Effects of Magnetic Dispersion on Axion and Dark Photon Emission

A case study of magnetic white dwarfs

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Collaborators: Ella Iles, Hugo Schérer, Katelin Schutz

Light Dark World Jul 28, 2026

Axions and Dark photons

Axion

$$\mathscr{L}_{a\gamma-f} \supset \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - \frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{af}}{2m_f}\partial_\mu a(\bar{f}\gamma^\mu\gamma^5 f)$$

David J E Marsh 2016

P. Carenza, M. Giannotti, J. Isern, A. Mirizzi, O. Straniero 2024

Dark Photon

$$\mathscr{L}_{A'\gamma} \supset -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu + \kappa m_{A'}^2 A'_\mu A^\mu$$

Bob Holdom 1986

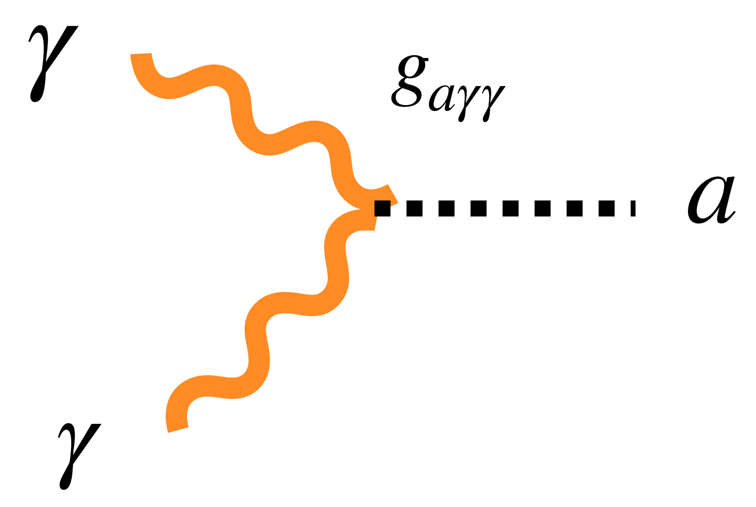
H.An, M.Pospelov & J.Pradler 2013

J. Redondo & G.G. Raffelt 2013

Axions and Dark photons

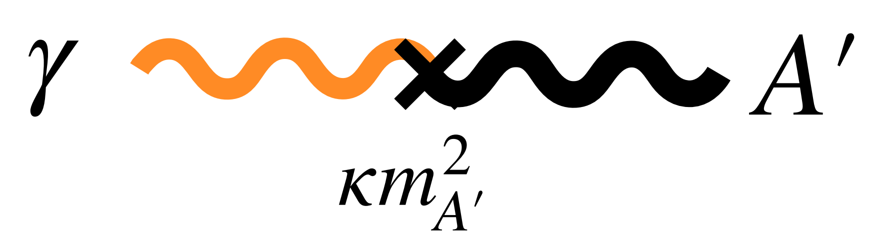
Axion

$$\mathcal{L}_{a\gamma-f} \supset \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 - \frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{af}}{2m_f}\partial_\mu a(\bar{f}\gamma^\mu\gamma^5 f)$$



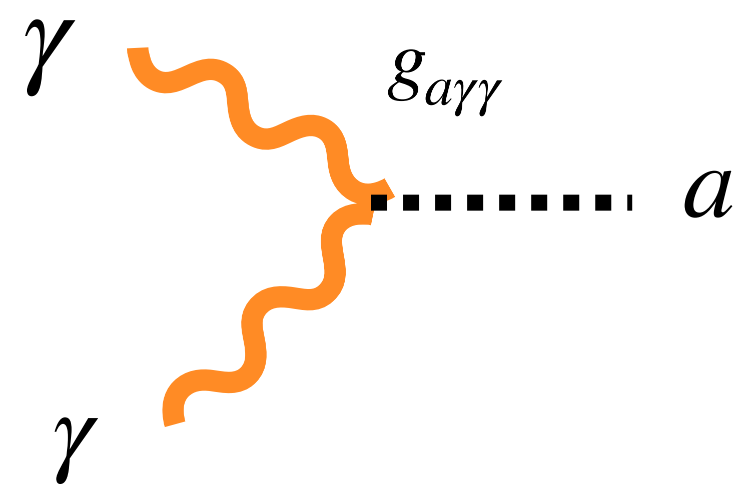
Dark Photon

$$\mathcal{L}_{A'\gamma} \supset -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} m_{A'}^2 A'_\mu A'^\mu + \kappa m_{A'}^2 A'_\mu A^\mu$$

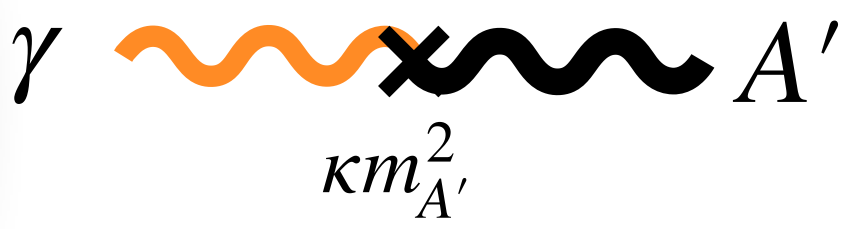


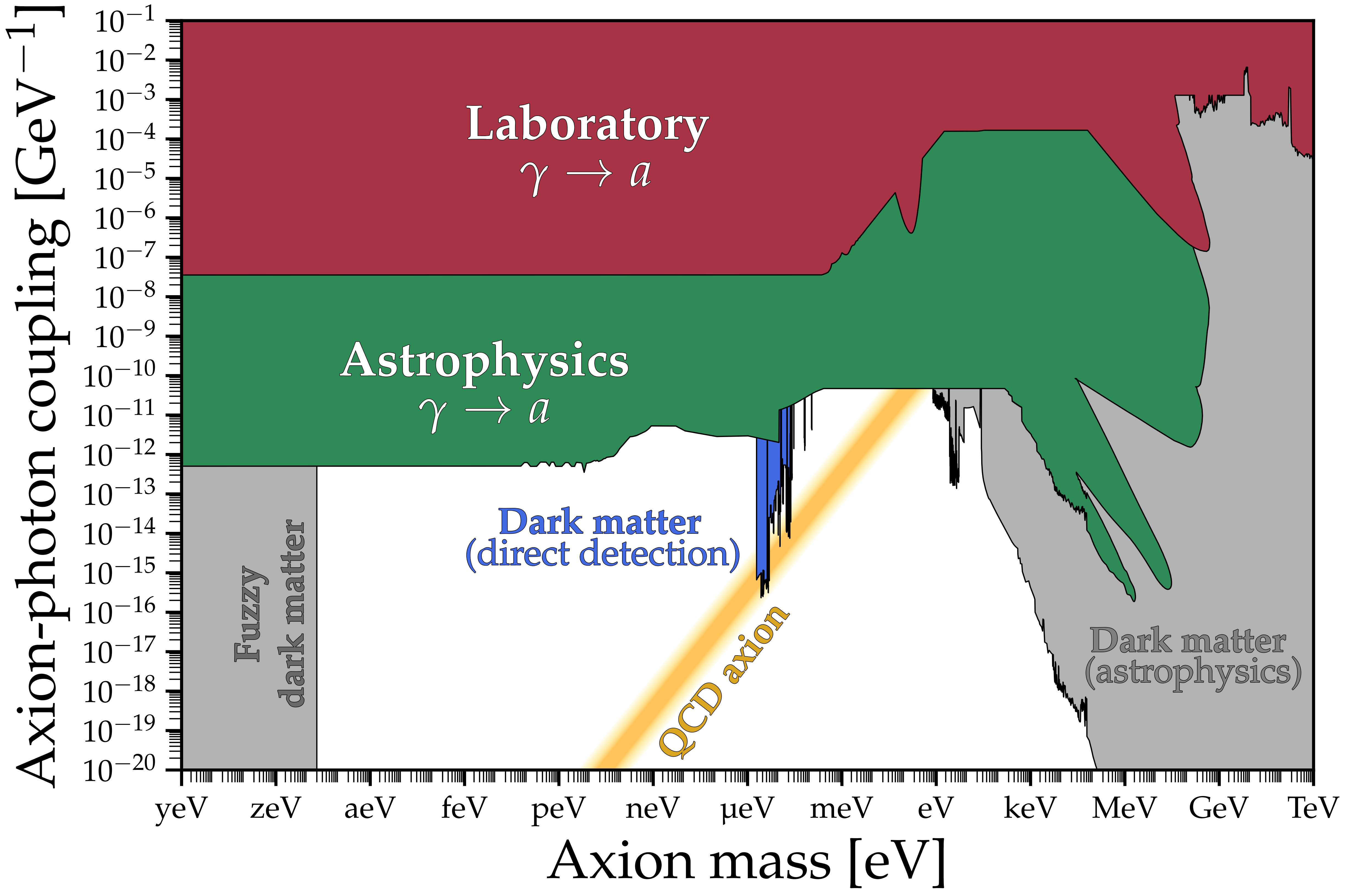
Axions and Dark photons

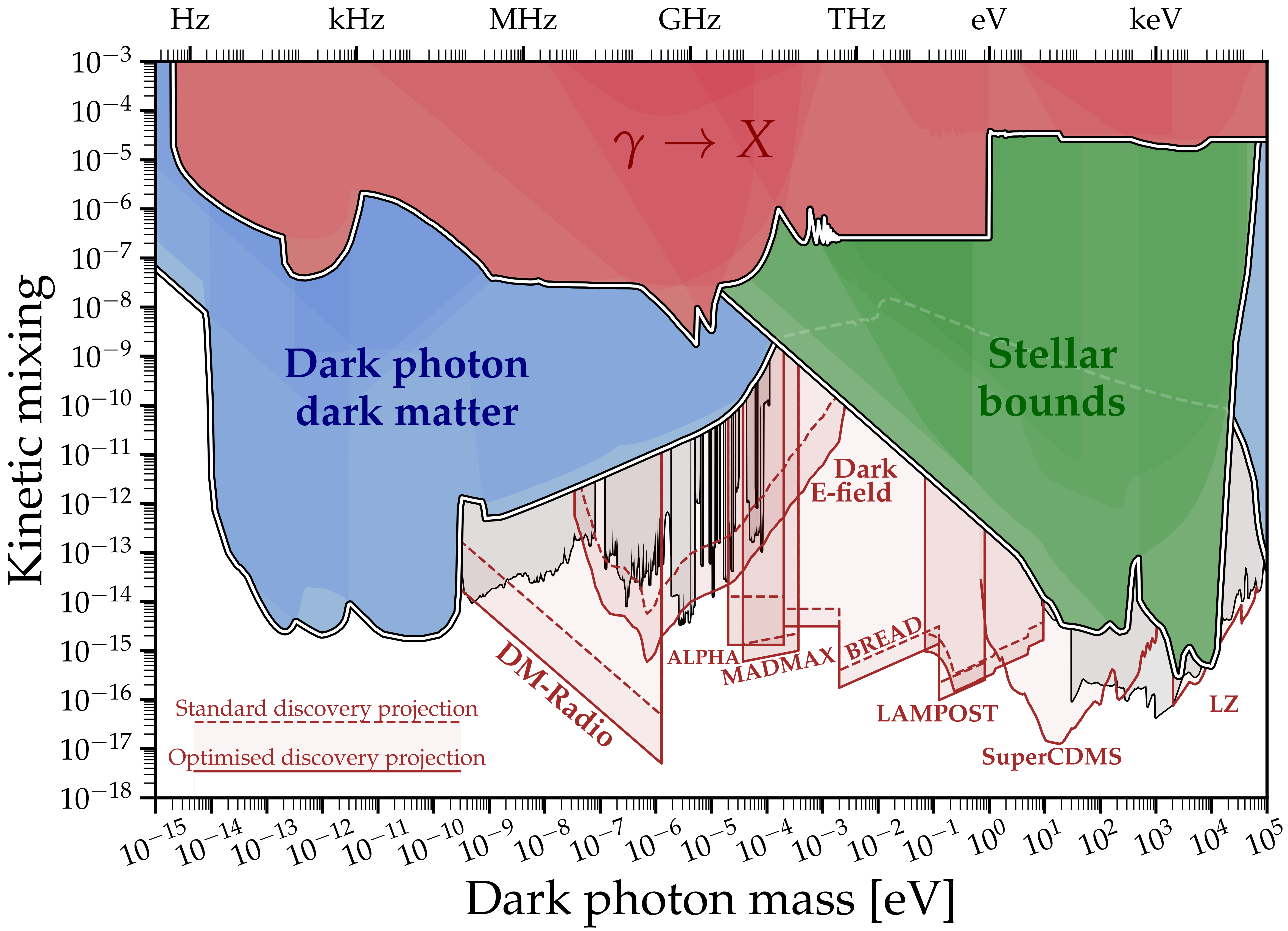
$$\mathscr{L}_{a\gamma} \supset g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$



$$\mathscr{L}_{A'\gamma} \supset \kappa m_{A'}^2 A^\mu A'_\mu$$



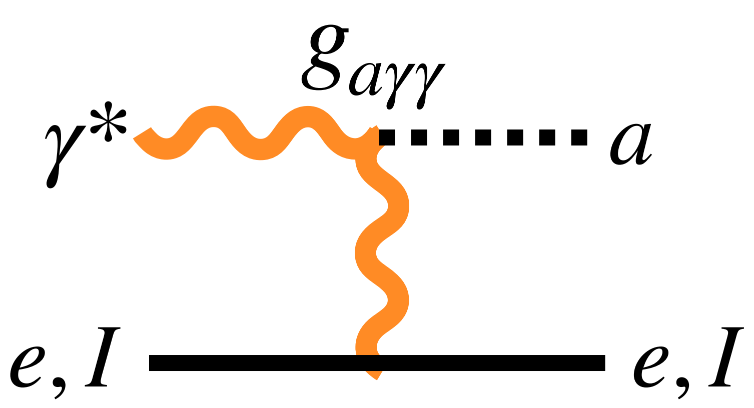




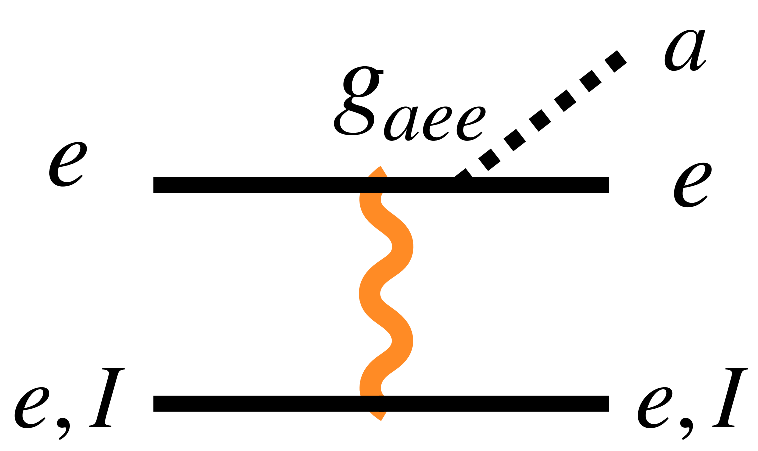
Ciaran O'Hare (2020) cajohare/AxionLimits:

How does the production of axions and dark photons change in the presence of magnetic field in a plasma?

Magnetic fields in astrophysical plasmas



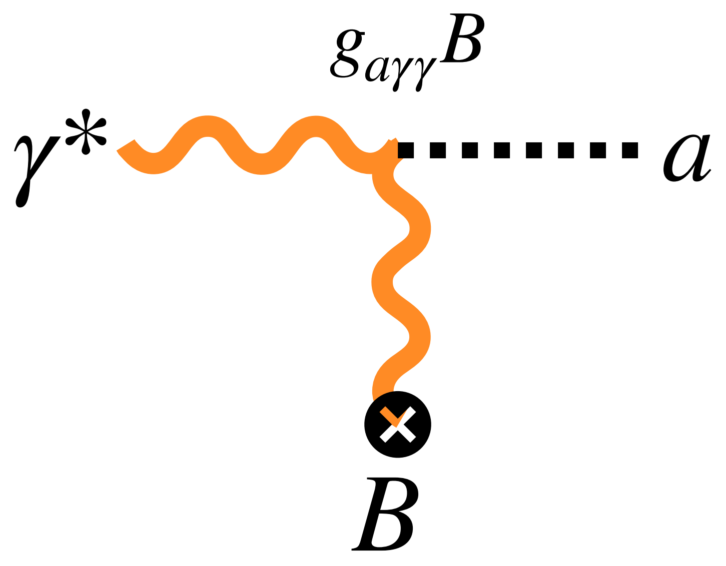
Primakoff



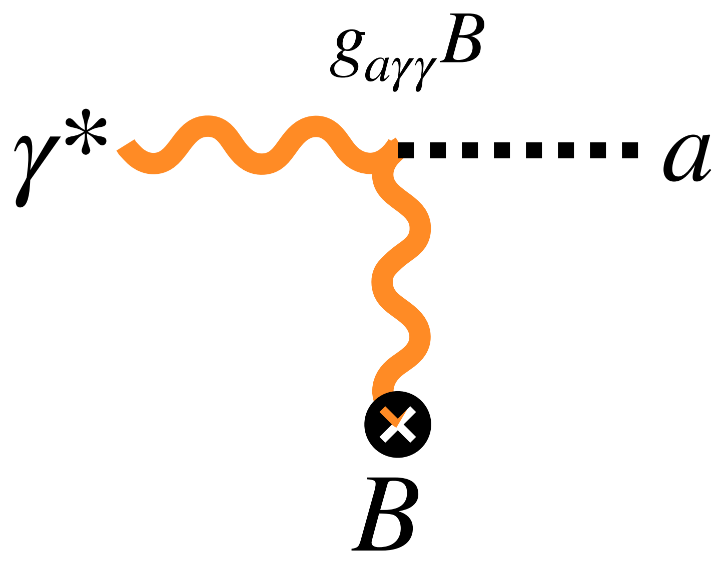
Bremsstrahlung

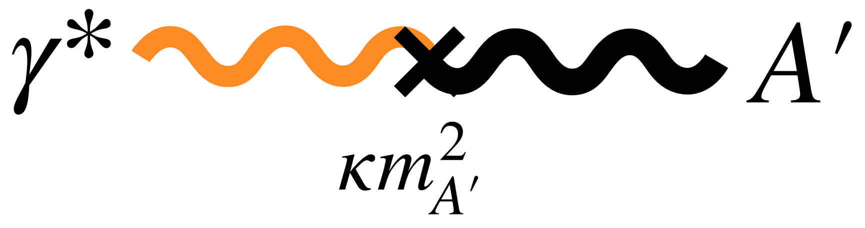
G.G. Raffelt *Physics Reports* 1990

Magnetic fields in astrophysical plasmas



Magnetic fields in astrophysical plasmas

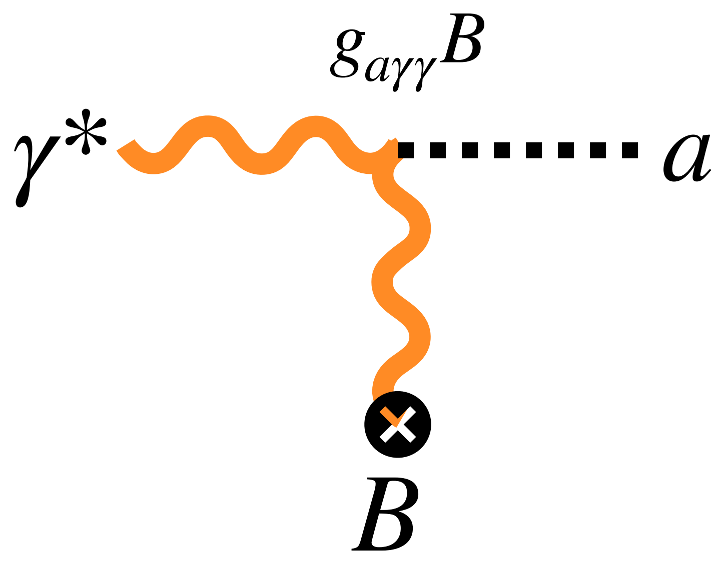


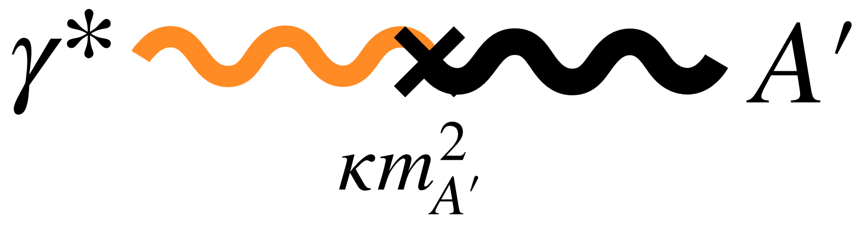


"Resonant or on-shell conversion"

$$m_X^2 = m_\gamma^2$$

Magnetic fields in astrophysical plasmas





"Resonant or on-shell conversion"

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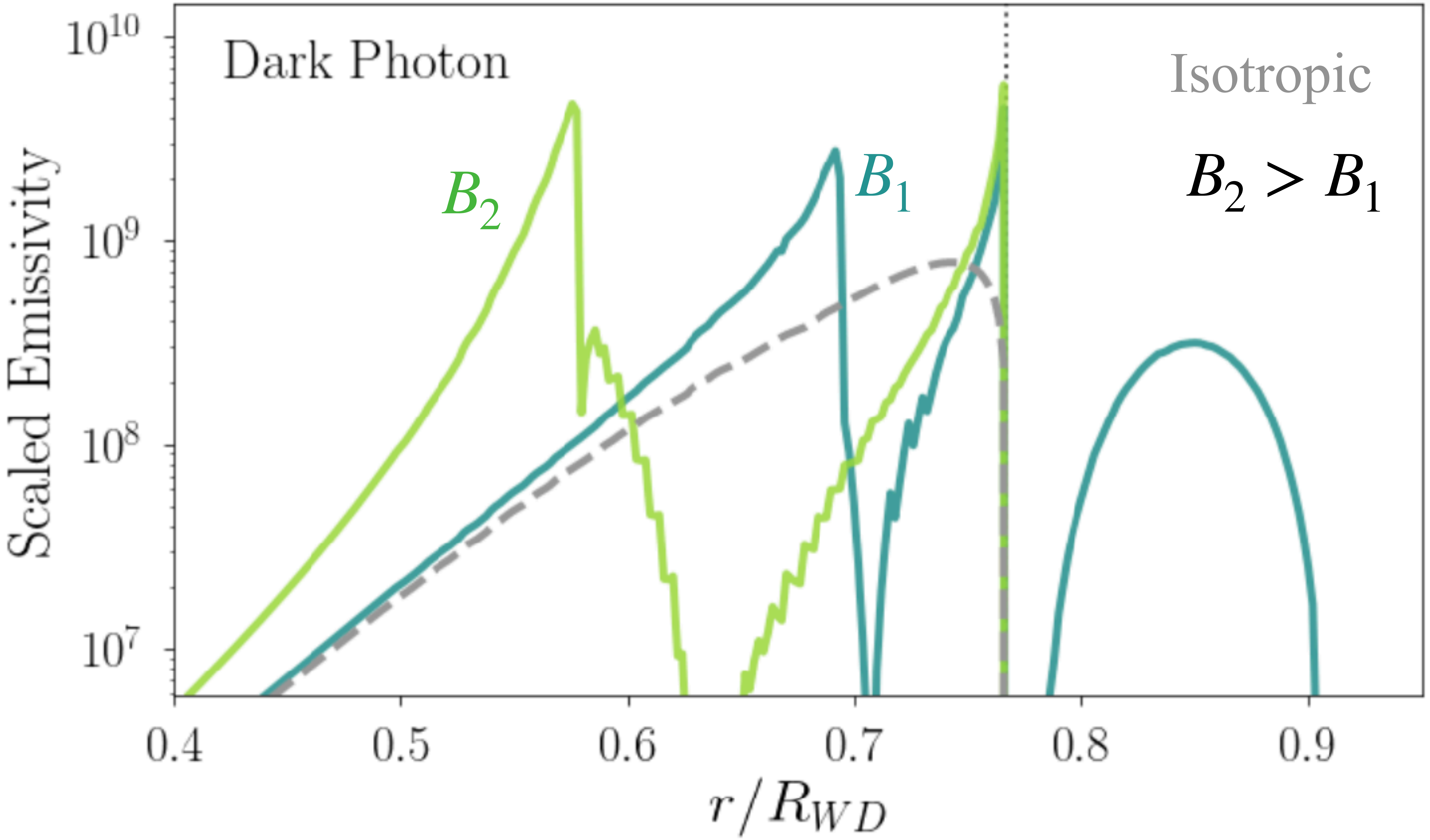
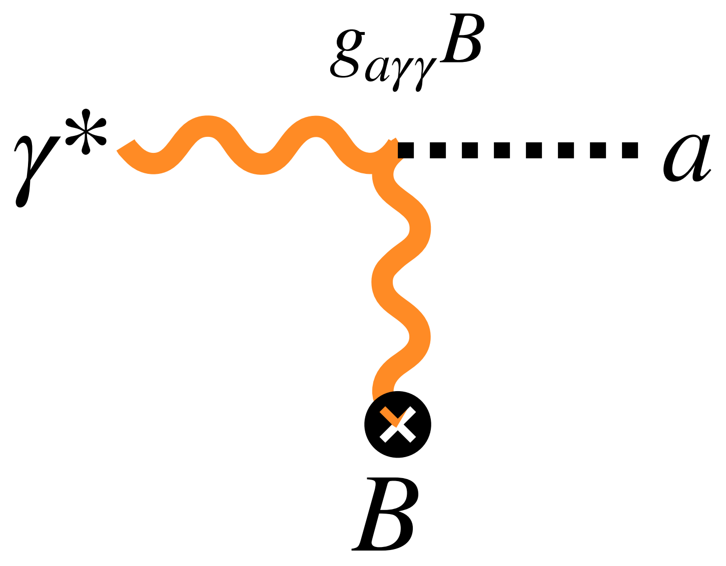
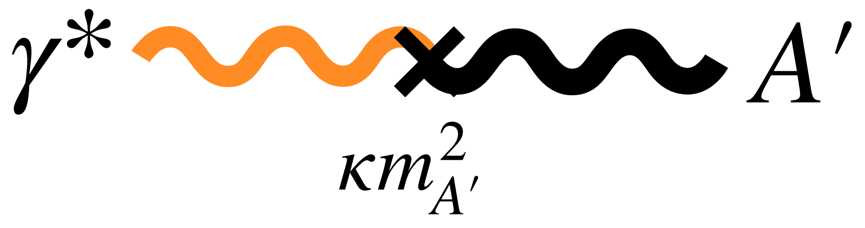
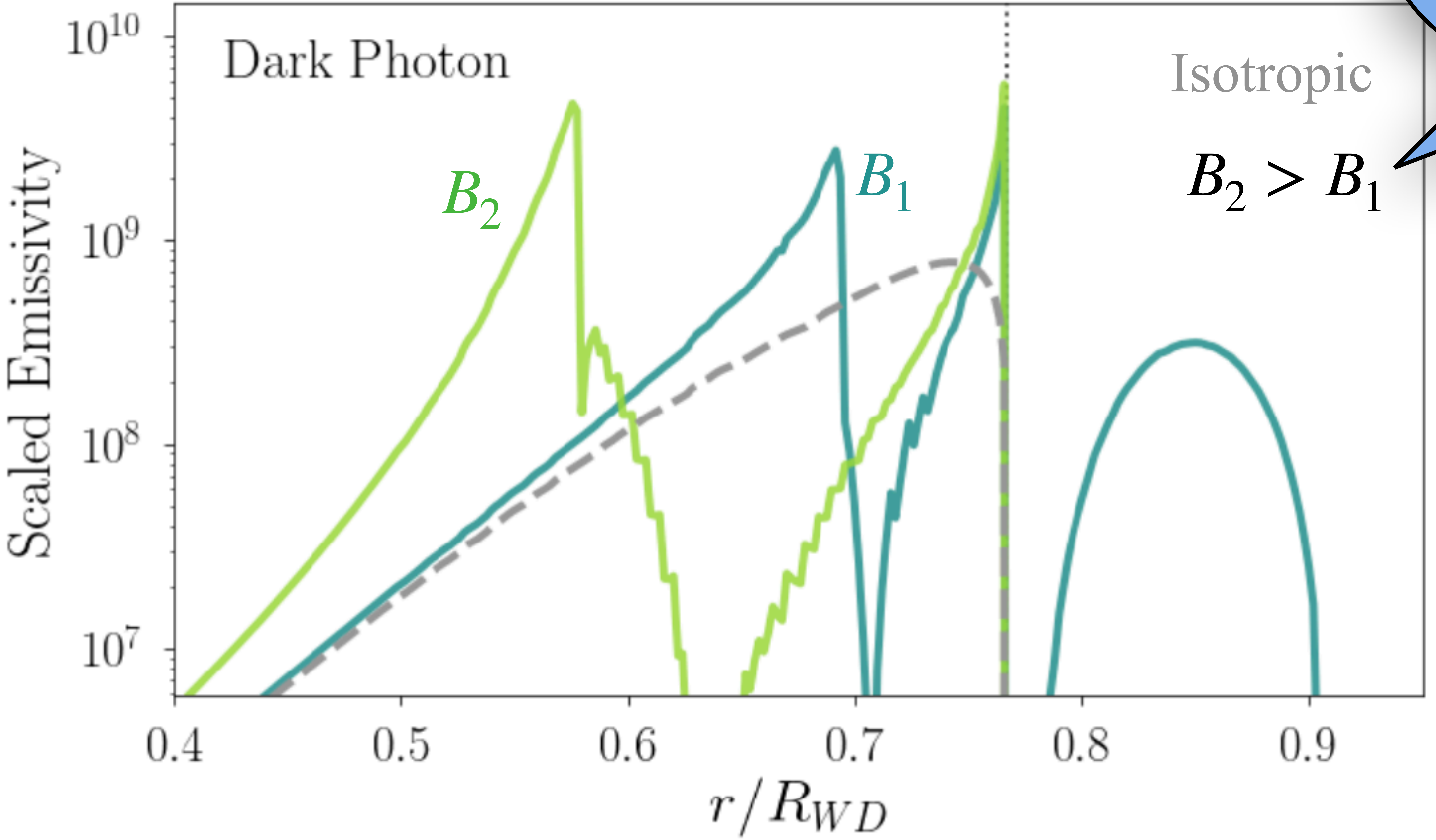


Magnetic fields in astrophysical plasmas

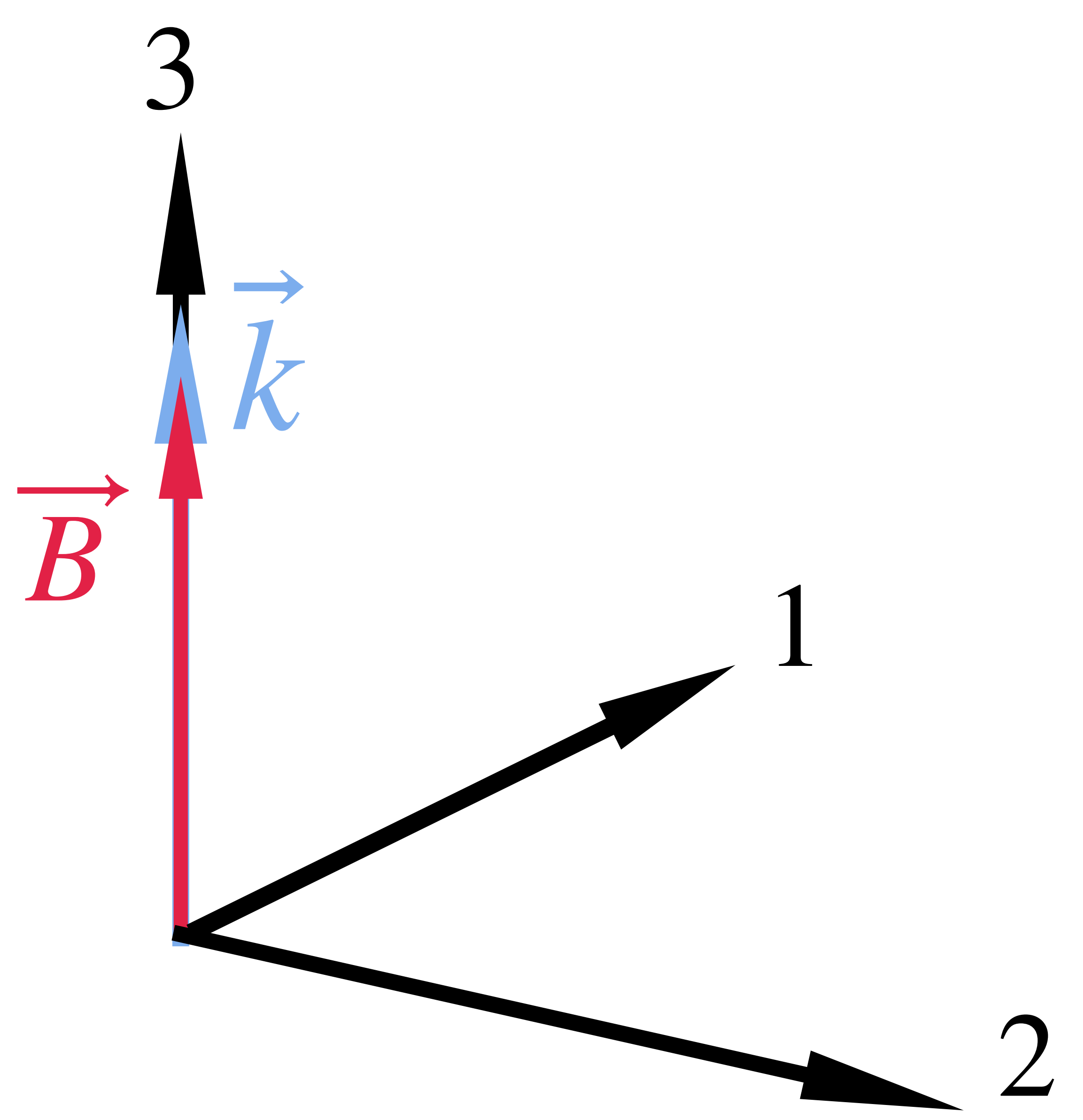


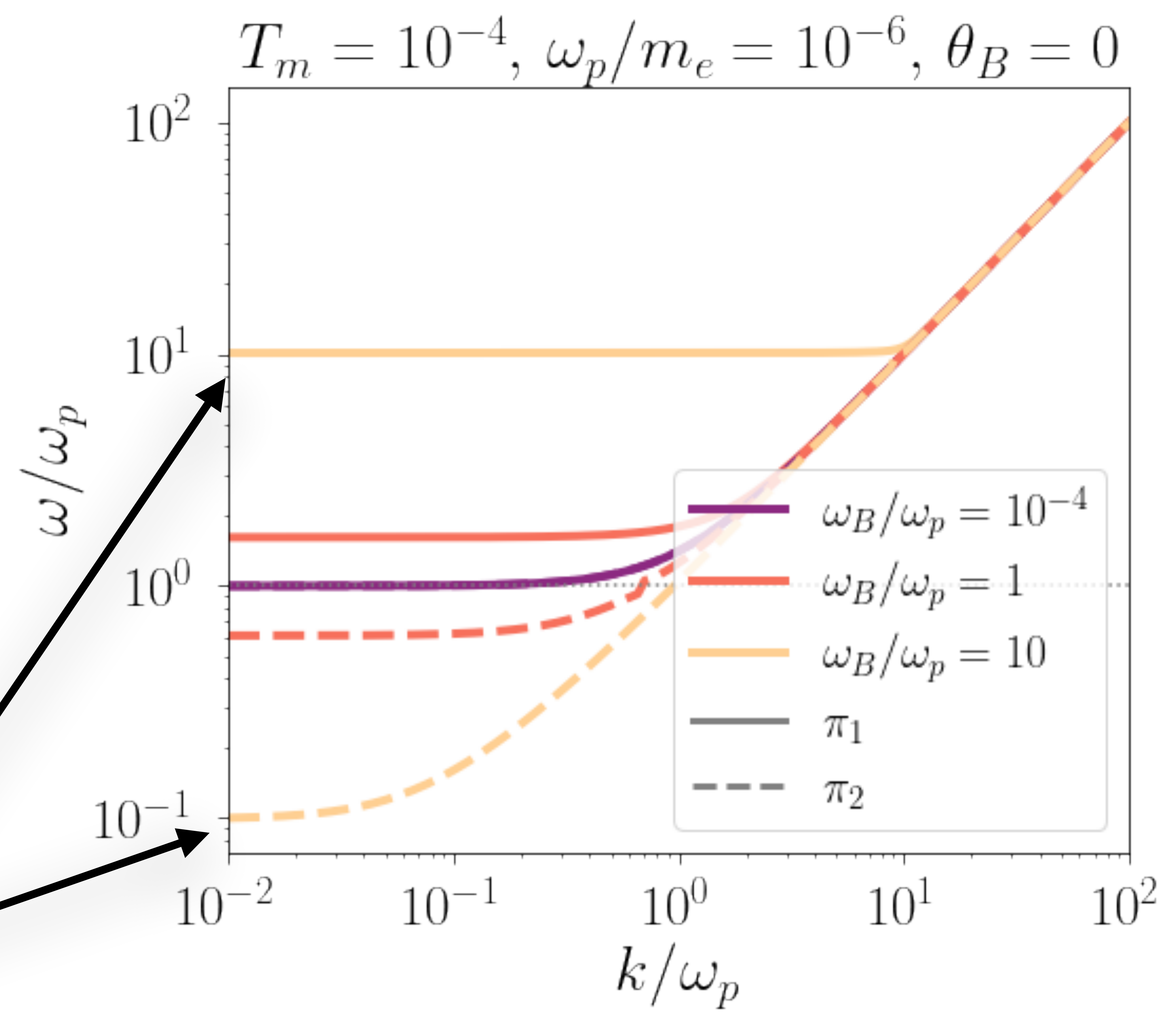




Very
different emission
with B-field !!

Dispersion in magnetized plasmas

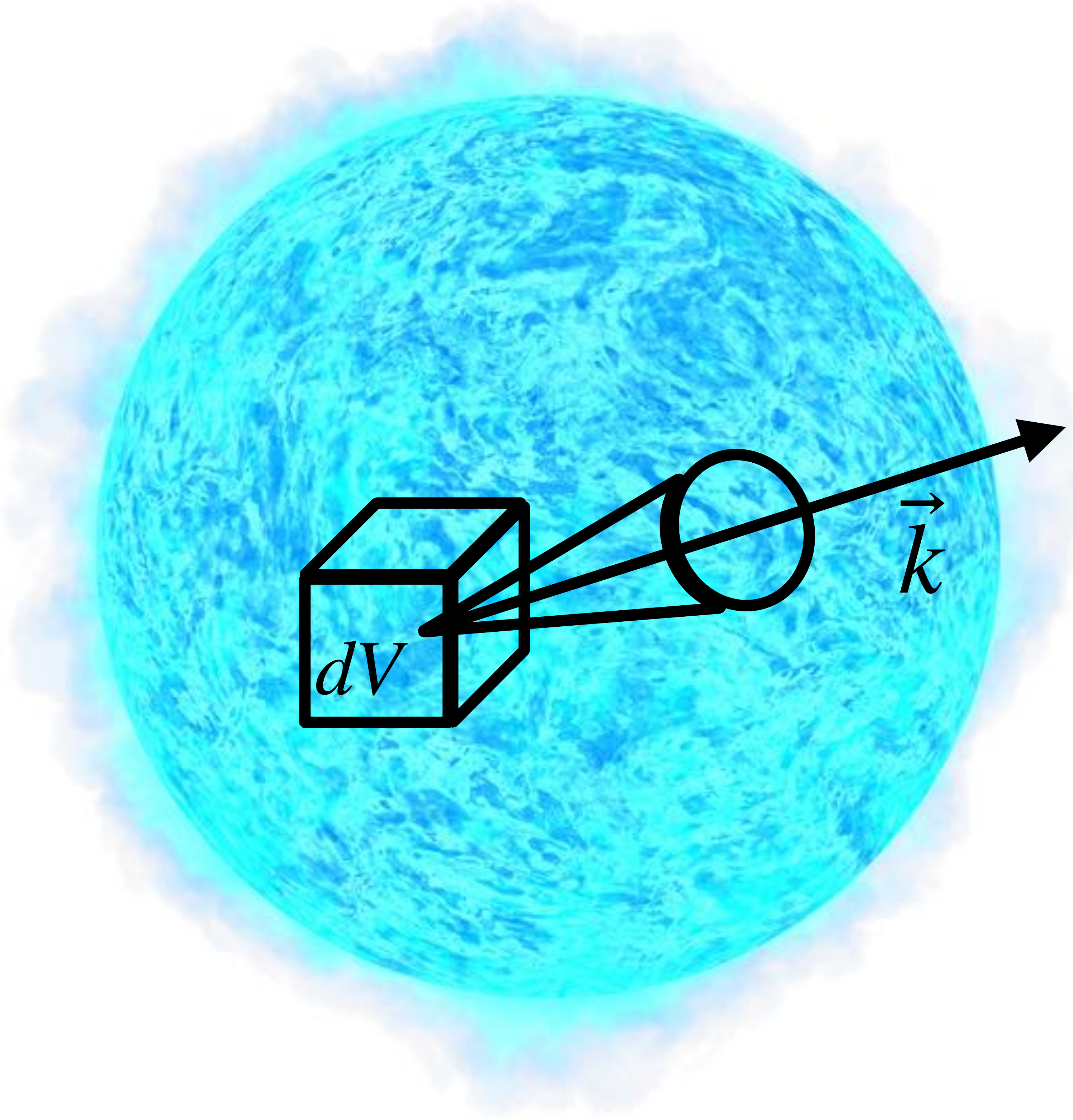




Transverse modes split (magnetic birefringence)

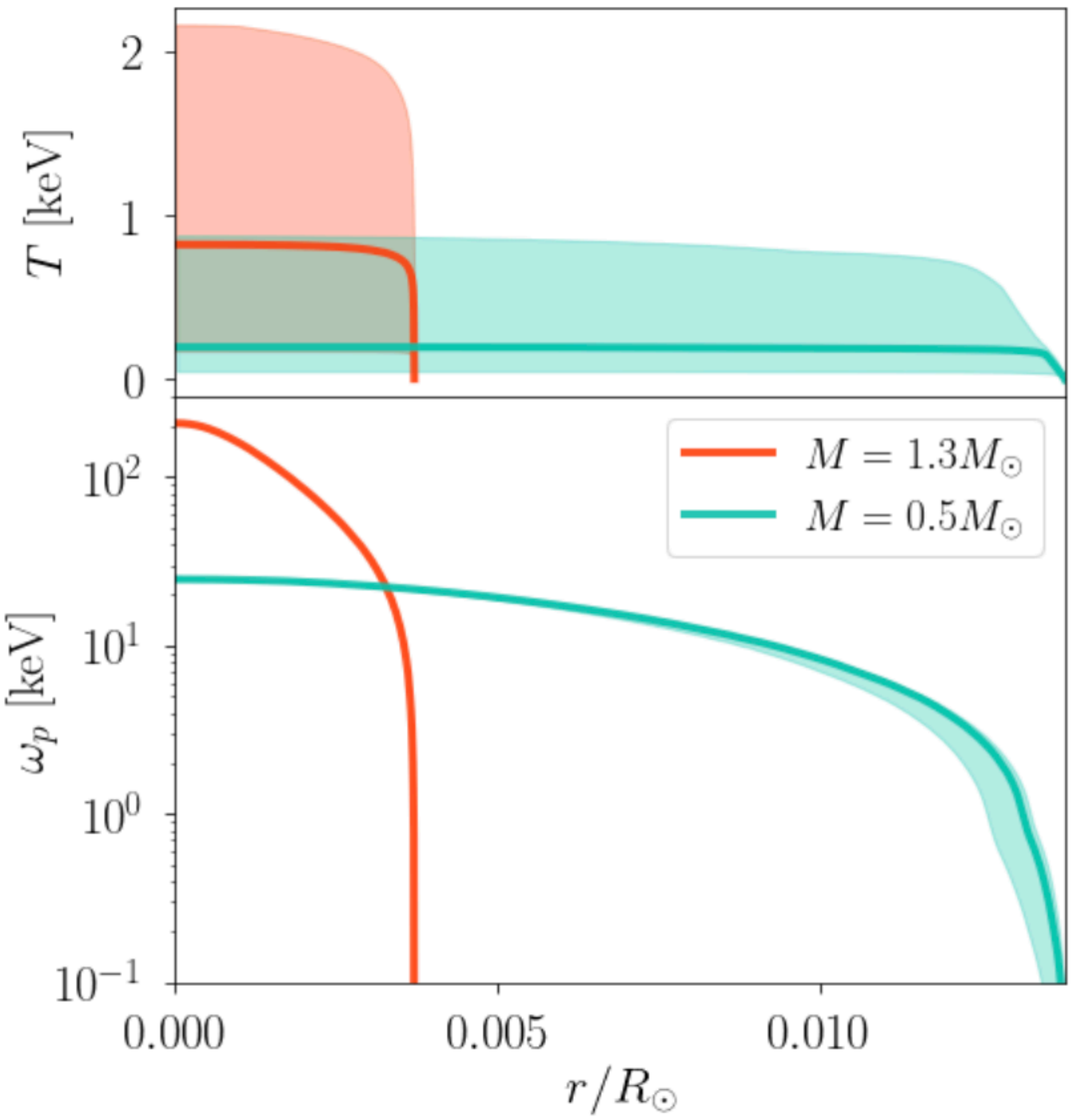
NB & K. Schutz JHEP *2024*

How does the magnetic dispersion affect BSM production in magnetic White Dwarfs?



$\frac{dQ_X}{d\Omega_k}$ (Angular emissivity)

Production in magnetized plasmas: *White Dwarf profiles*



M. C-Tapia, S. Zhang & A. Cumming *ApJ 2024*

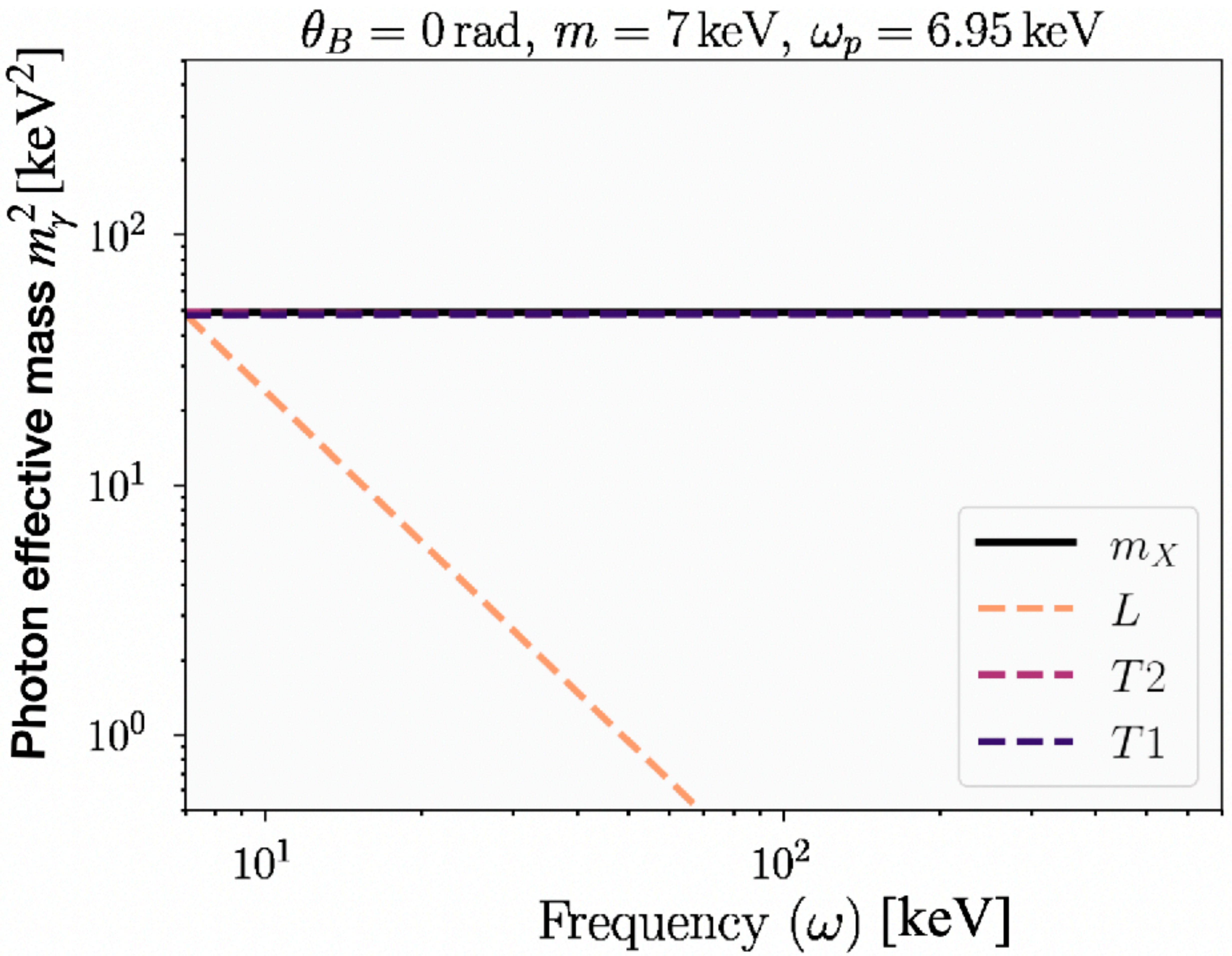
M. C-Tapia, M. Camisassa, S. Zhang
2508.09268

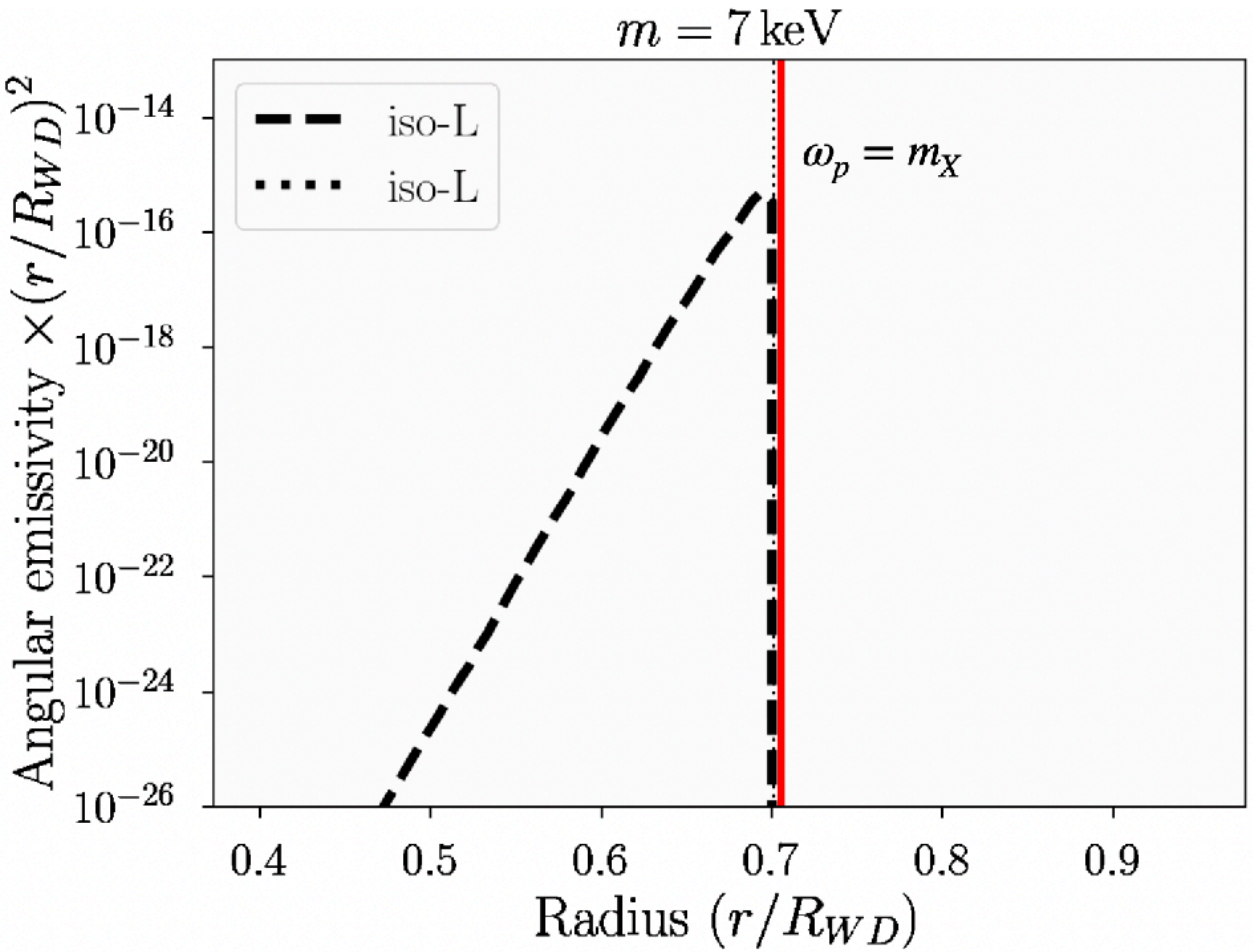
"Uniform B field profile"

Production in White Dwarfs: *Isotropic dispersion*

$\leftarrow \omega_p$ increases

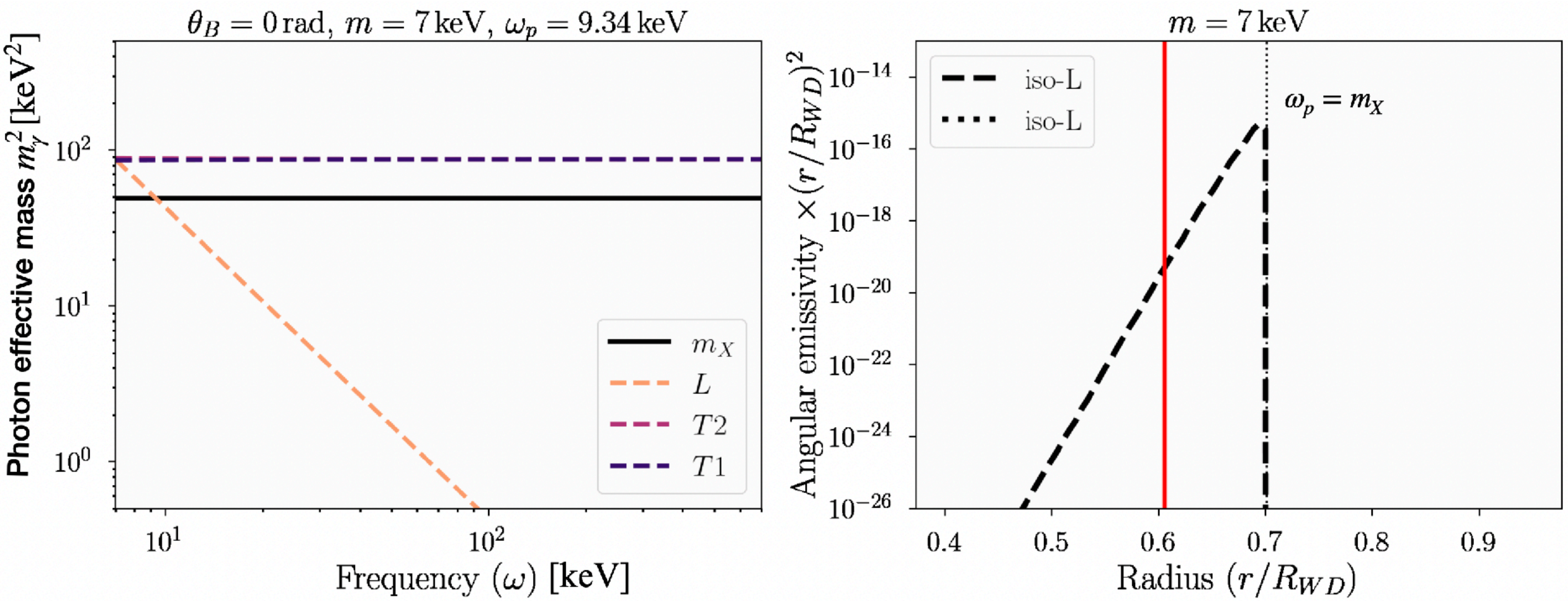
Production in White Dwarfs: *Isotropic dispersion*





$\leftarrow \omega_p$ increases

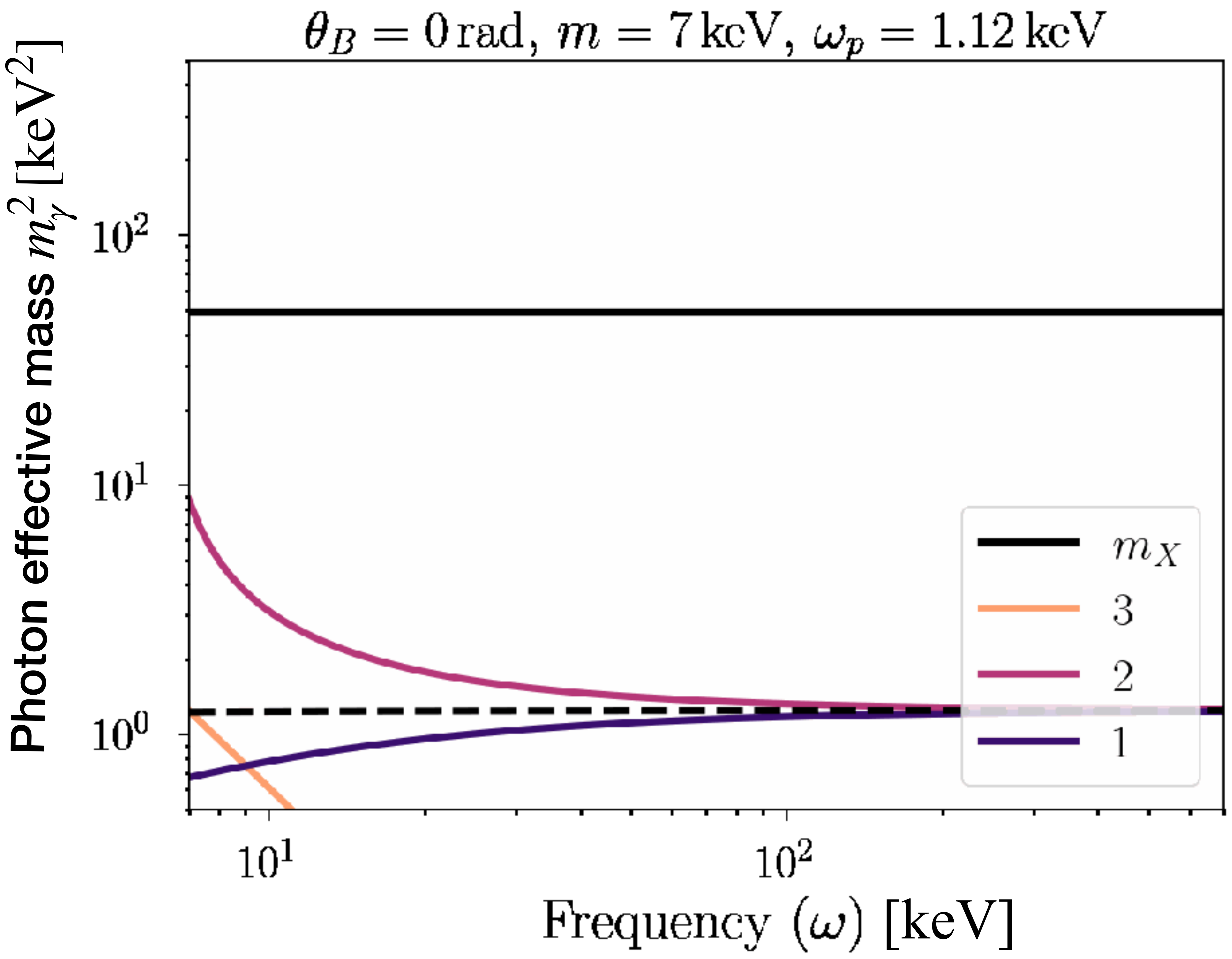
Production in White Dwarfs: *Isotropic dispersion*

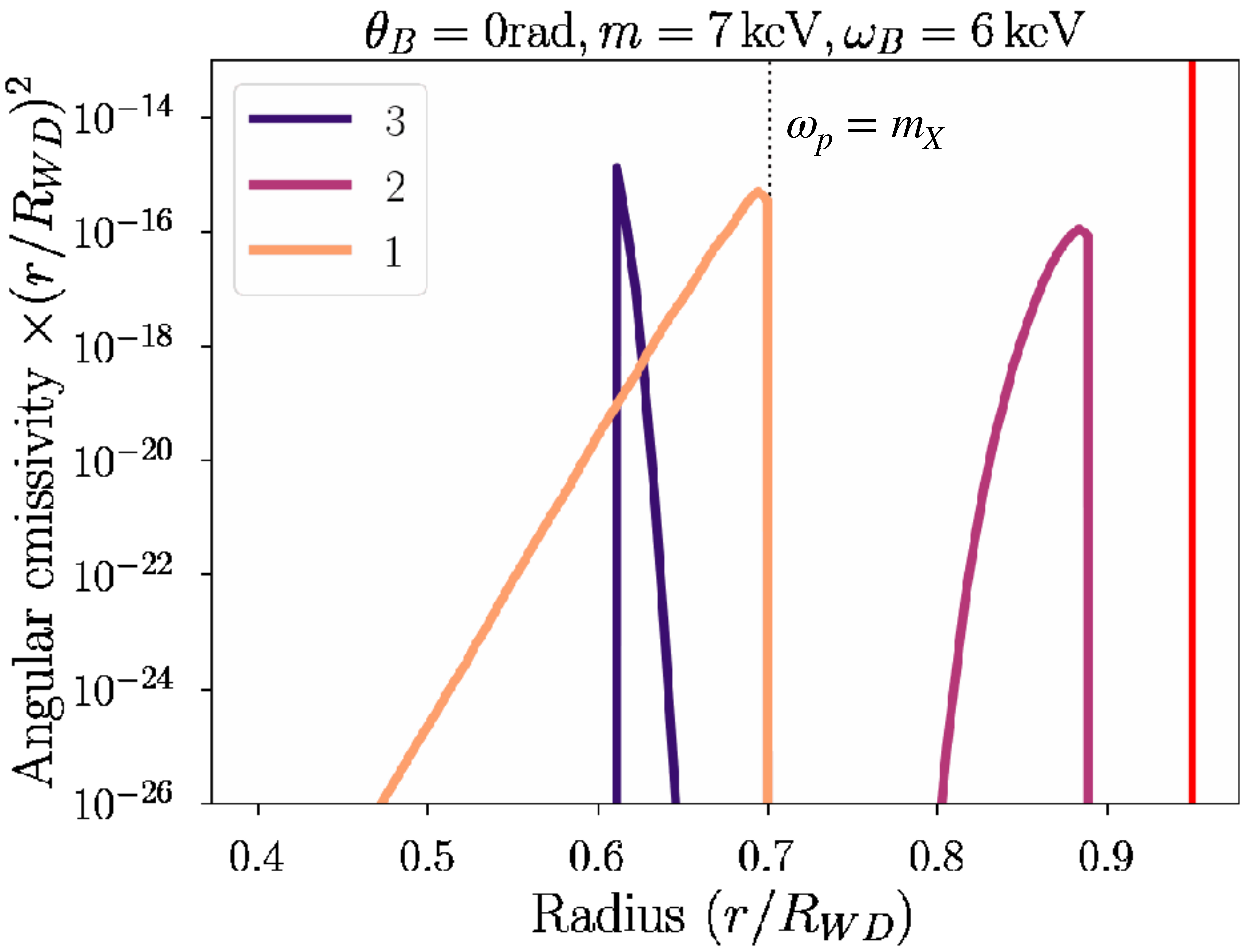


$\leftarrow \omega_p$ increases

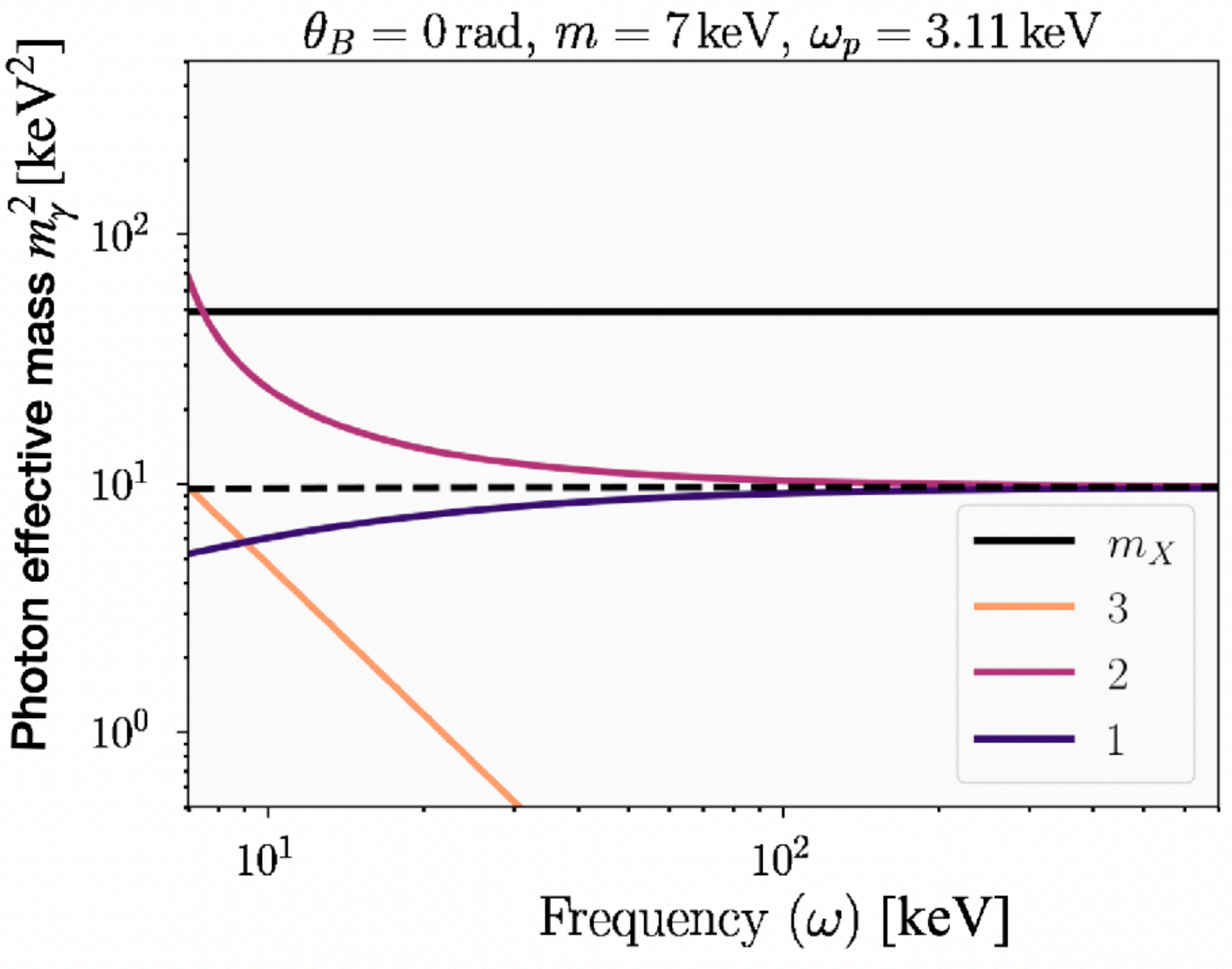
Production in magnetized plasmas: *Magnetic dispersion*

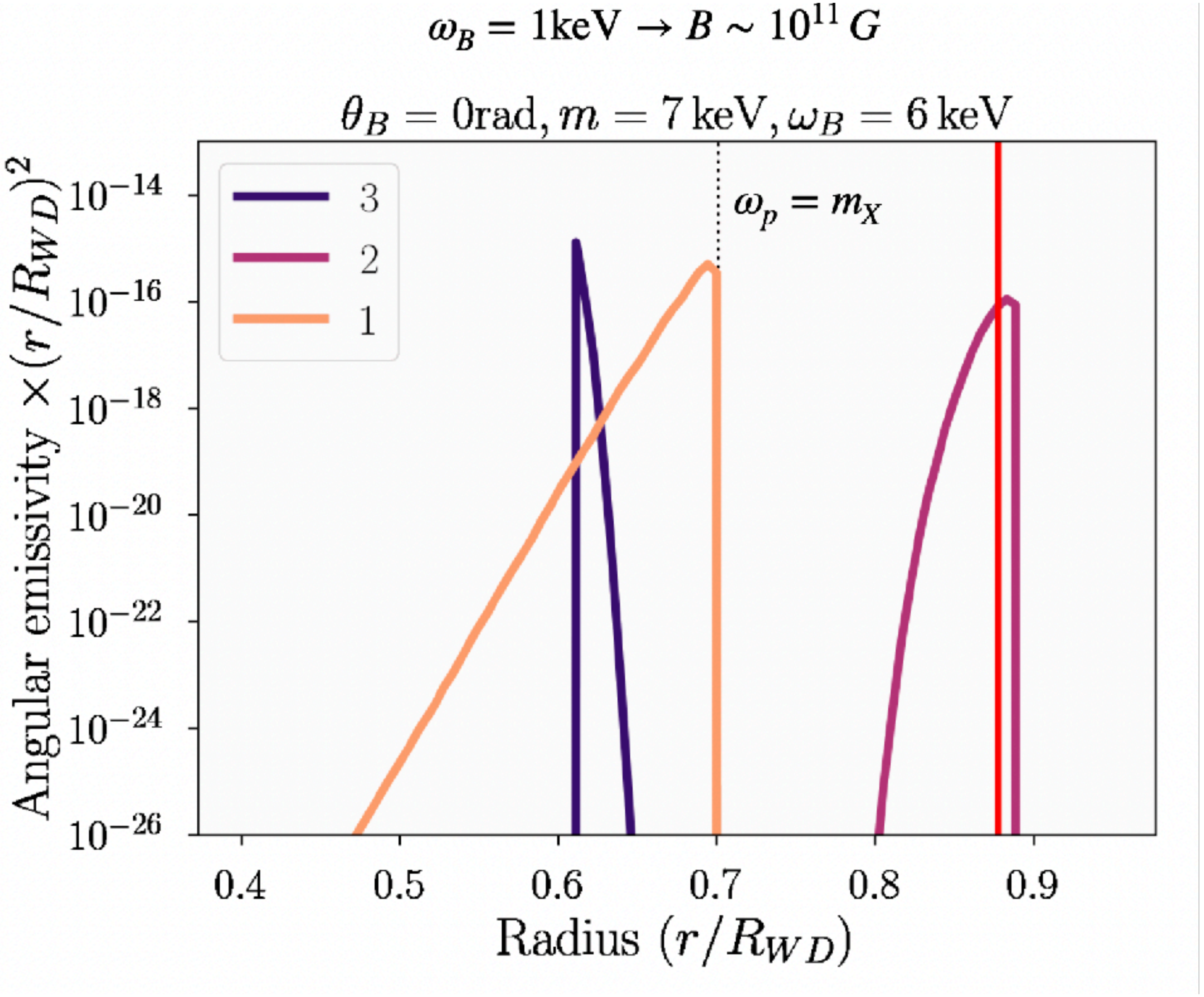
$\omega_B = 1\text{keV} \rightarrow B \sim 10^{11}\, G$



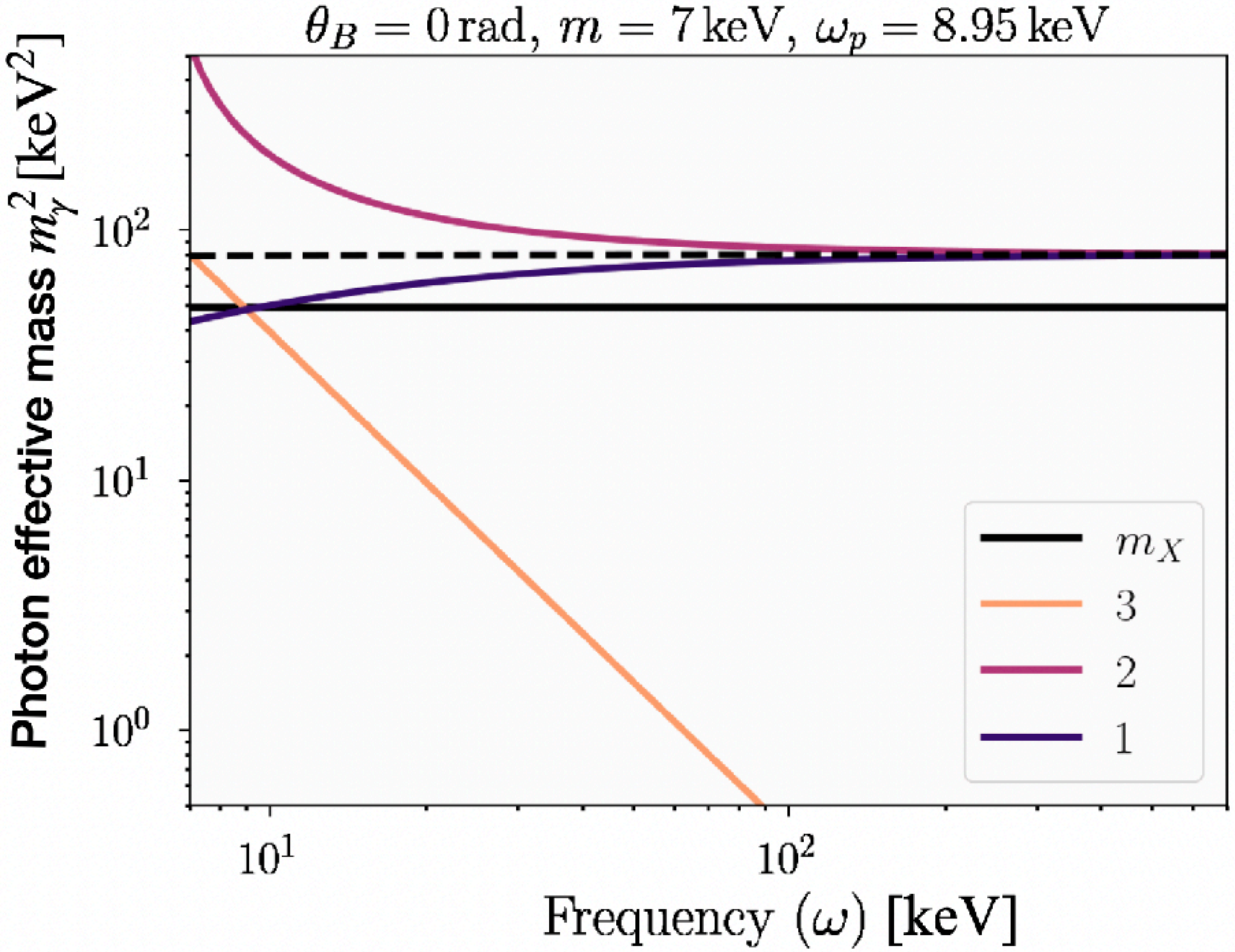


Production in magnetized plasmas: *Magnetic dispersion*





Production in magnetized plasmas: *Magnetic dispersion*



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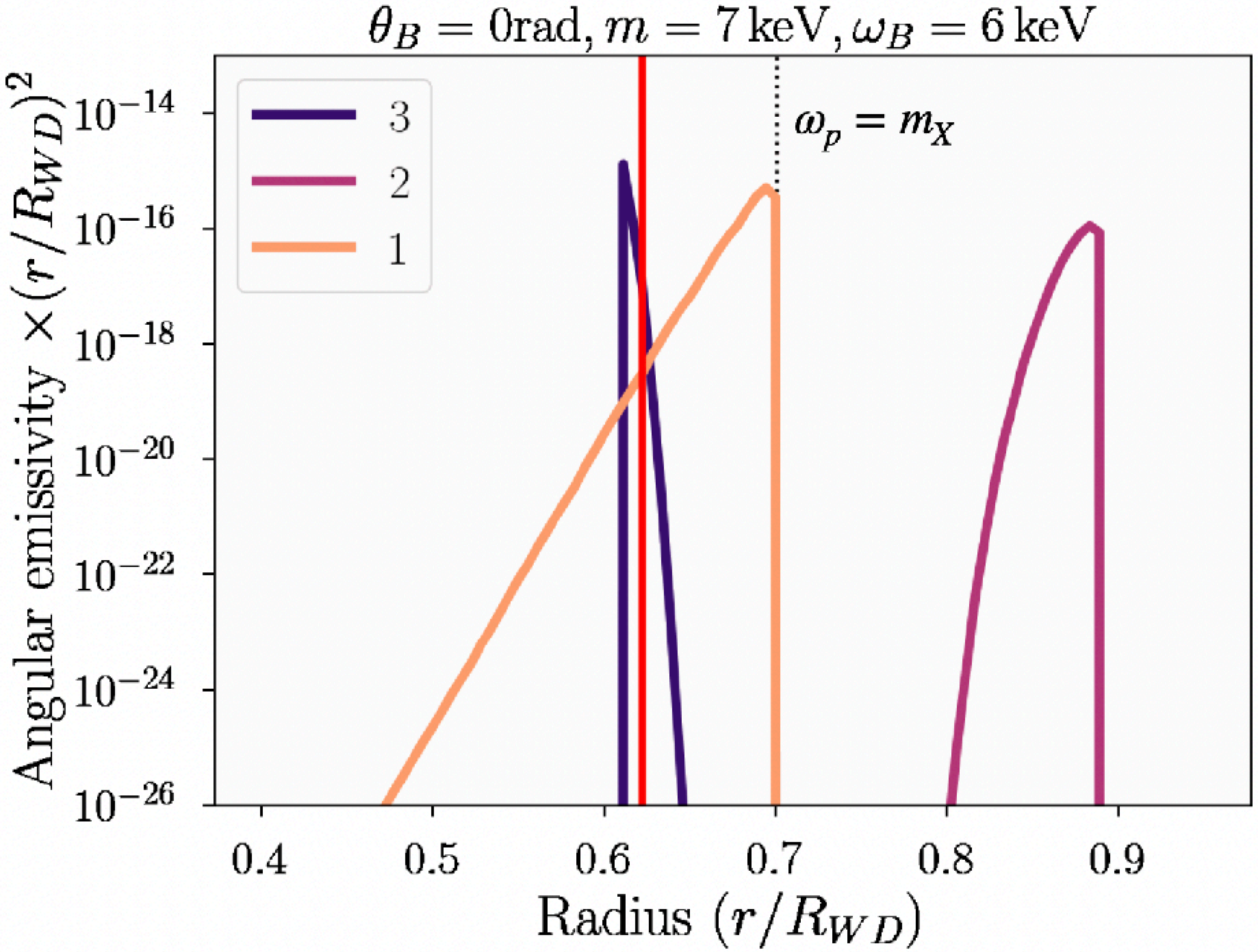

Production in magnetized plasmas: *Magnetic dispersion*

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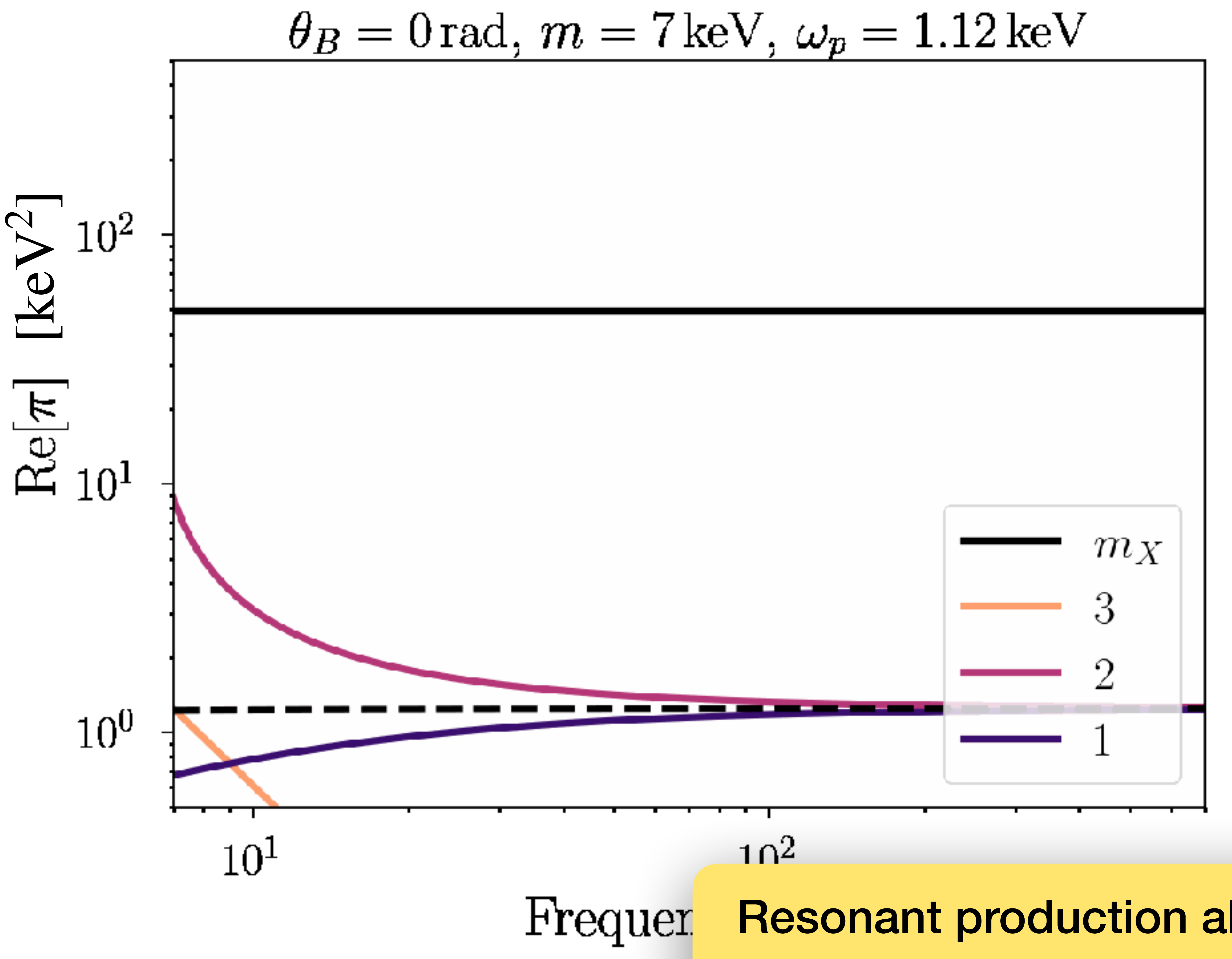
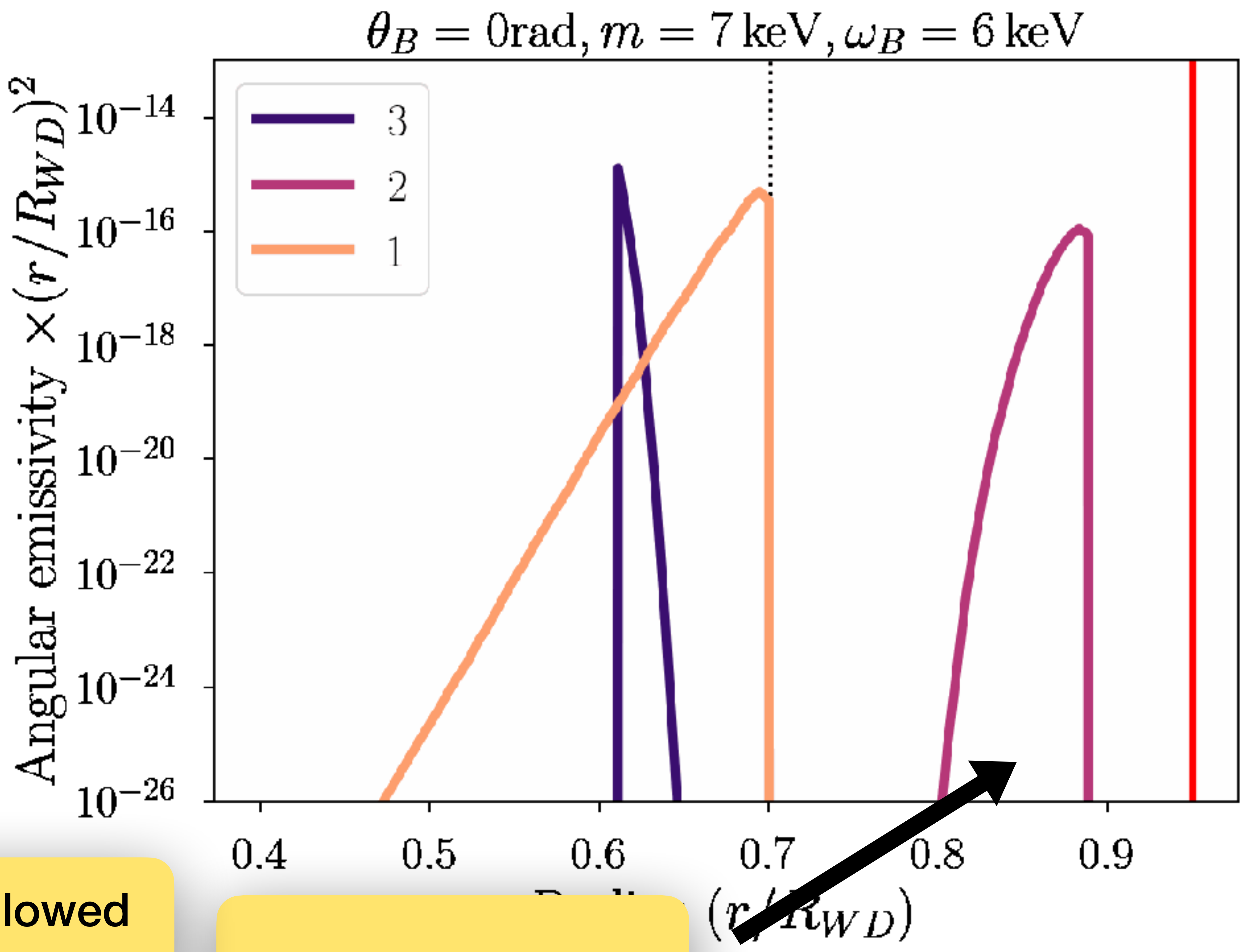
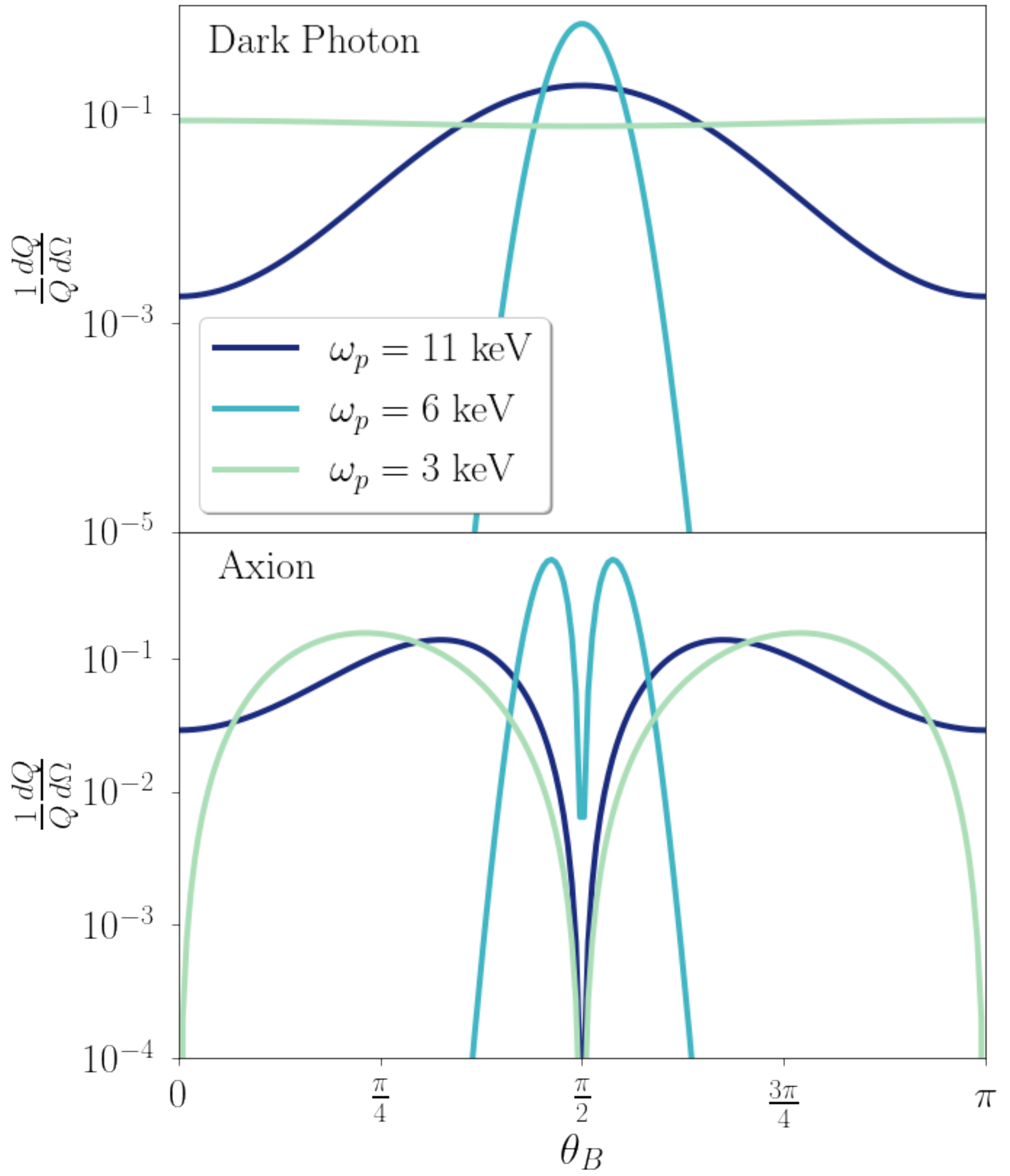


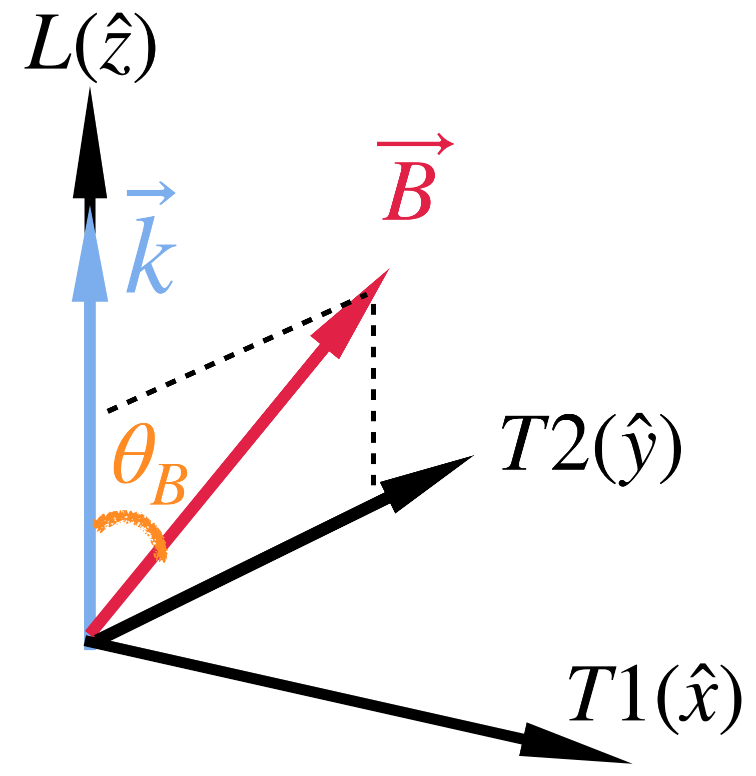


Resonant production allowed beyond plasma cutoff $\omega_p < m_X$

Larger shell volume

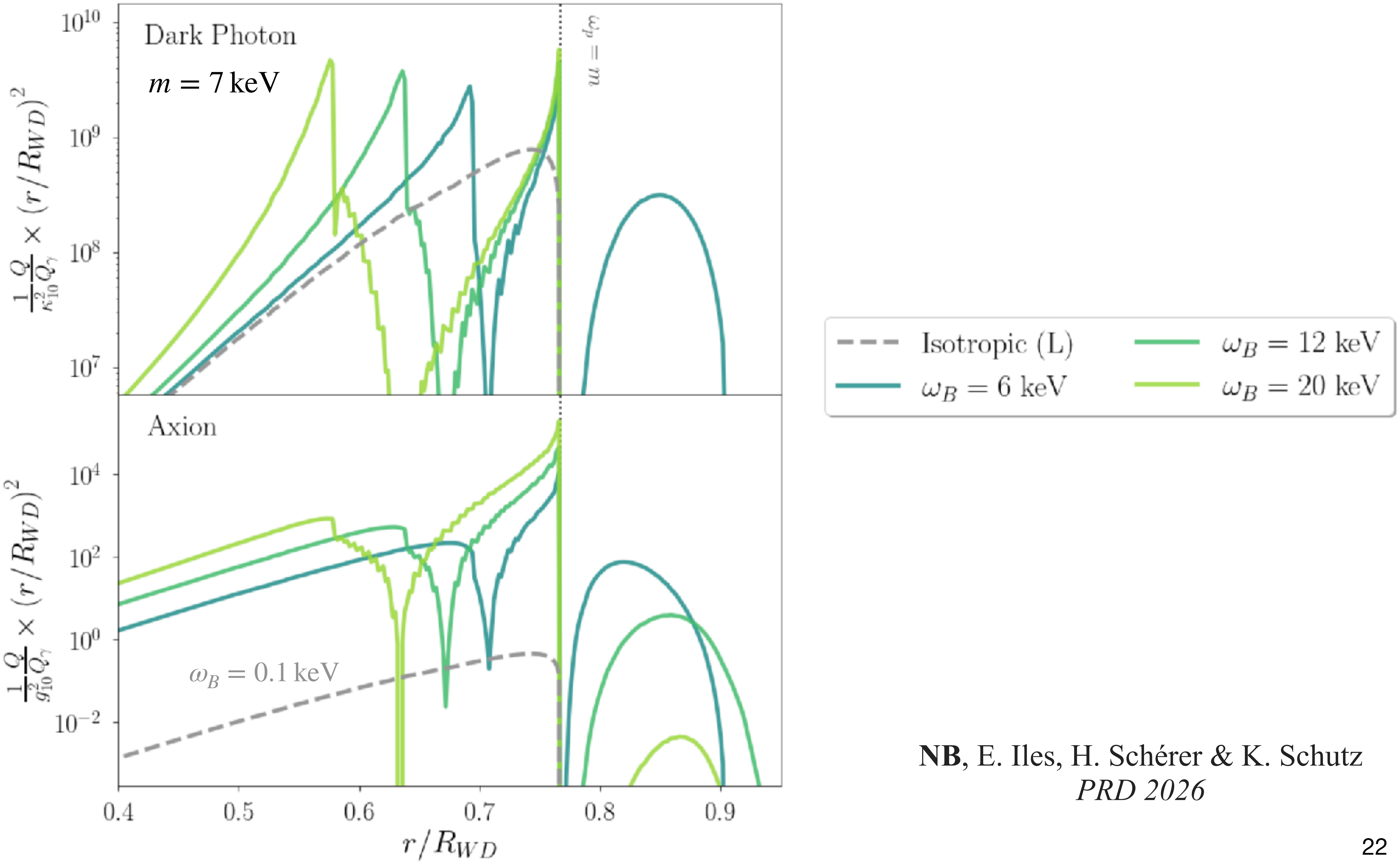
Production in magnetized plasmas: *Directional emission*





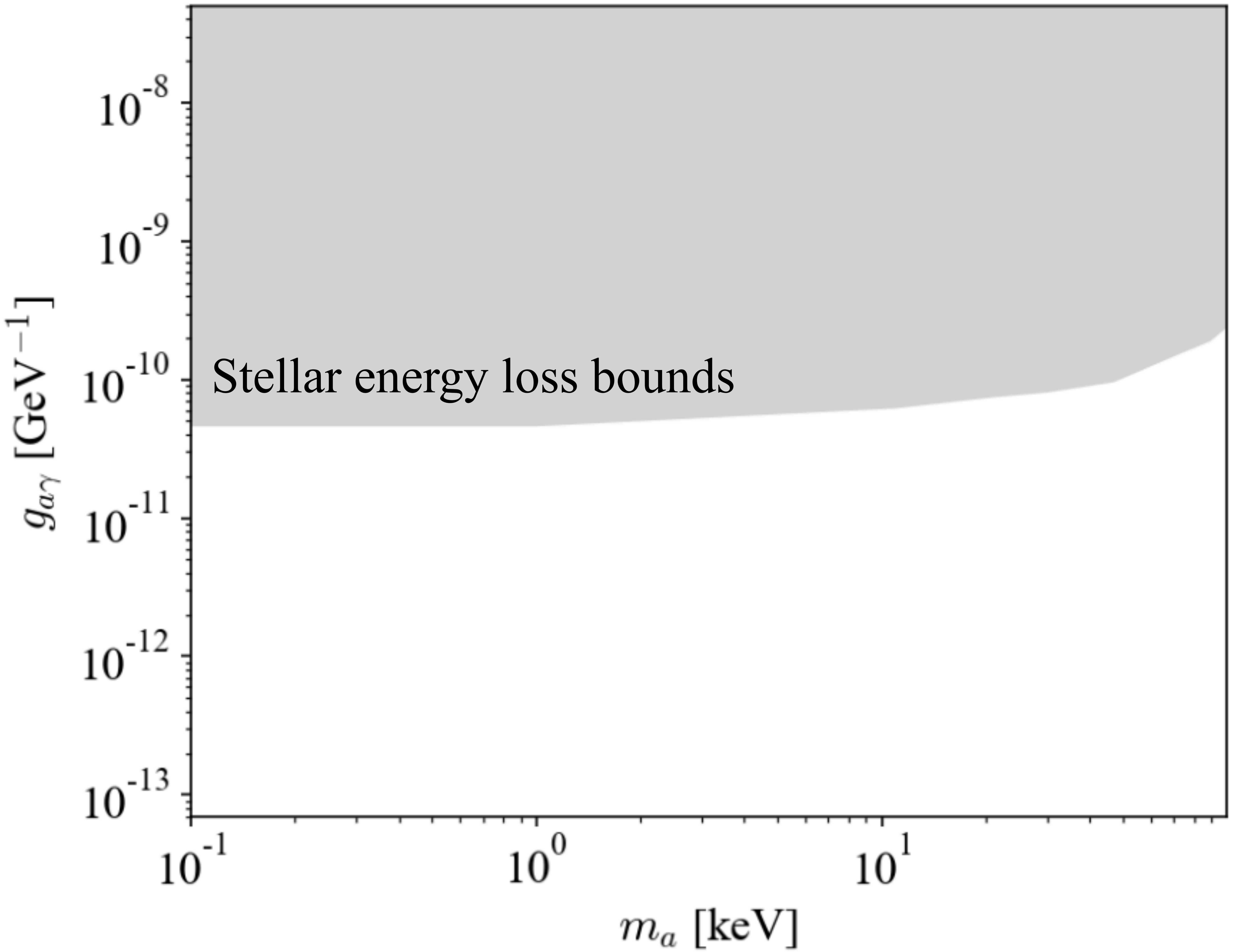
NB, E. Iles, H. Schérer & K. Schutz
PRD 2026

Production in magnetized plasmas: *Effect of magnetic dispersion*



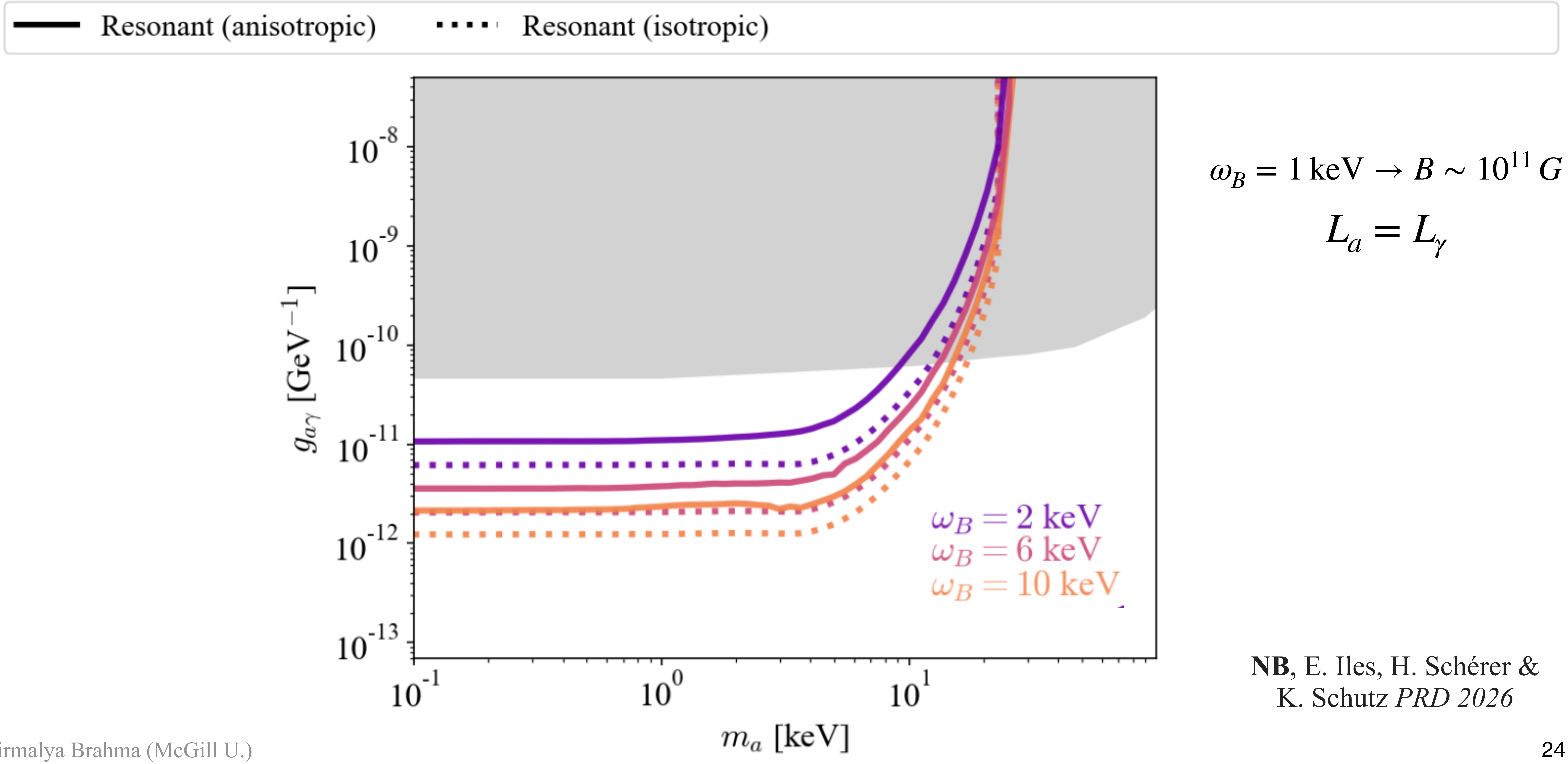
NB, E. Iles, H. Schérer & K. Schutz
PRD 2026

Magnetized white dwarf cooling: *Axions*



NB, E. Iles, H. Schérer & K. Schutz *PRD 2026*

Magnetized white dwarf cooling: *Axions*

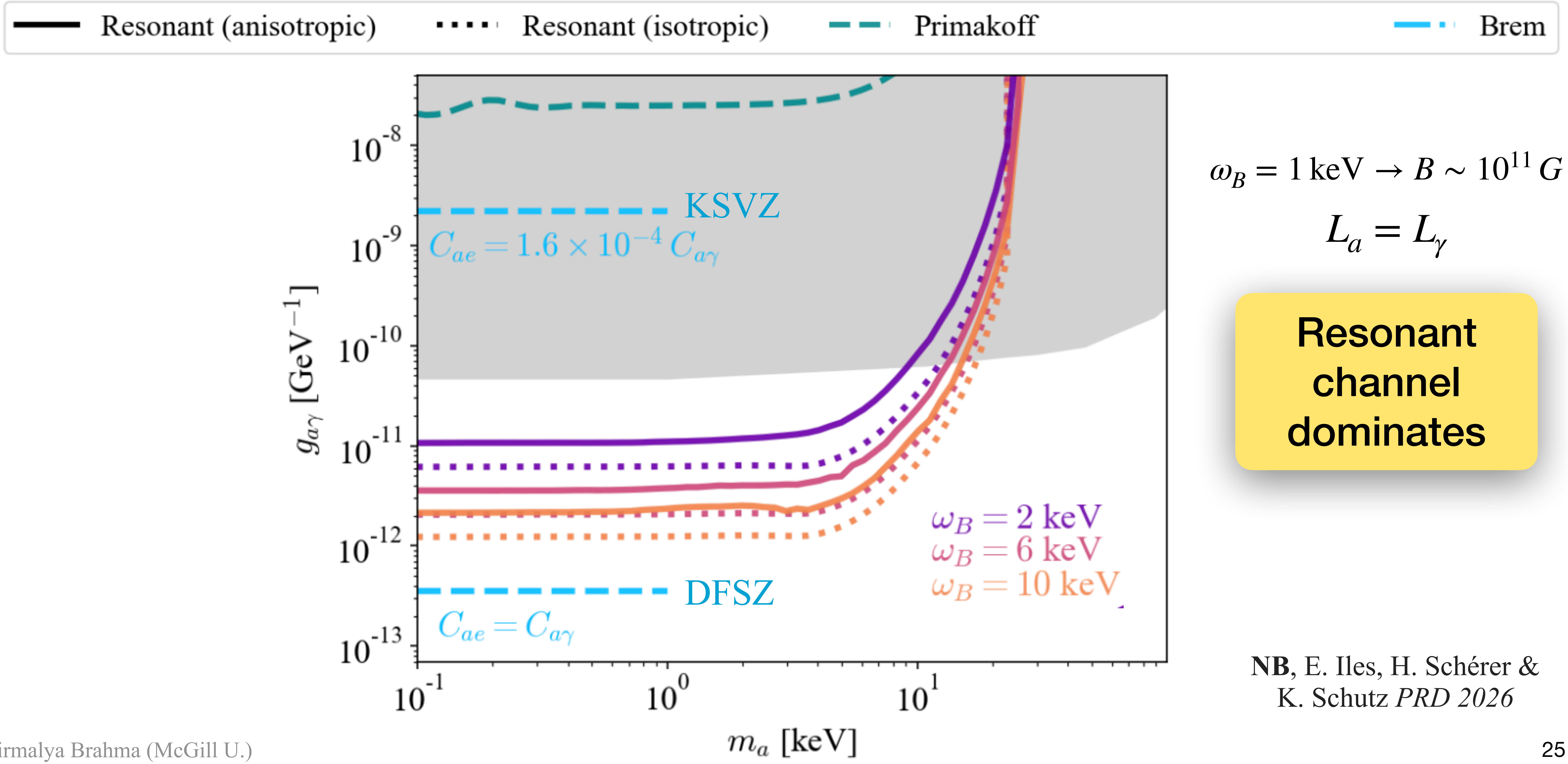


$\omega_B = 1\,\text{keV} \rightarrow B \sim 10^{11}\,G$

$$L_a = L_\gamma$$

NB, E. Iles, H. Schérer & K. Schutz *PRD 2026*

Magnetized white dwarf cooling: *Axions*



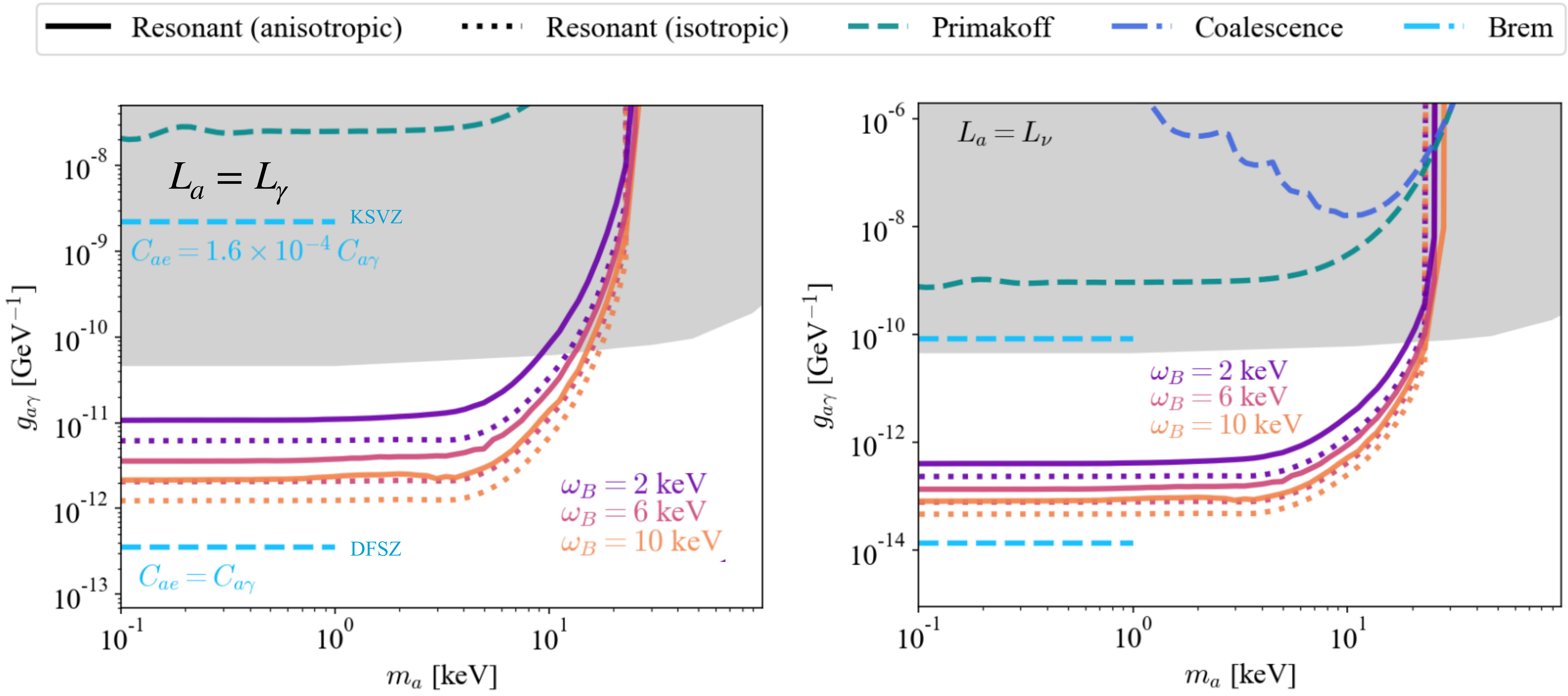
$\omega_B = 1\,\mathrm{keV} \rightarrow B \sim 10^{11}\, G$

$L_a = L_\gamma$

Resonant channel dominates

NB, E. Iles, H. Schérer & K. Schutz *PRD 2026*

Magnetized white dwarf cooling: *Axions*



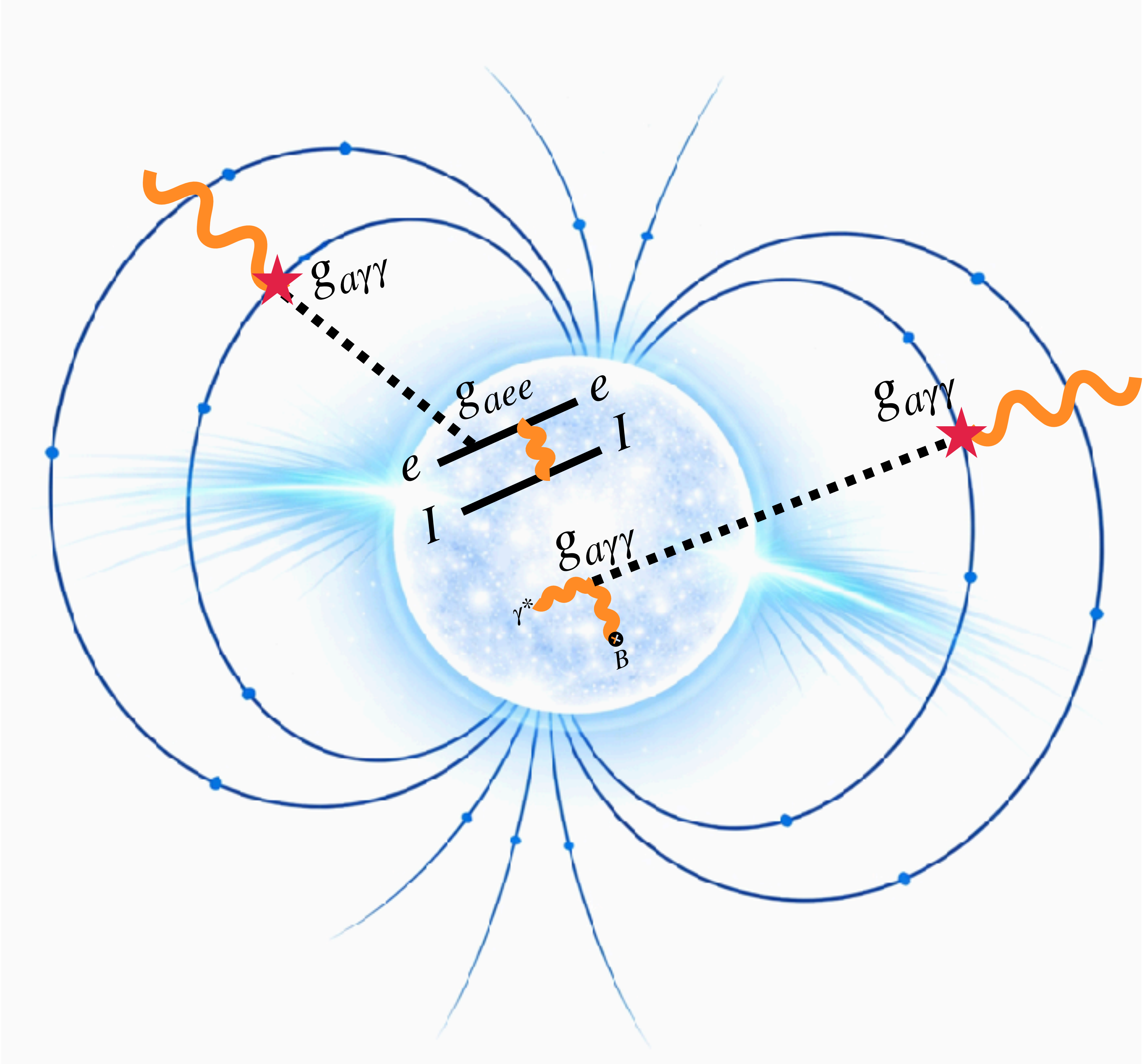
Conclusion and Future prospects

- Accounting for magnetic plasma dispersion leads to qualitatively new behaviour like the position shifts of resonances, directional emission, and provide new avenues to constrain BSM physics.

- Incorporating self consistent internal magnetic field profiles for MWDs computed with MESA.

- Produced axions can also reconvert outside the star in the stellar magnetosphere. In MWDs, this has been used to put (*C.Dessert, A.J.Long & B.R.Safdi 2021*) world leading bounds on $g_{a\gamma\gamma} \times g_{aee}$. Break degeneracy and enhance sensitivity by producing and reconverting via $g_{a\gamma\gamma}$ only.

- Extend this study to diverse astrophysical systems (with $\omega_B \gtrsim m_X \sim \omega_p$) like Neutron Stars, Quasars, solar chromosphere and other light bosons like B-L, scalar etc.



Thank you

Backup slides

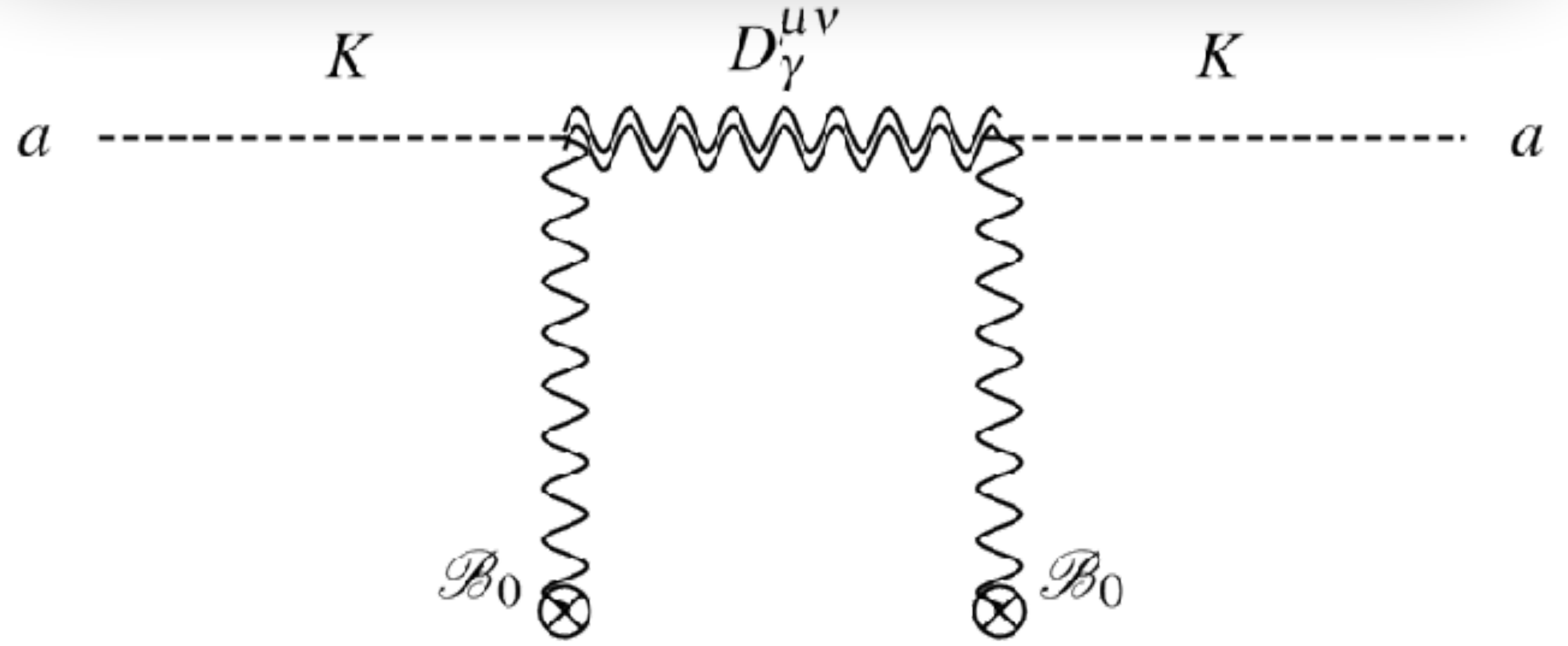
Production in an *isotropic thermal* medium

$$\frac{dn_X}{dt} = \int \frac{d^3\vec{k}}{(2\pi)^3}\Gamma_X^P = \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{-\mathrm{Im}\,\Pi_X}{\omega\left(e^{\omega/T}-1\right)}$$

Axions

$$\mathcal{L}_{a\gamma} \supset g_{a\gamma}A_\mu \tilde{F}_0^{\mu\nu}\,\partial_\nu a$$

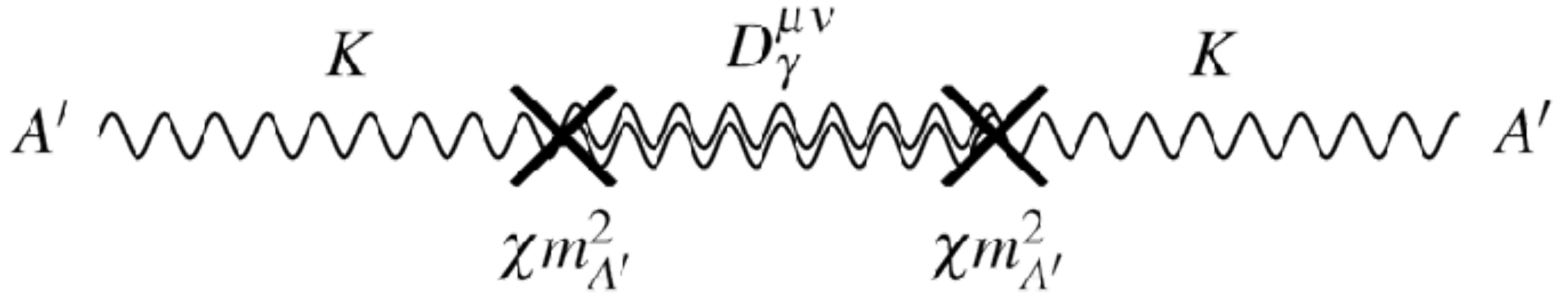
$$\Pi_a(K) = m_a^2 + g_{a\gamma\gamma}^2 K^\mu K^\nu \tilde{F}^0_{\mu\alpha}\tilde{F}^0_{\nu\beta}D_\gamma^{\alpha\beta}$$



Dark Photons

$$\mathcal{L}_{\mathcal{S}\gamma} \supset \frac{\chi m_{A'}^2}{2}A^\mu \mathcal{S}_\mu$$

$$\Pi_{A'}^{\mu\nu}(K) = m_{A'}^2\delta^{\mu\nu} + \chi^2 m_{A'}^4 D_\gamma^{\mu\nu}$$



In-medium photon propagator

$$\left(D_\gamma\right)^{-1}_{\mu\nu} = \left(D^0_\gamma\right)^{-1}_{\mu\nu} - \Pi_{\mu\nu}$$

$$\left(D^0_\gamma\right)^{-1}_{\mu\nu} = K^2 g_{\mu\nu} + \text{gauge terms}$$

In isotropic medium:

$$\Pi^{\mu\nu}_{iso}(K) = \pi_T P^{\mu\nu}_{T1} + \pi_T P^{\mu\nu}_{T2} + \pi_L P^{\mu\nu}_L$$

$$P^{\mu\nu}_I = \epsilon^\mu_I (\epsilon^\nu_I)^* \quad I = T1, T2, L$$

Projecting into the (T,L) basis:

$$\left(d_\gamma\right)^{-1}_{IJ} \equiv -(\epsilon^\mu_I)^* \left(D_\gamma\right)^{-1}_{\mu\nu} \epsilon^\nu_J = K^2 \delta_{IJ} - \pi_{IJ}$$

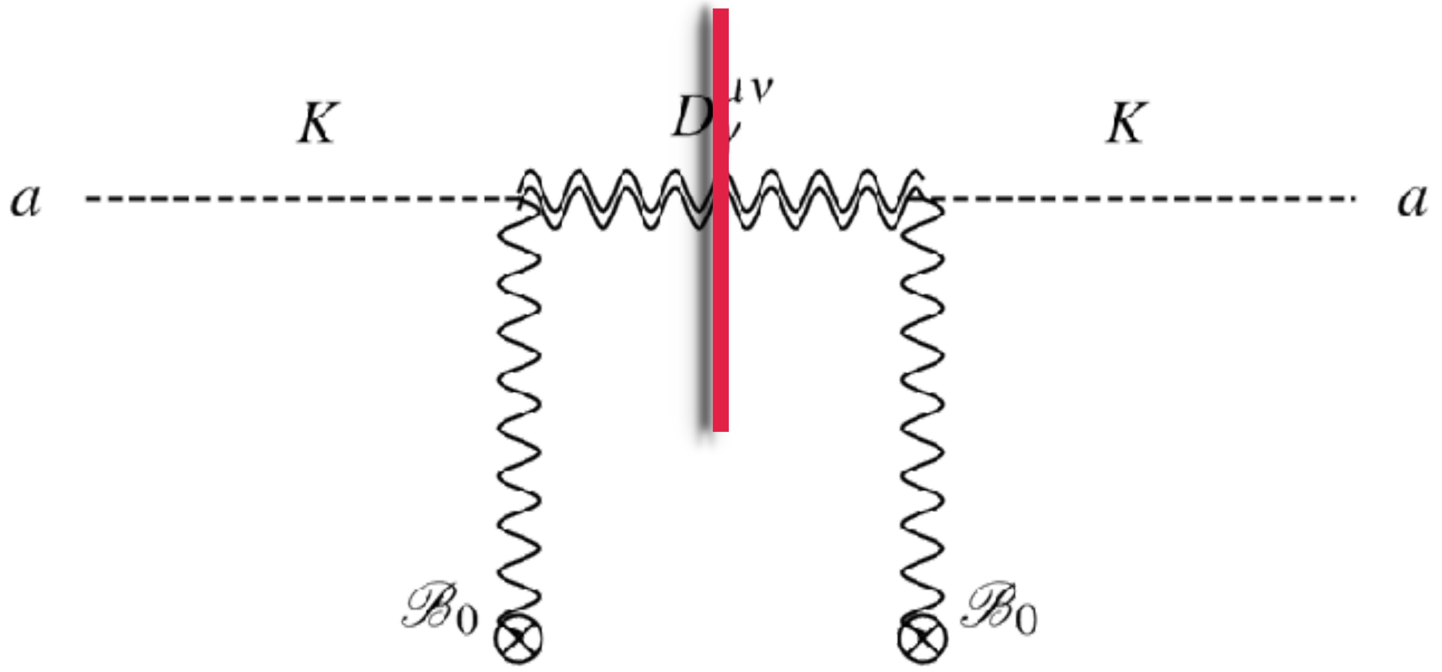
$$\pi_{IJ} \equiv -(\epsilon^I_\mu)^* \Pi^{\mu\nu} \epsilon^J_\nu$$

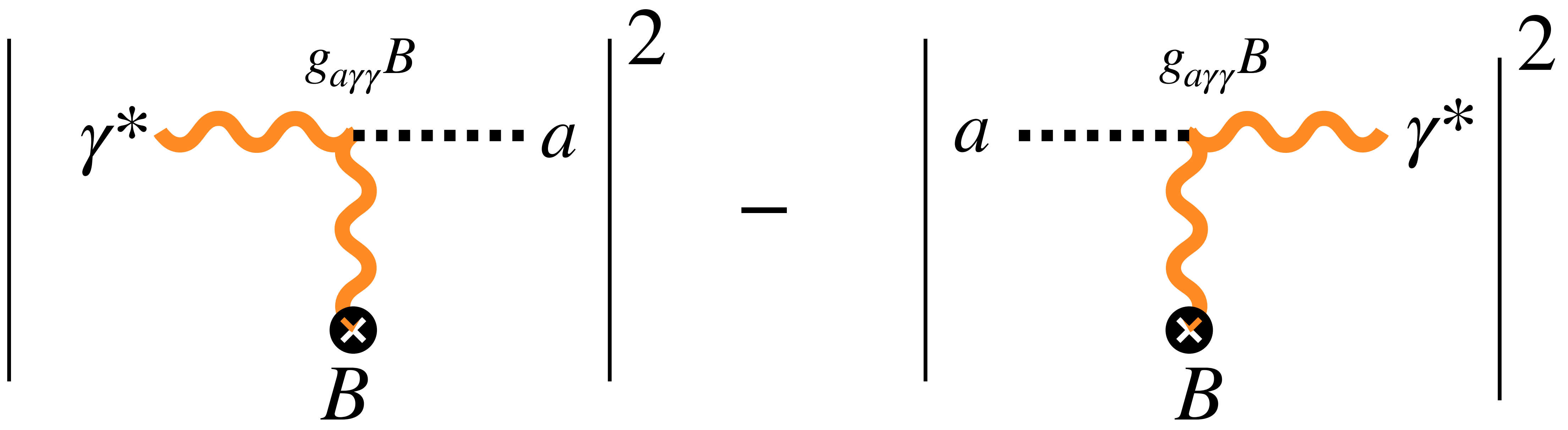
"Plasma mixing matrix"

Production in *thermal* medium

Production rate:

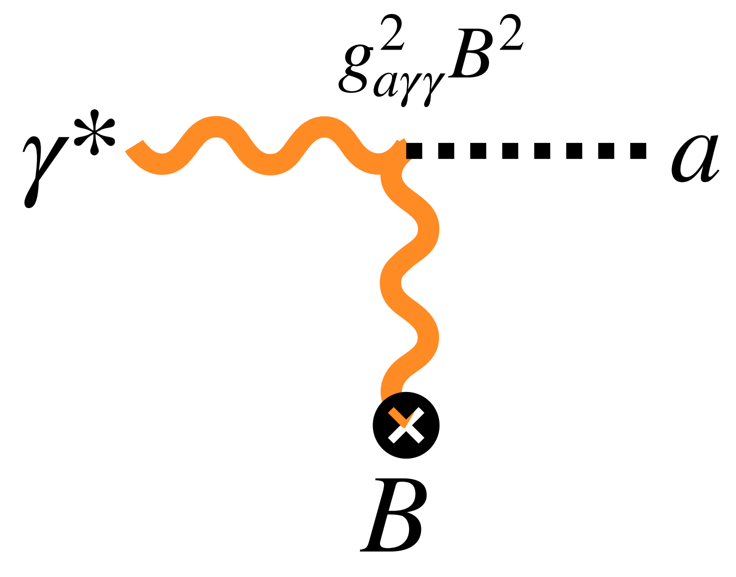
$$\Gamma_X^{prod} = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = \frac{-\mathrm{Im}\,\Pi_X}{\omega\left(e^{\omega/T}-1\right)}$$

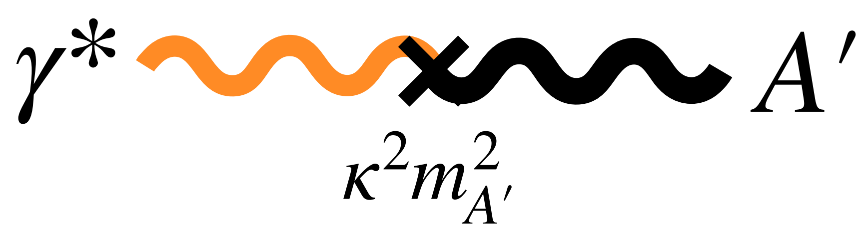
$\mathrm{Im}\,\Pi_X =$ 

$=$ 

$$= \omega\left(\Gamma_X^{prod} - \Gamma_X^{abs}\right)$$

Production in *thermal* medium





Production rate: $$\Gamma_X = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = P_X f_{\gamma^*}$$

- Thermal photon density: $$f_{\gamma^*} = \frac{1}{e^{\omega/T} - 1}$$

- Conversion probability rate: $$P_X = \frac{G_X^2 \Gamma_\gamma}{\left(m_X^2 - m_\gamma^2\right)^2 + \left(\omega \Gamma_\gamma\right)^2}$$

Damping

$\Gamma_\gamma = -\mathrm{Im}[\pi_\gamma]/\omega$

Dispersion

$m_\gamma = \mathrm{Re}[\pi_\gamma]$

- Couplings: $$G_X^2 = \begin{cases} g_{a\gamma}^2 K^2 |\epsilon \cdot B_0|^2 & X = \text{axion} \\ \chi^2 m_{A'}^4 & X = \text{dark photon} \end{cases}$$

In-medium photon propagator

In isotropic medium:

$$\Pi^{\mu\nu}_{iso}(K) = \pi_T P^{\mu\nu}_{T1} + \pi_T P^{\mu\nu}_{T2} + \pi_L P^{\mu\nu}_{L}$$

Plasma mixing matrix:

$$\pi_{IJ} \equiv -(\epsilon^I_\mu)^* \Pi^{\mu\nu} \epsilon^J_\nu = \begin{pmatrix} \pi_T & 0 & 0 \\ 0 & \pi_T & 0 \\ 0 & 0 & \pi_L \end{pmatrix}$$

"Diagonal in isotropic plasma"

Photon propagator in the (T,L) basis:

$$d^\gamma_{IJ} = \begin{pmatrix} (K^2-\pi_T)^{-1} & 0 & 0 \\ 0 & (K^2-\pi_T)^{-1} & 0 \\ 0 & 0 & (K^2-\pi_L)^{-1} \end{pmatrix}$$

Production rate per mode in an isotropic plasma

In isotropic medium:

$$\Pi^{\mu\nu}_{iso}(K) = \pi_T P^{\mu\nu}_{T1} + \pi_T P^{\mu\nu}_{T2} + \pi_L P^{\mu\nu}_L$$

$$d^{\gamma}_{II}(K) = \frac{1}{K^2 - \pi_I} \qquad I = T, L$$

Axions

$$\frac{dn_a^{A^{T,L}\to a}}{dt} = \int \frac{k^2 dk}{2\pi^2} \frac{g_{a\gamma}^2 \{\omega^2 (B_0^T)^2, K^2 \left(B_0^L\right)^2\}}{\left(m_a^2 - \mathrm{Re}[\pi^{T,L}]\right)^2 + \left(\mathrm{Im}[\pi^{T,L}]\right)^2} \frac{\mathrm{Im}[\pi^{T,L}]}{\omega \left(e^{\omega/T} - 1\right)}$$

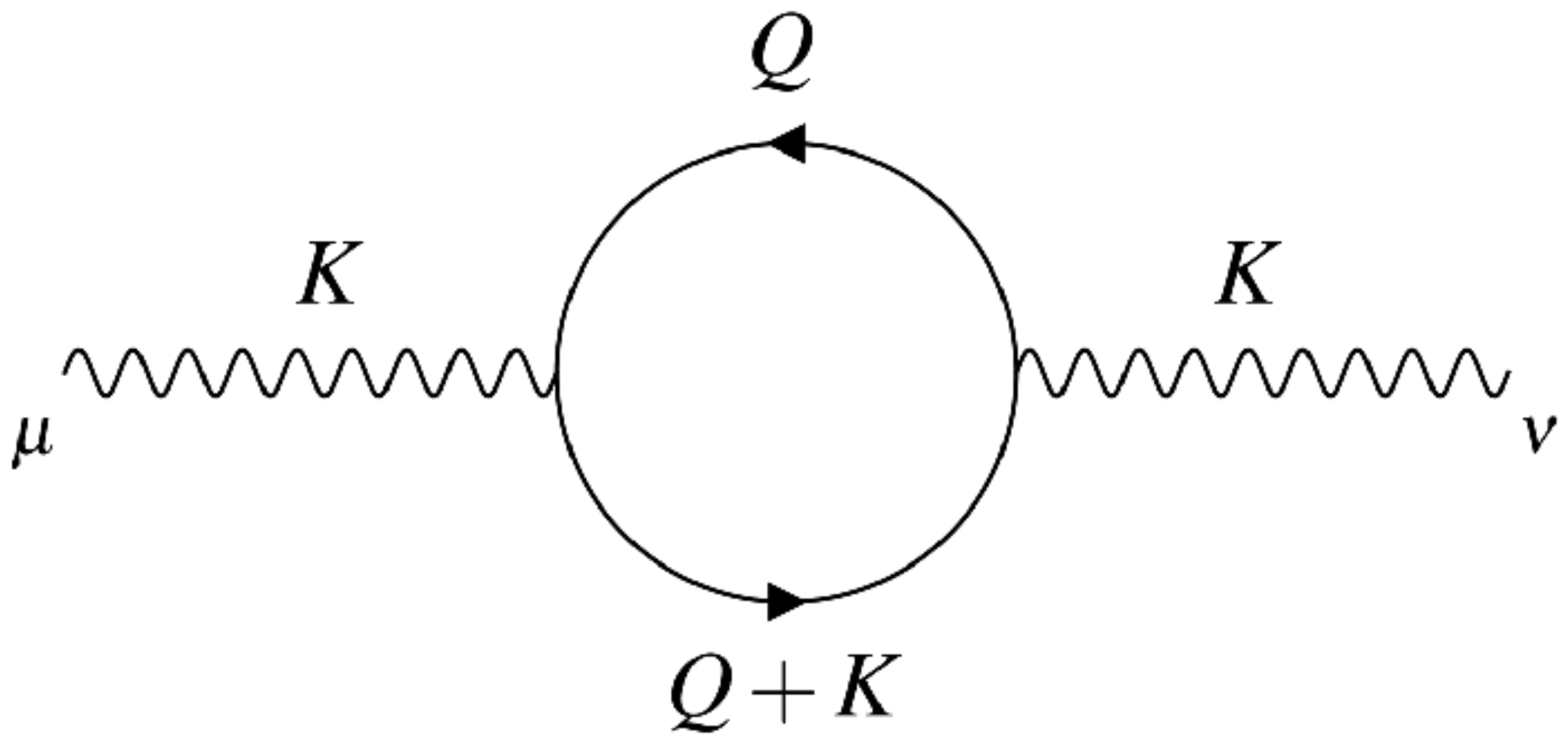
"Resonant or on-shell production"

Dark Photons

$$\frac{dn_{\mathcal{S}}^{A^{T,L}\to \mathcal{S}^{T,L}}}{dt} = \int \frac{k^2 dk}{2\pi^2} \frac{\chi^2 m_{A'}^4}{\left(m_{A'}^2 - \mathrm{Re}[\pi^{T,L}]\right)^2 + \left(\mathrm{Im}[\pi^{T,L}]\right)^2} \frac{\mathrm{Im}[\pi^{T,L}]}{\omega \left(e^{\omega/T} - 1\right)}$$

Photon self energy in an isotropic plasma

$$\Pi^{\mu\nu}_{(11)}(K) = -ie^2 \int \frac{d^4Q}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu iS^0_{(11)}(K+Q)\gamma^\nu iS^0_{(11)}(Q)\right]$$



$$iS^0_{(11)}(Q) = iS^0_V(Q) + iS^0_{\mathrm{th}}(Q)$$

"Vacuum part"

$$iS^0_V(Q) = i\frac{\gamma^\mu Q_\mu + m}{Q^2 - m^2 + i\epsilon}$$

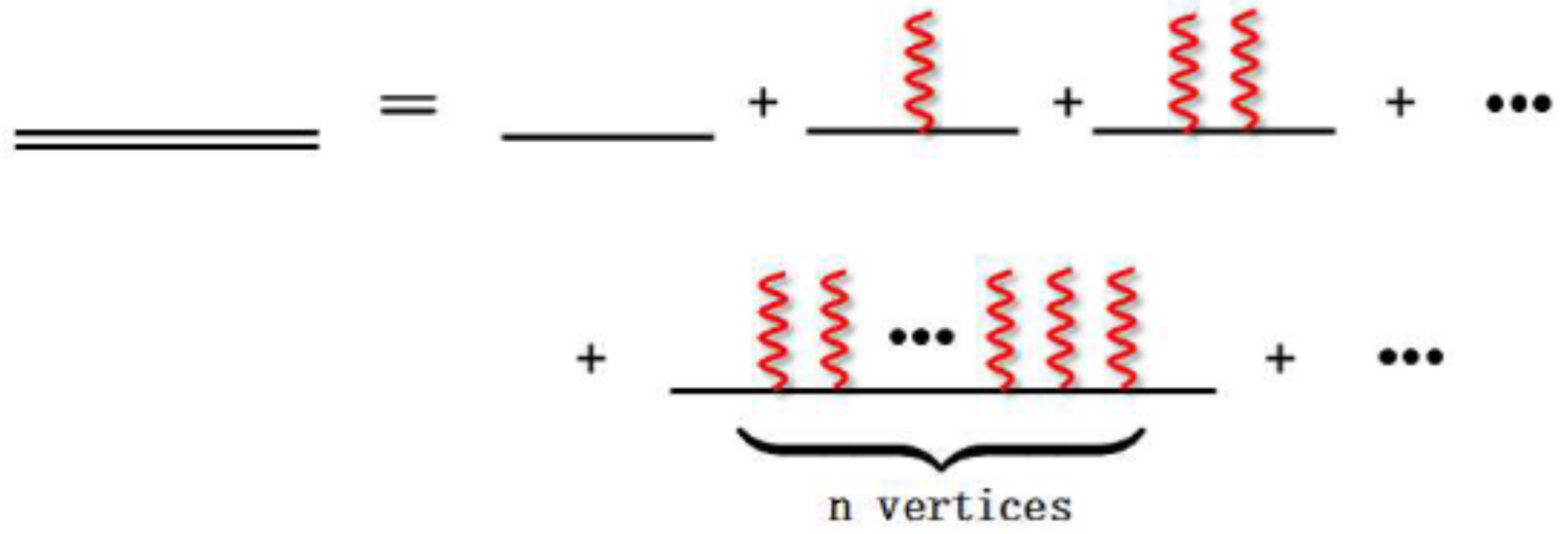
"Thermal part"

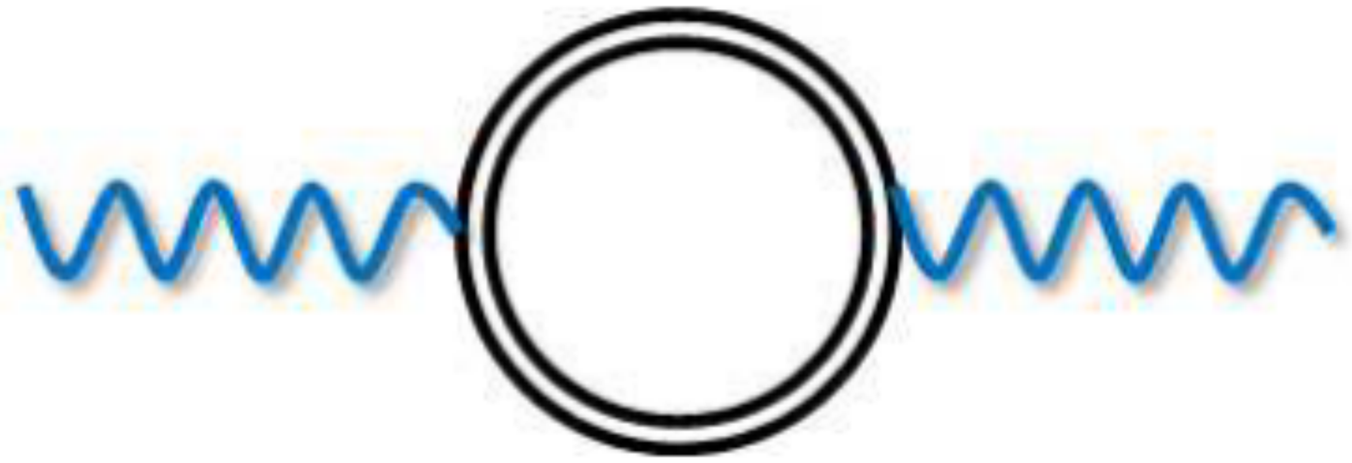
$$\begin{aligned} iS^0_{\mathrm{th}}(Q) &= -N(Q\cdot u)[iS^0_V(Q) - i\bar{S}^0_V(Q)] \\ &= -2\pi N(Q\cdot u)(\gamma^\mu Q_\mu + m)\,\delta(Q^2 - m^2) \end{aligned}$$

Photon self energy in a magnetized plasma

$$\Pi^{\mu\nu}_{(11)}(K) = -ie^2 \int \frac{d^4Q}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu iS^B_{(11)}(K+Q)\gamma^\nu iS^B_{(11)}(Q)\right]$$

$$iS^B_{(11)}(Q) = iS^0_{BV}(Q) + iS^0_{\mathrm{th}}(Q)$$





Magnetized vacuum part:

$$iS^0_{BV}(Q) = i\sum_{n=0}^{\infty} \frac{(-1)^n e^{-\alpha_q}\mathscr{D}_n(Q)}{q_\parallel^2 - M_n^2 + i\epsilon}$$

Thermal part:

$$iS^0_{\mathrm{th}}(Q) = \sum_{n=0}^{\infty} (-1)^n e^{-\alpha_q}\mathscr{D}_n(Q)\left[-2\pi N\left(q_\parallel \cdot u\right)\delta\left(q_\parallel^2 - M_n^2\right)\right]$$

“Landau representation”

Photon self energy in a magnetized plasma

$$\left(\Pi^B\right)^{\mu\nu}_{(11)}(K) = -ie^2 \int \frac{d^4Q}{(2\pi)^4} \mathrm{Tr}\left[\gamma^\mu iS^B_{(11)}(K+Q)\gamma^\nu iS^B_{(11)}(Q)\right]$$

$$iS^B_{(11)}(Q) = iS^0_{BV}(Q) + iS^0_{\mathrm{th}}(Q)$$

Magnetized vacuum part:

$$iS^0_{BV}\left(Q\right) = i\sum_{n=0}^{\infty} \frac{(-1)^n\, e^{-\alpha_q}\mathscr{D}_n\left(Q\right)}{q_\parallel^2 - M_n^2 + i\epsilon}$$

Thermal part:

$$iS^0_{\mathrm{th}}(Q) = \sum_{n=0}^{\infty} (-1)^n\, e^{-\alpha_q}\mathscr{D}_n\left(Q\right)\left[-2\pi N\left(q_\parallel \cdot u\right)\delta\left(q_\parallel^2 - M_n^2\right)\right]$$

$$\mathscr{D}_n(Q) = -4\gamma_\mu q_\perp^\mu L^1_{n-1}\left(2\alpha_q\right) + \left(\gamma_\mu q_\parallel^\mu + m\right)\left[\mathcal{S}^+ L_n\left(2\alpha_q\right) - \mathcal{S}^- L_{n-1}\left(2\alpha_q\right)\right]$$

$$E_n^2 \equiv q_\parallel^2 + M_n^2$$

$$M_n^2 \equiv m^2 + 2neB$$

Form-factors (FF) in magnetized plasmas

- Considered the Hermitian/real part of the self-energy to which the resonances are very sensitive
- Worked in the long wavelength limit (LWL), $k \ll \omega/v_e$, ignoring the effects of back-reaction and diffusion

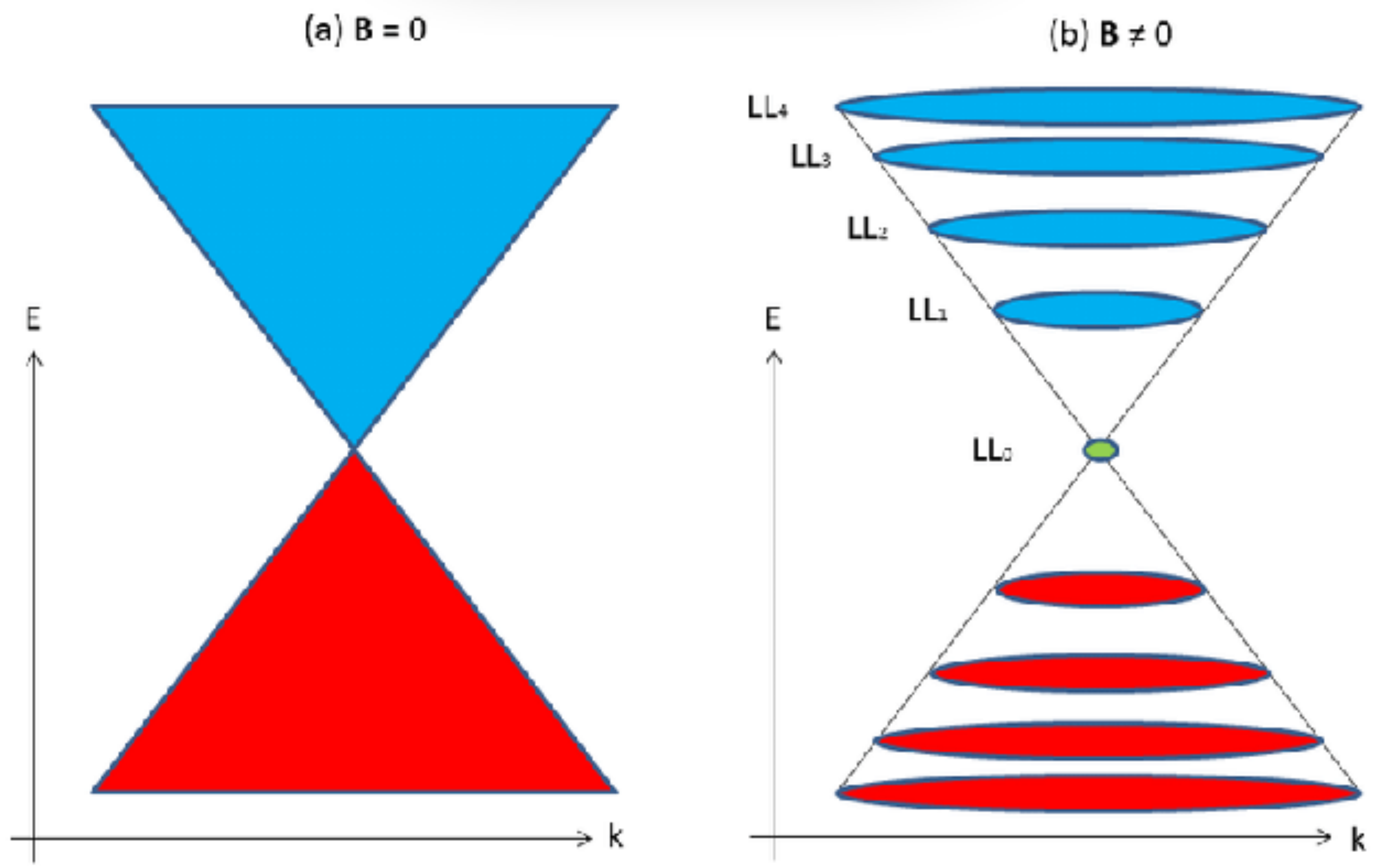
$$\mathrm{Re}[\pi_i(\omega, B)] = \frac{e^3 B}{4\pi} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dq_{\parallel,b}\ \pi_i^{(n)}(\omega, B, q_{\parallel,b}), \quad i = \perp , \parallel , \times$$

$$E_n = \pm\sqrt{m_e^2 + q_{\parallel,b}^2 + 2neB}$$

$$g_n = 2 - \delta_0^n$$

Doing this sum is hard!!

Let's take various limits :)

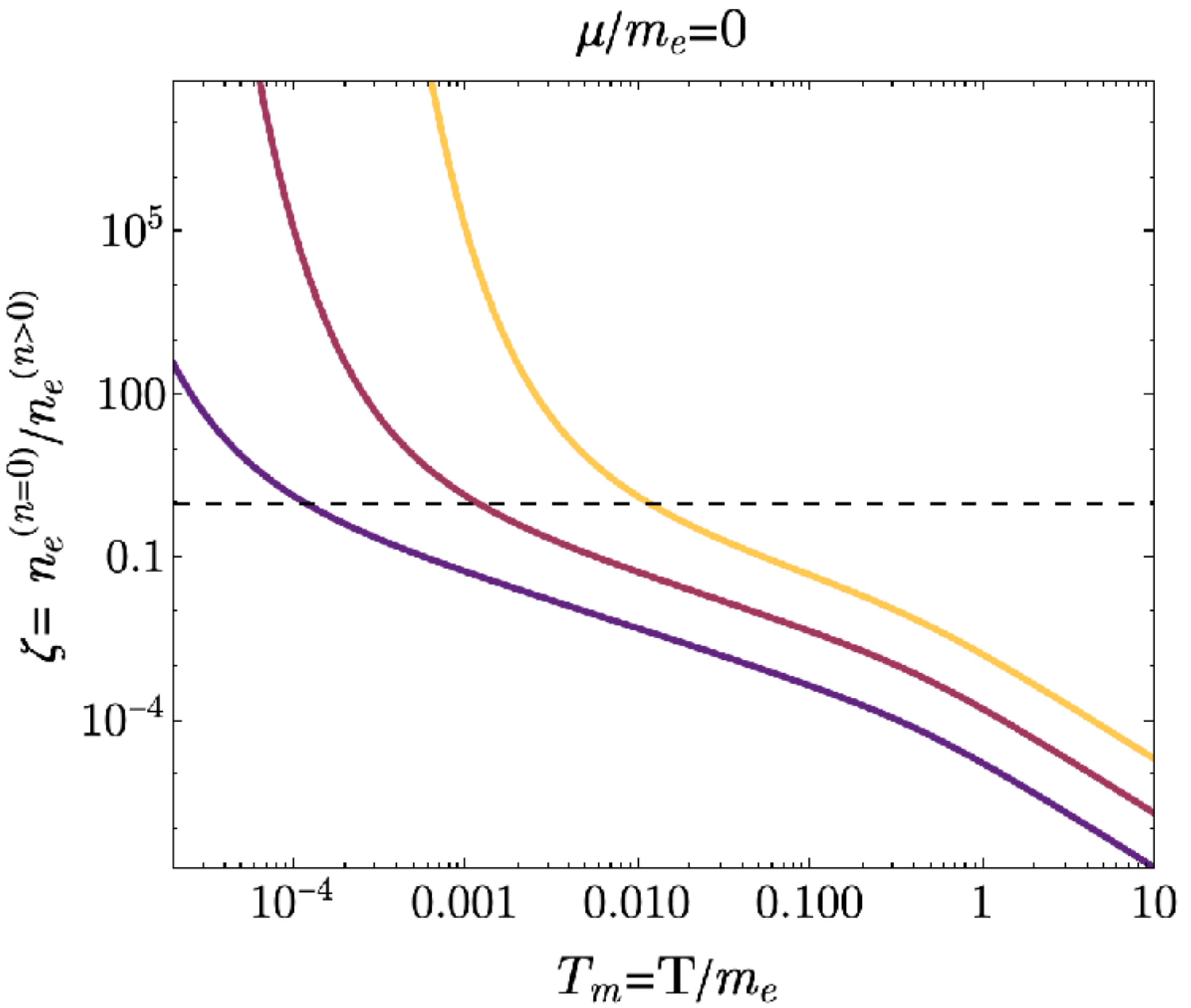


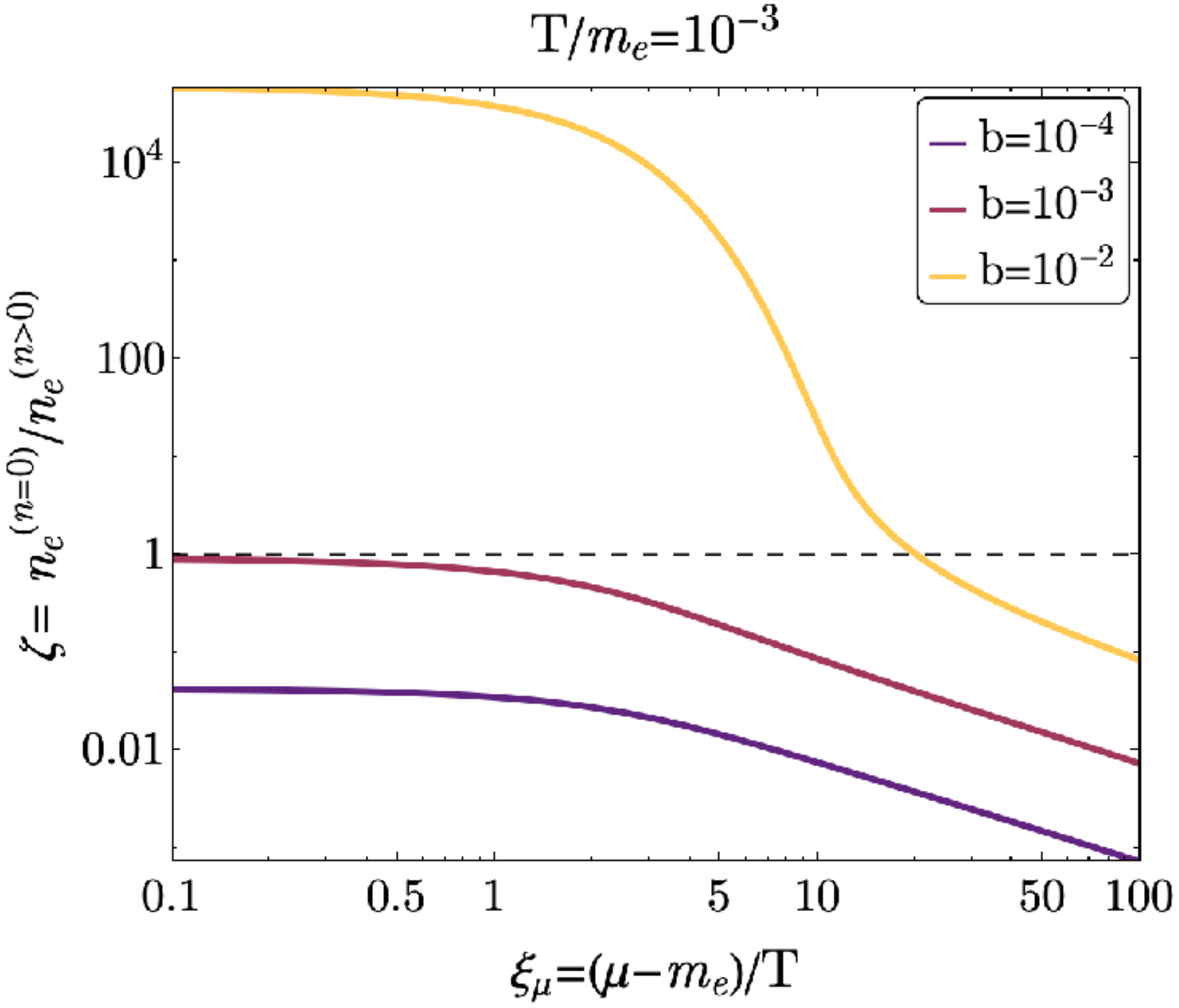
FF in magnetized plasmas: *Sub-critical* B-field

$$n_e^{(n=0)} = \frac{eB}{2\pi}\int_{-\infty}^{\infty}\frac{dq_{\parallel,b}}{2\pi}f_e(E_0)$$

$$n_e^{(n>0)} = 2\int_{\sqrt{2eB}}^{\infty}\frac{d^2\vec{q}_\perp}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{dq_{\parallel,b}}{2\pi}f_e(E)$$

$$\zeta \equiv \frac{n_e^{(n=0)}}{n_e^{(n>0)}}$$





FFs in magnetized plasmas: *Quasi-isotropic* limit

- The expressions for the FFs simplifies to 1-D integrals

$$\pi \propto \int dq[\cdots]\frac{d}{dq}(f_e \pm f_{\bar{e}})$$

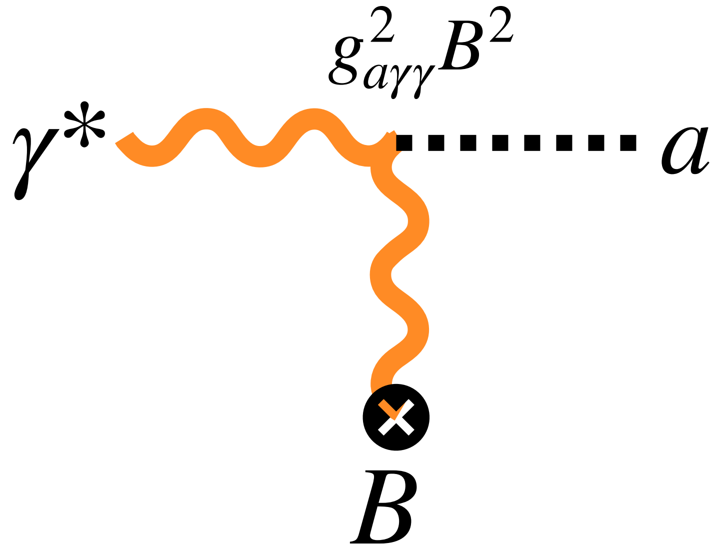
- The function $\frac{d}{dq}(f_e \pm f_{\bar{e}})$ peaks at $v_* = \frac{\omega_1}{\omega_p}$, where

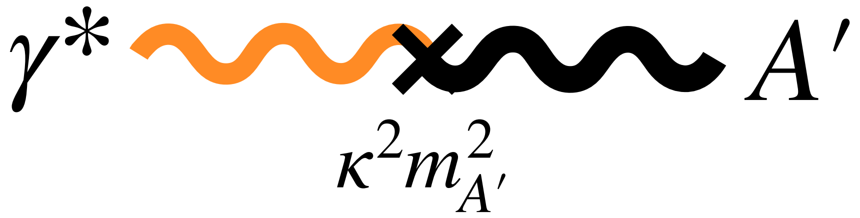
$$\omega_1^2 = m_e^2 \frac{e^2}{\pi^2} \int_0^1 dv \, \frac{v^2}{(1-v^2)^2} \left(\frac{5}{3}v^2 - v^4\right) \left(f_e + f_{\bar{e}}\right)$$

E. Braaten & D.Segel 1993

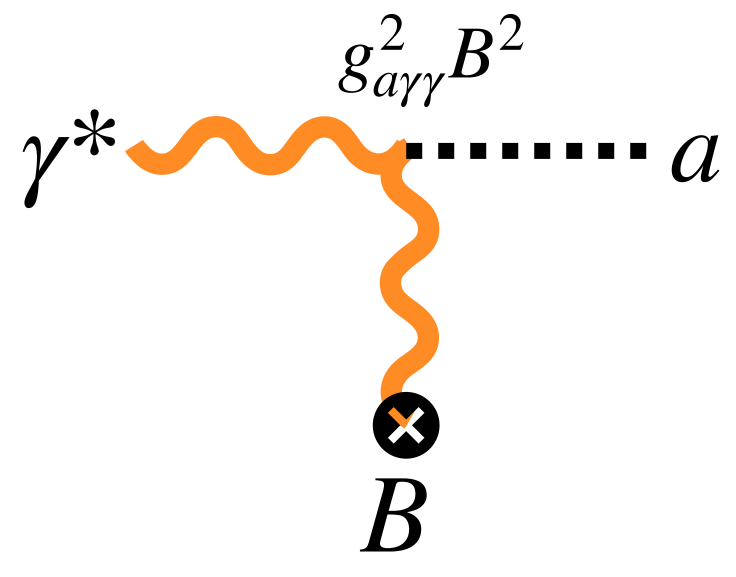
Production in *thermal* medium

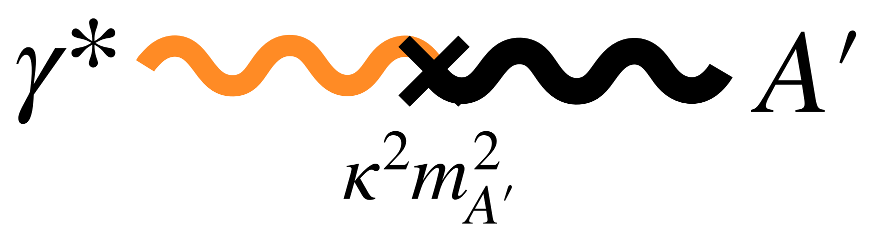
Production rate: $$\Gamma_X^{prod} = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = P_X f_{\gamma^*}$$





Production in *thermal* medium





Production rate: $$\Gamma_X^{prod} = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = P_X f_{\gamma^*}$$

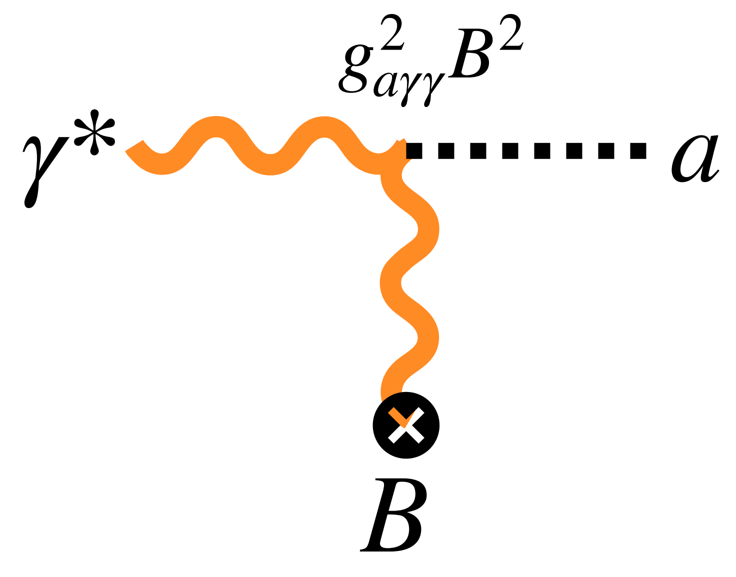
- Thermal photon density: $$f_{\gamma^*} = \frac{1}{e^{\omega/T} - 1}$$

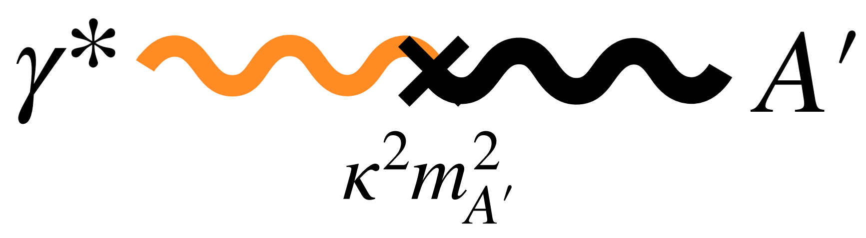
- Conversion probability rate: $$P_X = \frac{G_X^2 \Gamma_\gamma}{\left(m_X^2 - m_\gamma^2\right)^2 + \left(\omega\Gamma_\gamma\right)^2}$$

- Couplings: $$G_X^2 = \begin{cases} g_{a\gamma}^2 K^2 |\epsilon \cdot B_0|^2 & X = \text{axion} \\ \chi^2 m_{A'}^4 & X = \text{dark photon} \end{cases}$$

Production in *thermal* medium

Production rate: $$\Gamma_X^{prod} = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = P_X f_{\gamma*}$$





- Thermal photon density: $$f_{\gamma*} = \frac{1}{e^{\omega/T} - 1}$$

- Conversion probability rate: $$P_X = \frac{\pi G_X^2}{\omega}\delta\left(m_X^2 - m_\gamma^2\right)$$

Weak Damping

$\Gamma_\gamma \ll \omega$

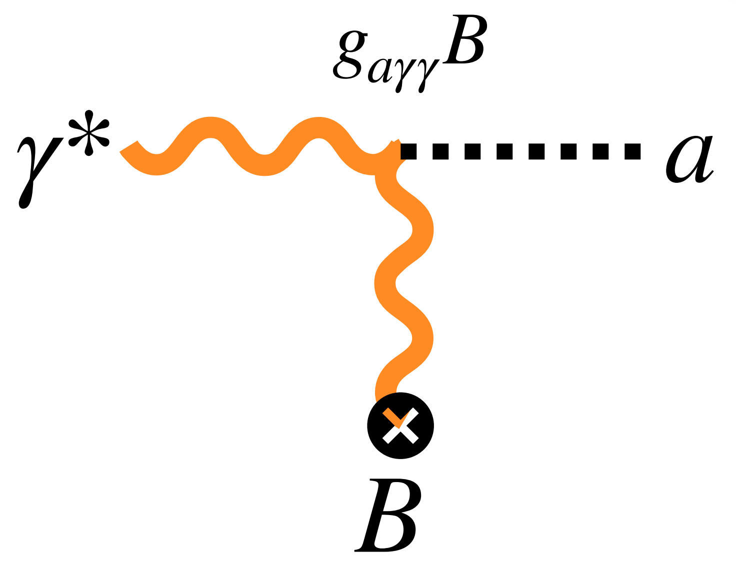
- Couplings: $$G_X^2 = \begin{cases} g_{a\gamma}^2 K^2 |\epsilon \cdot B_0|^2 & X = \text{axion} \\ \chi^2 m_{A'}^4 & X = \text{dark photon} \end{cases}$$

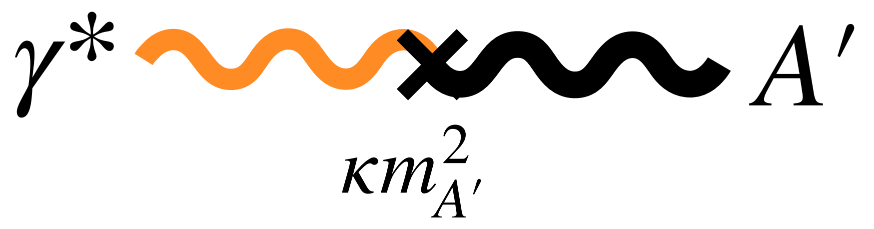
Resonant production in *thermal* medium

Production rate: $$\Gamma_X^{prod} = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = \frac{\pi G_X^2}{\omega(e^{\omega/T}-1)} \delta\left(m_X^2 - m_\gamma^2\right)$$

"Resonant or on-shell conversion"

$$m_X^2 = m_\gamma^2$$





Couplings: $$G_X^2 = \begin{cases} g_{a\gamma}^2 K^2 |\epsilon \cdot B_0|^2 & X = \text{axion} \\ \chi^2 m_{A'}^4 & X = \text{dark photon} \end{cases}$$

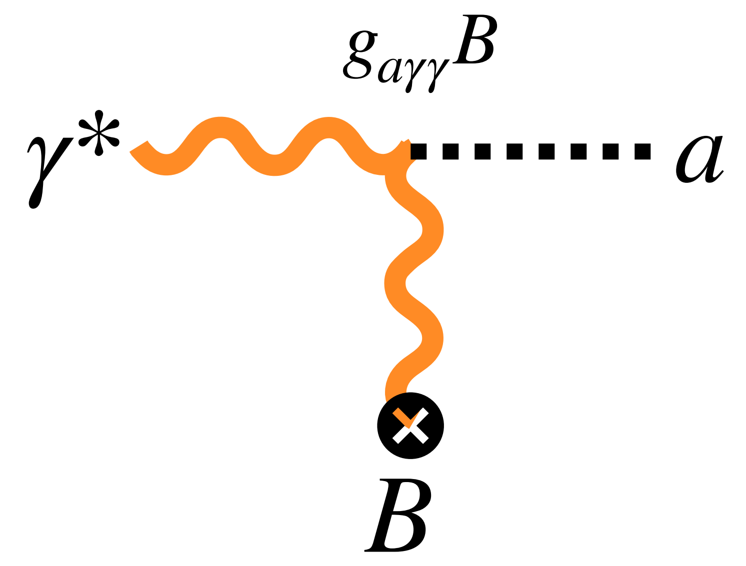
Resonant production in *thermal* medium

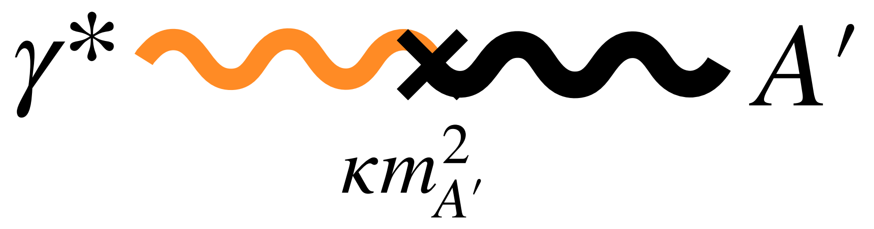
Production rate: $$\Gamma_X = (2\pi)^3 \frac{dN_X}{dt\,dV\,d^3\vec{k}} = \frac{\pi G_X^2}{\omega(e^{\omega/T}-1)}\delta\left(m_X^2 - m_\gamma^2\right)$$

"Resonant or on-shell conversion"

$$m_X^2 = m_\gamma^2$$

Photon effective mass





Couplings: $$G_X^2 = \begin{cases} g_{a\gamma}^2 K^2 |\epsilon \cdot B_0|^2 & X = \text{axion} \\ \chi^2 m_{A'}^4 & X = \text{dark photon} \end{cases}$$

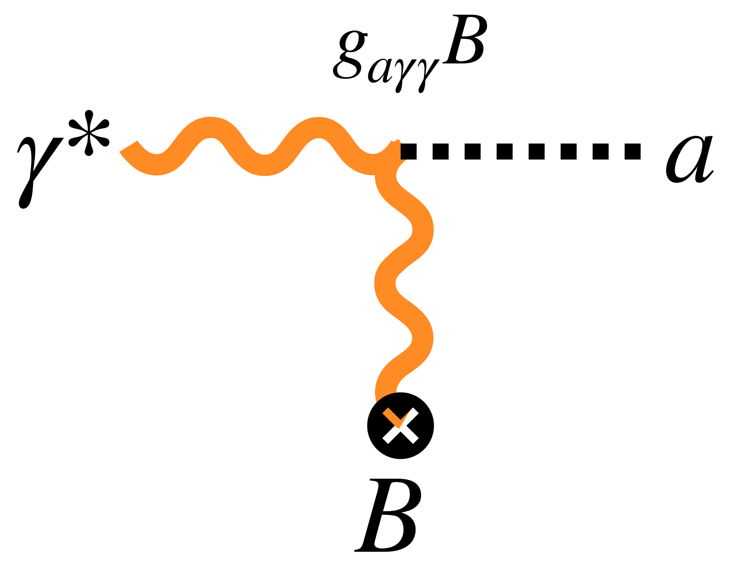
In-medium polarization vectors

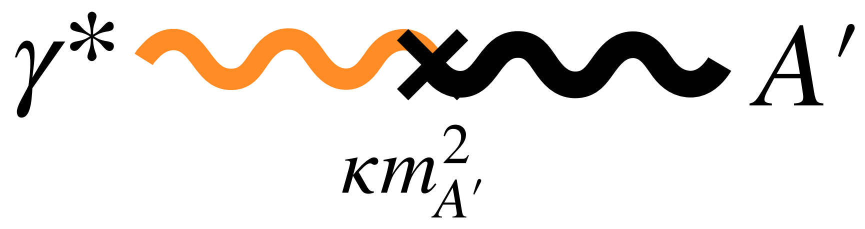
Resonant production in *thermal* medium

Production rate: $$\Gamma_X = (2\pi)^3 \frac{dN_X}{dt\, dV\, d^3\vec{k}} = \frac{\pi G_X^2}{\omega(e^{\omega/T}-1)}\delta\left(m_X^2 - m_\gamma^2\right)$$

"Resonant or on-shell conversion"

$$m_X^2 = m_\gamma^2$$



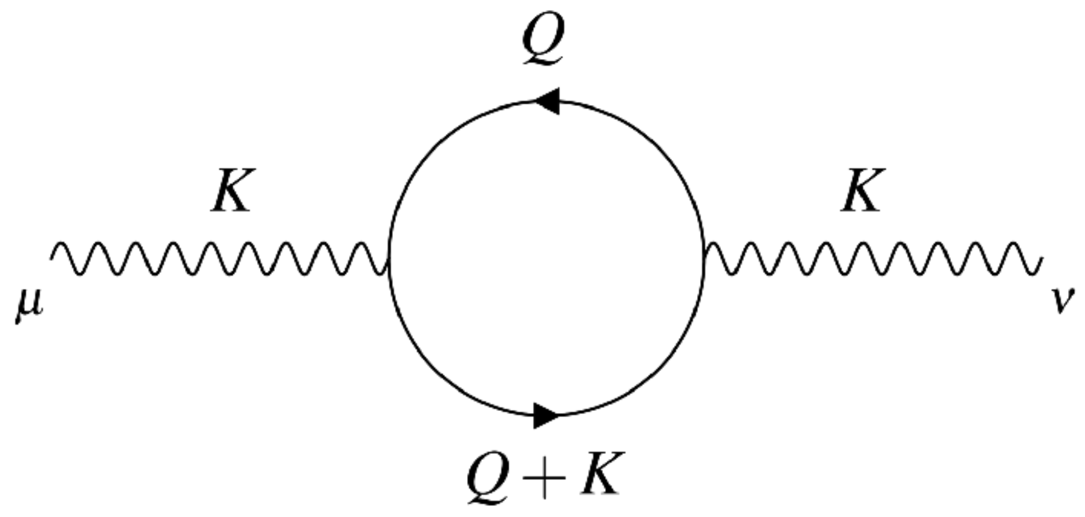


Couplings: $$G_X^2 = \begin{cases} g_{a\gamma}^2 K^2 |\epsilon \cdot B_0|^2 & X = \text{axion} \\ \chi^2 m_{A'}^4 & X = \text{dark photon} \end{cases}$$

Photon effective mass (Dispersion) $$m_\gamma^2 = \pi(\omega, n_e, T, B, ..)$$

Step 1: Calculate photon polarization tensor

Photon polarization tensor: $\Pi^{\mu\nu} =$



Isotropic

$$\Pi^{\mu\nu} = \pi_T P_T^{\mu\nu} + \pi_L P_L^{\mu\nu}$$

Transverse to $\vec{k}$

Longitudinal to $\vec{k}$

Magnetized

$$\left(\Pi^B\right)^{\mu\nu} = \pi_\perp P_\perp^{\mu\nu} + \pi_\parallel P_\parallel^{\mu\nu} + \pi_\times P_\times^{\mu\nu}$$

Transverse to B-field

Longitudinal to B-field

Gyrotropic term

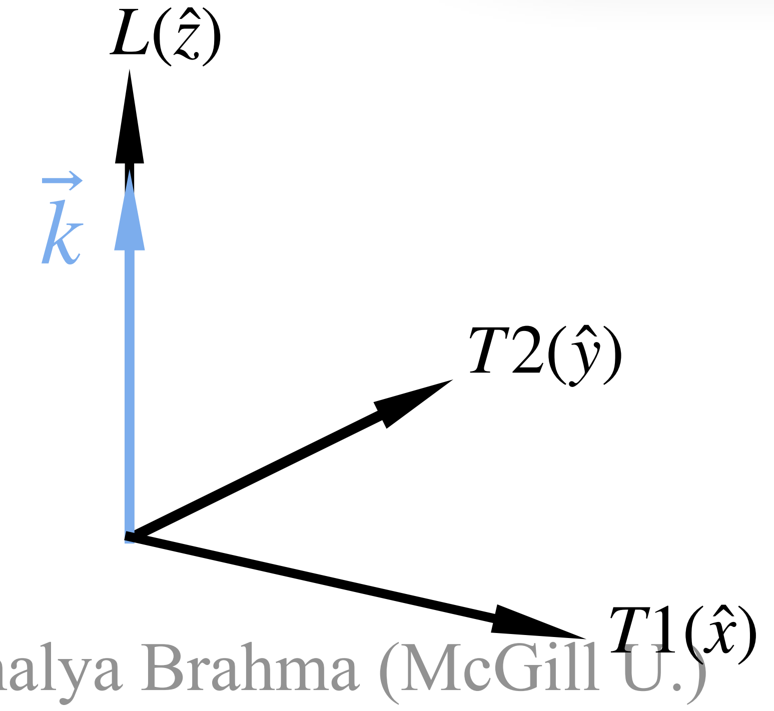
Step 2: Calculate plasma mixing matrix

Plasma mixing matrix: $\pi^{IJ} \equiv -(\epsilon^I_\mu)^* \Pi^{\mu\nu} \epsilon^J_\nu, \quad I, J = T1, T2, L$

Isotropic

$$\Pi^{\mu\nu} = \pi_T P_T^{\mu\nu} + \pi_L P_L^{\mu\nu}$$

$$\pi^{IJ} = \begin{pmatrix} \pi_T & 0 & 0 \\ 0 & \pi_T & 0 \\ 0 & 0 & \pi_L \end{pmatrix}$$



"Diagonal"

Magnetized

$$\left(\Pi^B\right)^{\mu\nu} = \pi_\perp P_\perp^{\mu\nu} + \pi_\parallel P_\parallel^{\mu\nu} + \pi_\times P_\times^{\mu\nu}$$

$$\pi^{IJ} = \begin{pmatrix} \pi_\perp & -i\pi_\times c_\theta & -i\frac{\sqrt{K^2}}{\omega}\pi_\times s_\theta \\ i\pi_\times c_\theta & \pi_\perp c_\theta^2 + \pi_\parallel s_\theta^2 & \frac{\sqrt{K^2}}{\omega}\left(\pi_\perp - \pi_\parallel\right) c_\theta s_\theta \\ i\frac{\sqrt{K^2}}{\omega}\pi_\times s_\theta & \frac{\sqrt{K^2}}{\omega}\left(\pi_\perp - \pi_\parallel\right) c_\theta s_\theta & \frac{K^2}{\omega^2}\left(\pi_\parallel c_\theta^2 + \pi_\perp s_\theta^2\right) \end{pmatrix}$$

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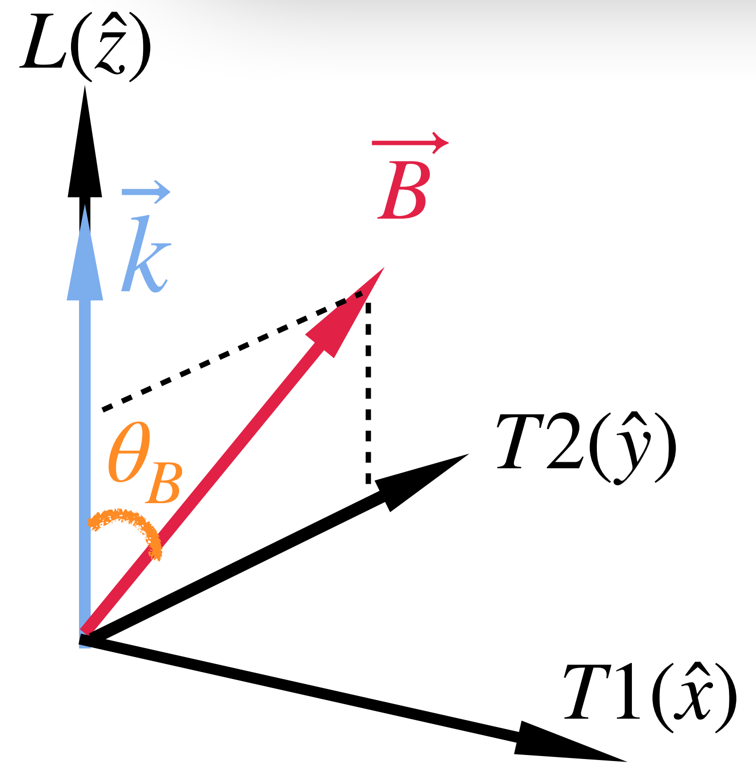


"Non-Diagonal"

Step 3: Diagonalize to obtain plasma normal modes

Isotropic

$$\Pi^{\mu\nu} = \pi_T P_T^{\mu\nu} + \pi_L P_L^{\mu\nu}$$

Classical plasma

$$\pi^{IJ} = \begin{pmatrix} \omega_p^2 & 0 & 0 \\ 0 & \omega_p^2 & 0 \\ 0 & 0 & \frac{K^2}{\omega^2}\omega_p^2 \end{pmatrix}$$

Dispersion:

$$\omega^2 = \vec{k}^2 + \omega_p^2$$

$$\omega^2 = \omega_p^2$$

Polarization vectors:

$$\{\epsilon^\mu_{T1}, \epsilon^\mu_{T2}, \epsilon^\mu_L\}$$

H.An, M.Pospelov & J.Pradler 2013

R.Lasenby & K.V.Tilburg 2020

A.Caputo, A.J.Millar & E.Vitagliano 2020

A.Caputo, H.Liu, S.M-Sharma & J.T.Ruderman 2020

M.J.Dolan, F.J.Hiskens & R.R.Volkas 2023

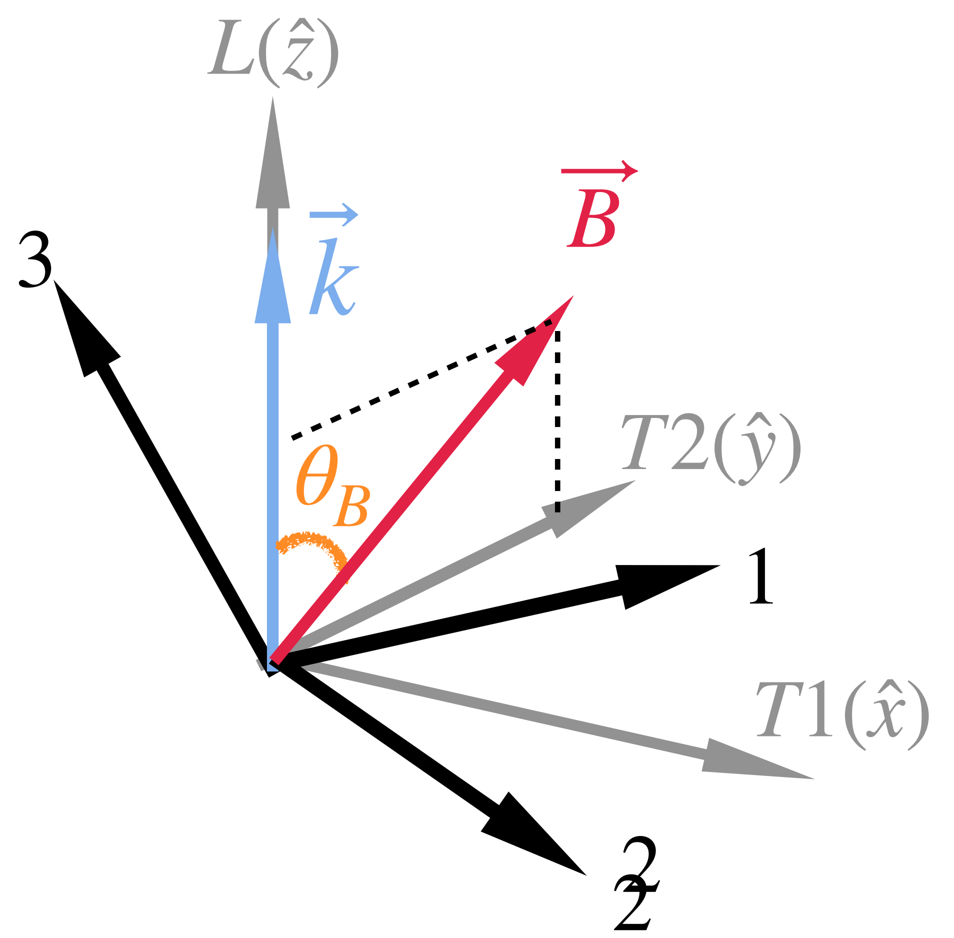
Magnetized

$$\left(\Pi^B\right)^{\mu\nu} = \pi_\perp P_\perp^{\mu\nu} + \pi_\parallel P_\parallel^{\mu\nu} + \pi_\times P_\times^{\mu\nu}$$

$$\pi^{I'J'} = \begin{pmatrix} \pi_1 & 0 & 0 \\ 0 & \pi_2 & 0 \\ 0 & 0 & \pi_3 \end{pmatrix}$$

$\pi_1 \neq \pi_2$

Magnetic birefringence



New Dispersion:

$$\omega^2 = \vec{k}^2 + \pi_{I'}, \quad I' = 1,2,3$$

New Polarization vectors:

$$\{\epsilon^\mu_1, \epsilon^\mu_2, \epsilon^\mu_3\}$$

Response of a magnetized plasma: *Form-factors*

$$\pi_\perp \sim \frac{\omega_p^2\omega^2}{\omega^2-\omega_B^2}, \quad \pi_\times \sim \frac{\omega_p^2\omega\omega_B}{\omega^2-\omega_B^2}, \quad \pi_\parallel \sim \omega_p^2$$

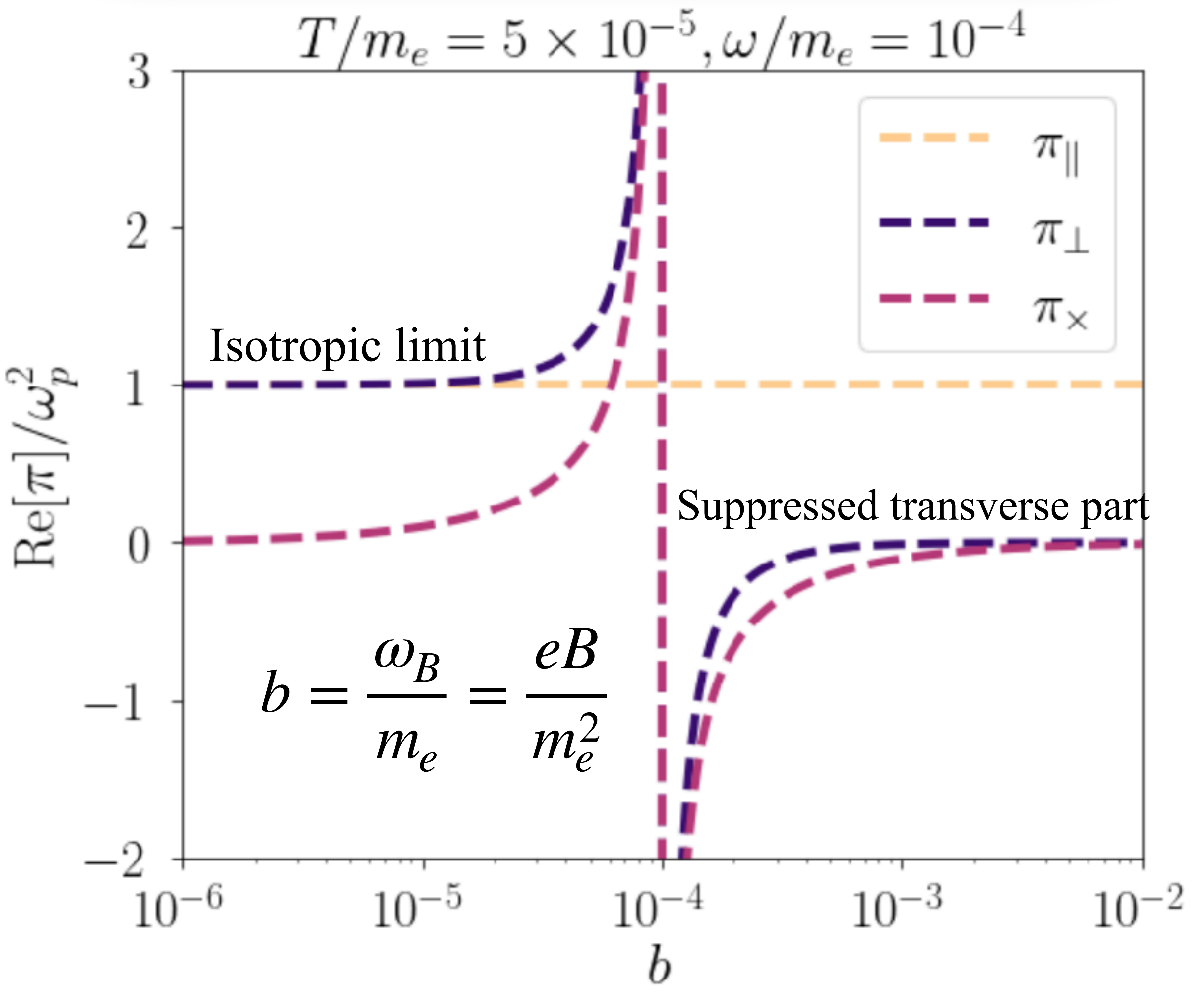
Cold, weakly magnetized plasma

"Classical kinetic theory"

Cyclotron resonance:

$$b \sim \omega_m$$

$$\omega_B \sim \omega$$

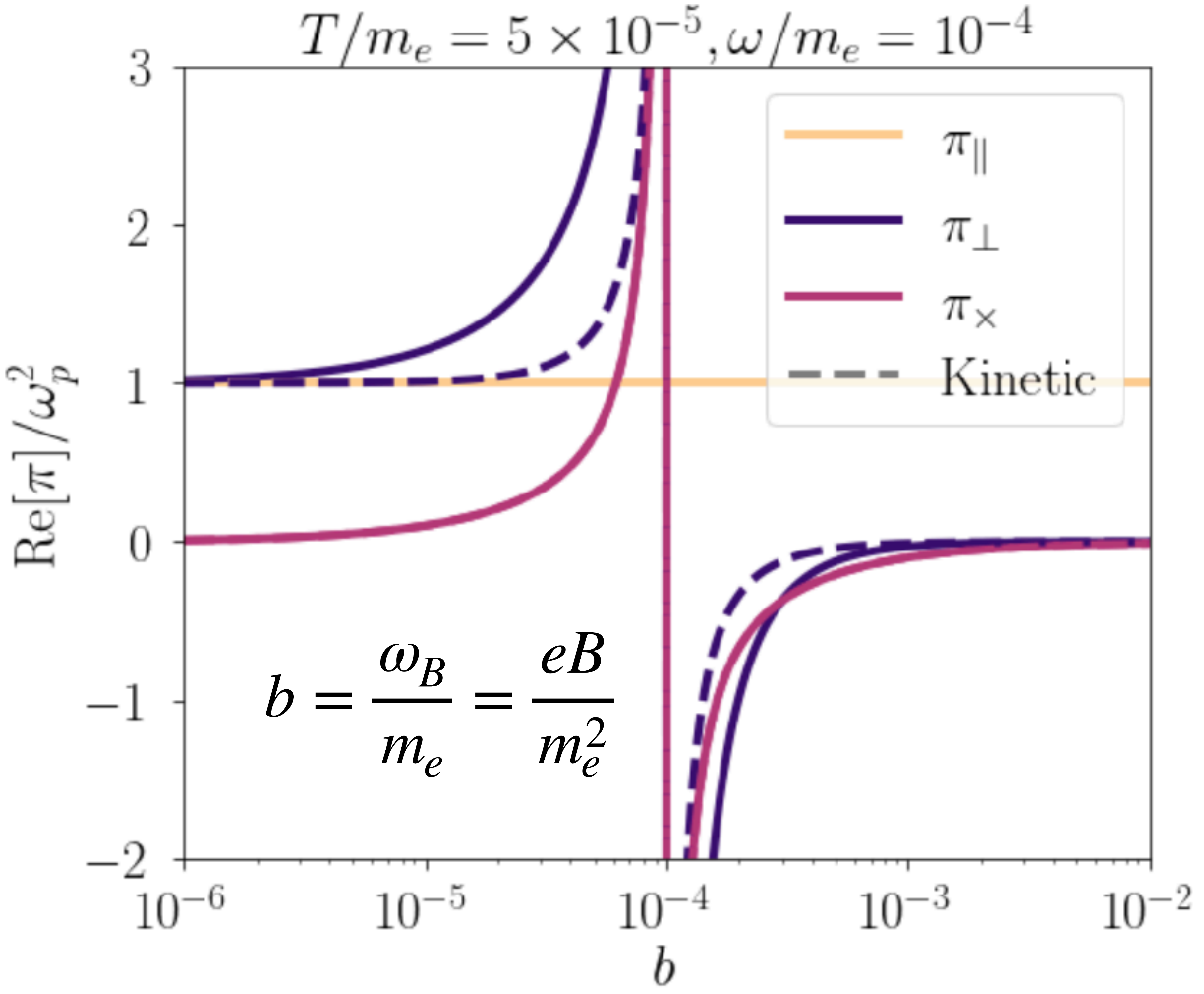


Response of a magnetized plasma: *Form-factors*

Cyclotron resonance:

$b \sim \omega_m$

$\omega_B \sim \omega$



$$b = \frac{\omega_B}{m_e} = \frac{eB}{m_e^2}$$

"Finite temperature field theory"

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Response of a magnetized plasma: *Form-factors*

$$\delta\pi^I \equiv \left| \left(\pi^I_{AA}\right)^{\text{FTFT}} / \left(\pi^I_{AA}\right)^{\text{kin}} - 1 \right|$$

All panels:

$k/\omega = 10^{-2}, \theta_B = \pi/4$

Left panel:

$T_m = 10^{-4}$

Right panel:

$\omega_m = 10^{-4}$

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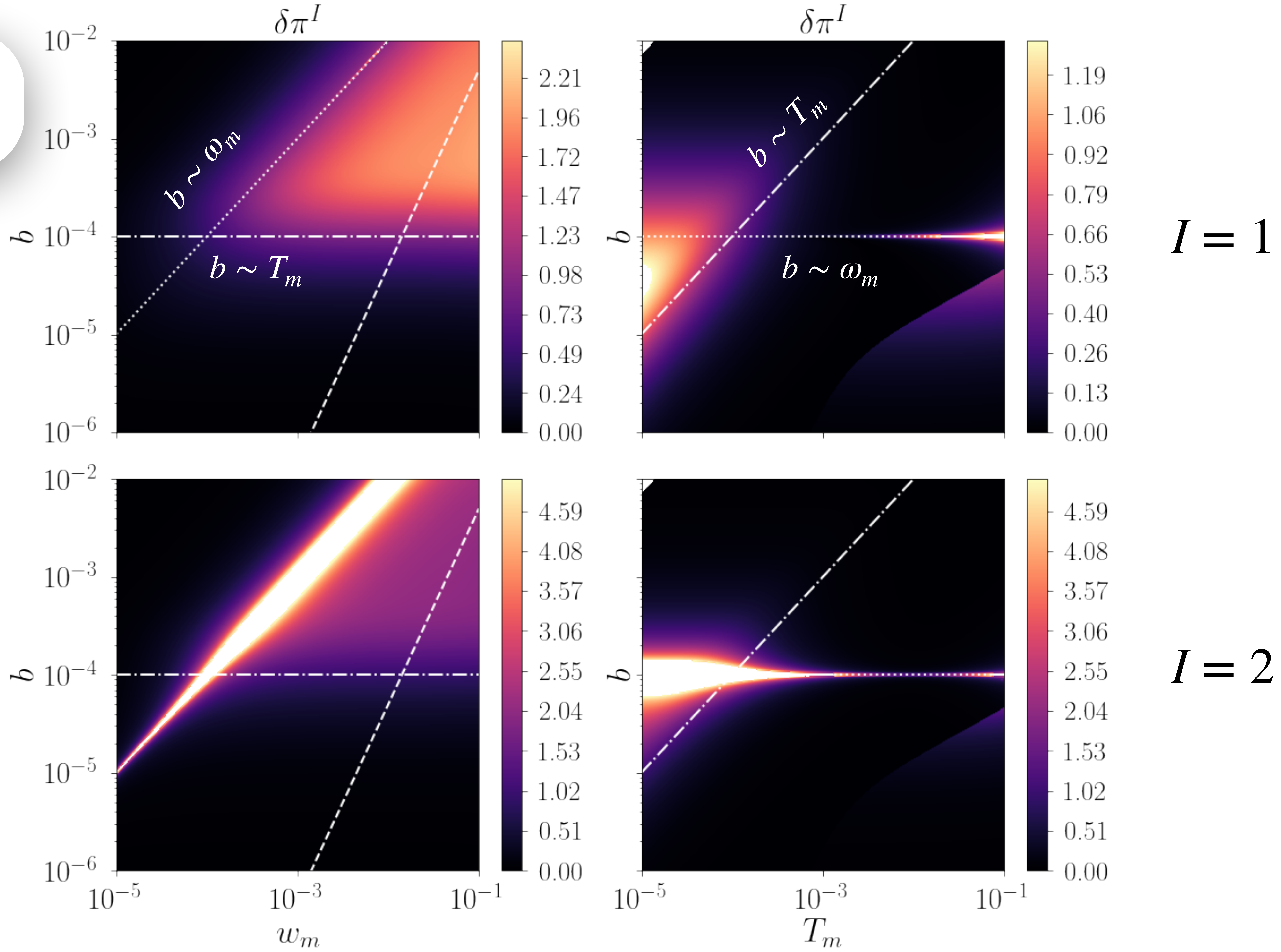
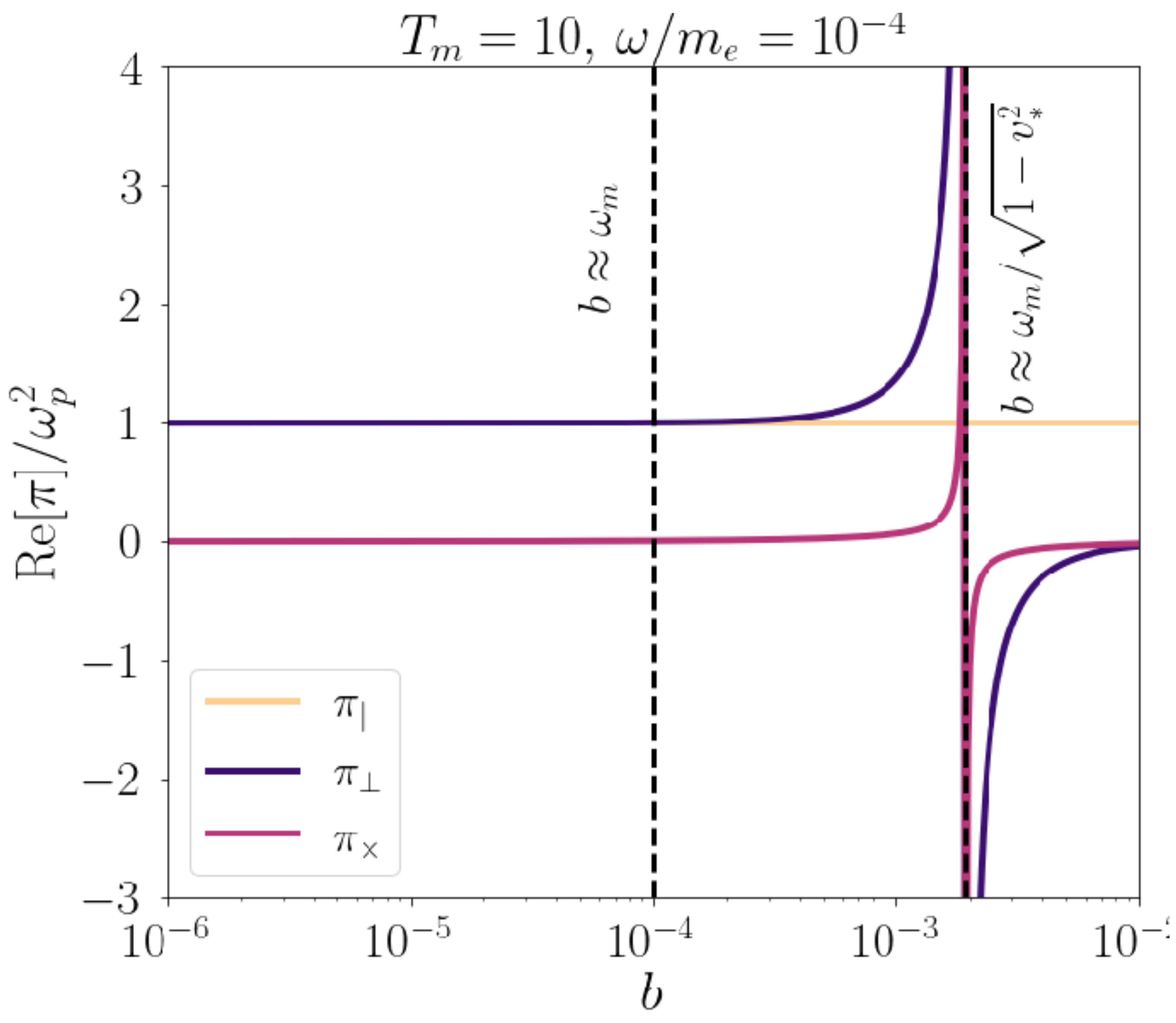


Response of a magnetized plasma: *Form-factors*



Cyclotron resonance:

$b \sim \omega_m/\sqrt{1-v_*^2}$

Degenerate

$\mu_m = 1/\sqrt{1-v_*^2}$

$\omega \sim \omega_B/\mu_m$

Classical

$v_* \to 0$

$\omega \sim \omega_B$

Relativistic

$\gamma = 1/\sqrt{1-v_*^2}$

$\omega \sim \omega_B/\gamma_*$

$$v_* = \frac{\omega_1}{\omega_p};\quad \omega_1^2 = m_e^2 \frac{e^2}{\pi^2} \int_0^1 dv\, \frac{v^2}{(1-v^2)^2} \left(\frac{5}{3}v^2 - v^4\right)(f_e + f_{\bar{e}})$$

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Dispersion in magnetized plasmas: *Sub-critical* B-field & *Classical*

All panels:

$k/\omega = 10^{-2}, \theta_B = \pi/4$

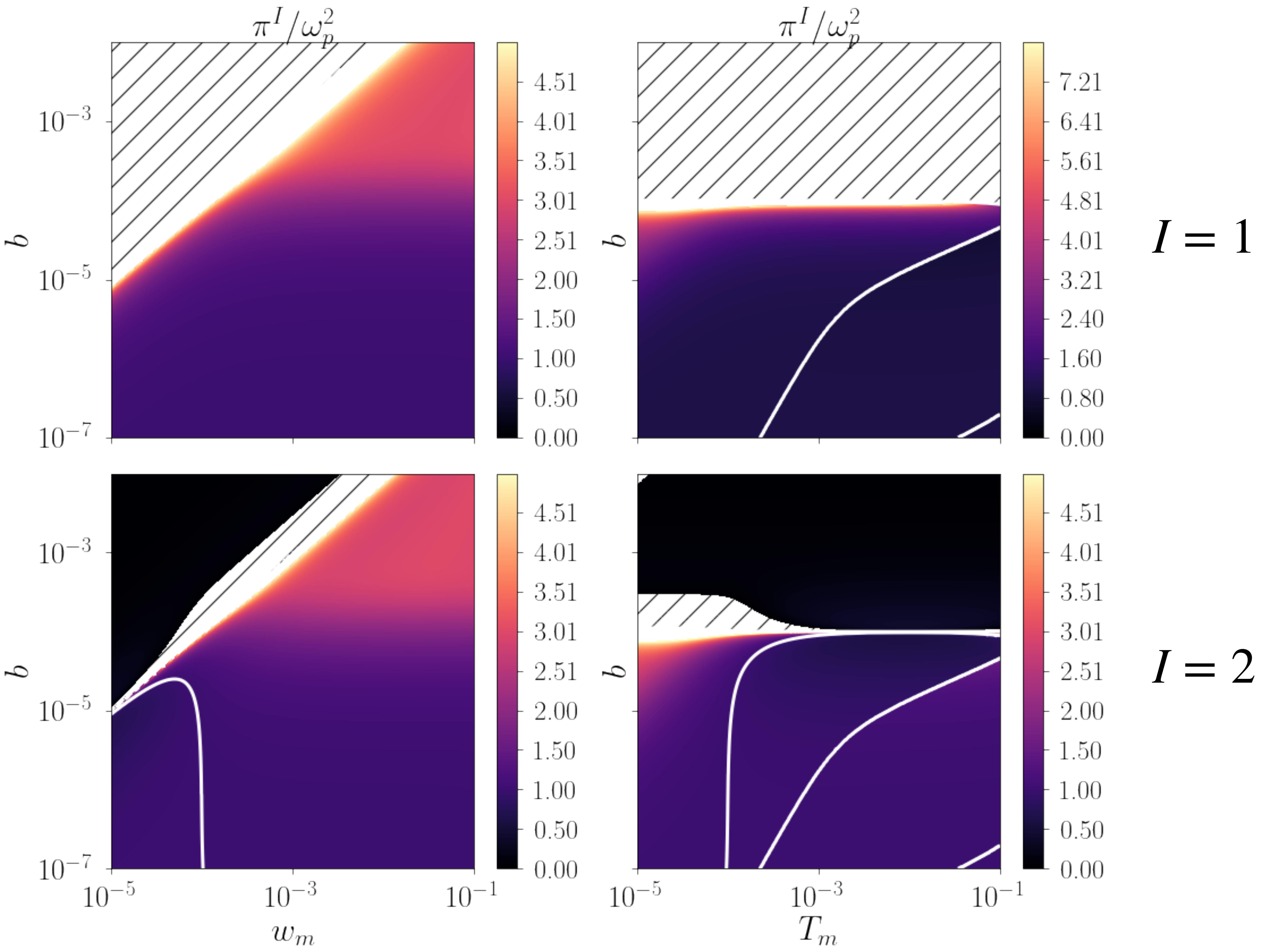
Left panel:

$T_m = 10^{-4}$

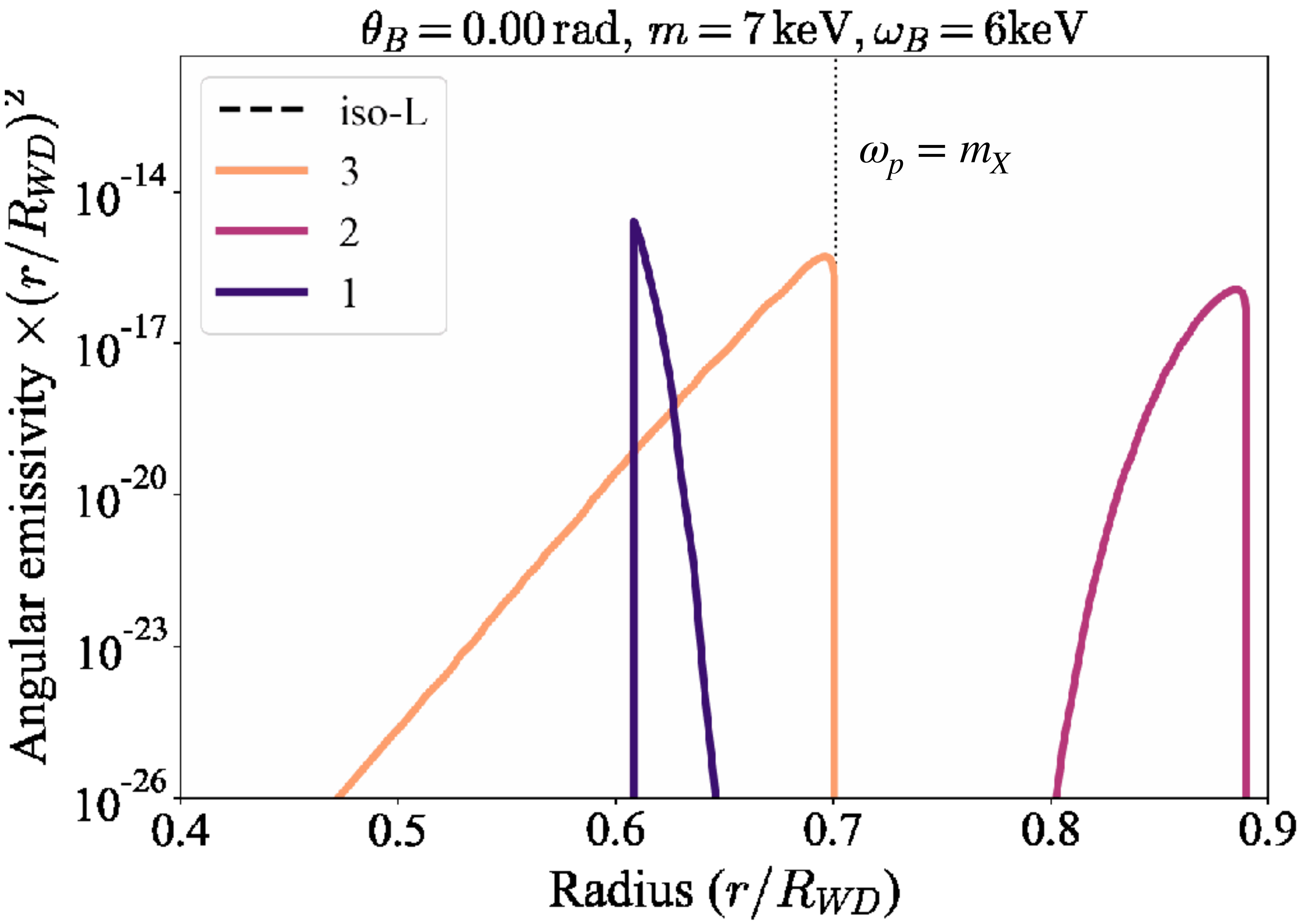
Right panel:

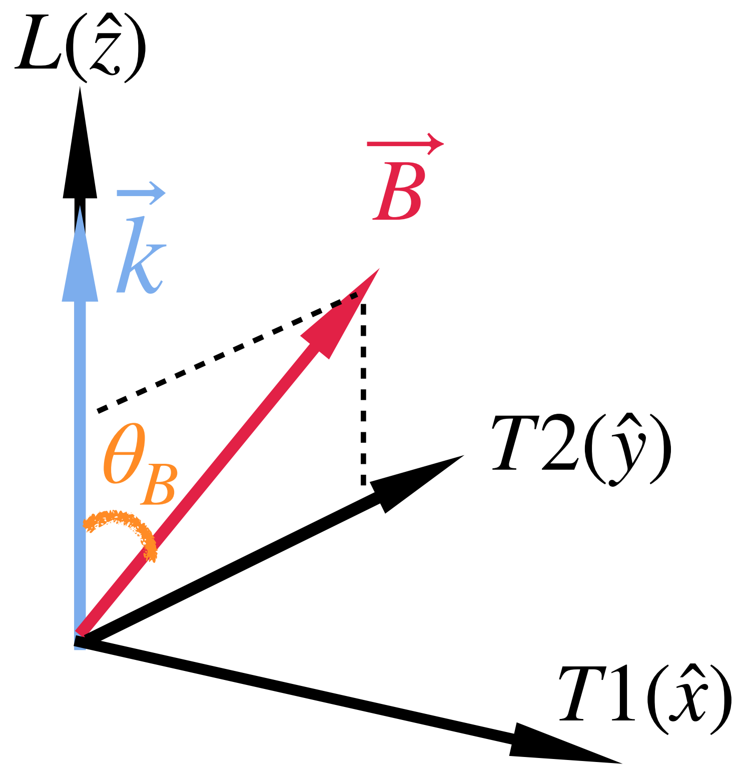
$\omega_m = 10^{-4}$

NB & Schutz 2410.14771

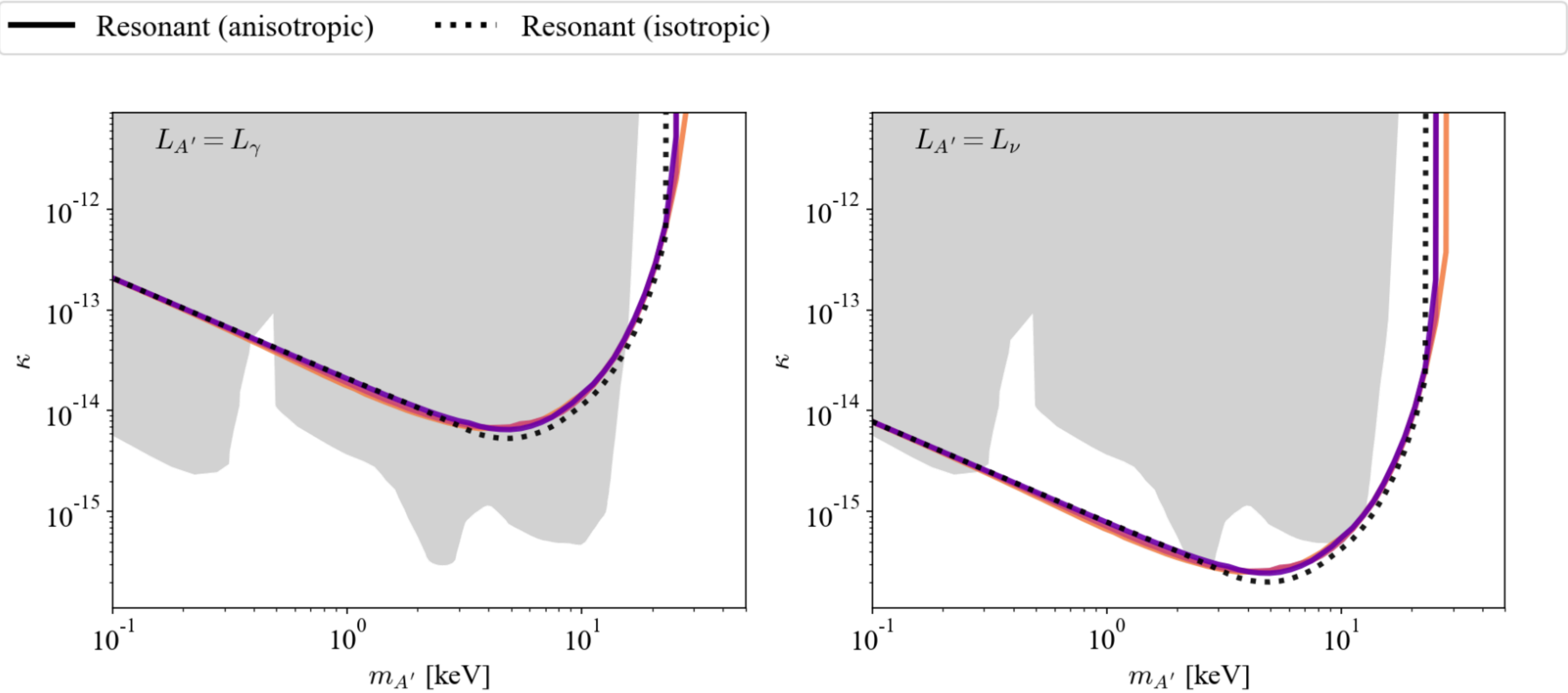


Production in magnetized plasmas: *Magnetic dispersion*





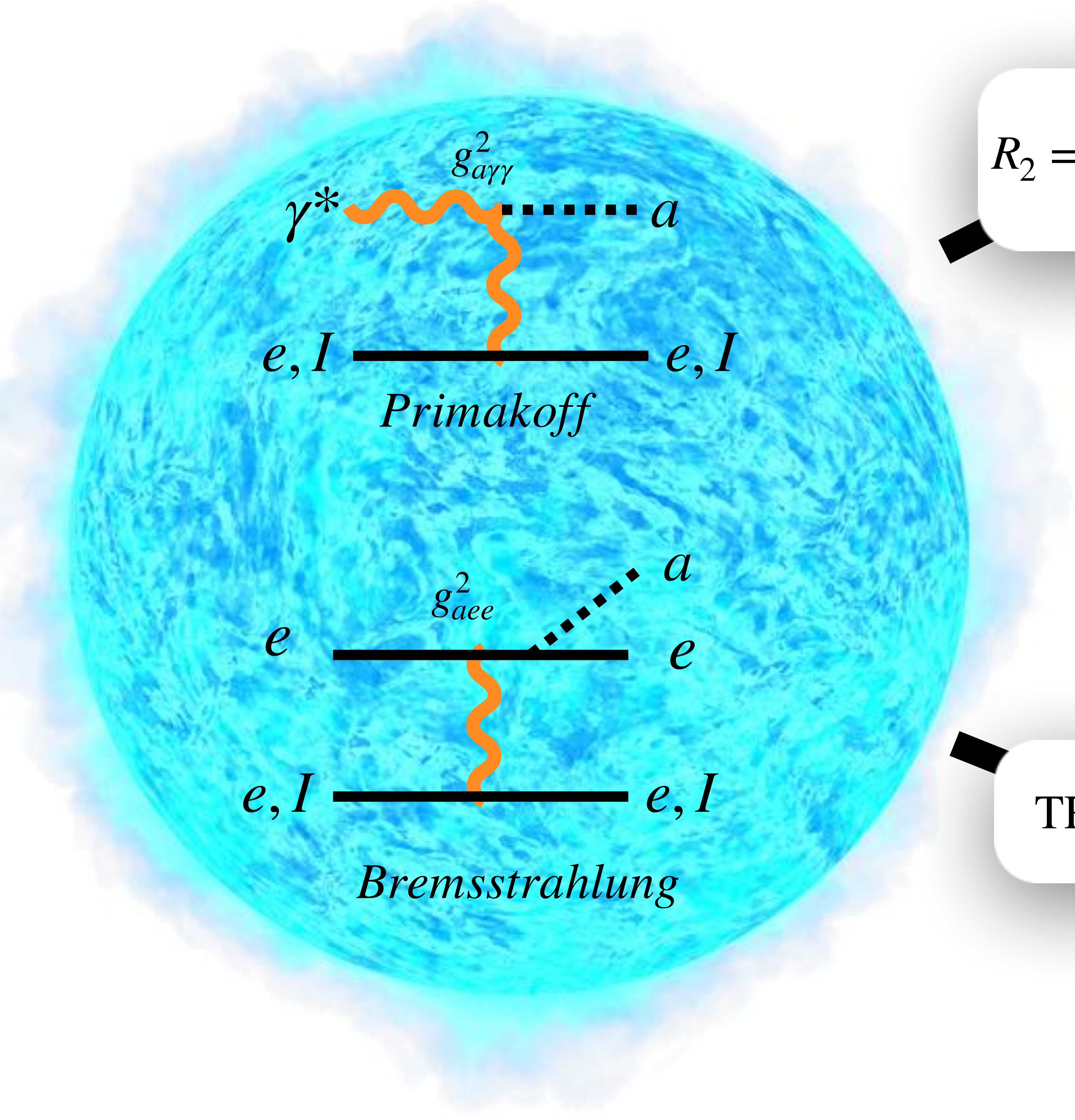
Magnetized white dwarf cooling: *Dark photons*



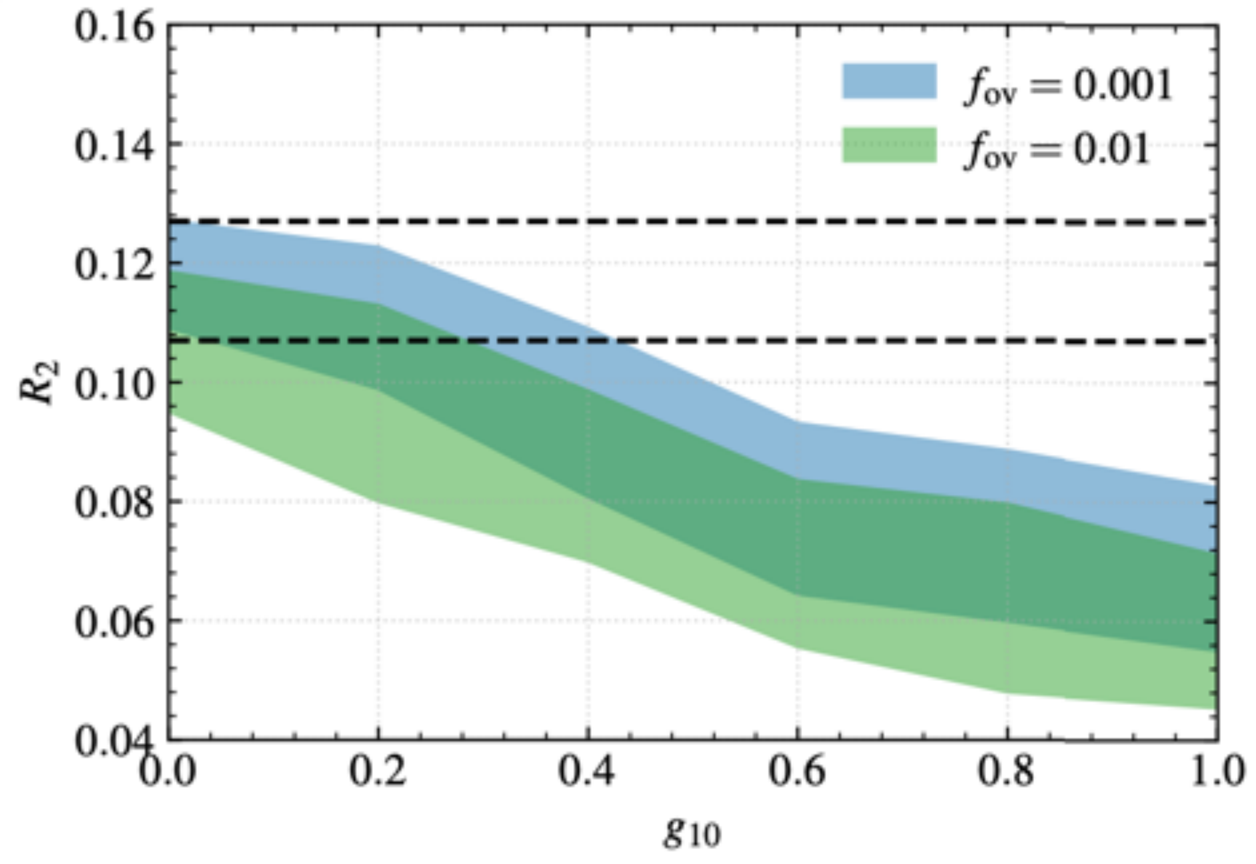
Axions and Dark photons

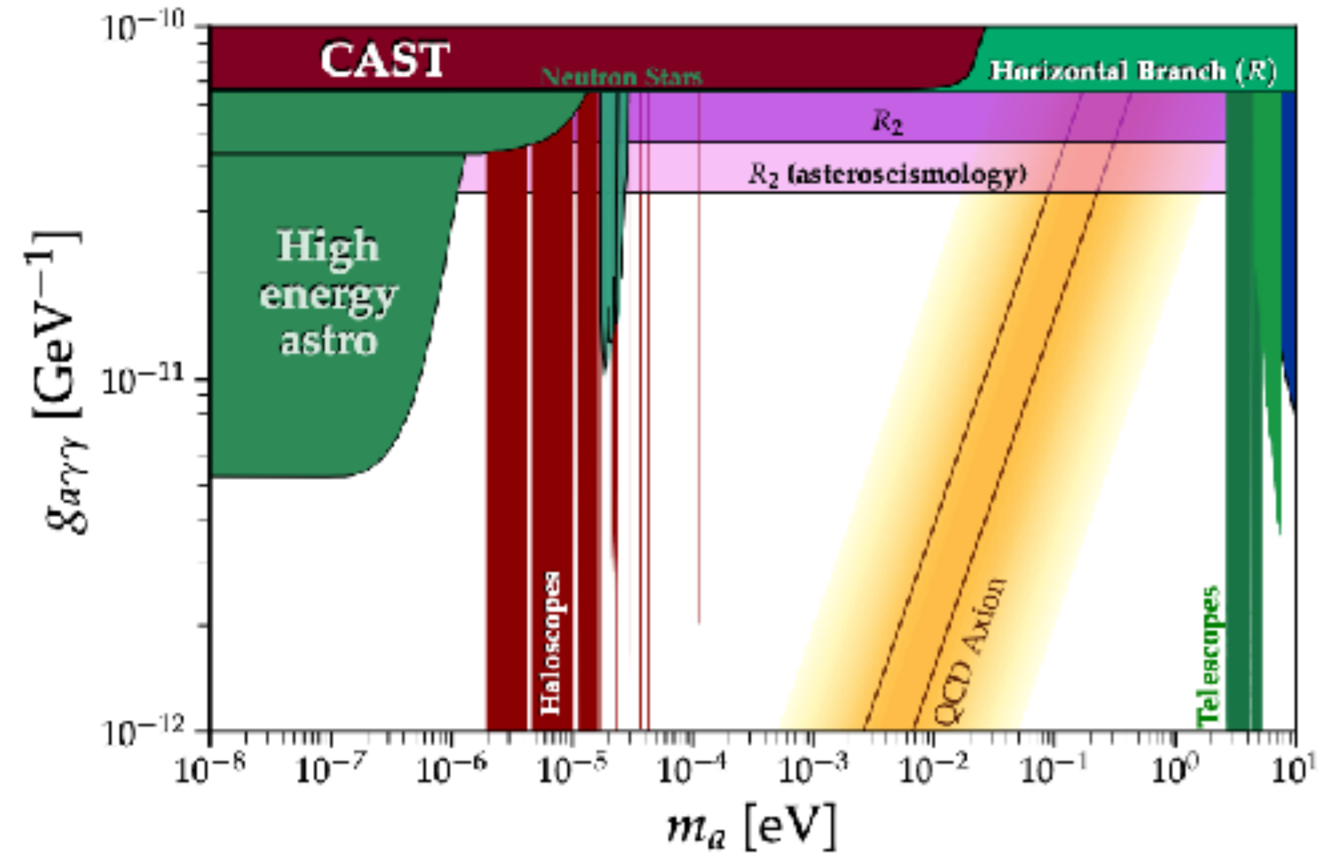
$$\mathscr{L}_{a\gamma} \supset -\frac{g_{a\gamma\gamma}}{4} a\, F_{\mu\nu}\, \tilde{F}^{\mu\nu}$$

$$\mathscr{L}_{ae} \supset -\frac{g_{aee}}{2m_e} (\partial_\mu a)\, \bar{e}\gamma^\mu \gamma^5 e$$

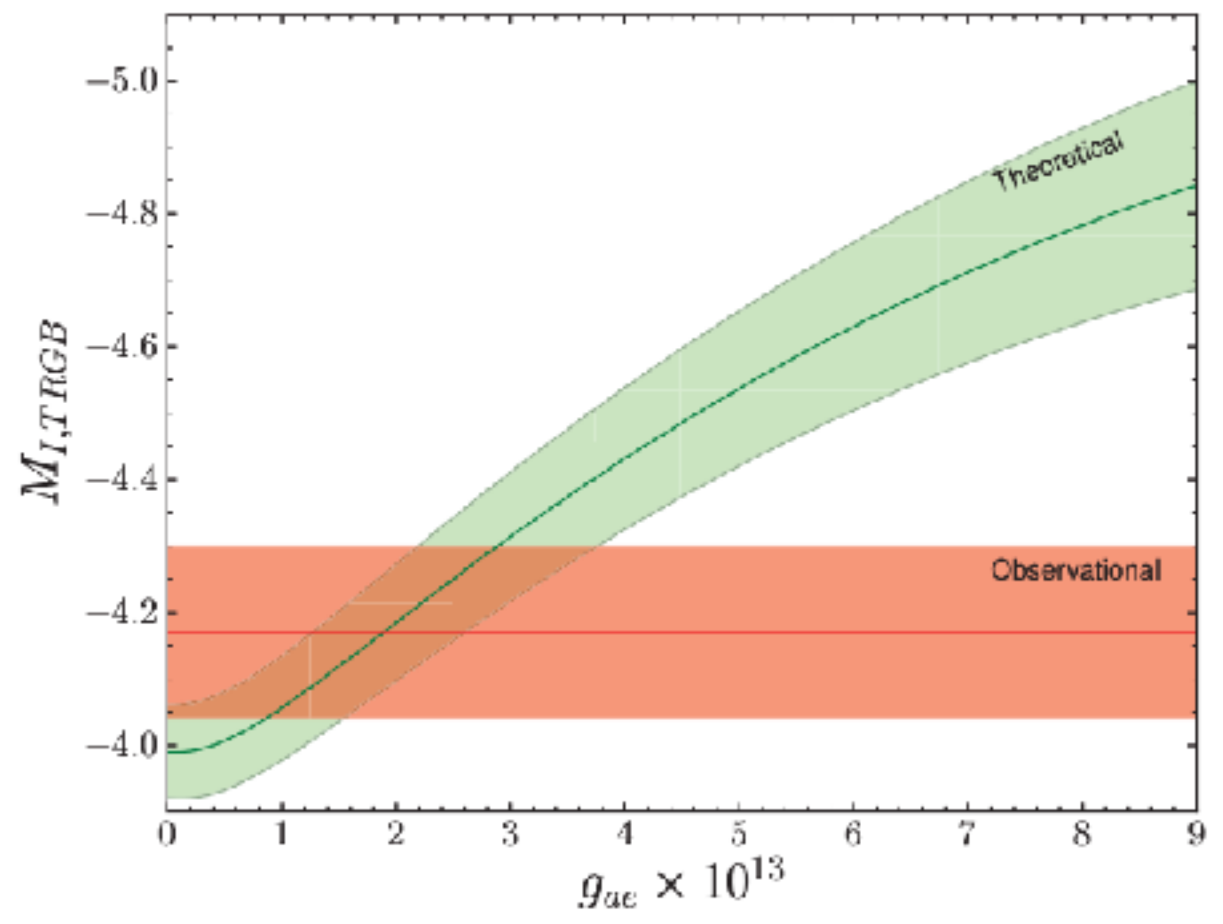


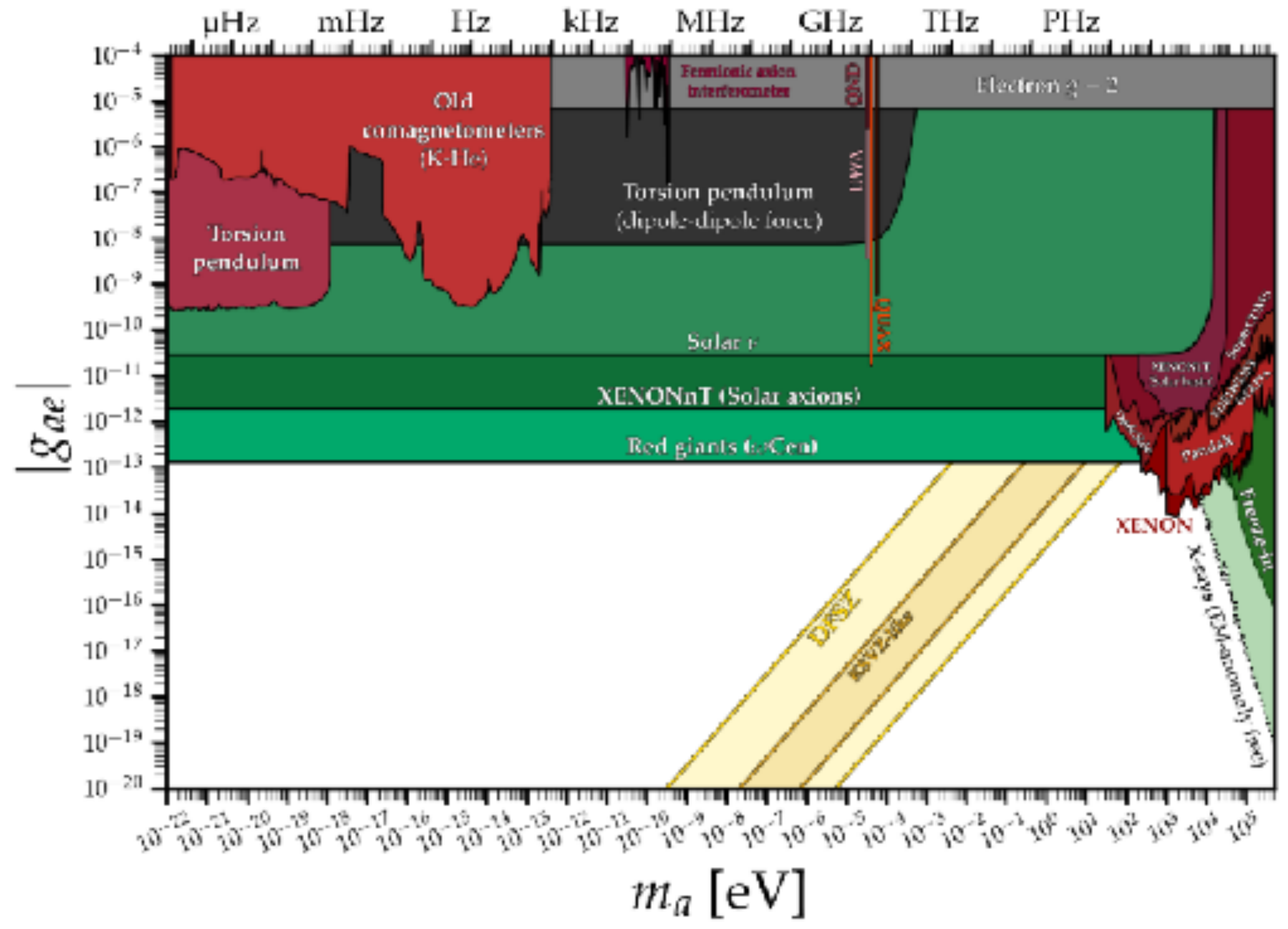
$$R_2 = \frac{N_{AGB}}{N_{HB}}$$



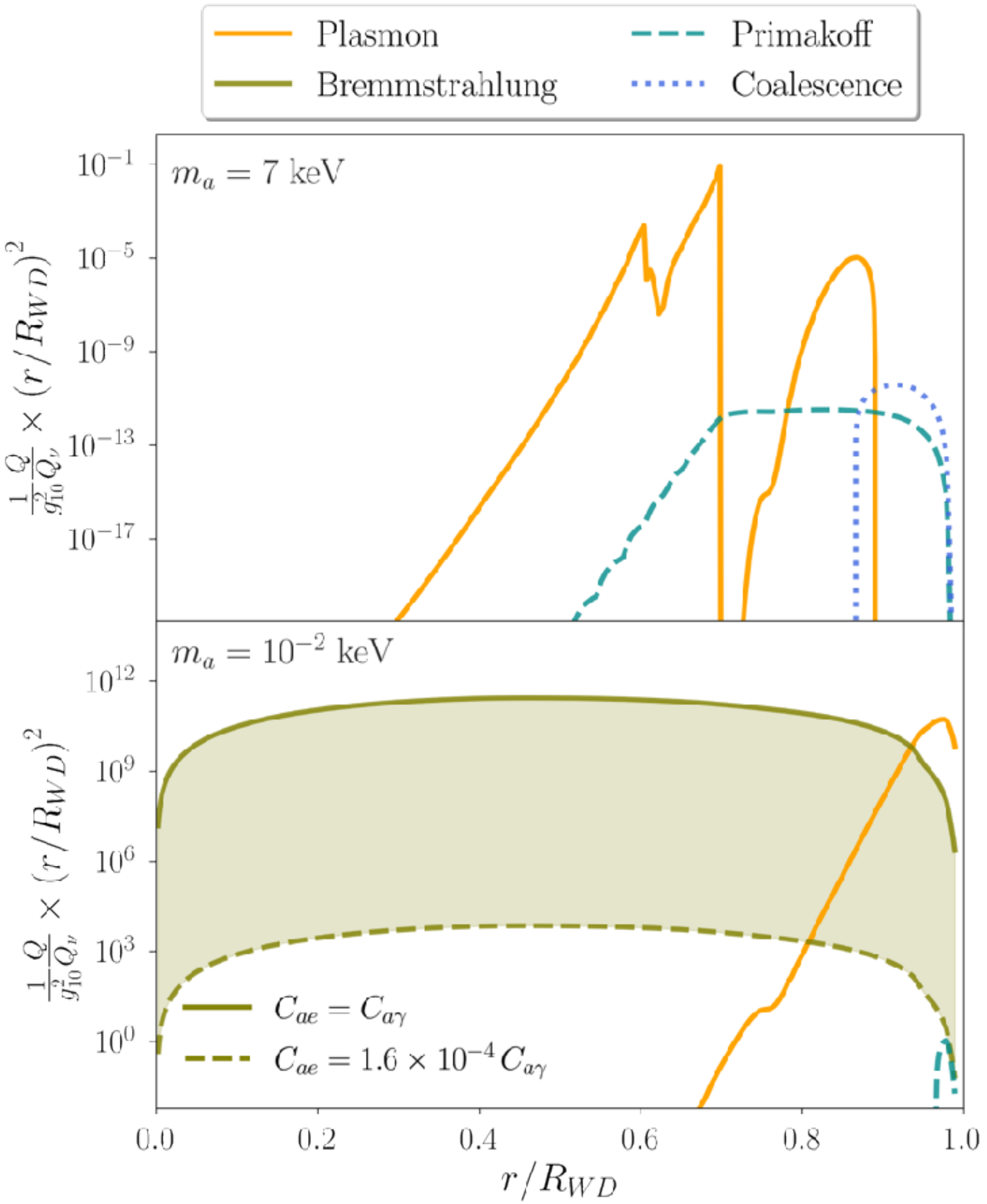


TRGB

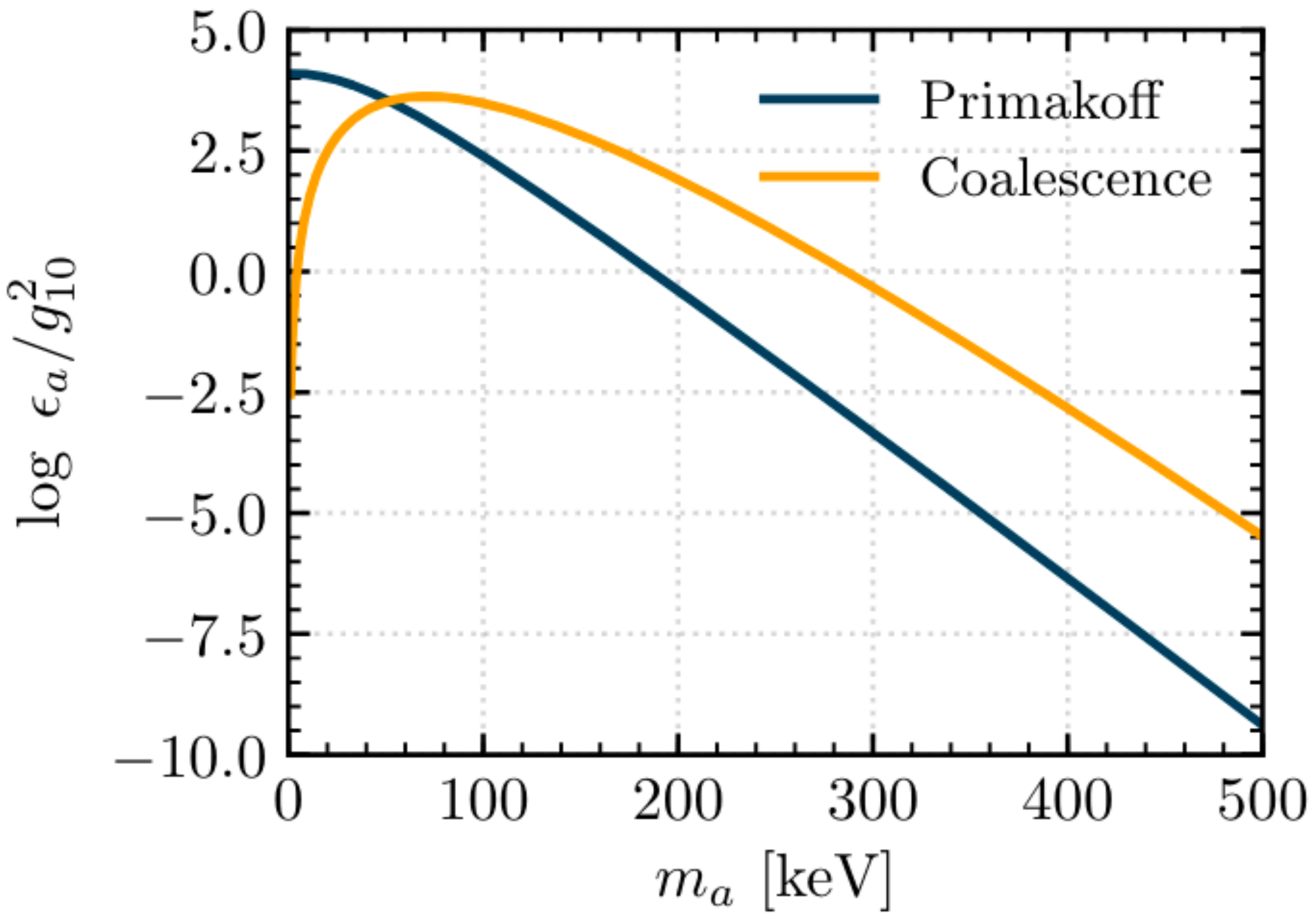


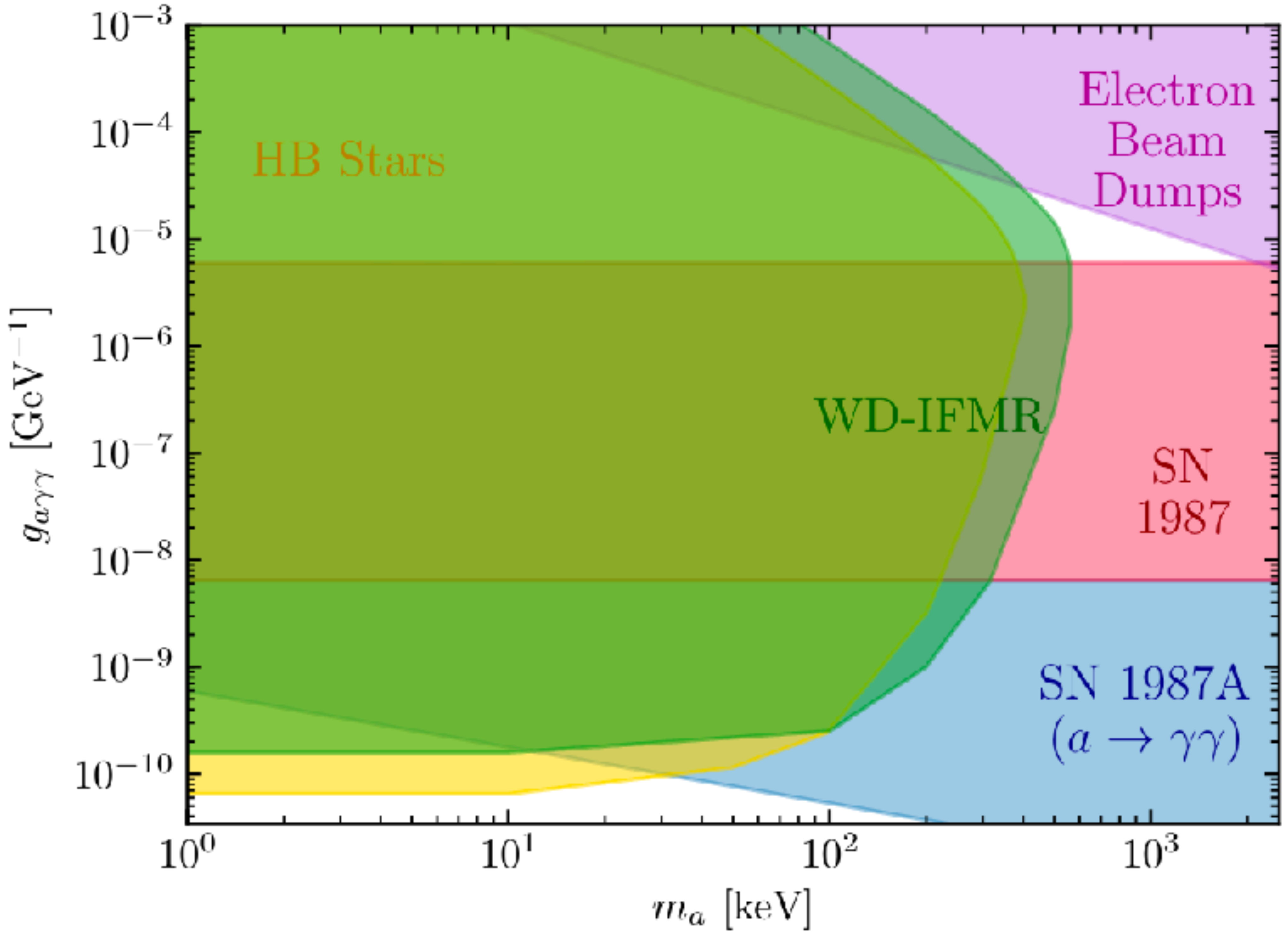


Dispersion in magnetized plasmas: *Axion channels*



Dispersion in magnetized plasmas: *Axion channels*





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