

Finite temperature and density effects and BSM phenomenology

Hugo Schérer

In collaboration with Nirmalya Brahma, Saniya Heeba, Katelin Schutz (arXiv:2507.14277)

Light Dark World (July 30th 2026)

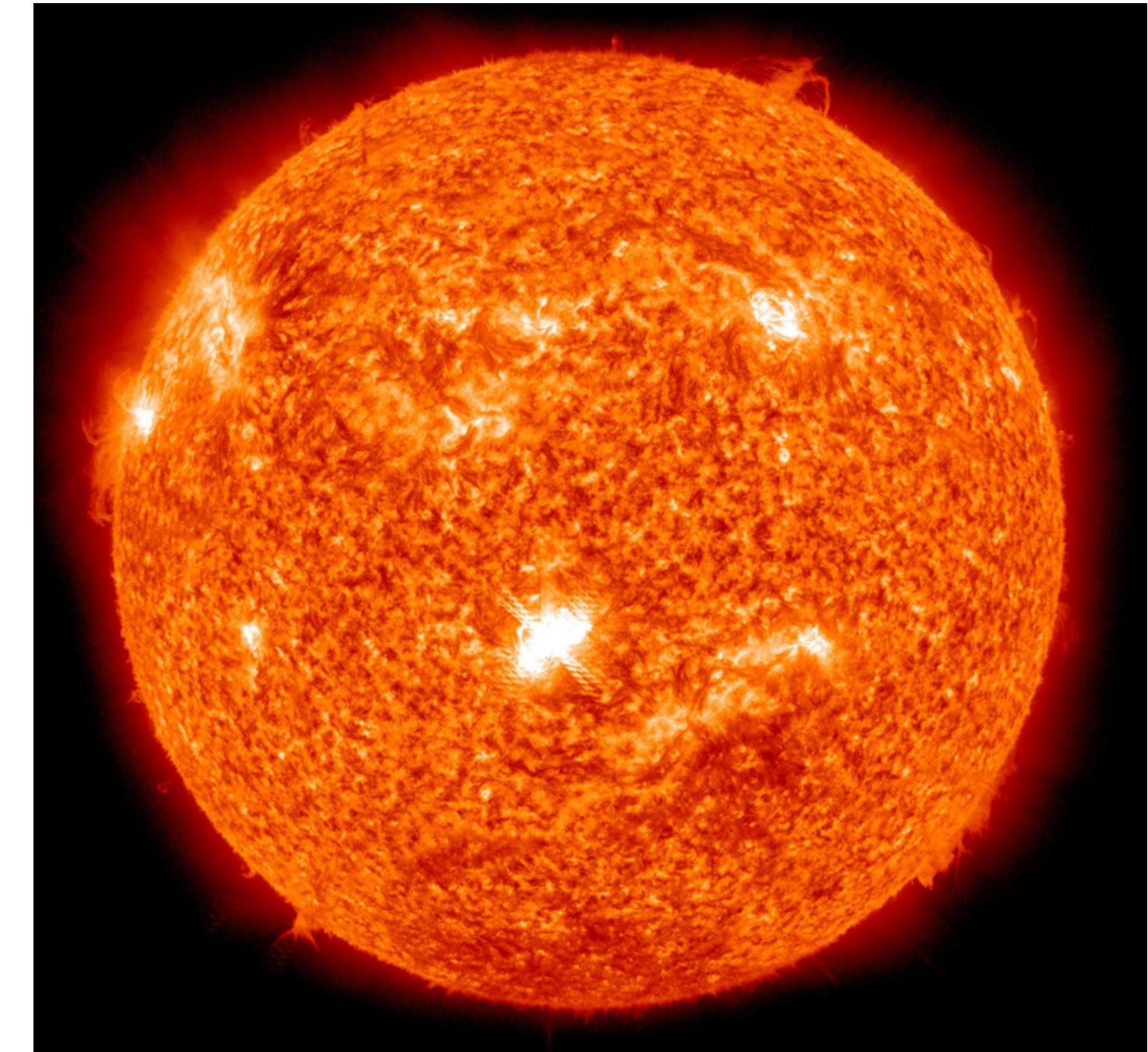
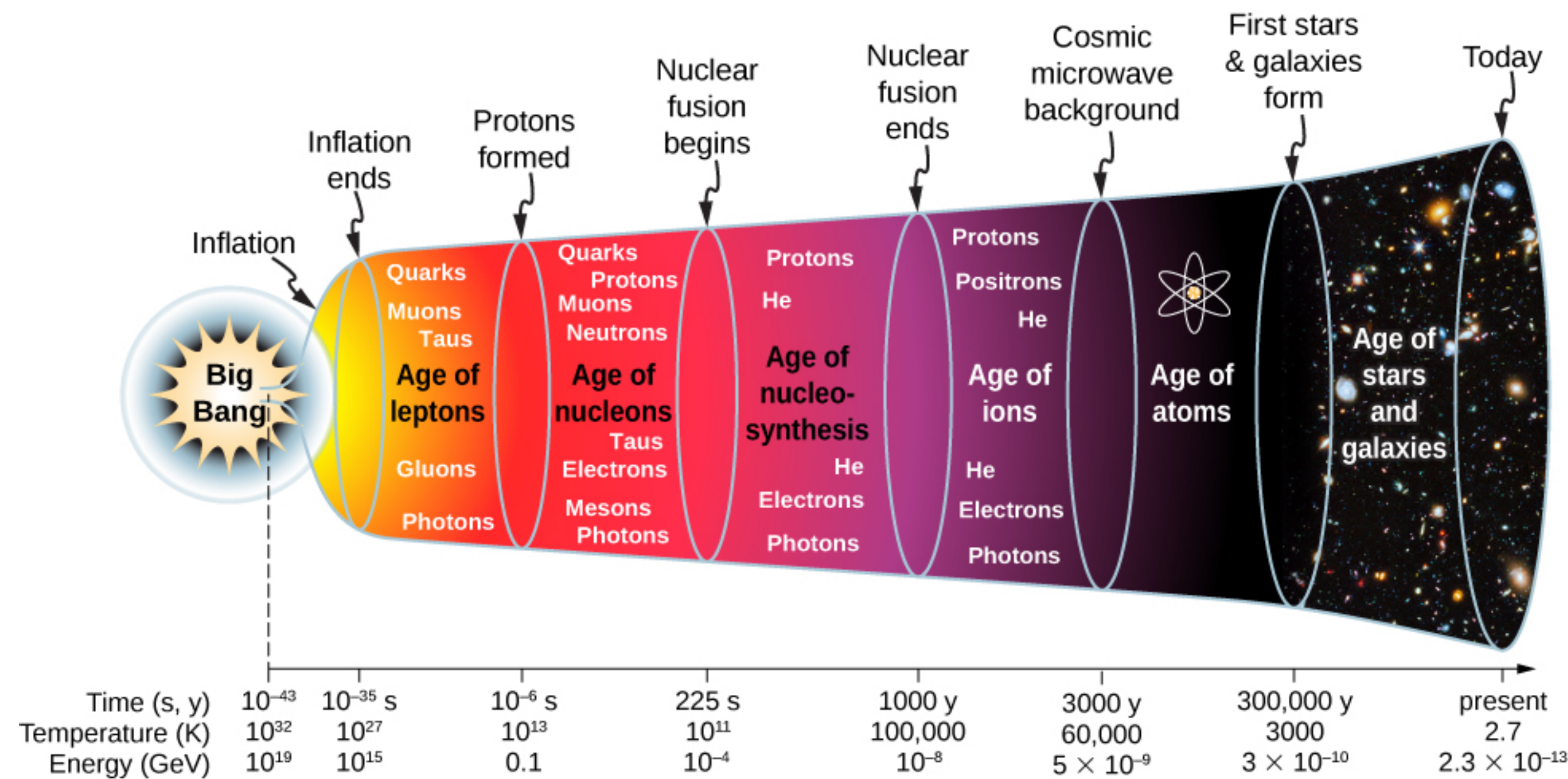


 **We should not neglect
a thermal effect**

Thermal effects are *essential* for accurate BSM predictions

Finite temperature and density effects

- The universe is not a vacuum!
- Many environments where we look for BSM are hot and dense medium



The self-energy Π

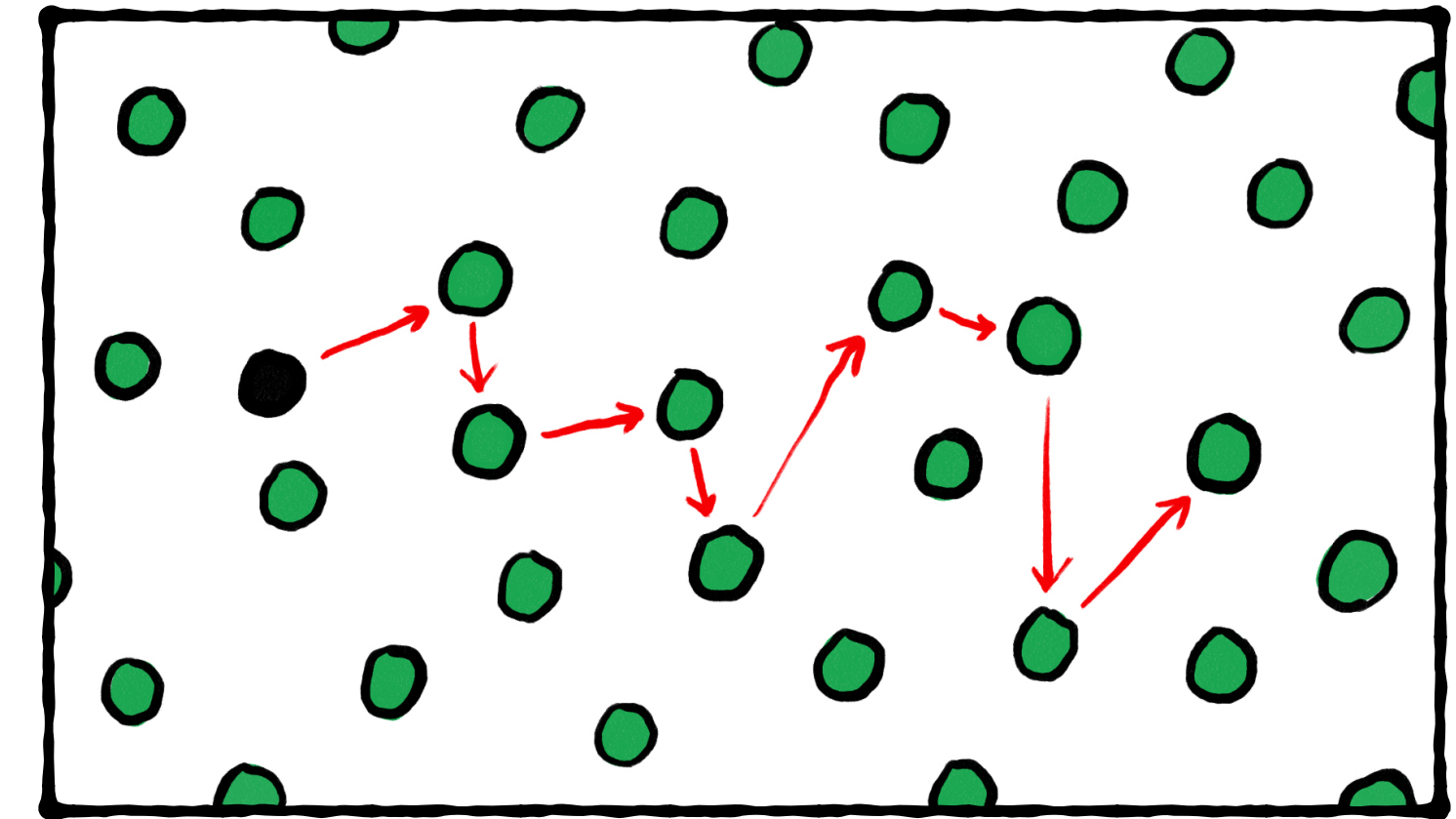


The self-energy Π



- **Real part:** mass (squared) correction to the dispersion

$$\omega^2 - \mathbf{k}^2 = m_\Phi^2 + \text{Re } \Pi_\Phi$$

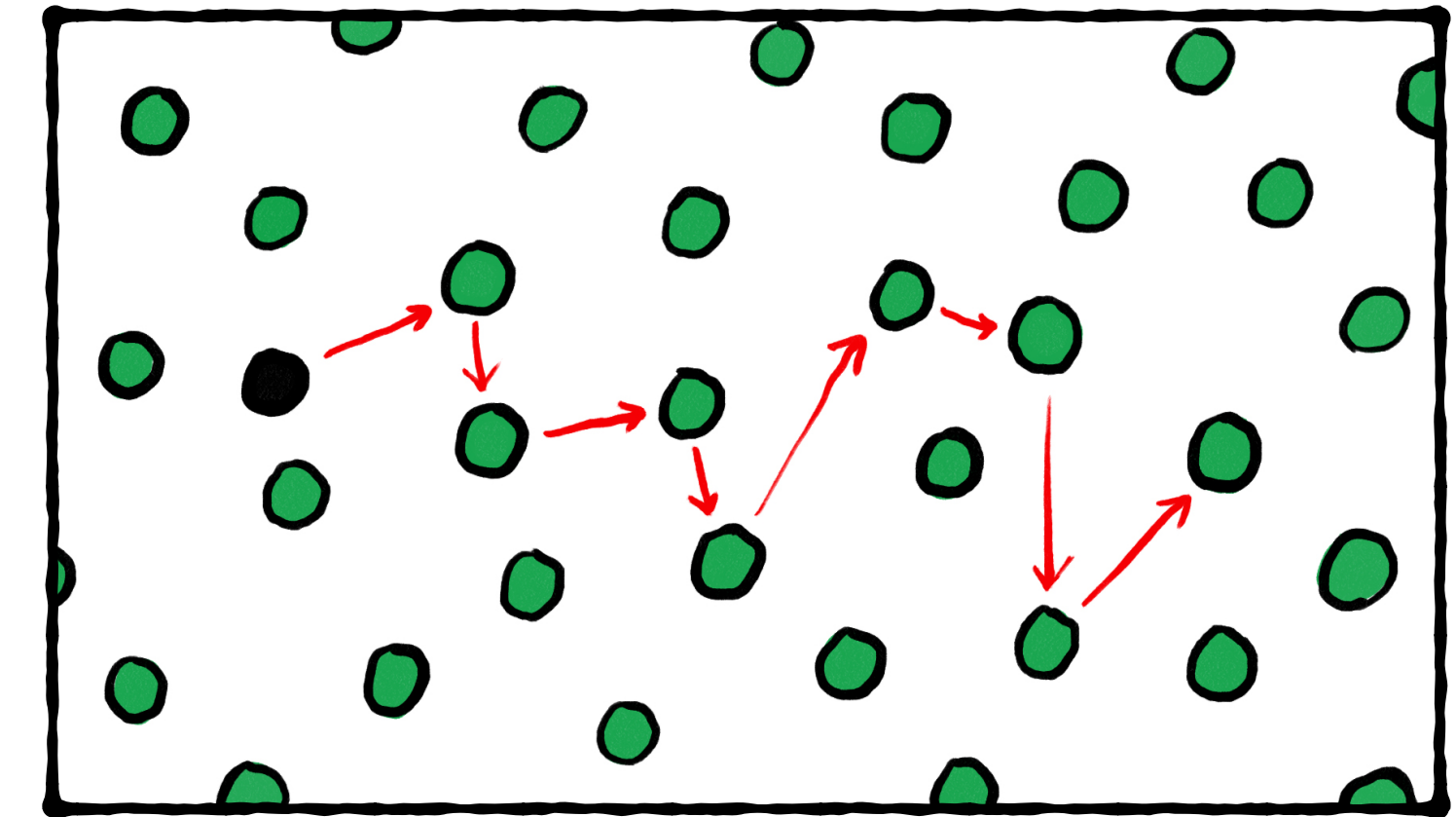


The self-energy Π



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- **Imaginary part:** particle production and absorption rate

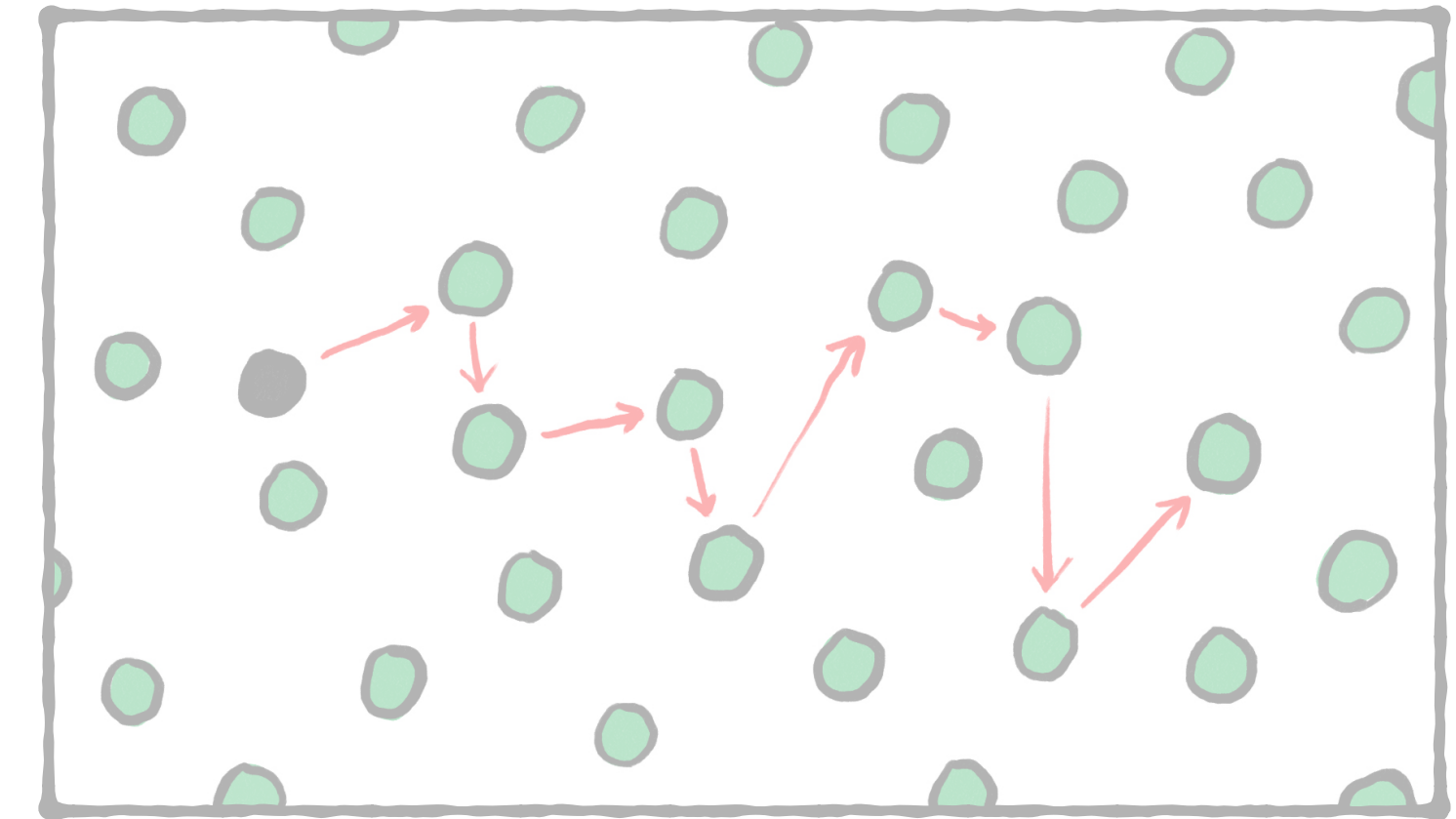
$$\frac{dn_\Phi}{dt} = - \int \frac{d^3 k}{(2\pi)^3 2\omega} (f_\Phi^{\text{eq}} - f_\Phi) 2 \text{Im } \Pi_\Phi$$

The self-energy Π



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Similar to optical theorem —
weighted by thermal phase space
distributions

Finite Temperature Field Theory (FTFT)

- **What is FTFT:** Quantum Field Theory at non-zero temperature and/or density
- **Equilibrium:** imaginary-time formalism

$$\rho(T) = e^{-H/T} \quad \leftrightarrow \quad U(t) = e^{-itH}$$

- **How to do FTFT:**

Same diagrammatic rules, but discrete imaginary frequencies

$$\omega_n = i2n\pi T \text{ (boson)} \qquad \omega_n = i(2n+1)\pi T \text{ (fermion)}$$

$$\int \frac{d^4 p}{(2\pi)^4} f(p_0, \mathbf{p}) \rightarrow T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} f(\omega_n + \mu, \mathbf{p})$$

Finite Temperature Field Theory (FTFT)

- What
- Equilib
- How t

density



 ***Spectral surgery in a heat bath:***
user-friendly cutting rules
that cut complexity, but not corners!

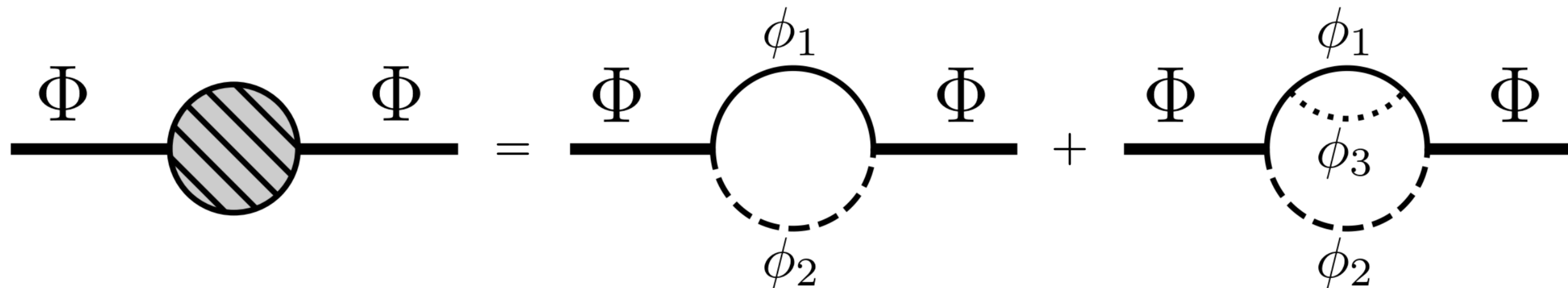
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Im Π : Spectral Surgery in a Heat Bath (SSHB)

- Toy model with four scalar particles

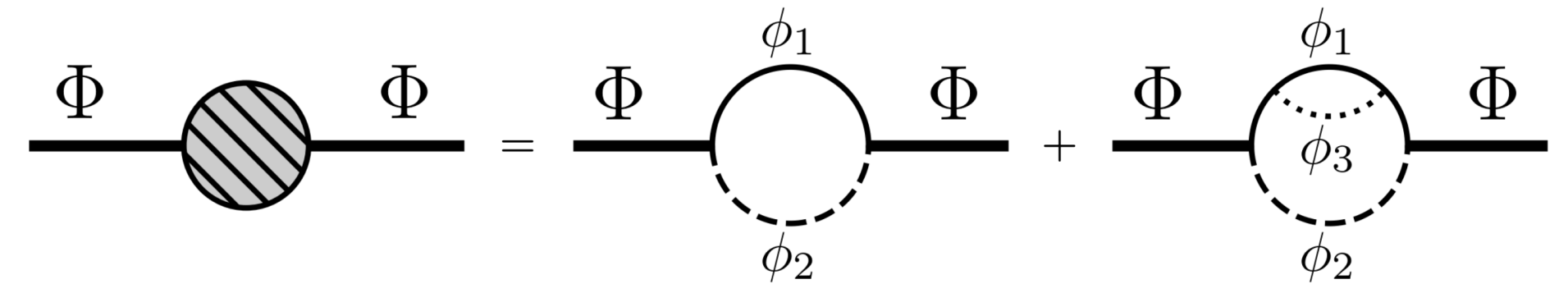
$$\mathcal{L}_{\text{int}} = g\Phi\phi_1\phi_2 + \frac{1}{2}\lambda\phi_1^2\phi_3$$

- Self-energy up to two loops



Im Π : Spectral Surgery in a Heat Bath (SSHB)

- Step 0: Self-energy diagrams

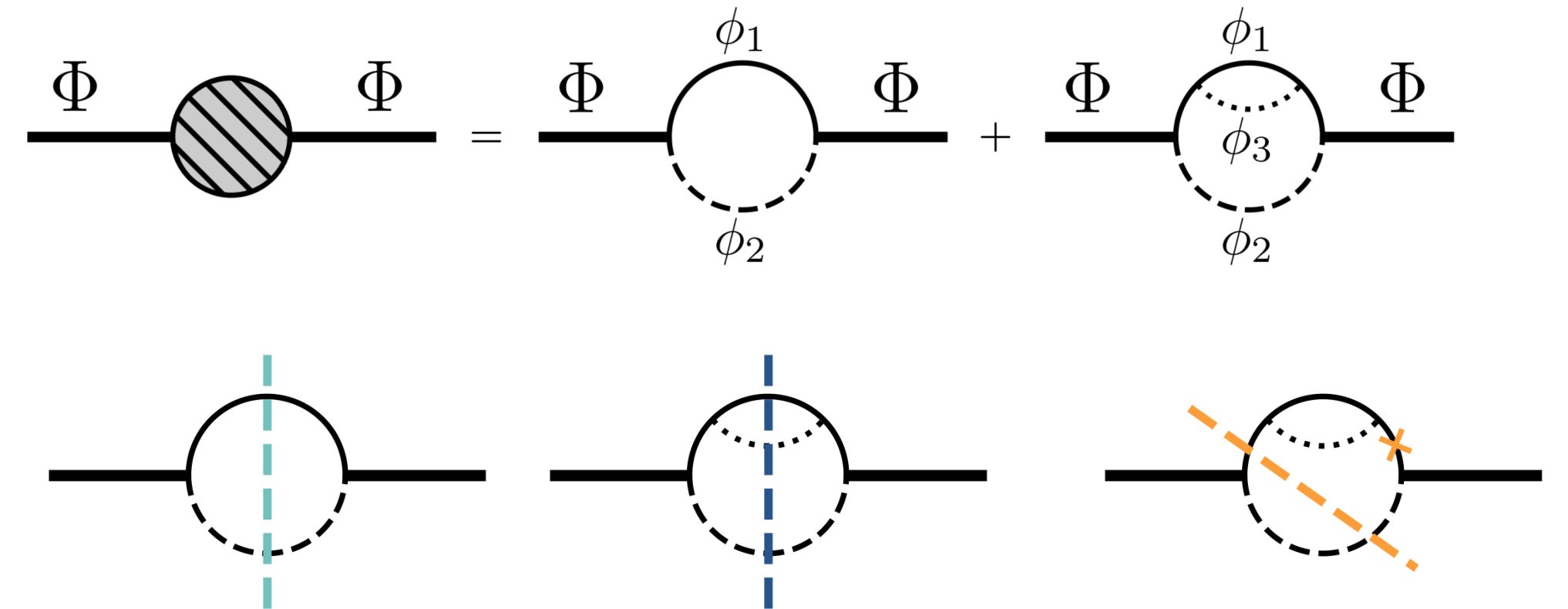


The diagram shows an equation for the imaginary part of the self-energy. On the left, a horizontal line with a shaded circle in the middle is flanked by Φ labels. This is equal to the sum of two terms. The first term is a horizontal line with a dashed circle in the middle, flanked by Φ labels; the dashed circle has ϕ_1 at the top and ϕ_2 at the bottom. The second term is a horizontal line with a dashed circle in the middle, flanked by Φ labels; the dashed circle has ϕ_1 at the top, ϕ_2 at the bottom, and a dotted line with ϕ_3 in the center.

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3}$$

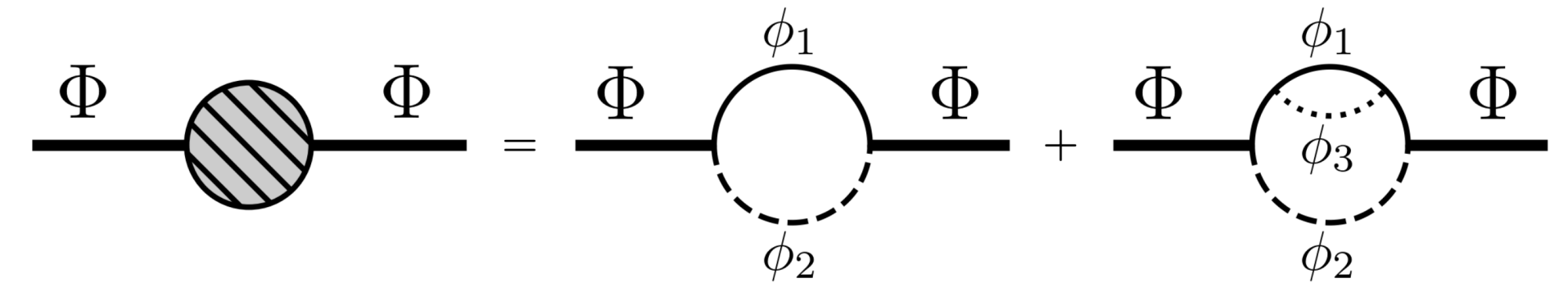
Im Π : Spectral Surgery in a Heat Bath (SSHB)

- Step 0: Self-energy diagrams
- Step 1: Bisections (full cuts)

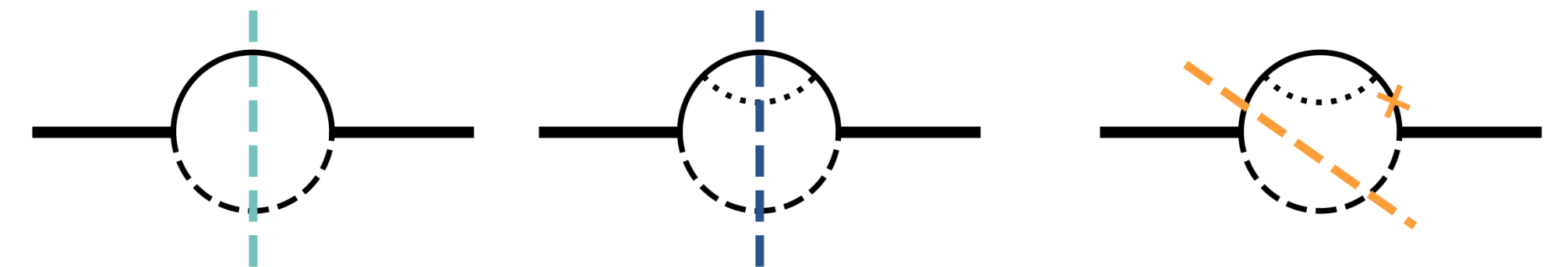


Im Π : Spectral Surgery in a Heat Bath (SSHB)

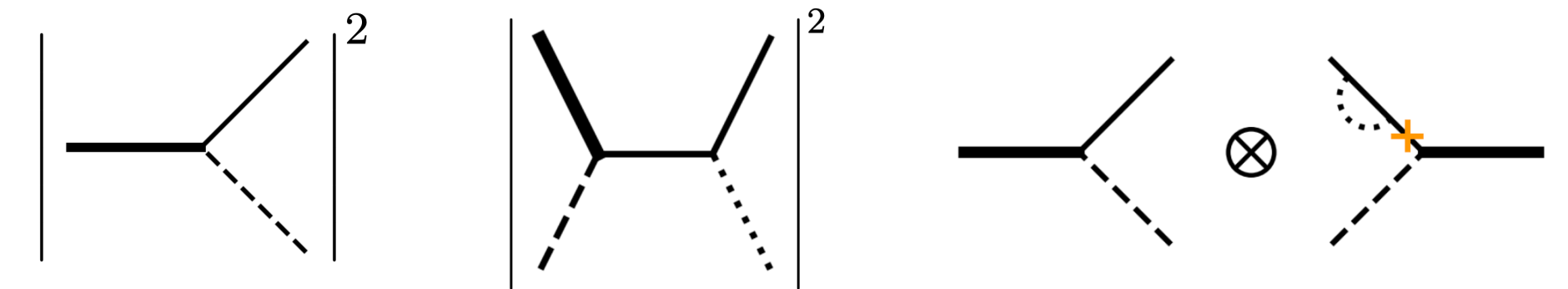
- Step 0: Self-energy diagrams



- Step 1: Bisections (full cuts)

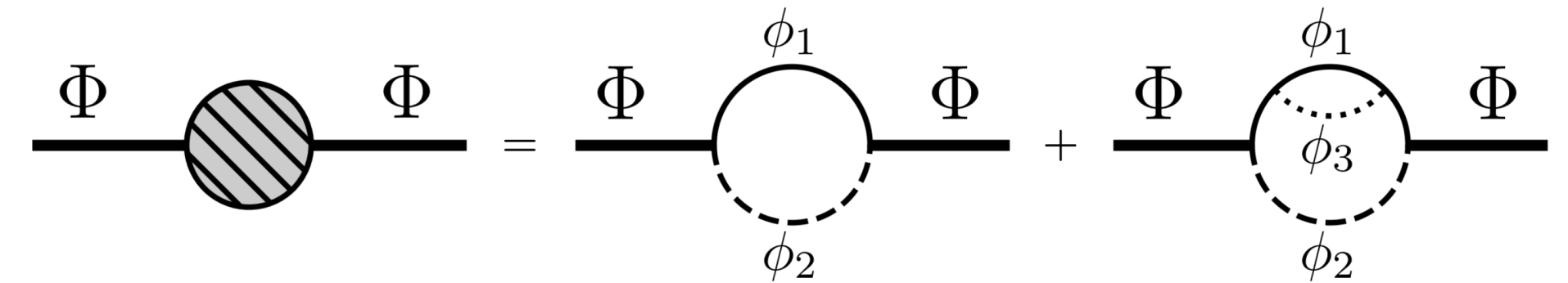


- Step 2: Converting bisections into processes

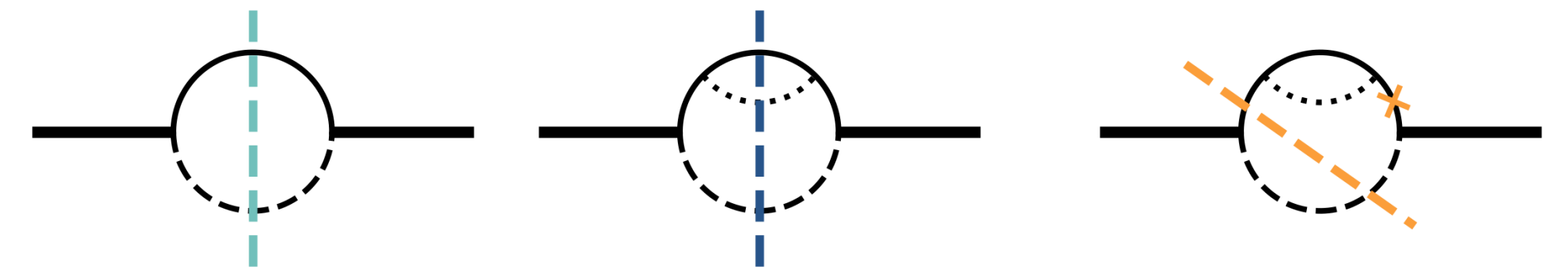


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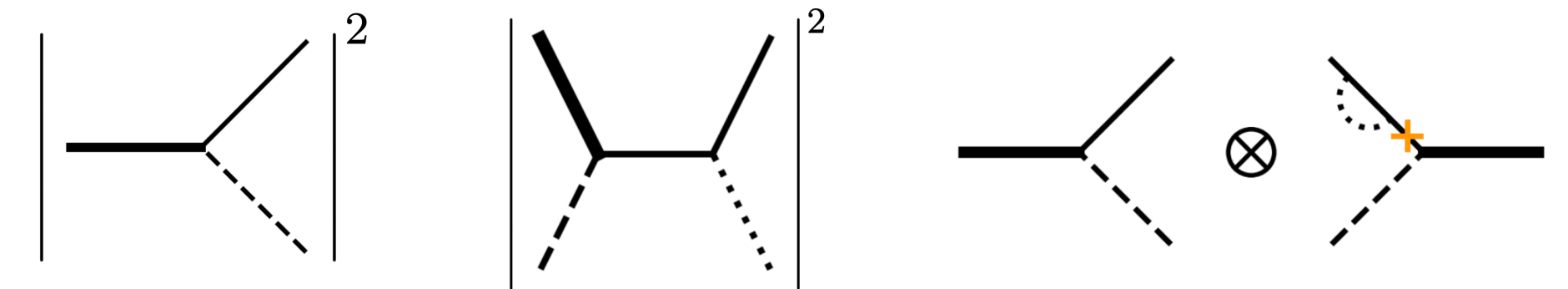
- Step 0: Self-energy diagrams



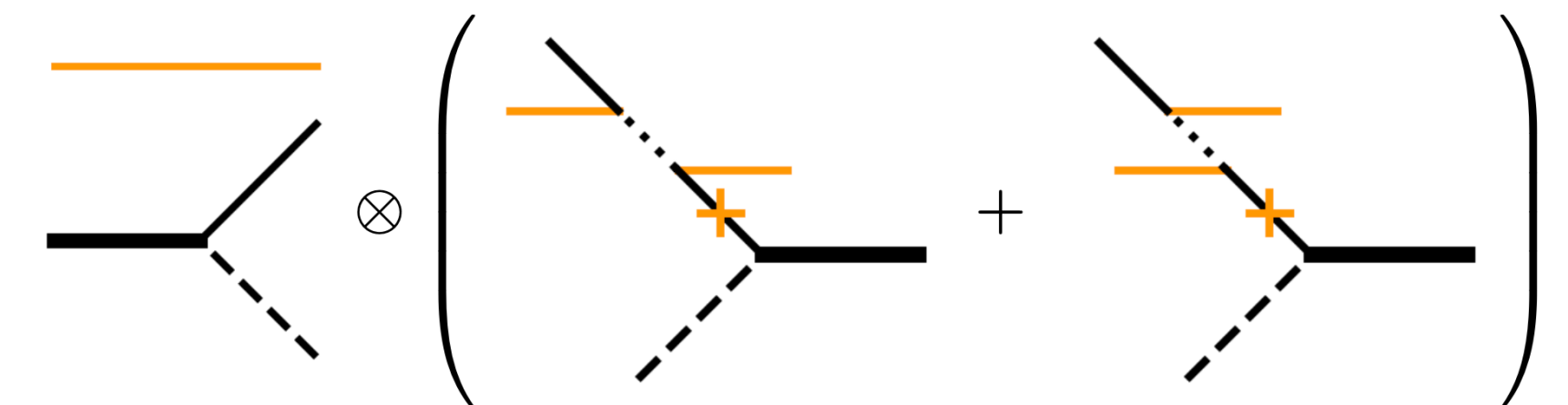
- Step 1: Bisections (full cuts)



- Step 2: Converting bisections into processes



- Step 3: Loopectomy (internal loop partial cuts)

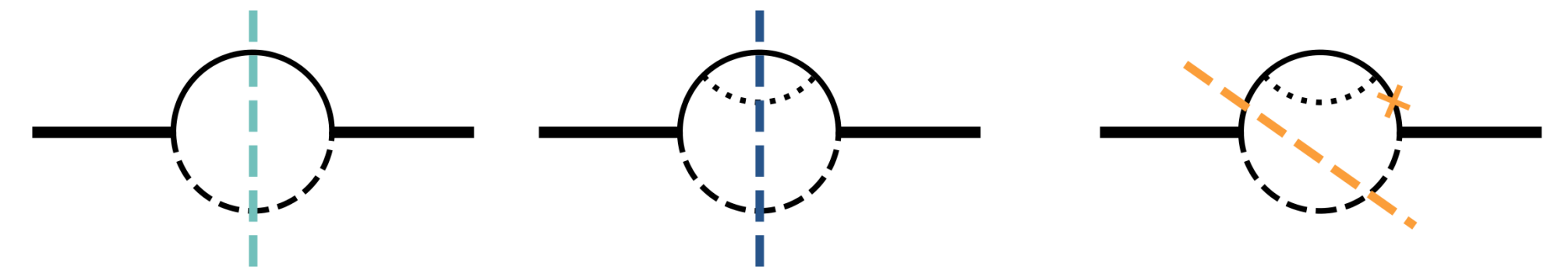


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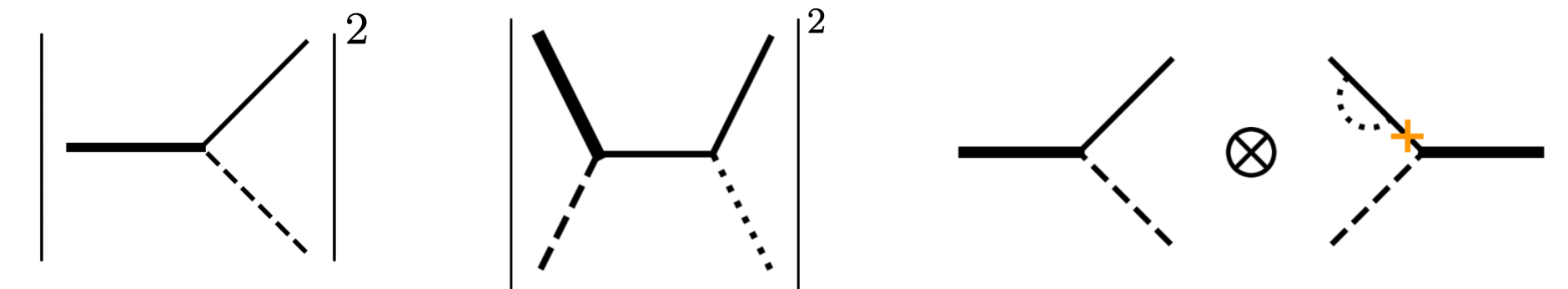
- Step 0: Self-energy diagrams

$$\text{Diagram with shaded circle and } \Phi \text{ lines} = \text{Diagram with circle, } \Phi \text{ lines, and } \phi_1, \phi_2 \text{ lines} + \text{Diagram with circle, } \Phi \text{ lines, and } \phi_1, \phi_2, \phi_3 \text{ lines}$$

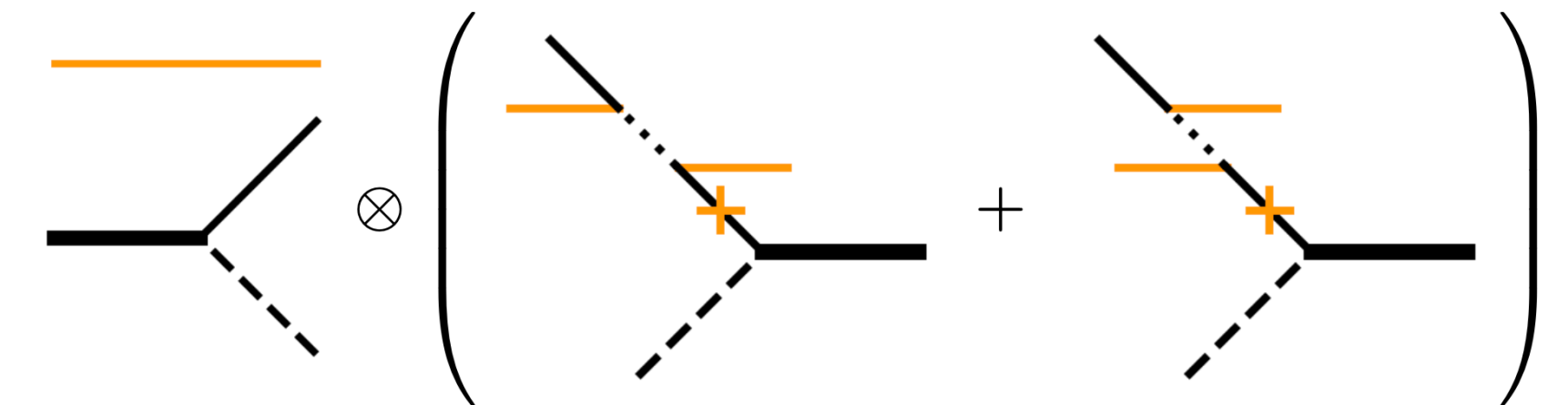
- Step 1: Bisections (full cuts)



- Step 2: Converting bisections into processes



- Step 3: Loopectomy (internal loop partial cuts)



- Step 4: Sum over all these tree processes

Cut = on-shell

Phase space distributions

Im Π : Spectral Surgery in a Heat Bath (SSHB)

- Example: one-loop contribution

Cut propagators
are on-shell

Phase space factors,
absorption - emission

$$\text{Im} \left(\text{Feynman diagram} \right) = -\frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2}$$

Usual momentum
conservation

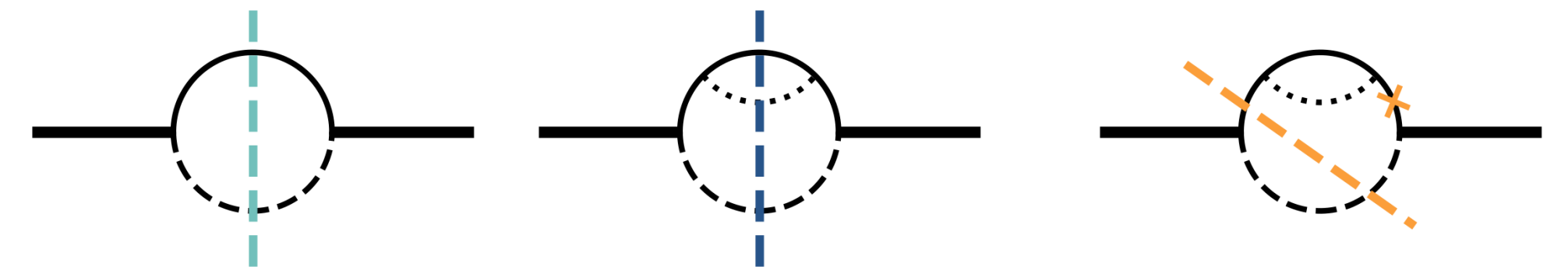
$$\times \left\{ \begin{aligned} & (2\pi)^4 \delta^{(4)}(k - p_1 - p_2) \left| \text{diagram} \right|^2 [(1 + f_1)(1 + f_2) - f_1 f_2] \\ & + (2\pi)^4 \delta^{(4)}(k + p_1 - p_2) \left| \text{diagram} \right|^2 [f_1(1 + f_2) - f_2(1 + f_1)] \\ & + (2\pi)^4 \delta^{(4)}(k - p_1 + p_2) \left| \text{diagram} \right|^2 [f_2(1 + f_1) - f_1(1 + f_2)] \\ & + (2\pi)^4 \delta^{(4)}(k + p_1 + p_2) \left| \text{diagram} \right|^2 [f_1 f_2 - (1 + f_2)(1 + f_1)] \end{aligned} \right\}$$

Im Π : Spectral Surgery in a Heat Bath (SSHB)

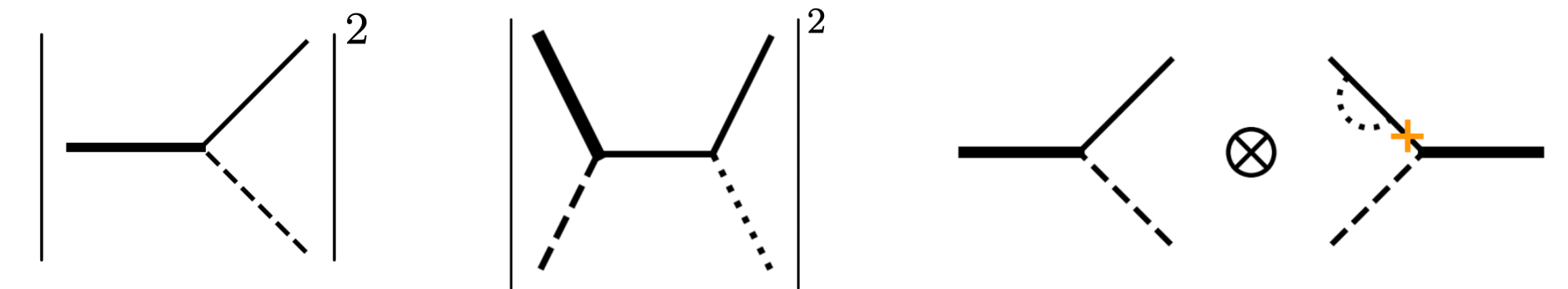
- Step 0: Self-energy diagrams

$$\text{Diagram with shaded circle} = \text{Diagram with dashed circle and } \phi_1, \phi_2 \text{ labels} + \text{Diagram with dashed circle and } \phi_1, \phi_2, \phi_3 \text{ labels}$$

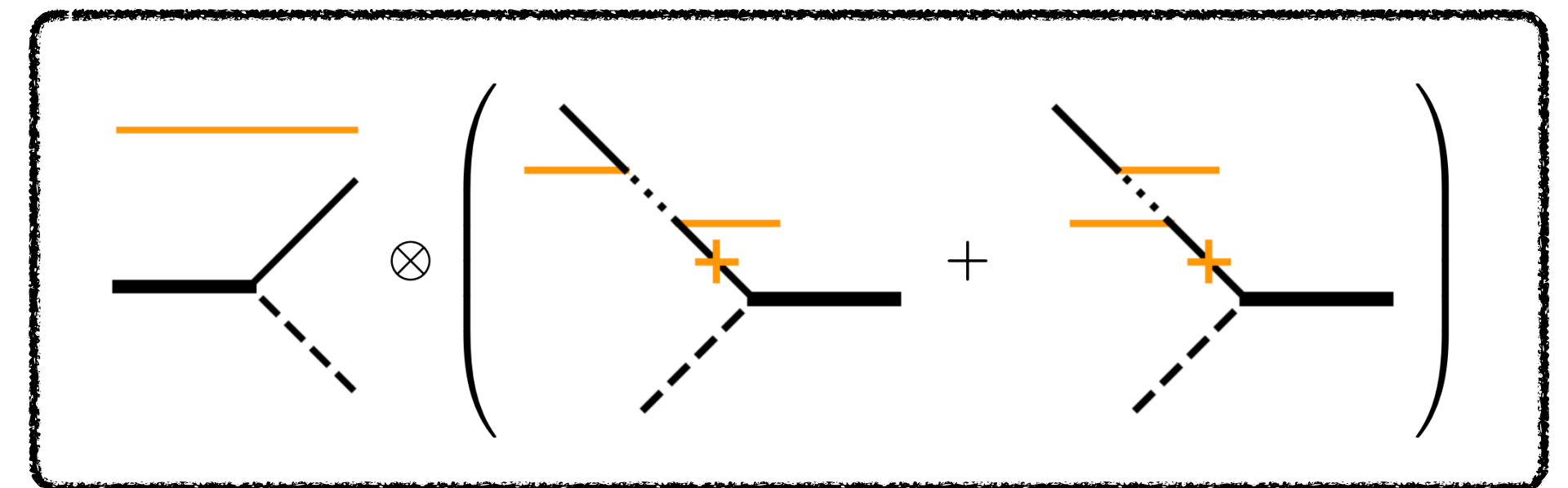
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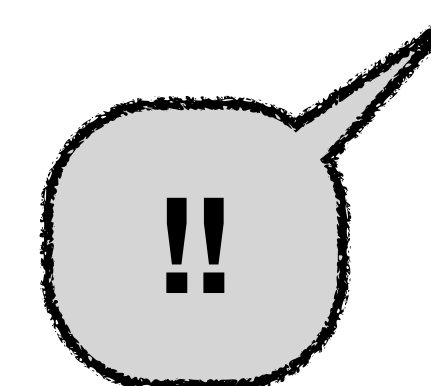
- Step 3: Loopectomy (internal loop partial cuts)



- Step 4: Sum over all these tree processes

Cut = on-shell

Phase space distributions



👉 Thermal *interference* processes,
from *asymmetric* cuts,
can significantly impact
production rates

Im Π : Spectral Surgery in a Heat Bath (SSHB)

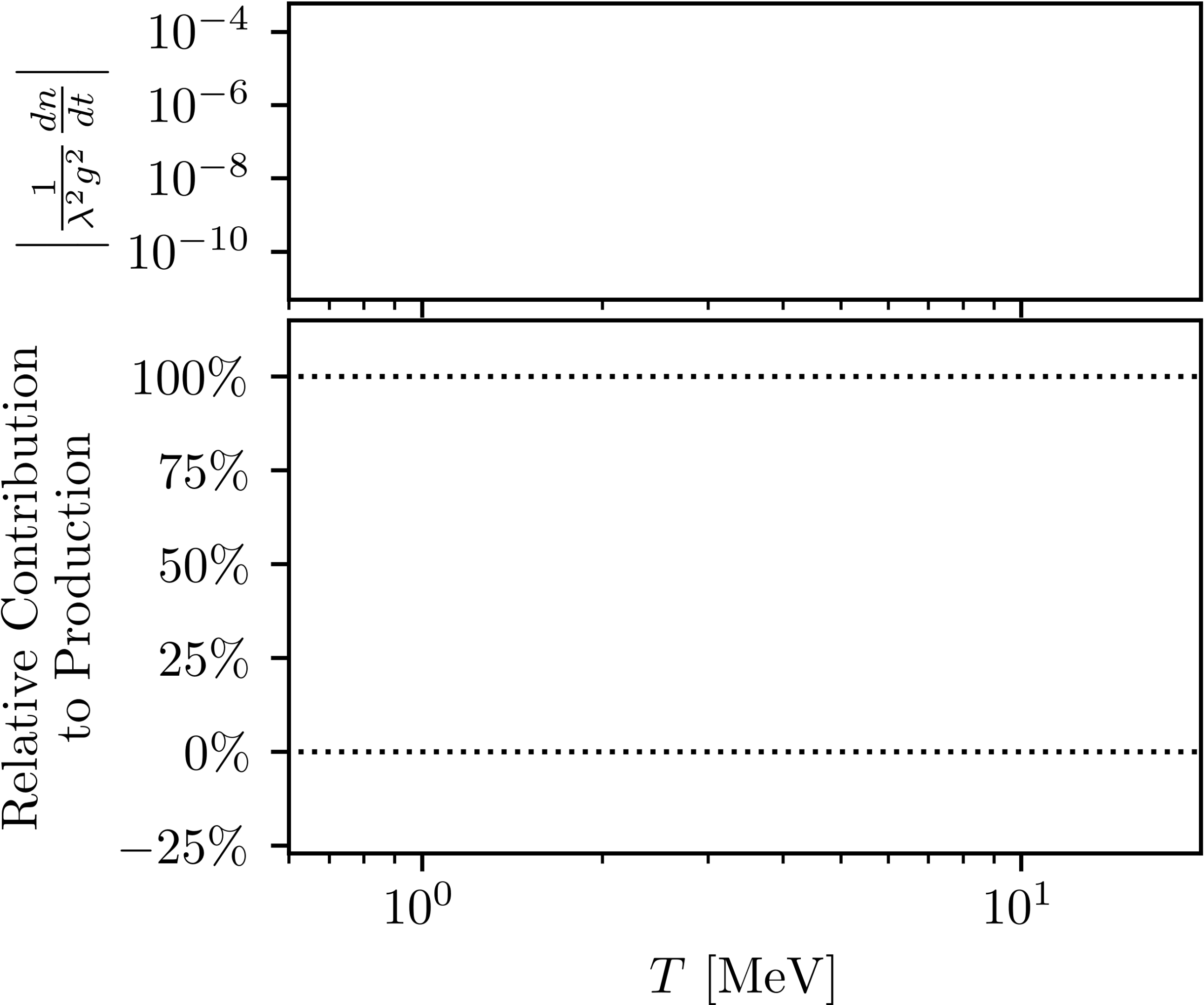
- Two-loop contribution, asymmetric cuts
- Interference: $1 \rightarrow 2$ times $2 \rightarrow 3$ amplitudes

The diagrammatic equation represents the imaginary part of the two-loop contribution to the self-energy, $\text{Im } \Pi$, in a heat bath (SSHB). The left side shows a circle with two external legs labeled Φ . The circle is divided into two regions by two dashed orange lines representing cuts, labeled ϕ_1 and ϕ_2 . A third dashed orange line, labeled ϕ_3 , is also shown. The right side of the equation is a sum of two terms, each representing a different cut configuration. Each term consists of a tree-level amplitude (a vertex with two external legs, one solid black and one dashed black) multiplied by a two-loop diagram (a vertex with two external legs, one solid black and one dashed black, and two internal lines, one solid black and one dashed black, forming a loop). The two-loop diagrams are shown in two different configurations, separated by a plus sign, and are enclosed in large parentheses. The entire equation is set against a white background.

All-scalar model production

$$\mathcal{L}_{\text{int}} = g\Phi\phi_1\phi_2 + \frac{1}{2}\lambda\phi_1^2\phi_3$$

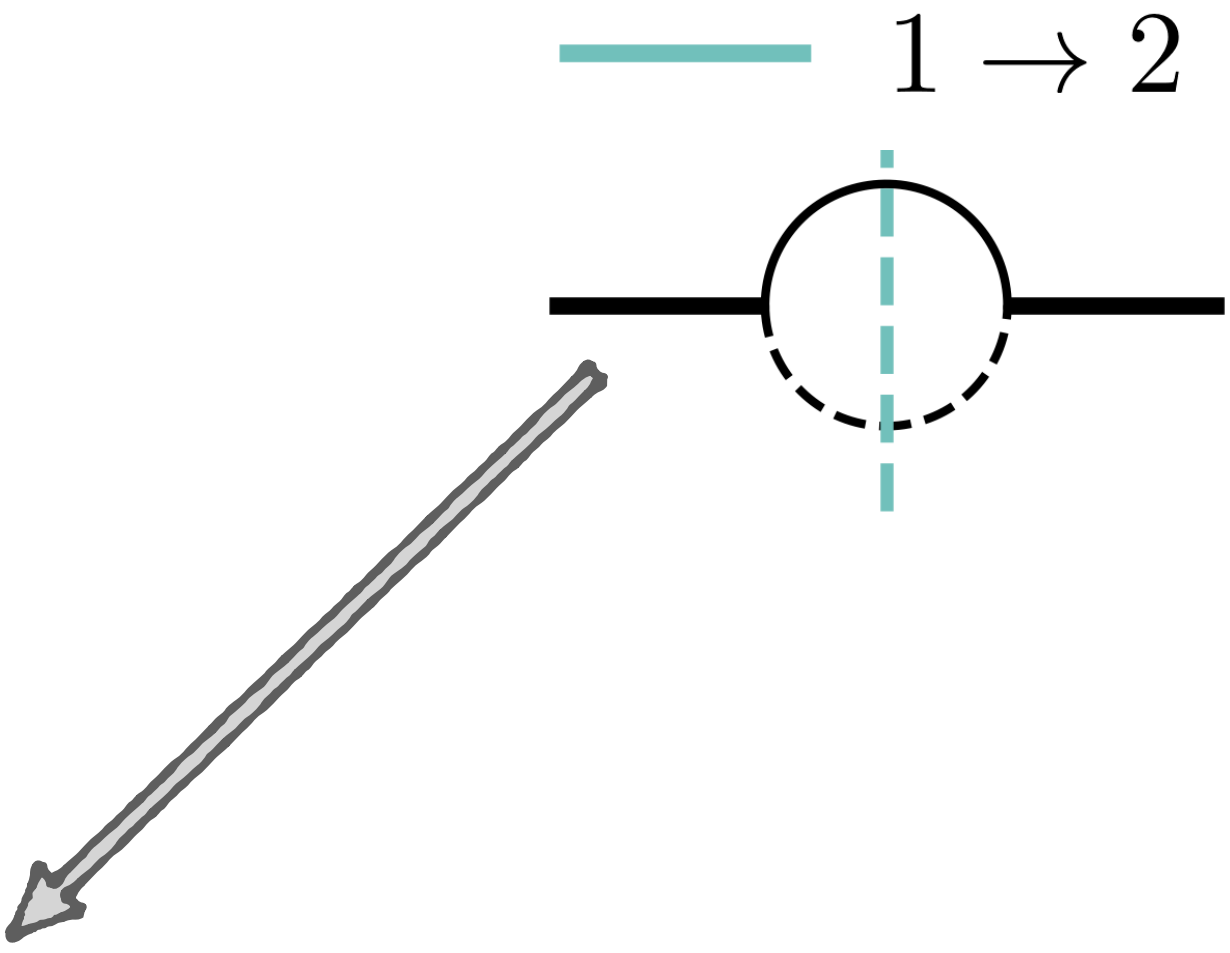
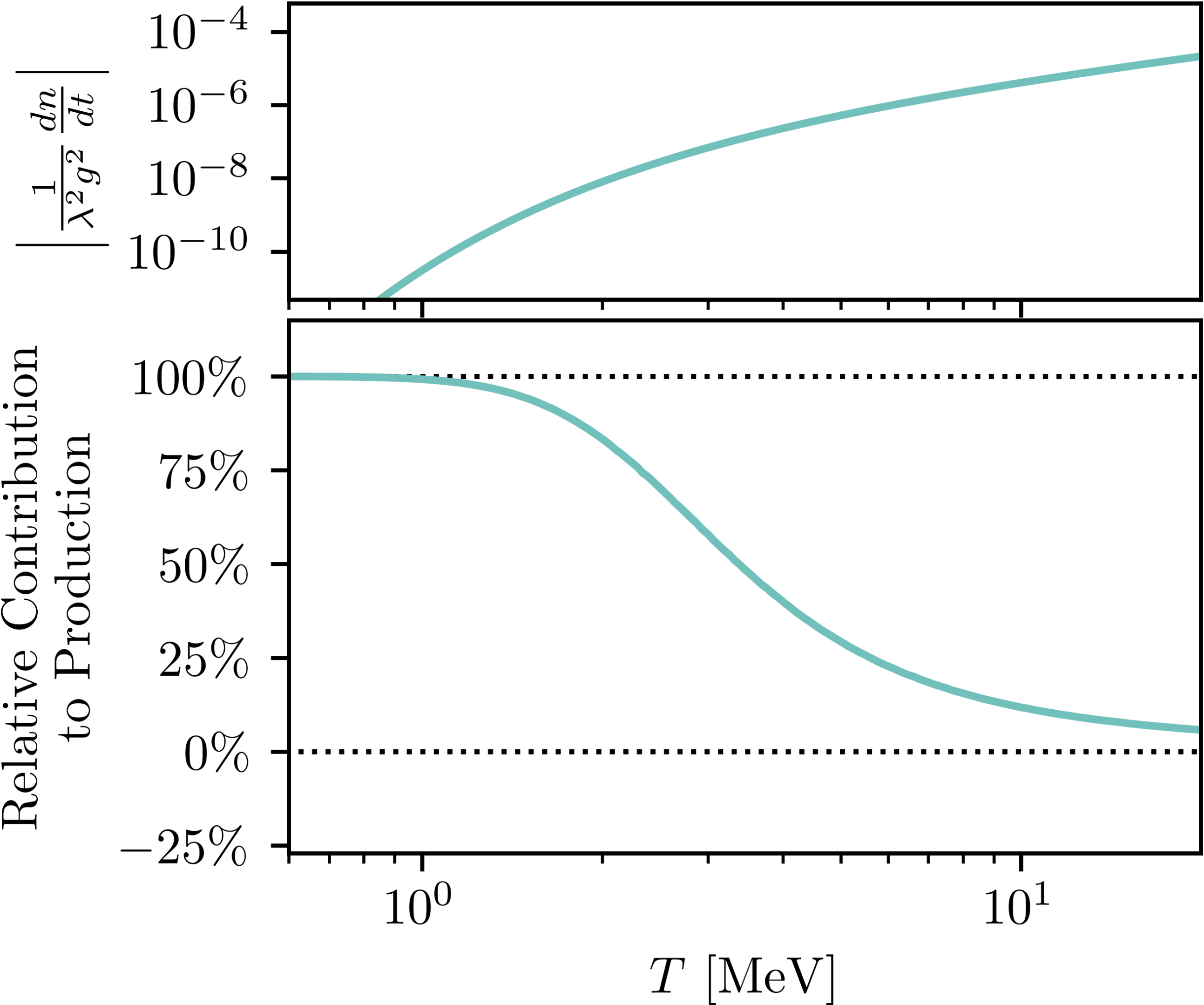
$$\begin{aligned} m_\Phi &= 9 \text{ MeV} \\ m_{\phi_1} &= 4 \text{ MeV} \\ m_{\phi_2} &= 3 \text{ MeV} \\ m_{\phi_3} &= 6 \text{ MeV} \\ \lambda &= 100 \text{ MeV} \end{aligned}$$



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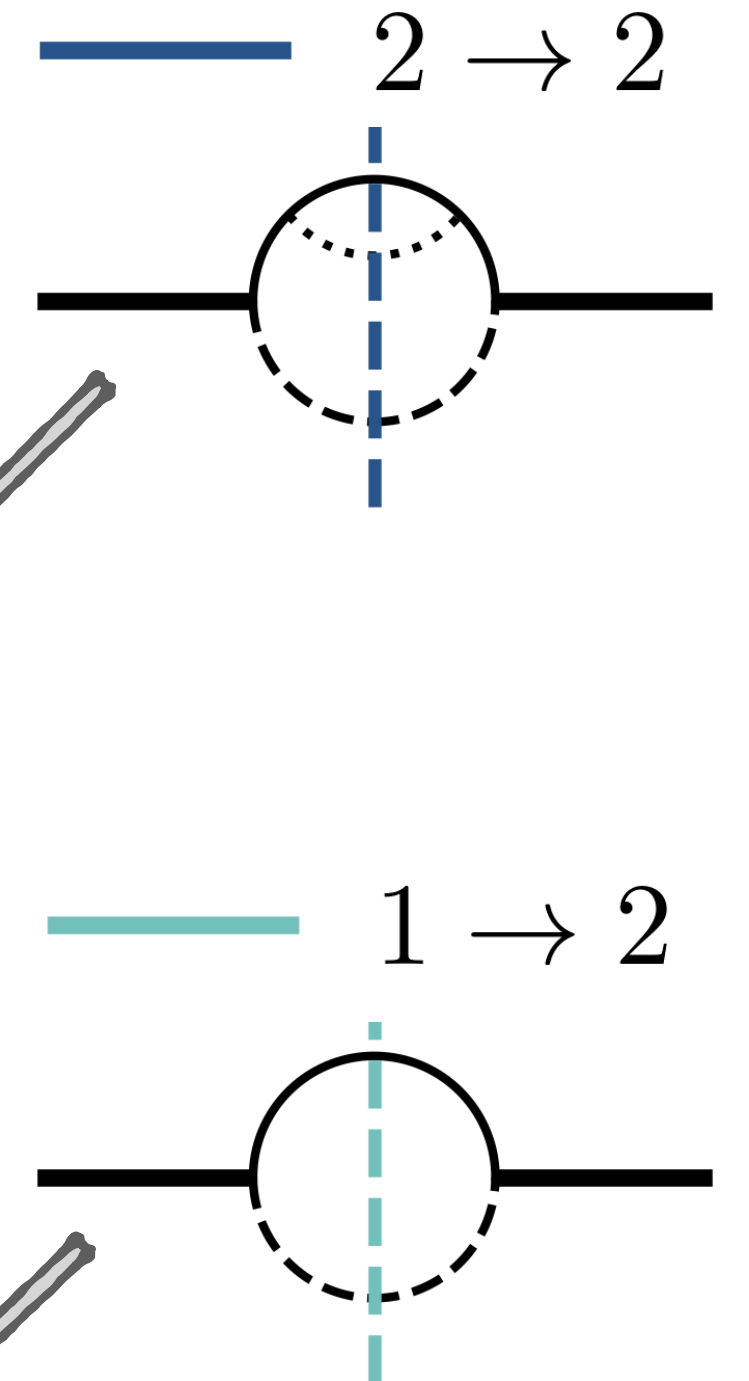
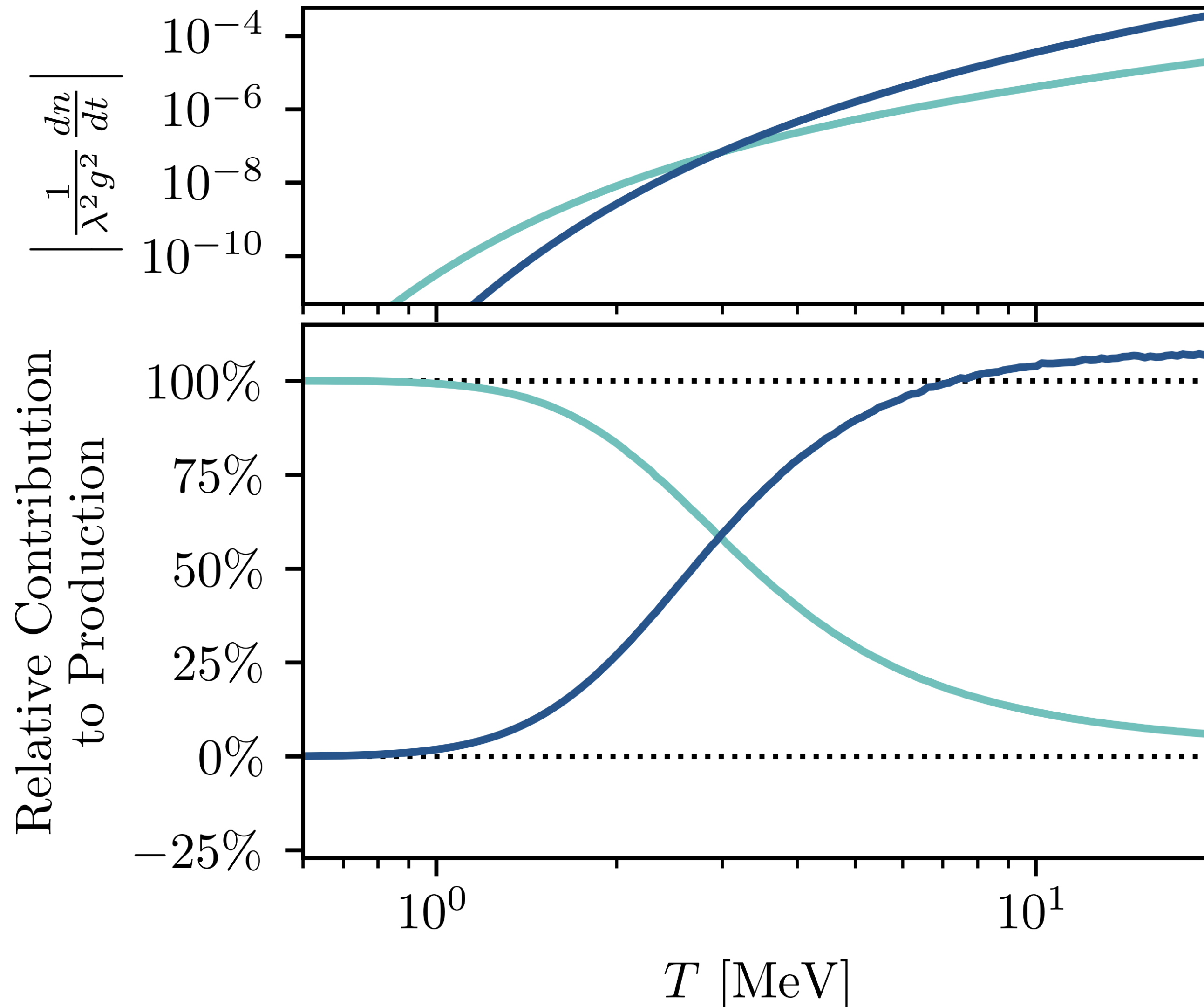
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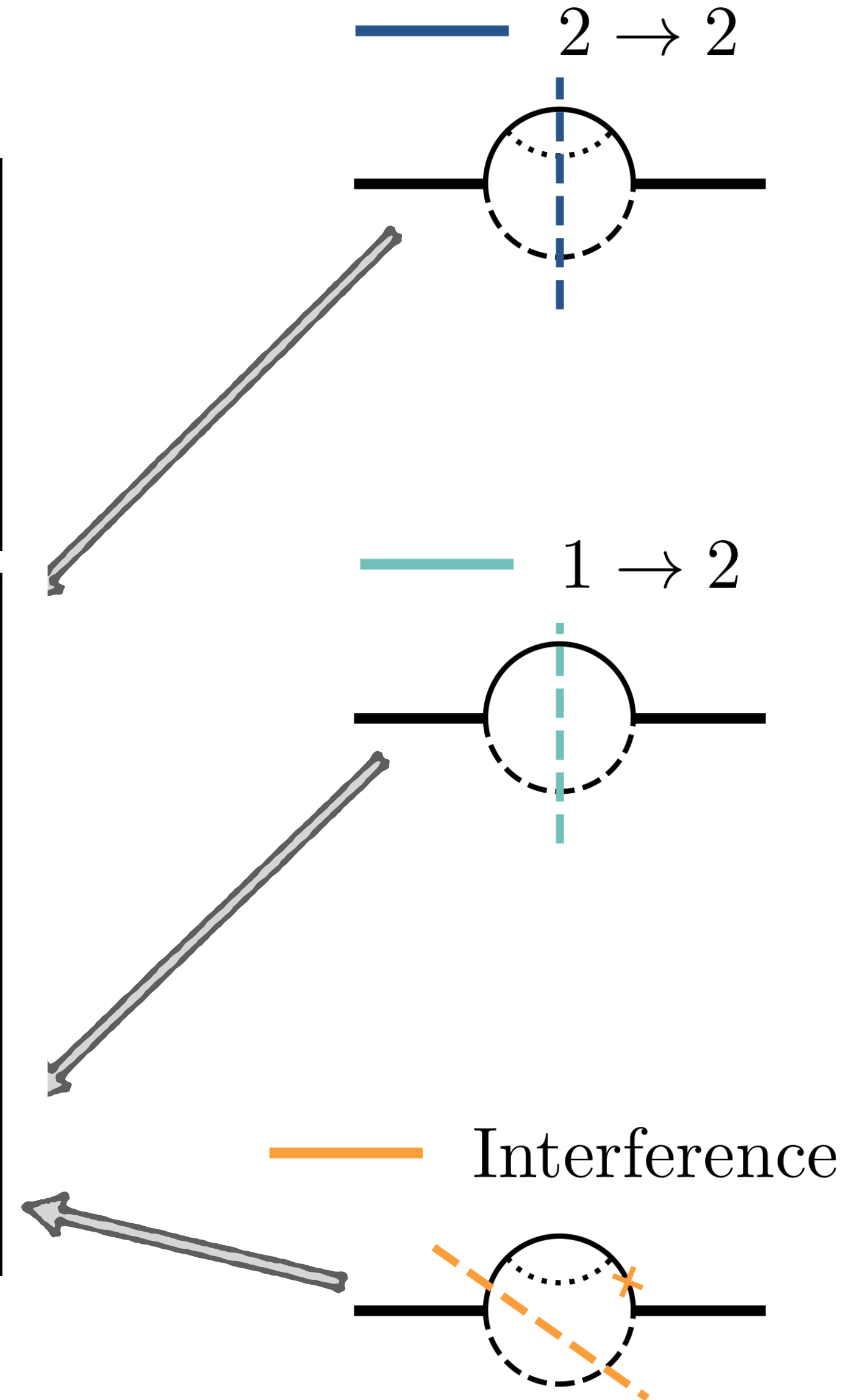
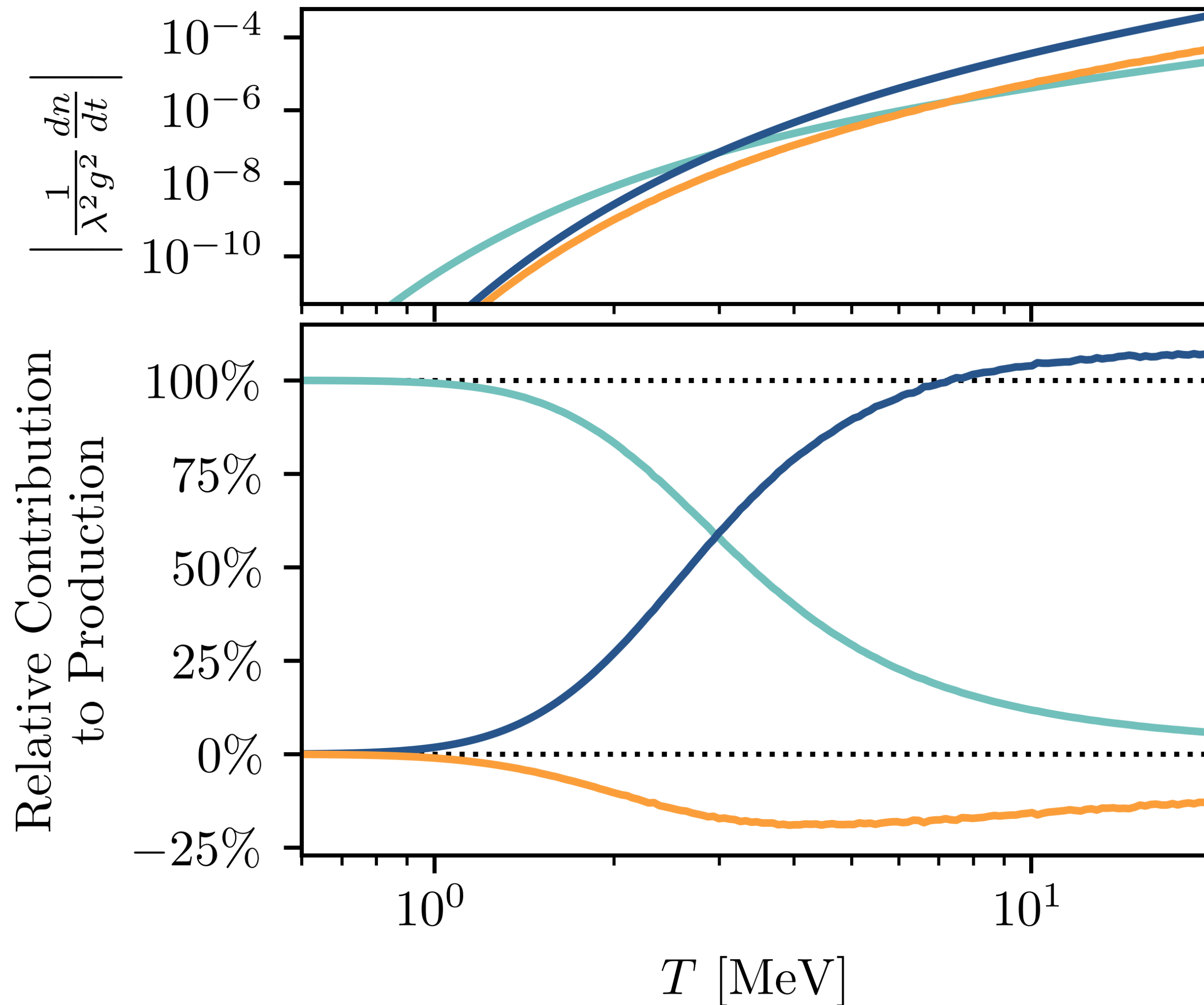
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Using these cutting rules

☞ Axion/Dark Photon production
in magnetic white dwarfs



see the talk from Dr. Nirmalya Brahma

2509.07085

☞ Light scalar production
in the core of red giants



see the talk from Natnael Debru

26XX.XXXXX

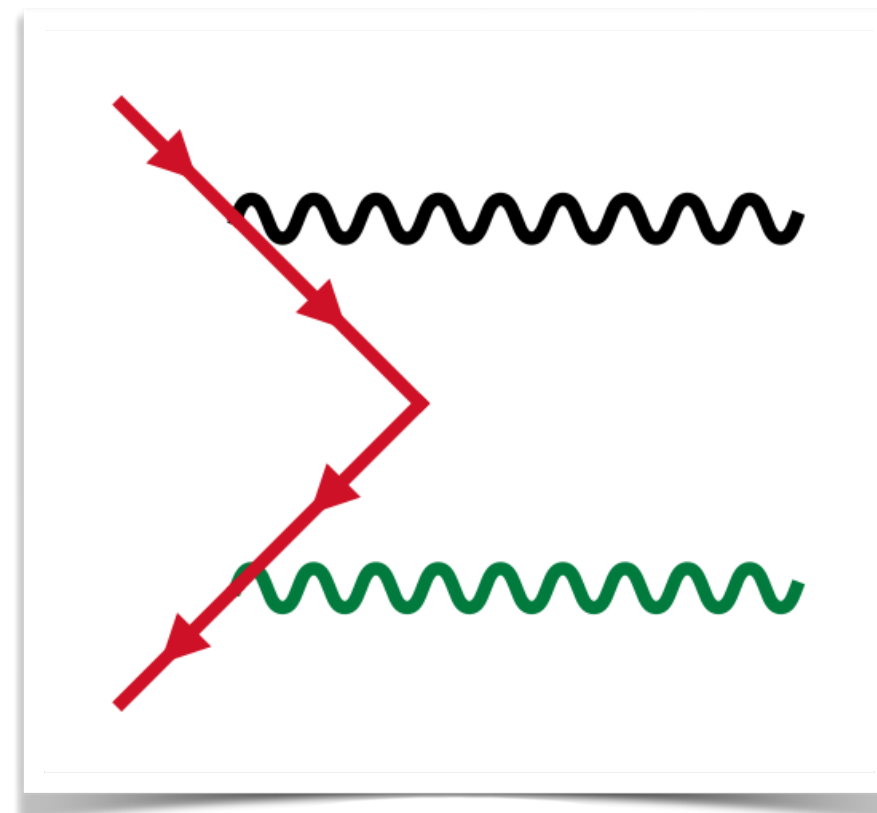
Takeaways

- ☞ We should not neglect a thermal effect:
Thermal effects are *essential* for accurate BSM predictions
- ☞ ***Spectral surgery in a heat bath:***
user-friendly cutting rules that cut complexity, but not corners!
- ☞ Thermal *interference* processes can significantly impact production rates

Want to draw some Feynman diagrams:

```
> pip install feynndraw
```

(or: <https://github.com/schhug/feyndraw>)



More at
arXiv:2507.14277



Back up slides

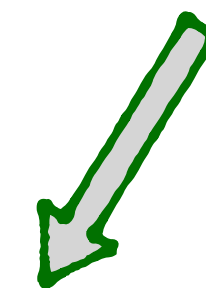
All-scalar model production

- One-loop contribution

$$\text{Im} \left(\text{Feynman diagram} \right) = -\frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \times \left\{ \begin{aligned} & (2\pi)^4 \delta^{(4)}(k - p_1 - p_2) \left| \text{diagram 1} \right|^2 [(1 + f_1)(1 + f_2) - f_1 f_2] \\ & + (2\pi)^4 \delta^{(4)}(k + p_1 - p_2) \left| \text{diagram 2} \right|^2 [f_1(1 + f_2) - f_2(1 + f_1)] \\ & + (2\pi)^4 \delta^{(4)}(k - p_1 + p_2) \left| \text{diagram 3} \right|^2 [f_2(1 + f_1) - f_1(1 + f_2)] \\ & + (2\pi)^4 \delta^{(4)}(k + p_1 + p_2) \left| \text{diagram 4} \right|^2 [f_1 f_2 - (1 + f_2)(1 + f_1)] \end{aligned} \right\}$$

Cut propagators
are on-shell

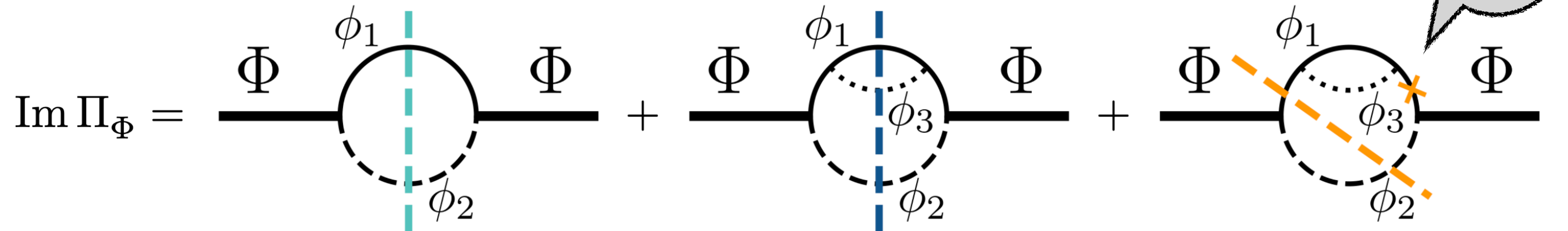
Phase space factors,
absorption - emission



All-scalar model production

- Assume Maxwell-Boltzmann distributions
- Assume freeze-in

$$\frac{dn_{\Phi}}{dt} = - \int \frac{d^3 k}{(2\pi)^3 2\omega} f_{\Phi}^{\text{eq}} 2 \text{Im} \Pi_{\Phi}$$



All-scalar model production

- One-loop contribution

$$\begin{aligned}
 & \text{Diagram 1} = \left| \text{Triangle (solid top, solid left, dashed right)} \right|^2 + \left| \text{Triangle (solid top, dashed left, solid right)} \right|^2 \\
 & + \left| \text{Triangle (dashed top, solid left, solid right)} \right|^2 + \left| \text{Triangle (dashed top, dashed left, dashed right)} \right|^2
 \end{aligned}$$

All-scalar model production

- Two-loop contribution, symmetric cuts

The diagram illustrates the decomposition of a two-loop scalar production process into nine terms. On the left, a scalar Φ (solid line) enters a loop from the left, and another scalar Φ (solid line) exits to the right. The loop is a circle with a vertical dashed line through its center. The top arc of the loop is labeled ϕ_1 , the bottom arc is labeled ϕ_2 , and the vertical dashed line is labeled ϕ_3 . This is followed by an equals sign and a sum of nine terms, each enclosed in vertical bars with a superscript 2, indicating the squared magnitude of the amplitude.

The nine terms are arranged in a 3x3 grid, separated by plus signs. Each term represents a different two-loop diagram with various internal line types (solid, dashed, dotted) and topologies. The first row contains three terms with a central horizontal line. The second row contains three terms with a central vertical line. The third row contains two terms with a central vertical line and one term with a central horizontal line.

All-scalar model production

- Two-loop contribution, asymmetric cuts

The diagram illustrates the decomposition of a two-loop self-energy contribution in an all-scalar model. On the left, a thick horizontal line represents an incoming scalar Φ , which enters a circular loop. The loop contains two internal scalar lines labeled ϕ_1 and ϕ_2 , and a dashed line labeled ϕ_3 . Two orange dashed lines with 'x' marks indicate asymmetric cuts. The loop is connected to another thick horizontal line representing an outgoing scalar Φ . This is set equal to the sum of four terms, each representing a different cut configuration. Each term consists of a tree-level vertex (represented by a thick line splitting into two thinner lines) multiplied by a two-loop diagram (represented by a circle with an 'x' mark), which is then multiplied by another tree-level vertex. The four terms correspond to the four possible cut configurations of the two-loop diagram.

All-scalar model production

$$\begin{array}{c} \Phi \end{array} \begin{array}{c} \phi_1 \\ \text{---} \bigcirc \text{---} \\ \phi_2 \end{array} \begin{array}{c} \Phi \end{array} = \left| \begin{array}{c} \text{---} \diagup \diagdown \text{---} \end{array} \right|^2$$

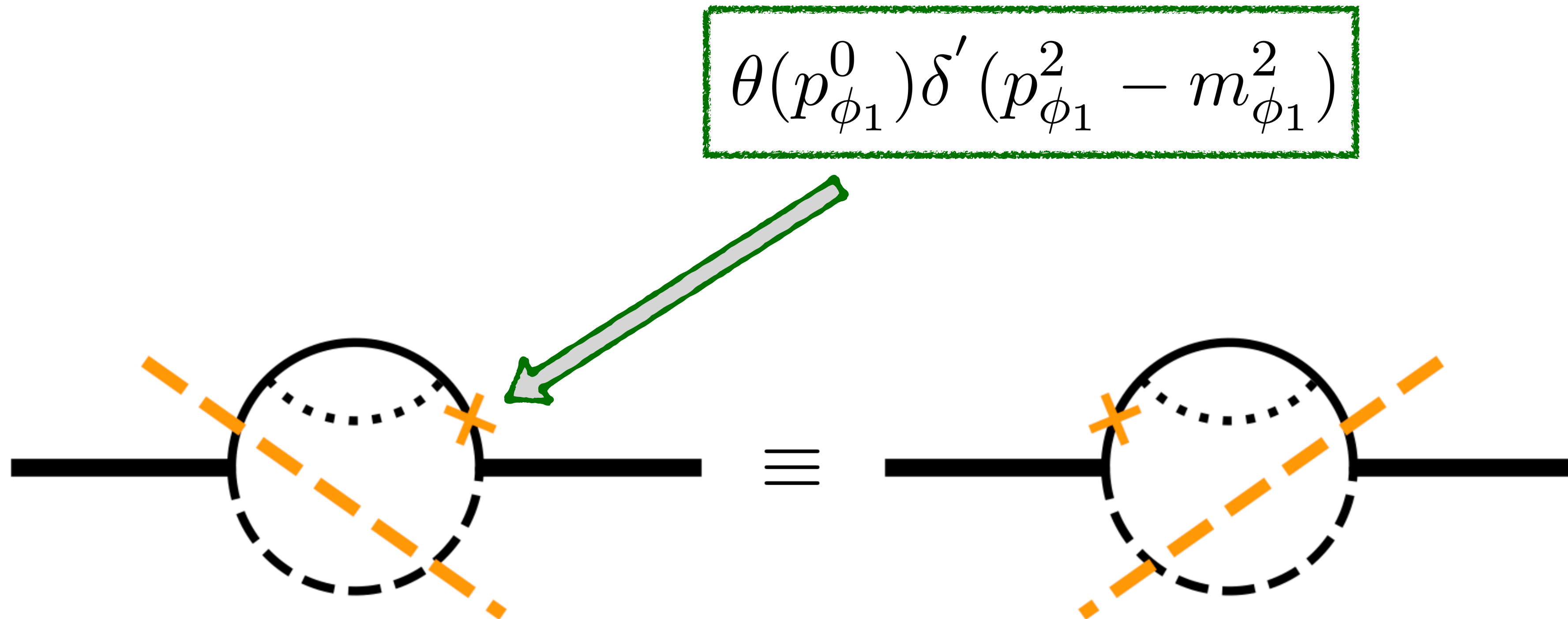
$$\begin{aligned}
 m_{\phi_1} + m_{\phi_2} &< m_{\Phi} \\
 m_{\Phi} &< m_{\phi_1} + m_{\phi_2} + m_{\phi_3} \\
 m_{\phi_3} &< m_{\phi_1} + m_{\phi_2} + m_{\Phi}
 \end{aligned}$$

$$\begin{array}{c} \Phi \end{array} \begin{array}{c} \phi_1 \\ \text{---} \bigcirc \text{---} \\ \phi_2 \end{array} \begin{array}{c} \Phi \end{array} = \left| \begin{array}{c} \text{---} \diagup \diagdown \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \diagup \diagdown \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \diagup \diagdown \text{---} \end{array} \right|^2$$

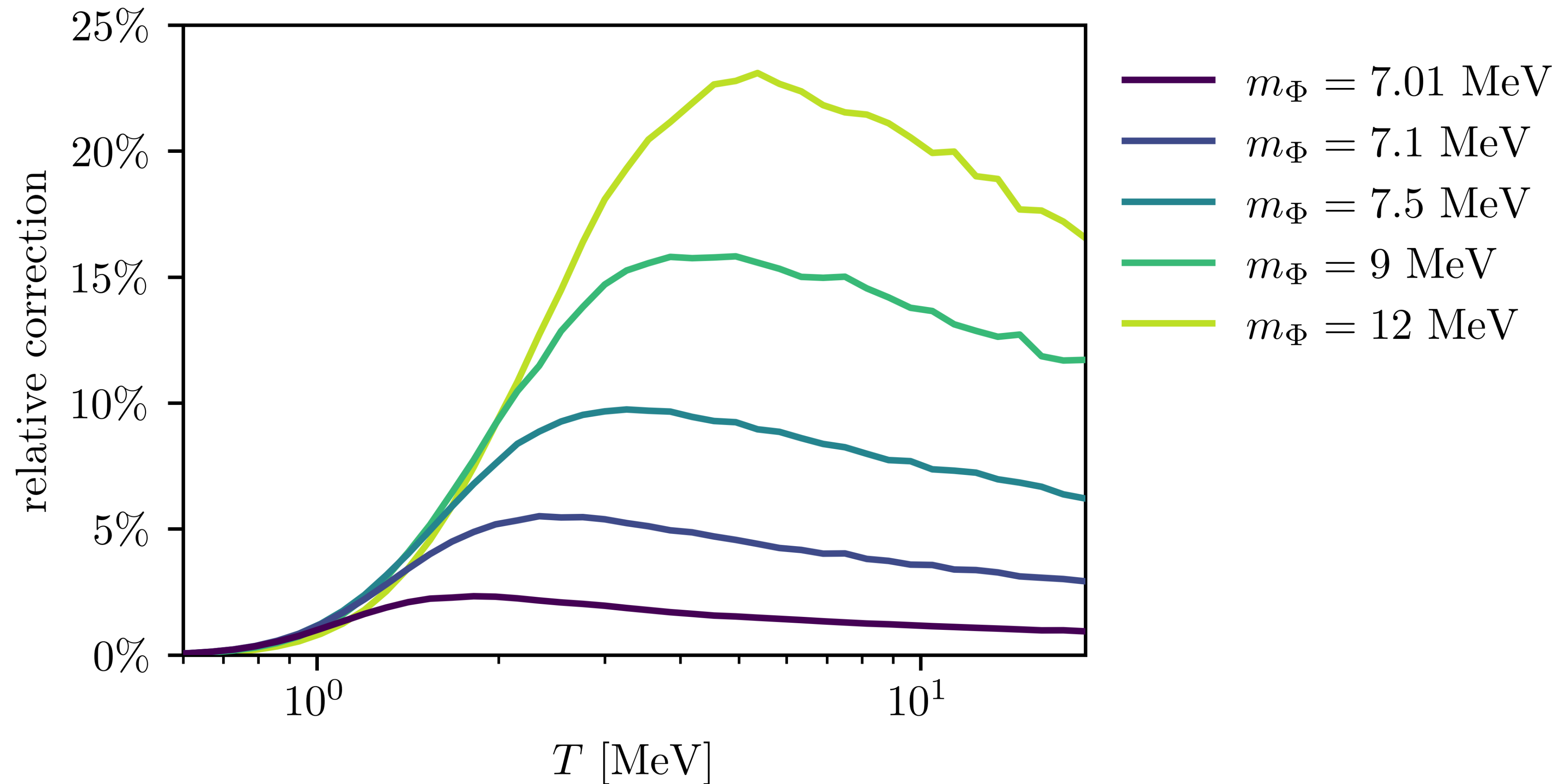
$$\begin{array}{c} \Phi \end{array} \begin{array}{c} \phi_1 \\ \text{---} \bigcirc \text{---} \\ \phi_2 \end{array} \begin{array}{c} \Phi \end{array} = \begin{array}{c} \text{---} \diagup \diagdown \text{---} \end{array} \otimes \begin{array}{c} \text{---} \diagup \diagdown \text{---} \end{array}$$

Im Π : Spectral Surgery in a Heat Bath (SSHB)

- Those are the same bisection (full cut)



All-scalar model production



Scalar-fermion-vector Toy Model

- Toy model with a fermion, a massive scalar and a massive boson

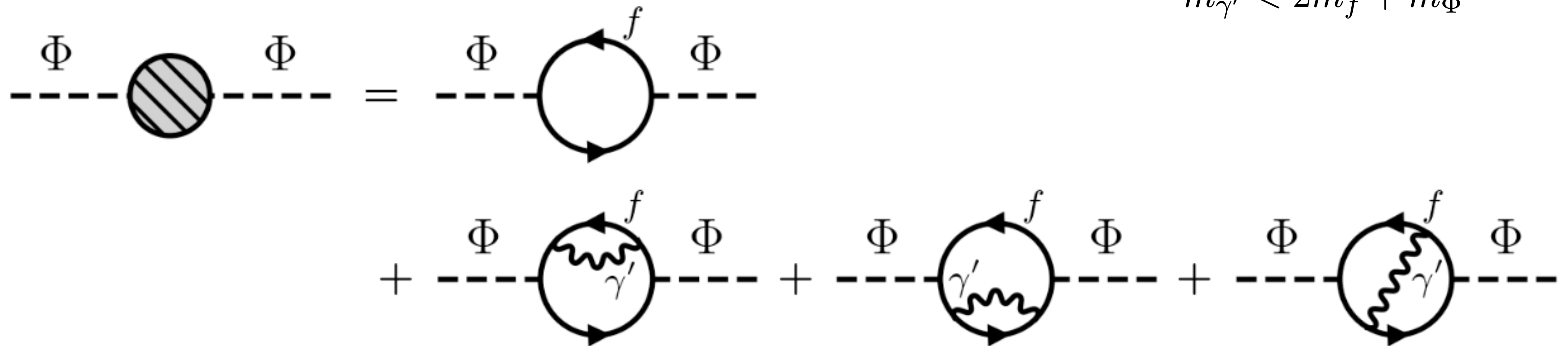
$$\mathcal{L}_{\text{int}} = g\Phi\bar{f}f - eA'_\mu\bar{f}\gamma^\mu f$$

- Self-energy up to two loops

$$2m_f < m_\Phi$$

$$m_\Phi < 2m_f + m_{\gamma'}$$

$$m_{\gamma'} < 2m_f + m_\Phi$$



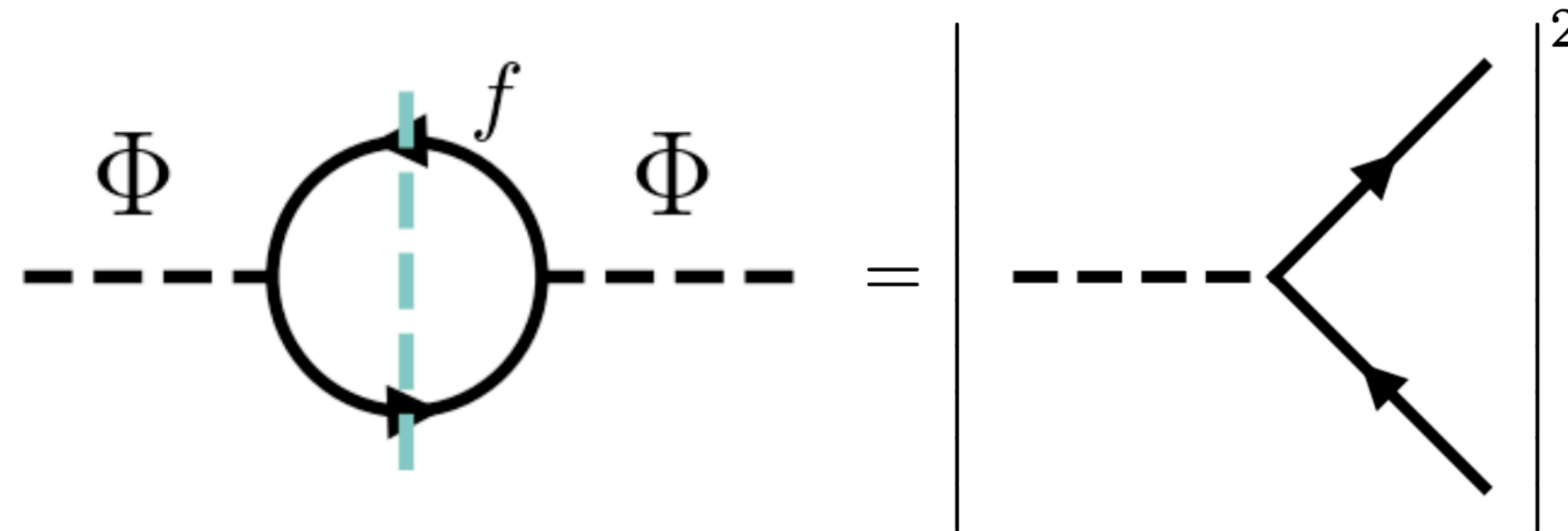
Scalar-fermion-vector Toy Model

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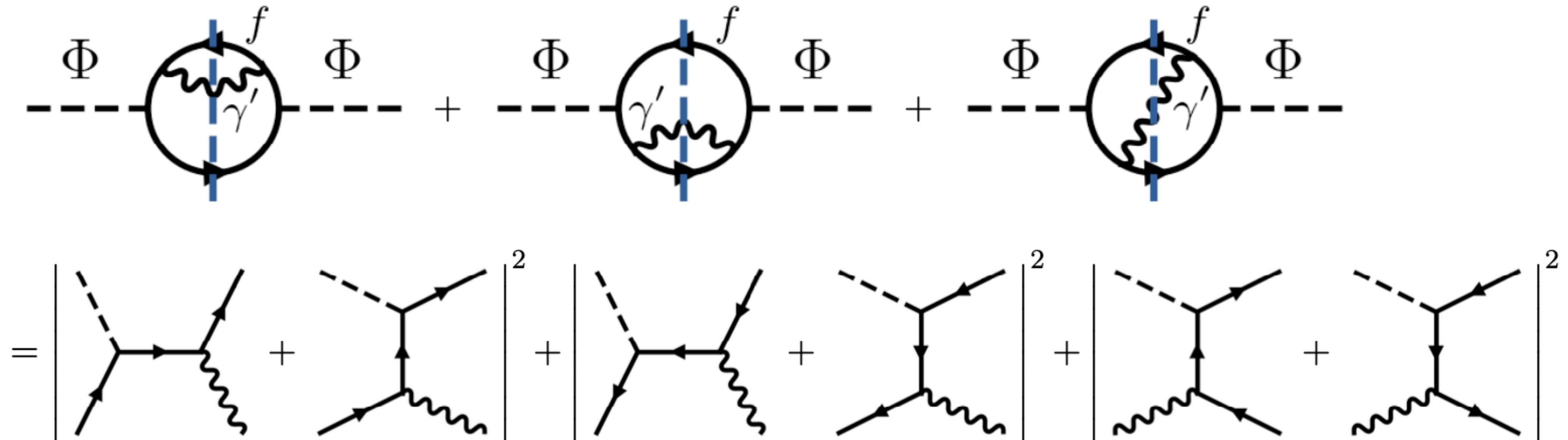
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- Two-loop contribution (symmetrical)



Scalar-fermion-vector Toy Model

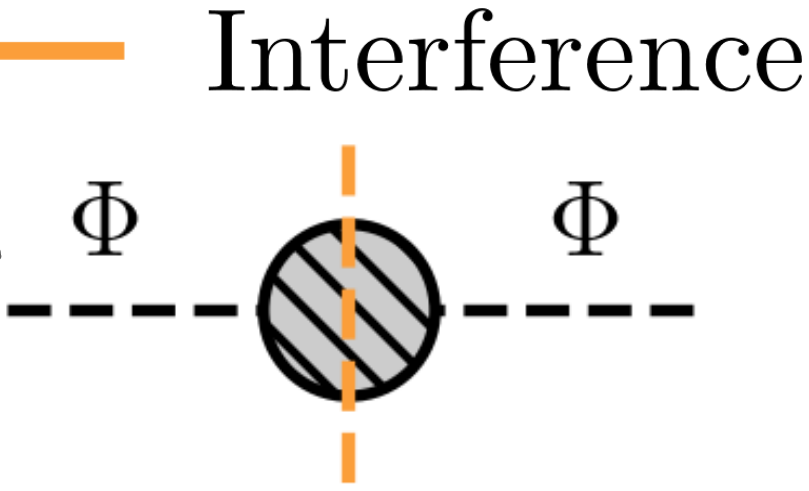
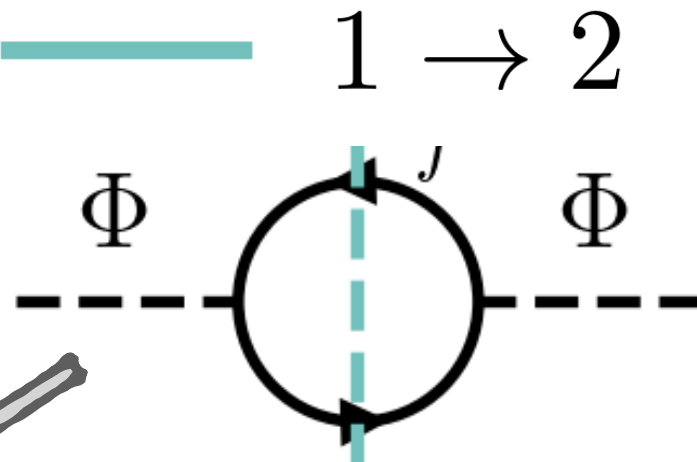
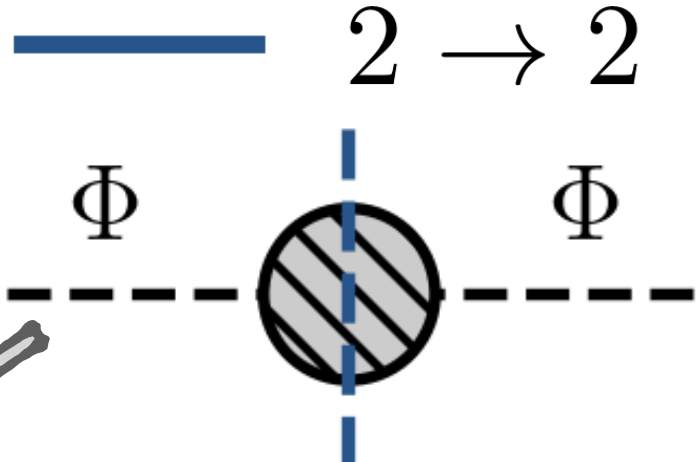
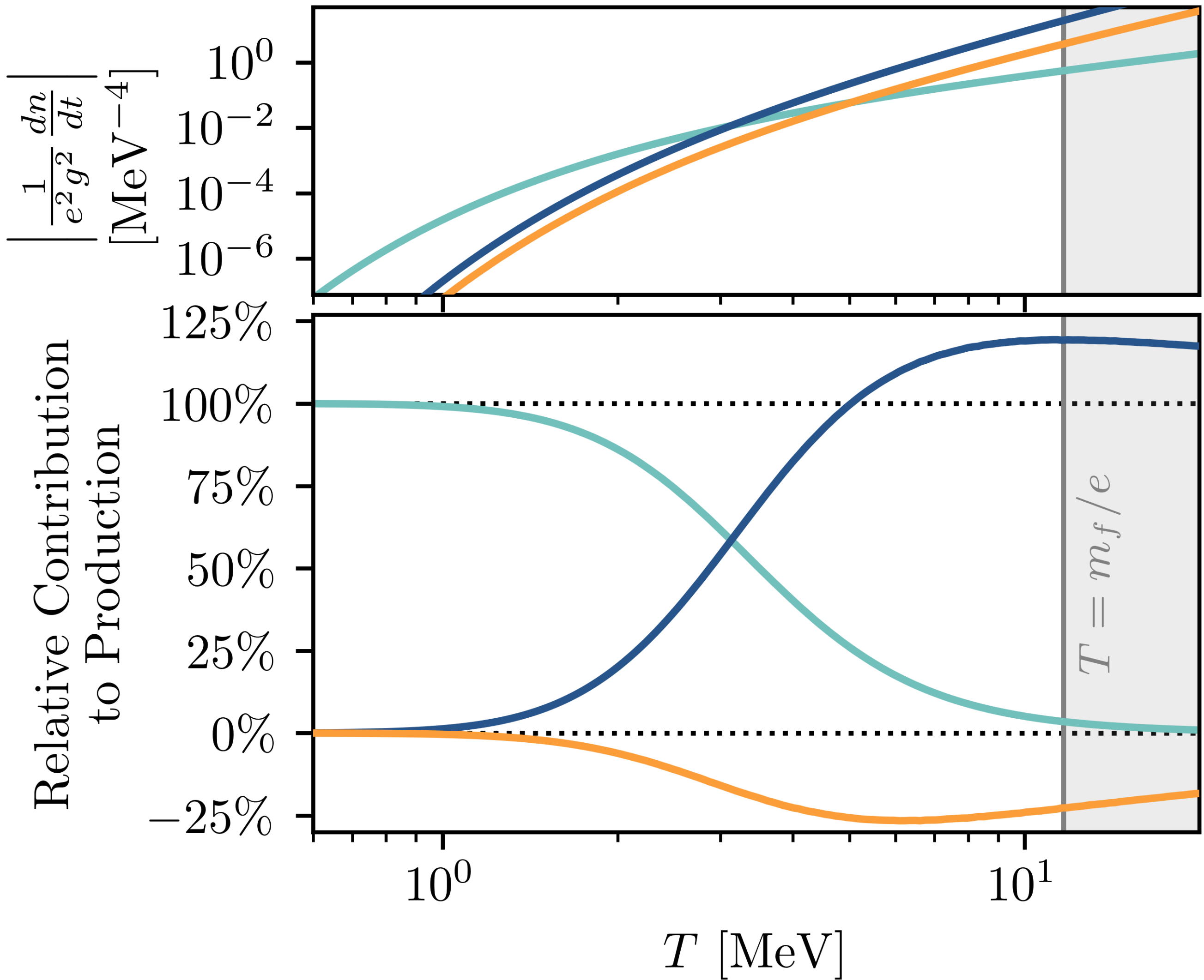
- Two-loop contribution (asymmetrical)

The diagram illustrates the two-loop contribution (asymmetrical) in a scalar-fermion-vector toy model. It shows a sum of four diagrams, each representing a two-loop contribution, followed by an equals sign. The diagrams are arranged in two rows. The top row shows four diagrams, each with a dashed line labeled Φ entering and exiting a loop. The loop contains a fermion line labeled f and a vector line labeled γ' . The diagrams are separated by plus signs. The bottom row shows two terms, each representing a sum of diagrams. The first term is a tensor product (indicated by $\otimes$) of a vertex diagram (a dashed line meeting two solid lines) and a sum of diagrams in parentheses, which are then raised to a power (indicated by $*$). The second term is a sum of a tensor product of a vertex diagram and a sum of diagrams in parentheses (raised to a power), plus another tensor product of a vertex diagram and a sum of diagrams in parentheses.

Scalar-fermion-vector Model Production

$$\mathcal{L}_{\text{int}} = g\Phi\bar{f}f - eA'_\mu\bar{f}\gamma^\mu f$$

$$\begin{aligned} m_\Phi &= 7.1 \text{ MeV} \\ m_f &= 3.5 \text{ MeV} \\ m_{\gamma'} &= 6 \text{ MeV} \\ e &= 0.3 \\ g &= 10^{-6} \end{aligned}$$

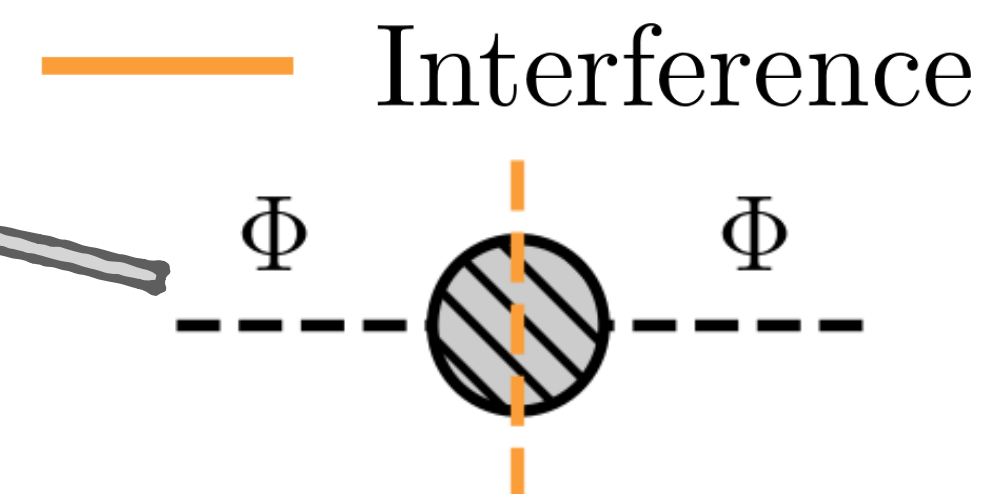
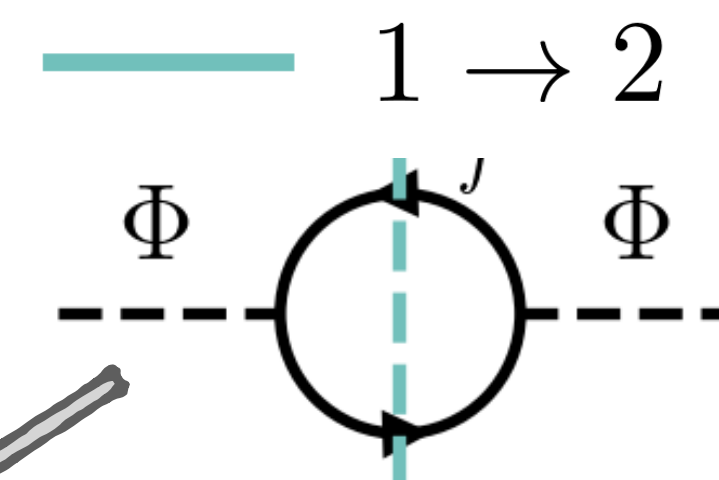
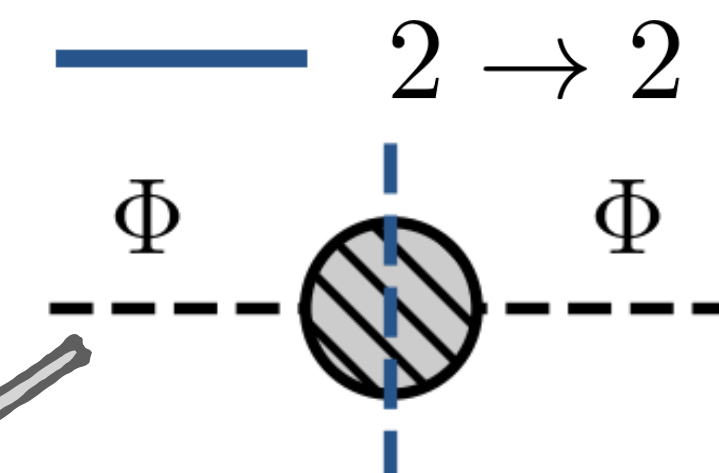
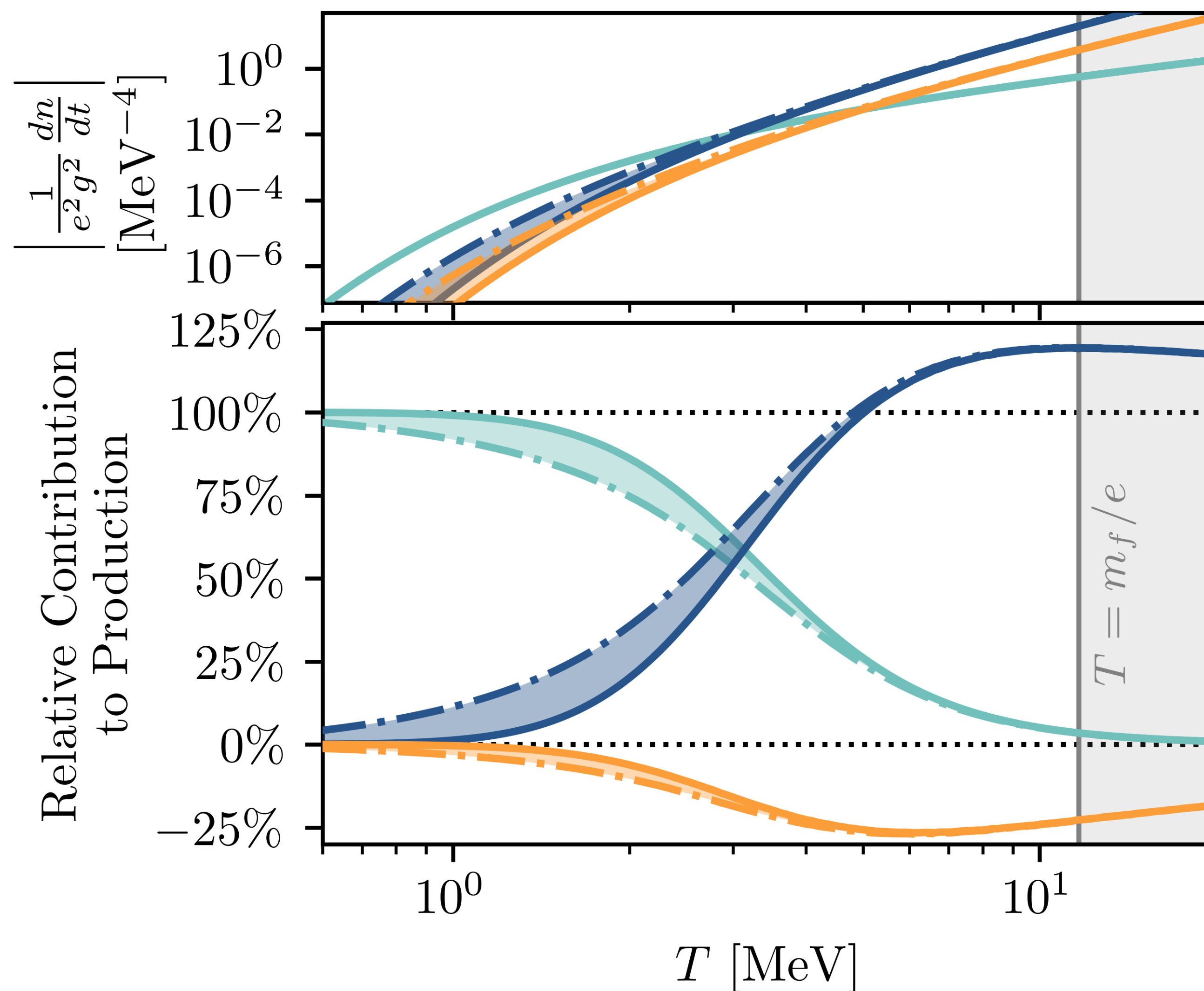


Scalar-fermion-vector Model Production

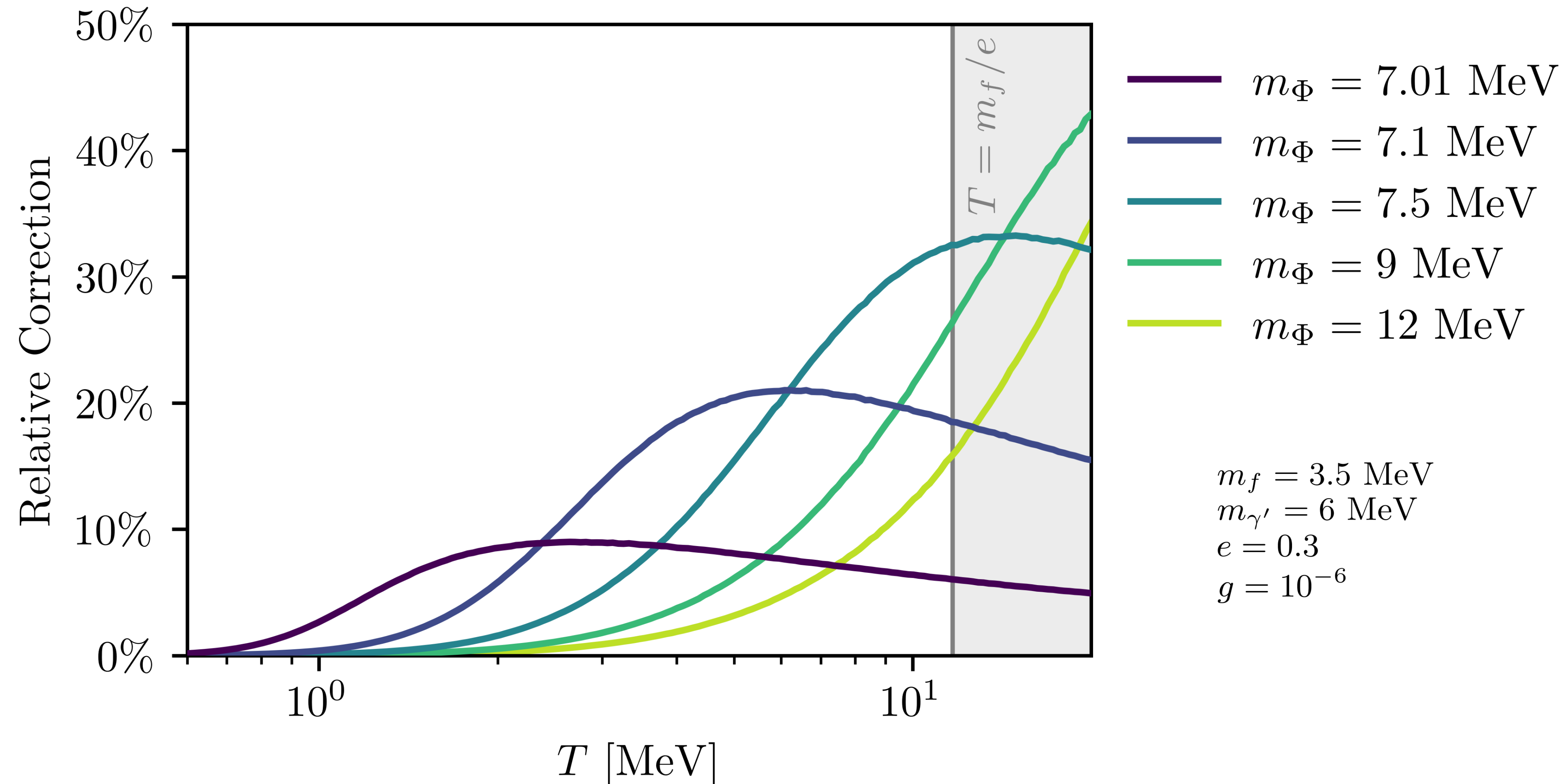
$$\mathcal{L}_{\text{int}} = g\Phi\bar{f}f - eA'_\mu\bar{f}\gamma^\mu f$$

$$\begin{aligned} m_\Phi &= 7.1 \text{ MeV} \\ m_f &= 3.5 \text{ MeV} \\ m_{\gamma'} &= 6 \text{ MeV} \\ e &= 0.3 \\ g &= 10^{-6} \end{aligned}$$

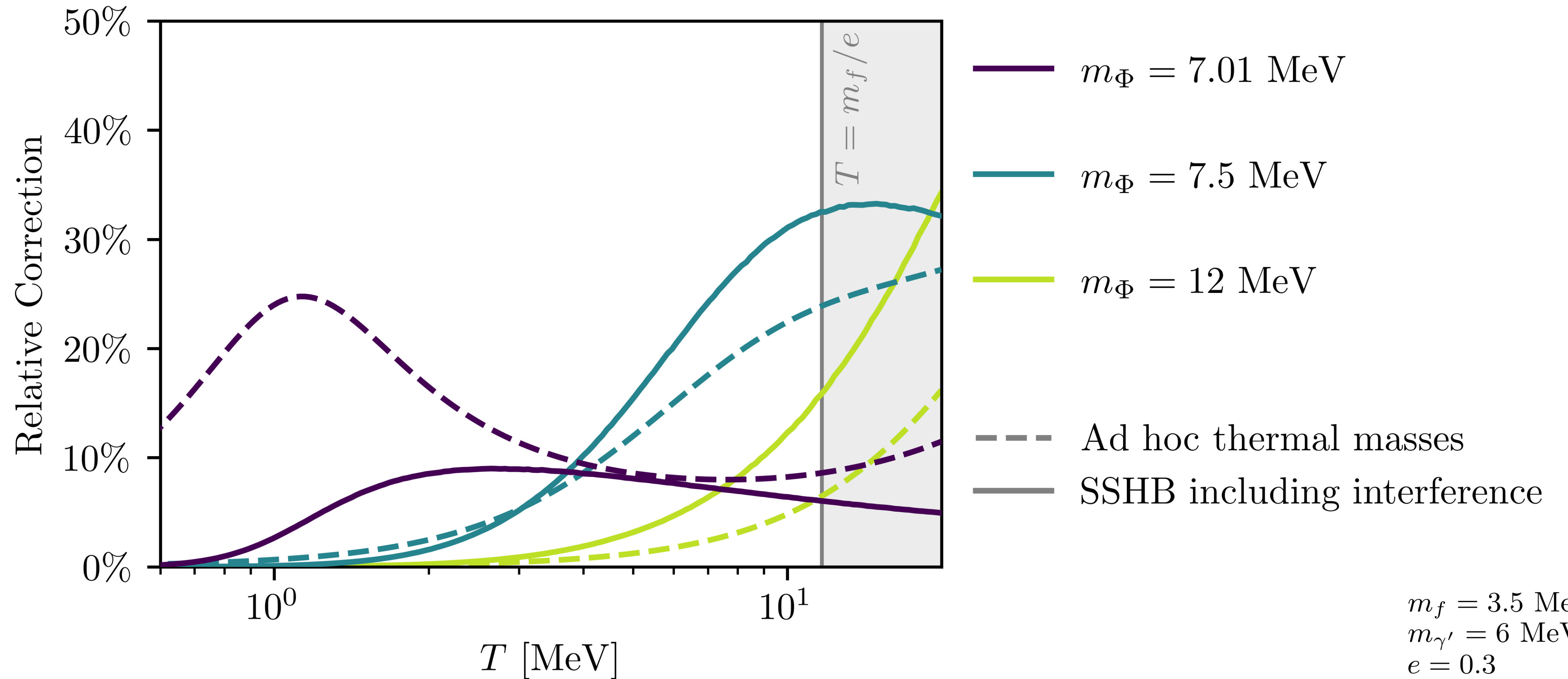
$$\begin{aligned} \text{---} & \mu_f = 0 \text{ MeV} \\ \text{---} \cdot \text{---} & \mu_f = 3 \text{ MeV} \end{aligned}$$



Scalar-fermion-vector Model Production



Scalar-fermion-vector Model Production



$m_f = 3.5$ MeV
 $m_{\gamma'} = 6$ MeV
 $e = 0.3$
 $g = 10^{-6}$