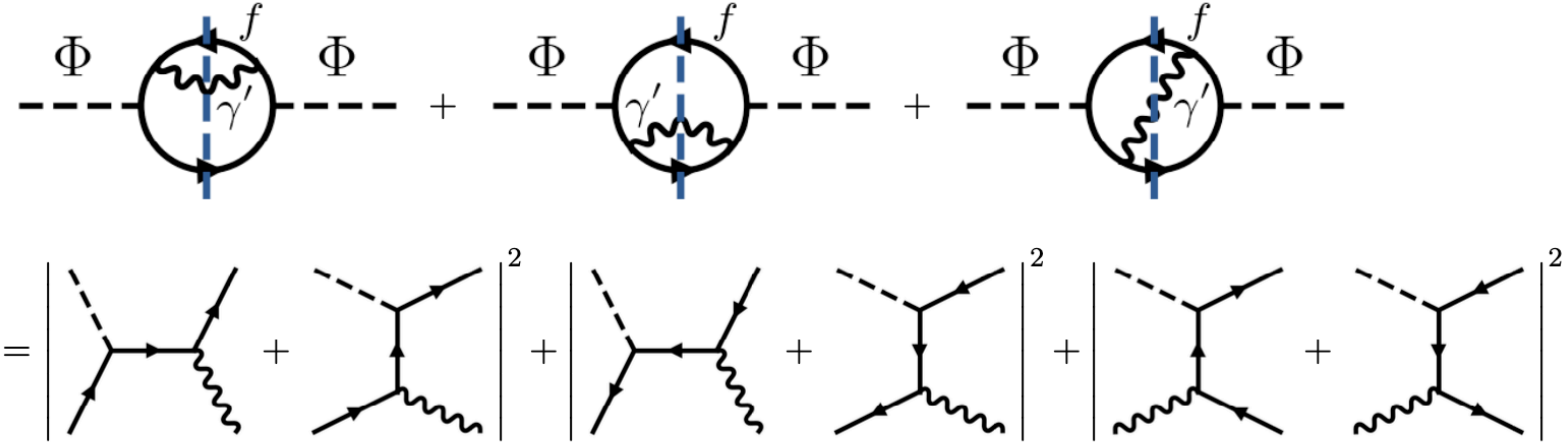


2507.14277

Scalar-fermion-vector Toy Model

$$2m_f < m_\Phi$$
$$m_\Phi < 2m_f + m_{\gamma'}$$
$$m_{\gamma'} < 2m_f + m_\Phi$$

- Two-loop contribution (symmetrical)



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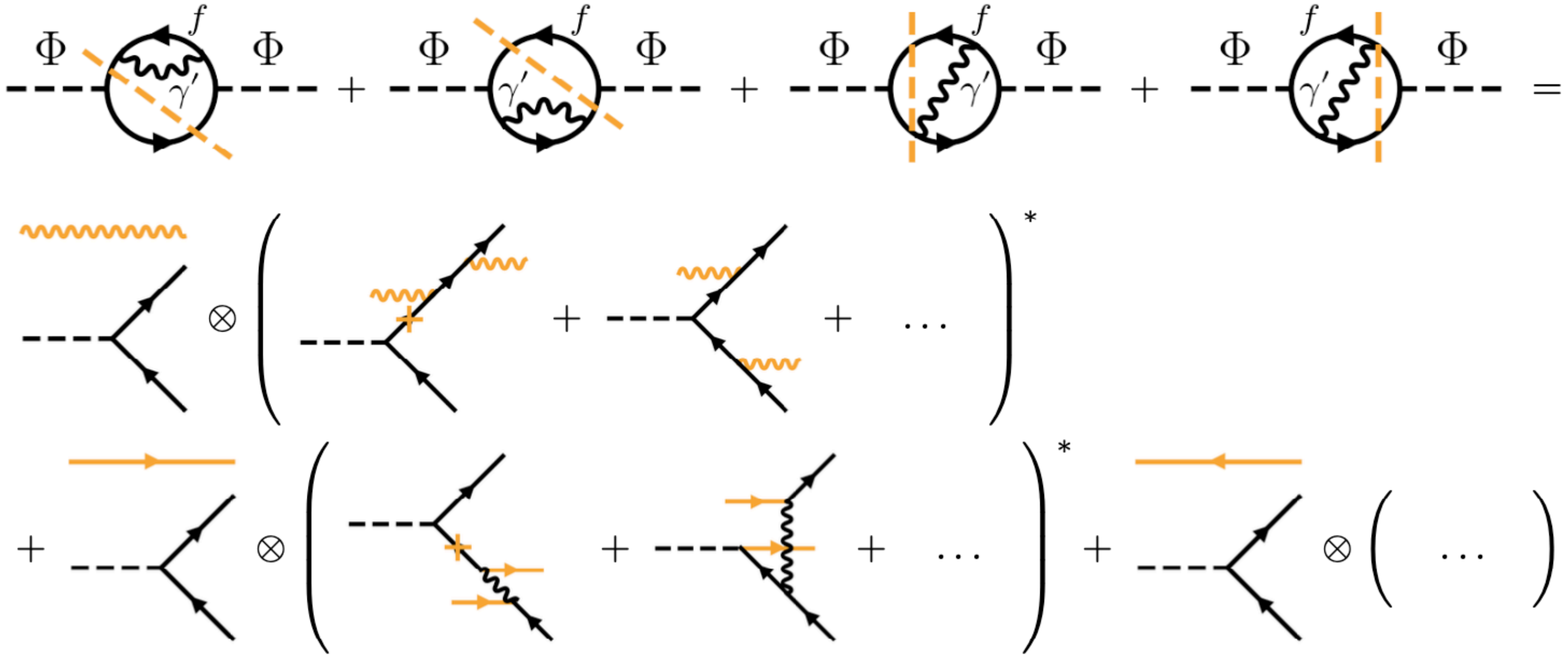
Scalar-fermion-vector Toy Model

- Two-loop contribution (asymmetrical)



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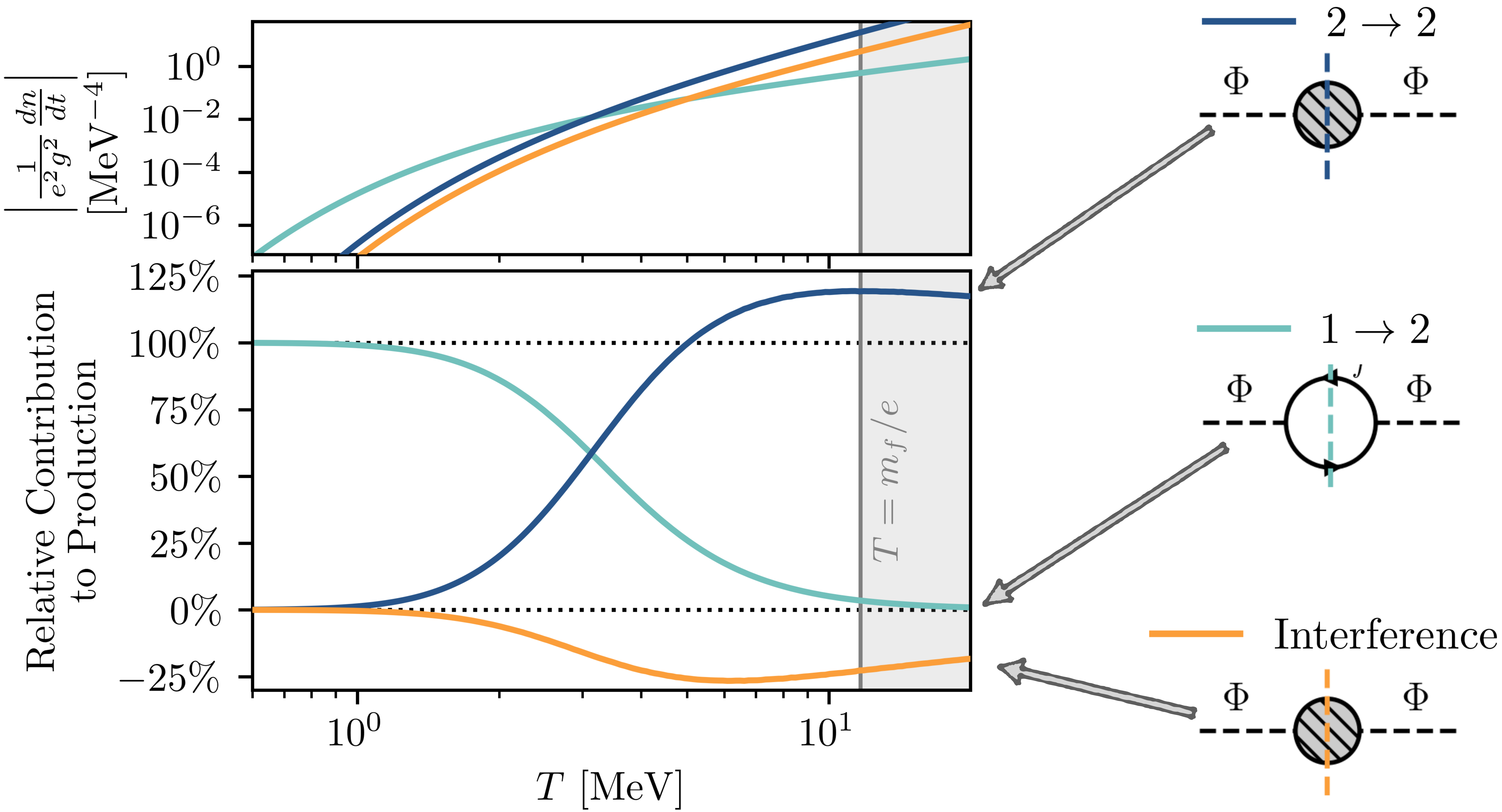
Scalar-fermion-vector Model Production

$$\mathcal{L}_{\text{int}} = g\Phi\bar{f}f - eA'_\mu\bar{f}\gamma^\mu f$$

$m_\Phi = 7.1$ MeV
$m_f = 3.5$ MeV
$m_{\gamma'} = 6$ MeV
$e = 0.3$
$g = 10^{-6}$



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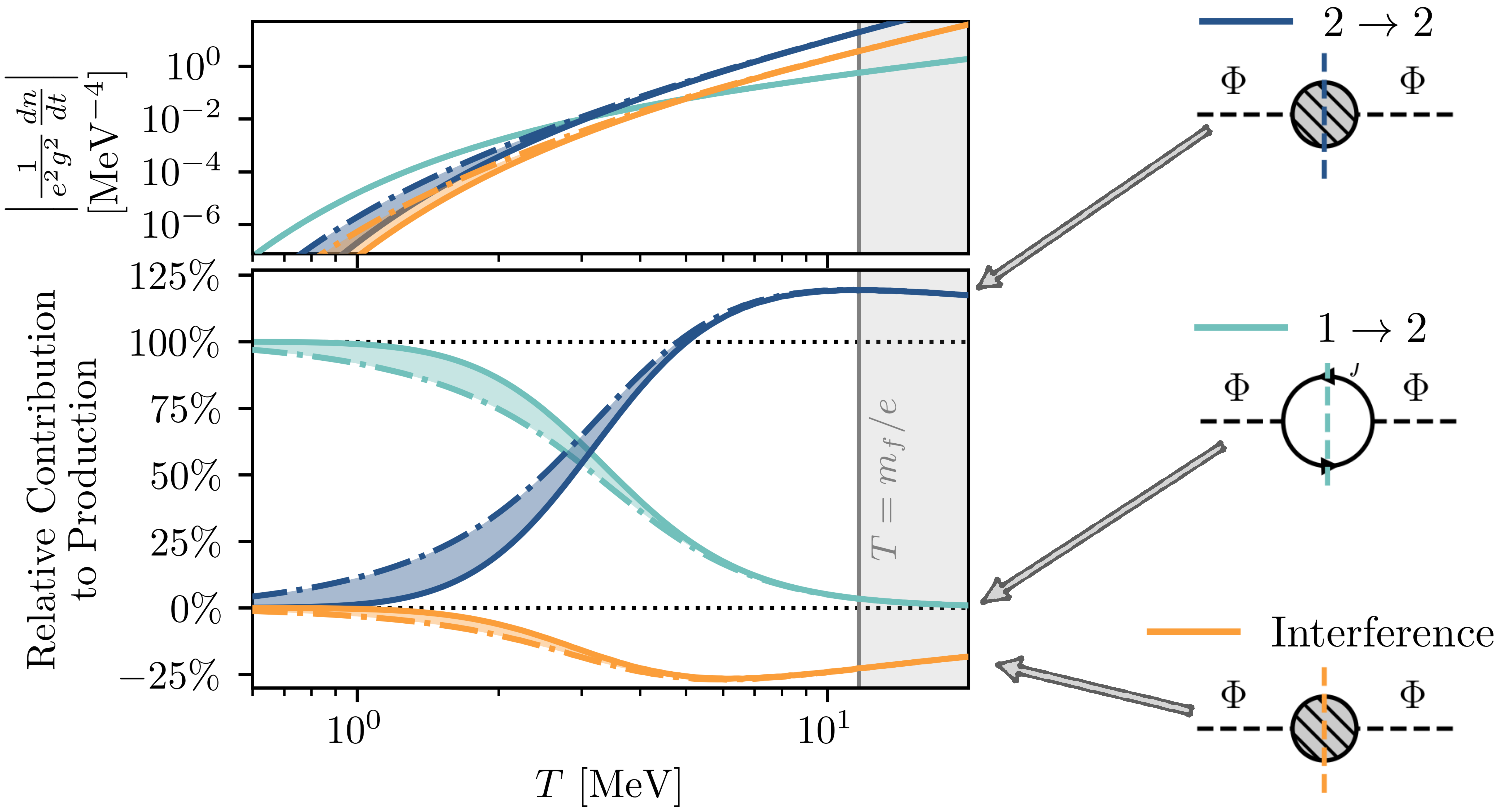
Scalar-fermion-vector Model Production

$$\mathcal{L}_{\rm int} = g\Phi\bar{f}f - eA'_\mu\bar{f}\gamma^\mu f$$

$m_\Phi = 7.1$ MeV
$m_f = 3.5$ MeV
$m_{\gamma'} = 6$ MeV
$e = 0.3$
$g = 10^{-6}$

$\mu_f = 0$ MeV
$\mu_f = 3$ MeV



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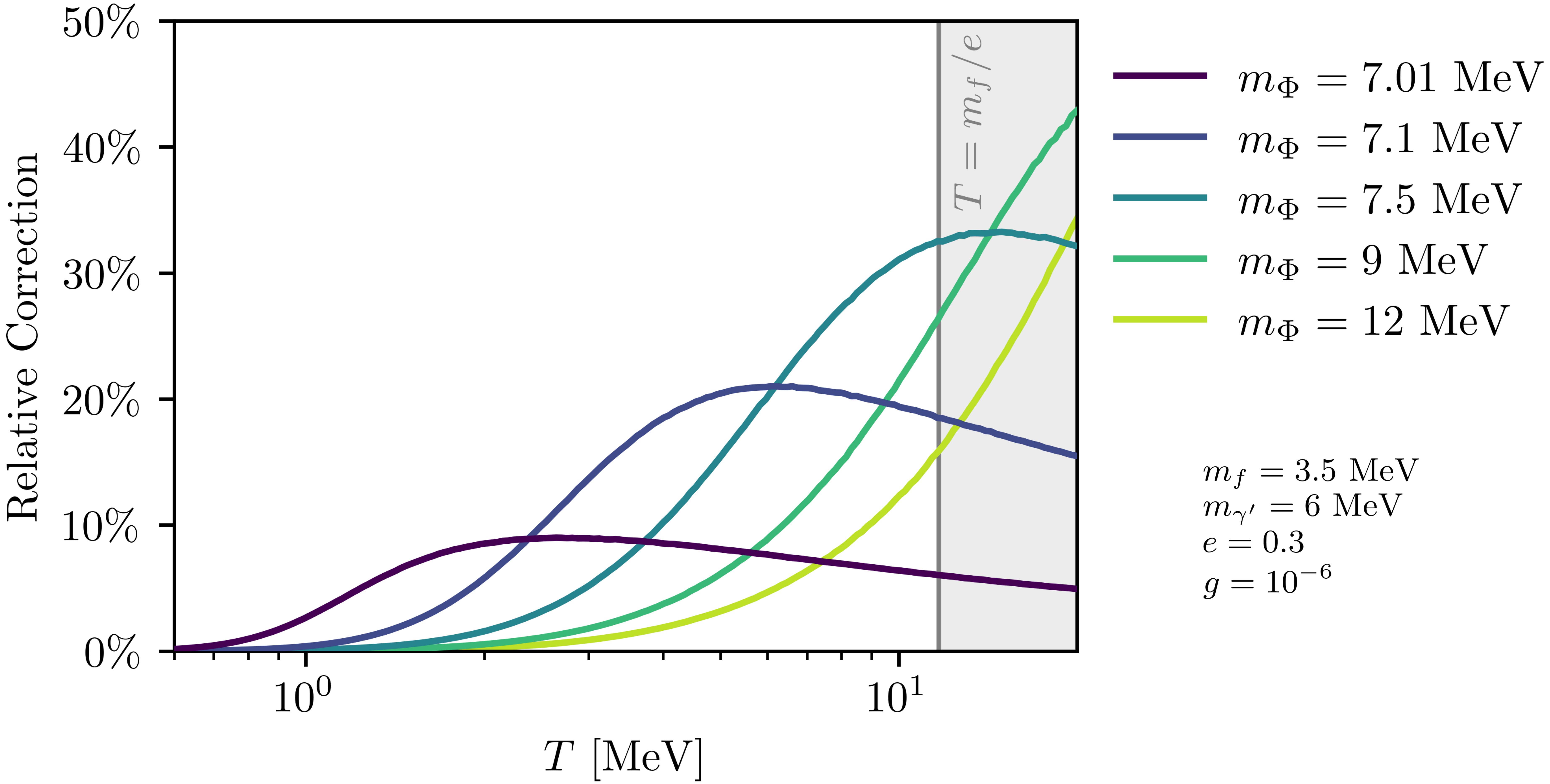
Scalar-fermion-vector Model Production



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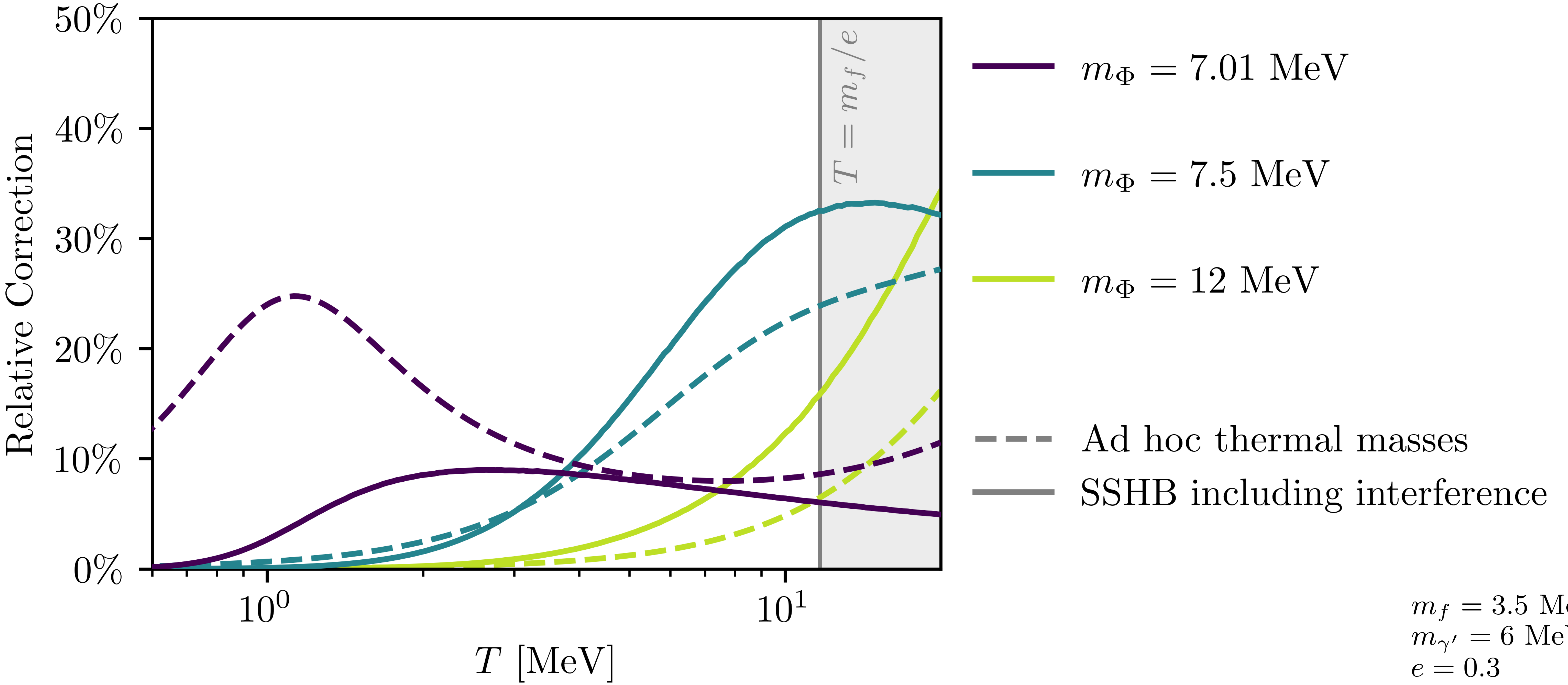
Scalar-fermion-vector Model Production



$m_f = 3.5$ MeV
$m_{\gamma'} = 6$ MeV
$e = 0.3$
$g = 10^{-6}$

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